

EFFECTS OF RED SHIFTS ON THE DISTRIBUTION OF NEBULAE

EDWIN HUBBLE*

ABSTRACT

Two new surveys of nebulae, to $m_{pg}=18.47$ and 21.03 , are described which, together with three earlier surveys to $m_{pg}=19$, 19.4 , and 20 , indicate the apparent distribution in depth. The numbers of nebulae per square degree, N , to limiting magnitudes m , are represented by the relation

$$\log N = 0.6 (m - \Delta m) - 9.052 \pm 0.005,$$

where

$$\log \Delta m = 0.2 (m - \Delta m) - 4.239 \pm 0.008.$$

The corrections, Δm , are interpreted as apparent departures from large-scale uniform distribution due to the effect of red shifts on apparent luminosities.

The effects to be expected are calculated on the two assumptions that red shifts are (a) velocity shifts and (b) not velocity shifts. With an average $T_0=6000^\circ$ assumed for integrated nebular radiation, $\Delta m=4.0 \, d\lambda/\lambda$ (velocity shifts) or $3.0 \, d\lambda/\lambda$ (not velocity shifts). The empirical expression, derived from the apparent distribution, together with the velocity-magnitude relation, is $\Delta m=2.94 \, d\lambda/\lambda$.

If red shifts are not velocity shifts, the apparent distribution agrees with that in an Einstein static model of the universe or an expanding homogeneous model with an inappreciable rate of expansion, provided spatial curvature is negligible.

If red shifts are velocity shifts which measure the rate of expansion, the expanding models are definitely inconsistent with the observations unless a large positive curvature (small, closed universe) is postulated. The maximum value of the present radius of curvature would be of the order of 4.7×10^8 light-years; and the mean density, of the general order of 10^{-26} . The high density suggests that the expanding models are a forced interpretation of the observational results.

Surveys with large reflectors on Mount Wilson and Mount Hamilton show that the distribution of nebulae¹ over the sky down to limiting magnitudes² between 19 and 20 is moderately uniform except as affected by local obscuration within the galactic system and by the occasional great clusters. The distribution in depth, as indicated by the rate of increase of the numbers of nebulae to successive limiting magnitudes, also appears to be approximately uni-

* *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 557.

¹ *Mt. W. Contr.*, No. 485; *A p. J.*, **79**, 9, 1934; *Lick Obs. Bull.*, **16**, 177 (No. 458), 1934.

² Unless otherwise stated, the terms "nebula" and "magnitude" in the present discussion refer to extragalactic nebulae and to photographic magnitudes on the international scale, respectively.

form when tentative corrections are applied for the effect of red shifts on apparent luminosity. The second conclusion is less reliable than the first and requires further investigation before it can be used with confidence in cosmological theory. The following contribution is a re-examination of the subject, carried out in greater detail and with the aid of additional material.

The observational data are five homogeneous groups of nebular counts, each giving the number of nebulae per unit area to a particular limiting magnitude. The five pairs of values determine an empirical relation, $\log N_m = f(m)$, which diverges systematically from the theoretical relation representing uniform distribution. The departures from uniformity, expressed as corrections to the observed magnitudes, are then formulated as a relation $\Delta m = f(m)$, which sums up the observational results of the investigation. This relation includes the effect of red shifts, in addition to observational errors, space absorption, and real departures from uniform distribution; hence its interpretation is of critical importance for cosmological theory.

I. OBSERVATIONAL DATA

PREVIOUS SURVEYS

Three groups of nebular counts are available from previous surveys. These represent the 1-hour exposures on Eastman 40 plates with the 100-inch and 60-inch reflectors already discussed by the writer,³ and the 1-hour exposures on Imperial Eclipse plates with the 36-inch Crossley reflector discussed by Mayall.⁴ Since the details for individual plates are already published, they will not be repeated here. The limiting magnitudes were 20.0, 19.4, and 19.0, with uncertainties of the estimated order of 0.1 mag. The mean values of $\log N$ per plate were derived from all available material in the polar caps ($\beta > 40^\circ$), reduced to standard exposures of 1 hour by corrections derived from empirical relations between $\log N$ and $\log E$. The results were reduced to mean $\log N$ per square degree with the aid of corrections for coma and for plate area.

For the present investigation the data were used as follows: The 1-hour exposures with the 60-inch and 100-inch reflectors (214 and

³ *Mt. W. Contr.*, No. 485; *Ap. J.*, **79**, 8, 1934.

⁴ *Lick Obs. Bull.*, **16**, 177 (No. 458), 1934.

228 plates, respectively) were extracted from the Mount Wilson surveys and reduced separately. Mayall's values, representing all the 36-inch material, were adopted as published, except that latitude corrections were applied in order to make the data comparable with the Mount Wilson counts. These corrections⁵ increased the mean $\log N$ by the increment 0.032 without materially affecting the dispersion. The results are included in Table IV.

NEW SURVEYS

The two additional groups of counts represent 2-hour exposures⁶ with the 100-inch reflector and 20-minute exposures with the 60-inch, both on Eastman 40 plates. The fields, as before, are restricted to the polar caps; and the counts are reduced to $\log N$ per square degree in the same manner as in the earlier surveys. Each plate was examined three times, the last examination being a continuous review of each group as a unit. It seems probable that a more critical standard was attained in the identification of threshold nebulae. This advantage, combined with the appreciably higher sensitivity of recent emulsions, led to relatively fainter limits, and hence larger numbers of nebulae, than would be expected for corresponding exposures in the former surveys. Details for individual plates are listed in Tables I and II.

LIMITING MAGNITUDES

The limiting magnitudes for the earlier surveys were given to the nearest 0.1 mag., and the published values are adopted without change. Since the new surveys represent extreme cases and play important rôles in the correlation of $\log N_m$ and m , the limits are given to another decimal place. The limiting magnitude of the counts on the 20-minute exposures was derived from about fifteen plates of six fields in which nebular magnitudes down to $m=19.0$

⁵ Mayall concluded that "some sort of latitude effect is probably involved in the Crossley material" but that "the data are insufficient to establish the nature of the variations." The more extensive material in the Mount Wilson surveys appears to establish a cosecant law.

⁶ Thirteen of the exposures with the 100-inch are not precisely 2 hours but range from 100 minutes to 150 minutes. The corrections reducing $\log N$ to the standard exposure range from 0.02 to 0.13 and average 0.066. The uncertainties in these small corrections are believed to be negligible.

TABLE I
SURVEY FIELDS
100-Inch Reflector; 2-Hour Exposures

β	λ	E^*	Z	Q	N_I	$\log N^\dagger$
+73°.....	276°	135	29°	FG	157	2.33
63.....	0	150	40	GE	363	2.54
60.....	280	150	36	GE	250	2.37
53.....	17	120	19	GE	238	2.47
52.....	14	110	31	G	121	2.30
51.....	22	120	28	E	328	2.58
49.....	39	120	36	G	177	2.44
47.....	15	120	36	GE	216	2.47
47.....	159	135	20	FG	146	2.35
47.....	164	105	19	GE	203	2.50
46.....	329	110	45	G	142	2.43
45.....	20	100	34	G	185	2.58
+40.....	143	120	35	GE	234	2.53
-40.....	50	130	24	G	200	2.45
40.....	57	120	34	G	157	2.41
40.....	70	130	25	E	214	2.38
40.....	130	120	22	GE	190	2.41
42.....	31	115	43	FG	160	2.53
43.....	30	120	38	GE	222	2.51
43.....	32	120	34	G	116	2.27
43.....	61	120	28	FG	174	2.51
44.....	34	120	43	G	154	2.42
45.....	45	120	32	G	144	2.35
45.....	160	120	41	G	175	2.46
47.....	79	120	21	G	150	2.35
47.....	104	120	30	GE	241	2.51
47.....	102	120	33	GE	182	2.39
49.....	38	115	47	G	185	2.52
50.....	50	120	30	GE	192	2.40
50.....	70	120	32	FG	198	2.56
50.....	120	120	31	FG	172	2.50
55.....	34	110	41	GE	132	2.30
55.....	100	120	28	GE	223	2.44
55.....	140	120	37	G	210	2.49
58.....	82	120	34	E	294	2.54
58.....	123	135	39	F	145	2.45
59.....	89	120	37	GE	246	2.49
60.....	50	120	43	G	171	2.41
60.....	110	120	35	GE	260	2.52
62.....	95	120	36	FG	231	2.60
-72.....	40	135	51	GE	301	2.56

* E = Exposure-time in minutes.

$\dagger \log N$ is reduced to the standard exposure, $E=120$, by the corrections $\Delta \log N = 1.33 \log E$. (*Mt. W. Contr.*, No. 485; *A.p. J.*, 79, 8, 1934).

TABLE II
SURVEY FIELDS
60-Inch Reflector; 20-Minute Exposures

β	λ	Z	Q	N_I	$\log N$	β	λ	Z	Q	N_I	$\log N$
+75°.....	320°	32°	E	18	1.30	-43°.....	61°	41°	GE	30	1.65
74°.....	315	28	GE	10	1.07	43°.....	66	34	E	16	1.31
73°.....	313	25	E	18	1.29	44°.....	41	34	GE	12	1.23
63°.....	323	24	GE	24	1.46	44°.....	55	38	E	19	1.40
63°.....	325	24	E	15	1.22	44°.....	66	40	E	14	1.27
62°.....	338	28	GE	23	1.44	44°.....	80	20	E	46	1.74
62°.....	340	33	E	16	1.25	45°.....	40	39	GE	18	1.41
60°.....	20	31	E	10	1.05	45°.....	55	39	E	8	1.01
60°.....	135	32	G	14	1.30	45°.....	80	24	E	28	1.53
60°.....	136	35	G	17	1.39	45°.....	102	28	E	38	1.66
60°.....	138	42	G	6	0.96	45°.....	109	33	E	17	1.32
60°.....	170	38	G	5	0.87	47°.....	79	29	E	27	1.51
59°.....	24	30	E	21	1.37	47°.....	102	25	GE	14	1.27
59°.....	28	37	E	21	1.38	49°.....	38	38	GE	27	1.57
58°.....	34	42	GE	6	0.91	49°.....	39	35	GE	21	1.45
58°.....	36	31	GE	13	1.21	49°.....	61	31	E	37	1.65
58°.....	40	38	GE	5	0.82	49°.....	99	32	E	52	1.80
55°.....	21	38	E	13	1.19	49°.....	149	39	E	8	1.00
54°.....	110	37	GE	24	1.50	50°.....	60	35	GE	28	1.58
54°.....	111	39	GE	23	1.49	50°.....	75	24	GE	6	0.89
54°.....	193	25	GE	25	1.50	50°.....	81	32	E	8	0.98
53°.....	195	23	GE	40	1.69	50°.....	99	35	E	13	1.20
53°.....	196	21	GE	8	0.99	50°.....	150	39	E	18	1.36
53°.....	225	40	GE	20	1.43	51°.....	75	24	G	16	1.36
53°.....	294	42	GE	17	1.37	51°.....	81	38	GE	19	1.41
51°.....	247	43	GE	10	1.14	51°.....	89	28	E	7	0.91
50°.....	140	31	G	6	0.96	51°.....	160	44	E	15	1.29
50°.....	141	34	G	8	1.09	51°.....	162	45	E	10	1.11
46°.....	133	28	E	23	1.44	52°.....	81	42	E	14	1.25
46°.....	134	30	GE	27	1.56	52°.....	89	32	E	17	1.30
46°.....	136	34	E	26	1.52	52°.....	163	48	E	14	1.28
42°.....	122	42	GE	8	1.07	53°.....	52	32	E	19	1.35
42°.....	123	45	GE	4	0.78	53°.....	100	26	E	19	1.34
41°.....	121	39	GE	7	1.02	53°.....	110	27	E	29	1.52
40°.....	141	30	G	16	1.41	54°.....	36	42	GE	11	1.18
40°.....	142	34	FG	4	0.90	54°.....	51	34	E	13	1.19
+40°.....	143	39	G	26	1.65	54°.....	100	29	E	32	1.56
-41°.....	171	47	E	9	1.11	54°.....	111	28	E	13	1.17
42°.....	62	36	GE	18	1.41	54°.....	129	34	E	16	1.28
-42°.....	172	47	E	7	1.00	-55°.....	35	45	E	13	1.21

TABLE II—*Continued*

β	λ	Z	Q	N_I	$\log N$	β	λ	Z	Q	N_I	$\log N$
-55°.....	50°	37°	E	26	1.49	-61.....	70°	42°	GE	10	1.12
55.....	73	29	GE	21	1.41	61.....	95	37	E	12	1.14
55.....	77	33	E	9	1.01	62.....	95	42	E	41	1.69
55.....	100	31	E	22	1.40	63.....	99	36	E	20	1.36
55.....	111	30	E	27	1.49	63.....	118	41	E	22	1.42
55.....	125	32	E	22	1.40	64.....	100	36	E	33	1.58
55.....	126	34	E	38	1.65	64.....	119	41	E	18	1.34
55.....	130	36	E	17	1.30	65.....	100	38	E	34	1.60
56.....	42	39	GE	9	1.07	65.....	103	42	E	25	1.48
56.....	73	32	E	55	1.80	65.....	120	40	E	21	1.39
56.....	77	31	E	18	1.32	66.....	85	40	E	16	1.26
56.....	127	36	E	37	1.64	66.....	104	39	E	43	1.69
57.....	40	46	E	17	1.34	67.....	85	38	E	13	1.17
57.....	41	42	GE	11	1.17	68.....	64	43	E	26	1.49
57.....	60	36	G	9	1.12	69.....	63	45	E	16	1.28
57.....	123	33	GE	31	1.59	-70.....	62	48	E	11	1.14
58.....	60	39	GE	16	1.32						
58.....	123	35	E	26	1.49						
59.....	51	40	E	18	1.33						
59.....	122	39	E	18	1.33						
59.....	123	38	E	18	1.33						
59.....	150	44	GE	13	1.24						
59.....	152	45	GE	24	1.51						
60.....	50	44	E	12	1.17						
-60.....	71	38	GE	26	1.53						

or 19.2 have been measured from schraffierkassette images. The scale was determined from the North Polar Sequence and three Selected Areas in which the scale has been extended by Baade beyond the reliable ranges in the *Mount Wilson Catalogue of Photographic Magnitudes*.⁷ The fifteen plates were scattered at intervals throughout the period of time required to complete the survey. Since the total range in the limits of the individual plates is only 0.25 mag., the mean value, 18.47 ± 0.03 , is believed to be reliable except for uncertainties in the scale of the comparison stars and in the zero point of the scale of nebular magnitudes.

The estimation of the limiting magnitude for 2-hour exposures necessarily involved considerable extrapolation. A dozen 20-minute exposures with the 100-inch telescope, scattered throughout the

⁷ Seares and others, *Carnegie Institution of Washington Publication*, No. 402, 1930.

group, were centered on the same fields as were used in calibrating the 20-minute exposures with the 60-inch. The mean limiting magnitude of these plates, reduced to the zenith and to excellent quality, is 19.11 ± 0.02 , very close to the limit of the reliable nebular magnitudes determined with the schraffierkassette. Hence, for 2-hour exposures, the limiting magnitude should be

$$19.11 + 2.5 p \log \frac{120}{20} = 19.11 + 1.94p ,$$

where p is the Schwarzschild exponent in the reciprocity law.

The value of p for surface images of equal density on fully developed Eastman 40 plates increases as the density diminishes and approaches unity at the threshold itself. The value unity was used for deriving magnitudes in the previous surveys, and observational data were presented which justified the procedure. A re-examination of the material with special reference to very small surface images suggests values of p ranging from 0.95 to 1.0 for the particular conditions under discussion, and indicates that the mean should give the desired results to a close approximation. The limiting magnitude for the 2-hour exposures is then 21.00 ± 0.04 , with uncertainties of much the same order as those for the nebular magnitudes around $m=19$ in the standard fields.

A second method of calibration, also applied in the previous surveys, depends upon a known relation between diameters and surface brightness of threshold images.⁸ Several 2-hour exposures on Selected Areas, in which Baade has extended the scale of stellar magnitudes, were made under conditions of poor seeing. Stars with known magnitudes were thus smeared out into surface images similar to, although considerably larger than, nebular images at the threshold of identification on plates of excellent quality. The same sorts of results are obtained with small extra-focal images, although the shadow of the Newtonian flat introduces uncertainties. The relative diameters of the threshold images of these stars and the threshold images of nebulae on good-seeing plates with equal exposures indicate the relative surface brightness and hence the difference in the integrated magnitudes. For instance, a 2-hour exposure

⁸ *Mt. W. Contr.*, No. 453; *Ap. J.*, **76**, 106, 1932.

of S.A. 68, with diameters of faint images about $8''.5$, registered stars down to about 19.9. Diameters of threshold nebulae on the sharpest survey plates average about $3''.2$. From the relative areas, the nebular images should be 2.12 mag. fainter than the stellar images, but the correction for surface brightness as a function of diameter reduces the increment to 1.15 mag.; hence the threshold magnitude for the nebulae is 21.05.

Four 2-hour exposures are available and, in addition, six 1-hour exposures and five schraffierkassette plates (images $16'' \times 16''$) with

TABLE III
LIMITING MAGNITUDE FOR 120-MINUTE EXPOSURES

MEAN EXP.	NO. OF PLATES	MEAN DIAM.	m_e		REMARKS
			Observed	Reduced*	
Min.					
20.....	12	$3''.2$	19.11 ± 0.02	21.00 ± 0.02	Good seeing
60.....	6	6	19.57 .06	21.02 .05	Poor seeing
120.....	4	7	20.20 .06	21.02 .06	Poor seeing
145.....	5	16	19.08 ± 0.06	21.08 ± 0.06	Schraffierkassette
Mean.....	21.03 ± 0.03

* The reduced values of m_e refer to nebulae with diameters of $3''.2$ at the threshold of identification on 120-minute exposures at the zenith, made with the 100-inch reflector on Eastman 40 plates. Exposures with the schraffierkassette range from 110 to 180 minutes, $\bar{E} = 145$, $\log \bar{E} = 2.164$. Mean diameters are approximate; the reduced limiting magnitudes are the means of those for the individual plates in each group.

exposures ranging from 2 to 3 hours. Results from the latter two groups involve extrapolations in exposure times as well as in diameters. Limiting magnitudes from the three groups, together with the limit from the 20-minute exposures, are listed in Table III. The limiting magnitudes, m_e , refer to nebulae with diameters of $3''.2$ at the threshold of identification on 2-hour exposures at the zenith. The simple mean, 21.03 ± 0.03 , is adopted as the most probable value. The small probable error, of course, merely indicates the consistency of the data and does not fully eliminate the possibility of systematic errors.

The series of five limiting magnitudes form a consistent system in which the scale appears to approximate closely the conventional scale over the range involved, and the accidental errors are less than

0.1 mag. This conclusion is drawn from examination of the Mount Wilson data alone, but Mayall's data for the Crossley counts are thoroughly consistent and may be included in the general statement.

The zero point of the scale is uncertain by the amount of the errors in the standard nebular magnitudes between 18.5 and 19.0. These arise from two sources. One, the uncertainties in the scale of the standard sequences of stellar magnitudes, is believed to be small since it involves the North Polar Sequence and three independent extensions of that scale from about $m = 17.0$ downward. The other arises from the method of measuring nebular images and involves the question of the relative amount of luminosity contributed by extremely faint but relatively large outer regions of nebulae. A rough rule is that reliable total magnitudes may be derived by smearing the nebulae and comparison stars over areas with diameters three times the diameters of the nebulae, as estimated by simple inspection of focal plates.⁹ These conditions were fulfilled in measuring the very faint nebulae in the standard regions, and hence the uncertainties arising from outer regions are believed to be less than 0.1 mag.

For many purposes, including investigations of distribution and associated problems, the zero point is relatively unimportant; the significant feature is the scale. The present series of magnitudes can be represented to the nearest 0.1 mag. by extrapolations from $m = 19.0$, based on the reciprocity law with an exponent p between 0.95 and 1.0. The reliability of the scale thus depends largely on the value of p . Since it is improbable that p should be greater than unity, any appreciable revision would probably reduce the range in the observed magnitudes and make the faint magnitudes brighter with respect to the bright magnitudes.

SIGNIFICANCE OF $\log \bar{N}$

The mean values, $\overline{\log \bar{N}}$ and $\log \bar{N}$, for the five surveys are listed in Table IV, together with the dispersions, σ , calculated from the residuals relative to $\overline{\log \bar{N}}$. The three quantities should satisfy the relation

$$\log \bar{N} = \overline{\log \bar{N}} + 1.151\sigma^2,$$

⁹ The rule is based on a comparison of magnitudes from extrafocal and schraffierkas-sette images with those derived by Stebbins and Whitford from accurate measures with photoelectric cells, made directly at the foci of reflecting telescopes.

provided that the frequency distributions of $\log N$ are normal error-curves; the small differences, C—O, show that this condition is very closely approximated.

The surveys represent random sampling with average samples of five different sizes. The significance of the dispersions is somewhat obscured by the fact that the sizes of samples do not correspond to the adopted $\log N$ but represent the numbers of nebulae actually identified on the plates before the counts were reduced to standard conditions. The latter data are included in the table and show that the dispersion tends to diminish as the size of the sample increases.

TABLE IV
DATA FROM INDIVIDUAL SURVEYS*

Tel.	E	NP	\bar{N}_1	m	$\overline{\log N}$	σ	$\log \bar{N}$	$\log \bar{N}$ (obs)
60-in.	20	121	19	18.47	1.829	0.224	1.887	1.885
36-in.	60	284	35	19.0	2.103	.225	2.161
60-in.	60	214	41	19.4	2.312	.156	2.340	2.342
100-in.	60	228	52	20.0	2.650	.176	2.686	2.685
100-in.	120	41	200	21.03	3.154	0.084	3.162	3.162

* Tel.=telescope; E =exposure time in minutes; NP =number of plates; \bar{N}_1 =average number of nebulae actually identified per plate; m =limiting magnitude of survey; $\overline{\log N}$ =mean logarithm of number of nebulae per square degree; σ =dispersion in $\log N$; $\log \bar{N} = \overline{\log N} + 1.151\sigma$; $\log \bar{N}$ (obs)=logarithm of the mean number of nebulae per square degree (each plate reduced to standard conditions).

This tendency, together with the close agreement of the results for the two galactic hemispheres and the absence of conspicuous systematic variations in latitude or longitude, emphasizes once more the random character of the large-scale distribution of nebulae throughout the sample of the universe available to inspection.

While the large-scale distribution appears to be essentially uniform, the small-scale distribution is very appreciably influenced by the well-known tendency toward clustering. The phenomena might be roughly represented by an originally uniform distribution from which the nebulae have tended to gather about various points until now they are found in all stages from random scattering, through groups of various sizes, up to the occasional great clusters.¹⁰ The

¹⁰ This representation is purely formal and has no genetic implications. For purposes of speculation, the reverse development seems preferable, namely, that nebulae were formed in great clusters whose gradual evaporation has populated the general field.

tendency appears to operate on a relatively modest scale—at least no clusters are known with more than a few hundred members; hence irregularities tend to average out among samples that are large compared to a single cluster.¹¹ In small samples irregularities are conspicuous, and, as an empirical fact, they lead to a random distribution of $\log \bar{N}$ rather than of N . Accidental errors of observation and reduction also tend to introduce random scattering of percentage deviations (since they are generally equivalent to variations in limiting magnitude), but the effects are relatively small and are presumably constant for all the surveys.

The significant quantity, for the present purpose, is the arithmetical mean, \bar{N} , related to the geometrical mean by the expression already given, in which σ represents the combined effects of irregularities and accidental errors. If the errors are small compared to the irregularities, as in the surveys using small samples, the relation may be used as it stands, or \bar{N} may be calculated directly from the data. On the other hand, if the samples are so large that the dispersion is dominated by errors rather than by irregularities, the term including σ may be ignored, and \bar{N} derived from the simple relation

$$\log \bar{N} \text{ (distribution)} = \overline{\log N} \text{ (observed)} .$$

The dispersion arising from accidental errors alone is probably of the order of 0.06 in $\log N$ (0.1 mag) and about equal to that arising from irregularities in the survey using the largest samples. Appropriate corrections for accidental errors would consist of a constant term, $1.151 (0.06)^2 = 0.004$, to be subtracted from each $\log \bar{N}$ in Table IV. However, the term is much smaller than the various probable errors, and it may be neglected or thrown in with the uncertainties of the zero point of the magnitude scale. It appears, therefore, that the tabulated $\log \bar{N}$ may be used directly for comparing the numbers of nebulae to the successive limits of the various surveys.

¹¹ Samples in the regions of great clusters were rejected in the present surveys; and, for this reason, irregularities tend to average out for samples that are large compared to single groups of nebulae.

COMPLETENESS OF THE COUNTS

Completeness of the counts was investigated by comparing plates of the same regions made with different apertures and exposures, the 2-hour exposures with the 100-inch being taken as standards. In this way it was possible to count all the nebulae actually recorded, regardless of whether or not they were identified in the course of the survey, on plates of 20-minute and 1-hour exposures with the 60-inch and of 1-hour exposures with the 100-inch. The numbers actually recorded, N_2 , were then compared with the numbers previously identified, N_1 , reduced to quality E , corrected by the coma factor, and then designated as N_c . Since the investigations were restricted to the central regions of the plates, the two corrections (for quality and coma) concern the identification, and not the registration, of nebular images. The differences, $N_2 - N_c$, represent the numbers of nebulae recorded below the threshold of identification. This threshold is above the threshold of registration by the amount

$$\Delta m_n = \frac{\log N_2 - \log N_c}{0.6}.$$

Additional evidence is furnished by similar comparisons of 20-minute and 1-hour exposures with the 60-inch, the 1-hour exposures with the 100-inch being used as standards; and also by comparisons of the 20-minute exposures with the 1-hour exposures with the 60-inch as standards.

The available material, much of which is listed in the surveys, includes about fifteen comparisons of 20-minute exposures, about ten of the 1-hour exposures with the 60-inch, and six of the 1-hour exposures with the 100-inch, and leads to values for Δm_n of 0.60 ± 0.04 , 0.55 ± 0.04 , and 0.57 ± 0.06 , respectively. The last value, representing 1-hour exposures with the 100-inch, is necessarily the least reliable. These direct tests of the completeness of the counts are consistent with the conclusion derived from measures of nebular magnitudes, namely, that the value of Δm_n is of the general order of half a magnitude.

The results for the three surveys indicate a satisfactory degree of homogeneity and suggest no conspicuous systematic variation de-

pending upon limiting magnitude. For these reasons it is assumed that the Crossley counts and the 2-hour survey with the 100-inch behave consistently, although no direct controls are available. Corrections for incompleteness would probably make the limiting magnitudes in Table IV slightly brighter, but the amounts would be very small and approximately constant throughout the series. Therefore, they may be included among the uncertainties of the zero point of the magnitude scale.

Finally, the effect of systematic variations in plate densities arising from the different exposure times has been investigated. It seemed possible that an optimal density for the identification of faint nebular images might exist, which would affect the counts systematically. Fortunately, the extensive material in the 1-hour exposures with the Mount Wilson reflectors exhibits a range in plate densities sufficient to test this possibility. The results indicated that no appreciable systematic effects would be expected except in a few of the densest 2-hour exposures. On these latter plates the counts may be slightly smaller than normal, but the effect on $\log \bar{N}$ for the survey would probably be negligible.

The examination of possible sources of systematic errors strongly suggests that the data in Table IV exhibit a maximum range in m and a minimum range in $\log \bar{N}$. The conclusion has an important bearing on the interpretation of the data.

RELATION BETWEEN $\log N$, m , AND Δm

In Figure 1, values of $\log \bar{N}$, hereafter designated as "log N ," are plotted against the corresponding limiting magnitude m . The five points determine the linear relation

$$\log N = 0.501m - 7.371,$$

probably within the uncertainties of the data. Since uniform distribution is expressed by

$$\log N = 0.6(m - \Delta m) + C,$$

where Δm is a correction to the observed magnitude required by red shifts, etc., the deviation from uniformity, expressed by Δm , is

$$\Delta m = 0.165 m + \frac{7.371 + C}{0.6}.$$

This relation represents the differential corrections over the observed range but is not suitable for extrapolation much beyond that range. The constant C is known to be of the order of -9 ; hence the correction is zero in the vicinity of $m = 16.5$ and negative for brighter

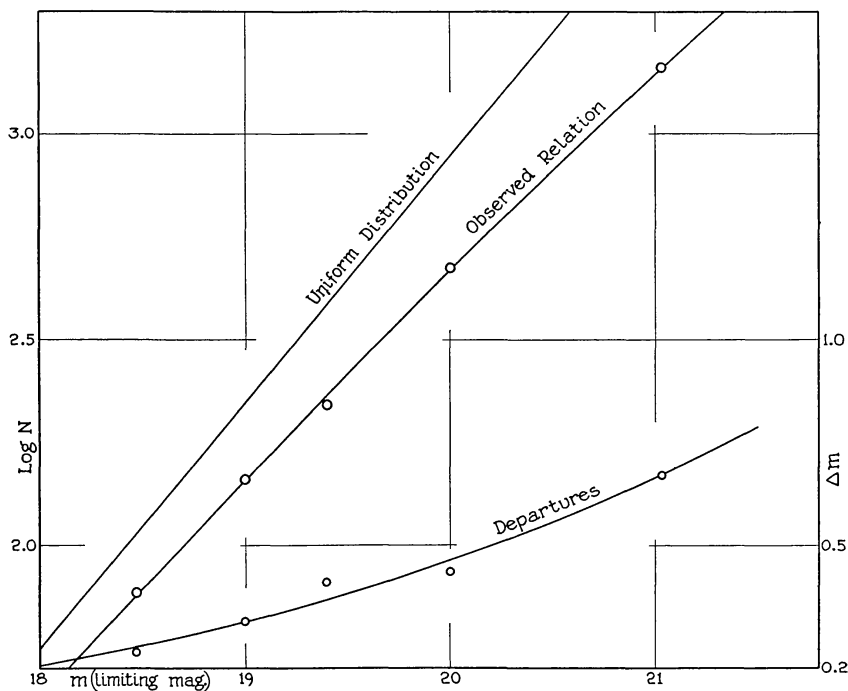


FIG. 1.—Apparent distribution of nebulae in depth. The straight line marked “Uniform Distribution” is the relation $\log N$ (per square degree) $= 0.6m - 9.052$, which represents uniform distribution with the density equal to that observed in the vicinity of the galactic system. The curve marked “Observed Relation” is a least-squares solution representing the mean results of the five surveys, on the assumption that apparent departures from uniformity, when expressed as corrections to the observed limiting magnitudes, are linear functions of the distance. The solution is $\log N = 0.6(m - \Delta m) - 9.052$, where $\log \Delta m = 0.2(m - \Delta m) - 4.239$. The curve marked “Departures” exhibits the corrections, Δm , as a function of the observed limiting magnitudes, m .

limits. For the Harvard survey of bright nebulae ($m = 12.9$) the correction would be about -0.6 mag., which seems to be definitely inconsistent.

A more significant representation is derived as follows: the correction to the observed magnitudes introduced by uniform space absorption would be a linear function of the distance. Corrections

due to red shifts will also approximately follow a linear function, because red shift itself is a linear function of distance; and the corrections, expressed in terms of magnitudes, approximate linear functions of red shifts. Finally, any systematic deviations from uniform distribution would be partially compensated by corrections to the magnitudes, which were linear functions of distance. Hence, as a first approximation, we may assume that Δm is a linear function of distance.¹² Then

$$\log \Delta m = 0.2(m - \Delta m) + b,$$

and the corresponding expression for uniform distribution is

$$\log N = 0.6(m - 10^{0.2(m - \Delta m) + b}) + C.$$

The values of b and C are readily found by successive approximations when the equation is written in a form suitable for least-squares solutions. As a first approximation, Δm in the exponent may be ignored. As a second step, the values of Δm derived in the first step are introduced in the exponent. The third approximation, with values of Δm derived in the second, appears to be sufficient; further approximations do not change the significant figures in the results. The solution, expressed as the mean of the regression-curves, is¹³

$$C = -9.052 \pm 0.005,$$

$$b = -4.239 \pm 0.008.$$

¹² The term "assume" in the statement should be stressed because the empirical data are insufficient to distinguish definitely a linear relation between Δm and distance, from a linear relation between Δm and m . As usual, curves of many different forms would satisfactorily represent the observed points in Fig. 1, and the selection is guided by information from other sources. The apparent distribution must evidently be corrected for known effects on apparent luminosities. The chief, if not the only, effects known at the moment are those due to red shifts, whose calculated values closely approximate the selected relation $\Delta m = \text{Constant} \times \text{Distance}$, over the range of the surveys. Later it will be shown that these effects alone account for apparent departures. If other effects are postulated, they must be arbitrarily adjusted to compensate one another. This evidence justifies the selection of the particular expression for departures, although the expression does not emerge directly from the observational data of the five surveys.

¹³ Values of C previously derived were -9.12 , by the writer, and -9.2 , by Mayall. The present revision is influenced by the method of analysis rather than by the addition of new data.

The solution adjusts C and b to fit a particular set of data; errors in one constant are compensated by adjustment of the other. Independent confirmation of either constant not only would be important in itself but would inspire greater confidence in the other. A check on C would be afforded by counts of bright nebulae, for which the corrections Δm are negligible. Surveys to $m = 14$ to 15 are under way with this end in view, but the only significant data available at the moment are those in the Harvard survey.¹⁴

These counts are believed to be complete over the entire sky to the tabulated magnitude, 12.9 . Accurate photoelectric-cell measures by Stebbins and Whitford¹⁵ indicate that the limiting magnitude should be corrected by -0.1 mag. and that the dispersion is not more than 0.2 mag. The polar caps offer two samples, but one (the northern) is dominated by the great Virgo cluster and the group south-following, which Shapley has called the "Extension to the Coma-Virgo cluster." The influence of the aggregation may be avoided by omitting the regions, R.A. $= 12^h$ to 13^h , Dec. $= 0^\circ$ to $+20^\circ$, and R.A. $= 12^h 5$ to $13^h 5$, Dec. $= 0^\circ$ to -15° . The two samples combined then contain 592 nebulae ($m = 12.8$) scattered over $14,270$ square degrees. Therefore, $\log N = -1.38$ and $C = -9.06$.

The uncertainties are difficult to estimate because the data are rather meager. The mean latitude correction, which would reduce the numerical value of $\log N$ by about 0.03 , is probably compensated by clustering outside the omitted regions. The evidence for such clustering is fairly strong. The polar caps furnish two samples of the general order of those in the 2-hour survey, in which $\sigma = 0.084$. Therefore, the probable error for the mean of the two caps might be estimated as of the order of 0.04 or slightly less.

Another way of presenting the data is to say that the solution from the five surveys indicates a limiting magnitude of 12.78 for the Harvard counts, which agrees with that actually observed, well within the uncertainties of the data. It follows that the inclusion

¹⁴ Shapley and Ames, "A Survey of the External Galaxies Brighter than the Thirteenth Magnitude," *Harvard Ann.*, **88**, No. 2, 1932.

¹⁵ *Annual Report of the Mount Wilson Observatory, 1935-1936.*

of the Harvard survey in the general solution would not materially alter the values of C and b as derived from the five surveys alone.¹⁶

RELATION BETWEEN Δm AND RED SHIFTS

The corrections Δm represent observed departures from apparent uniformity arising from all sources. Nevertheless, they are so small that, for a preliminary discussion, space absorption and real departures from uniformity may be ignored and the corrections referred to effect of red shifts alone. Since red shifts, $d\lambda/\lambda$, and the corrections, Δm , are both approximately linear functions of distance (over the observed range), one can be expressed as a linear function of the other. The velocity-magnitude relation¹⁷ is

$$\log v = 0.2(m - \Delta m) + 0.77,$$

or

$$\log \frac{d\lambda}{\lambda} = 0.2(m - \Delta m) - 4.707,$$

and

$$\log \Delta m = 0.2(m - \Delta m) - 4.239.$$

Hence

$$\Delta m = 2.94 \frac{d\lambda}{\lambda}.$$

The probable error of the coefficient in the last relation is relatively small; but, in view of the possibility of systematic errors, a rough estimate of the uncertainties is more significant. The uncertainty of the constant in the first two equations has been previously estimated as not more than 0.02; and, if the same reliability is assigned to the constant b , the uncertainty in the coefficient will be not more than 10 per cent (about three times the probable error).

The most questionable datum is probably the zero point of the magnitude scale; but, fortunately, errors in this quantity do not seriously affect the coefficient. For instance, arbitrary revisions by

¹⁶ With $\log N = -1.38$ and $m = 12.8$ for the Harvard counts, a least-squares solution for the six groups leads to $C = -9.053$, $b = -4.241$.

¹⁷ *Mt. W. Contr.*, No. 549; *Ap. J.*, **84**, 270, 1936.

as much as 0.3 mag. in either direction change the coefficient by less than 15 per cent. In the past, revisions of nebular magnitudes have rather consistently reduced the numerical values. A similar revision of the present data would increase the coefficient; but the change, it is believed, could not be very great.

Other factors involved in the calculations have already been mentioned. The observed range in limiting magnitudes is probably a maximum, and any revision would be expected to reduce the coefficient. The completeness of the nebular counts appears to be comparable in the various surveys. The linear velocity-distance relation has been extrapolated well beyond the observed range; but three of the surveys fall within the observed range, and these alone furnish a value of the coefficient which is comparable with that derived from all five surveys as a group.

The relations

$$\log N = 0.6(m - \Delta m) - 9.052,$$

$$\log \Delta m = 0.2(m - \Delta m) - 4.239,$$

$$\Delta m = 2.94 \frac{d\lambda}{\lambda}$$

summarize the observational results. They are formally consistent with the assumption that apparent departures from large-scale uniformity, Δm , are linear functions of distance and represent effects of red shifts on apparent luminosity. Therefore a comparison of the observed value of B in the relation $\Delta m = B d\lambda/\lambda$ with values calculated on various possible interpretations of red shifts may be expected to indicate which interpretation is the most probable.

II. EFFECTS OF RED SHIFTS ON APPARENT LUMINOSITY

The apparent luminosity of a nebula is measured by the rate at which its light-quanta reach the observer and by the energy in the quanta. Each of these quantities may be influenced by red shifts, and the results are known as the "number effect" and "energy effect," respectively.

Number effect.—If red shifts are velocity shifts, the recession of the nebulae reduces the rates at which the quanta reach the observer, and hence the apparent luminosities, by the factor $1 + v/c$. This fac-

tor, the number effect, operates *only* in the case of actual motion. It is constant throughout a given spectrum because it merely reduces the vertical, or intensity, scale of the energy distribution-curves. Since it is non-selective, the number effect increases apparent magnitudes on any system—bolometric, visual, photographic, etc.—by the same amount, namely,

$$\Delta m_N = 2.5 \log \left(1 + \frac{v}{c} \right).$$

In expanding models of the universe obeying the relativistic law of gravitation, the ratio v/c is replaced by $d\lambda/\lambda$.

Energy effect.—The energy effect follows from the familiar relation $E\lambda = \text{constant}$. Red shifts, regardless of their interpretation, evidently reduce the energy of individual quanta and, consequently, the energy of the sum total of all the quanta that reach the earth's atmosphere, by the factor $1 + d\lambda/\lambda$. The energy effect is selective, for it changes the horizontal or wave-length scale of the energy-distribution curves. Bolometric magnitudes are increased by the increment $\Delta m_E = 2.5 \log (1 + d\lambda/\lambda)$, but the effect must be traced through the atmosphere (selective absorption) and the telescope (selective reflection) to the photographic plate (selective sensitivity) in order to be evaluated in terms of photographic magnitudes. For convenience, the reduction factor, expressed as a magnitude increment, may be termed K . Then, the total effect of red shift on apparent photographic magnitude is to increase the magnitude by the increment

$$\Delta m = 5 \log \left(1 + \frac{d\lambda}{\lambda} \right) + K$$

or by

$$\Delta m = 2.5 \log \left(1 + \frac{d\lambda}{\lambda} \right) + K,$$

depending upon whether or not red shifts are velocity shifts.

EVALUATION OF K

The evaluation of K depends upon the distribution of intensity in nebular spectra. The precise forms of the curves are not well known, but throughout the photographic region they resemble that

of the solar spectrum. The agreement suggests that nebular radiation may be discussed in terms of black-body radiation with about the same degree of confidence that is inspired by similar treatments of solar and stellar radiation. Red shifts distort an original black-body curve for temperature T_0 into new black-body curves corresponding to $T = T_0 / (1 + d\lambda/\lambda)$. The lower temperatures introduce increments to the effects of atmospheric absorption, to the heat index, and to the color index corresponding to the original radiation at T_0 ; and the sums of the increments are the desired values of K . Thus,

$$K = \Delta(\Delta m_r) + \Delta HI + \Delta CI.$$

The increments are readily evaluated from curves representing Δm_r (reduction to no atmosphere, including reflection in the telescope), HI , and CI , for stars, as functions of effective temperature.¹⁸

The values of Δm_r vary less than 0.1 mag. over a range in black-body temperatures from 6000° to 4000°; hence uncertainties in T_0 will introduce only small uncertainties in the increments $\Delta(\Delta m_r)$. The temperature scale for color indices differs widely from the scales derived from heat indices and from water-cell absorptions; but the spectral type is a common denominator by means of which T_0 , the zero point for the present purpose, may be transferred from one scale to another. The increments, ΔHI and ΔCI , are then relatively insensitive to the particular scale employed. Actually the scale derived from water-cell absorptions was used for deriving ΔHI ; and another, the familiar black-body scale,¹⁹ was used for deriving ΔCI in order that the results for K might, with some justification, be regarded as minimum values. Table V lists values of K determined in this manner for various values of T_0 and of the red shift.

Another, and more purely computational, method of evaluating K may also be employed.²⁰ A black-body curve for temperature T_0

¹⁸ The data for Δm_r and HI are given in Tables IV and V of "Stellar Radiation Measurements," by Pettit and Nicholson, *Mt. W. Contr.*, No. 369; *Ap. J.*, **67**, 279, 1928. Color indices are conveniently listed on p. 734 of *Astronomy* (1927) by Russell, Dugan, and Stewart. The values of Δm_r , reduction to no atmosphere, include the reflections in the telescope—two silver surfaces.

¹⁹ Russell, Dugan, and Stewart, *op. cit.*, Table XXX.

²⁰ The method is described by de Sitter, *B.A.N.*, **7**, 205 (No. 261), 1934, and by Hubble and Tolman, *Mt. W. Contr.*, No. 527; *Ap. J.*, **82**, 302, 1935.

is distorted by factors representing atmospheric absorption, reflections in the telescope, and plate sensitivity. The area under the distorted curve represents the intensity actually recorded on the plate. The original curve is then displaced by red shifts and later distorted

TABLE V
K AS A FUNCTION OF T_0 AND $d\lambda/\lambda$

$d\lambda/\lambda$	T_0			
	6500°	6000°	$6000^{\circ*}$	5500°
0.05.....	0.08	0.09	0.09	0.12
.075.....	.11	.13	.14	.18
.10.....	.15	.18	.20	.25
.125.....	.19	.23	.25	.31
.15.....	.24	.28	.31	.38
.175.....	.28	.34	.36	.44
.20.....	.33	.40	.42	.52
.225.....	.38	.47	.48	.60
0.25.....	0.44	0.53	0.55	0.68

* De Sitter, *B.A.N.*, 7, 205 (No. 261), par. 4, 1934.

by the factors previously applied. The ratios of the various recorded intensities to the intensity for zero red shift indicate the energy effects,

$$\Delta m_E = 2.5 \log \left(1 + \frac{d\lambda}{\lambda} \right) + K,$$

from which the values of K are readily derived.

This method was used by de Sitter²¹ in computing Δm_E in terms of pg, pv, and vis magnitudes for $T_0 = 6000^\circ$ and 5000° . The results were expressed in the form of power series, those for photographic magnitudes being

$$(T_0 = 6000^\circ) \quad \Delta m = 2.90 \frac{d\lambda}{\lambda} + \left(\frac{d\lambda}{\lambda} \right)^2 + \dots,$$

$$(T_0 = 5000^\circ) \quad \Delta m = 4.05 \frac{d\lambda}{\lambda} + 0.8 \left(\frac{d\lambda}{\lambda} \right)^2 + \dots$$

²¹ *B.A.N.*, 7, 205 (No. 261), 1934. De Sitter assumed one reflection from silver and one from aluminum, but the difference from the two silver reflections used in the surveys is negligible.

The two methods of determining K lead to results which are in substantial agreement. The greatest uncertainties are those introduced by the assumption of black-body radiation and the neglect of line absorption. Appropriate revisions will be suggested after the characteristics of integrated nebular radiation have been discussed.

The data in Table V could be expressed as power series, following de Sitter's example; but for the present purpose, only the first terms are required. Out to the limits of the surveys, say to $d\lambda/\lambda = 0.22 \pm$, the three quantities, K , Δm_E , and Δm_N are all nearly linear functions of $d\lambda/\lambda$. Approximate values of the coefficient B in the relation

TABLE VI
 B^* AS A FUNCTION OF T_0

T_0	B	
	$K + \Delta m_E^\dagger$	$K + \Delta m_E + \Delta m_N^\ddagger$
7500°.....	2.0	3.0
7000.....	2.4	3.4
6500.....	2.65	3.65
6000.....	3.0	4.0
5500.....	3.6	4.6
5000.....	4.45	5.45

* B is the coefficient in the relation $\Delta m = B d\lambda/\lambda$.

† If red shifts are not velocity shifts, $\Delta m = K + \Delta m_E = K + 2.5 \log (1 + d\lambda/\lambda)$.

‡ If red shifts are velocity shifts, $\Delta m = K + \Delta m_E + \Delta m_N = K + 5 \log (1 + d\lambda/\lambda)$.

$\Delta m = B d\lambda/\lambda$ are readily obtained from the values of Δm for a given red shift; at $d\lambda/\lambda = 0.2$, for instance, $B = 5 \Delta m$. These values are listed in Table VI for red shifts interpreted both as velocity shifts and not as velocity shifts and for various effective temperatures.

EFFECTIVE TEMPERATURE OF NEBULAR RADIATION

The choice of an effective temperature for integrated nebular radiation is rather arbitrary. Spectra, in general, are recorded on small scales and are confined to the nuclear regions. Spectral types vary systematically with nebular types; but the range is small, the large majority lying between F5 and G5. Dwarf characteristics are conspicuous in the two spectra (M 31 and M 32) which have been

recorded on considerable scales, and are presumed to be normal features.²²

Colors of nebulae also vary systematically with the nebular types, and, except for the later spirals, indicate pronounced color excesses, only a fraction of which can be attributed to selective absorption within the galactic system.²³ The measures, by Stebbins and Whitford, were, in general, confined to the central regions of the nebulae and did not include the outer portions.

Mean spectral types (nuclear regions), mean color classes (central regions), and relative frequencies for different nebular types are

TABLE VII
SPECTRAL TYPES AND COLOR CLASSES OF NEBULAE

Nebular Type	Relative Frequency*	Spectral Type†	T_0 ‡	Color Class¶	T_0
Eo-E7.....	17	G3.6	5710°	g6	4580°
Sa, SBa.....	19	G3.4	5730	g5	4700
Sb, SBb.....	26	G1.6	5870	g4	4860
Sc, SBc.....	38	F8.8	6120	f7	5800
Mean.....	G1.2	5910	g1.9	5140

* The frequencies are derived from types estimated on large reflector plates of about 650 objects in the Harvard survey of bright nebulae. The rare Irr nebulae are omitted.

† The spectral types are presumably dwarfs. Data by Humason, *Mt. W. Contr.*, No. 531; *Ap. J.*, **83**, 10, 1936.

‡ T_0 in the fourth column indicates the effective temperature corresponding to the spectral type, while T_0 in the last column indicates the temperature corresponding to the color class.

¶ The color classes are on the scale of giant stars. Data by Stebbins and Whitford, *Annual Rept. of the Mt. Wilson Obs.*, 1935-1936.

listed in Table VII, together with the corresponding effective temperatures. The average nebula in the general field is about Sb, spectral type dG1.2, and color class g1.9 (on the scale of giants). The mean effective temperatures, on the particular scale employed, are 5910° from the spectral types and 5140° from the color classes. These data permit a second approximation to the equivalent black-

²² Exceptions are found in the rare nebulae whose nuclei appear stellar or highly condensed and give emission or early-type spectra.

²³ Investigations of the colors of globular clusters by Stebbins and Whitford (*Mt. W. Contr.*, No. 547; *Ap. J.*, **84**, 132, 1936) suggest a cosecant relation between color excess and galactic latitude, but the selective effect would be a small fraction of the total obscuration. It is not impossible that the correlation may be with position within clouds rather than with latitude.

body temperature that should be used in calculating effects of red shifts.

The data in Tables V and VI indicate the qualitative behavior of Δm as a function of T_0 , but purely empirical revisions may be made in order to adapt the results to the actual conditions of the surveys. The revisions involve variations in reflectivity of silver surfaces, departures from black-body radiation, and color excess.

The tabulated values of Δm were based upon reflectivity of freshly burnished silver surfaces, while the surveys were made with mirrors which were resilvered about every six months and burnished about every month. Reflectivity in the violet falls rapidly for a short time and then approaches a fairly steady state. Precise information on variations of the Mount Wilson mirrors (during the era of silver surfaces) is not available, but measures of laboratory mirrors by Strong²⁴ indicate the order of the deterioration. Curves for reflectivity as a function of λ for silver surfaces 13 and 19 days after burnishing are closely parallel, and the curve for the longer period is believed to approximate the actual conditions of the surveys.²⁵

Departures from black-body radiation have been determined in detail only in the case of the sun. Measures of the spectral region in question—from about 6000 Å to 3000 Å—by Abbot, Pettit, and others furnish an empirical curve for the intensity distribution, which is the curve for the continuous radiation as depressed by line absorption.²⁶ In the nebulae, the line absorption is presumably comparable with that in the sun, but the distribution of continuous radiation is altered by the observed color excess—of the order of

²⁴ *Ap. J.*, **83**, 40, 1936, supplemented by unpublished data very generously furnished to the writer.

²⁵ It makes little difference whether the 13- or the 19-day curve is used, because the significant feature, for the present purpose, is the form of the curve rather than the precise values of the ordinates. Some evidence in favor of the longer interval was derived from a silvered mirror on which Humason selected a region that, in his opinion, appeared to resemble the average state of the large mirrors on Mount Wilson. Strong very kindly measured this selected region and found a curiously close agreement with the 19-day curve.

²⁶ Abbot, Fowle, and Aldrich, *Smithsonian Misc. Coll.*, **74**, No. 7, 1923. Pettit, *Mt. W. Contr.*, No. 445; *Ap. J.*, **75**, 185, 1932. See also a discussion of these and other data by Mulders, *Zs. f. Ap.*, **11**, 132, 1935.

0.15 mag. (see Table VII). The source of the excess²⁷ is unknown, but the effect will clearly reduce the equivalent black-body radiation for the nebulae. The effect may be approximated, following a suggestion by Baade, by using observed intensity-curves near the limb of the sun instead of the curve for the integrated light.

The graphical analysis, based on Pettit's measures for integrated sunlight and Abbot's reduction factors for a distance from the center of 0.95 times the radius (which approximates the desired color excess), together with Strong's reflectivity-curve and a table of sensitivities of Eastman 40 plates furnished by the Eastman Kodak Laboratory, gives a relation between the energy effect, Δm_E , and $d\lambda/\lambda$, which is not precisely linear but which fairly approximates the relation for black-body radiation at $T_0 = 5750^\circ$, over the range in $d\lambda/\lambda$ represented in the surveys. The value of B_E is of the order of 3.25.

Abbot's measures of intensities lead to a slightly larger value of B_E , while Mulders' data give a considerably smaller value—less than 3.0. For these reasons it is concluded that the correct value lies between 3.0 and 3.25, and, in view of the prevalence of early-type giant stars in the outer region of open spirals, is probably nearer the lower limit than the upper. Therefore, the round number 3.0 is adopted, corresponding to an equivalent black-body temperature of 6000° , with a recognized uncertainty of the order of 200° .

Thus, for provisional discussions, the data in Tables V and VI may be used directly with some assurance that the errors are within the uncertainties of the investigation. The uncertainties refer mainly to the unknown origin of the color excess and to the possibility that the

²⁷ The natural assumption of scattering by finely divided material in the nebulae leads to difficulties in interpreting the appearance of the systems as seen from a distance. Whipple's ingenious attempt to account for the color excess as an effect of a mixture of stellar types in nebulae (*Harvard Circ.*, No. 404, 1935) is not confirmed when the method of calculation is applied to globular clusters. Whipple concluded that the data for the one cluster then available were not inconsistent; but several clusters have been observed by Humason, using the standard nebular spectrograph, and these data, together with colors by Stebbins and his associates, are inconsistent. Moreover, Smith (*Annual Report of the Mount Wilson Observatory, 1935-1936*) reports that the distribution of intensities over the wide range of 8000 Å to 3100 Å, in elliptical nebulae (where the color excess is most pronounced), show no appreciable indications of mixtures of types.

effect of early giants in the outer regions of spirals has been underestimated—that abnormal intensities exist in the ultraviolet which, when shifted into the photographic region, would materially reduce the tabulated values of K . The former source of uncertainty is probably not important, because different assumptions lead to comparable results. The latter source has been investigated in two ways: First, an ultraviolet spectrum (by Smith) of a nearby, intermediate spiral, extending far out from the nucleus, showed no evidence of conspicuous excess intensities. Second, the relative colors of elliptical nebulae and spirals, in clusters as distant as $d\lambda/\lambda = 0.13$, were found to be appreciably the same as those of similar nebulae in the neighborhood of the galactic system. Since the elliptical nebulae are known to be homogeneous and their spectra have been recorded in the distant clusters, it follows that the spirals also behave normally—that the excess intensities in the ultraviolet are not sufficient to affect seriously the integrated radiation. The observations do not eliminate the possibility of small effects at the limit of the deepest survey; they merely indicate that such effects, if present, are probably not greater than a fraction of 0.1 mag.

III. COMPARISON OF OBSERVATION AND THEORY

It is evident that the observed result, $B = 2.94$, is accounted for if red shifts are not velocity shifts. The comparison is based on an effective temperature, T_0 , of 6000° , but the uncertainties cover the range down to about $T_0 = 5750^\circ$. The interpretation is consistent with the data whether the nebulae are pictured as scattered through an old-fashioned, infinite universe or whether a homogeneous model obeying relativistic laws of gravitation is adopted, in which the expansion and spatial curvature are either negligible or zero. The conclusion assumes (a) that the large-scale distribution is uniform, (b) that nebulae maintain fairly constant luminosities over the time interval involved in the observations, and (c) that red shifts are an approximately linear function of distance over the range of the surveys.

The argument for real uniformity follows from the small values of the observed departures from apparent uniformity. They are no greater than the effects of the red shifts calculated under minimum conditions. These calculated corrections, when applied to the obser-

vations, lead to uniformity; further corrections would indicate that the real distribution density increased outward, steadily and symmetrically, in all directions. The latter picture is unwelcome because it implies a curiously unique position for the observer.²⁸ For this reason uniform distribution is postulated and space absorption is assumed to be negligible. The whole of the observed departures is then referred to effects of red shifts alone.

As for the constancy of nebular luminosities, the question is whether or not luminosities of spirals change materially (say 10 per cent, or 0.1 mag.) during the time required for light to travel from the limit of the deepest survey to the limit of the shallowest survey—an interval of the order of 250 million years. No definite information is available, of course; and probabilities depend upon speculative arguments. Nevertheless, very few students will hesitate to adopt the assumption that systematic variation in so short an interval will be inappreciable. Nebulae are enormous systems, and it is reasonable to suppose that their evolution is correspondingly slow. Theories of stellar radiation, especially of stars with spectral types comparable with those of nebulae, suggest relatively little loss of mass or luminosity over such intervals. Moreover, the sequence of nebular classification may be an evolutionary sequence; and it can be stated with some confidence, from the study of nearby nebulae, that the range in mean photographic luminosities, from one end of the sequence to the other, is not more than 0.2 mag. Finally, the velocity-distance relation is sensibly linear over an observed range comparable with the interval in question.

²⁸ If the nebulae are assumed to be receding with constant velocities in Euclidean space (as a cloud expands in a vacuum), those counted in a given survey would be scattered through a volume V_0 when the light left the limit of the survey, but would be scattered through a larger volume, $V_n = V_0(1+v/c)^3$, when the light reached the observer. Appropriate corrections to the various surveys would then be necessary in order to derive the momentary distribution at a given epoch. The momentary distribution could be uniform at one, and only one, instant. The fact that the data indicate moderate uniformity near the epoch of the surveys—thus formally accounting for an outwardly increasing apparent density—is a remarkable coincidence that immediately suggests the operation of some principle of relativity. This circumstance encourages the recourse to models obeying the relativistic laws of gravitation, the justification for which is discussed in the preliminary contributions on principles and methods by Tolman and the writer, *Mt. W. Contr.*, No. 527; *Ap. J.*, **82**, 302, 1935.

The most impressive evidence for the linearity of the velocity-distance relation is that offered by the great clusters at distances ranging out to about 250 million light-years, or approximately two-thirds of the distance to the limit of the deepest survey. The expression

$$\log \frac{d\lambda}{\lambda} = 0.2(m - \Delta m) - 4.457,$$

where m is the observed magnitude of the fifth brightest nebula and $\Delta m = 3 d\lambda/\lambda$, very closely represents the data for the ten clusters

TABLE VIII

DEPARTURES FROM LINEARITY IN THE VELOCITY-DISTANCE RELATION WHEN RED SHIFTS ARE INTERPRETED AS DOPPLER EFFECTS*

Cluster	$d\lambda/\lambda$	$m_o - \Delta m_\beta$	$\log d\lambda/\lambda - 0.2m_c$
Virgo.....	0.0041	10.50	-4.483
Pegasus.....	.0127	12.92	.470
Perseus.....	.0174	13.53	.452
Coma.....	.0245	14.30	.451
U Ma I.....	.0517	16.27	.498
Leo.....	.0653	16.53	.439
Cor. Bor.....	.0707	16.75	.445
Boötes.....	.1307	18.28	.436
U Ma II.....	0.1403	18.15	-4.371

* The data are extracted from Table I, *Mt. W. Contr.*, No. 549; *Ap. J.*, **84**, 270, 1936, where full details are given. The Gemini cluster ($\lambda = 150^\circ$, $\beta = +20^\circ$) has been omitted because the latitude correction, Δm_β , is known to be very uncertain. Magnitudes refer to the fifth brightest nebulae in the clusters. The corrected magnitudes, m_c , include the effects of red shifts interpreted as velocity shifts: $m_c = m_o - \Delta m_\beta - 4d\lambda/\lambda$. The two Ursa Major clusters have minimum weight because they are each represented by a single spectrogram of a single nebula.

that have been investigated. The only residuals as large as 0.1 mag. involve recognized uncertainties, and there are no indications of any systematic trend away from the linear relation. Therefore, a linear extrapolation to distances 50 per cent greater than the observed range will probably serve as a fair approximation. Otherwise, a rather sharp departure from linearity must be postulated, immediately beyond the limits of the observations.²⁹

If the corrections $\Delta m = 3d\lambda/\lambda$ led to a precisely linear relation,

²⁹ The most recent discussion of the relation, in the form $\log v = 0.2(m - \Delta m) + c$, is found in *Mt. W. Contr.*, No. 549; *Ap. J.*, **84**, 270, 1936.

the larger corrections, $4d\lambda/\lambda$, which should be used if red shifts are assumed a priori to be velocity shifts, would evidently introduce departures from linearity. The data are now sufficient to demonstrate the existence of such departures, although the numerical values are

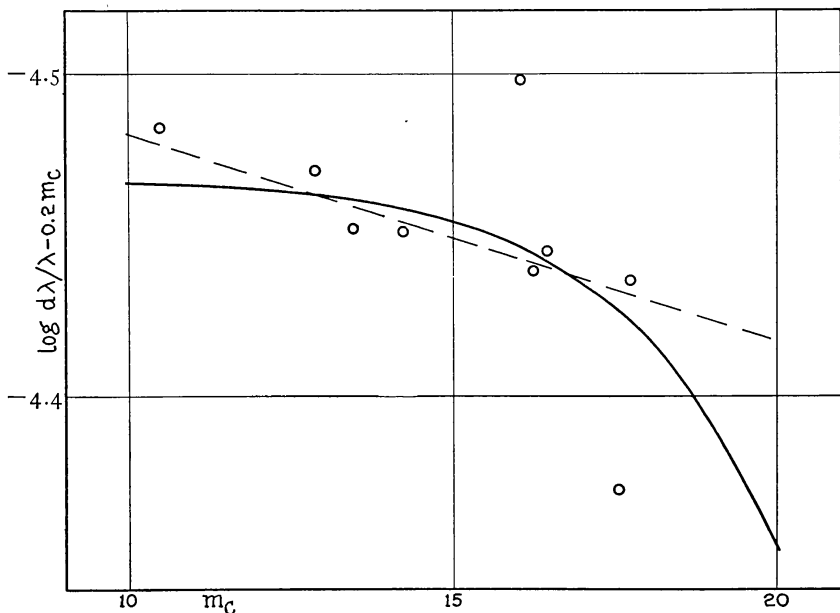


FIG. 2.—Departures from linearity in the velocity-distance relation when red shifts are interpreted as velocity shifts. The points represent mean red shifts, $d\lambda/\lambda$, and corrected magnitudes of the fifth brightest nebula in nine clusters (Table VIII). The observed magnitudes have been reduced to the galactic poles and corrected for red shifts interpreted as velocity shifts, $m_c = m_o - \Delta m_\beta - 4d\lambda/\lambda$. If the velocity-distance relation were linear (as it is when red shifts are referred to some effect other than recession, and the appropriate corrections are $3d\lambda/\lambda$), the differences, $\log d\lambda/\lambda - 0.2m_c$, would be constant, and the points in the diagram would determine a horizontal line. The systematic variation of the differences with m_c indicates the departure from linearity. The straight line corresponds to a linear velocity-magnitude relation, $\log d\lambda/\lambda = 0.2064m_c - 4.545$, adjusted to the data, and the curved line to the power series, $d\lambda/\lambda = 1.75 \times 10^{-9}r + 2.7 \times 10^{-18}r^2$, where the distance, r , is expressed in parsecs. The two outstanding residuals refer to the two Ursa Major clusters, which have minimum weight because they are each represented by a single spectrogram of a single nebula.

still uncertain. Mean red shifts and observed magnitudes (reduced to the galactic poles) of the fifth brightest nebulae in nine clusters are listed in Table VIII. The significant correlation is between $d\lambda/\lambda$ and the corrected magnitudes, $m_c = (m_o - \Delta m_\beta) - 4d\lambda/\lambda$.

Departures from the linear velocity-distance relation, $\log d\lambda/\lambda - 0.2 m_c = C$, are shown in Figure 2, where C is plotted against m_c . The straight line corresponds to a linear velocity-magnitude relation,

$$\log \frac{d\lambda}{\lambda} = 0.2064m_c - 4.545.$$

The curved line represents the expression of red shifts as a power series of the distance,

$$\frac{d\lambda}{\lambda} = 1.75 \times 10^{-9}r + 2.7 \times 10^{-18}r^2,$$

where r is expressed in parsecs.³⁰ The two outstanding residuals refer to the two Ursa Major clusters which have minimum weight.

The power series will be adopted in the following discussions because it conforms with the notation in theoretical investigations of expanding models of the universe. Since the second-order term is definitely positive, the possible models are restricted to those in which the rate of expansion has been diminishing during the past several hundred million years. The numerical value of the coefficient is rather narrowly limited (probably between 2.4 and 3.4×10^{-18}) over the range of the observed clusters, and extrapolations to the limit of the deepest survey (about $m_c = 18.8$ in Fig. 2) should not introduce serious errors. Further extrapolations rapidly become very uncertain. In view of the importance of the second-order term for theoretical investigations, it may be emphasized that the empirical value is rather precisely the effect of recession which has been introduced as an additional assumption not necessarily demanded by the observations themselves.

HOMOGENEOUS UNIVERSE OF GENERAL RELATIVITY

A detailed comparison may now be made between observational results and world-pictures for homogeneous models of the universe obeying the relativistic laws of gravitation. The principles and

³⁰ Since the mean absolute magnitude of the fifth nebula in clusters is about $\bar{M}_{pg} = -16.45$, the distances of the clusters in Table VIII are $\log r = 0.2 m_c + 4.29$. For isolated nebulae, where $\bar{M} = -15.15$, $\log r = 0.2 m_c + 4.03$. The first term in the power series is essentially that derived in the earlier investigation from resolved nebulae in the general field.

methods have been discussed in a preliminary contribution³¹ by Tolman and the writer, where the theoretical formulae derived by Tolman³² were adapted for application to observational data. It was found that the number of nebulae, N , to the observed limiting magnitude, m_0 , would be given by the relation

$$\log N = 0.6(m_0 - \Delta m) + F + C,$$

where

$$\Delta m = 5 \log \left(1 + \frac{d\lambda}{\lambda} \right) + K$$

or

$$\Delta m = 2.5 \log \left(1 + \frac{d\lambda}{\lambda} \right) + K,$$

depending upon whether or not red shifts are velocity shifts measuring the rate of expansion of the model. Further,

$$F = \log \left[\frac{3}{2x^3} (\sin^{-1} x - x\sqrt{1-x^2}) \right],$$

where $x = r/R_0$ represents the distance, r , expressed as a fraction of a constant, R_0 , commonly identified as the radius of curvature of space. In expanding models, R_0 refers to a particular epoch.

TABLE IX
EFFECTS OF SPATIAL CURVATURE EXPRESSED AS
MAGNITUDE INCREMENTS

r/R_0	$F/0.6$	r/R_0	$F/0.6$
0.05.....	0.001	0.55.....	0.075
.1.....	.002	.6.....	.092
.15.....	.005	.65.....	.112
.2.....	.010	.7.....	.135
.25.....	.014	.75.....	.162
.3.....	.020	.8.....	.196
.35.....	.028	.85.....	.238
.4.....	.037	.9.....	.292
.45.....	.048	0.95.....	.373
0.5.....	0.061	1.0.....	0.620

³¹ *Mt. W. Contr.*, No. 527; *Ap. J.*, **82**, 302, 1935. The discussion includes a preliminary statement of the observational findings, which are now presented in detail.

³² *Relativity, Thermodynamics and Cosmology*, Clarendon Press, 1934.

The term F is equivalent to a magnitude increment, $F/0.6$, the values of which are listed in Table IX for successive values of $x = r/R_0$. An inspection of the table indicates that the effects of curvature may be neglected for values of x less than about one-third and that they do not become important until x is greater than one half. Moreover, for $x = 0.9$ and greater, the effects dominate the apparent distribution. These features suggest caution in resorting to curvature to account for observed residuals. No significant contributions can be expected unless R_0 is of the same general order as the penetrating power of existing telescopes.

TABLE X
RED SHIFTS NOT VELOCITY SHIFTS
 $\log N - 0.6 (m_0 - 3d\lambda/\lambda) = \text{Constant}$

m_0	$d\lambda/\lambda^*$	$0.6 \times$ $(m_0 - 3d\lambda/\lambda)$	$\log N$	CONSTANT		
				Cal.	Obs.	C-O
21.03	0.230	12.204	3.162	-9.042	-9.050	+0.008
20158	11.728	2.686	.042	.033	-0.009
19.4125	11.414	2.342	.072	.077	+0.005
19.0107	11.207	2.161	.046	.051	+0.005
18.47	0.086	10.927	1.886	-9.041	-9.044	+0.003

* Calculated from the velocity-magnitude relation, $\log v = 0.2 (m - \Delta m) + 0.77$, from which $5 \log d\lambda/\lambda + 3d\lambda/\lambda = m_0 - 23.535$.

Case I. Red shifts not velocity shifts.—The model employed is Einstein's static universe which, for the purpose, is equivalent to an expanding model whose rate of expansion is inappreciable. The data in Table X show that the model is thoroughly consistent with the observations ($B = 3.0$, as compared with 2.94), provided curvature is negligible. The comparison is based on $T_0 = 6000^\circ$; but, as previously mentioned, the uncertainties permit a considerable range in T_0 .

Case II. Red shifts are velocity shifts and measure the rate of expansion of the universe.—If curvature is negligible, these models are definitely inconsistent with the observations. The values of B are 4.0 (calculated) and 2.94 (observed). The residuals, $C - O = d\lambda/\lambda$ (approximately), range up to 0.25 mag. and are equivalent to the

number effects introduced by the assumption that red shifts measure actual motion.

If the discrepancies are due to curvature,³³ the effects must be just sufficient to balance the number effects. The curvature must be real and positive, and very great—the model must be closed and small. The calculation of R_0 starts from the value, $C = -9.05$, derived from the direct, least-squares solution of the empirical data, supported by the Harvard survey of bright nebulae and consistent with the static model with negligible curvature. The empirical corrections, Δm_0 , are then compared with the corrections for red shifts, calculated from the relations

$$\Delta m_c = \frac{4d\lambda}{\lambda},$$

$$\frac{d\lambda}{\lambda} = 1.75 \times 10^{-9}r + 2.7 \times 10^{-18}r^2,$$

$$\log r = 0.2(m - \Delta m_c) + 4.03.$$

The differences, $\Delta m_c - \Delta m_0$, which are included in Table XI, are the residuals which must be compensated by curvature.

The next step is to plot the curvature increments, $F/0.6$ (Table IX), as a function of $\log r/R_0$. The curve is shown in Figure 3. The range in $\log r$ for the limits of the various surveys (Table XI) is about 0.40. Therefore, the curve in Figure 3 may be searched for some range of the order of 0.40 in $\log r/R_0$ which will furnish the desired series of corrections, $F/0.6$.

No segment of the curve will fit the tabulated residuals exactly. Nevertheless, a segment does exist, $\log r/R_0 = -0.11$ for the deepest survey, which furnishes increments about equal to the residuals *minus* a constant, 0.075 mag. This fit (Table XI), which is rather narrowly limited, may be accepted as a possible solution on the assumption that the small constant, 0.075 mag., reflects a considerable

³³ Since the formulae for the world-pictures were derived by means of co-moving space-like co-ordinates, with respect to which the nebulae have no relative systematic motion, the differences in time required for light to reach the observer from the limits of the various surveys is already taken into account (*Mt. W. Contr.*, No. 527; *Ap. J.*, 82, 302, 1935).

error in the zero point of the magnitude scale (true magnitudes brighter than observed magnitudes).

TABLE XI
OBSERVATIONS CORRECTED FOR SPATIAL CURVATURE
IN AN EXPANDING UNIVERSE

m_c	$d\lambda/\lambda$	Δm_c^*	Δm_o	$D = \Delta m_c - \Delta m_o$	$\log r \dagger$	$F/0.6 \ddagger$	$D - F/0.6$
21.03....	0.231	0.924	0.676	0.248	8.051	0.180	0.068
20.....	.158	.630	.468	.162	7.904	.077	.085
19.4....	.124	.500	.368	.132	7.810	.047	.085
19.....	.106	.423	.314	.109	7.746	.034	.075
18.47....	0.085	0.338	0.253	0.085	7.656	0.022	0.063
Mean..	0.075

* $\Delta m_c = 4 \, d\lambda/\lambda$.
† $\log r$ (in parsecs) refers to the observed limiting magnitude of the surveys.
‡ Corrections for curvature are taken from Table VIII on the assumption that $\log r/R_0 = -0.11$ for the limit of the deepest survey ($m_o = 21.03$).

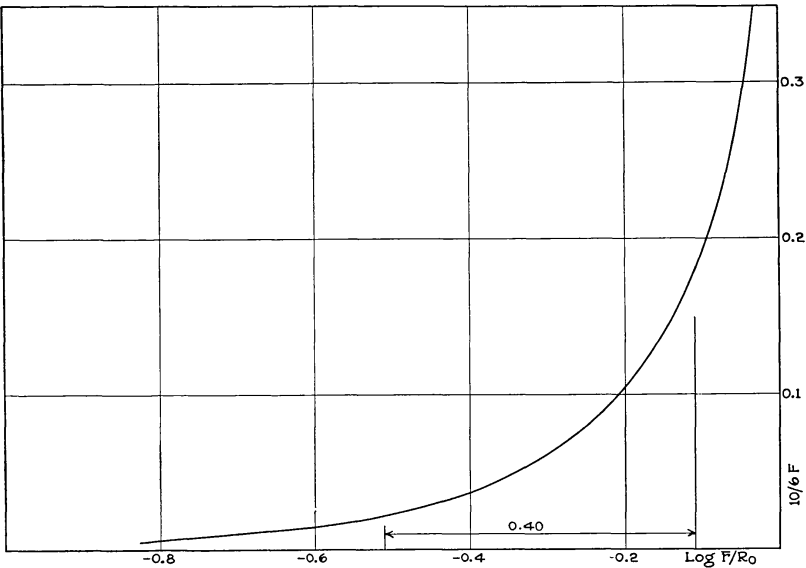


FIG. 3.—Corrections for spatial curvature. The curve shows the corrections to nebular counts, expressed as increments to the limiting magnitudes, required by spatial curvature whose radius is R_0 , in a homogeneous universe obeying the relativistic laws of gravitation. The magnitude increments (Table IX) are exhibited as functions of $\log r/R_0$. The section from $\log r/R_0 = -0.11$ to -0.51 has been selected as furnishing the best fit with the observational results when red shifts are interpreted as velocity shifts measuring the rate of expansion of the universe. The corresponding value of R_0 (at the present epoch) is 4.7×10^8 light-years.

The positive curvature, as previously mentioned, indicates a closed, finite universe. The radius, R_0 , would be of the order of 1.45×10^8 parsecs or 4.7×10^8 light-years—just less than the estimated mean distance of the faintest nebulae recorded with the 100-inch reflector under optimal conditions. If the radius of curvature were of this order, it would follow that we now observe a large fraction of the universe.³⁴

The mean density of the model would be of the general order of 10^{-26} gm/cm³. Any significant revision of the empirical data would probably reduce R_0 and, consequently, increase the density. Thus the validity of the solution, as a representation of the actual uni-

³⁴ The spatial curvature and the two terms in the power series for red shifts determine a particular model of the universe among the many homogeneous expanding models which obey the relativistic laws of gravitation. Tolman's calculations of the possible ranges within which the various constants must lie (*Relativity, Thermodynamics and Cosmology*, pp. 472-474) can now be pushed to a much closer approximation. In his notation, the unit of distance is the light-year and

$$\frac{d\lambda}{\lambda} = k\bar{r} - l\bar{r}^2 + \left(\frac{k}{6R^2} + \frac{kl}{3} + m \right) \bar{r}^3 + \dots$$

The co-moving co-ordinate, \bar{r} , is equivalent to the co-ordinate r used in the present discussion, namely, the distance derived from apparent magnitudes when the latter are corrected for red shifts. The new data give the following values of the constants:

$$k = 5.37 \times 10^{-10} \text{ (lt. yr.)}^{-1},$$

$$l = -2.54 \times 10^{-19} \text{ (lt. yr.)}^{-2}$$

$$\frac{1}{R_0^2} = 4.52 \times 10^{-18} \text{ (lt. yr.)}^{-2},$$

$$\Lambda = (4.4 \text{ to } 6.9) \times 10^{-18} \text{ (lt. yr.)}^{-2}.$$

The cosmological constant, Λ , is determined between upper and lower limits by the assumptions: (a) that pressure cannot be less than zero, and (b) that the density of matter is greater than zero. The constant m is undetermined; but if it counteracts, rather than adds to, the second-order term in the power series, it will be negative, and the numerical value will be not much greater than

$$2.7 \times 10^{-28} \text{ (lt. yr.)}^{-3}.$$

Since the model is closed and the value of Λ is evidently greater than that for a possible Einstein universe, the model is probably of the type known as a "monotonic universe of the first kind, type M_1 ." It "would be an ever-expanding type which proceeds from some singular state at $R_s \geq 0$ to the final state of an empty de Sitter universe as $R \rightarrow \infty$ " (*loc. cit.*, p. 399), in other words, the type that will always be associated with the name of Lemaître.

verse, may depend rather nicely on the question of density. In the neighborhood of the sun the mean density is believed to be of the order of $10^{-23.5}$, and is probably abnormally high because of the local cluster. Since the galactic system, by analogy with other spirals, fades outward to undefined boundaries, it is reasonable to suppose that the mean density of matter in internebular space is thousands of times less than the local density. The smoothed-out density of material concentrated in nebulae is believed to lie between 10^{-28} and 10^{-30} ; evidence from various sources³⁵ is fairly consistent within this range. There is evidently no reason for assuming a mean density of 10^{-26} unless the nebulae are imbedded in a relatively dense, pervading medium which escapes detection.

Photometric evidence is summarized by the statement that space absorption over a light-path of 3.5×10^{26} cm (limit of the deepest survey) is less than 0.1 mag., and probably less than 0.05 mag.³⁶ A density of 10^{-26} corresponds to an absorption of less than 1 mag. by about 35 (or 70) grams of matter per square centimeter cross-section. This condition immediately rules out many forms of matter (for instance, dust, meteoritic material, and highly ionized gas), but certain forms (for instance, large chunks and non-ionized gas similar to the earth's atmosphere) would still be possible. The permissible forms, however, are probably not those which would be a priori postulated for the contents of internebular space.

CONCLUSION

The observations may be fitted into either of two quite different types of universes. If red shifts are velocity shifts, the model is closed, small, and dense. It is rapidly expanding, but over a long

³⁵ The limits are set by Smith's value, Mass of the Virgo cluster \div Number of nebulae in cluster = 2×10^{11} suns per nebula (*Mt. W. Contr.*, No. 532; *Ap. J.*, **83**, 23, 1936), and the writer's value, 2×10^9 suns, derived from spectrographic rotations (*The Realm of the Nebulae*, Yale University Press, 1936). Since the present investigation leads to a density of about one nebula per 1.4×10^{17} cubic parsecs, the smoothed-out densities are 10^{-28} and 10^{-30} , respectively.

³⁶ This conclusion is necessary in order to avoid a density distribution for the nebulae which increases systematically with distance. The smaller limit is probably permissible because any appreciable absorption would increase the curvature, and hence the density, required to compensate the residuals in Table XI.

period the rate of expansion has been steadily diminishing. Existing instruments range through a large fraction of the entire volume and, perhaps, through a considerable fraction of past time since the expansion began.

On the other hand, if red shifts are not primarily due to velocity shifts, the observable region loses much of its significance. The velocity-distance relation is linear; the distribution of nebulae is uniform; there is no evidence of expansion, no trace of curvature, no restriction of the time scale. The sample, it seems, is too small to indicate the particular type of universe we inhabit.

Thus the surveys to about the practical limits of existing instruments present as alternatives a curiously small-scale universe or a hitherto unrecognized principle of nature. A definitive choice, based upon observational criteria that are well above the threshold of uncertainty, may not be possible until results with the 200-inch reflector become available.

Meanwhile, certain features of the two solutions may be emphasized. The one outstanding objection to the assumption that red shifts are not velocity shifts is, of course, the fact that no other satisfactory interpretation has yet been formulated. Nevertheless, the large-scale characteristics of nature are known only from the kind of data whose interpretation is now in question. The assumption that red shifts measure the rate of expansion of the universe is a long extrapolation from the familiar small-scale Doppler effects. Clearly, the data should be analyzed with the aid of as few assumptions as possible. The observations must be corrected for energy effects, regardless of the origin of red shifts. The interpretation of the corrected data is then direct, economical, and very simple. The observable region is homogeneous and is evidently an insignificant fraction of the universe.

The unexpected and truly remarkable features are introduced by the additional assumption that red shifts measure recession. The velocity-distance relation deviates from linearity by the exact amount of the postulated recession. The distribution departs from uniformity by the exact amount of the recession. The departures are compensated by curvature which is the exact equivalent of the

recession. Unless the coincidences are evidence of an underlying, necessary relation between the various factors, they detract materially from the plausibility of the interpretation.

The high density required by the curvature is another disturbing feature. Although certain specific forms of matter are not ruled out by the photometric data now available, it may be emphasized that the observations merely set upper limits to possible densities and otherwise have no bearing on the actual amount of matter. Dynamical arguments may eventually be developed which will be valid regardless of the form of material. Finally, the small scale of the expanding model, both in space and in time, is a novelty, and as such will require rather decisive evidence for its acceptance.

CARNEGIE INSTITUTION OF WASHINGTON

MOUNT WILSON OBSERVATORY

August 1936