

EECE2010: Circuit Theory I

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Office Hours: MW: 10am-12pm

<http://faculty.uml.edu/thu/16.201/material.htm>

http://faculty.uml.edu/thu/16.201/lecture_note.htm

Recitation Instructors:

Dr. Ryan Mcpherson;

Dr. Rachel Wakim

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L1

§1.1 Introduction: About this course:

- Foundation to all branches of electrical and computer engineering
 - **Communication systems** : all information is encoded in electrical signals (digital, like 010101, analog, like a sinusoid)
 - **Computer systems**: all operation and data handling are performed via electrical variables – Everything is composed of 0 and 1.
 - **Control systems**: physical quantities, such as speed, force, temperature, pressure, are transformed into electrical signals. Control algorithms process these signals and figure out the optimal actuation strategy, like how much force to apply, which direction.
 - **Power systems**: all forms of energy (mechanical, nuclear, wind, chemical, hydraulic) are converted into or from electrical power, since electrical energy can be easily stored (in battery, supercapacitor) and transported.
 - **Signal and image processing**: all forms of information converted into electrical signals, and processed via computer, digital/analog filter, for pattern recognition, comparison, identification.

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- Also important to other engineering systems, in actuation, sensing, instrumentation and various purposes for automation
 - **Mechanical and aerospace systems**
 - **Chemical process**, actuation, sensing, instrumentation. To adjust the temperature, pressure, density, flow rate.
 - **Biomedical systems**, electrical devices are widely used in diagnosing and treatment
- Many physical quantities are transformed into electrical signals because electrical signals are easy to transfer and manipulate,
 - e.g., to be processed by computers
- Imagine how your life will be changed without electricity?
- It directly impacts your career: You will not be qualified as an electrical engineer if you fail this course: you cannot move on to other core courses.

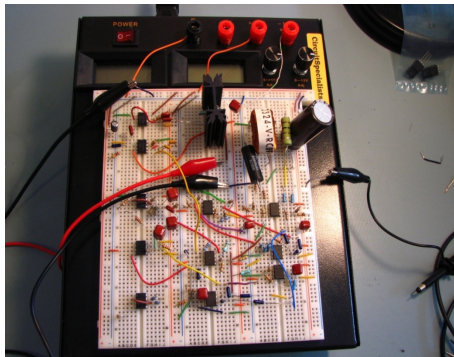
Most importantly:

It is a lot of fun to build a useful circuit!

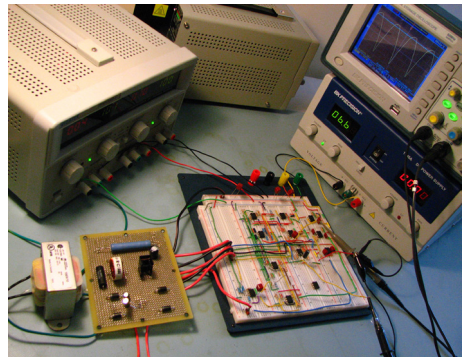
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A buck-boost
Converter with
Control circuit

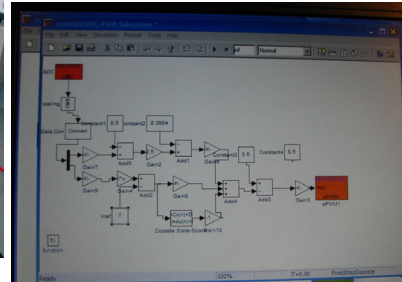
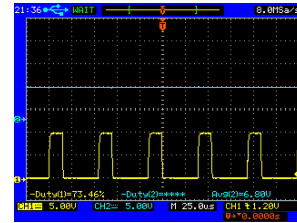
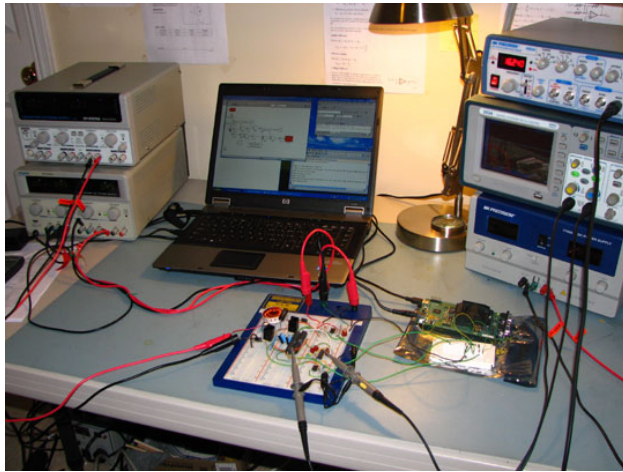


An AC-DC
Converter with
Control circuit

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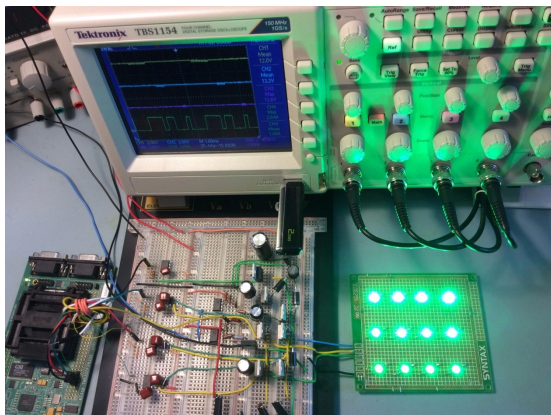


A boost converter controlled by a microcontroller
 The controller is constructed using Matlab/Simulink ,
 Then written into the microprocessor

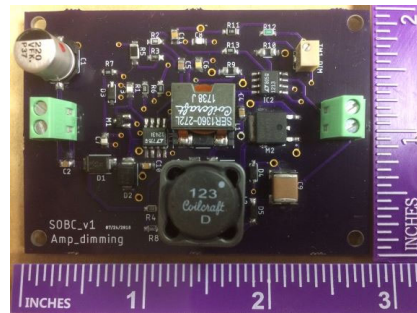
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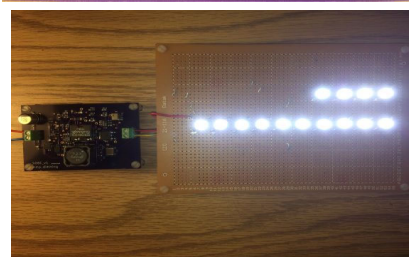
Last year's project:
 A low cost high performance
 LED driver



My most recent project:

A high efficient high performance
 LED driver with dimming control

The paper by my PhD student and myself
 will be published by
 IEEE Transactions on Power Electronics.
 Funded by National Science Foundation



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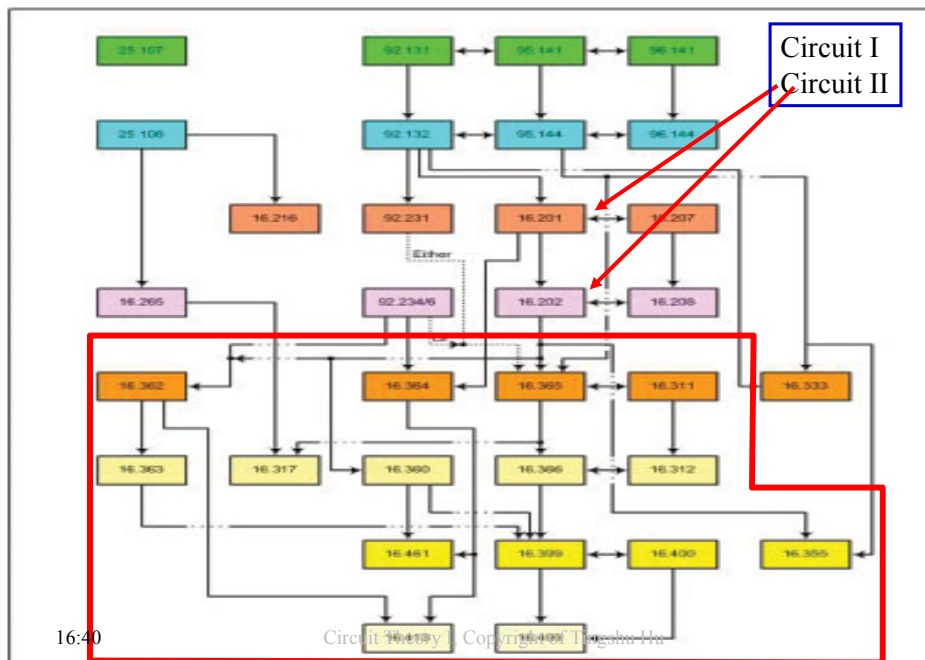
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The courses inside the red box all need this course as prerequisite

L1

Curriculum check sheet of BS in EE



L1

Textbook:

C. K Alexander and M. N. O. Sadiku, Fundamentals of Electric Circuits, edition 6, 5, or 3,4, 2 (McGraw Hill).

GRADING POLICY:

Homework+Attendance: 11%

3 Tests: 54%

Final Exam: 20%

Quizzes: 15%

HOMEWORK POLICY:

- Late homework is NOT accepted.
- Homework should be clear, concise, and complete.

Attendance: Will be taken every class. Positive attitude is a key to success. Being half hearted is a waste of time.

Course materials: <http://faculty.uml.edu/thu/16.201/material.htm>
(homework assignment/solution, sample tests/solutions)

Lecture notes: http://faculty.uml.edu/thu/16.201/lecture_note.htm

Prerequisite:

- Calculus II with grade C or better

Circuit Theory I Tentative Class schedule (Spring 2019)

Period 1: Chapter 1,2: Basic concepts, Basic laws

1/23(W): Course overview, (1.2), Charge & Current(1.3), reference direction of current
 1/28(M): ref. dir of current; voltage(1.4), power & energy(1.5), circuit elements(1.6)
 1/30(W): Ohm's law (2.2); Nodes, branches, loops(2.3), KCL, KVL(2.4)
 2/4(M): Use basic laws to solve circuit problems (2.4)
 2/6(W): Series resistors & voltage division (2.5); Parallel resistors & current division(2.6),
 2/11(M): Solving circuit problems using basic laws and tools
 2/13(W): More practice problems
 2/19(Tu): **Test 1 (no cheat sheet)**

Period 2: Chapter 3, Methods of analysis

2/20(W): Linear algebra review, Nodal analysis (2.2)
 2/25(M): Nodal analysis with voltage sources (2.3)
 2/27(W): Mesh analysis (2.4); Mesh with current sources (2.5)
 3/4(M): More mesh analysis problems
 3/18(M): **Test 2 (no cheat sheet)**

Please show clear steps in Quizzes, Tests and Exams.

If major steps are missing, or wrong steps with correct final answer, you may be asked to retake the Test, with some changes.

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Period 3: Chapter 4, Circuit Theorems

3/6(W): Linearity (4.2), Superposition without dependent source(4.3)
 3/20(W): Superposition with dependent source (4.3), Source transformation (4.4)
 3/25(M): Thevenin's theorem (4.5)
 3/27(W): Norton's theorem (4.6), Maximum power transfer (4.7)
 4/1(M): **Test 3 (no cheat sheet)**

Period 4: Chapters 6,7,8

4/3(W): Chapter 6, capacitors and inductors
 4/8(M): Source free RC (7.2)
 4/10(W): Source free RL (7.3), Singularity functions (7.4)
 4/17(W): Step resp. of RC (7.5), Step resp. of RL (7.6)
 4/19(F): Solving 2nd-order differential equations
 4/22(M): Finding initial values (8.2), Step resp. of a series RLC circuit(8.5), source free RLC(8.3)
 4/24(W): Step resp. of a series RLC circuit(8.5), Briefly mention other 2nd-order circuits (8.6,8.7)
 4/29(M): Review
 5/1(W): Office hours

Final Exam will cover Chapters 7,8

All tests and exam are closed-book, closed-notes.

Makeup test/exam will only be given in *extreme* emergency or illness (Evidence required).
 Ask for help before the final exam, NOT after! NO extra work to raise your grade after final!

Attention: You need to pass Circuit I with grade C- or better to take Circuit II.

A sample score and grading table from a previous term

ID	Name:	Section	20x	Test 1	Test 2	Test 3	Qz	hw/att	FE	opt1	opt2	score	Final grade
		100%	20	20	20	15	11	20		100	100	100	
1			13.8	17.5	9.4	9	9.19	13.3		68.12	67.59	68.1	C-
2			16	17.5	15.2	10.5	10	17.5		81.83	84.36	84.4	B+
3			19	19.7	19	14.3	10.4	20		96.56	98.09	98.1	A
4			12.7	16.3	13.5	9.5	8.82	15.6		72.89	75.22	75.2	C+
5			12	15	6.6	7.76	4.44	5.1		47.54	35.33	47.5	F
6			16.9	12.1	17.8	11.3	5.68	11.4		70.45	62.05	70.5	C
7			8.4	14.4	12	11.7	8.71	12.7		67.72	67.51	67.7	C-
8			13.7	0.5		3.26	2.98			19.02	9.548	19	W
9			18.6	18.8	18.3	12.5	2.72	12		77.35	64.21	77.3	B-
10			19.8	16.2	19.2	11.3	7.79	7.8		76.52	55.32	76.5	C+
11			11	16	10.8	12.8	7.76	8.3		62.83	54.23	62.8	D
12			16.4	8.9	14.3	9	3.6	5		53.24	36.84	53.2	F
13			19	19.1	16.5	12.8	9.0	16.7		87.63	81.96	87.6	A-
14			16.2	18.3	14.4	10	3.82	17		74.84	76.23	76.2	C+
15			19.2	18	19.1	13	8.02	18.3		90	89.06	90	A
16			18	9.8	13.7	6.38	1.91	15.7		58.2	55.65	58.2	D
17			12	11.5	10	7.76	9.06	15.2		62.17	70.23	70.2	C
18			9	11.3	6	8	7.16	2.9		41.73	29.99	41.7	F
19			15.6	19.3	15.5	11.3	9.66	17		83.27	83.67	83.7	B+

Note: Need C- or better to take Circuit II Questions?

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§1.2 International Systems of units (SI)

Units: Standard measurements of physical quantities.

- facilitate international communication

Principal units (basic SI units):

Quantity	Basic unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Temperature	kelvin	K
Luminous Intensity	candela	cd

Other quantities, like, force, speed, torque, power, energy, are derived from these.

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Prefixes based on the power of 10

Multiplier	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	k
100	hecto	h
10	deka	da
0.1	deci	d
0.01	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

Under certain situations, the quantities are within a particular range, e.g., 0.0000001s to 0.0001s; It would be more convenient to say $1\mu\text{s}$ to $100\mu\text{s}$.

Sometimes, 1 meter may be too large, e.g., to measure the length of an atom, we may use nm or pm.

In other situations, 1 meter may be too small, e.g., to measure the distance between planets, we may like to use Em or Pm

In the rest of Chapter 1, we review 5 basic concepts

- Charge
- Current
- Voltage
- Power
- Energy

We will deal with these concepts more rigorously.

Pay attention to:

- Reference direction of current
- Polarity of voltage
- Passive sign convention
- Active sign convention
- Dependent voltage source
- Dependent current source

§1.3 Charge and Current

Charge: Fundamental concept for explaining all electrical phenomena, such as, light, heat, signal, data,...

Charge is an electrical property of atomic particles of which matter consists, measured in Coulombs (C)

Notation of charge: q

Charge on one electron, $q = -1.602 \times 10^{-19} \text{ C}$.

Charge on one proton, $q = +1.602 \times 10^{-19} \text{ C}$.

Charge on an atom = charge on electrons + charge on protons, $q = 0$.

1 C is a huge quantity.

We need 6.24×10^{18} electrons (or protons) to make 1C of charge. Quantities to the order of pC, or nC are usually used in the lab.

$-1\text{pC} = 6.24 \times 10^6$ electrons, $1\text{nC} = 6.24 \times 10^9$ electrons.

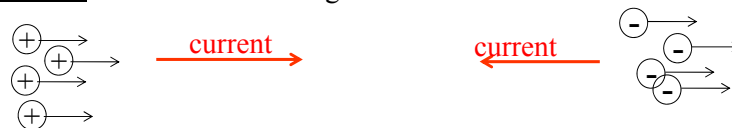
Charge can neither be created nor destroyed. It can only be transferred.

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Current: the motion of charges.



Direction of current:

the direction of motion of positive charges.

In metallic conductors, current is caused by the motion of electrons.

Direction of the current: opposite to the motion of electrons

All electrical phenomena are caused by the flow of charges:

- Heating
- Light
- Force - In a solenoid. Make a coil of wire around an iron core.
 - Fundamental concept behind electro-mechanics
 - Electrical energy \leftrightarrow Mechanical Energy

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Definition of current: Current is the time rate of change of charge, measured in ampere (A). Notation: i

$$i \triangleq \frac{dq}{dt}$$

Also, current is the amount of charge flowing through a substance, or a cross sectional area, in 1 second.

1 ampere = 1 Coulomb/second;

1A=1C/s, ampere is a derived unit

Given the current as a function of time, $i(t)$, for $t \in [t_0, t]$, the total charge flowing through a substance over $[t_0, t]$ is

$$q(t) - q(t_0) = \int_{t_0}^t i(\tau) d\tau \quad \longleftrightarrow \quad q(t) = q(t_0) + \int_{t_0}^t i(\tau) d\tau$$

Example: $q(t) = \sin 2t \text{ C}$, $i(t) = dq/dt = 2 \cos 2t \text{ A}$

$q(t) = \cos 3t \text{ (mC)}$, $i(t) = dq/dt = -3 \sin 3t \text{ (mA)}$

A list of $q(t)$, $i(t)$

$q(t)$	$i(t) = dq/dt$
$\sin \omega t$	$\omega \cos \omega t$
$\cos \omega t$	$-\omega \sin \omega t$
e^{at}	ae^{at}
t^k	kt^{k-1}

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Product Rule: Let $f(t) = f_1(t)f_2(t)$

$$\text{Then } \frac{df(t)}{dt} = \frac{df_1(t)}{dt} f_2(t) + f_1(t) \frac{df_2(t)}{dt}$$

Example: $q(t) = t^2 e^{-3t} \text{ C}$, $i(t) = ?$ $\frac{dt^2}{dt} = 2t$; $\frac{de^{-3t}}{dt} = -3e^{-3t}$

$$\text{Let } f_1(t) = t^2, f_2(t) = e^{-3t},$$

$$\begin{aligned} i(t) = dq/dt &= \frac{dt^2}{dt} e^{-3t} + t^2 \frac{de^{-3t}}{dt} = (2t - 3t^2)e^{-3t} \text{ A} \\ &= 2te^{-3t} + t^2(-3)e^{-3t} \end{aligned}$$

Given current $i(t)$, how to obtain charge $q(t)$?

Example: $i(t) = \sin 2t \text{ A}$, $q(t) = ?$

$$q(t) = \int i(t) dt = \int \sin 2t dt = -\frac{1}{2} \cos 2t + \text{constant}$$

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Example: $i(t) = \sin 2t$ A, $q(t) = ?$

$$q(t) = \int i(t) dt = \int \sin 2t dt = -\frac{1}{2} \cos 2t + \text{constant} \quad \text{constant} = ?$$

You need the value of $q(t)$ at one time instant to uniquely determine $q(t)$. Suppose $q(0)=0.5C$.

Method 1: Plug in $t=0$. $\rightarrow 0.5 = q(0) = -\frac{1}{2} \cos 0 + \text{constant}$

$$0.5 = q(0) = -\frac{1}{2} \times 1 + \text{constant} \quad \rightarrow \quad \text{constant} = 1$$

$$q(t) = -\frac{1}{2} \cos 2t + 1 C$$

Method 2:

$$q(t) = q(0) + \int_0^t \sin 2t dt = 0.5 + \left(-\frac{1}{2} \cos 2t\right) \Big|_0^t$$

Notation:

$$f(t) \Big|_a^b = f(b) - f(a)$$

$$= 0.5 - \frac{1}{2} \cos 2t - \left(-\frac{1}{2} \cos 0\right)$$

$$16:40 \quad = -\frac{1}{2} \cos 2t + 1 C \quad \text{Circuit Theory I, Copyright of Tingshu Hu}$$

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Practice problem 1: Determine the current $i(t)$ flowing through an element if the charge is given by:

$$q(t) = e^{-3t}(3t^2 - 2t + 2 \sin 4t) C$$

Practice problem 2: Determine the charge $q(t)$ flowing through an element if $q(0)=2C$ and the current is given by

$$i(t) = (-4t - 4 \cos 2t + 2e^{-2t}) A$$

Practice problem 3: Determine the current $i(t)$ flowing through an element if the charge is given by:

$$q(t) = t^3(e^{-2t} + 4 \sin(\frac{1}{2}t + \pi))C$$

Practice problem 4: Determine the charge $q(t)$ flowing through an element if $q(0)=1.5C$ and the current is given by

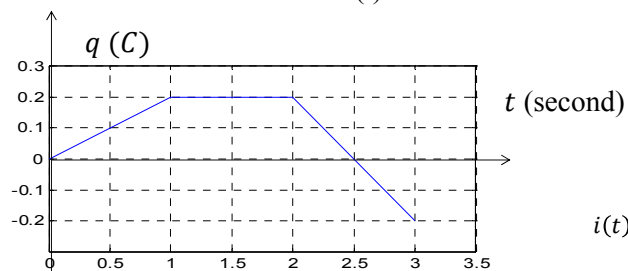
$$i(t) = \left(2e^{-4t} + 3t^2 + 4 \sin(2t + \frac{\pi}{2})\right)A$$

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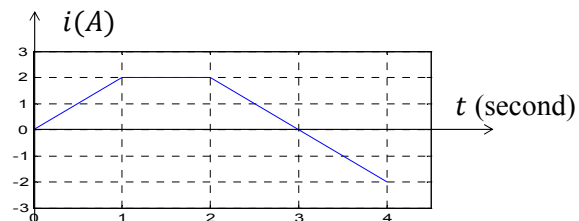
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Practice 5: The charge $q(t)$ is given by a piecewise linear function below. Find the current $i(t)$.



$$i(t) = \begin{cases} ?, & t \in [0,1] \\ ?, & t \in [1,2] \\ ?, & t \in [2,3] \end{cases}$$

Practice 6: The current $i(t)$ is given by a piecewise linear function below. Find the total charge over the interval $[0,4]$



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Last time:

Relationship between charge q and current i :

$$i \triangleq \frac{dq}{dt}$$

→ Current is the amount of charge flowing through a substance, or a cross sectional area, in 1 second.

1 ampere = 1 Coulomb/second;
1A=1C/s, ampere is a derived unit

Given the current as a function of time, $i(t)$, for $t \in [t_0, t]$, the total charge flowing through a substance over $[t_0, t]$ is

$$q(t) - q(t_0) = \int_{t_0}^t i(\tau) d\tau \quad \longleftrightarrow \quad q(t) = q(t_0) + \int_{t_0}^t i(\tau) d\tau$$

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More concepts:

L2

Direct current (DC): a current that remains constant with time, i.e., $i(t) = I$ for all t . Denoted as I .

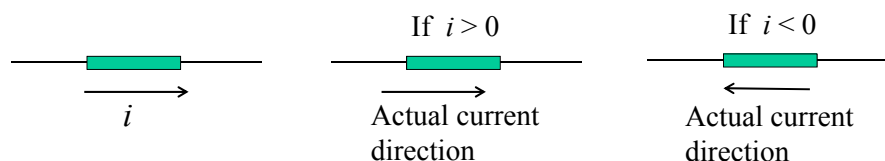
Alternating current (AC): a current varying sinusoidally with time: $i(t) = I_m \sin(\omega t + \varphi)$ A. Will be studied in Circuit II.

Next is a very important concept in circuit analysis

Reference direction of current: A direction that is assigned to any current variable i .

If $i > 0$, then the actual current direction is the same as the reference direction;

If $i < 0$, then the actual direction is opposite to the reference direction.



Reference direction can be arbitrarily assigned, but must be assigned. For a complex circuit, it may be hard to tell which way the current go. Ref. Dir. makes it more convenient to solve a circuit problem.

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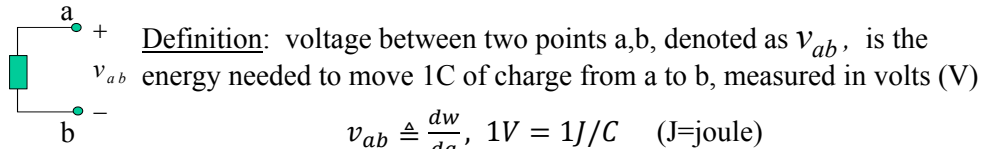
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1.4 Voltage

L2

-- Why electrons move? The driving force is the voltage.



$$v_{ab} \triangleq \frac{dw}{dq}, 1V = 1J/C \quad (J=\text{joule})$$

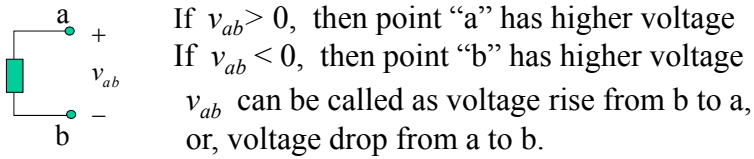
$$1J = 1V \times 1C = 1\text{volt} \times 1\text{Coulomb}$$

Recall in mechanics, $1J = 1\text{ Meter} \times 1\text{ Newton}$

Voltage can also be considered as the potential difference between two points.

We spend energy if we move positive charge from low potential to high potential

Voltage polarity: Assigned with “+” and “-”

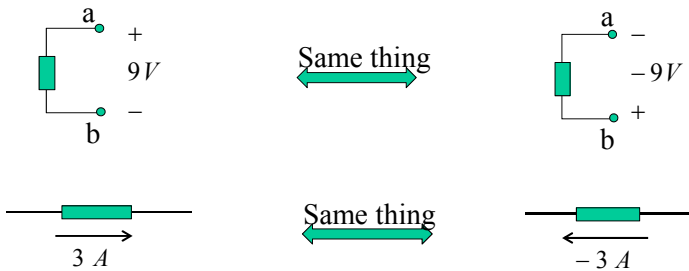


Voltage polarity can be arbitrarily assigned, but must be assigned.

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L2

DC volatge: $v(t) = \text{constant}$

AC voltage: $v(t) = V_m \sin(\omega t + \phi)$ V

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1.5 Power and energy

We need energy to run a circuit.

Energy: total amount of work done over a period of time, measured in joules (J). Notation w .

Power: amount of work done in one unit of time, typically, in 1 second, measured in watts (W). Notation: p .

Definition: power is the time rate of absorbing or generating energy.

$$p \triangleq \frac{dw}{dt}, \quad 1W = 1J/s$$

What is the relationship between power, current and voltage?

Recall: $v = \frac{dw}{dq}, \quad i = \frac{dq}{dt}$

$$v \times i = \frac{dw}{dq} \frac{dq}{dt} = \frac{dw}{dt} = p$$

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In summary:

$$\begin{aligned} p &= vi, \\ w &= \int p dt = \int vidt \\ w(t_2) - w(t_1) &= \int_{t_1}^{t_2} vidt \end{aligned}$$

In a circuit, an element may absorb power or generate power.

→ p also has a sign

If $p > 0$, the element absorbs power;

If $p < 0$, the element generates power.

How to tell if an element is absorbing or generating power?

- The reference direction of current and the polarity of voltage are needed.

The passive sign convention and active sign convention will be introduced

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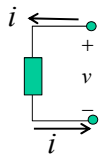
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The passive sign convention and active sign convention:

Case 1: the current enters an element through “+” terminal.

Called the passive sign convention



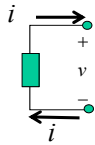
$$p = vi$$

If $p > 0$, the element absorbs power

If $p < 0$, the element generates power

Case 2: the current enters an element through “-” terminal.

Called the active sign convention



$$p = -vi$$

If $p > 0$, the element absorbs power

If $p < 0$, the element generates power

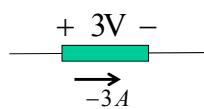
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With reference direction of current and voltage polarity arbitrarily assigned, both v and i can be positive or negative.

Examples: Determine the power consumed by each element



(a)

Passive sign:

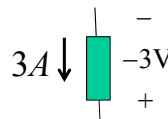
$$p = vi$$

$$v = 3V, i = -3A$$

$$\begin{aligned} p &= vi \\ &= 3 \times (-3) \\ &= -9W \end{aligned}$$

Generates
power

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(b)

Active sign:

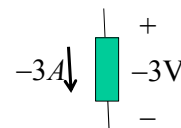
$$p = -vi$$

$$v = -3V, i = 3A$$

$$\begin{aligned} p &= -vi \\ &= -(-3) \times 3 \\ &= 9W \end{aligned}$$

Absorbs
power

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(c)

Passive sign:

$$p = vi$$

$$v = -3V, i = -3A$$

$$\begin{aligned} p &= vi \\ &= (-3) \times (-3) \\ &= 9W \end{aligned}$$

Absorbs
power

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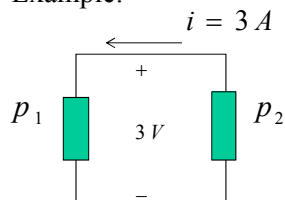
Law of conservation of energy:

In any circuit,
the power absorbed = the power generated.

Equivalently, The total amount of power absorbed = 0

$$\sum_{n=1}^N p_n = 0$$

Example:



$$p_1 = 3 \times 3 = 9W ;$$

$$p_2 = -3 \times 3 = -9W$$

$$p_1 + p_2 = 0$$

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1.6 Circuit elements

A circuit is an interconnection of elements to realize a certain function, e.g., amplifier, filter, power conversion.

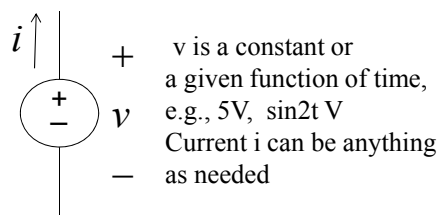
Two types of elements:

- Passive element: consumes or stores energy, such as resistors, capacitors, inductors. Never generates power
- Active element: generates energy, such as batteries, generators. May absorb energy, e.g., when a battery is charged.

Active elements include current source and voltage source.

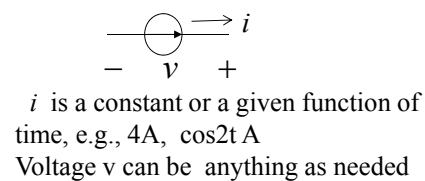
Independent source: a voltage/current source that provide specific voltage or current that is independent of the other elements.

Independent voltage source:



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Independent current source:



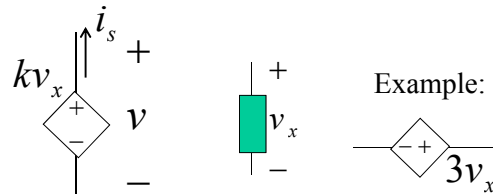
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Dependent source (controlled source): an active element where the source quantity is controlled by another voltage or current.

Four types of dependent sources:

1. Voltage controlled voltage source (VCVS)



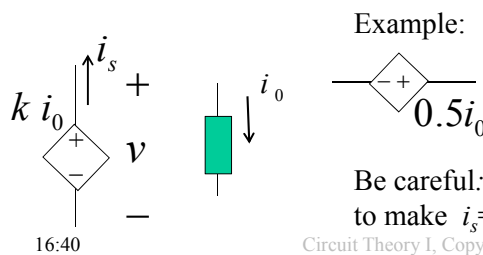
$$v = kv_x, \text{ e.g., } k = 3$$

$$\text{If } v_x = 2V,$$

$$\text{then } v = k \times 2 = 3 \times 2 = 6V$$

The coefficient k has no unit

2. Current controlled voltage source (CCVS)



$$v = ki_0, \text{ e.g., } k = 0.5$$

$$\text{If } i_0 = 4A,$$

$$\text{then } v = k \times 4 = 0.5 \times 4 = 2V$$

k 's unit is V/A

Be careful: ki_0 is a voltage. It is a huge mistake to make $i_s = ki_0$.

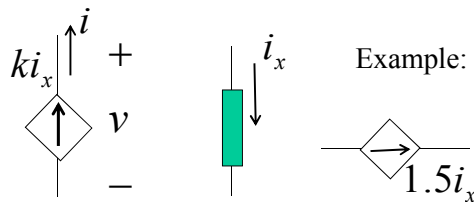
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$$i_s \neq ki_0$$

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3. Current controlled current source (CCCS)



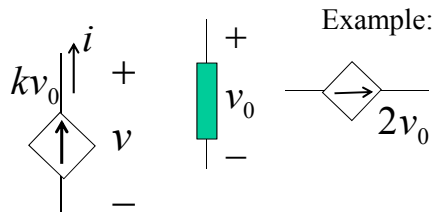
$$i = ki_x, \text{ e.g., } k = 1.5$$

$$\text{If } i_x = 6A,$$

$$\text{then } i = k \times 6 = 1.5 \times 6 = 9A$$

k has no unit

4. Voltage controlled current source (VCCS)



$$i = kv_0, \text{ e.g., } k = 2$$

$$\text{If } v_0 = 0.5V,$$

$$\text{then } i = k \times 0.5 = 2 \times 0.5 = 1A$$

k 's unit is A/V

Be careful: kv_0 is a current. It is a huge mistake to make $v = kv_0$.

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$$v \neq kv_0$$

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All the variables we have learned so far: q , i , v , p , w . Their relationship:

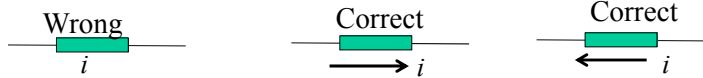
$$i = \frac{dq}{dt}, \quad q(t) = q(t_0) + \int_{t_0}^t i dt; \quad v = \frac{dw}{dq}$$

$$p = vi \text{ (if passive sign), } p = -vi \text{ (if active sign)}$$

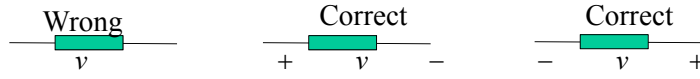
$$w(t) = w(t_0) + \int_{t_0}^t vi dt$$

Important details to remember:

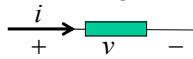
- Current i should always be assigned with a reference direction



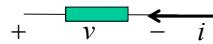
- Voltage should always be assigned with polarity



Passive sign convention



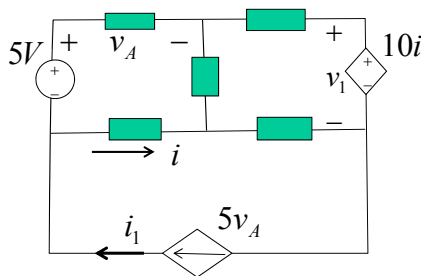
Active sign convention



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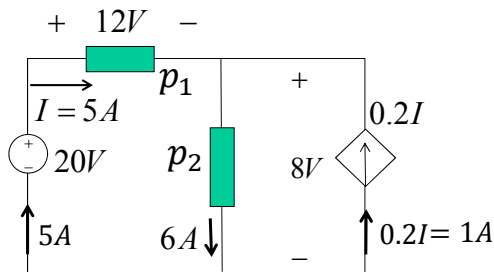
Example 1:



If $v_A = 0.5V$,
then $i_1 = 5v_A = 2.5A$

If $i = 0.2A$,
then $v_1 = 10i = 2V$

Example 2: Compute the power of each element



$$p_1 = 12 \times 5 = 60W$$

$$p_2 = 8 \times 6 = 48W$$

$$p_{20V} = -20 \times 5$$

$$= -100W$$

$$p_{0.2I} = -8 \times 0.2I$$

$$= -8 \times 1$$

$$= -8W$$

$$\text{Total power} = 60 + 48 - 100 - 8 = 0$$

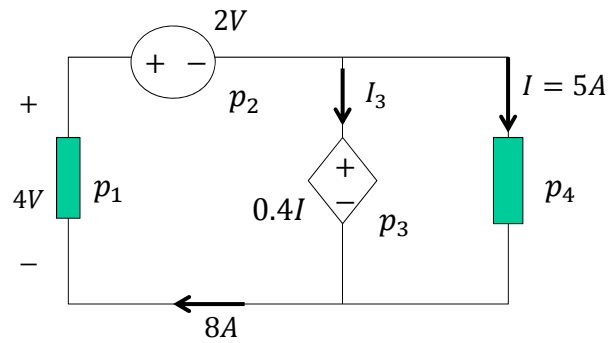
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Practice 7: Given $v(t) = \cos 3t$ V, $i(t) = \sin 3t$ A.

Find the total energy over time period $[0, 0.2]$ second. Assume passive sign convention.

Practice 8: Find the power of each element. $I_3 = ?$

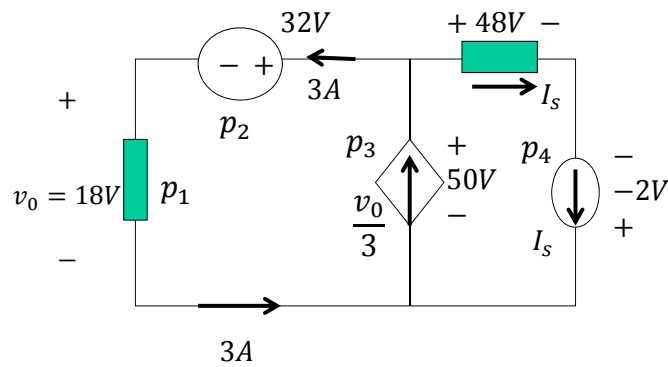


Hint: use the law of conservation of energy.

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Practice 9: Find the power of each element. $I_s = ?$



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