Bipolar Junction Transistor Transport Model

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This supplement presents the bipolar junction transistor Transport model, explains how it predicts DC BJT behavior, and shows how to simplify the model for four common modes of BJT operation: forward active, saturation, reverse active, and cut-off.

***Index Terms*—Bipolar transistors, bipolar integrated circuits, integrated circuit modeling, semiconductor device modeling, semiconductor devices.**

I. Introduction and Learning Objectives

THE Ebers-Moll[1] and Gummel-Poon[2] models predict detailed large- and small-signal BJT behavior to support hand analysis or computer simulations. The Transport model offers a slightly simpler approach while preserving intuition and accuracy. The Transport model circuit topology derives deductively from the Ebers-Moll model via circuit theory. Alternately, superposing forward active and reverse active mode equivalent circuits permits assembling the Transport model equivalent circuit inductively [3]. The latter strategy, conveniently proceeds from the EE 306 text treatment [4].

This supplement informs the following learning objectives:
(a) an ability to draw the Transport model equivalent circuit
(b) an ability to use the Transport model to predict BJT terminal voltages and currents
(c) an ability to draw simplified Transport model equivalent circuits for the forward active, saturation, reverse active, and cut-off operation modes.

Table I outlines the supplement organization. Section II presents the Transport model equivalent circuits and circuit equations for the NPN and PNP model versions. Section III simplifies the Transport model for the four BJT operation modes. An appendix describes the Ebers-Moll model.

II. Assembling the Transport Model

A. NPN BJT

We begin with an overview. Equations 1 relate the BJT terminal currents (i_C, i_E, and i_B) to the Base-Emitter (V_{BE}) and Base-Collector (V_{BC}) voltage drops at a given thermal voltage (V_T). The saturation current (I_S), the forward current gain (β_F), and the reverse current gain (β_R) describe a given transistor. Figure 1 contains the Transport model equivalent circuit for an NPN-BJT. Equations 2 define the transport current (i_T), forward current (i_F), and reverse current (i_R).

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TABLE I
SUPPLEMENT OUTLINE

$$i_C = I_S\left[\exp\left(\frac{V_{BE}}{V_T}\right) - \exp\left(\frac{V_{BC}}{V_T}\right)\right] - \frac{I_S}{\beta_R}\left[\exp\left(\frac{V_{BC}}{V_T}\right) - 1\right] \quad (1C)$$

$$i_E = I_S\left[\exp\left(\frac{V_{BE}}{V_T}\right) - \exp\left(\frac{V_{BC}}{V_T}\right)\right] + \frac{I_S}{\beta_F}\left[\exp\left(\frac{V_{BE}}{V_T}\right) - 1\right] \quad (1E)$$

$$i_B = \frac{I_S}{\beta_F}\left[\exp\left(\frac{V_{BE}}{V_T}\right) - 1\right] + \frac{I_S}{\beta_R}\left[\exp\left(\frac{V_{BC}}{V_T}\right) - 1\right] \quad (1B)$$

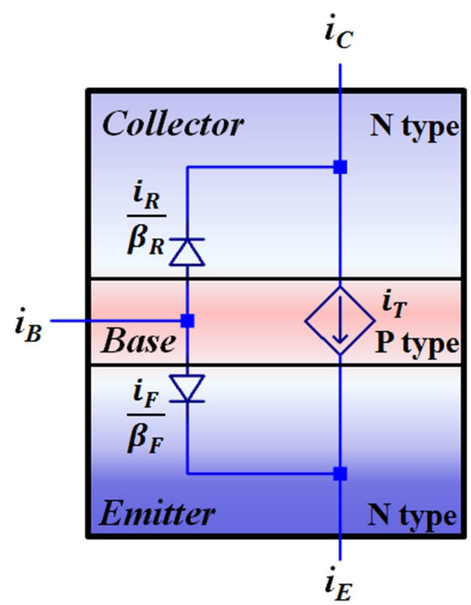


Fig. 1. NPN–BJT Transport model equivalent circuit juxtaposed on the NPN-BJT cross section. Per convention, positive base current (i_B) and collector current (i_C) flow into the device. Positive emitter current (i_E) leaves the device.

$$i_T = i_F - i_R \quad \text{(2T)}$$

$$i_F = I_S[\exp\left(\frac{V_{BE}}{V_T}\right) - 1] \quad \text{(2F)}$$

$$i_R = I_S[\exp\left(\frac{V_{BC}}{V_T}\right) - 1] \quad \text{(2R)}$$

Applying Kirchhoff's Current Law at the Collector, Emitter, and Base terminals permits translation between Equations 1 and 2. The NPN transistor formulation assumes positive currents flow into the base and collector terminals, while positive current flows out the emitter terminal. See Figure 2A.

B. PNP BJT

The PNP transistor formulation assumes positive currents opposite to those in the NPN BJT. Positive current flows in the emitter terminal, while positive currents flow out the base and collector terminals. Figure 2B shows the current conventions.

Equations 3 relate the BJT terminal currents (i_E, i_C, and i_B) to the Emitter-Base (V_{EB}) and Collector-Base (V_{CB}) voltage drops at a given thermal voltage (V_T). The saturation current (I_S), the forward current gain (β_F), and the reverse current gain (β_R) describe a given transistor. Figure 3 contains the Transport model equivalent circuit for a PNP-BJT. Equations 4 define the transport current (i_T), forward current (i_F), and reverse current (i_R).

$$i_C = I_S[\exp\left(\frac{V_{EB}}{V_T}\right) - \exp\left(\frac{V_{CB}}{V_T}\right)] - \frac{I_S}{\beta_R}[\exp\left(\frac{V_{CB}}{V_T}\right) - 1] \quad \text{(3C)}$$

$$i_E = I_S[\exp\left(\frac{V_{EB}}{V_T}\right) - \exp\left(\frac{V_{CB}}{V_T}\right)] + \frac{I_S}{\beta_F}[\exp\left(\frac{V_{EB}}{V_T}\right) - 1] \quad \text{(3E)}$$

$$i_B = \frac{I_S}{\beta_F}[\exp\left(\frac{V_{EB}}{V_T}\right) - 1] + \frac{I_S}{\beta_R}[\exp\left(\frac{V_{CB}}{V_T}\right) - 1] \quad \text{(3B)}$$

$$i_T = i_F - i_R \quad \text{(4T)}$$

$$i_F = I_S[\exp\left(\frac{V_{EB}}{V_T}\right) - 1] \quad \text{(4F)}$$

$$i_R = I_S[\exp\left(\frac{V_{CB}}{V_T}\right) - 1] \quad \text{(4R)}$$

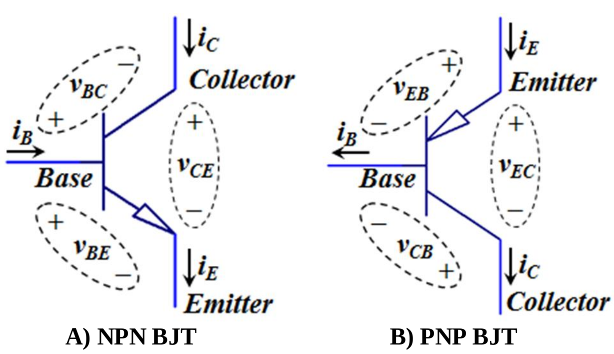


Fig. 2. BJT circuit symbols showing positive current and voltage naming conventions. For the NPN BJT, positive base current (i_B) and collector current (i_C) flow into the device, while positive emitter current (i_E) leaves the device. For the PNP BJT, positive emitter current (i_E) flows into the device, while positive base current (i_B) and collector current (i_C) leave the device. Note the voltage naming conventions use appropriate subscripts to indicate the voltage polarities. NPN and PNP devices use reversed subscripts.

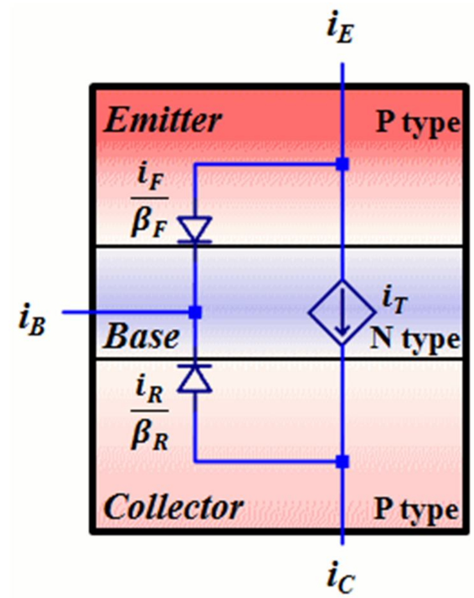


Fig. 3. PNP–BJT Transport model equivalent circuit juxtaposed on the PNP-BJT cross section. Per convention, positive emitter current (i_E) flows into the device. Positive base current (i_B) and collector current (i_C) leave the device.

III. Simplifying the Transport Model

For general BJT operation, applying Equations 1 or 3 permits calculating unknown BJT terminal voltages or currents. Many practical circuit applications bias BJTs so that the equations simplify into one of the four modes of operation shown in Table II. This section covers how to simplify the Transport model for these four operation modes. The strategy takes advantage of the relative magnitudes of the exponential terms in Equations 1. Following the convention that a Silicon *pn* junction "turns on" when its voltage drop reaches 0.5 V to 0.9 V means at room temperature that the exponential term $\exp(V_{BE}/V_T)$ or $\exp(V_{BC}/V_T)$ exceeds 1 by somewhere between 8 and 15 orders of magnitude. Such a great difference in magnitudes justifies ignoring small or negligible terms. Simplifying the Transport model equations leads to correspondingly simplified equivalent circuits and dramatically simplified circuit analysis.

This section only present results for the NPN BJT, since the PNP version proceeds so similarly.

TABLE III
Simplified BJT Operation Modes

	BC Junction **Off**	BC Junction **On**
BE Junction **Off**	**Cut-off** $I_C \approx I_E \approx I_B \approx 0$	**Reverse Active** $I_E = -\beta_R I_B$
BE Junction **On**	**Forward Active** $I_C = \beta_F I_B$	**Saturated** $I_C < \beta_F I_B$

A. Forward Active Mode

In Forward Active Mode (*FA*), the Base-emitter junction (*BE* junction) turns on, and the Base-collector junction (*BC* junction) turns off. The large *BE* junction exponential terms greatly exceed the negligible *BC* junction terms and –1 terms, so Equations 1 simplify as follows for *FA*:

$$i_C = I_S[\exp\left(\frac{V_{BE}}{V_T}\right) - \cancel{\exp\left(\frac{V_{BC}}{V_T}\right)}] - \frac{I_S}{\beta_R}[\cancel{\exp\left(\frac{V_{BC}}{V_T}\right)} - \cancel{1}]$$
$$= I_S[\exp\left(\frac{V_{BE}}{V_T}\right)] \quad (5C)$$

$$i_E = I_S[\exp\left(\frac{V_{BE}}{V_T}\right) - \cancel{\exp\left(\frac{V_{BC}}{V_T}\right)}] + \frac{I_S}{\beta_F}[\exp\left(\frac{V_{BE}}{V_T}\right) - \cancel{1}]$$
$$= I_S[\exp\left(\frac{V_{BE}}{V_T}\right)]\left(\frac{\beta_F+1}{\beta_F}\right) \quad (5E)$$

$$i_B = \frac{I_S}{\beta_F}[\exp\left(\frac{V_{BE}}{V_T}\right) - \cancel{1}] + \frac{I_S}{\beta_R}[\cancel{\exp\left(\frac{V_{BC}}{V_T}\right)} - \cancel{1}]$$
$$= \frac{I_S}{\beta_F}[\exp\left(\frac{V_{BE}}{V_T}\right)] \quad (5B)$$

Equations 5 support the equivalent circuit simplification shown in Figure 4. As required, Kirchhoff's Current Law applies in *FA* at the emitter terminal:

$$i_E = i_B + i_C = (\beta_F + 1)i_B \quad (6)$$

B. Reverse Active Mode

RA operation occurs less frequently in practice than the other operation modes, primarily due to the low reverse current gain (β_R) compared to forward current gain (β_F). β_R usually has a value less than 10 and often less than 1, whereas β_F usually exceeds 20 and often exceeds 100.

In Reverse Active Mode (*RA*), the Base-emitter junction (*BE* junction) turns off, and the Base-collector junction (*BC* junction) turns on. The large *BC* junction exponential terms greatly exceed the negligible *BE* junction terms and –1 terms, so Equations 1 simplify into Equations 7 as follows:

$$i_C = I_S[\cancel{\exp\left(\frac{V_{BE}}{V_T}\right)} - \exp\left(\frac{V_{BC}}{V_T}\right)] - \frac{I_S}{\beta_R}[\exp\left(\frac{V_{BC}}{V_T}\right) - \cancel{1}]$$
$$= -I_S[\exp\left(\frac{V_{BC}}{V_T}\right)]\left(\frac{\beta_R+1}{\beta_R}\right) \quad (7C)$$

$$i_E = I_S[\cancel{\exp\left(\frac{V_{BE}}{V_T}\right)} - \exp\left(\frac{V_{BC}}{V_T}\right)] + \frac{I_S}{\beta_F}[\cancel{\exp\left(\frac{V_{BE}}{V_T}\right)} - \cancel{1}]$$
$$= -I_S[\exp\left(\frac{V_{BC}}{V_T}\right)] \quad (7E)$$

$$i_B = \frac{I_S}{\beta_F}[\exp\left(\frac{V_{BE}}{V_T}\right) - \cancel{1}] + \frac{I_S}{\beta_R}[\cancel{\exp\left(\frac{V_{BC}}{V_T}\right)} - \cancel{1}]$$
$$= \frac{I_S}{\beta_R}[\exp\left(\frac{V_{BC}}{V_T}\right)] \quad (7B)$$

Because the negative signs on the collector and emitter terminals can make circuit analysis *more* confusing than necessary, it often proves more convenient to redefine the directions of positive collector current (i_C) and positive emitter current (i_E). Therefore, ***for reverse active mode***, we define

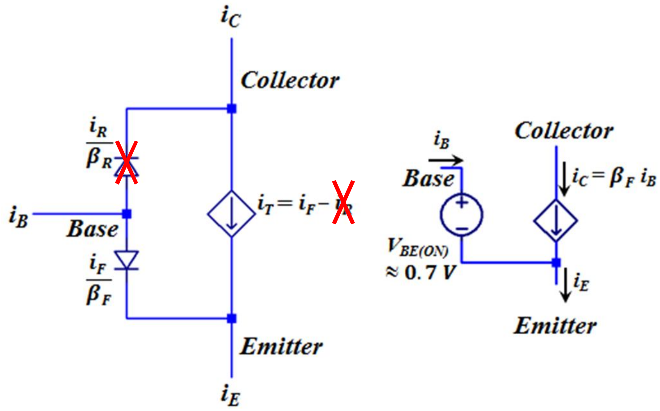


Fig. 4. Simplified Transport model for *Forward Active* mode. With the *BE* junction on and the *BC* junction off, the reverse current term, i_R, becomes negligible, and the Transport model on the left simplifies to the equivalent circuit on the right. The equivalent circuit on the right corresponds to Equations 5.

positive emitter current flowing ***into*** the emitter terminal and positive collector current flowing ***out*** the collector terminal. This convention, shown in Figure 5, does ***not*** agree with the convention used for forward active mode shown in Figure 4.

Rewriting Equations 7 using the new convention yields Equations 8 for *RA*.

$$i_C = I_S[\exp\left(\frac{V_{BC}}{V_T}\right)]\left(\frac{\beta_R+1}{\beta_R}\right) \quad (8C)$$

$$i_E = I_S[\exp\left(\frac{V_{BC}}{V_T}\right)] \quad (8E)$$

$$i_B = \frac{I_S}{\beta_R}[\exp\left(\frac{V_{BC}}{V_T}\right)] \quad (8B)$$

Equations 8 support the equivalent circuit simplification shown in Figure 5. As required, Kirchhoff's Current Law

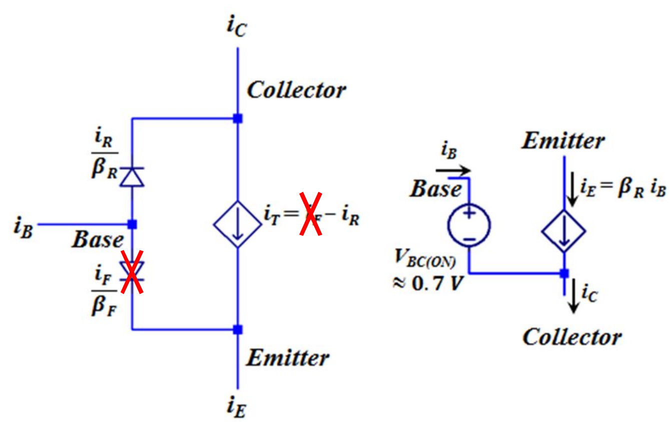


Fig. 5. Simplified Transport model for *Reverse Active* mode. With the *BC* junction on and the *BE* junction off, the forward current term, i_F, becomes negligible, and the Transport model on the left simplifies to the equivalent circuit on the right. The equivalent circuit on the right corresponds to Equations 8. It uses a convention for positive collector and emitter current ***opposite*** to that used for the other three BJT modes.

applies in *RA* at the collector terminal:

$$i_C = i_B + i_E = (\beta_R + 1)i_B \tag{9}$$

C. Cut-off Mode

In Cut-off Mode (*CO*), both junctions turn off. With both junctions reversed biased, the small exponential terms become negligible compared to the –1 terms, so Equations 1 simplify as follows for *CO*, leaving only leakage currents:

$$i_C = I_S\left[\exp\left(\frac{V_{BE}}{V_T}\right) - \exp\left(\frac{V_{BC}}{V_T}\right)\right] - \frac{I_S}{\beta_R}\left[\exp\left(\frac{V_{BC}}{V_T}\right) - 1\right] = \frac{I_S}{\beta_R} \tag{10C}$$

$$i_E = I_S\left[\exp\left(\frac{V_{BE}}{V_T}\right) - \exp\left(\frac{V_{BC}}{V_T}\right)\right] + \frac{I_S}{\beta_F}\left[\exp\left(\frac{V_{BE}}{V_T}\right) - 1\right] = -\frac{I_S}{\beta_F} \tag{10E}$$

$$i_B = \frac{I_S}{\beta_F}\left[\exp\left(\frac{V_{BE}}{V_T}\right) - 1\right] + \frac{I_S}{\beta_R}\left[\exp\left(\frac{V_{BC}}{V_T}\right) - 1\right] = -\frac{I_S}{\beta_F} - \frac{I_S}{\beta_R} \tag{10B}$$

Equations 10 support the equivalent circuit simplification shown in the left pane of Figure 6. Typically, hand calculations ignore the negligible leakage currents and use the following approximation:

$$i_B \approx i_C \approx i_E \approx 0 \tag{11}$$

In *CO*, the BJT behaves as an open circuit. Figure 6 shows the Equation 11 open circuit approximation in its right pane.

D. Saturation Mode

In Saturation Mode (*SAT*), both junctions turn on, so the –1 terms in Equations 1 become negligible compared to the exponential terms. Equations 1 simplify as follows for *SAT*:

$$i_C = I_S\left[\exp\left(\frac{V_{BE}}{V_T}\right) - \exp\left(\frac{V_{BC}}{V_T}\right)\right] - \frac{I_S}{\beta_R}\left[\exp\left(\frac{V_{BC}}{V_T}\right)\right] \tag{12C}$$

$$i_E = I_S\left[\exp\left(\frac{V_{BE}}{V_T}\right) - \exp\left(\frac{V_{BC}}{V_T}\right)\right] + \frac{I_S}{\beta_F}\left[\exp\left(\frac{V_{BE}}{V_T}\right)\right] \tag{12E}$$

$$i_B = \frac{I_S}{\beta_F}\left[\exp\left(\frac{V_{BE}}{V_T}\right)\right] + \frac{I_S}{\beta_R}\left[\exp\left(\frac{V_{BC}}{V_T}\right)\right] \tag{12B}$$

In Saturation Mode, $\beta_F i_B \geq i_C$, so hand analysis proceeds more conveniently by considering terminal voltages. Combining Equations 12C and 12B produces

$$V_{BE(SAT)} = V_T \ln\left[\frac{(\beta_R+1)i_B+i_C}{I_S}\frac{\beta_F}{\beta_F+\beta_R+1}\right] \tag{13}$$

and

$$V_{BC(SAT)} = V_T \ln\left[\frac{\beta_F i_B - i_C}{I_S}\frac{\beta_R}{\beta_F+\beta_R+1}\right]. \tag{14}$$

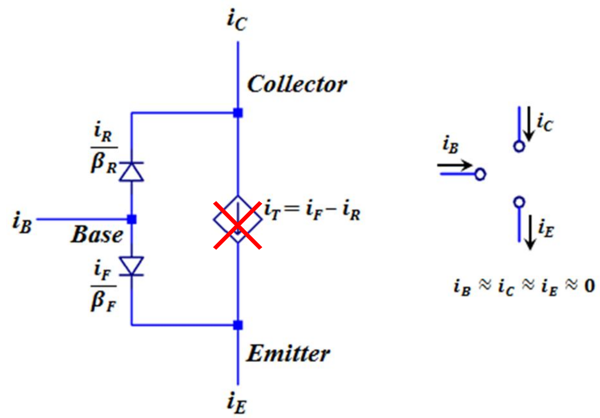


Fig. 6. Simplified Transport model for *Cut-off* mode. With both junctions off, the transport current term, i_T, becomes negligible, leaving the Transport model on the left. Ignoring leakage currents produces the equivalent circuit on the right. The open circuit equivalent circuit on the right corresponds to Equation 11.

Applying Kirchhoff's Voltage Law around the BJT gives

$$V_{CE(SAT)} = V_{BE(SAT)} - V_{BC(SAT)}. \tag{15}$$

Inserting Equations 13 and 14 into Equation 15 produces

$$V_{CE(SAT)} = V_T \ln\left[\frac{\beta_F}{\beta_R}\frac{(\beta_R+1)i_B+i_C}{\beta_F i_B - i_C}\right]. \tag{16}$$

The preceding formulations use the common-emitter current gain, β. Equivalent versions based on the common-base current gain, α, also exist. To translate from one to the other, use

$$\alpha = \frac{\beta}{\beta+1} \tag{17}$$

or

$$\beta = \frac{\alpha}{1-\alpha}. \tag{18}$$

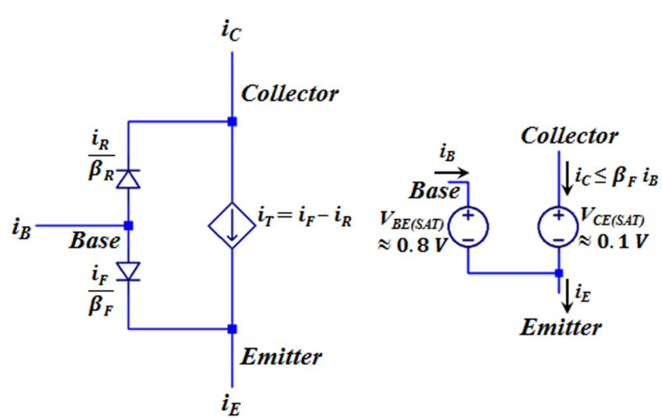


Fig. 7. Simplified Transport model for *Saturation* mode. With both junctions on, the Transport model on the left simplifies to the equivalent circuit on the right. The equivalent circuit on the right corresponds to Equations 14 – 16.

E. *Edge of Conduction and Edge of Saturation*

Figure 8 shows equivalent circuits for the Edge of Conduction (*EOC*) and Edge of Saturation (*EOS*) modes. *EOC* describes operation at the corner case between *CO* and *FA* modes as the BJT almost turns on or just turns off. At *EOC*, the *BE* junction drops almost enough voltage to cause non-negligible base current, but neither junction turns on or conducts significant current. In Figure 8A, the voltage, $V_{BE(EOC)}$, recognizes the *BE* junction voltage drop, and the open circuit recognizes the lack of current.

EOS describes operation at the corner case between *FA* and *SAT* modes as the BJT operates with the collector and base currents obeying the *FA* relation $i_C = \beta_F\ i_B$, and the terminal voltages reach their saturation values. In Figure 8B, the voltage drops $V_{BE(EOS)}=V_{BE(SAT)}$, and $V_{CE(EOS)} = V_{CE(SAT)}$ capture the saturation behavior.

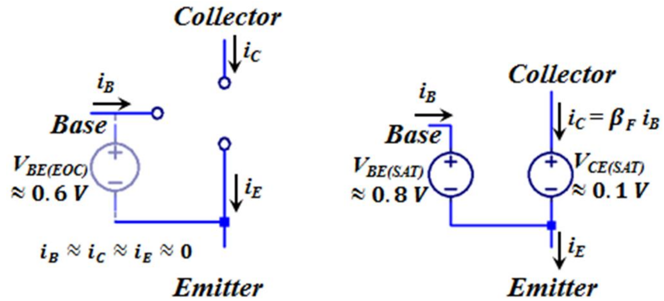
Table IV places *EOC* on the boundary between *CO* and *FA*, while *EOS* sits on the boundary between *FA* and *SAT*. Table V shows typical values for Silicon BJT parameters applied to the equivalent circuits derived from the Transport Model [5].

TABLE IV
SIMPLIFIED BJT OPERATION MODES PLUS EOC AND EOS

	BC Junction **Off**		BC Junction **On**
BE Junction **Off**	**Cut-off** $I_C \approx I_E \approx I_B \approx 0$		**Reverse Active** $I_E = -\beta_R I_B$
	EOC		
BE Junction **On**	**Forward Active** $I_C = \beta_F I_B$	**EOS**	**Saturated** $I_C < \beta_F I_B$

TABLE V
TYPICAL SILICON BJT PARAMETERS FOR HAND CALCULATIONS [5]

$V_{BE(EOC)}$	0.6 V
$V_{BE(ON)}$	0.7 V
$V_{BE(SAT)} = V_{BE(EOS)}$	0.8 V
$V_{CE(SAT)} = V_{CE(EOS)}$	0.1 V
$V_{BC(ON)} = V_{BC(SAT)}$	0.7 V



A) Edge of Conduction (EOC) **B) Edge of Saturation (EOS)**

Fig. 8. *EOC* and *EOS* equivalent circuits. A) At *EOC*, the *BE* junction almost has a large enough voltage drop to turn on, but neither junction conducts significant current. The voltage, $V_{BE(EOC)}$, recognizes the *BE* junction voltage drop, and the open circuit recognizes the lack of current. B) At *EOS*, the transistor just leaves Forward Active Mode and enters Saturation. *FA* preserves the current relation $i_C = \beta_F\ i_B$, and *SAT* accounts for the $V_{BE(EOS)}$, and $V_{CE(EOS)}$ voltage drops.

APPENDIX—THE EBERS-MOLL BJT MODEL [6]

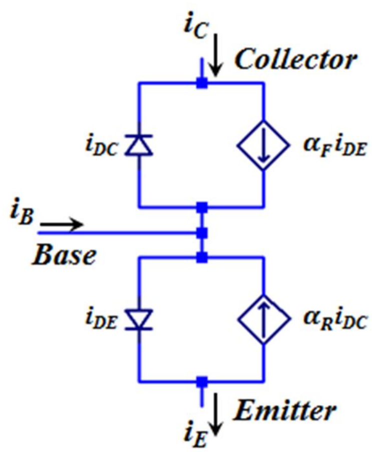


Fig. A1. NPN–BJT Ebers-Moll model equivalent circuit. Per convention, positive base current (i_B) and collector current (i_C) flow into the device. Positive emitter current (i_E) leaves the device. The equivalent circuit corresponds to Equations A3.

Because the Transport model derives from the Ebers-Moll model, this appendix presents the Ebers-Moll model following the Hodges and Jackson treatment [6]. Figure A1 shows the model in terms of the diode currents, i_{DE}, and i_{DC} shown in Equations A1:

$$i_{DE} = I_{ES}[\exp\left(\frac{V_{BE}}{V_T}\right) - 1] \quad \text{(A1E)}$$

$$i_{DC} = I_{CS}[\exp\left(\frac{V_{BC}}{V_T}\right) - 1] \quad \text{(A1C)}$$

Applying Kirchhoff's Current Law at the collector and emitter terminals in Figure A1 gives Equations A2:

$$i_E = i_{DE} - \alpha_R i_{DC} \quad \text{(A2E)}$$

$$i_C = \alpha_F i_{DE} - i_{DC} \quad \text{(A2C)}$$

Inserting Equations A1 into Equations A2 gives the general Ebers-Moll Equations:

$$i_E = I_{ES}\left[\exp\left(\frac{V_{BE}}{V_T}\right) - 1\right] - \alpha_R I_{CS}[\exp\left(\frac{V_{BC}}{V_T}\right) - 1] \quad \text{(A3E)}$$

$$i_C = \alpha_F I_{ES}\left[\exp\left(\frac{V_{BE}}{V_T}\right) - 1\right] - I_{CS}[\exp\left(\frac{V_{BC}}{V_T}\right) - 1] \quad \text{(A3C)}$$

$$i_B = i_E - i_C \quad \text{(A3B)}$$

Note that the parameters I_{ES}, I_{ES}, α_F, and α_R describe BJT behavior for the Ebers-Moll model.

ACKNOWLEDGMENT

This supplement uses LTspice IV to create the circuit diagrams [7].

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