(Effective) Field Theory and Emergence in Condensed Matter

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Effective field theory in condensed matter physics

Microscopic models (e.g, Hubbard/t-J, lattice spin Hamiltonians, etc)

`Low energy' effective field theory

`Low energy' experiments/
phenomenology

Effective field theory:*minimal* requirements/ challenges

1. **`Tractable’:** Must be simpler to understand than original microscopic models and relate to experiments

- continuum field theory often useful but not necessarily of the kind familiar from high energy physics.

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1. **`Tractable':** Must be simpler to understand than original microscopic models and to relate to experiments

- continuum field theory often useful but not necessarily of the kind familiar from high energy physics.

2. **`Emergeable':** A proposed low energy field theory must (at the very least) be <u>*capable of emerging*</u> from microscopic lattice models in the *`right' physical Hilbert space* with the *right symmetries*.

- demonstrate by calculations on `designer' lattice Hamiltonians.

Designer Hamiltonians do not need to be realistic to serve their purpose.

Conventional condensed matter physics

Hartee-Fock + fluctuations

Structure of effective field theory:

Landau quasiparticles + broken symmetry order parameters (if any).

`Exotic' quantum matter

Quantum spin liquids, Landau-forbidden quantum critical points, non-fermi liquid metals........

What are the useful degrees of freedom for formulating an effective field theory?

Field theory not necessarily in terms of electrons + Landau order parameters.

Emergeability

A crucial constraint on effective field theories of condensed matter systems

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Emergeability

Microscopic model (UV theory) : We often have a very good idea of the physical Hilbert space and global symmetries of the UV theory if not the detailed Hamiltonian.

Effective field theory (IR theory): To be emergable all its local operators must live in physical UV Hilbert space.

Global symmetries must be ``non-anomalous".

A trivial example

UV theory: Lattice model of charge-e electrons
Symmetries: Charge conservation,....

IR theory:
Non-emergable: Field theory of charge-e bosons

In physical Hilbert space all bosons must have even charge.

Emergable: Field theory of charge-2e bosons

(eg: Ginzburg-Landau theory of superconductors).

An almost trivial example

1. UV theory: Lattice model of spins/bosons

2. IR theory:

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Emergable: Free Dirac fermions coupled to Z_2 gauge field.

Now fermions are not local....all local operators are bosonic.

Theory of a quantum spin liquid: Can demonstrate emergability of this particular theory through solvable spin models.

Remarks

Fractional quantum numbers/fractional statistics excitations can never be local objects even if they are good IR quasiparticles.

Coupling them to gauge fields in IR is a way to `hide' them from UV.

Gauge fields deconfined => effective theory of a non-trivial phase/phase transition (eg: quantum spin liquids/non-fermi liquids/Landau-forbidden criticality).

A non-trivial example

UV theory: Lattice model of spins with U(1) x time reversal in d = 2 space dimensions

IR field theory: massless QED_3 with N_f fermions.

$$\mathcal{L} = \bar{\psi}\left(\gamma^{\mu}(i\partial_{\mu} - a_{\mu})\right)\psi + \frac{1}{2e^2} f_{\mu\nu}^2 \tag{1}$$

Whether this is emergable or not depends on how symmetry is implemented.
Naive global symmetries:

1. $SU(N_f)$:

$$\psi \to U\psi$$

2. $U(1)$: If a_{μ} is non-compact, magnetic flux is conserved and generates a 'dual' $U(1)$.

(Non)-emergability of massless QED3 in XY spin systems

Emergable: Physical $U(1)$ is subgroup of $SU(N_f)$.

Hermele, TS, Fisher, Lee, Nagaosa, Wen, 04

Field theory must include terms that break all other symmetries (e.g.: instantons).

These may be irrelevant at IR fixed point (=>emergent IR symmetries).

Example of a gapless quantum spin liquid

Non-emergable: Physical U(1) = `dual' U(1) of non-compact gauge field*.

`Anomalous' implementation of U(1) x T.

Cannot emerge in any 2+1-d spin system but can only emerge at the surface of a 3+1-d (interacting) topological insulator (Wang, TS, 13)

*Proposed in spin liquid literature in 2007.

A very non-trivial example

UV theory: Lattice model of bosons/spins with no symmetry in 3 space dimensions.

IR theory: Massive QED

Field content: (i) Gapless photon

(ii) Gapped electric charge

(iii) Gapped magnetic monopole.

Emergable photons

Such theories are emergable from lattice bosons.

Many `designer' examples (Motrunich, TS, 02, Hermele, Balents, Fisher 04, Levin, Wen 05, ...)

Currently active experimental search (`quantum spin ice' materials)

In designer models, gapped (emergent) electric charge may be boson or fermion.

Gapped (emergent) monopole is boson.

Non-emergable photons

Are there lattice boson models in a photon phase where both electric charge and magnetic charge are fermions?

No!!

Massive QED with fermion statistics for both e and m forbidden in strict 3+1-d.

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Proof (biproduct of recent classification of interacting electronic topological insulators): Wang, Potter, TS, Science 2014 (Appendix).

Key idea: Can think of such a phase as a (gauged) putative topological insulator of fermionic e particles.

Show such a putative topological insulator does not have a consistent surface in the right Hilbert space.

Open question: Can such a theory arise as boundary of 4+1-d theory?