EE595S: Class Lecture Notes
Chapter 14: Induction Motor Drives

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Fall 2005

Overview of Strategies

- Volts-Per-Hertz Control
- Constant Slip Control
- Field-Oriented Control

Volts-Per-Hertz Control

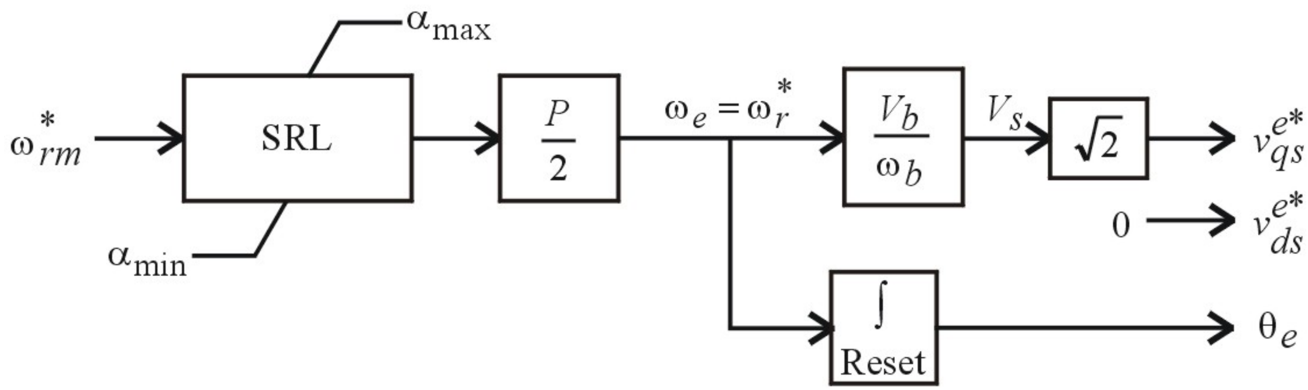
- History
- Advantages
- Disadvantages

Volts-Per-Hertz Control

- Basic Idea
  - Machine will operate close to synchronous speed, thus speed controlled with frequency
  - We must avoid saturation
- On Avoiding Saturation
  - $v_{as} = r_s i_{as} + p\lambda_{as}$ (14.2-1)
  - $V_s = \omega_e \Lambda_s$ (14.2-2)

Volts-Per-Hertz Control

- Elementary Volts Per Hertz Control

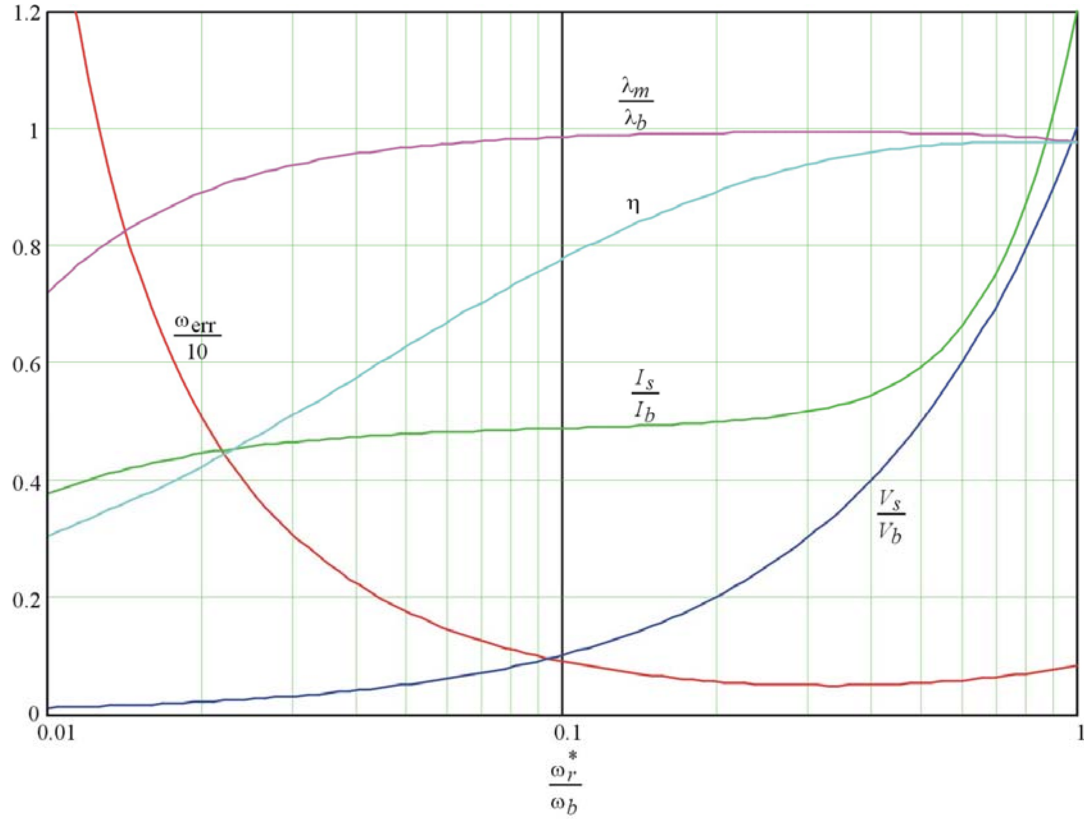


Volts-Per-Hertz Control

- Example System
  - 50 Hp Machine
  - Rated for 1800 rpm, 460 V l-l rms, P=4, 60 Hz
  - r_s=72.5 mΩ; r_r'=41.3 mΩ
  - L_M=30.1 mH; L_{ls}=L_{lr}=1.32 mH
  - $$T_L = T_b\left(0.1S(\omega_{rm}) + 0.9\left(\frac{\omega_{rm}}{\omega_{bm}}\right)^2\right) \qquad (14.2\text{-}3)$$

Volts-Per-Hertz Control

- Performance Characteristics



Volts-Per-Hertz Control

- Comments on Steady-State Performance

Volts-Per-Hertz Control

- Problem 1: Resistance At Low Speed

- Solution: Set Voltage So Slope is Invarient
  - $$T_e = \frac{3\left(\frac{P}{2}\right)\frac{\omega_e}{\omega_b}\left(\frac{X_M^2}{\omega_b}\right) r_r' s \left|\tilde{V}_{as}\right|^2}{\left[r_s r_r' + s\left(\frac{\omega_e}{\omega_b}\right)^2 (X_M^2 - X_{ss}X_{rr}')\right]^2 + \left(\frac{\omega_e}{\omega_b}\right)^2 (r_r' X_{ss} + s r_s X_{rr}')^2} \qquad (4.9\text{-}19)$$
  - $$V_s = V_b \sqrt{\frac{r_{s,est}^2 + \omega_e^2 L_{ss,est}^2}{r_{s,est}^2 + \omega_b^2 L_{ss,est}^2}} \qquad (14.2\text{-}4)$$

Volts-Per-Hertz Control

- Further Improvement – Current Feedback
  - Let's approximate torque as
    - $T_e = K_{tv}(\omega_e - \omega_r)$ (14.2-5)
    - $K_{tv} = -\left.\dfrac{\partial T_e}{\partial \omega_r}\right|_{\omega_r = \omega_e}$ (14.2-6)
  - Using (14.2-4) we have
    - $K_{tv} = \dfrac{3\left(\dfrac{P}{2}\right)L_M^2 r_r' V_b^2}{r_r'^2 (r_s^2 + \omega_b^2 L_{ss}^2)}$ (14.2-7)
    - This is not a function of synchronous speed

Volts-Per-Hertz Control

- Next, consider torque. We have
  - $T_e = \frac{3}{2}\frac{P}{2}(\lambda_{ds}^{e} i_{qs}^{e} - \lambda_{qs}^{e} i_{ds}^{e})$ (14.2-8)
- For steady-state conditions
  - $\lambda_{ds}^{e} = \frac{v_{qs}^{e} - r_s i_{qs}^{e}}{\omega_e}$ (14.2-9)
  - $\lambda_{qs}^{e} = -\frac{v_{ds}^{e} - r_s i_{ds}^{e}}{\omega_e}$ (14.2-10)
- Thus
  - $T_e = \frac{3}{2}\frac{P}{2}\frac{1}{\omega_e}(v_{qs}^{e*} i_{qs}^{e} - 2 r_s I_s^2)$ (14.2-11)
- Where
  - $I_s = \frac{1}{\sqrt{2}}\sqrt{i_{qs}^{e2} + i_{ds}^{e2}}$ (14.2-12)

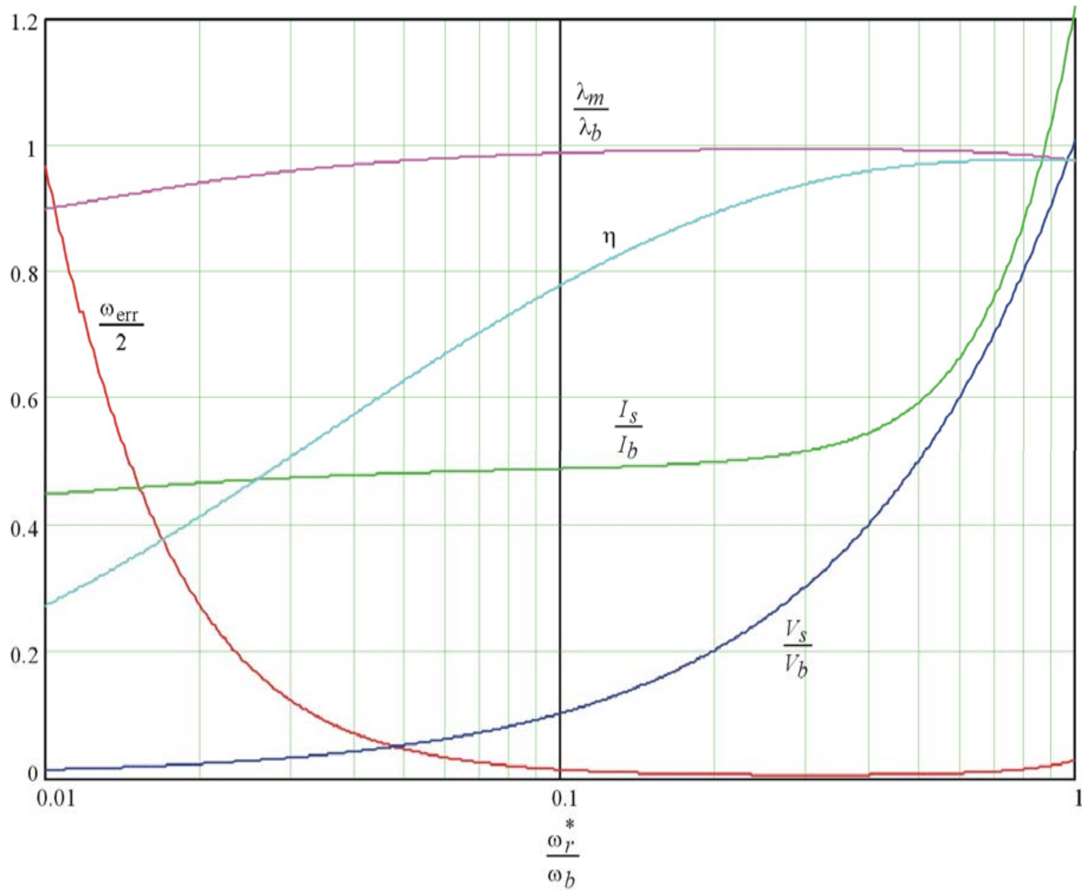
Volts-Per-Hertz Control

- Equating (14.2-7) and (14.2-11)
  - $\omega_e = \dfrac{\omega_r^* + \sqrt{\omega_r^{*2} + 3P(v_{qs}^{e*} i_{qs}^{e} - 2r_s I_s^2)/K_{tv}}}{2}$ (14.2-13)
- Or
  - $\omega_e = \dfrac{\omega_r^* + \sqrt{\max(0, \omega_r^{*2} + \mathrm{X}_{corr})}}{2}$ (14.2-14)
- Where
  - $\mathrm{X}_{corr} = H_{LPF}(s)\chi_{corr}$ (14.2-15)
  - $\chi_{corr} = 3P(v_{qs}^{e*} i_{qs}^{e} - 2r_s I_s^2)/K_{tv}$ (14.2-16)

Volts-Per-Hertz Control

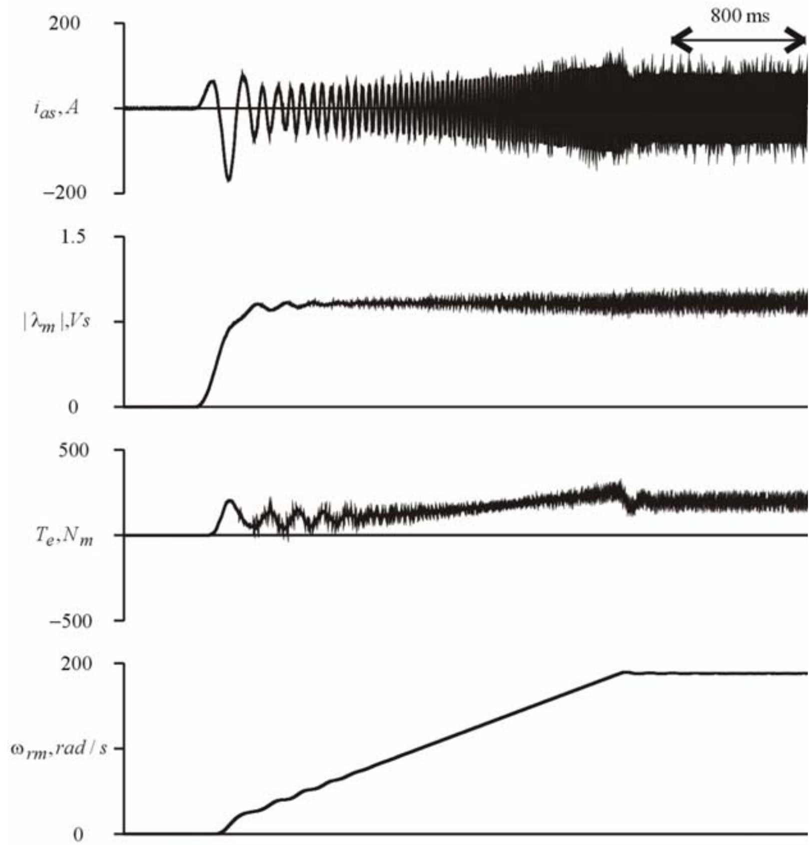
Volts-Per-Hertz Control

- Performance of Compensated Volts-Per-Hertz Control



Volts-Per-Hertz Control

- Start-Up Performance of Compensated Volts Per Hertz Control



Overview of Strategies

- Volts-Per-Hertz Control
- **Constant Slip Control**
- Field-Oriented Control

Constant Slip Control

- Basic Idea
  - Definition of Slip Frequency
  - $\omega_s = \omega_e - \omega_r$ (14.3-1)
  - Our Strategy

Derivation of Constant Slip Control

- Consider Our Expression for Electromagnetic Torque
- $$T_e = \frac{3\left(\frac{P}{2}\right)\omega_s L_M^2 I_s^2 r_r'}{(r_r')^2 + (\omega_s L_{rr}')^2} \qquad (14.3\text{-}2)$$
- Solving for the Current Yields
- $$I_s = \sqrt{\frac{2\,|T_e^*|\,(r_{r,est}'^2 + (\omega_s L_{rr,est}')^2)}{3P\,|\omega_s|\,L_{M,est}^2 r_{rr,est}'}} \qquad (14.3\text{-}3)$$
- The Plan

Avoiding Saturation

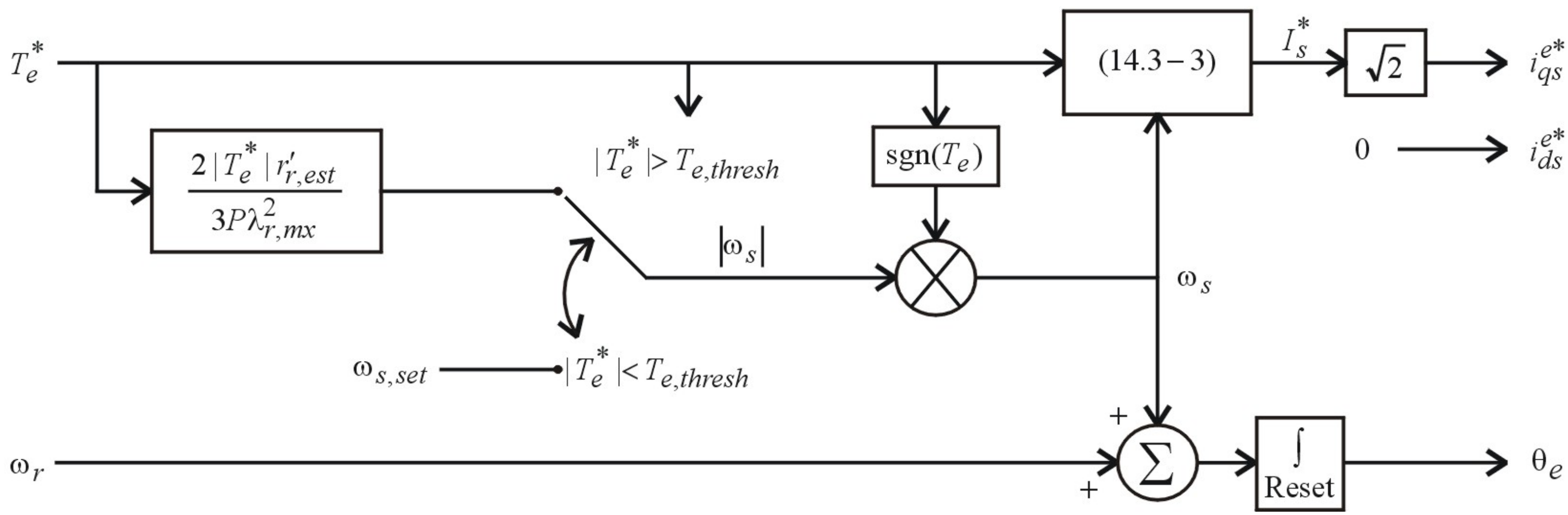
- Before Proceeding …

- Consider the Steady-State Equivalent Circuit
  - $\tilde{\lambda}_{ar} = L_{lr}\tilde{I}'_{ar} + L_M(\tilde{I}_{as} + \tilde{I}'_{ar})$ (14.3-4)
  - $\tilde{I}'_{ar} = -\tilde{I}_{as}\dfrac{j\omega_e L_M}{j\omega_e L'_{rr} + r'_r / s}$ (14.3-5)
  - Thus
  - $\lambda'_r = I_s L_M \dfrac{r'_r}{\sqrt{\omega_s^2 L'^2_{rr} + r'^2_r}}$ (14.3-7)

Avoiding Saturation

- Taking the Magnitude
- $\lambda_r' = I_s L_M \dfrac{r_r'}{\sqrt{\omega_s^2 L_{rr}'^2 + r_r'^2}}$ (14.3-7)
- Combing (14.3-7) and (14.3-2)
- $T_e = 3\dfrac{P}{2}\dfrac{\omega_s \lambda_r'^2}{r_r'}$ (14.3-8)
- Thus
- $T_{e,thresh} = 3\dfrac{P}{2}\dfrac{\omega_{s,set}\lambda_{r,\max}^2}{r_{r,est}'}$ (14.3-9)
- So For Large Torque Commands
- $\omega_s = \dfrac{2T_e^* r_{r,est}'}{3P\lambda_{r,\max}^2}$ (14.3-10)

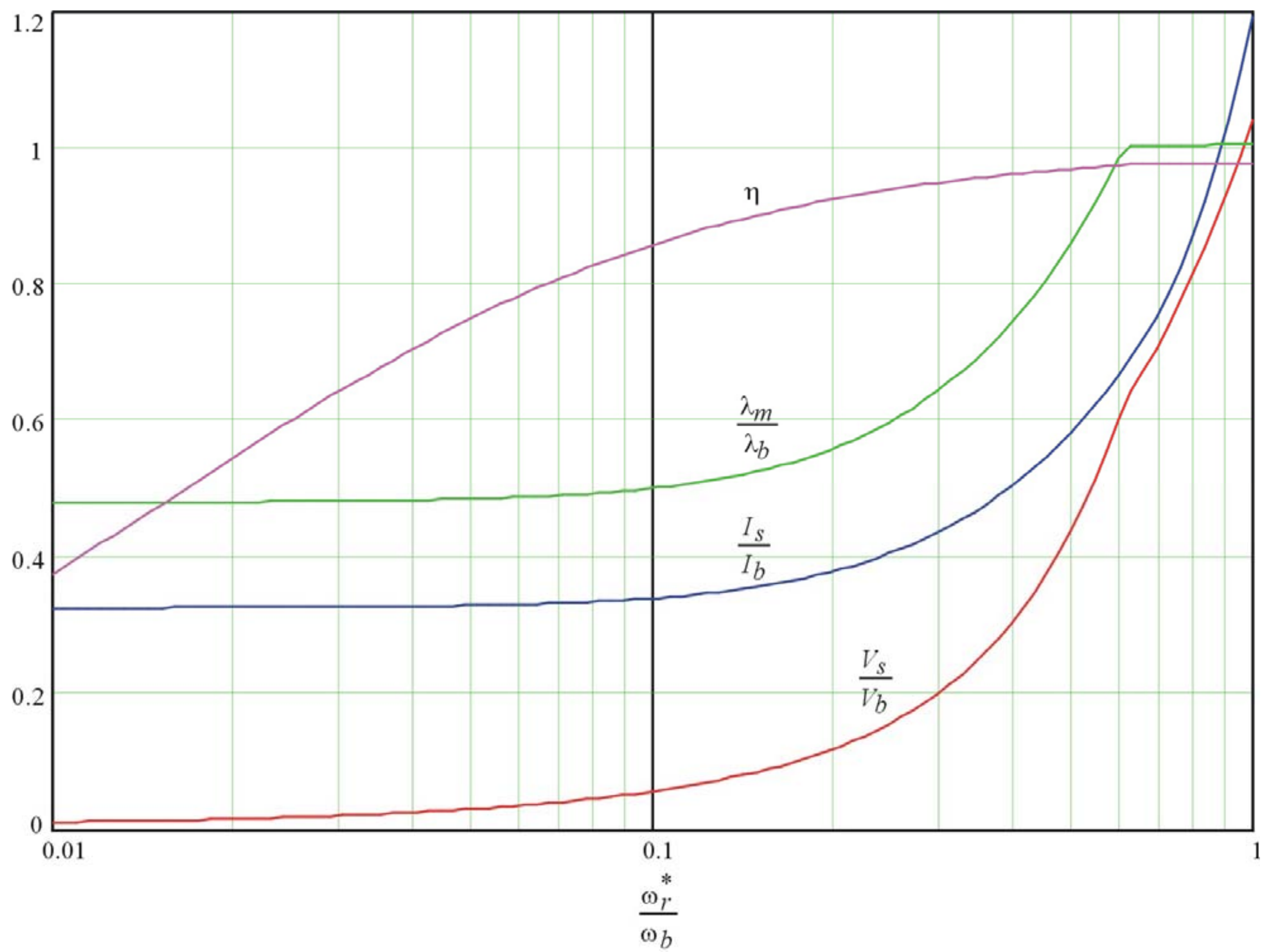
Constant Slip Control



Selection of Slip Frequency

- Control for Maximum Torque Per Amp
  - From (14.3-1)
  - $$\frac{T_e}{I_s^2} = \frac{3\left(\frac{P}{2}\right)\omega_{s,set} L_M^2 r_r'}{(r_r')^2 + (\omega_{s,set} L_r')^2} \qquad (14.3\text{-}11)$$
  - Optimizing with Respect to Slip Frequency
  - $$\omega_{s,set} = \frac{r'_{r,est}}{L'_{rr,est}} \qquad (14.3\text{-}12)$$

Maximum Torque Per Amp Control



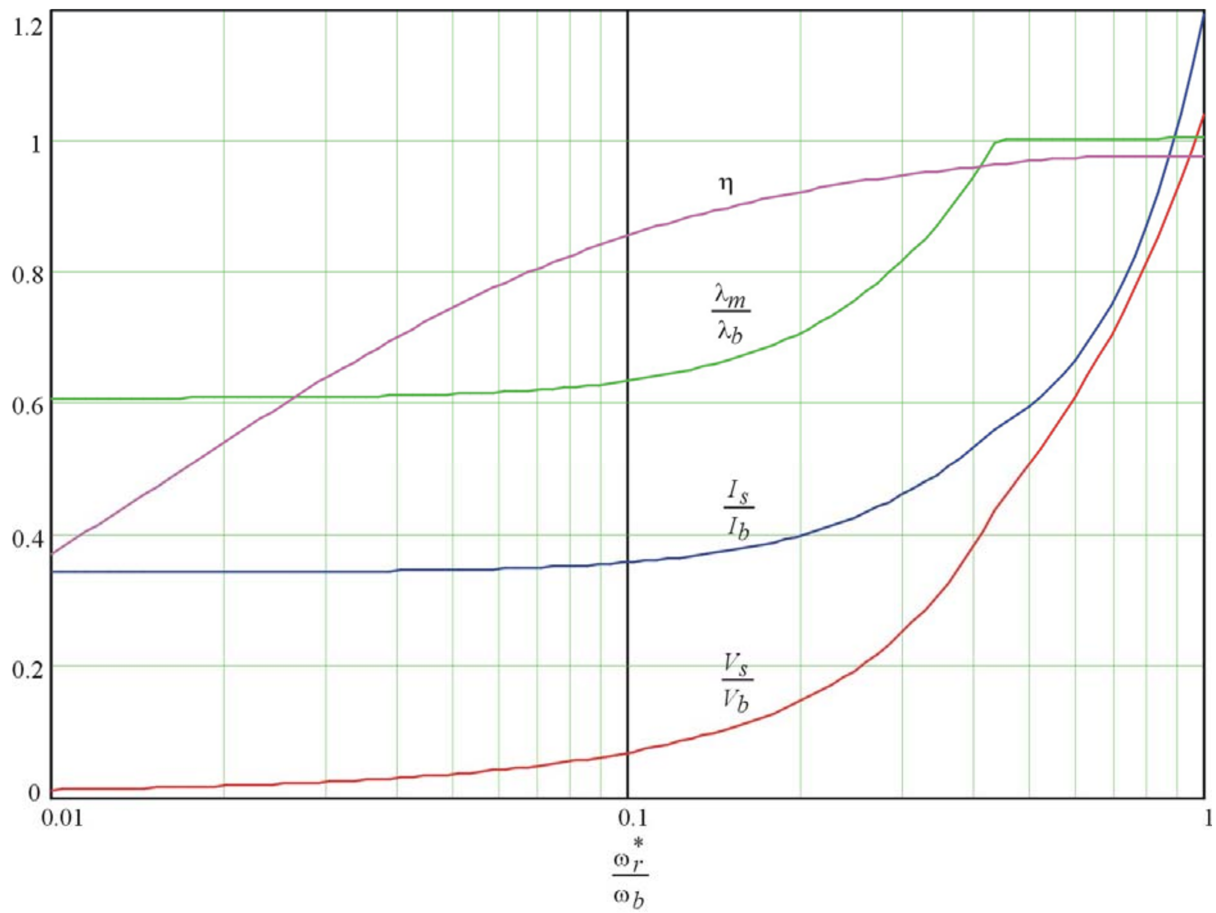
Maximum Efficiency Control

- To Optimize Efficiency Note That
  - $P_{in} = 3I_s \operatorname{Re}(\tilde{V}_{as})$ (14.3-13)
  - So
  - $P_{in} = 3r_s I_s^2 + \dfrac{3I_s^2 \omega_e L_M^2 \omega_s r_r'}{r_r'^2 + (\omega_s L_{rr}')^2}$ (14.3-14)
  - Comparison to (14.3-14) to (14.3-2)
  - $P_{in} = 3r_s I_s^2 + \dfrac{2}{P}\omega_e T_e$ (14.3-15)
  - Now
  - $P_{out} = \dfrac{2}{P}\omega_r T_e$ (14.3-16)

Maximum Efficiency Control

- So
- $P_{in} - P_{out} = 3r_s I_s^2 + \frac{2}{P} T_e \omega_s$ (14.3-17)
- Finally
- $P_{loss} = \frac{2}{P} T_e \left[\frac{r_r' r_s}{\omega_s L_m^2} + \frac{\omega_s r_s L_{rr}^2}{r_r' L_m^2} + \omega_s \right]$ (14.3-18)
- Minimizing Yields
- $\omega_{s,set} = \frac{r'_{r,est}}{L'_{rr,est}} \frac{1}{\sqrt{\frac{L_{m,est}^2}{L_{rr,est}'^2} \frac{r_{s,est}}{r'_{r,est}} + 1}}$ (14.3-19)

Maximum Efficiency Control

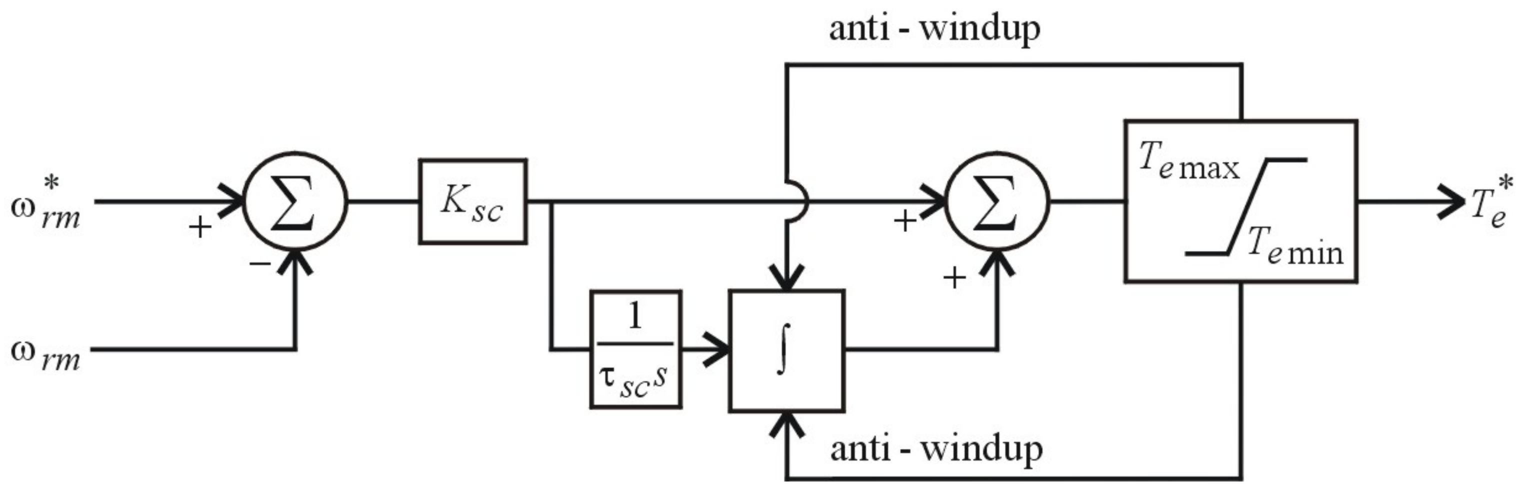


Comparison of Constant Slip Controls

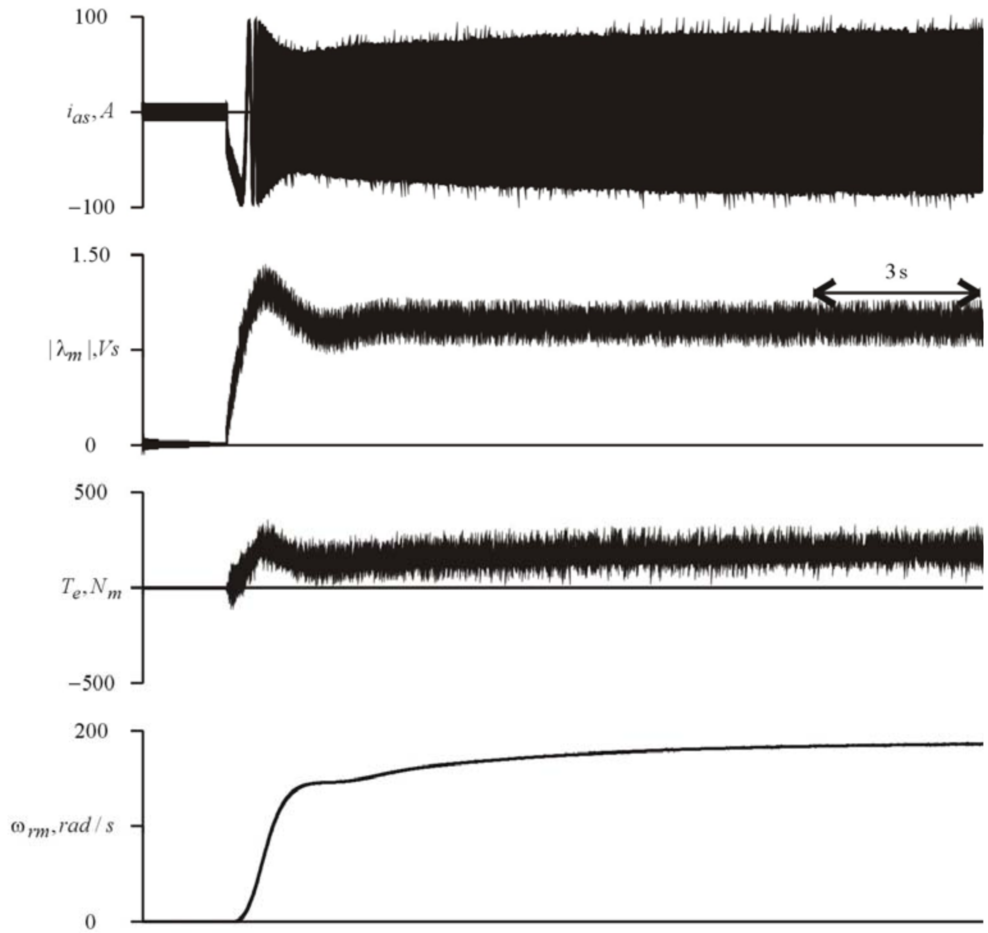
- Maximum Efficiency

- Maximum Torque Per Amp

Speed Control with Constant Slip Control



Transient Performance of Constant Slip Controlled Drive



Overview of Strategies

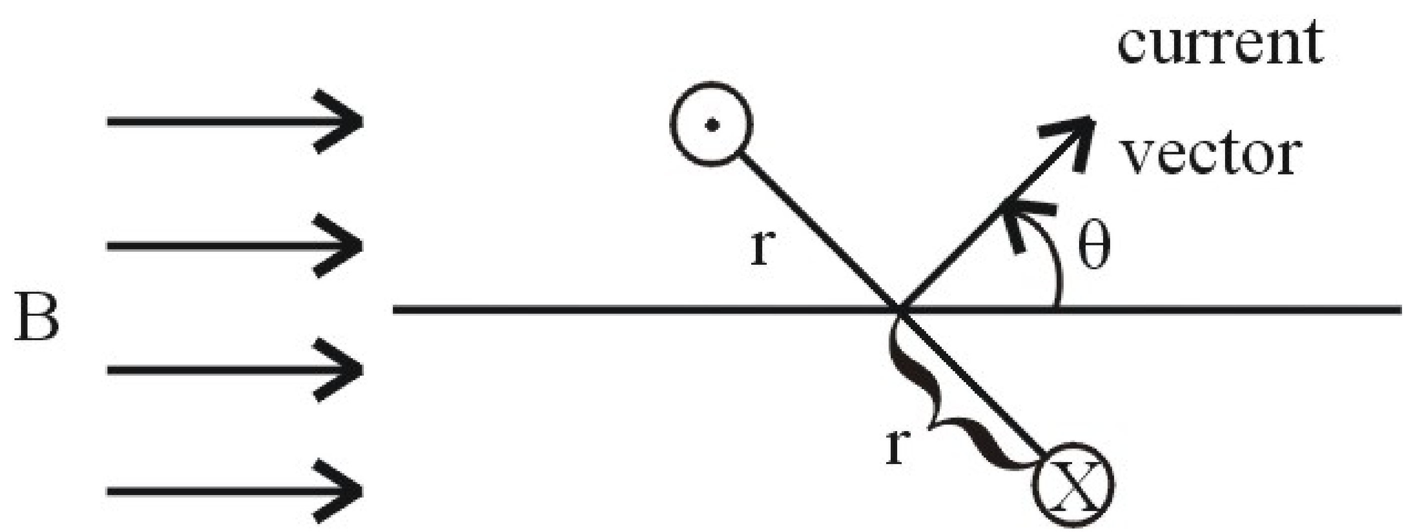
- Volts-Per-Hertz Control
- Constant Slip Control
- **Field-Oriented Control**

14.4 Field-Oriented Control

- Objective

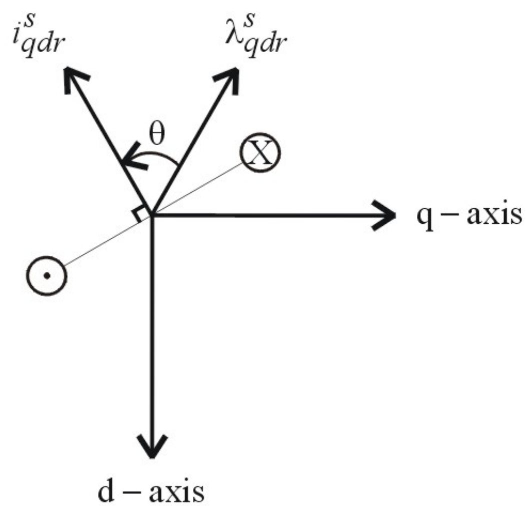
- Challenges

The Basic Idea



$$T_e = -2BiNLr\sin\theta$$

Torque Production in Induction Motor



- Now
  - $T_e = \frac{3}{2}\frac{P}{2}(\lambda'_{qr} i'_{dr} - \lambda'_{dr} i'_{qr})$ (14.4-2)
  - $T_e = -\frac{3}{2}\frac{P}{2} |\lambda'_{qdr}| |i'_{qdr}| \sin\theta$ (14.4-3)

The Plan

- Objective

- Observation
  - ➢In Steady-State, This is Always True
    - $i_{qr}^{e} = -\frac{1}{r_r'}(\omega_e - \omega_r)\lambda_{dr}^{e}$ (14.4-4)
    - $i_{dr}^{e} = \frac{1}{r_r'}(\omega_e - \omega_r)\lambda_{qr}^{e}$ (14.4-5)
    - $\lambda'^{e}_{qdr} \cdot i'^{e}_{qdr} = \lambda'^{e}_{qr} i'^{e}_{qr} + \lambda'^{e}_{dr} i'^{e}_{dr}$ (14.4-6)
  - ➢Thus

The Plan (Continued)

- But.. We need flux and current to be perpendicular all the time. To do this ..
  - $\lambda'^{e}_{qr} = 0$ (14.4-7)
  - $i'^{e}_{dr} = 0$ (14.4-8)
- Achieving (14.4-7)
  - By choice of θ_e

The Plan (Continued)

- Achieving (14.4-8)
  - All we need to do is to keep d-axis stator current constant. Proof:
    - Consider
      - $0 = r_r' i_{dr}'^e + (\omega_e - \omega_r)\lambda'^e_{qr} + p\lambda'^e_{dr}$ (14.4-9)
    - The q-axis rotor flux linkage is zero (by choice of reference frame)
      - $0 = r_r' i_{dr}'^e + p\lambda'^e_{dr}$ (14.4-10)
    - Substitution of d-axis rotor flux linkage equation into (14.4-10)
      - $pi_{dr}'^e = -\frac{r_r'}{L_{rr}} i_{dr}'^e - \frac{L_M}{L_{rr}} pi_{ds}^e$ (14.4-11)
    - Thus the d-axis rotor current goes to zero

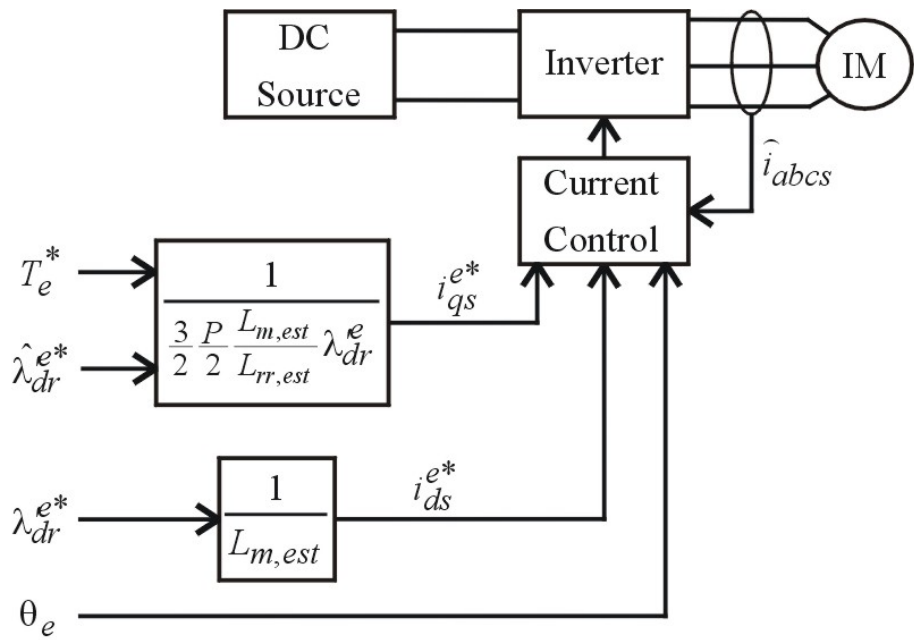
Some Observations

- Since the d-axis rotor current is zero
  - $\lambda_{ds}^{e} = L_{ss} i_{ds}^{e}$ (14.4-12)
  - $\lambda_{dr}^{\prime e} = L_{M} i_{ds}^{e}$ (14.4-13)
- Combining these results with our flux linkage equation
  - $T_e = -\frac{3}{2}\frac{P}{2}\lambda_{dr}^{\prime e} i_{qr}^{\prime e}$ (14.4-14)

Some Observations

- Since the q-axis rotor flux linkage is zero
  - $i'^e_{qr} = -\frac{L_M}{L_{rr}} i^e_{qs}$ (14.4-15)
- Thus
  - $T_e = \frac{3}{2}\frac{P}{2}\frac{L_M}{L_{rr}} \lambda'^e_{dr} i^e_{qs}$ (14.4-16)

Generic Rotor Flux-Oriented Control



Note: θ_e must be selected so as to keep q-axis rotor flux equal to zero.

Some “Minor” Details

- Determination of position of synchronous reference frame
- Determination of the rotor flux

Aside Types of Field Oriented Control

- Orientations
  - Stator
  - Magnetizing (Air-Gap)
  - **Rotor**
- Methods
  - **Direct**
  - **Indirect**
- Modifications
  - **Robust**

Direct Field Oriented Control

- Basic Idea
  - Measure the flux directly (more or less)
- Consider
  - $$\begin{bmatrix} \lambda_{qr}'^{e} \\ \lambda_{dr}'^{e} \end{bmatrix} = \begin{bmatrix} \cos\theta_e & -\sin\theta_e \\ \sin\theta_e & \cos\theta_e \end{bmatrix} \begin{bmatrix} \lambda_{qr}'^{s} \\ \lambda_{dr}'^{s} \end{bmatrix} \qquad (14.5\text{-}1)$$
- We need to set q-axis rotor flux to zero. Thus
  - $$\theta_e = angle(\lambda_{qr}'^{s} - j\lambda_{dr}'^{s}) + \frac{\pi}{2} \qquad (14.5\text{-}2)$$

Direct Field-Oriented Control

- As a Side Effect
  - $\lambda_{dr}'^{e} = \sqrt{(\lambda_{qr}'^{s})^2 + (\lambda_{dr}'^{s})^2}$ (14.5-3)

- The problem: We can measure the rotor flux linkages. We can measure the stator flux linkages.

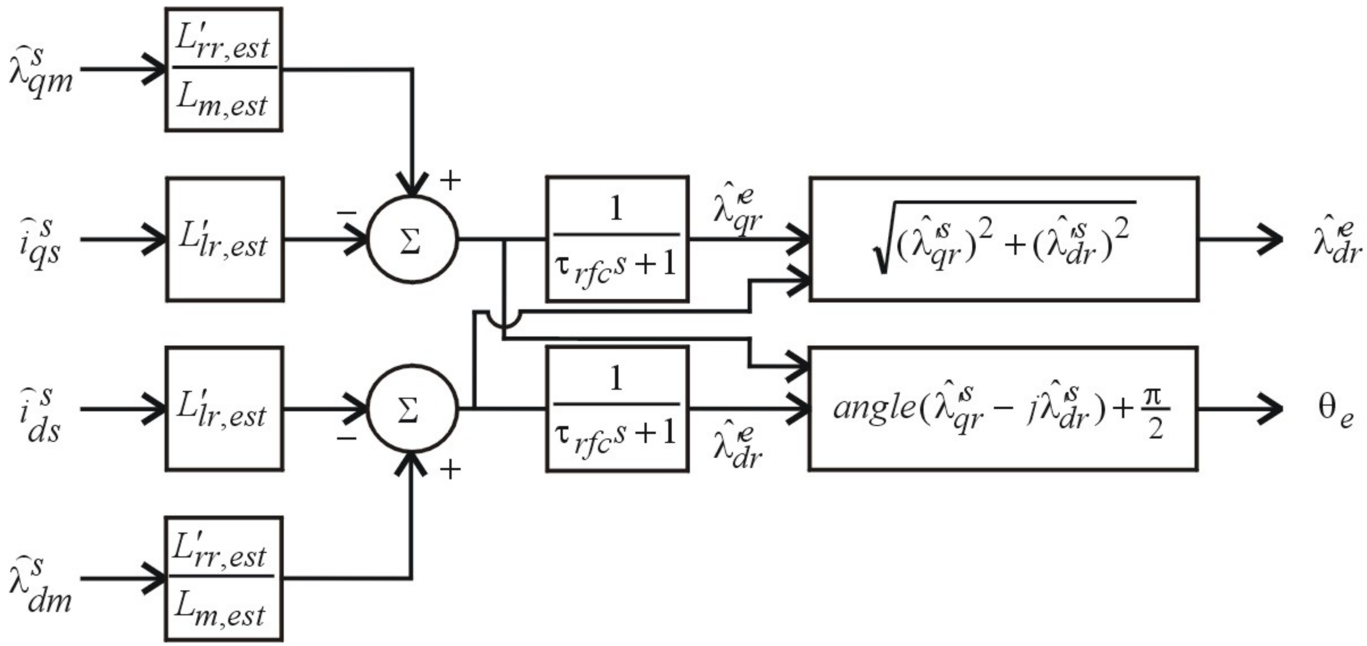
Estimation of Rotor Flux Linkages

- Consider
  - $\lambda_{qm}^{s} = L_M (i_{qs}^{s} + i_{qr}'^{s})$ (14.5-4)
- Thus
  - $i_{qr}'^{s} = \dfrac{\lambda_{qm}^{s} - L_M i_{qs}^{s}}{L_M}$ (14.5-5)
- Recall
  - $\lambda_{qr}'^{s} = L_{lr} i_{qr}'^{s} + L_M (i_{qs}^{s} + i_{qr}'^{s})$ (14.5-6)

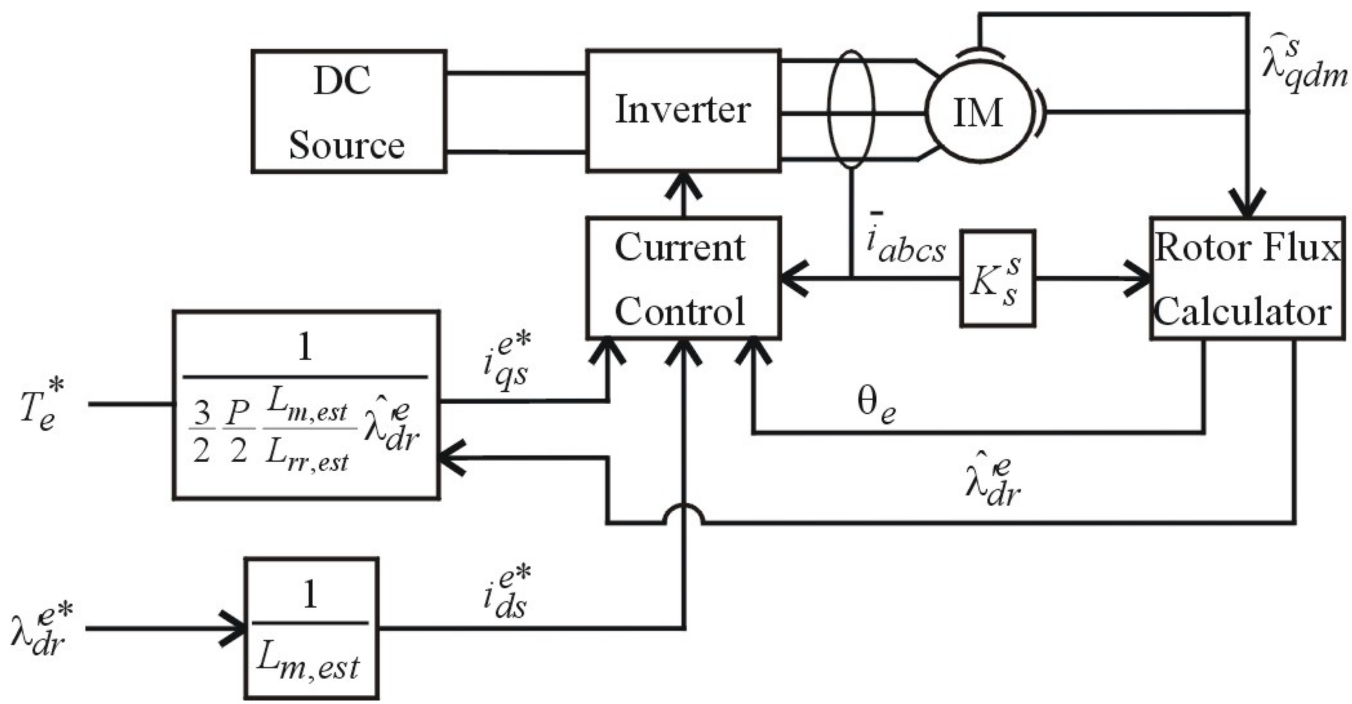
Estimation of Rotor Flux Linkages

- So
  - $\lambda'^{s}_{qr} = \frac{L'_{rr}}{L_M}\lambda^{s}_{qm} - L'_{lr} i^{s}_{qs}$ (14.5-7)
  - $\lambda'^{s}_{dr} = \frac{L'_{rr}}{L_M}\lambda^{s}_{dm} - L'_{lr} i^{s}_{ds}$ (14.5-8)

Rotor Flux Estimator



Direct Rotor Field-Oriented Control



Robust Direct Field-Oriented Control

- Sensitivity of Rotor Flux Estimator
  - $$\hat{\lambda}'^{e}_{qdr} = \frac{L'_{rr,est}}{L_{M,est}} \hat{\lambda}^{s}_{qd,m} - L'_{lr,est} \hat{i}^{s}_{qds} \qquad (14.6\text{-}1)$$

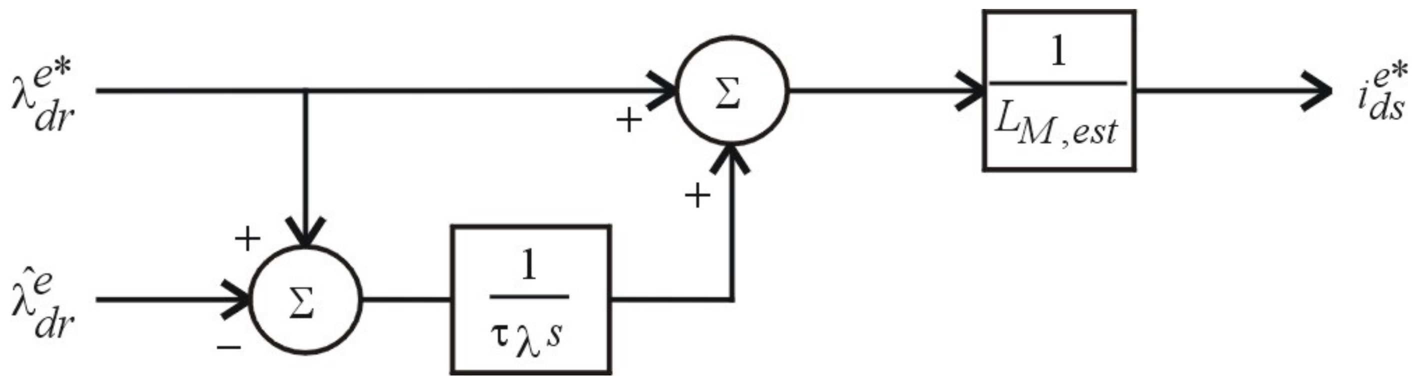
- Sensitivity of Q-Axis Current Calculation
  - $$i^{e*}_{qs} = \frac{T^{*}_{e}}{\frac{3}{2}\frac{P}{2}\frac{L_{M,est}}{L_{rr,est}}\hat{\lambda}'^{e}_{dr}} \qquad (14.6\text{-}2)$$

Robust Direct Field-Oriented Control

- Sensitivity of the D-Axis Current Injection
  - $$i_{ds}^{e*} = \frac{\lambda_{dr}^{e*}}{L_{M,est}} \qquad (14.6\text{-}3)$$

Flux Control Loop

- Since we have a flux estimator, consider



- It can be shown that

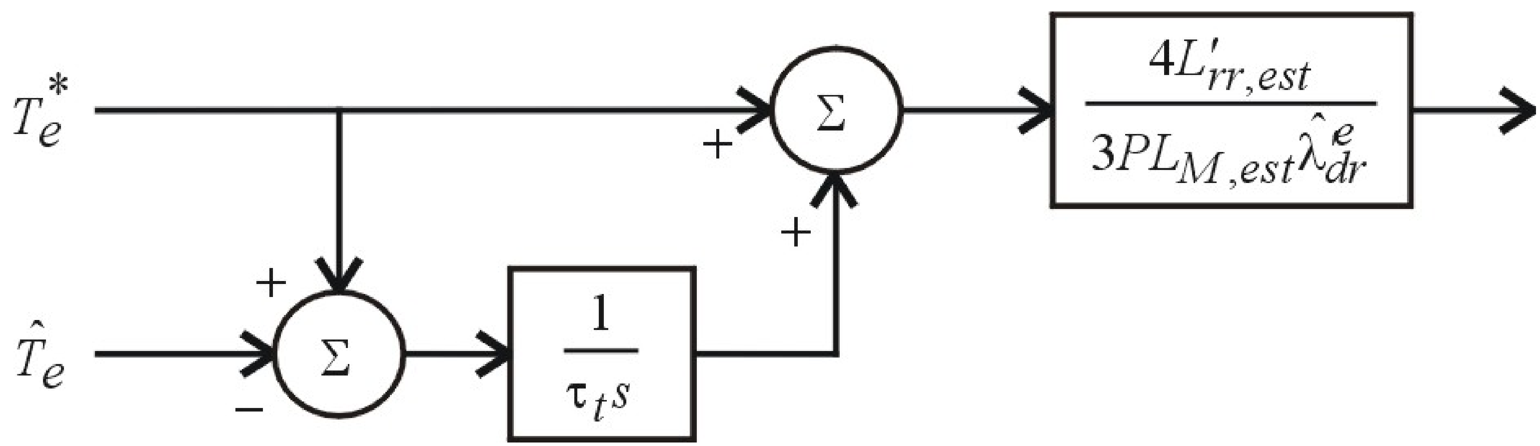
➢ $$\frac{\hat{\lambda}_{dr}^{e}}{\lambda_{dr}^{e*}} = \frac{\tau_\lambda s + 1}{\tau_\lambda \dfrac{L_{M,est}}{L_M} s + 1} \qquad (14.6\text{-}5)$$

Torque Control Loop

- Torque Estimation
  - Recall
  - $T_e = \frac{3}{2}\frac{P}{2}(\lambda_{ds}^{s} i_{qs}^{s} - \lambda_{qs}^{s} i_{ds}^{s})$ (14.6-6)
  - Now
  - $\lambda_{qds}^{s} = L_{ls} i_{qds}^{s} + \lambda_{qdm}^{s}$ (14.6-7)
  - So
  - $T_e = \frac{3}{2}\frac{P}{2}(\lambda_{dm}^{s} i_{qs}^{s} - \lambda_{qm}^{s} i_{ds}^{s})$ (14.6-8)
  - Which Suggests
  - $\hat{T}_e = \frac{3}{2}\frac{P}{2}(\hat{\lambda}_{dm}^{s} \hat{i}_{qs}^{s} - \hat{\lambda}_{qm}^{s} \hat{i}_{ds}^{s})$ (14.6-9)

Torque Control Loop

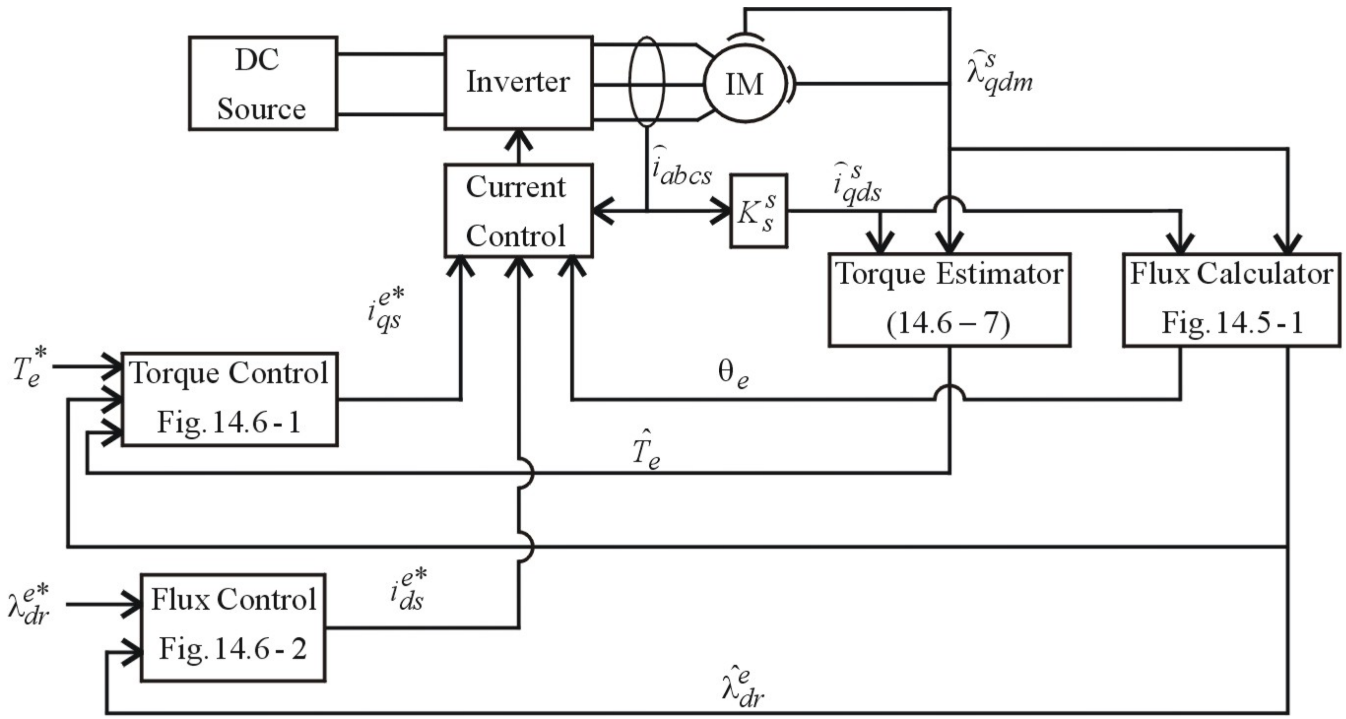
- Now that we can estimate torque, how about



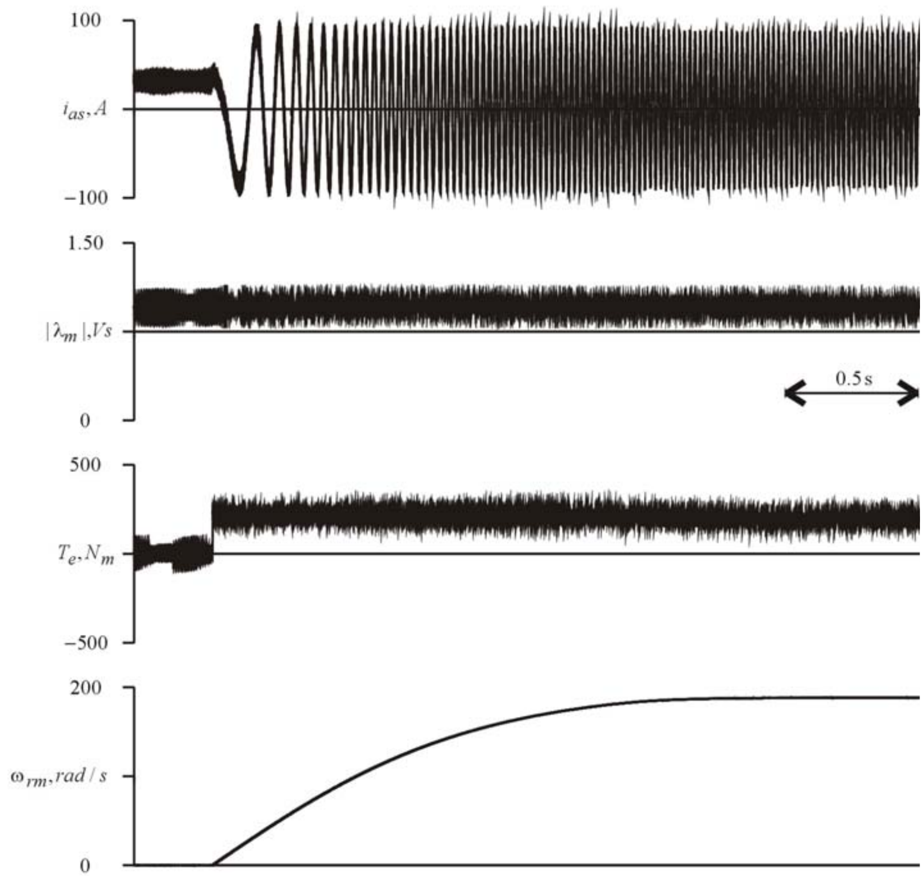
Torque Control Loop

- Analysis of Torque Control Loop
  - Define
  - $K_{t,est} = \frac{3}{2}\frac{P}{2}\frac{L_{M,est}}{L'_{rr,est}}\lambda'^{e*}_{dr}$ (14.6-10)
  - $K_t = \frac{3}{2}\frac{P}{2}\frac{L_M}{L'_{rr}}\lambda'^{e}_{dr}$ (14.6-12)
  - Now, assuming the control works
  - $T_e = K_t i^{e*}_{qs}$ (14.6-11)
  - This, assumption, coupled with our torque control loop yields
  - $\frac{T_e}{T_e^*} = \frac{\tau_t s + 1}{\tau_t \frac{K_{t,est}}{K_t} s + 1}$ (14.6-13)

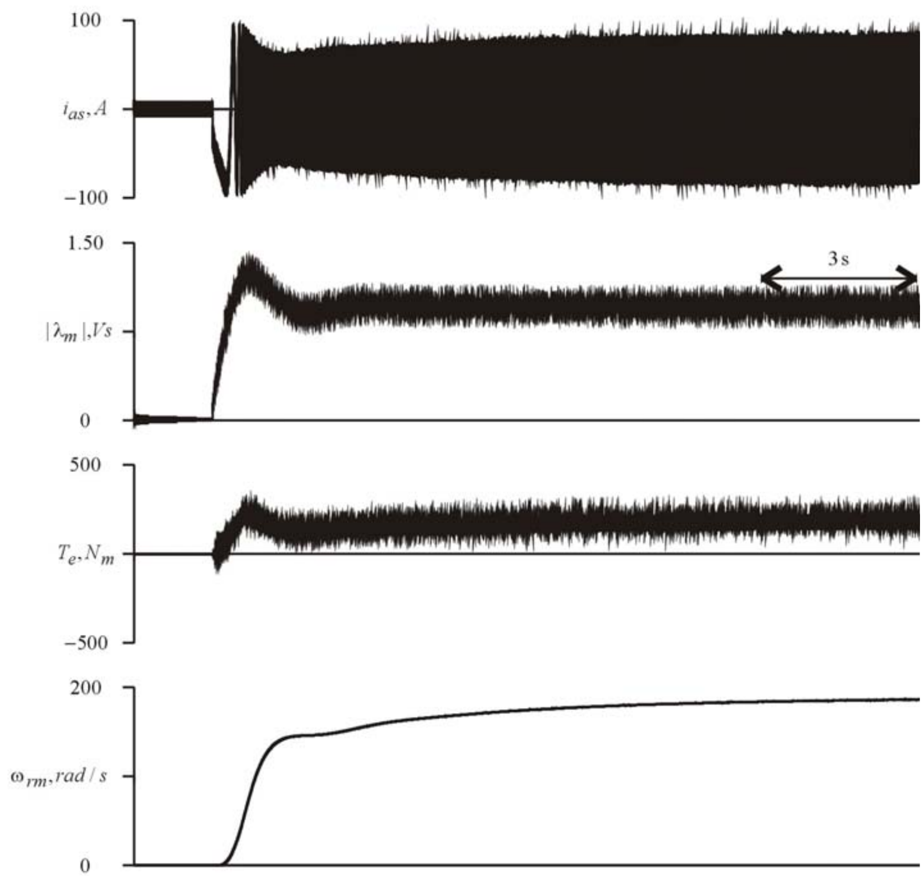
Robust Direct Field-Oriented Control



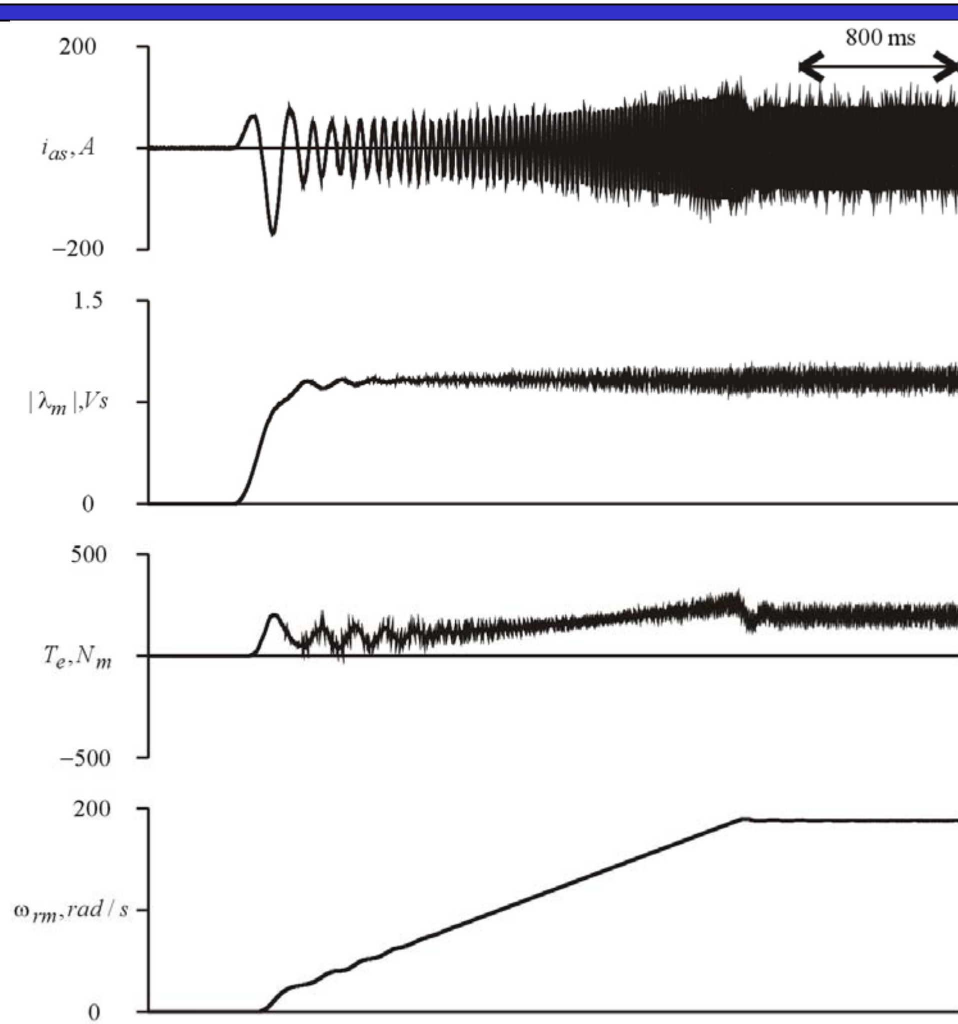
Performance of Robust Direct Field Oriented Control



Performance of Constant Slip Drive



Performance of Volts-Per-Hertz Drive



Indirect Field Oriented Control

- Consider
  - $0 = r'_{rr} i'^{e}_{qr} + (\omega_e - \omega_r)\lambda'^{e}_{dr} + p\lambda'^{e}_{qr}$ (14.7-1)
  - Since the q-axis rotor flux is zero
  - $\omega_e = \omega_r - r'_r \dfrac{i'^{e}_{qr}}{\lambda'^{e}_{dr}}$ (14.7-2)
  - Which may be expressed
  - $\omega_e = \omega_r + \dfrac{r'_r}{L'_{rr}} \dfrac{i^{e}_{qs}}{i^{e}_{de}}$ (14.7-3)

Indirect Field Oriented Control

- Thought
  - We know what the electrical frequency will be if we have field orientation (i.e. field orientation causes (14.7-1))
  - Does it work backward ? Will using (14.7-1) cause field-orientation ?
  - The answer: Yes !

Indirect Field Orientation

- Proof
  - Consider the rotor voltage equations
  - $0 = r_r' i_{qr}'^e + (\omega_e - \omega_r)\lambda_{dr}'^e + p\lambda_{qr}'^e$ (14.7-5)
  - $0 = r_r' i_{dr}'^e - (\omega_e - \omega_r)\lambda_{qr}'^e + p\lambda_{dr}'^e$ (14.7-6)
  - Now substitute in our expression for electrical frequency
  - $0 = r_r' i_{qr}'^e + \dfrac{r_r'}{L_{rr}'}\dfrac{i_{qs}^{e*}}{i_{de}^{e*}}\lambda_{dr}'^e + p\lambda_{qr}'^e$ (14.7-7)
  - $0 = r_r' i_{dr}'^e - \dfrac{r_r'}{L_{rr}'}\dfrac{i_{qs}^{e*}}{i_{de}^{e*}}\lambda_{qr}'^e + p\lambda_{dr}'^e$ (14.7-8)

Indirect Field Orientation

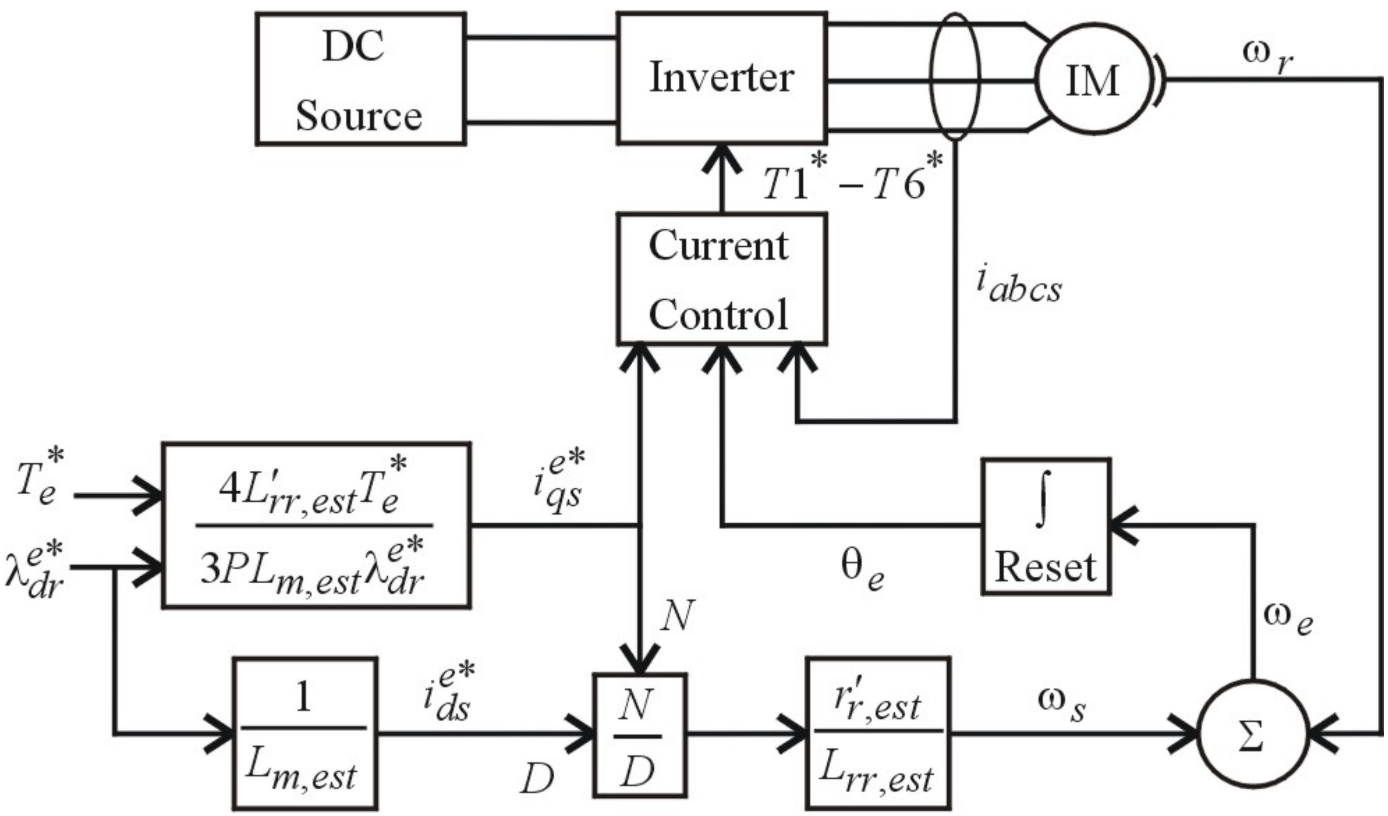
- Continuing On …
  - Now put in terms of q-axis rotor flux and d-axis rotor current (and stator currents)
  - This yields
  - $$0 = r_r'\left[\frac{\lambda_{qr}'^e - L_M i_{qs}^{e*}}{L_{rr}'}\right] + \frac{r_r'}{L_{rr}'}\frac{i_{qs}^{e*}}{i_{ds}^{e*}}\left[L_{rr}' i_{dr}'^e + L_M i_{ds}^{e*}\right] + p\lambda_{qr}'^e \qquad (14.7\text{-}9)$$
  - $$0 = r_r' i_{dr}'^e - \frac{r_r'}{L_{rr}'}\frac{i_{qs}^{e*}}{i_{ds}^{e*}}\lambda_{qr}'^e + p\left[L_{rr}' i_{dr}'^e + L_M i_{ds}^{e*}\right] \qquad (14.7\text{-}10)$$

Indirect Field Orientation

- Now, keeping the d-axis stator current fixed we have
  - $$p\lambda_{qr}^{\prime e} = -\frac{r_r'}{L_{rr}'}\lambda_{qr}^{\prime e} - r_r' \frac{i_{qs}^{e*}}{i_{ds}^{e*}} i_{dr}^{\prime e} \qquad (14.7\text{-}11)$$
  - $$pi_{dr}^{\prime e} = -\frac{r_r'}{L_{rr}'} i_{dr}^{\prime e} + \frac{r_r'}{(L_{rr}')^2} \frac{i_{qs}^{e*}}{i_{ds}^{e*}} \lambda_{qr}^{\prime e} \qquad (14.7\text{-}12)$$
- Comments

Indirect Field Oriented Control

- Thus, our control becomes



Indirect Field Oriented Control

- Advantages of Indirect Field Oriented Control over Direct Field Oriented Control

- Disadvantages of Indirect Field Oriented Control over Direct Field Oriented Control

Performance of Detuned Indirect Field Oriented Control

- Overestimate magnetizing inductance by 25% (we started to saturate machine)
- Underestimate rotor resistance by 25% (rotor resistance increased with temperature)

Performance of Detuned Field Oriented Control