Effect of Stokes Number on Turbulent Fluid-Solid Flows

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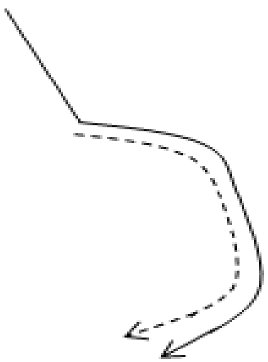
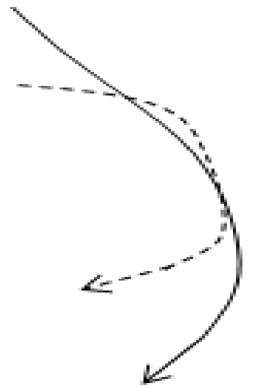
Fluid-Solid Flows

Viscous Flow Regime



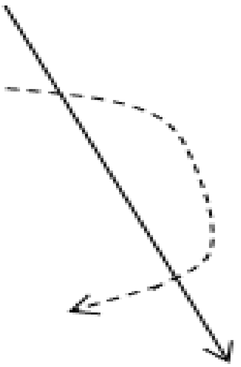
ST<5

Transitional Regime



5<ST<40

Inertia Flow Regime



ST>40

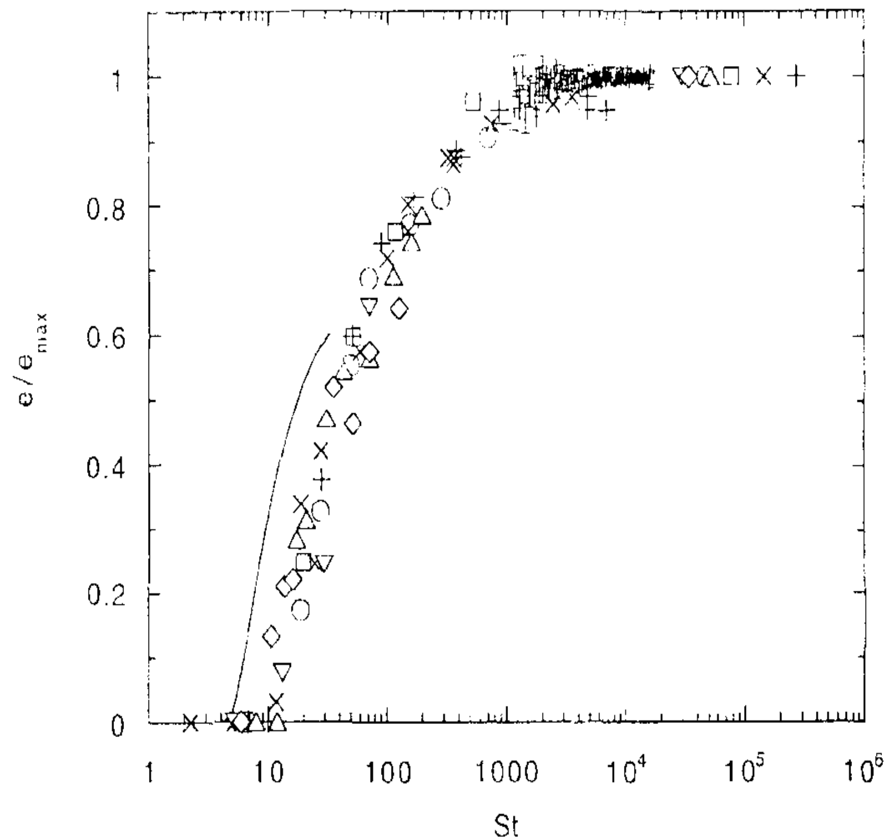
$$\text{Stokes \#} = \frac{\text{Solid Response time}}{\text{Fluid Response time}} = \left(\frac{\rho_s d^2}{18\mu_f}\right) \Big/ \left(\frac{D}{V_{fzcl}}\right)$$

Effect of St on Coefficient of Restitution

$$\frac{e_f}{e_s} = f(St)$$

$$St = \frac{\rho_s d_p v_{imp}}{9\mu_f}$$

- ρ_s = particle density
- d_p = particle diameter
- V_{imp} = impact velocity
- μ_f = fluid viscosity



Ratio of e_f/e_s as a function of St (Gondret *et al.*, 2002 and Joseph *et al.*, 2001)

Motivation

- Good amount of available data and modeling in inertia-dominated regime
  - Particle sizes > 100 microns in gas-solid flow
  - Many models available but there is no consensus

- Modeling in viscous-flow regime and some experimental data

  Low Re, neutrally-buoyant suspensions

  (Koh *et al.* 1994; Averbakh *et al*. 1997; Morris and Brady, 1998)

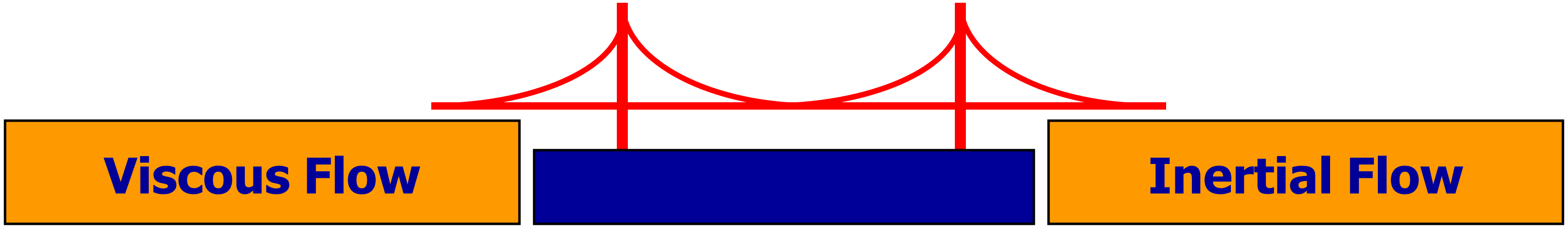
Motivation

Transitional regime:

Group A particles in gas-solid flows
Heterogeneous slurry flows

Particle-particle interactions but particle fluctuations are significantly affected by interstitial fluid

Significant lack of detailed, non-intrusive data (*mean and fluctuating velocity for both phases*) in the transitional flow regime for model development and validation



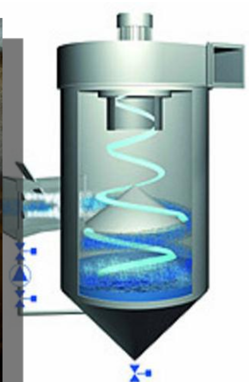
Applications

Numerous fossil fuel and energy applications
Work directly addresses Section 3 on Liquid-Solid Flows of DOE Multiphase Flow Roadmap (e.g. Fischer-Tropsch synthesis) and also addresses Sections 1 and 2 on Gas-Solid Flow regimes for intermediate Stokes numbers

Shell Oil is partially supporting this work as part of the NSF-I/UCRC center in Particulate and Surfactant Systems at the University of Florida

Prediction of the critical settling velocities in slurry lines, improvement in design of new lines and operating conditions on existing lines



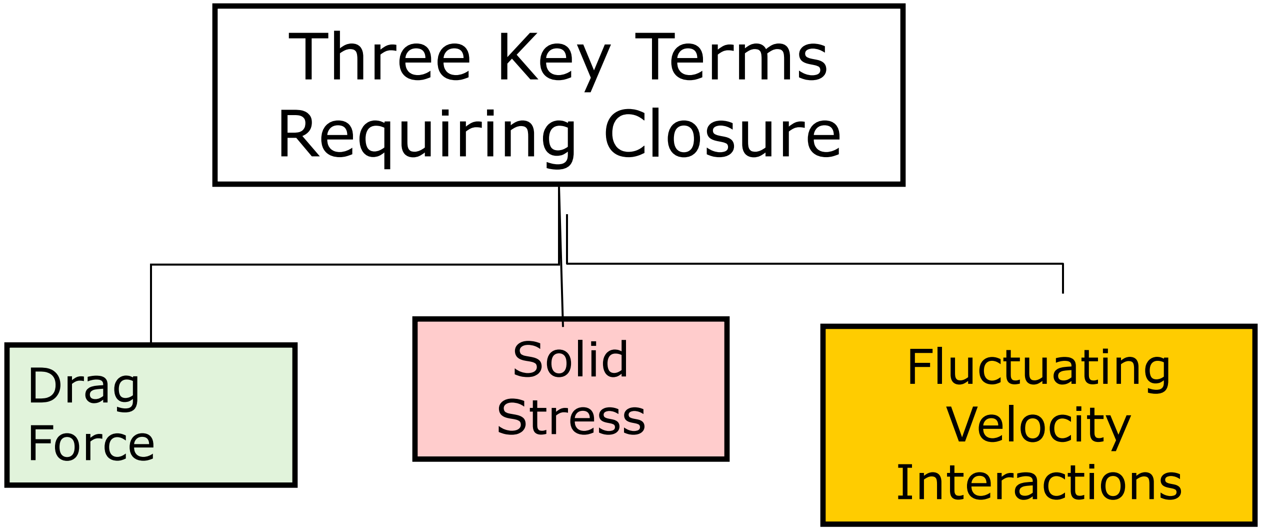




Dilute, Turbulent Gas-Solid Flows

Eulerian Two-Fluid Model

- For inertia-dominated flow regime
  - Particle-particle/particle-wall interactions dominate details of particle motion
  - Granular kinetic theory is used to describe the velocity fluctuations associated with these interactions
  - Gas turbulence k-ε turbulence model

Two-Fluid Model Closures



Gas-Phase Turbulence – k-ε Model

$$\rho_g(1-\nu)\left[\frac{\partial k}{\partial t}+V_g.\nabla k\right]=-\nabla\left[(1-\nu)\mu_{gk}^{t}\nabla k\right]+G_k-\varepsilon+I_k$$

$$\rho_g(1-\nu)\left[\frac{\partial \varepsilon}{\partial t}+V_g.\nabla \varepsilon\right]=-\nabla\left[(1-\nu)\mu_{g\varepsilon}^{t}\nabla \varepsilon\right]+c_1G_k\frac{\varepsilon}{k}-c_2\frac{\varepsilon^2}{k}+c_3I_k$$

Unsteady+Convection =Diffusion + Generation - Dissipation +Interaction

Interaction Terms

$$\rho_s\nu\left[\frac{\partial E_T}{\partial t}+V_s.\nabla E_T\right]=-\nabla.\left[\lambda\nabla T\right]+G_T-\gamma+I_T$$

Solid Stress

Granular Energy Balance

Interaction Terms & Fluctuation Velocity Cross-Correlation

$$I_k = -\beta(1-\nu)\left(2k - k_{sg}\right)$$

$$I_T = \beta(1-\nu)\left(k_{sg} - 3T\right)$$

Cross-correlation

- Time and Volume Based Averaging (TVBA)

$$2k = (\overline{v_g\,'.v_g\,'});\; 3T = (\overline{v_s\,'.v_s\,'});\; k_{sg} = (\overline{v_g\,'.v_s\,'})$$

Summary of Closure Models

Drag Force (F_D)	Solid Stress (μ_s, λ_s, Υ)	Interaction Term (I_k, I_T)	Cross - Correlation (k_{sg})
Wen & Yu (1966) Syamlal & O'Brien (1987) Hill *et. al.* (2001)	Lun *et. al.* (1984) Peirano & Leckner (1998)	TVBA	Louge *et al.* (1991) Koch & Sangani (1999) Wylie *et al.* (2003) Simonin (1996) Sinclair & Mallo (1998)

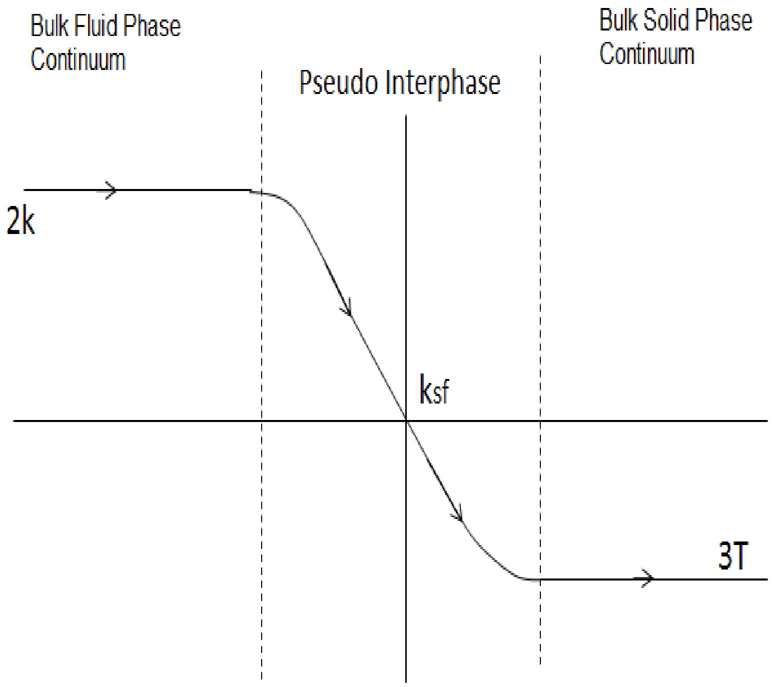
Comprehensive Evaluation of All Closure Models

A. Rao, J. Curtis, B. Hancock, and C. Wassgren, "Numerical Simulation and Validation of a Dilute Turbulent Gas-Particle Flow Model with Turbulence Modulation", *AIChE Journal*, in press (2011)

Fluctuation Energy Transfer (FET)

New Model for Fluctuating Velocity Interaction (*AIChE Journal*, 2011, in press)

Based on heat transfer analogy
Energy transfer occurs due to particle drag or particle collisions
Fluid turbulence enhancement and dissipation



$$I_k = -\frac{\rho_g}{\tau_{sg}}\left(2k - k_{sg}\right)$$

$$I_T = \frac{\rho_g}{\tau_{sg}}\left(k_{sg} - 3T\right)$$

τ_{sg} = *time scale for FET*

For particles with 40<St<100

τ_{sg} = Drag time scale

For particles with St>100

τ_{sg} = Collisional time scale

Summary of Closure Models

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		New Model for Fluctuating Energy Transfer	Sinclair & Mallo (1998)
Wen & Yu (1966)	Peirano & Leckner (1998)	New Model for Fluctuating Energy Transfer	Sinclair & Mallo (1998)

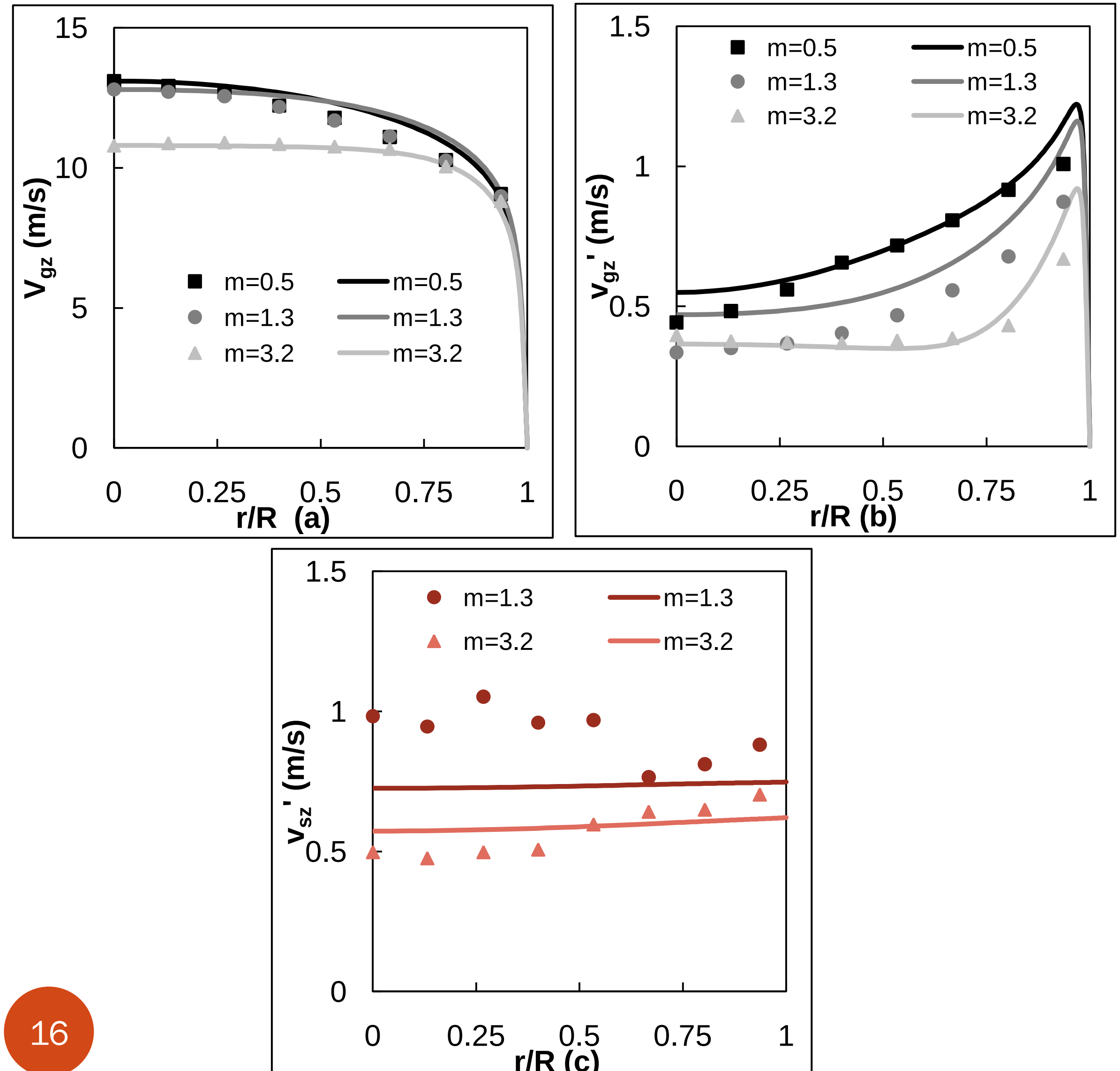
Dilute-Phase Gas-Solid Data
Inertia Regime

Experimental Data	Size	Density	~Re_p	~Stokes
	μm	kg/m^3	No.	No.
Tsuji *et al.* (1984)	243	1020	45	75
	500	1020	130	320
	1420	1030	730	2700
	2780	1020	1800	11300
Jones *et al* (2001)	70	2529	12	50
Lee and Durst (1982)	400	2500	90	170
	100	2500	5	120
Sheen *et al.* (1993)	275	1020	30	40
	450	1020	85	115
	800	1020	220	380

Models employ granular kinetic theory to describe particle velocity fluctuations when St>40

New Model – Data of Tsuji *et al.* (1984), 200μm

Dilute Gas-Solid Flow Through a Vertical Pipe St~65



For this case,

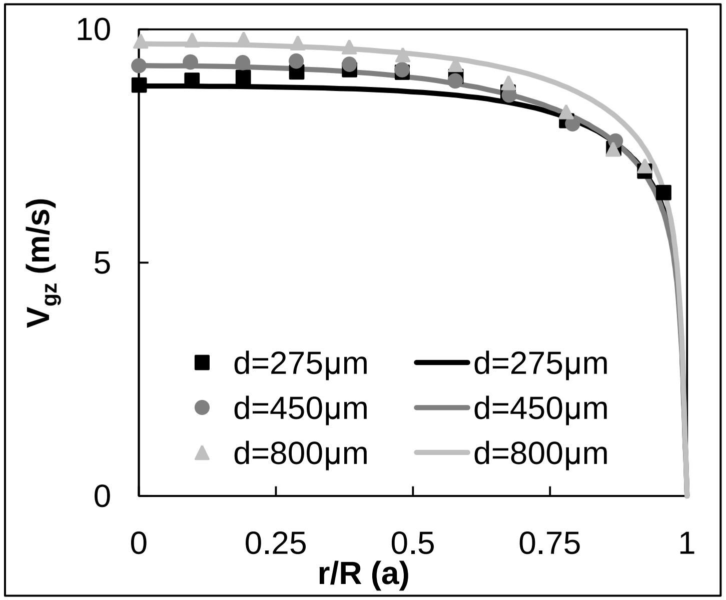
- τ_{sg} = Drag time scale

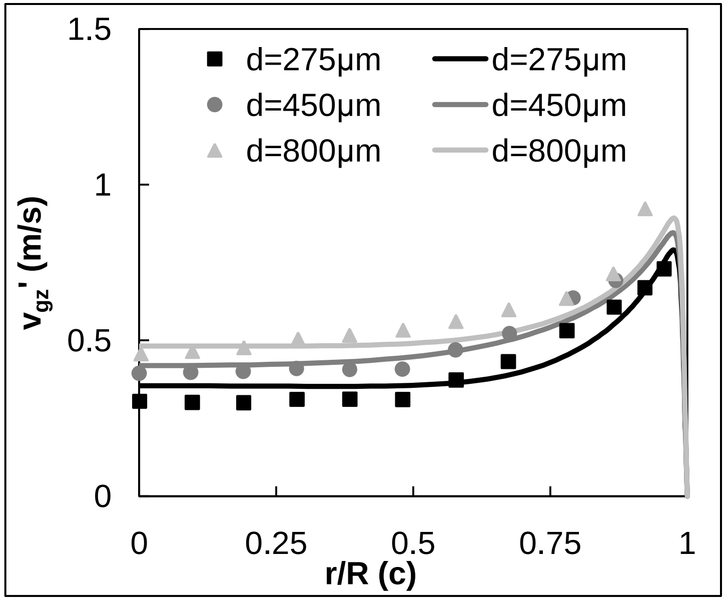
Wen & Yu Drag Force

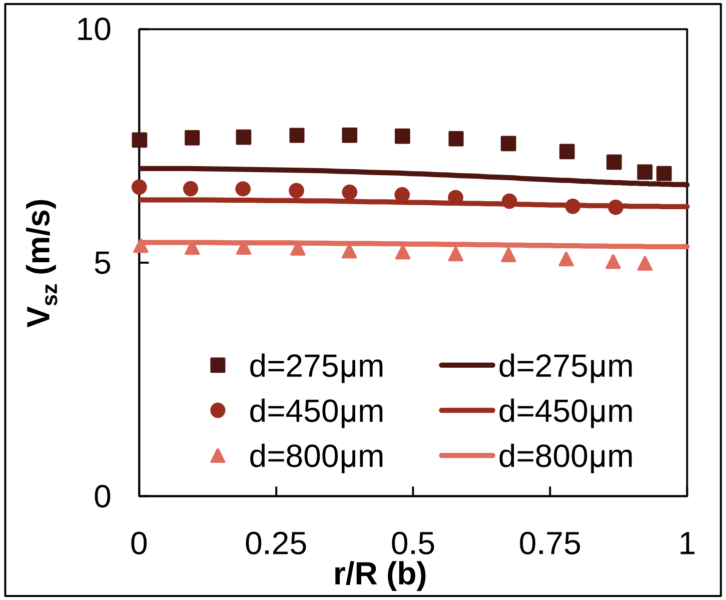
Peirano & Leckner Solids Stress

New Fluctuating Energy Transfer Model

New Model– Data of Sheen *et al.* (1993)



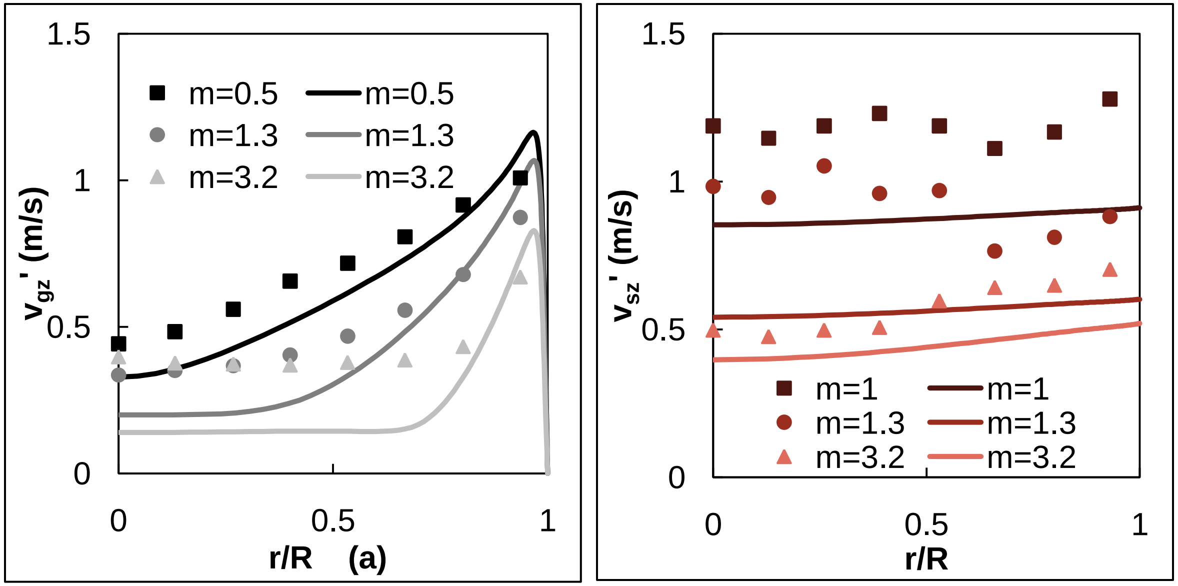




Large range of particle sizes

Effect of Using Koch (1990) Type Models for Cross-Correlation

TVBA+ Koch (1990) model – Tsuji *et al.*, 200μm



$$k_{sg} \sim \left|V_g - V_s\right|^2 \frac{\tau_c}{\tau_D}$$

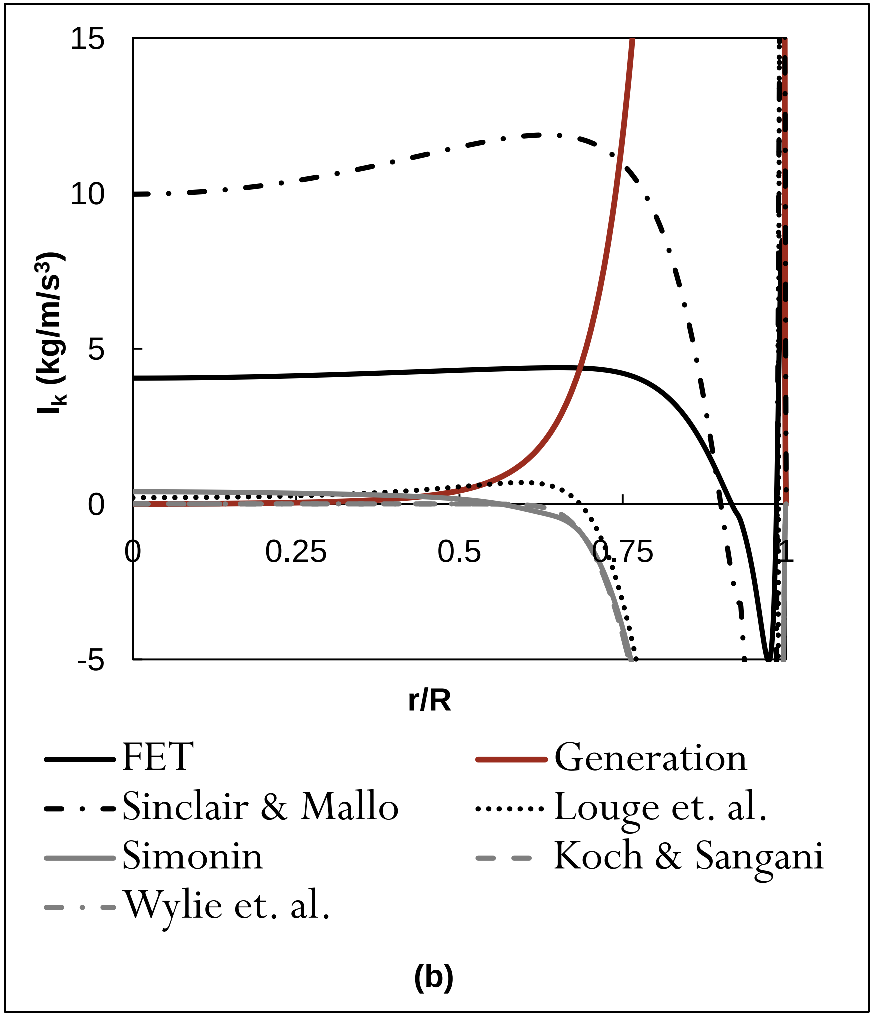
τ_D relates k_{sg} to drag

τ_c relates k_{sg} to $T^{0.5}$

I_k is too small

Comparison of Fluctuating Interaction

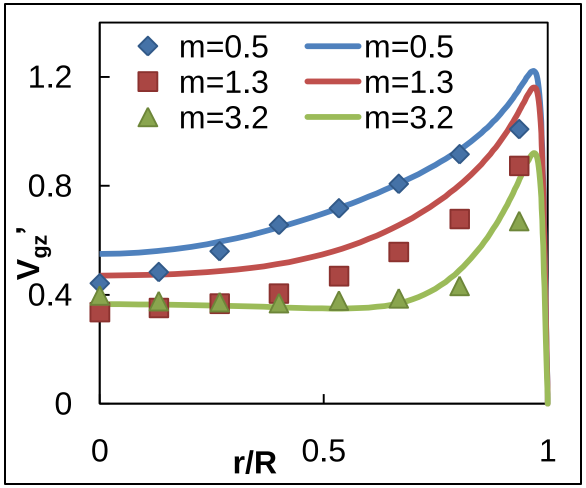
Tsuji *et al.*, d=200μm, m=3.2



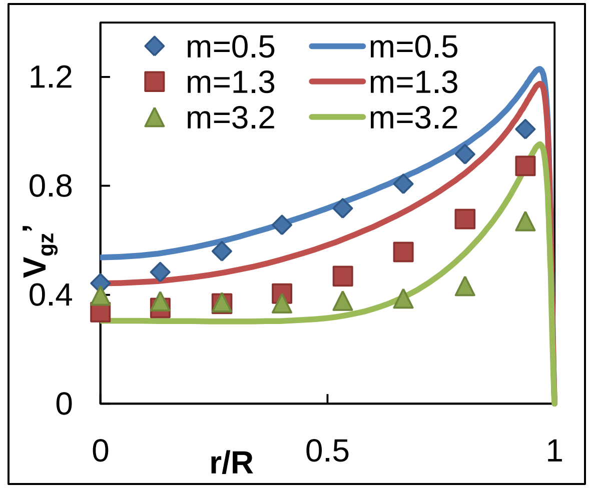
- Generation is small at pipe core
- I_k term can dominate there

- For most models
  - **Generation ~Interaction** at the core of the pipe

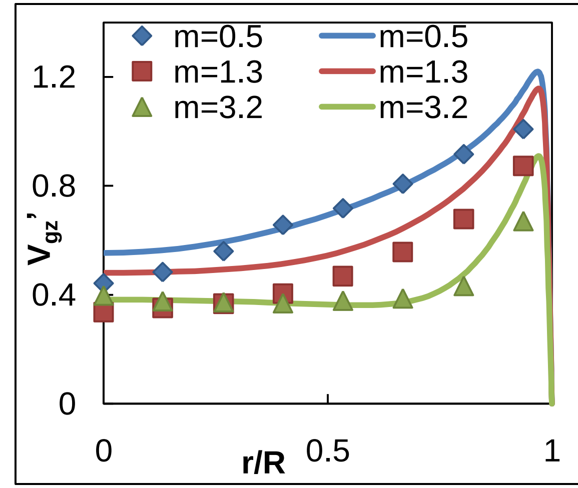
Different Drag Force Relations Data of Tsuji *et al.*, 200μm



Wen & Yu (1966)



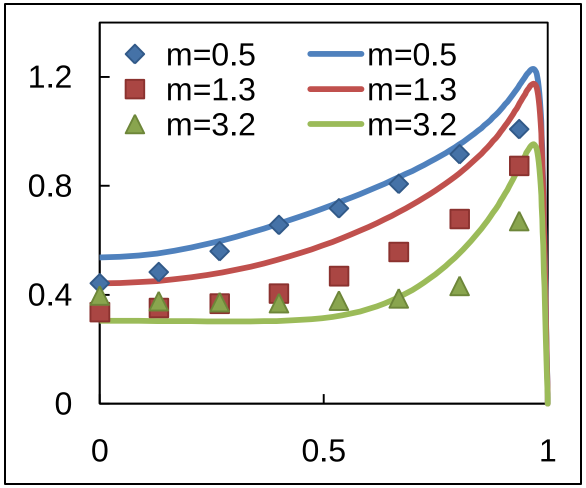
Hill *et al.* (2001)



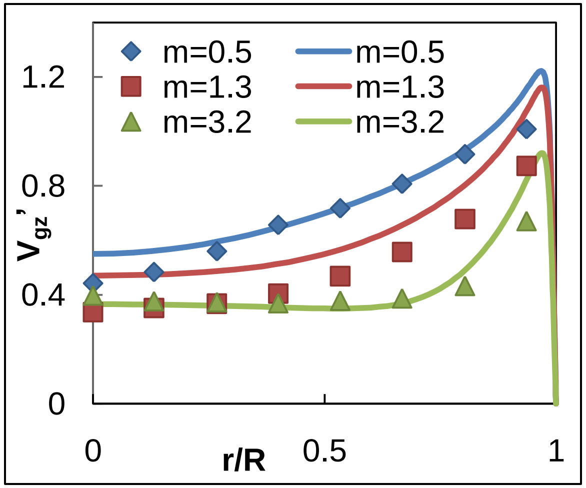
Syamlal & O'Brien (1987)

Changing the **Drag Model** has little effect on flow predictions

Different Solid Stress Descriptions   Tsuji *et al.*, 200μm



Lun *et al.* (1984) neglects fluid effects



Peirano & Leckner (1998) incorporates fluid effects

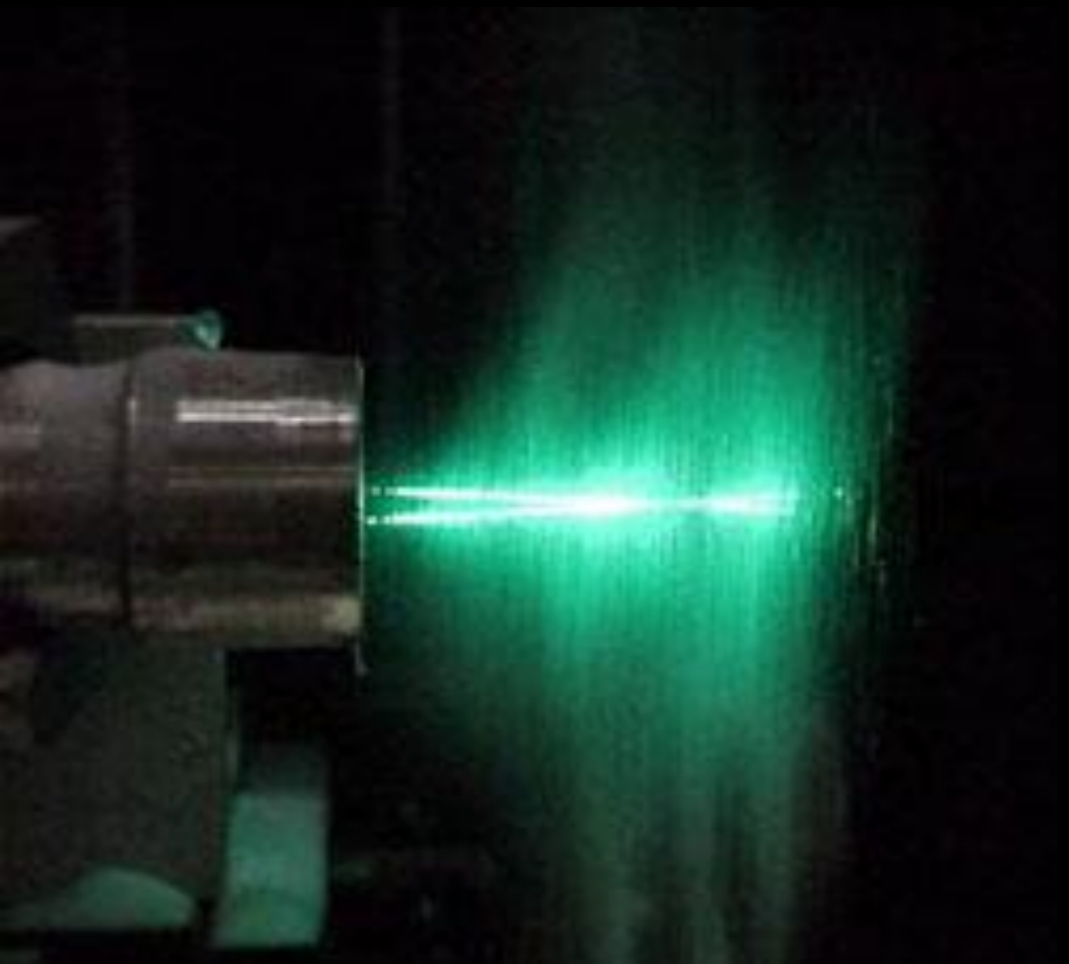
Changing the **Solid Stress Description** has little effect on flow predictions in gas-solid flow

Dilute, Turbulent Liquid-Solid Flows

Experimental Setup

- Pilot-scale slurry flow facility in the UF Particle Science and Technology Building high bay area
- Non-intrusive flow measurements via LDV/PDPA
- Can accommodate a wide range of flowrates, particle sizes and solids concentrations

EXPERIMENTAL TECHNIQUE: LASER DOPPLER VELOCIMETRY



Non-intrusive laser based technique

Allows for the instantaneous and average velocity measurement of fluid (via seed) and particles

Measures refractive light from particles in the flow and relates it to velocity – sizing from the Phase Doppler capability

LDV/PDPA

2-D backscatter with ~ 250, 500, 750 mm focal length

Movement controlled by traversing mechanism with accuracy of 1/10,000 inch

Approach distance to wall: r/R ~ 0.97

Seed:

Filtered tap water: matter < 5 µm or

10 micron hollow glass beads

Experimental Setup

- L/D before entering test section > 50
- Test section L/D > 10 (two sets of measurements)
- D/d > 50
- Fully-developed and axisymmetric flow verified for both single and two-phase flows
- Re range limited by minimum conveying velocity at low end and splashing at high end
- Explore range of St by varying flow velocity, solids fraction and particle size
- Index of refraction matching for dense-phase flow

Experimental Setup

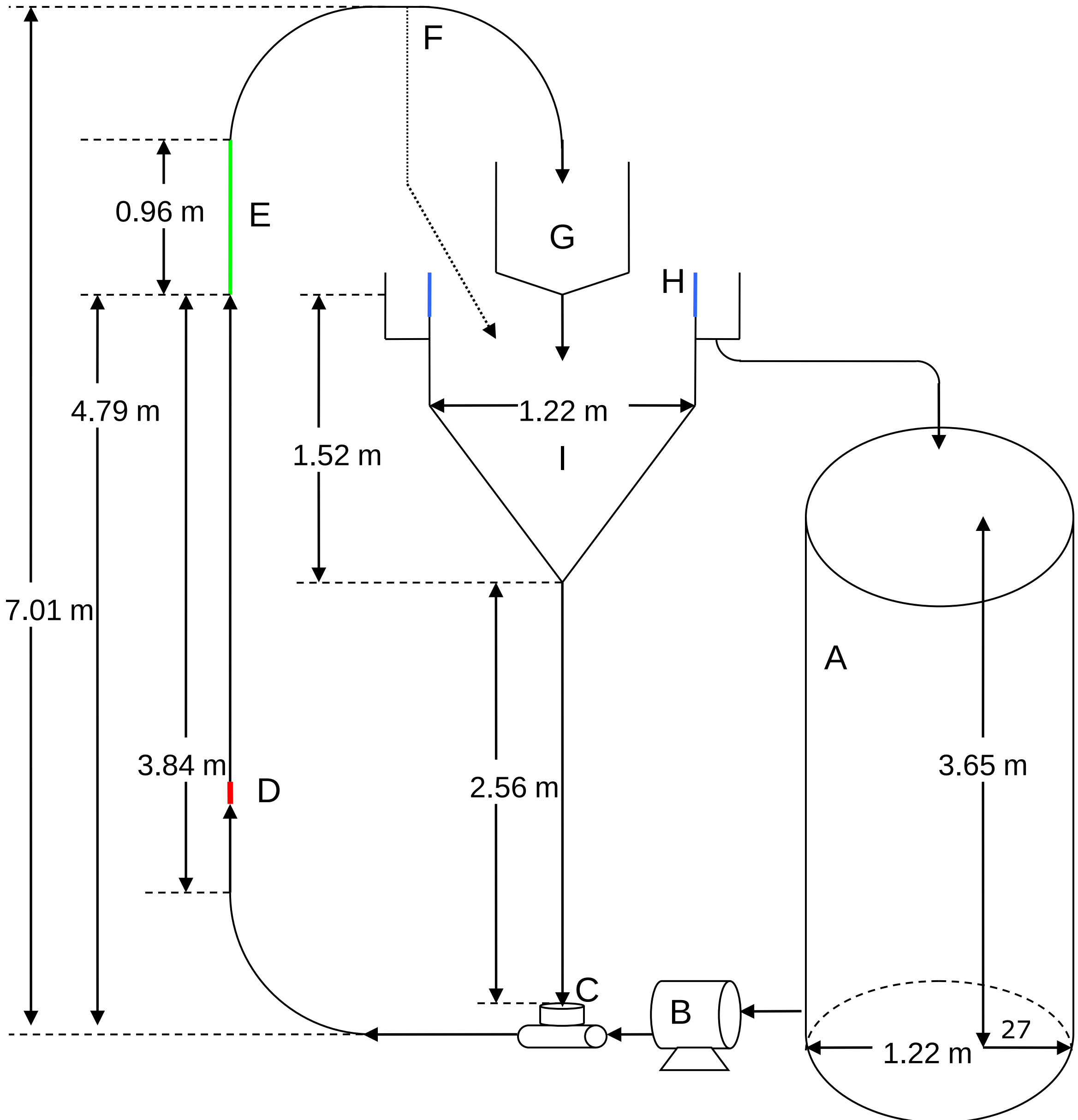
Pipe Diameter = 3 in.

- **Glass Particles:**
  - **0.5mm, St<5 (Viscous)**
  - **1mm, 5<St<10 (Transitional)**
  - **1.5mm, 10<St<30 (Transitional)**
  - **2.3mm, St>40 (Inertia)**

  (Alajbegovic *et al.*, 1994)

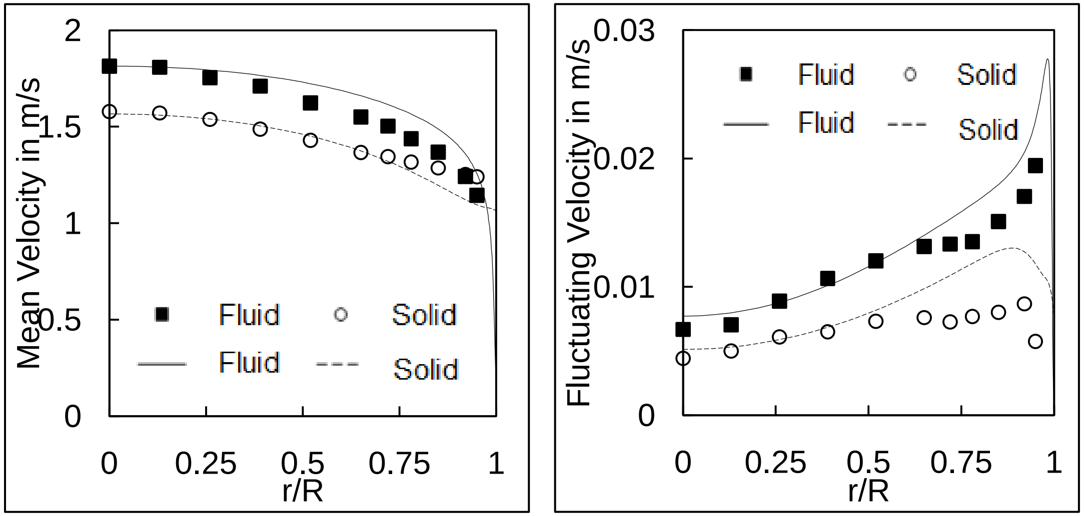
- **Reynolds Number:**
  - **$2x10^5$, $3.35x10^5$, $5x10^5$**

  27 sets of operating conditions

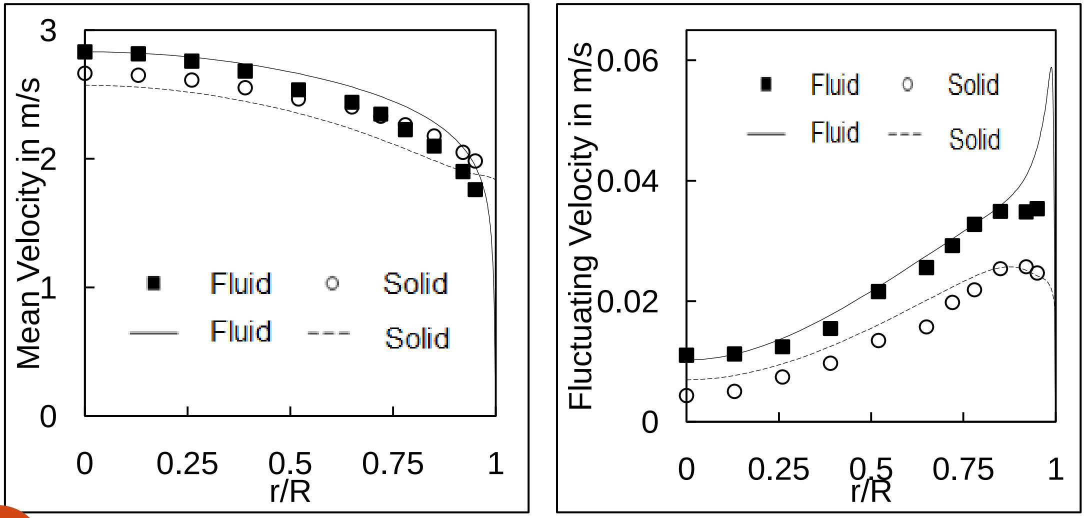


St > 40 2.32mm glass beads (Alajbegovic *et al.*, 1994)

$V_{fzcl} = 1.81$ m/s, m = 0.055, St = 43



$V_{fzcl} = 2.83$ m/s, m = 0.067, St = 68

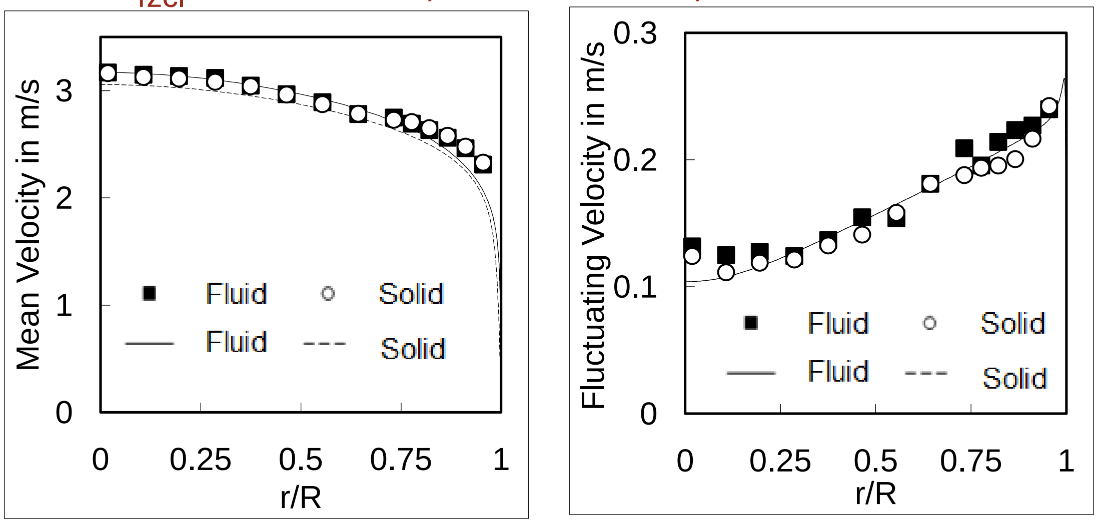


Inertia-Dominated Flow

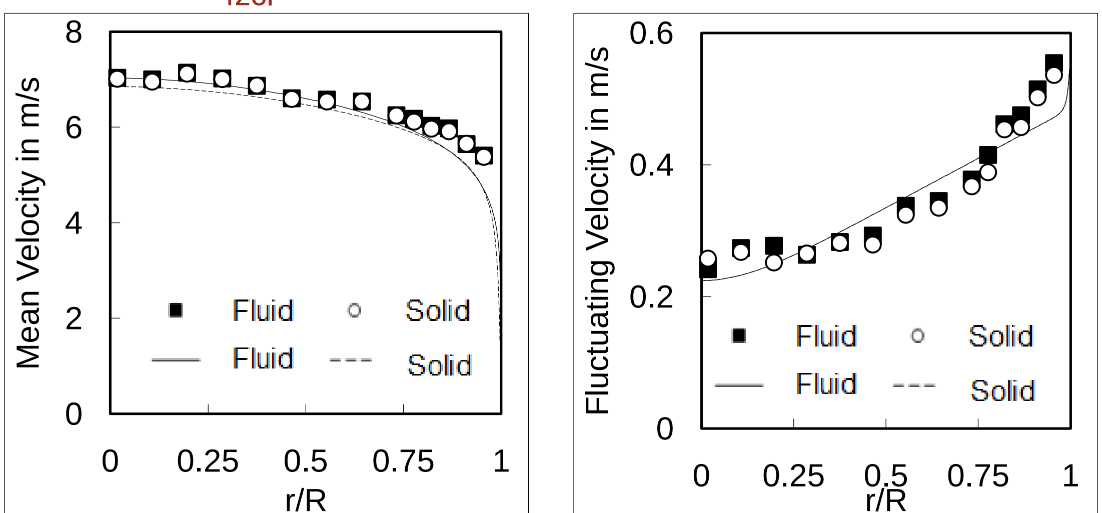
Same model as for gas-solid flow

St ~ 1 0.5 mm glass beads

V_{fzcl} = 3.17 m/s, m = 0.0425, St = 1.4

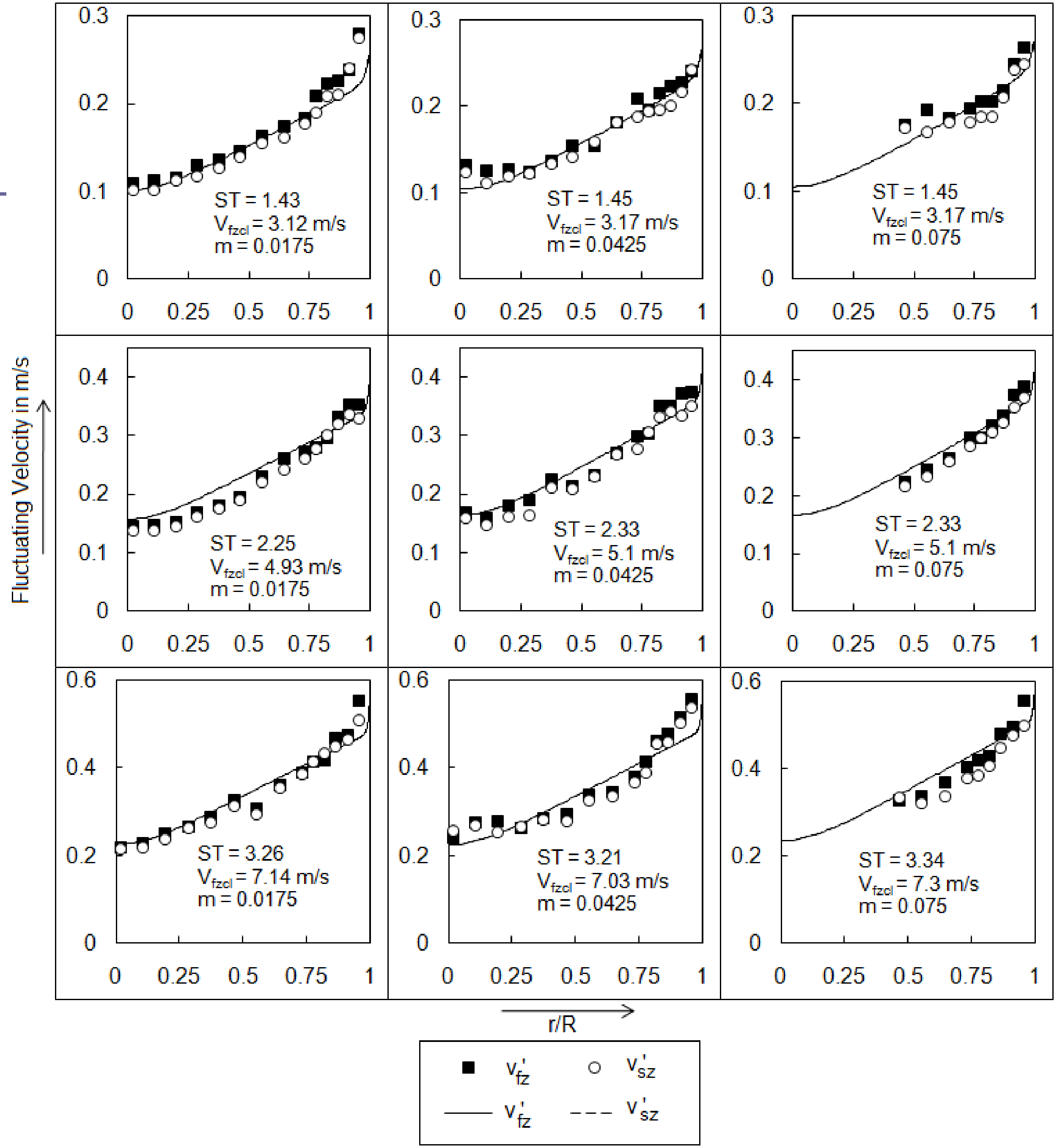


V_{fzcl} = 7.03 m/s, m = 0.0425, St = 3.2



Viscous-Dominated Flow Model of Chen & Wood (1985)

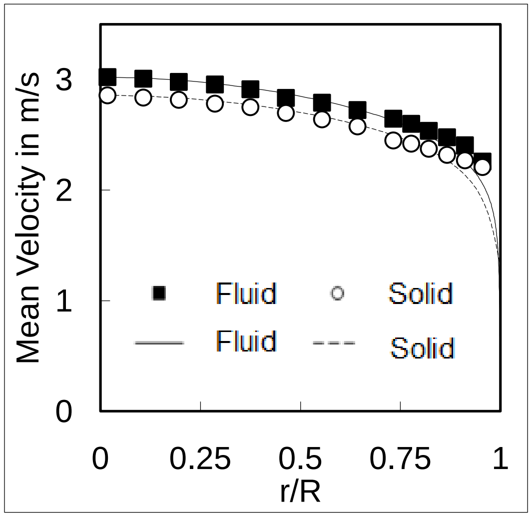
- Negligible relative mean velocity between the two phases
- Solid velocity fluctuations similar to fluid velocity fluctuations
- Direct relationship between solids viscosity and fluid viscosity
- Kinetic theory of granular flow not a good description for the solid-phase stress
- Number density issues associated with LDV measurements even for 3-4% solids fraction

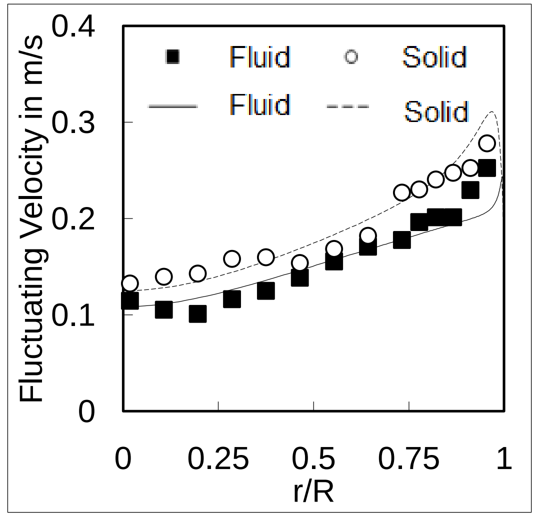


Fluctuating Velocity in m/s
r/R
ST = 1.43
V_fzcl = 3.12 m/s
m = 0.0175
ST = 1.45
V_fzcl = 3.17 m/s
m = 0.0425
ST = 1.45
V_fzcl = 3.17 m/s
m = 0.075
ST = 2.25
V_fzcl = 4.93 m/s
m = 0.0175
ST = 2.33
V_fzcl = 5.1 m/s
m = 0.0425
ST = 2.33
V_fzcl = 5.1 m/s
m = 0.075
ST = 3.26
V_fzcl = 7.14 m/s
m = 0.0175
ST = 3.21
V_fzcl = 7.03 m/s
m = 0.0425
ST = 3.34
V_fzcl = 7.3 m/s
m = 0.075
v'_fz
v'_sz
v'_fz
v'_sz

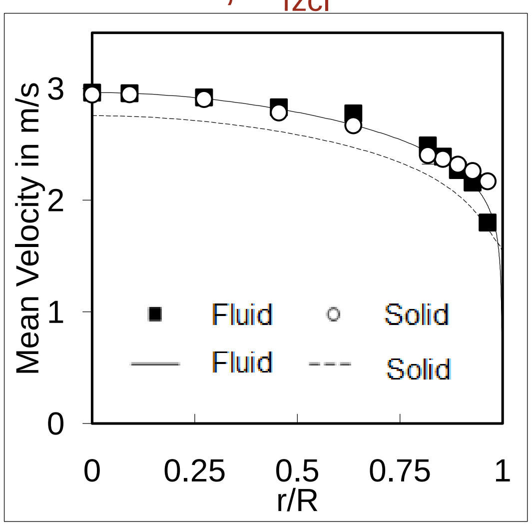
St ~ 5-30 1mm and 1.5mm glass beads

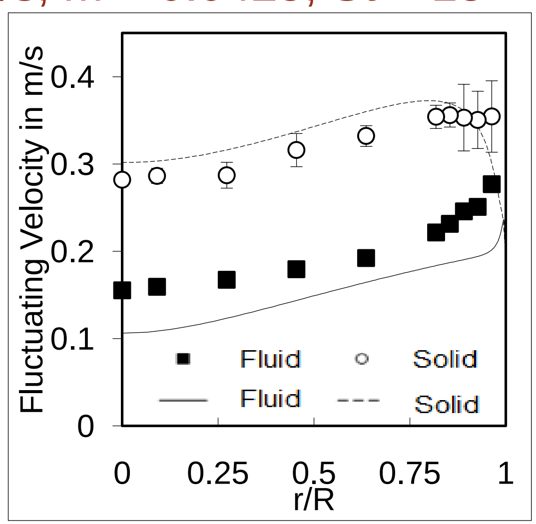
d = 1 mm, V_{fzcl} = 3.02 m/s, m = 0.0425, St = 6





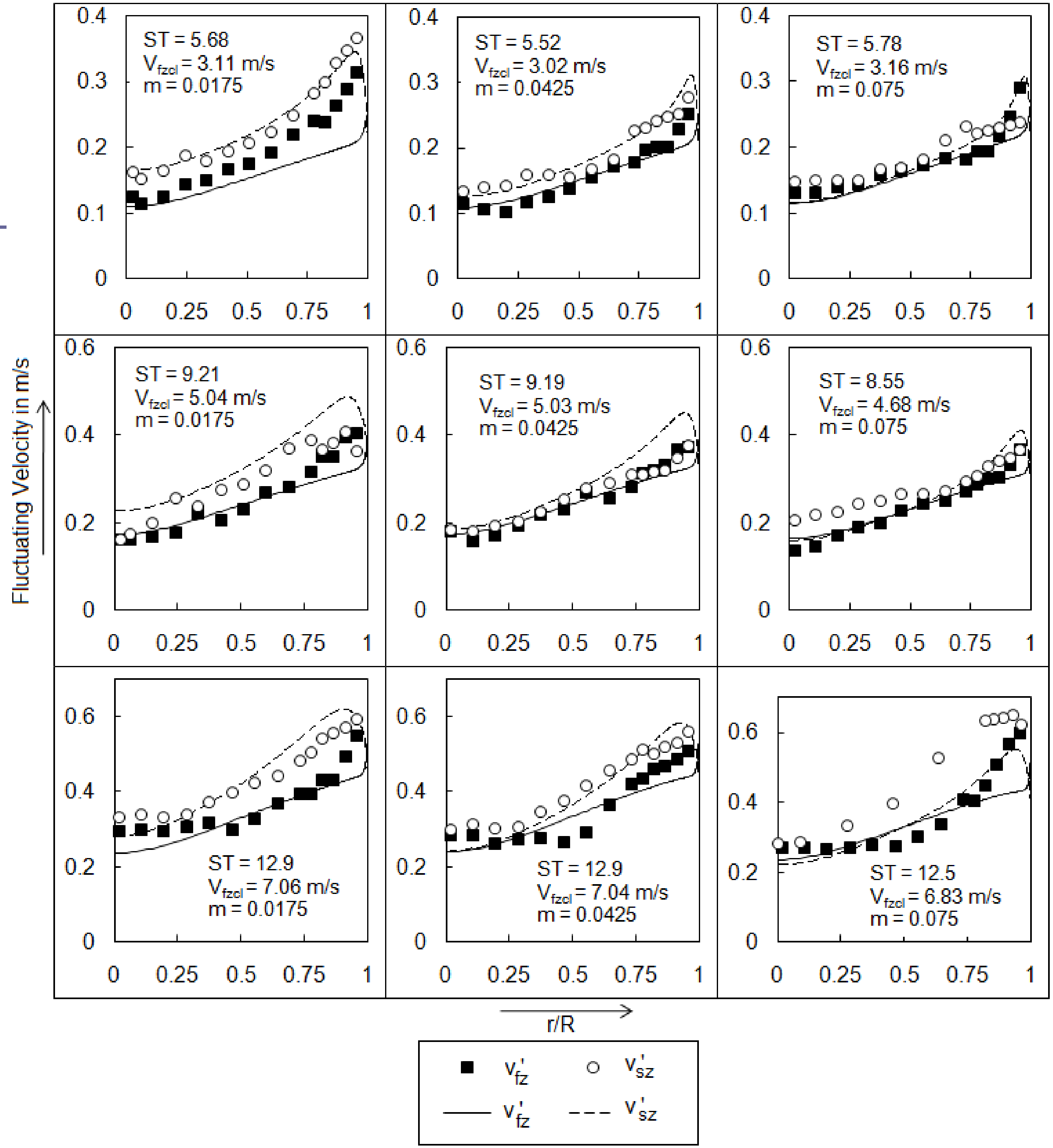
d = 1.5 mm, V_{fzcl} = 2.96 m/s, m = 0.0425, St = 13



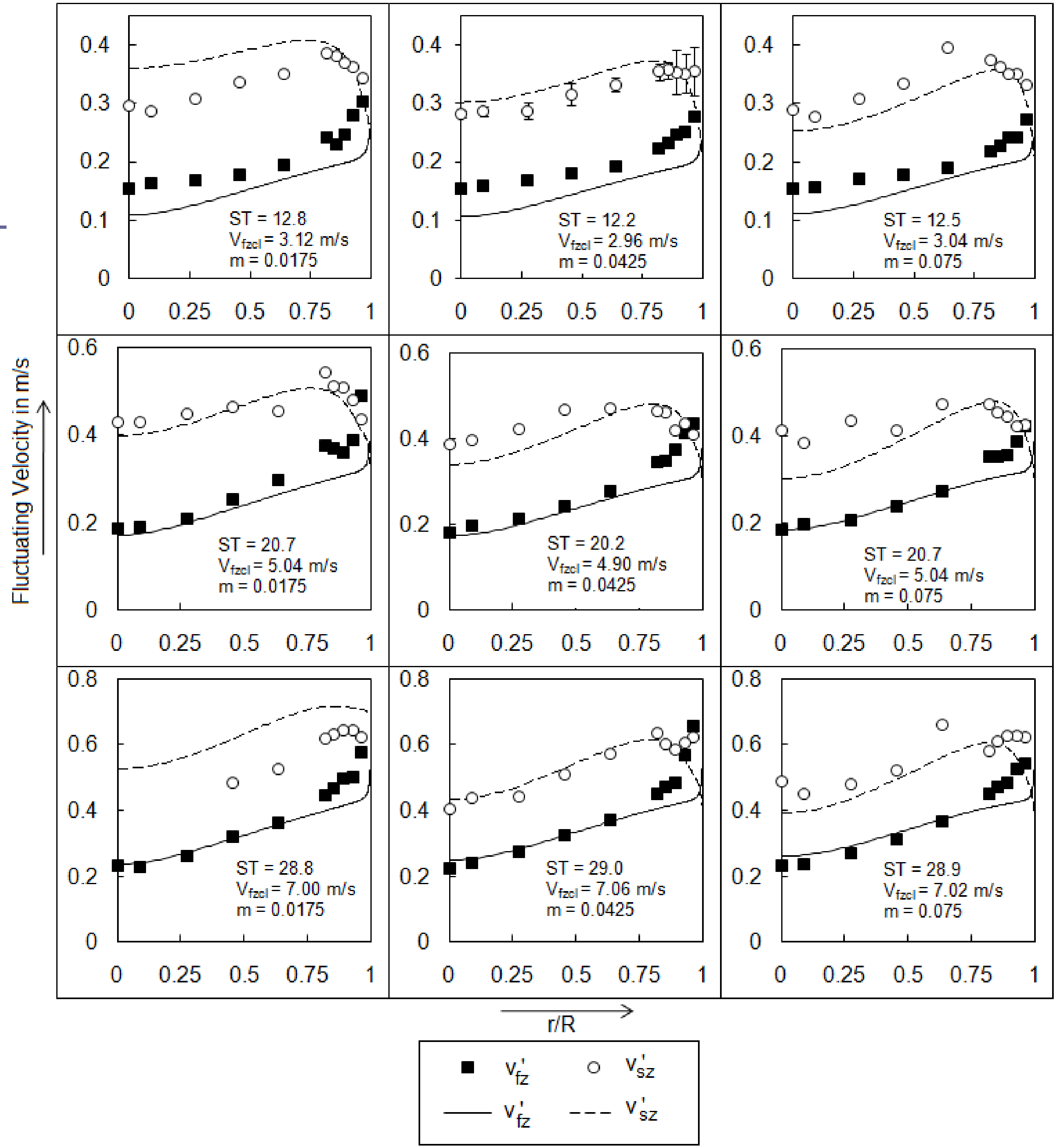


Transitional Flow

- Relative mean velocity increases with increasing St
- Solid fluctuations increase with increasing St
- Bridge model for solid-phase stress based on St weighting
- Fluctuating velocity correlation – time scale following observed experimental trends with Re, d, and solids loading



ST = 5.68
V_{fzcl} = 3.11 m/s
m = 0.0175
ST = 5.52
V_{fzcl} = 3.02 m/s
m = 0.0425
ST = 5.78
V_{fzcl} = 3.16 m/s
m = 0.075
ST = 9.21
V_{fzcl} = 5.04 m/s
m = 0.0175
ST = 9.19
V_{fzcl} = 5.03 m/s
m = 0.0425
ST = 8.55
V_{fzcl} = 4.68 m/s
m = 0.075
ST = 12.9
V_{fzcl} = 7.06 m/s
m = 0.0175
ST = 12.9
V_{fzcl} = 7.04 m/s
m = 0.0425
ST = 12.5
V_{fzcl} = 6.83 m/s
m = 0.075
Fluctuating Velocity in m/s
r/R
v'_{fz}
v'_{sz}
v'_{fz}
v'_{sz}



Fluctuating Velocity in m/s
r/R
ST = 12.8
V_fzcl = 3.12 m/s
m = 0.0175
ST = 12.2
V_fzcl = 2.96 m/s
m = 0.0425
ST = 12.5
V_fzcl = 3.04 m/s
m = 0.075
ST = 20.7
V_fzcl = 5.04 m/s
m = 0.0175
ST = 20.2
V_fzcl = 4.90 m/s
m = 0.0425
ST = 20.7
V_fzcl = 5.04 m/s
m = 0.075
ST = 28.8
V_fzcl = 7.00 m/s
m = 0.0175
ST = 29.0
V_fzcl = 7.06 m/s
m = 0.0425
ST = 28.9
V_fzcl = 7.02 m/s
m = 0.075
v'fz
v'sz
v'fz
v'sz

Summary

- New model for fluctuating velocity interactions
- Model development for transition regime
- Detailed, non-intrusive data for range of St
- Homogeneous flow conditions at lower St numbers
- Solid velocity fluctuations increase and become independent of fluid fluctuations with increasing St

A. Rao, J. Curtis, B. Hancock, and C. Wassgren, "Numerical Simulation and Validation of a Dilute Turbulent Gas-Particle Flow Model with Turbulence Modulation", AIChE Journal, in press (2011)

A. Rao, M. Pepple, D. Rangarajan, J. Curtis, B. Hancock, C. Wassgren and C. Yurteri, "Effect of Stokes Number on Dilute Turbulent Liquid-Solid Flow: An Experimental and Numerical Study", AIChE Journal, submitted (2011)

Next Steps

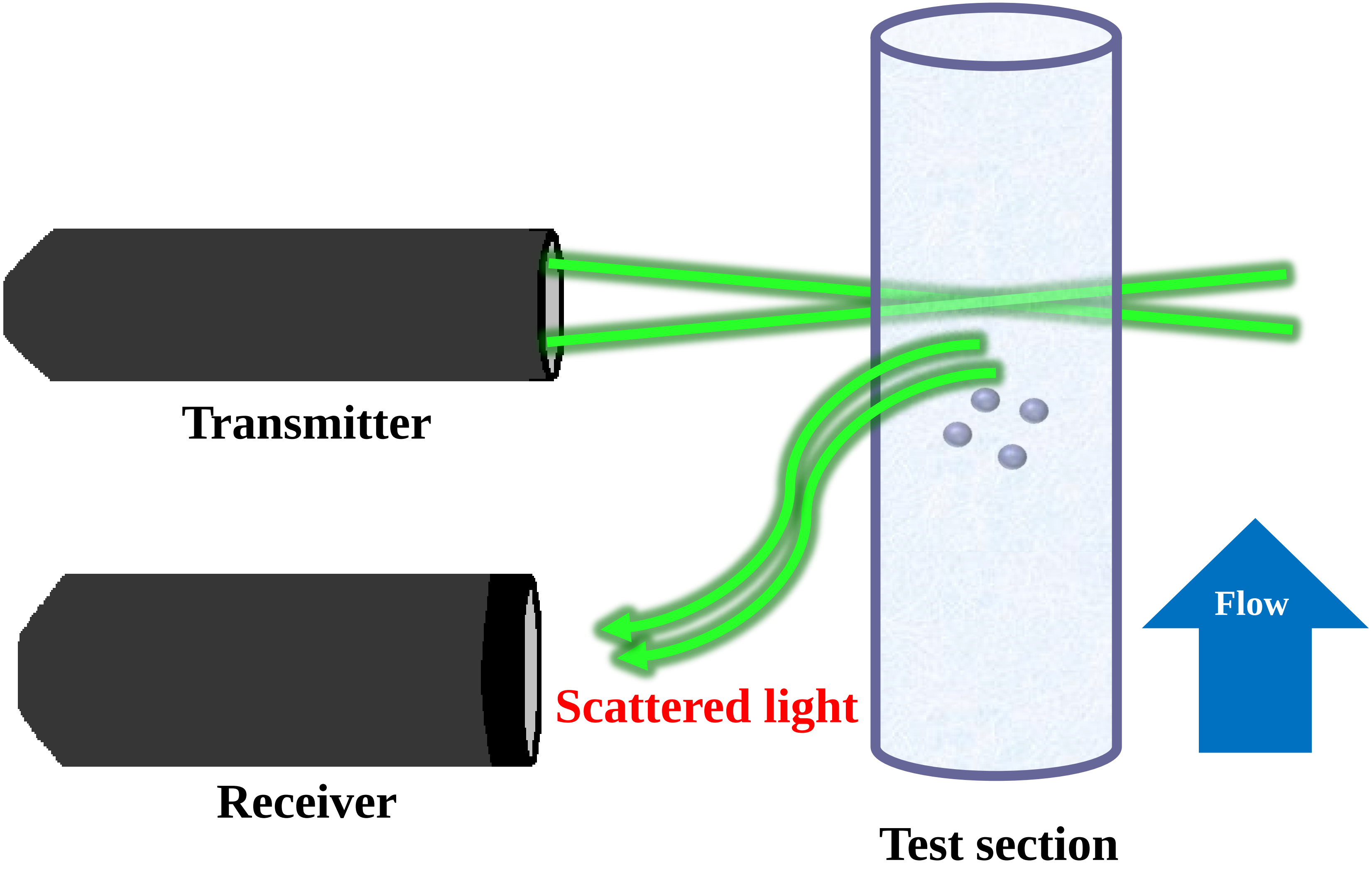
- Fluid and solid velocity data with 2mm glass beads ($25<St<45$)
- Non-spherical particles
- Dense-phase fluid-solid flow

Koh, C.J., P. Hookham and L.G. Leal, 1994, An Experimental Investigation Of Concentrated Suspension Flows In A Rectangular Channel, *J. Fluid Mech.* **266**, 1-32

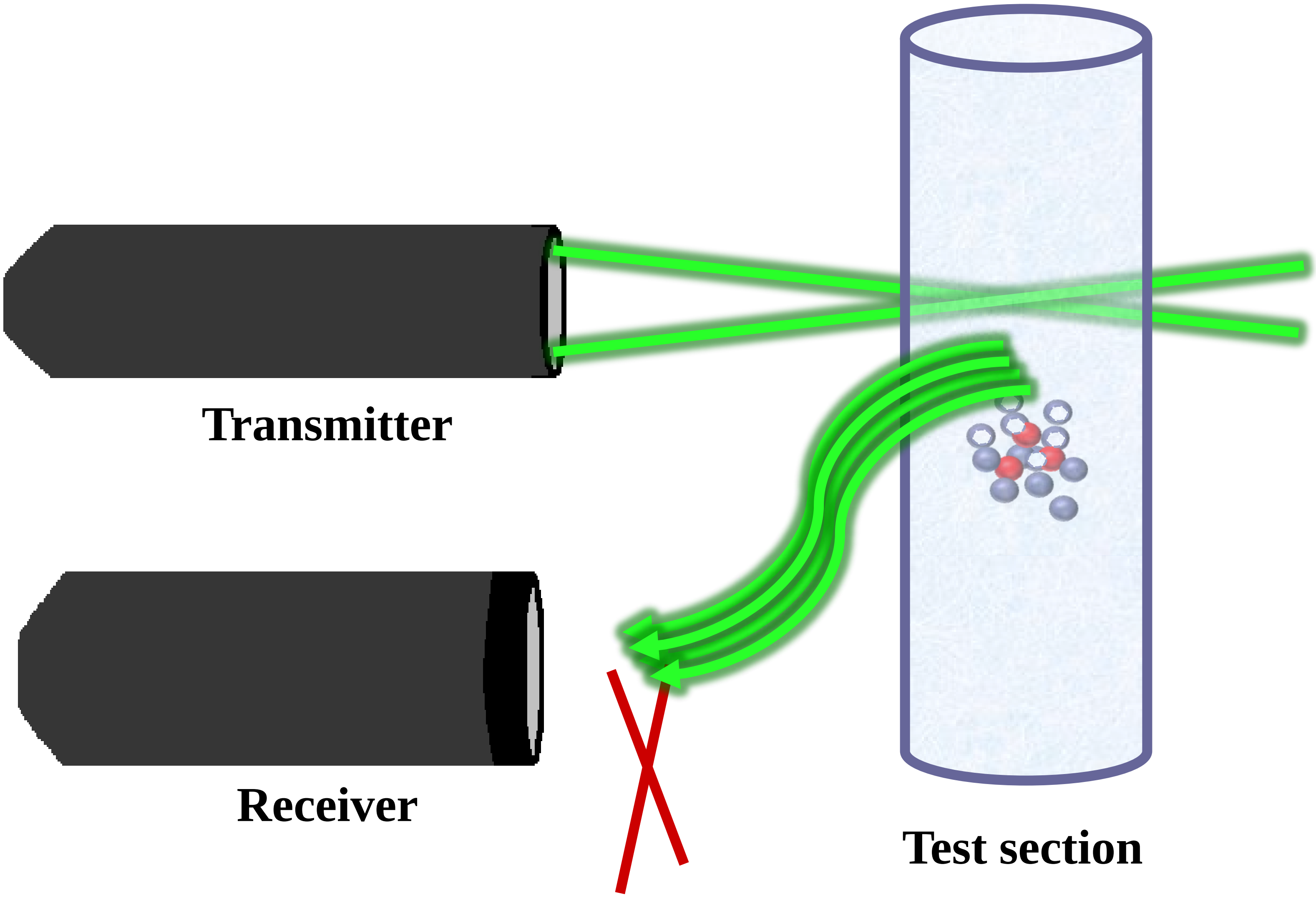
Lyon, M.K. and L.G. Leal, 1998, An experimental study of the motion of concentrated suspensions in two-dimensional channel flow Part 1: Monodisperse systems, *J. Fluid Mech.* **363**, 25-56

Low Re, neutrally-buoyant suspensions

LDV: DILUTE-PHASE OPERATION



LDV: DENSE-PHASE OPERATION



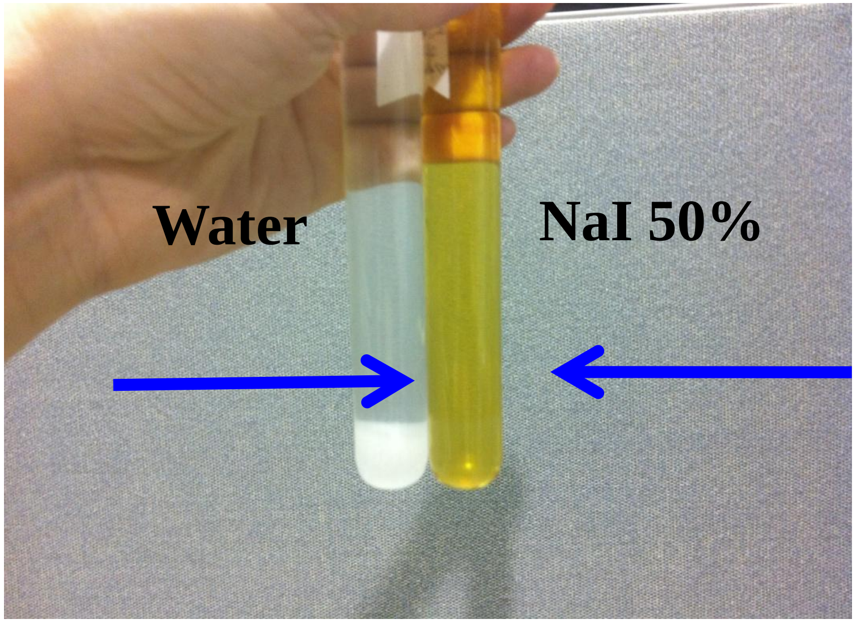
REFRACTIVE INDEX MATCHED SYSTEM - Candidates

Fluid (all percentage w/w)	Particles
Sodium iodide solution 60%	PMMA
Sodium iodide solution 54.5-55%	Pyrex glass
Sodium iodide solution 50%	Silica gel
Zinc chloride solution 50%	Fused silica
Penreco Drakeol	Fused quartz
Glycerin 37.1% NaCl 15% water 47.9%	Silicone rubber
Calcium chloride solution 30%	Silicone elastomer
Aqueous sucrose solution	Silicone elastomer
Olive oil	Pyrex glass
Soybean oil	Pyrex glass
Oilve oil	PMMA
Soybean oil	PMMA

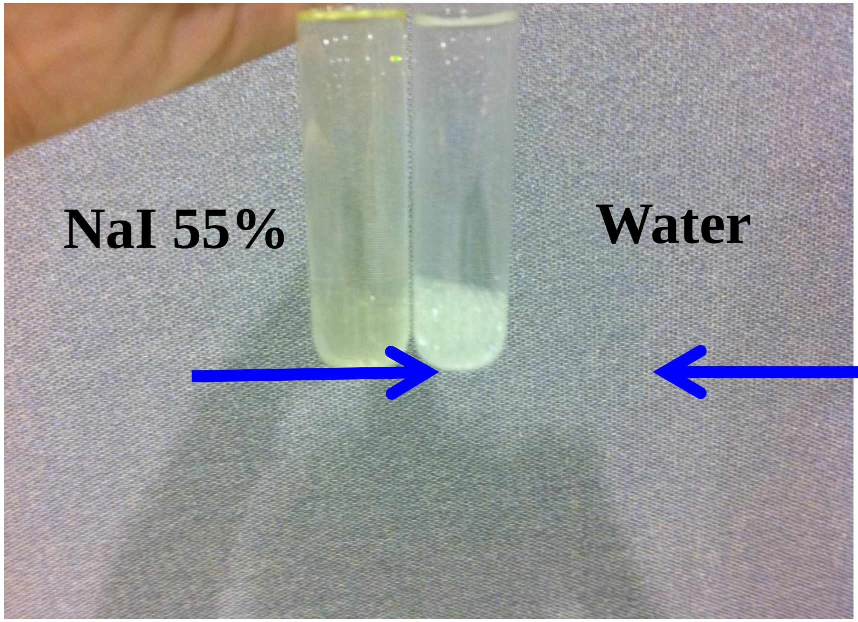
Need non-flammable, low viscosity fluid, no light sensitivity, non-toxic system

REFRACTIVE INDEX MATCHED SYSTEM

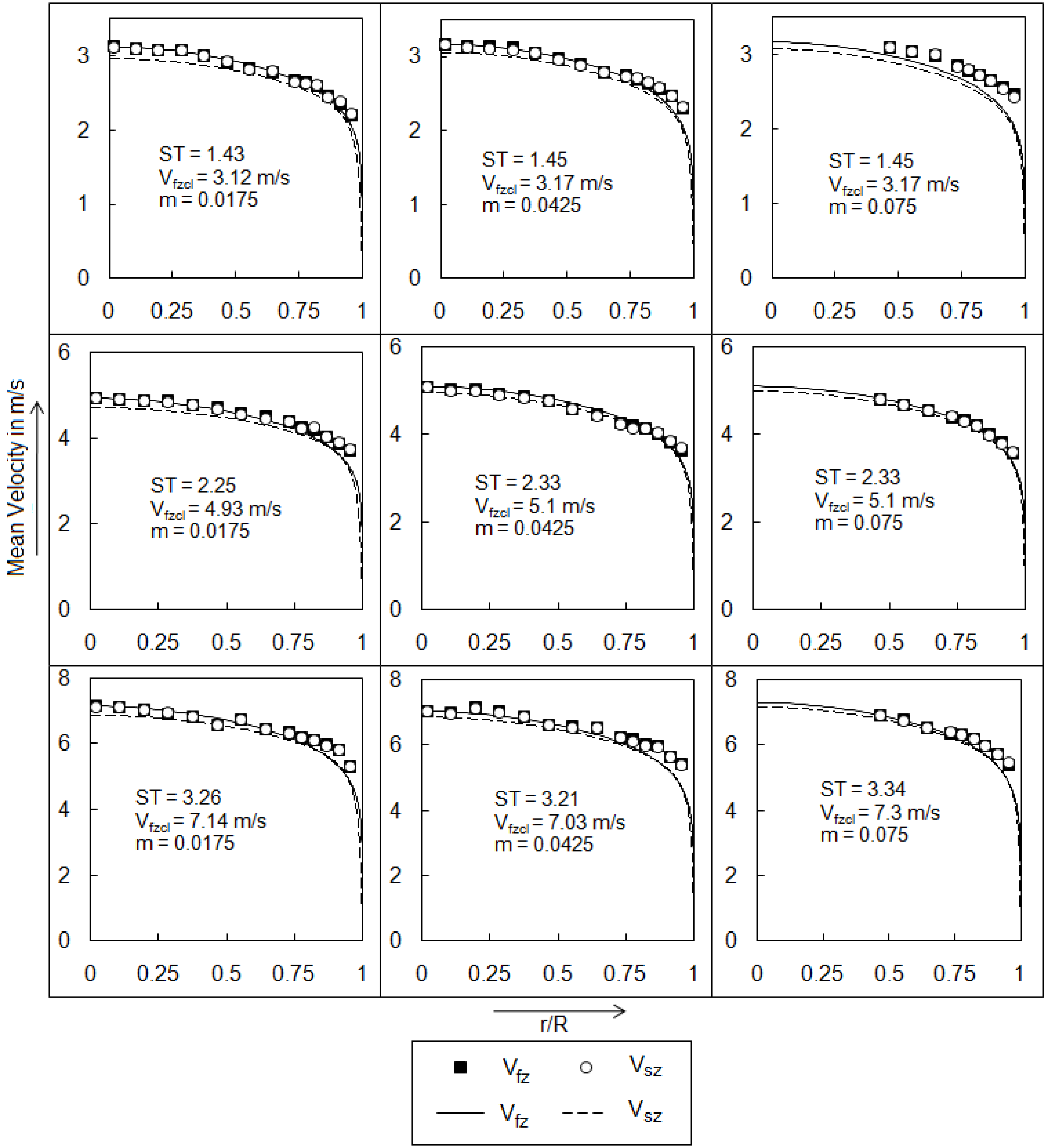
Silica gel particles



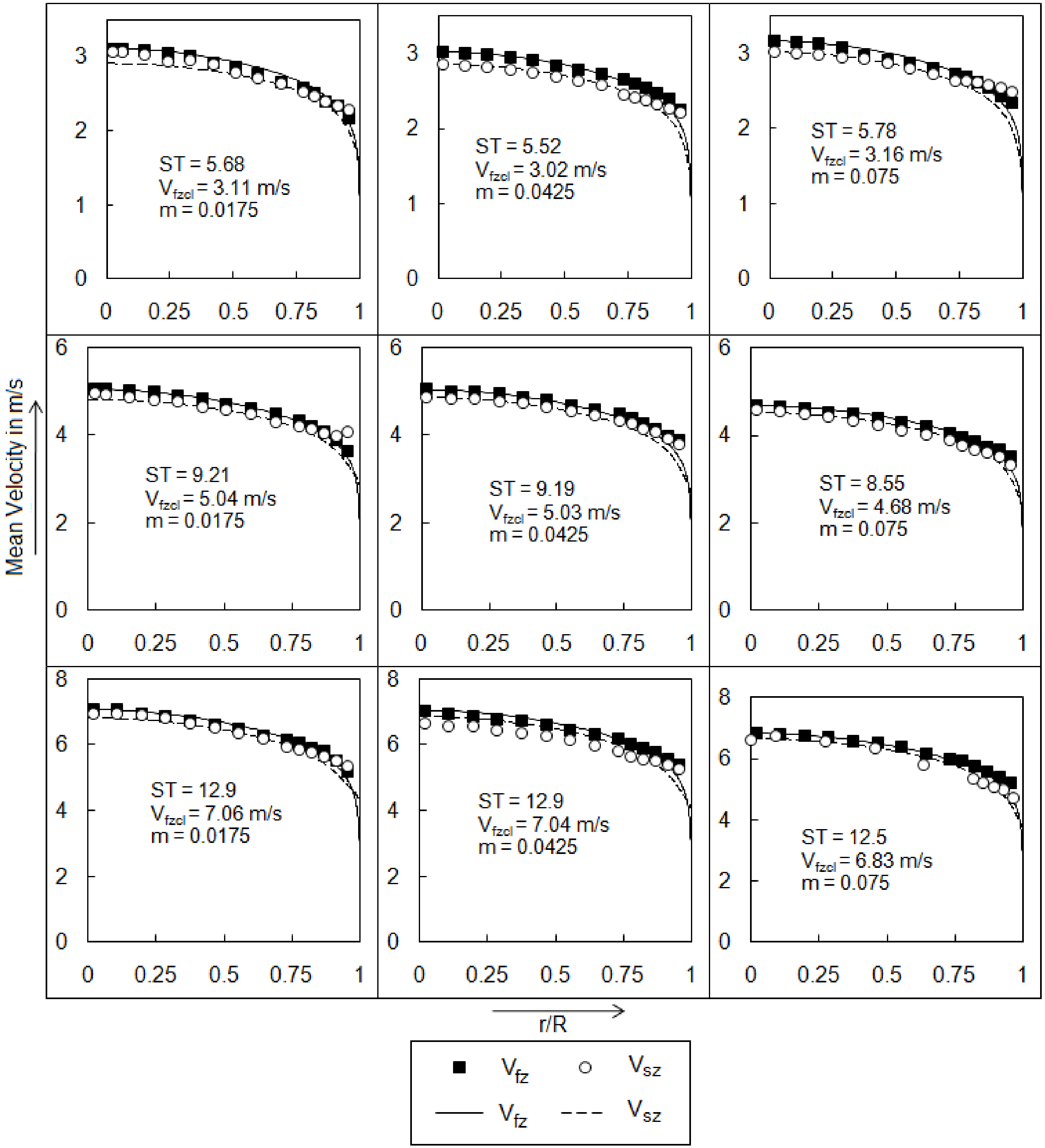
Pyrex glass particles



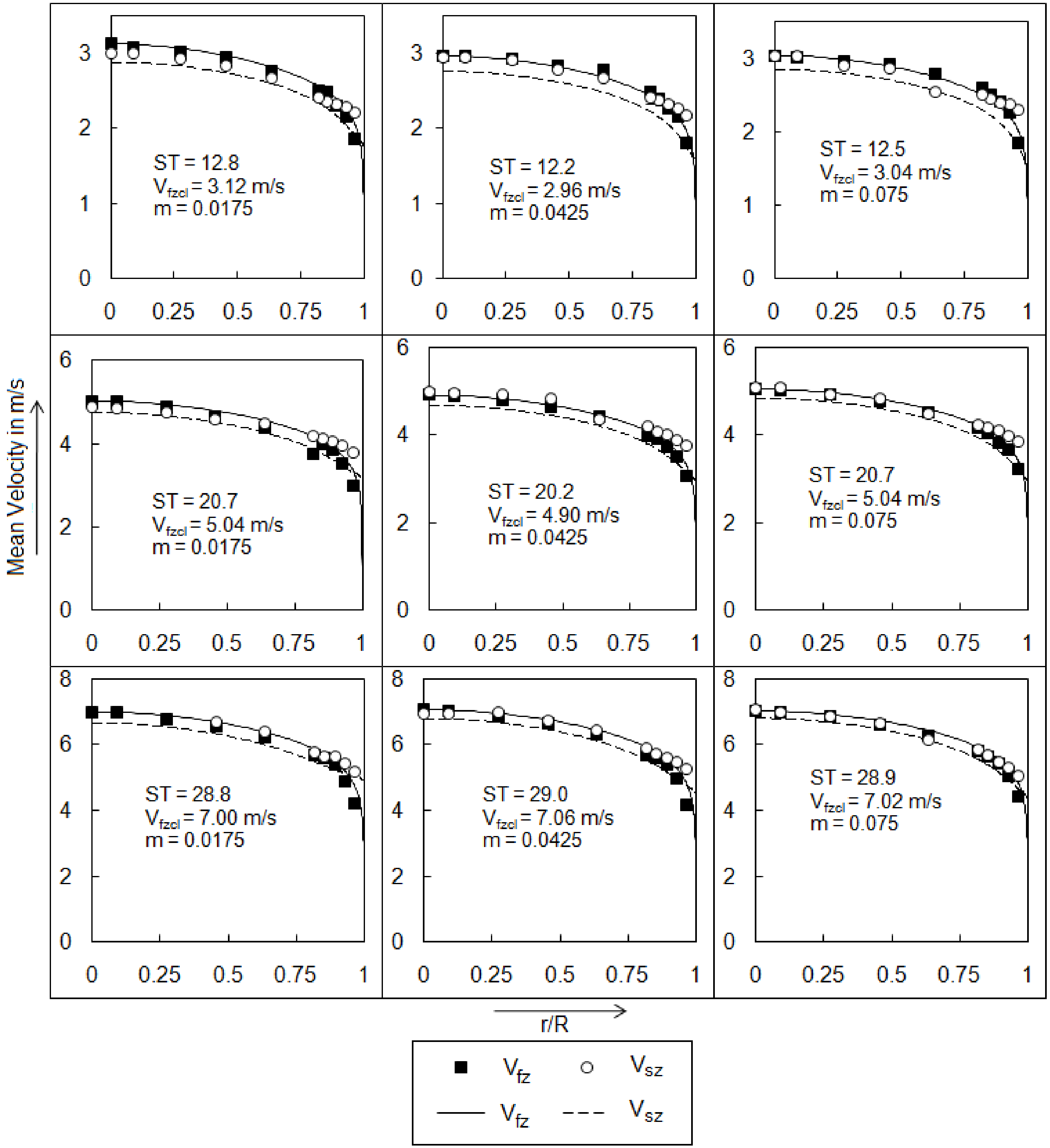
Minimal Temperature Sensitivity



Mean Velocity in m/s
ST = 1.43
V_{fzcl} = 3.12 m/s
m = 0.0175
ST = 1.45
V_{fzcl} = 3.17 m/s
m = 0.0425
ST = 1.45
V_{fzcl} = 3.17 m/s
m = 0.075
ST = 2.25
V_{fzcl} = 4.93 m/s
m = 0.0175
ST = 2.33
V_{fzcl} = 5.1 m/s
m = 0.0425
ST = 2.33
V_{fzcl} = 5.1 m/s
m = 0.075
ST = 3.26
V_{fzcl} = 7.14 m/s
m = 0.0175
ST = 3.21
V_{fzcl} = 7.03 m/s
m = 0.0425
ST = 3.34
V_{fzcl} = 7.3 m/s
m = 0.075
r/R
V_{fz}
V_{sz}
V_{fz}
V_{sz}



ST = 5.68
V_fzcl = 3.11 m/s
m = 0.0175
ST = 5.52
V_fzcl = 3.02 m/s
m = 0.0425
ST = 5.78
V_fzcl = 3.16 m/s
m = 0.075
ST = 9.21
V_fzcl = 5.04 m/s
m = 0.0175
ST = 9.19
V_fzcl = 5.03 m/s
m = 0.0425
ST = 8.55
V_fzcl = 4.68 m/s
m = 0.075
ST = 12.9
V_fzcl = 7.06 m/s
m = 0.0175
ST = 12.9
V_fzcl = 7.04 m/s
m = 0.0425
ST = 12.5
V_fzcl = 6.83 m/s
m = 0.075
Mean Velocity in m/s
r/R
V_fz
V_sz
V_fz
V_sz



ST = 12.8
V_{fzcl} = 3.12 m/s
m = 0.0175
ST = 12.2
V_{fzcl} = 2.96 m/s
m = 0.0425
ST = 12.5
V_{fzcl} = 3.04 m/s
m = 0.075
ST = 20.7
V_{fzcl} = 5.04 m/s
m = 0.0175
ST = 20.2
V_{fzcl} = 4.90 m/s
m = 0.0425
ST = 20.7
V_{fzcl} = 5.04 m/s
m = 0.075
ST = 28.8
V_{fzcl} = 7.00 m/s
m = 0.0175
ST = 29.0
V_{fzcl} = 7.06 m/s
m = 0.0425
ST = 28.9
V_{fzcl} = 7.02 m/s
m = 0.075
Mean Velocity in m/s
r/R
V_{fz}
V_{sz}
V_{fz}
V_{sz}

Extra Slides

Interaction Term & Fluctuation Velocity Cross-Correlation

- Time and Volume Based Averaging (TVBA)

$$I_k = -\beta(1-\nu)\left(2k - k_{sg}\right)$$

$$I_T = \beta(1-\nu)\left(k_{sg} - 3T\right)$$

Cross-correlation

- Volume Based Averaging (VBA) by Crowe (2000)

$$I_k = \beta(1-\nu)\left(3T - k_{sg}\right) + \beta(1-\nu)\left|V_g - V_s\right|^2$$

$$I_T = \beta(1-\nu)\left(k_{sg} - 3T\right)$$

Cross-correlation

$$2k = (\overline{v_g{}'.v_g{}'});\ 3T = (\overline{v_s{}'.v_s{}'});\ k_{sg} = (\overline{v_g{}'.v_s{}'})$$

Motivation: Modeling of Inertia-Dominated Flows

- Fluctuating Velocity Interaction Terms
  - Effect of Particles on Gas-Phase Fluctuations (k equation)

$$I_k = \beta(1-\nu)(2k - \overline{v_{gi}'v_{si}'})$$

$$I_k = \beta(1-\nu)\left|v_g - v_s\right|^2 + \beta(1-\nu)(\overline{v_{gi}'v_{si}'} - 3T)$$ Crowe (2000)

  - Effect of Fluid on Particle-Phase Fluctuations (T equation)

$$I_T = \beta\left(1-\nu\right)\left(\overline{v_{gi}'v_{si}'} - 3T\right)$$

Other relations proposed which neglect fluid turbulence or inertia

- New Model for Fluctuating Velocity Interaction Terms
  - Based on heat transfer analogy
  - Energy transfer occurs due to particle drag or particle collisions
  - Fluid turbulence enhancement and dissipation

  - Rao *et al.*, Circulating Fluidized Beds session, Thursday afternoon