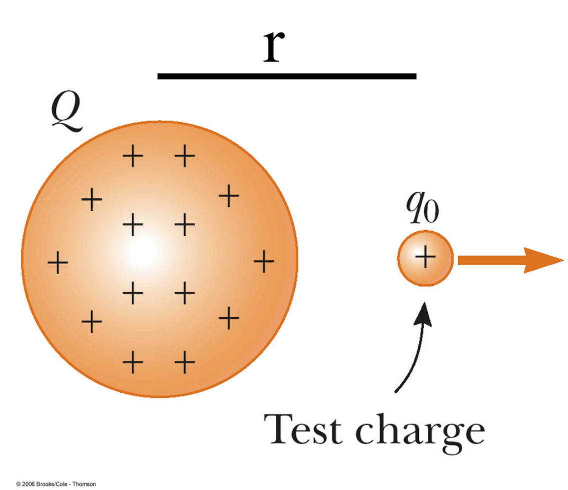
Electric Field due to a point charge

E-field exerts a force on other point charges

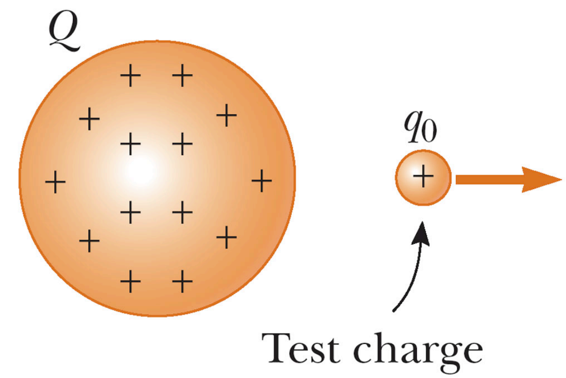
$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{\frac{k_e Q q_o}{r^2}}{q_0}$$

$$\vec{E} = \frac{k_e Q}{r^2}$$



$\vec{E}$ is a vector quantity

Magnitude & direction vary with position--but depend on object w/ charge Q setting up the field



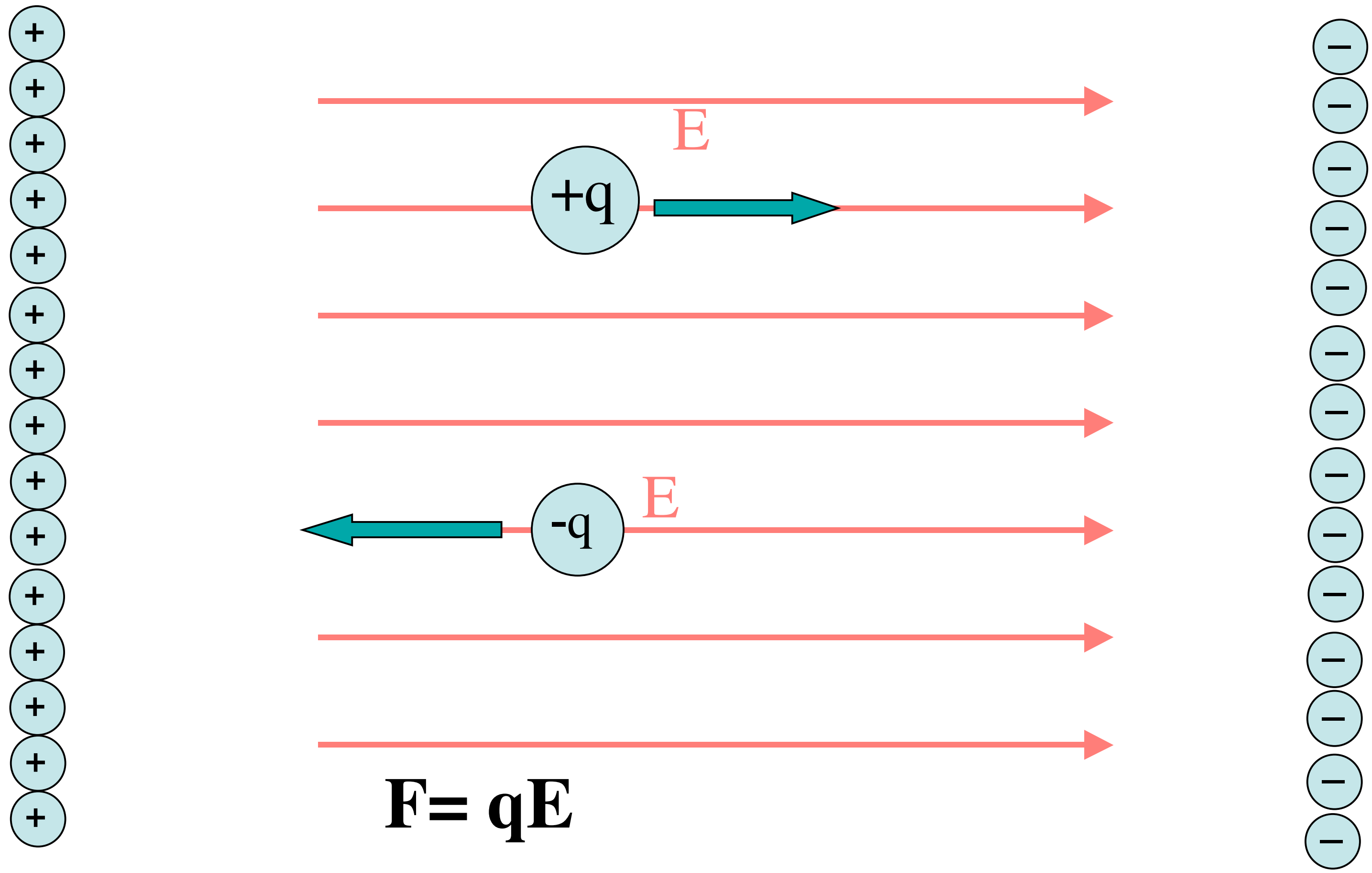
The electric field depends on Q, not q_0. It also depends on r.

If you replace q_0 with $-q_0$ or $2q_0$, the strength & magnitude of the E-field at that point in space remain the same

The electrostatic FORCE, however, depends on Q AND q_0 as well as r.

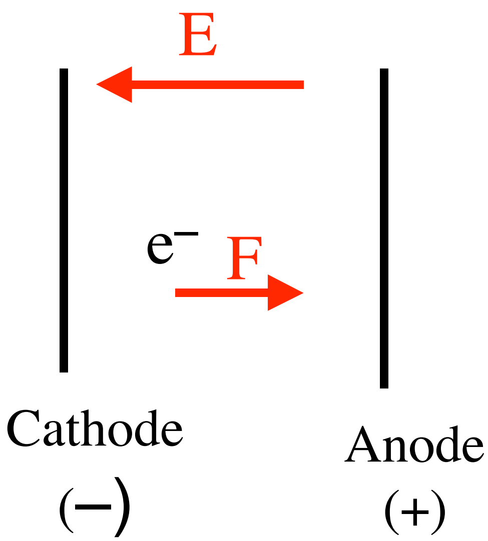
E-field exerts force on a charge

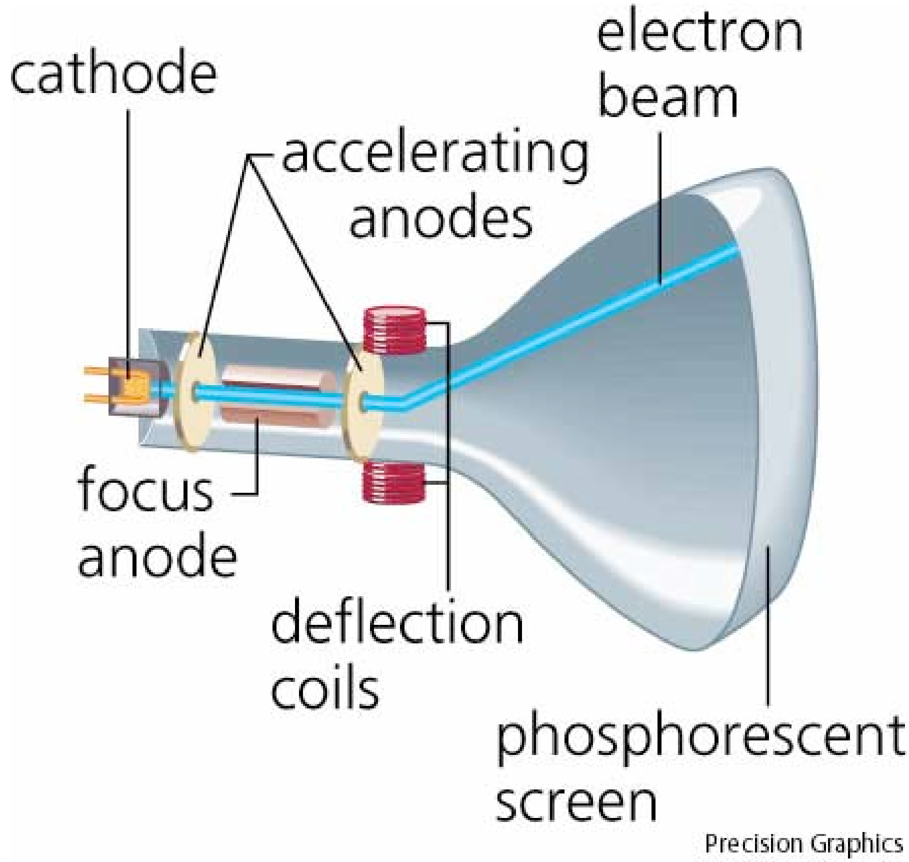
Consider an array of + charges and an array of – charges:



F= qE

Cathode Ray Tube

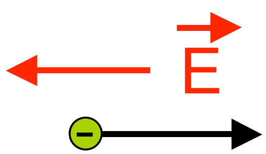




http://building.pbworks.com

Accelerating electrons in a constant E-field

A single electron is accelerated from rest in a constant electric field of 1000 N/C through a distance of 3 cm. Find the electric force on the electron, and calculate its final velocity (m_e = 9.1x10^{-31} kg)



$F = qE = m_e a$

$F = qE = (1.6\text{x}10^{-19}\ \text{C})(1000\text{N/C})$
$= 1.6\text{x}10^{-16}\ \text{N}$

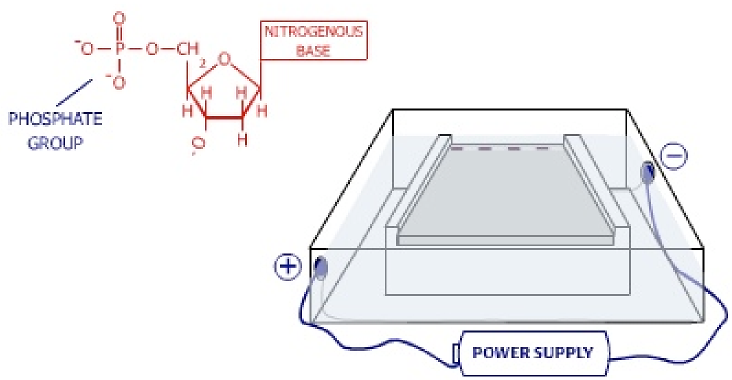
$$v^2 = v_0^2 + 2ad$$
$$\rightarrow v = \sqrt{2ad} = \sqrt{2(F/m_e)d} = \sqrt{2(qE/m_e)d}$$

$$v = \sqrt{\frac{2(1.6\times10^{-19}C)(1000N/C)0.03m}{9.1\times10^{-31}kg}}$$
$$v = 3.2 \times 10^6\ \text{m/s}$$

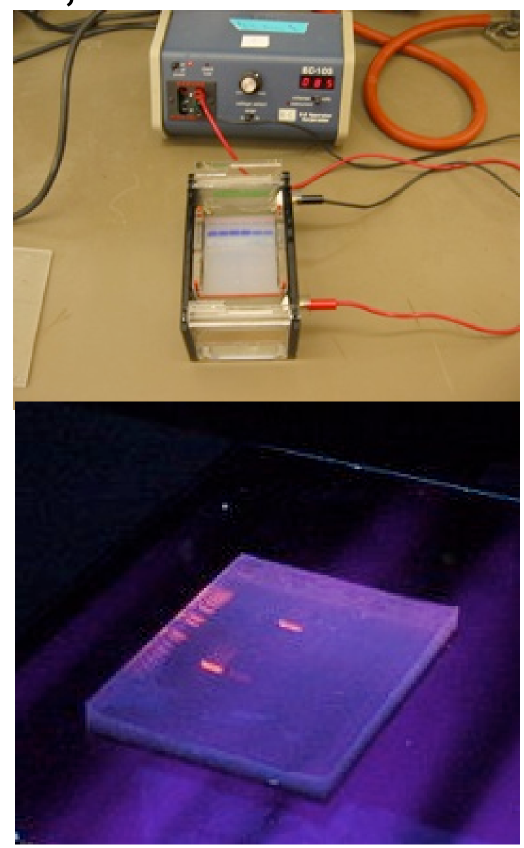
Electrophoresis

Separation of DNA segments (q ~ –1000 e due to O^{-}'s in phosphate backbone of DNA chain) in an E-field ~ 1000 N/C.

Moves through pores in gel towards anode; smaller segments travel further

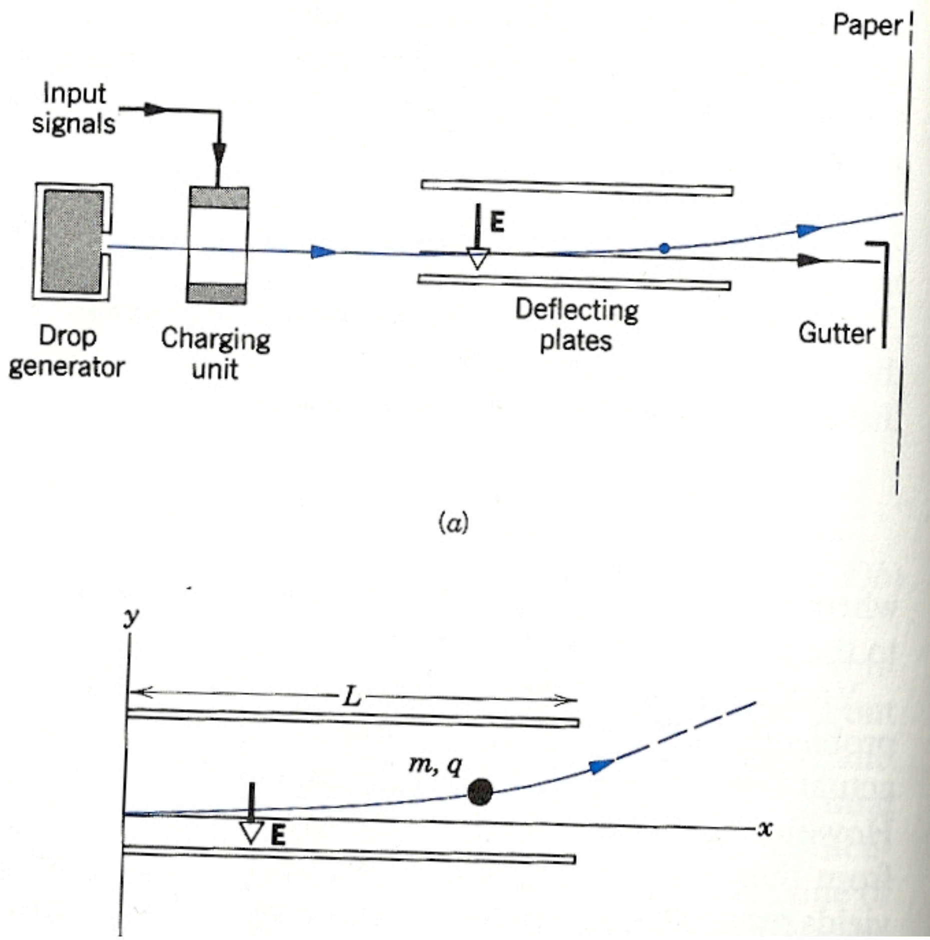


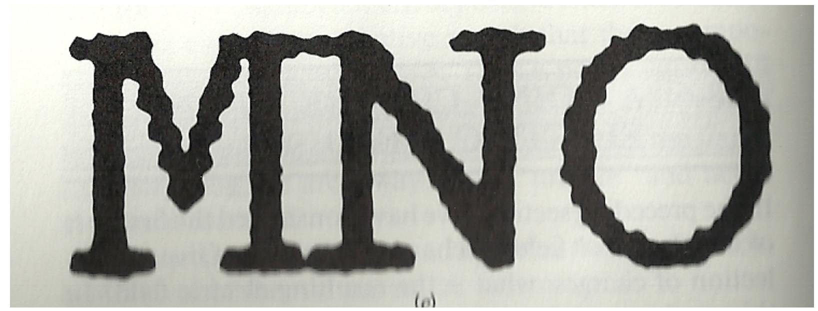
Source: http://dnalc.org



http://web.mit.edu/7.02/virtual_lab/RDM/RDM1virtuallab.html

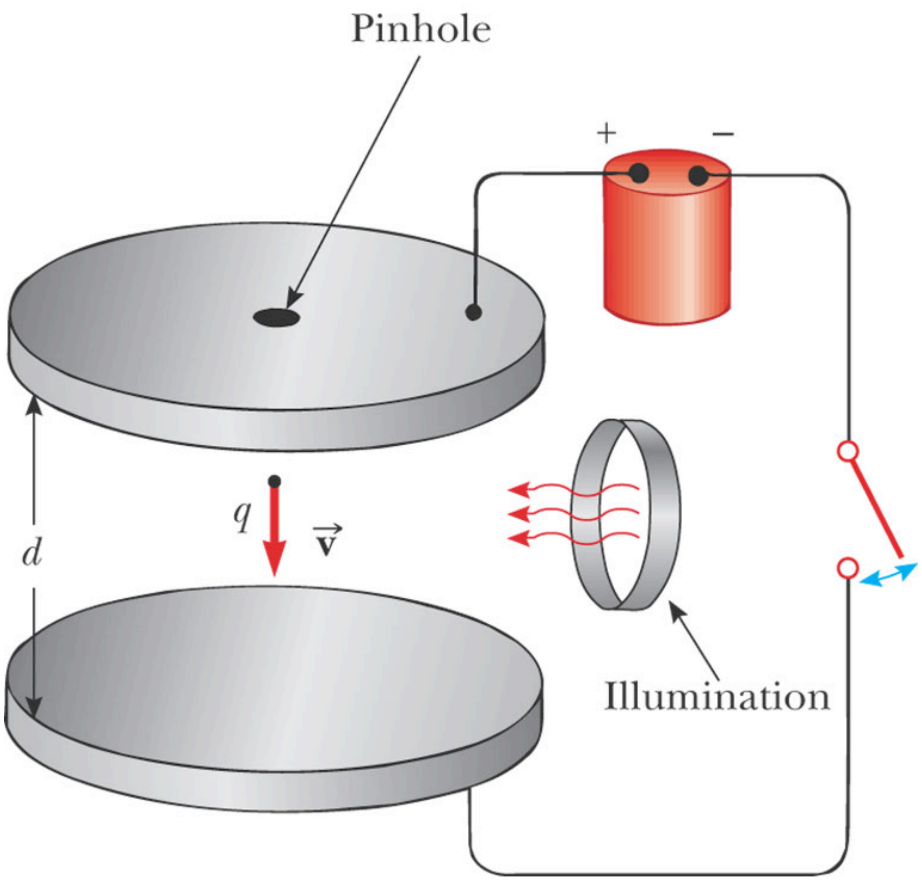
Application: Ink-jet printers

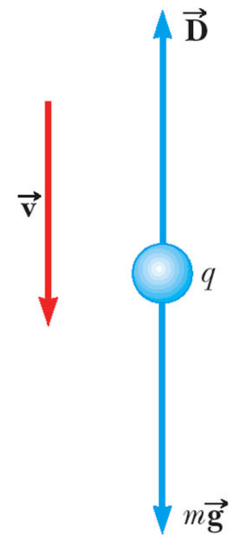




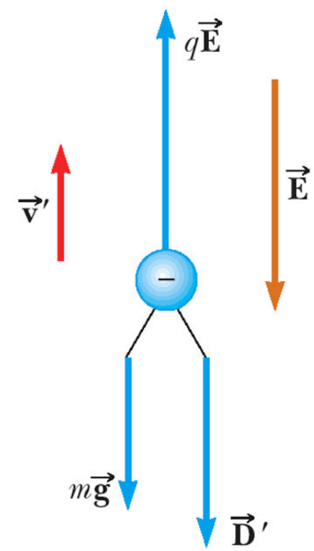
Tiny drop of ink is shot through charging unit, where a negative charge (typ. ~ –1000e) is applied. An E-field is then applied to deflect the drop through the proper angle.

Millikan’s Oil Drop Experiment





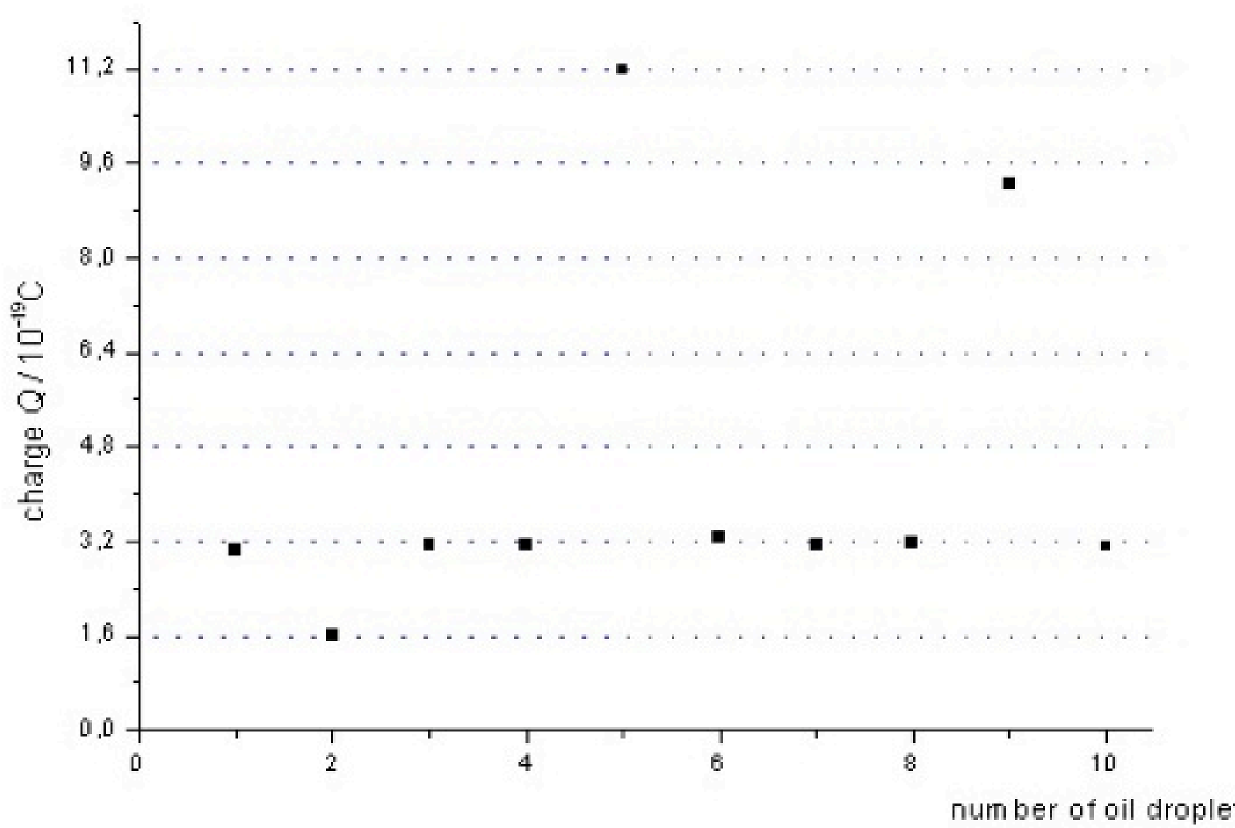
(a) Field off



(b) Field on

Millikan's Oil Drop Experiment

Every droplet contained an amount of charge equal to 0e, ±1e, ± 2e, ±3e,….



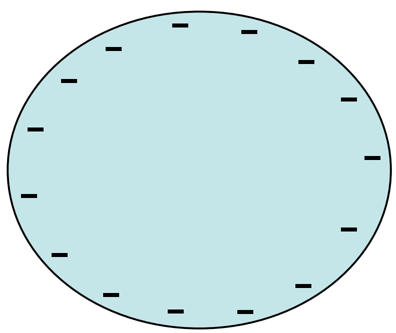
Conductors in Electrostatic Equilibrium

Like charges repel and can move freely along the surface.

In electrostatic equilibrium, charges are not moving

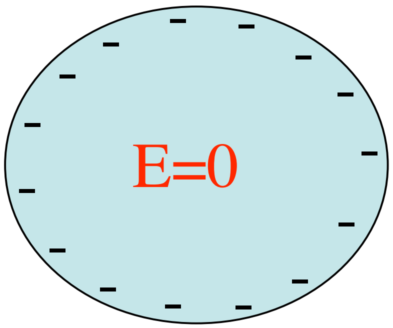
4 key properties:

1: Charge resides entirely on its surface (like charges move as far apart as possible)

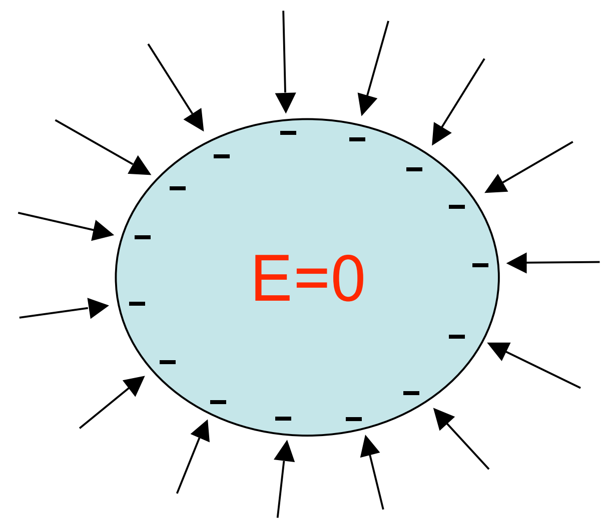


2: Inside a conductor, E-field is zero

(if there are charges, an E-field is established, and other charges would move, and conductor wouldn't be at equilibrium)

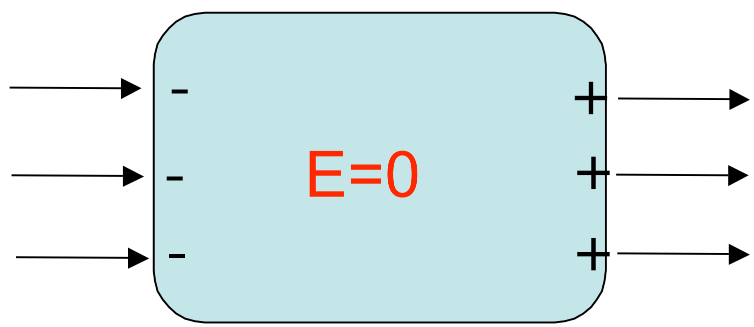


2: Inside a conductor, E-field is zero



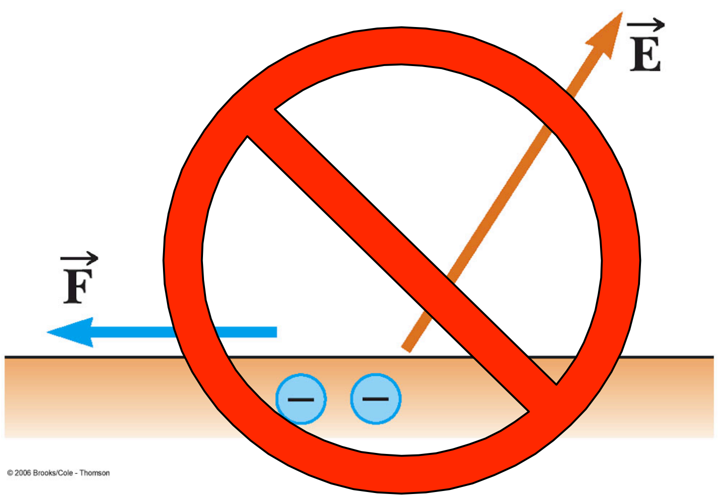
True for a conductor with excess charge

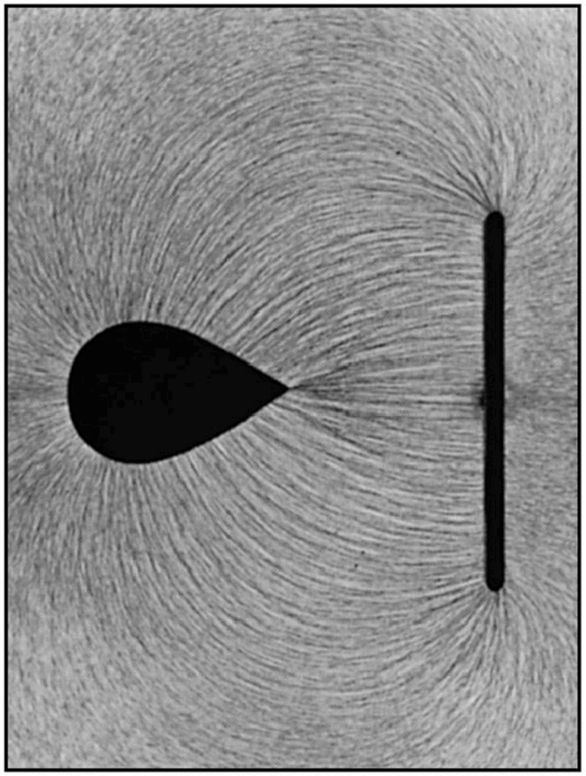
And for a conductor in an external E-field:



3: E-field just outside the conductor is perpendicular to its surface

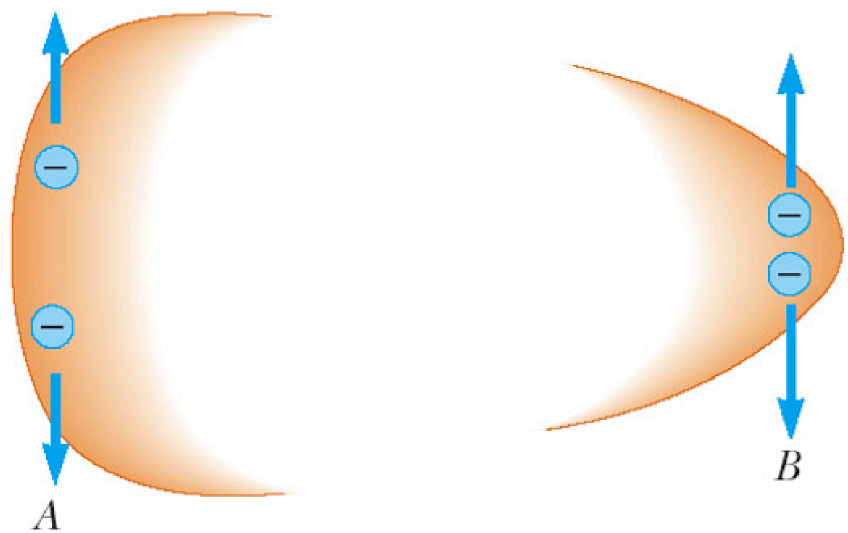
Any non-perpendicular component would cause charges to migrate, thereby disrupting equilibrium





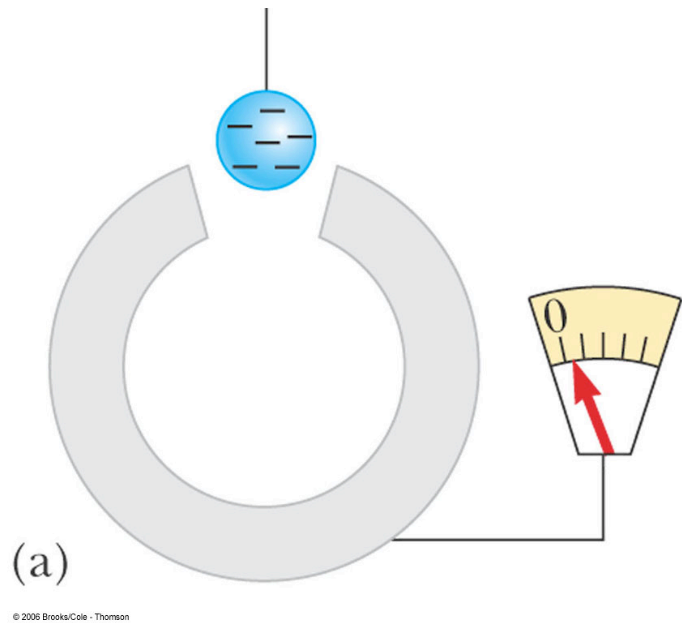
4: Charges accumulate at sharp points (smallest radius of curvature)

Here, repulsive forces are directed more away from surface, so more charges per unit area can accumulate



Faraday's "ice-pail" experiment

In a conductor: free charges reside on its surface

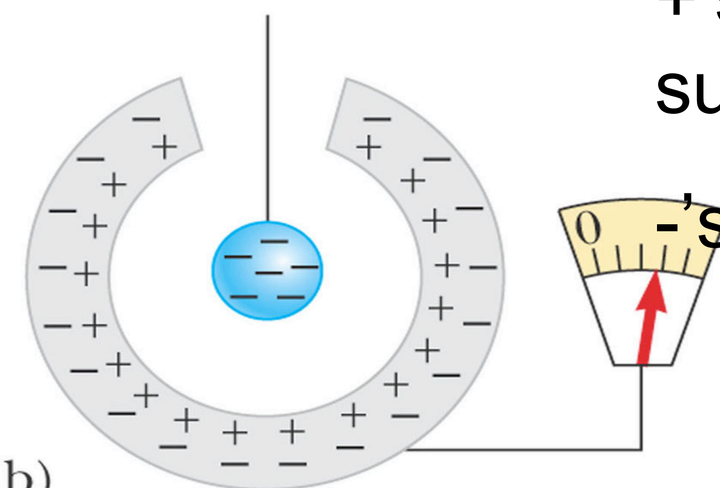


Electrometer attached to OUTER surface: measures amount of charge on outer surface

Metal ice-pail: insulated from ground

Faraday's "ice-pail" experiment

In a conductor: free charges reside on its surface



(b)

+'s attracted to inner surface

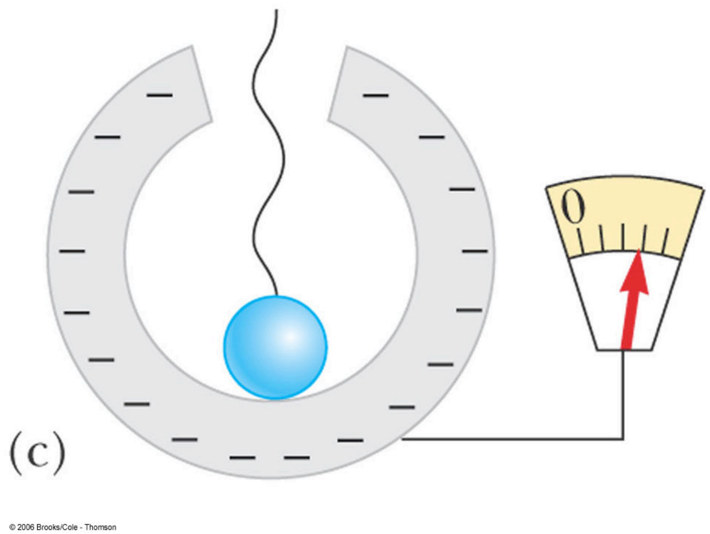
-'s repelled to outer surface

Charge on outer surface is same sign as charge on metal ball

Faraday's "ice-pail" experiment

In a conductor: free charges reside on its surface

CONTACT:



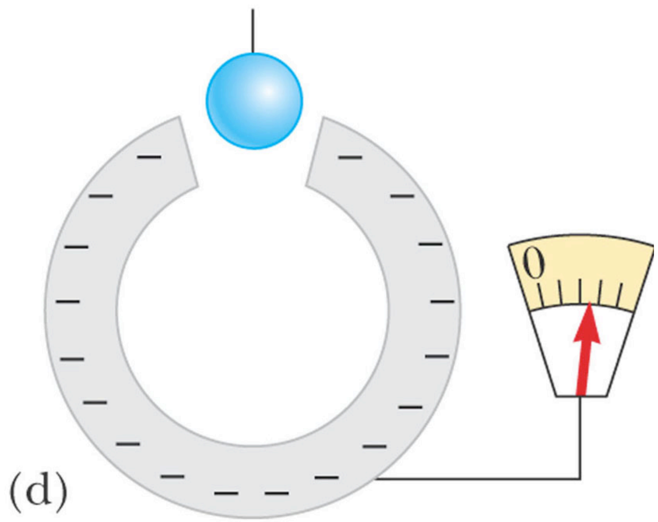
Needle on electrometer does not move!

Negative charge on ball and positive charge on inner surface neutralize each other

Faraday's "ice-pail" experiment

Remove ball:

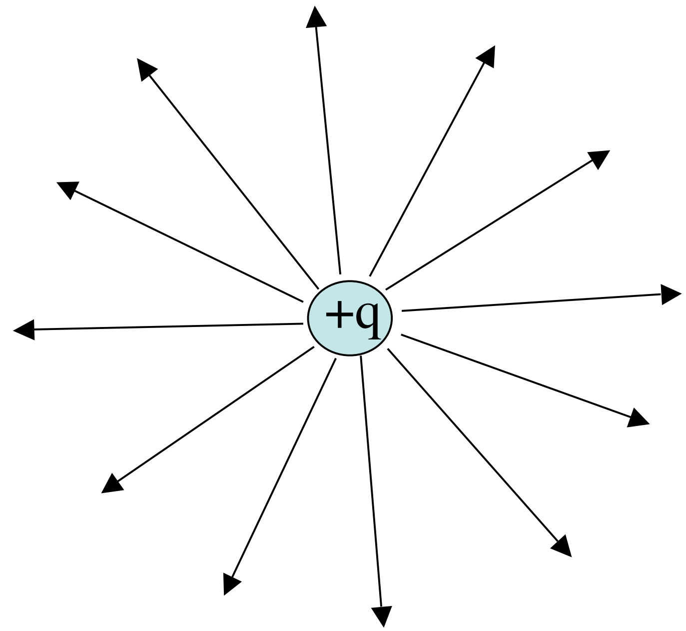
Needle on electrometer still does not move!



Conductors in Electrostatic Equilibrium

Suppose you had a point charge +q. You surround the charge with a conducting spherical shell.

What happens?

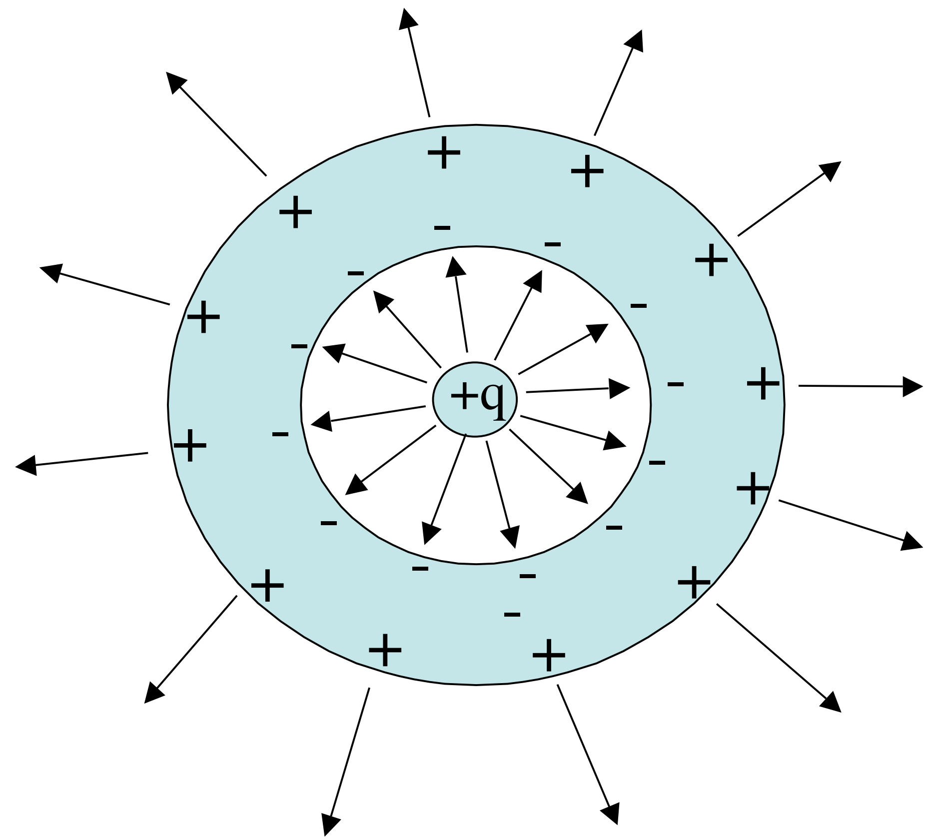


Conductors in Electrostatic Equilibrium

-'s accumulate on inner surface. +'s accumulate on outer surface

E-field within conductor is zero

From very far away, field lines look exactly as they did before

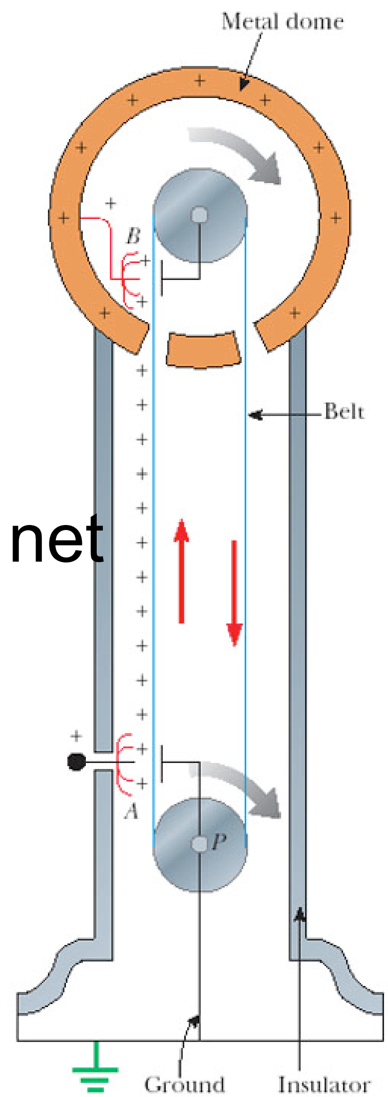


15.8 Van de graff Generators

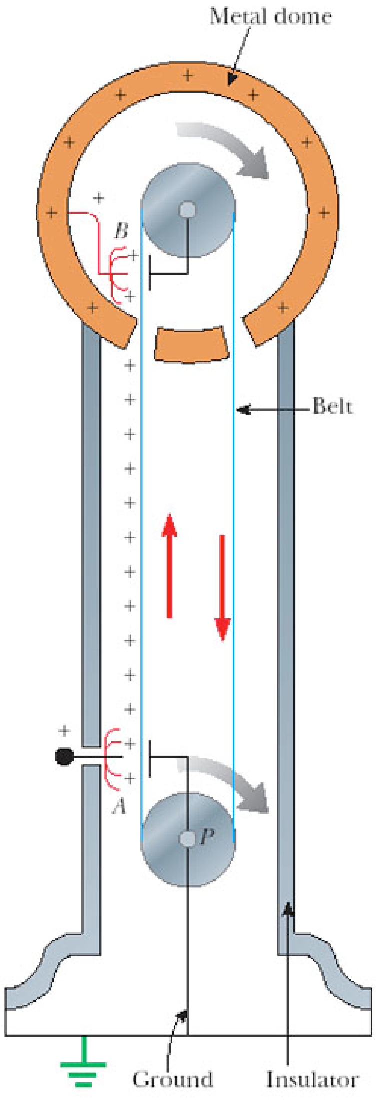
Positive charges transferred to conducting dome, accumulate, spread out

Left side of belt has net positive charge

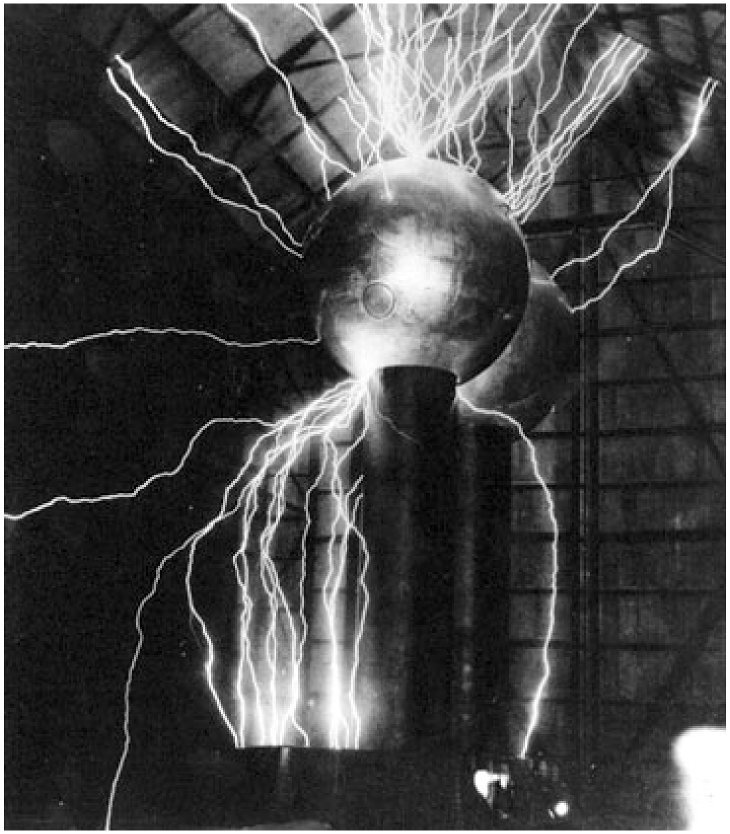
Positively-charged needles in contact w/ belt: pulls over e$^-$'s



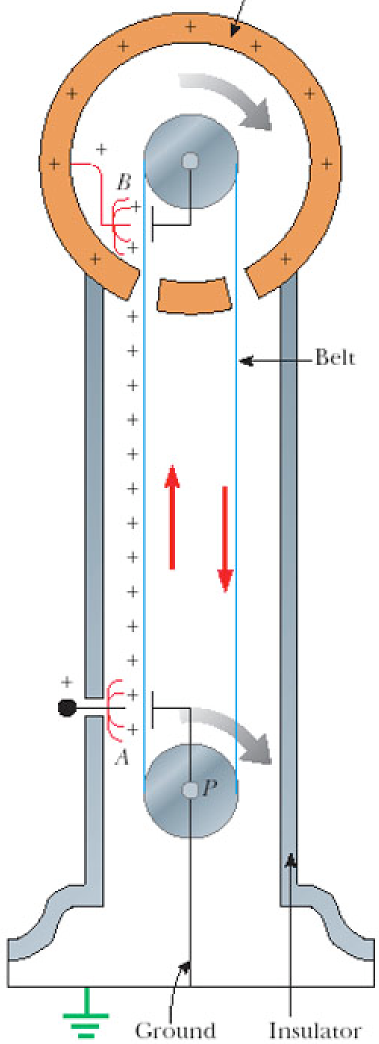
Positive charges transferred to conducting dome, accumulate, spread out



E-field eventually gets high enough to ionize air & increase its conductivity-- get mini-lightning bolts



Boston Museum of Science / M.I.T.



The electric field strength needed to ionize air and allow it to conduct electricity is 3×10^6 N/C

The maximum charge that can be accumulated on the dome WITHOUT having electrical discharge in the vicinity of the dome can be calculated via

$E_{max,VdG} = 3 \times 10^6 \text{ N/C} = k_e Q / r^2$

where r is the radius of the dome

VdG generator at Boston Museum of Science
(largest air-insulating VdG in the world):
lightning travels along outside of operator's conducting cage:
http://www.youtube.com/watch?v=PT_MJotkMd8
(fast forward to ~1:10)

Another example of a Faraday cage:
(Tesla coil, not VdG generator, used to generate the lightning):
http://www.youtube.com/watch?v=Zi4kXgDBFhw

Boston M.O.S. VdG demonstrations
http://www.youtube.com/watch?v=TTPBDkbiTSY
http://www.youtube.com/watch?v=rzbEPcD-DKM

15.9 Electric Flux & Gauss' Law

OVERVIEW:

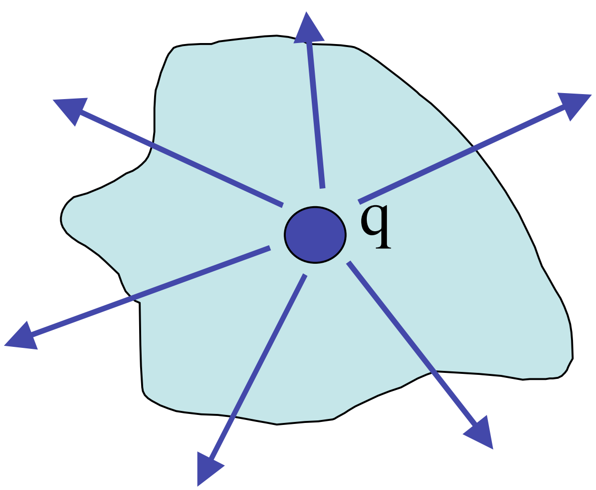
Gauss' Law: relates electric fields and the charges from which they emanate

Technique for calculating electric field for a given distribution of charge

Relates the total amount of charge to the "electric flux" passing through a closed surface surrounding the charge(s).

Electric Flux

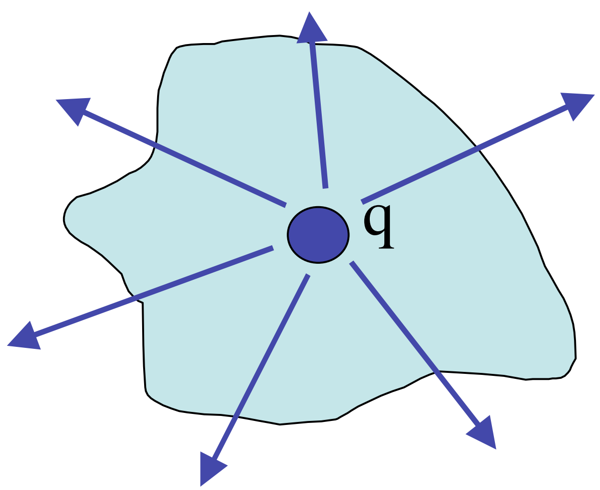
Reminder: Total number of field lines prop. to total charge. Density of E field lines in a given part of space is prop. to magnitude of $\vec{E}$



Electric Flux

Reminder: Total number of field lines prop. to total charge. Density of E field lines in a given part of space is prop. to magnitude of $\vec{E}$

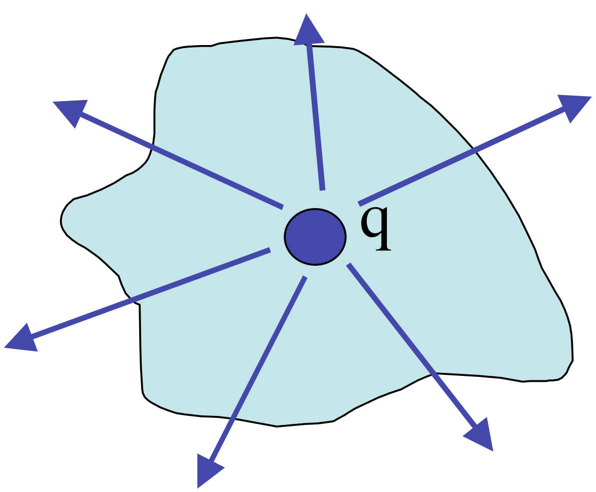
Electric flux: a measure of how much electric field vectors penetrate a given surface



Electric Flux

Reminder: Total number of field lines prop. to total charge. Density of E field lines in a given part of space is prop. to magnitude of $\vec{E}$

Electric flux: a measure of how much electric field vectors penetrate a given surface

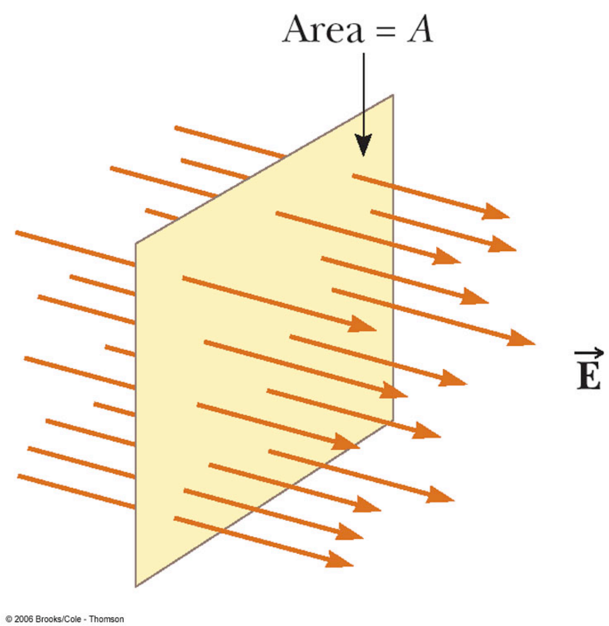


Gauss' Law (qualitative): Surround the charge by a closed surface. The density of E-field lines at the surface can be related to the enclosed charge

Electric Flux Φ_E

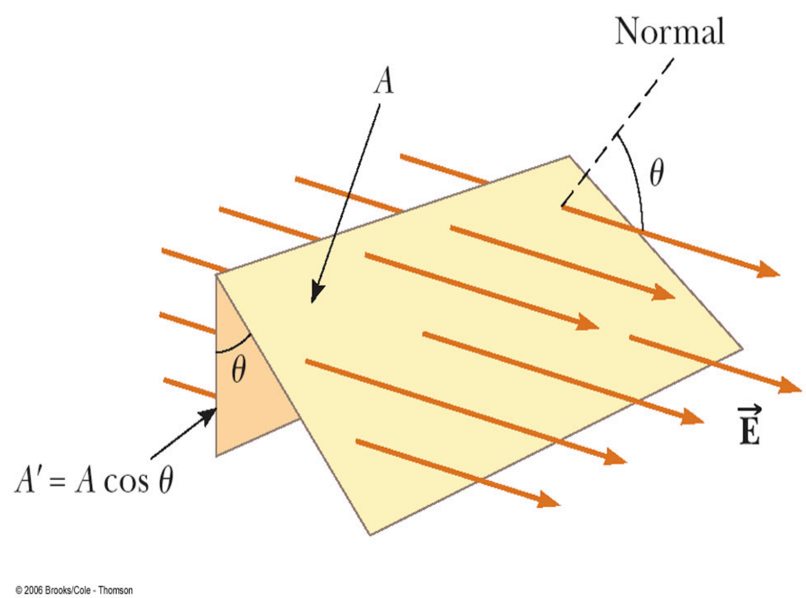
Consider a uniform E-field and an area A ⊥ to E-field lines:

$$\Phi_E = E\,A$$



If E-field lines make angle θ to normal of plane:

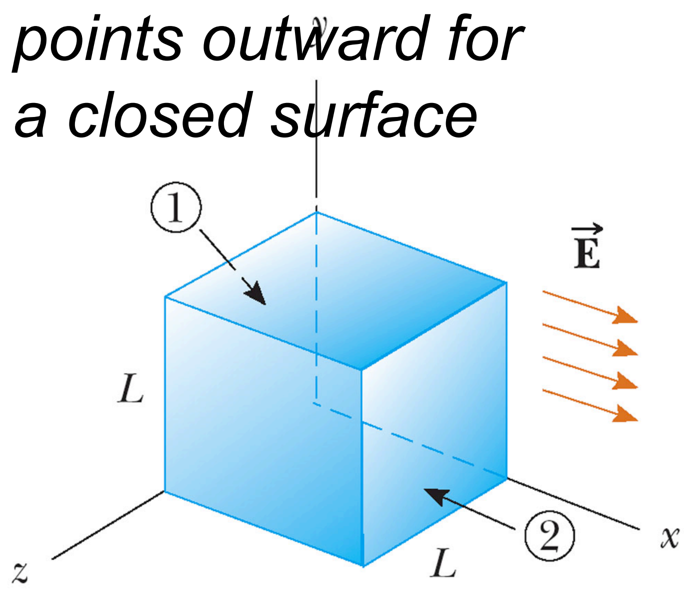
$$\Phi_E = E\,A\cos\theta$$



Electric Flux Φ_E Through a Cube

Uniform E-field parallel to x-axis: What's the net elec. flux Φ_E through the cube?

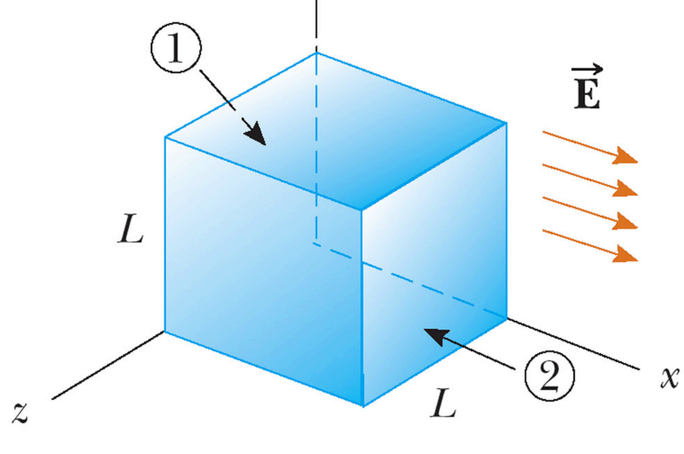
Normal vector points outward for a closed surface



Electric Flux Φ_E Through a Cube

Uniform E-field parallel to x-axis: What's the net elec. flux Φ_E through the cube?

Normal vector points outward for a closed surface



$$\Phi_E = E\ A \cos\theta$$

Top & Bottom:

$$\Phi_E = E\ A \cos(90°) = 0$$

Each side:

$$\Phi_E = E\ A \cos(90°) = 0$$

Surface2:

$$\Phi_E = E\ A \cos(0°) = +EL^2$$

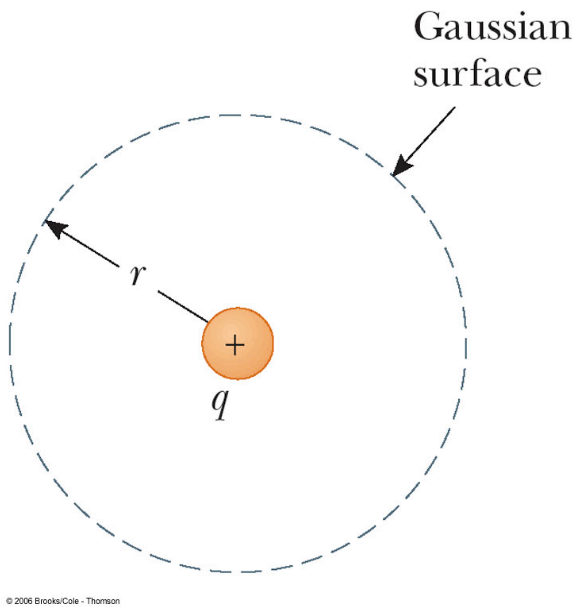
Surface1:

$$\Phi_E = E\ A \cos(180°) = -EL^2$$

Net $\Phi_E = 0+0+0+0+ EL^2 - EL^2 = \mathbf{0}$

The net electric flux through any closed surface will be zero if there is no charge enclosed inside!

Gauss' Law



At radius r: $E = \frac{k_e q}{r^2}$

$\Phi_E = E \times Area = \frac{k_e q}{r^2} \times (4\pi r^2)$

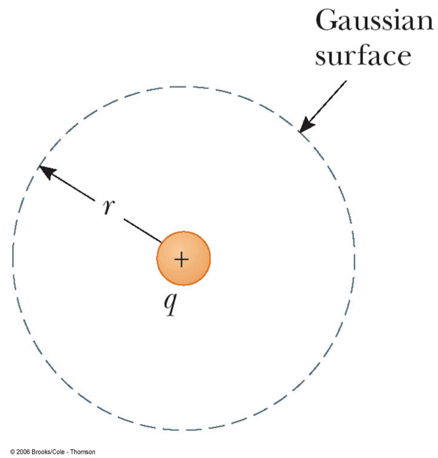
Define $\epsilon_0 = \frac{1}{4\pi k_e} = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

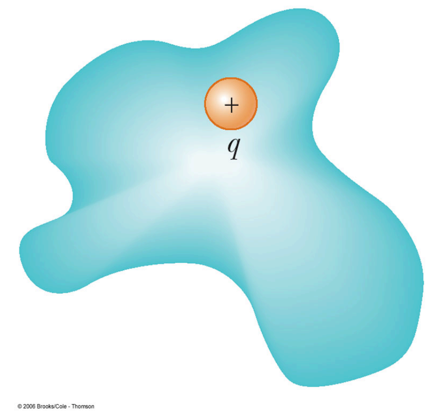
ϵ_0 = permittivity of free space

$$\Phi_E = Q_{encl} / \varepsilon_0$$

Φ_E through any closed surface is equal to the net charge enclosed, Q_{encl}, div. by ε_0

Gauss' Law





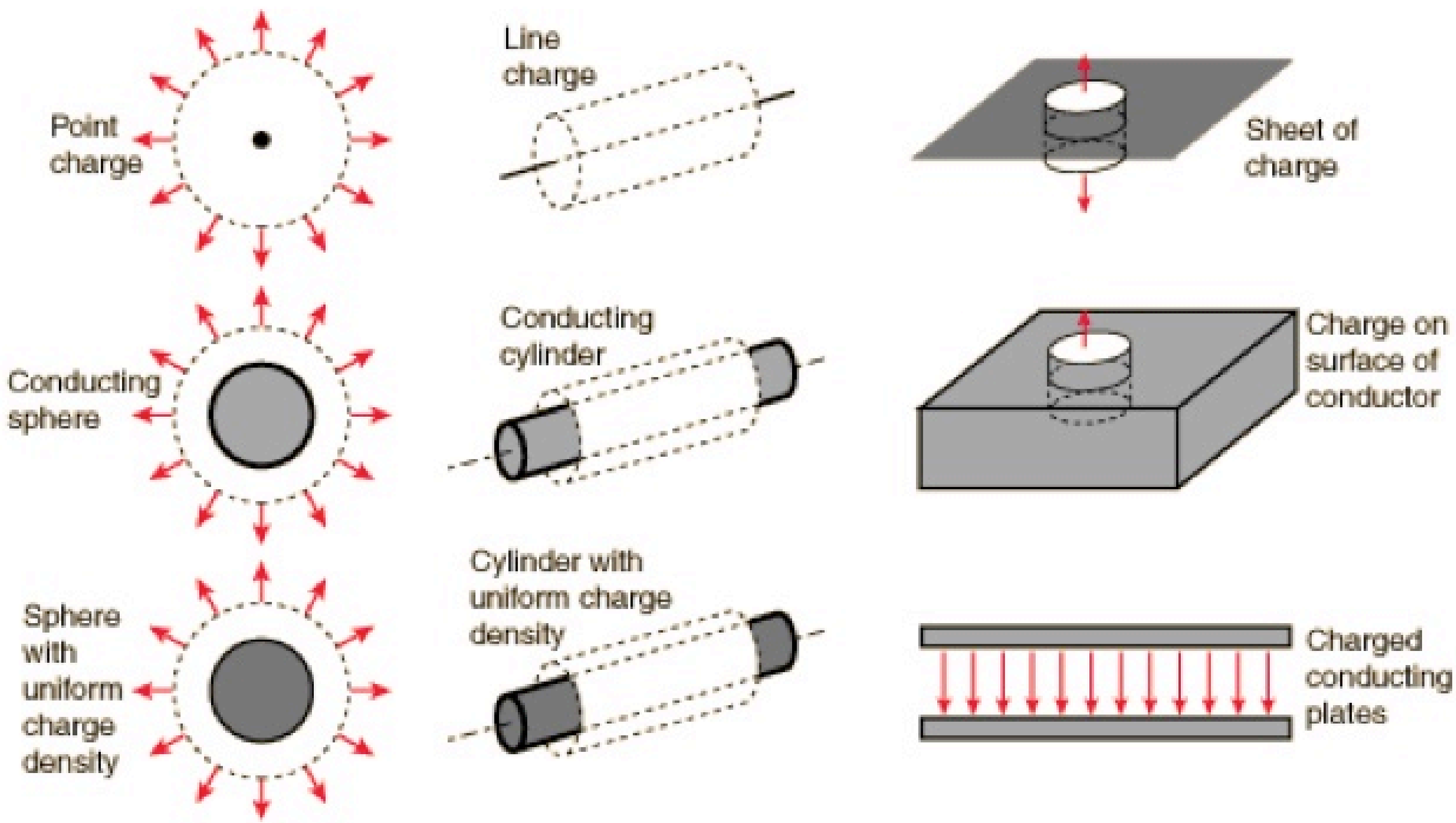
Gauss' Law: describes how charges create electric fields

Gaussian surfaces: not a real surface -- does not have to coincide with the surface of a physical object

Φ_E does not depend on radius of sphere: just the charge enclosed ($1/r^2$ dependence of E cancelled by r^2 dependence of A)

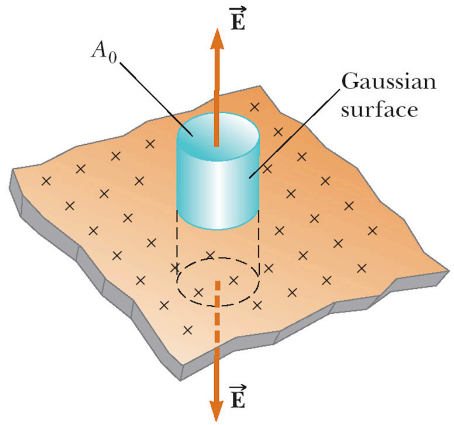
Sample Gaussian surfaces

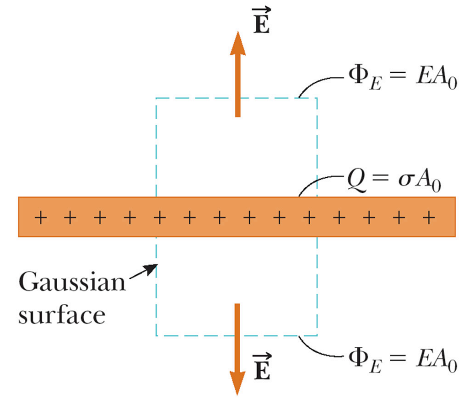
Hint: Choose surfaces such that $\vec{E}$ is ⊥ or || to surface!



Gauss' Law: A sheet of charge

Define σ = charge per unit area





$$\Phi_E = EA = Q_{encl}/\epsilon_0$$

A = area of top + bottom surfaces = 2 A_0

$$Q_{encl} = \sigma A_0$$

$$EA = \frac{\sigma A_0}{\epsilon_0}$$

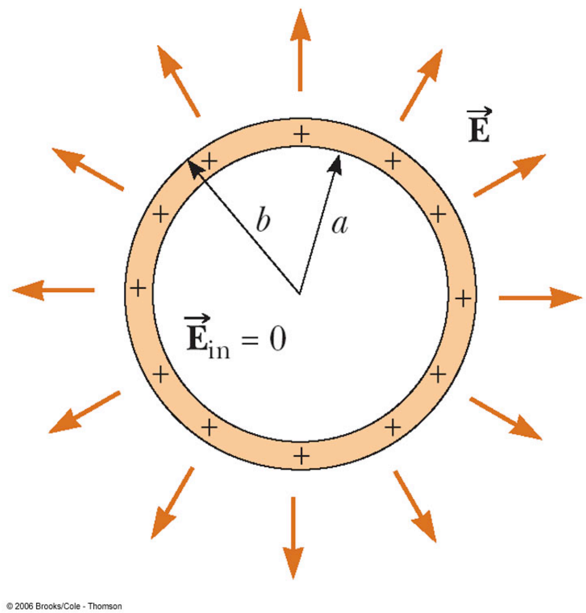
$$E = \frac{\sigma A_0}{2A_0\epsilon_0}$$

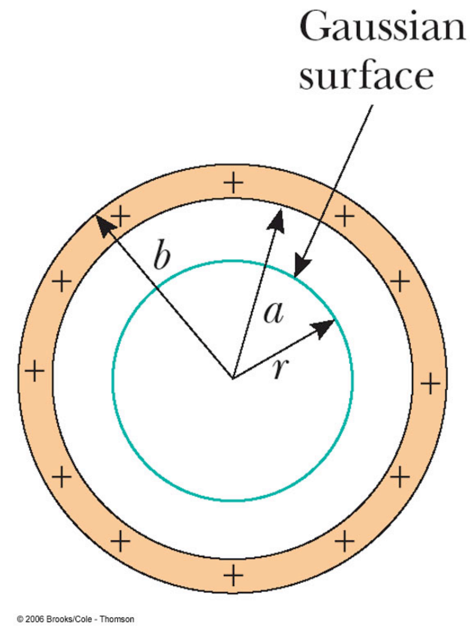
$$E = \frac{\sigma}{2\epsilon_0}$$

This is the magnitude of $\vec{E}$.
$\vec{E}$ points away from the the plane.
$\vec{E} = +\frac{\sigma}{2\epsilon_0}$ above the plane
$\vec{E} = -\frac{\sigma}{2\epsilon_0}$ below the plane

these following slides we did not get to on Wednesday
but we will view them on Thusday right after the quiz

Gauss' Law: Charged Spherical Shell

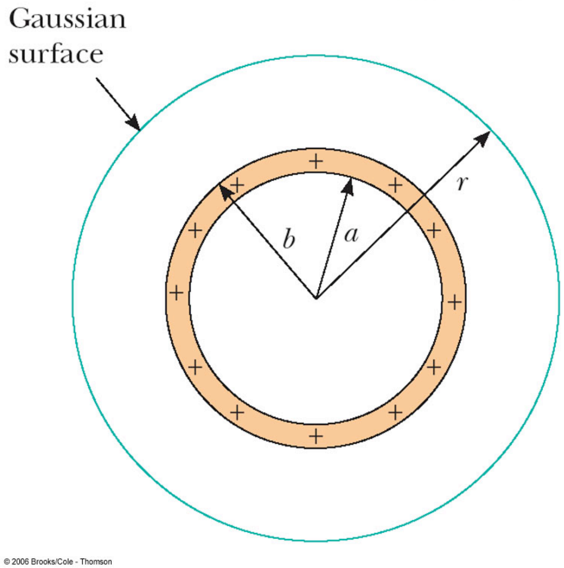




At r < a: $\vec{E} = 0$.

Gauss' Law: Charged Spherical Shell

$$\text{At } r > b,\ \Phi_E = EA = E4\pi r^2 = Q_{encl}/\epsilon_0$$

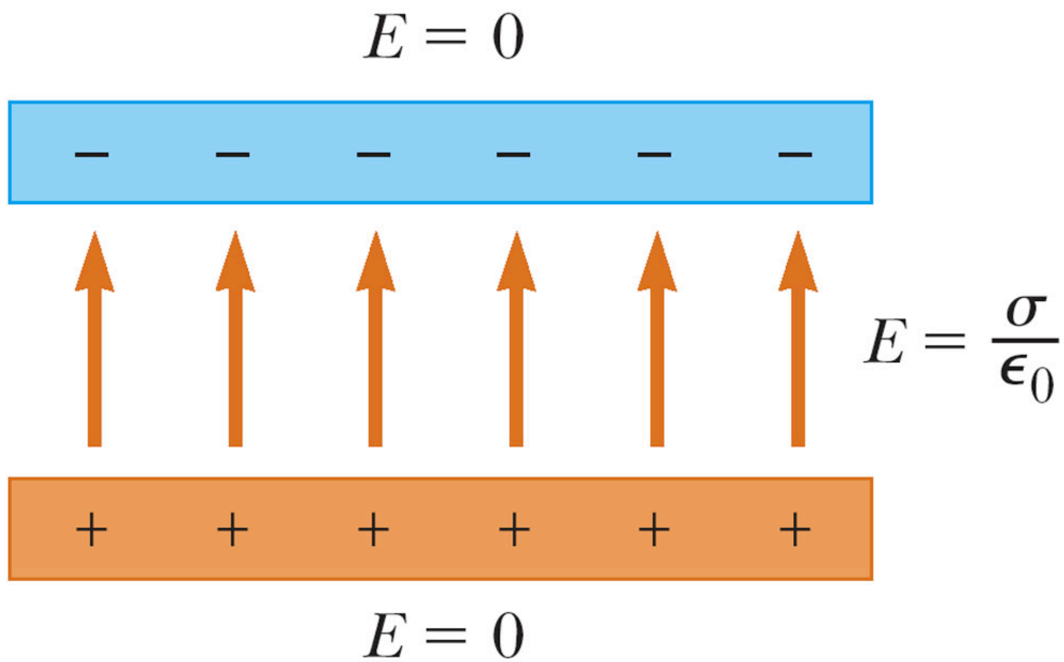


Divide both sides by area:

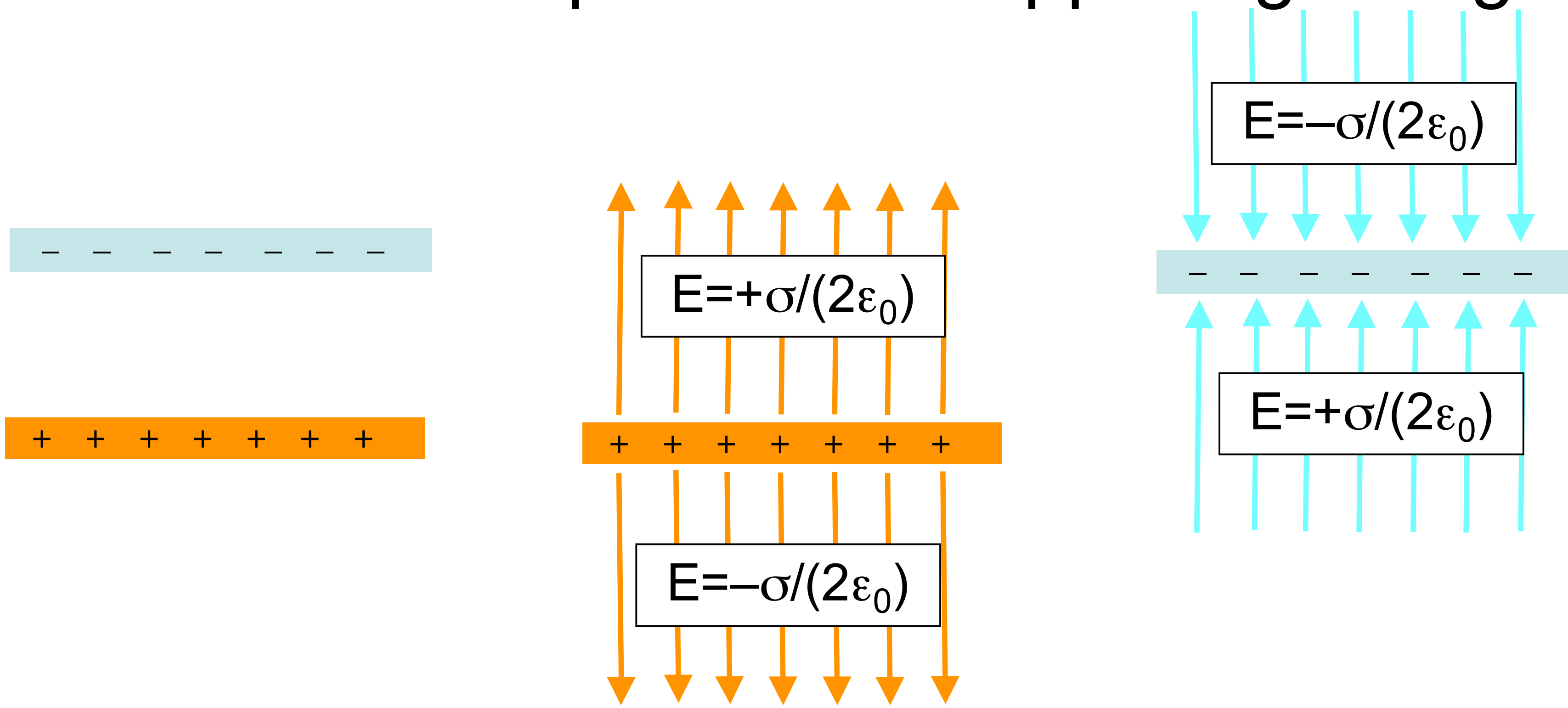
$$E = \frac{Q_{encl}}{4\pi\varepsilon_0 r^2}$$

At r > b, $\vec{E}$ looks like that from a single point charge Q

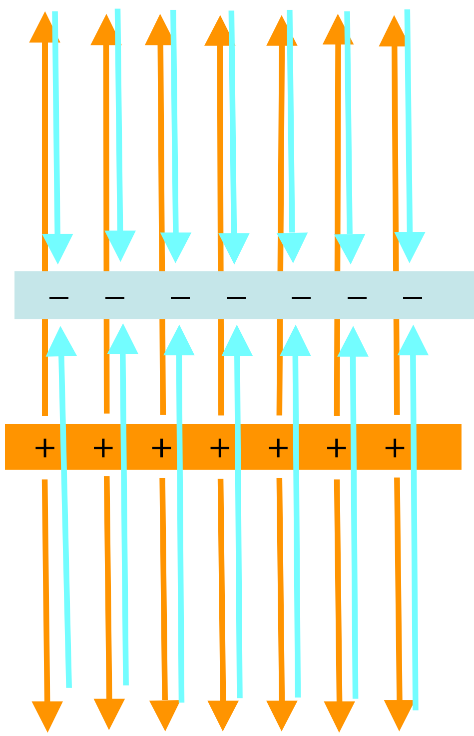
Gauss' Law: 2 planes with opposing charges



Gauss' Law: 2 planes with opposing charges



Gauss' Law: 2 planes with opposing charges



E=0 outside

Inside:

$E=+\sigma/(2\varepsilon_0)+\sigma/(2\varepsilon_0)$
$= \sigma/\varepsilon_0$

E=0 outside