

Electric Fields and Dipole

Announcements

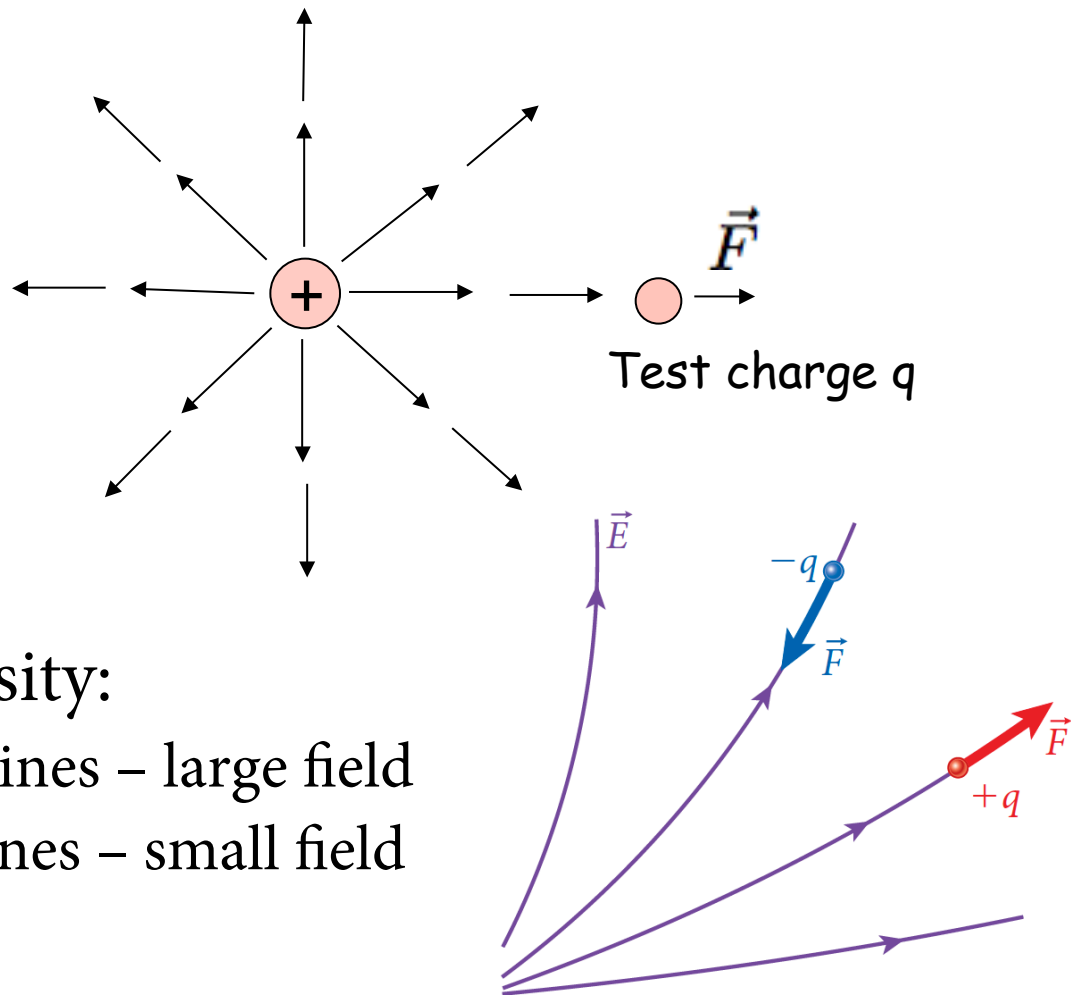
- Homework set 1 is due 1/20
- Lecture notes: linked from lon-cap, or directly at <http://www.pa.msu.edu/~schwier/courses/2014SpringPhy184/>
- Section 1 lecture notes: http://www.pa.msu.edu/~nagy_t/phy184/lecturenotes.html
- Strosacker learning center schedule – BPS 1248
 - Mo: 10am – noon, 1pm – 9pm
 - Tue: noon – 6pm
 - We: noon – 2pm
 - Th: 10am – 1pm, 2pm – 9pm
- Clicker:
 - If you didn't enter it on lon-cap, then email me your clicker number
 - Clickers only work for the section in which you are registered

The Electric Field

- The **electric field** is defined at any point in space as the net electric force on a charge, divided by that charge

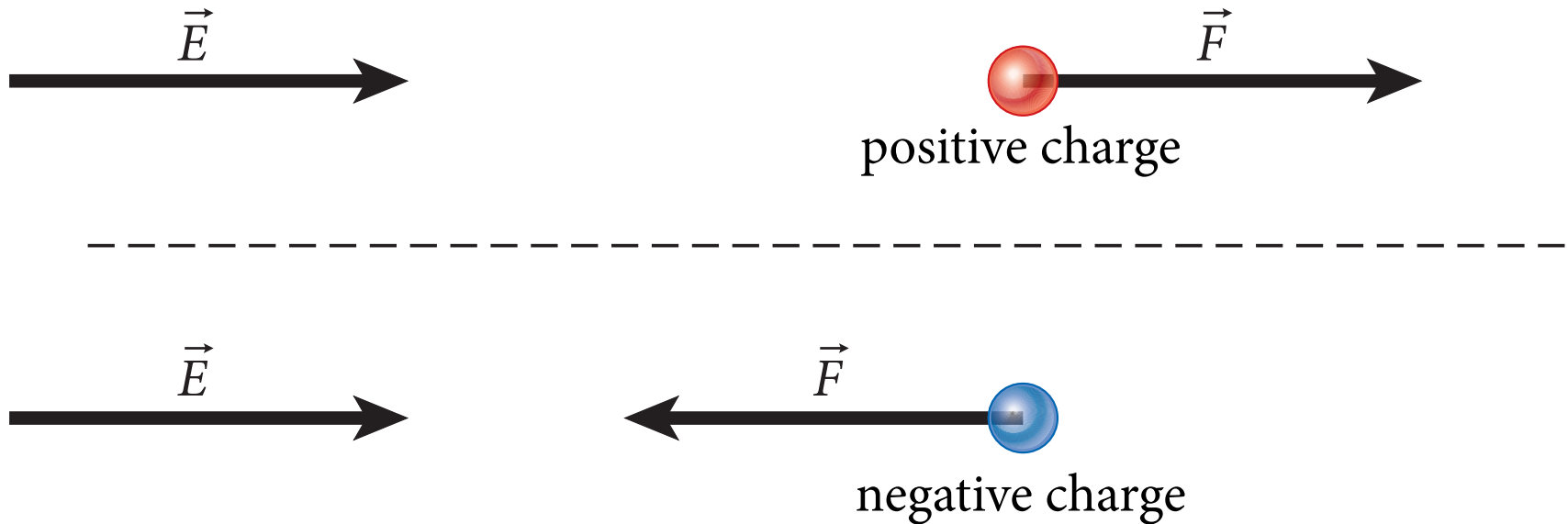
$$\vec{E}(r) = \frac{\vec{F}(\vec{r})}{q}$$

- Electric field lines start at positive charges and end at negative charges
- Electric field line density:
 - High density of field lines – large field
 - Low density of field lines – small field



The Electric Field and force

- The electric force on a charge is parallel or antiparallel to the electric field at that point



- The electric force is

$$\vec{F}(\vec{r}) = q\vec{E}(\vec{r}) \quad F = |q|E$$

Superposition of electric fields

- The total electric field at any point is the vector sum of all individual electric fields

$$\vec{E}_{net} = \sum_i \vec{E}_i$$

- Note that this means adding the x -, y -, and z - components separately

$$E_{net,X} = \sum_i E_{i,X}$$

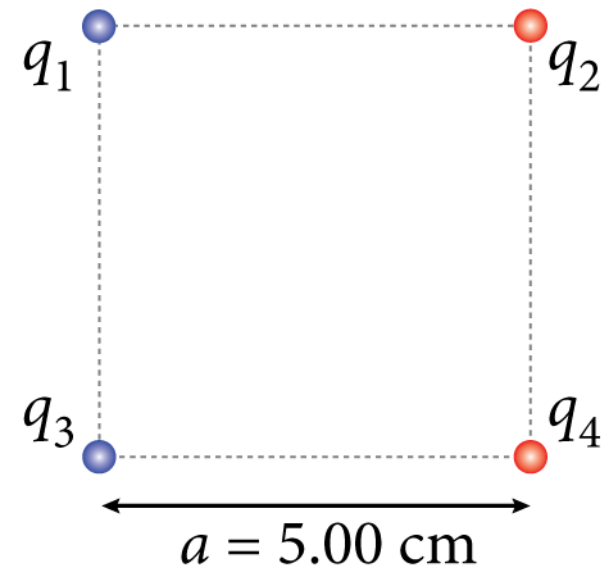
$$E_{net,Y} = \sum_i E_{i,Y}$$

$$E_{net,Z} = \sum_i E_{i,Z}$$

Electric Field from 4 Point Charges

PROBLEM

- We have four charges
 - $q_1 = 10.0 \text{ nC}$
 - $q_2 = -20.0 \text{ nC}$
 - $q_3 = 20.0 \text{ nC}$
 - $q_4 = -10.0 \text{ nC}$
- These charges form a square of edge length 5.00 cm
- What electric field do the particles produce at the square center?



Problem solving strategy

- Use this approach to solve problems, in particular if at first you have no clue.

Step 1
Think

Recognize the problem

What's going on?

Think

Step 2
Sketch

Describe the problem in terms of the field

What does this have to do with...?

Sketch

Step 3
Research

Plan a solution

How do I get out of this?

Research

Step 4
Execute

Execute the plan

Let's get an answer!

Simplify, Calculate, Round

Step 5
Double-check

Evaluate the solution

Can this be true?

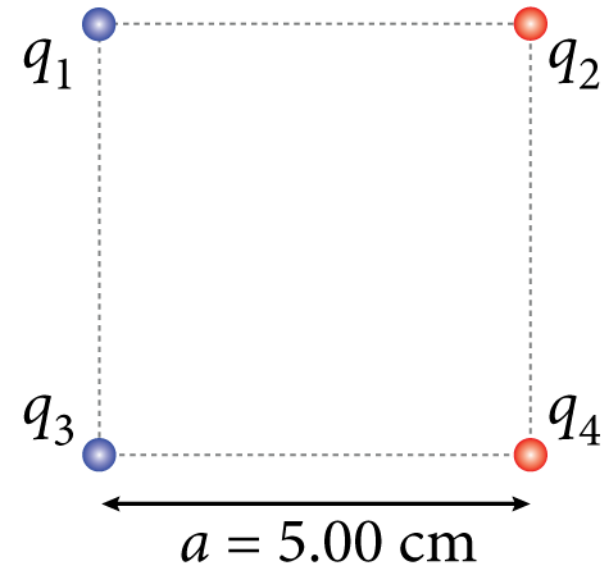
Double-check

- Draw a picture
- Phrase the question in your own words
- Relate the question to something you just learned
- Identify physics quantities, forces, fields, potentials,...
- Find a physics principle (symmetry, conservation, ...)
- Write down the equations
- Solve equations, starting with intermediate steps
- Check units, order-of-magnitude, insert into original question, ...

Electric Field from 4 Point Charges

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SOLUTION

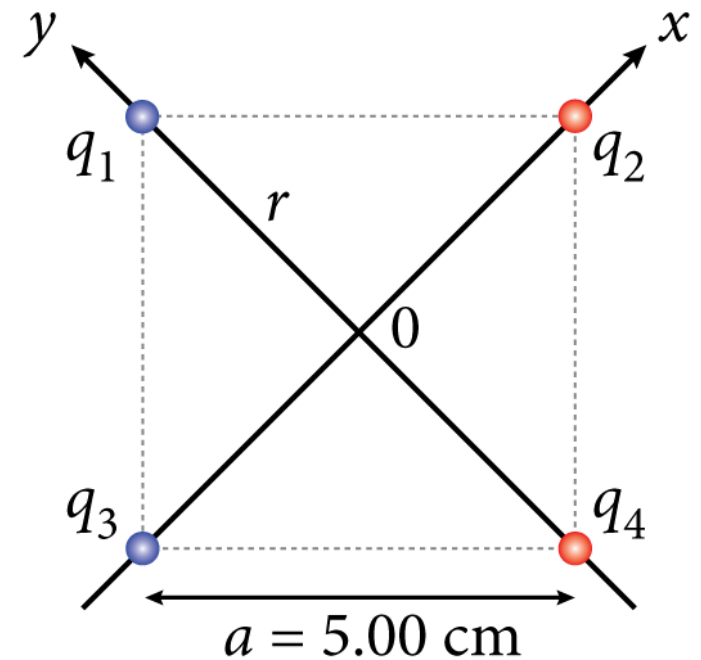
THINK

- Each of the four charges produces a field at the center
- We can use the principle of superposition
 - The electric field at the center of the square is the vector sum of the electric field from each charge

Electric Field from 4 Point Charges

SKETCH

- We define an x - y coordinate system
 - We place q_2 and q_3 on the x axis
 - We place q_1 and q_4 on the y axis
 - The center of the square is at $x = y = 0$
 - The sides of the square are $a = 5.00$ cm
 - The distance from each charge to the center is r



RESEARCH

- The electric field at the center of the square is given by the principle of superposition

$$\vec{E}_{\text{center}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$$

Electric Field from 4 Point Charges

- We can write the field in terms of the x - and y -components

$$E_{\text{center},x} = E_{1,x} + E_{2,x} + E_{3,x} + E_{4,x}$$

$$E_{\text{center},y} = E_{1,y} + E_{2,y} + E_{3,y} + E_{4,y}$$

- All four charges are a distance r from the center

$$r = \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$$

- The electric field component from each charge at the center of the square is given by

$$E_i = k \frac{q_i}{r^2}$$

SIMPLIFY

- Let's start with the x -components

$$E_{\text{center},x} = \cancel{E_{1,x}} + E_{2,x} + E_{3,x} + \cancel{E_{4,x}} = k \frac{q_2}{r^2} - k \frac{q_3}{r^2} = \frac{k}{r^2} (q_2 - q_3)$$

Electric Field from 4 Point Charges

- Now the y -components

$$E_{\text{center},y} = E_{1,y} + \cancel{E_{2,y}} + \cancel{E_{3,y}} + E_{4,y} = k \frac{q_1}{r^2} - k \frac{q_4}{r^2} = \frac{k}{r^2} (q_1 - q_4)$$

- The magnitude of the electric field at the center is

$$E_{\text{center}} = \sqrt{E_{\text{center},x}^2 + E_{\text{center},y}^2}$$

$$E_{\text{center}} = \frac{k}{r^2} \sqrt{(q_2 - q_3)^2 + (q_1 - q_4)^2} = \frac{2k}{a^2} \sqrt{(q_2 - q_3)^2 + (q_1 - q_4)^2} \quad \left(r^2 = \frac{a^2}{2} \right)$$

- The angle is given by

$$\theta = \tan^{-1} \left(\frac{E_{\text{center},y}}{E_{\text{center},x}} \right) = \tan^{-1} \left(\frac{\frac{k}{r^2} (q_1 - q_4)}{\frac{k}{r^2} (q_2 - q_3)} \right) = \tan^{-1} \left(\frac{q_1 - q_4}{q_2 - q_3} \right)$$

Electric Field from 4 Point Charges

CALCULATE

- Putting in our numerical values we get

$$E_{\text{center}} = \frac{2k}{a^2} \sqrt{(q_2 - q_3)^2 + (q_1 - q_4)^2}$$

$$(q_2 - q_3) = (-20.0 \text{ nC}) - (20.0 \text{ nC}) = -40.0 \text{ nC} = -40.0 \cdot 10^{-9} \text{ C}$$

$$(q_1 - q_4) = (10.0 \text{ nC}) - (-10.0 \text{ nC}) = 20.0 \text{ nC} = 20.0 \cdot 10^{-9} \text{ C}$$

$$E_{\text{center}} = \frac{2 \left(8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \right)}{(0.0500 \text{ m})^2} \sqrt{(-40.0 \cdot 10^{-9} \text{ C})^2 + (20.0 \cdot 10^{-9} \text{ C})^2}$$

$$E_{\text{center}} = 321636.0179 \text{ N/C}$$

Electric Field from 4 Point Charges

- For the angle we get

$$\theta = \tan^{-1} \left(\frac{q_1 - q_4}{q_2 - q_3} \right)$$

$$\theta = \tan^{-1} \left(\frac{10.0 - (-10.0)}{-20.0 - 20.0} \right) = \tan^{-1} (-0.500)$$

$$\theta = -26.56505118^\circ$$

ROUND

- We round our results to three significant figures

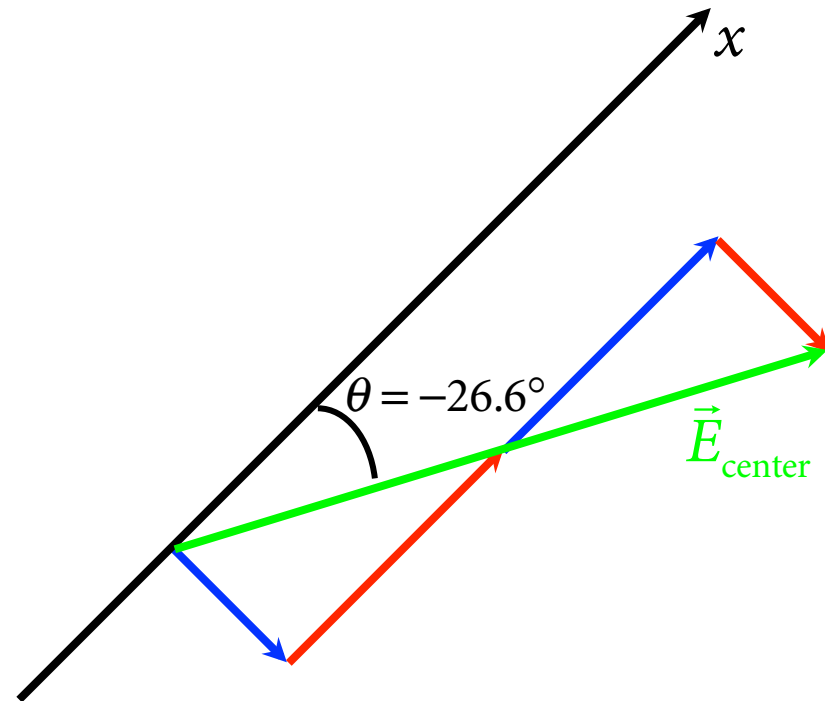
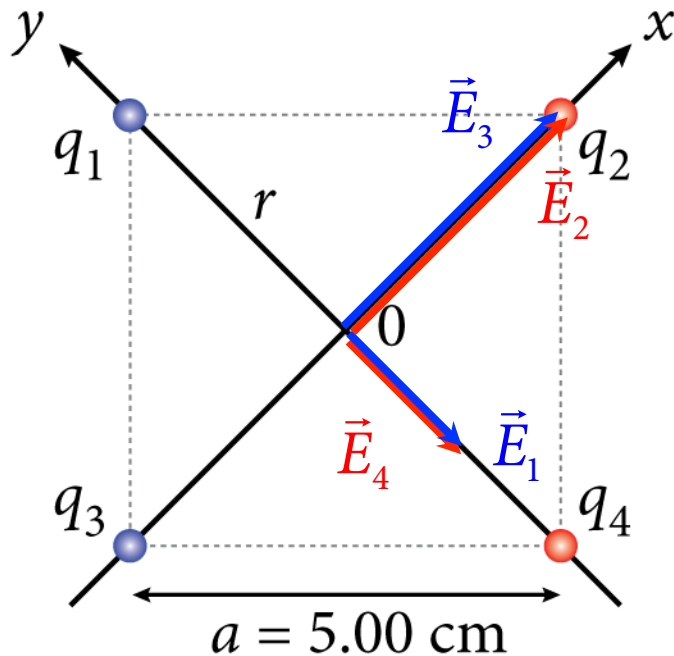
$$E_{\text{center}} = 3.22 \cdot 10^5 \text{ N/C}$$

$$\theta = -26.6^\circ$$

Electric Field from 4 Point Charges

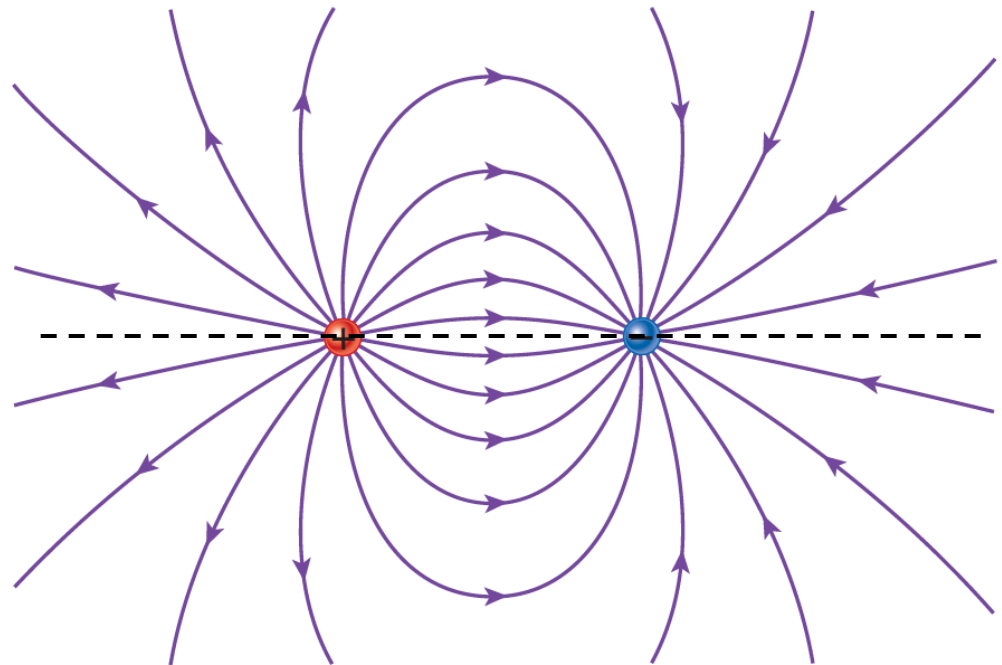
DOUBLE-CHECK

- What about the angle?



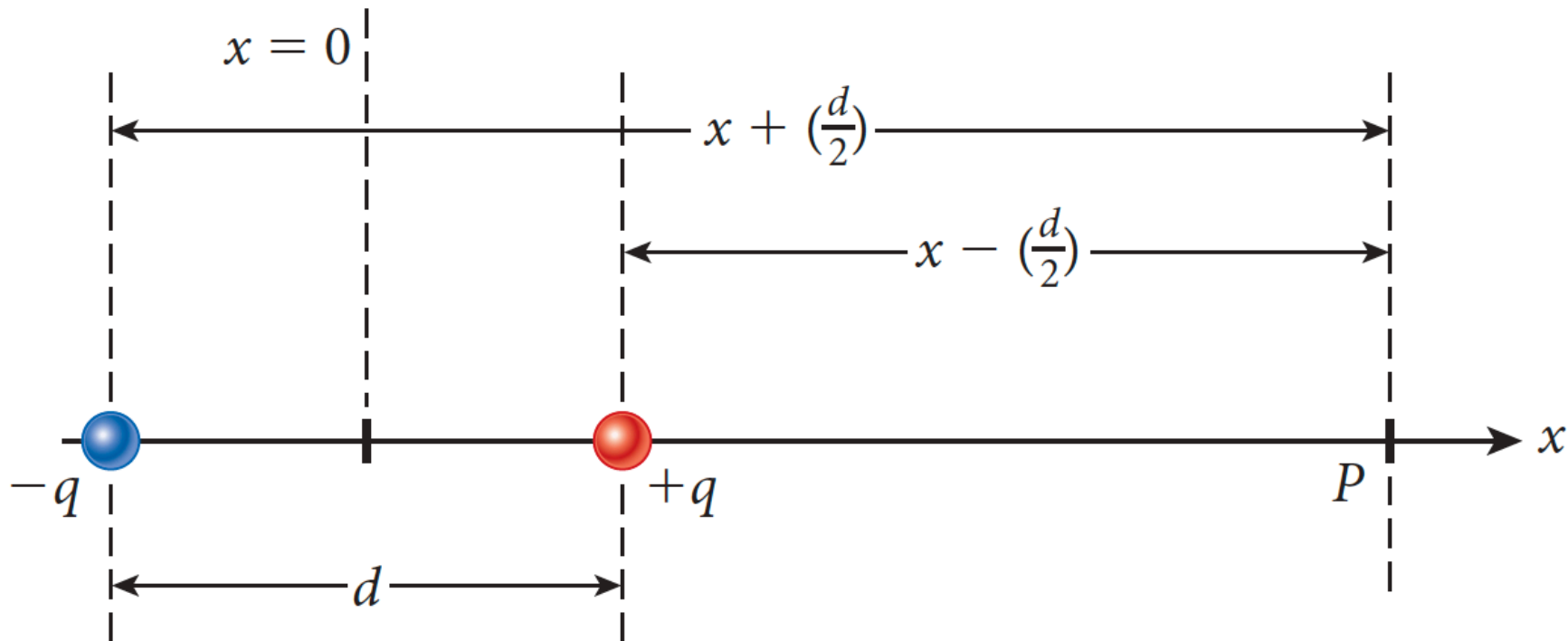
Electric Field from an Electric Dipole

- A system of two oppositely charged point particles is called an **electric dipole**
- The vector sum of the electric field from the two charges gives the electric field of the dipole
- We have shown the electric field lines from a dipole
- We will derive a general expression good anywhere along the dashed line and then get an expression for the electric field a long distance away from the dipole



Electric Field from an Electric Dipole

- Start with two charges on the x -axis a distance d apart
 - Put $-q$ at $x = -d/2$
 - Put $+q$ at $x = +d/2$
- Calculate the electric field at a point P a distance x from the origin



Electric Field from an Electric Dipole

- The electric field at any point x is the sum of the electric fields from $+q$ and $-q$

$$E = E_+ + E_- = \frac{1}{4\pi\epsilon_0} \frac{q}{r_+^2} + \frac{1}{4\pi\epsilon_0} \frac{-q}{r_-^2}$$

- Replacing r_+ and r_- we get the electric field everywhere on the x -axis (except for $x = \pm d/2$)

$$E = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{\left(x - \frac{1}{2}d\right)^2} - \frac{1}{\left(x + \frac{1}{2}d\right)^2} \right]$$

- Interesting limit far away along the positive x -axis ($x \gg d$)

Electric Field from an Electric Dipole

- We can rewrite our result as

$$E = \frac{q}{4\pi\epsilon_0 x^2} \left[\left(1 - \frac{d}{2x} \right)^{-2} - \left(1 + \frac{d}{2x} \right)^{-2} \right]$$

- We can use a binomial expansion or a Taylor expansion

$$\alpha = \frac{d}{2x}, \quad (1 \pm \alpha)^n \approx 1 \pm n\alpha + \dots$$

$$f(z) = (1 \pm z)^n$$

$$(1 - \alpha)^{-2} = 1 - (-2)\alpha + \dots = 1 + 2\frac{d}{2x} + \dots$$

$$f(z) = \sum_{n=0}^{\infty} \frac{f^{(n)}(z=0)}{n!} (z-0)^n$$

$$(1 + \alpha)^{-2} = 1 + (-2)\alpha + \dots = 1 - 2\frac{d}{2x} + \dots$$

For small $z=d/2x$, stop after $n=1$

- So we can write

$$E \approx \frac{q}{4\pi\epsilon_0 x^2} \left[\left(1 + \frac{d}{x} \right) - \left(1 - \frac{d}{x} \right) \right] = \frac{q}{4\pi\epsilon_0 x^2} \left[\frac{2d}{x} \right] = \frac{qd}{2\pi\epsilon_0 x^3}$$

Definition of Electric Dipole Moment

- We define the **electric dipole moment** as a vector that points from the negative charge to the positive charge

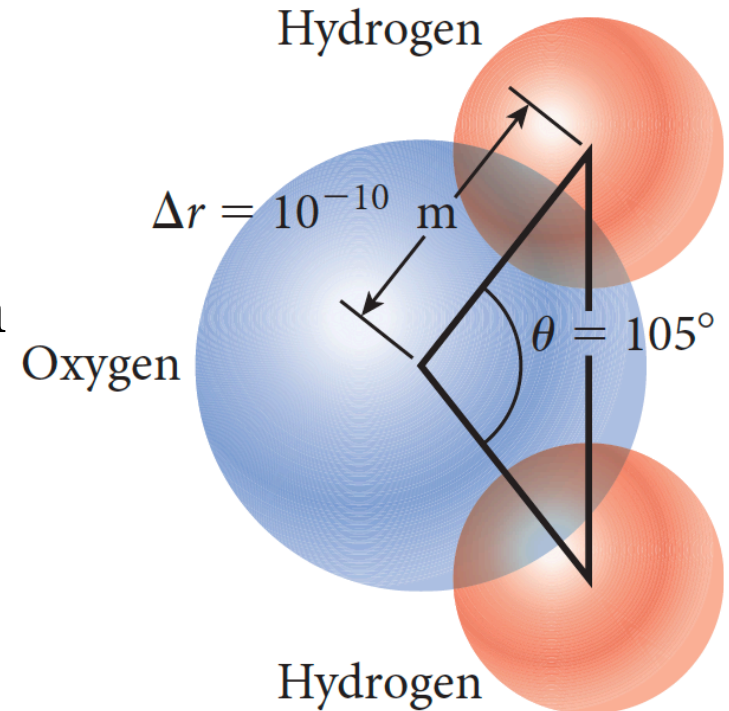
$$\vec{p} = q\vec{d}$$

- p is the magnitude of the dipole moment
 - q is the magnitude of one of the opposite charges
 - d is the distance between the charges
- Using this definition we can write the electric field far away from an electric dipole as

$$E = \frac{p}{2\pi\epsilon_0 x^3}$$

Electric Dipole Moment of Water

- Chemistry reminder
 - the H_2O molecule
- The distribution of electric charge in an H_2O molecule is non-uniform
- The more electronegative oxygen atom attracts electrons from the hydrogen atoms
- Thus, the oxygen atom acquires a partial negative charge and the hydrogen atoms acquire a partial positive charge
- The water molecule is “polarized”



Electric Dipole Moment of Water

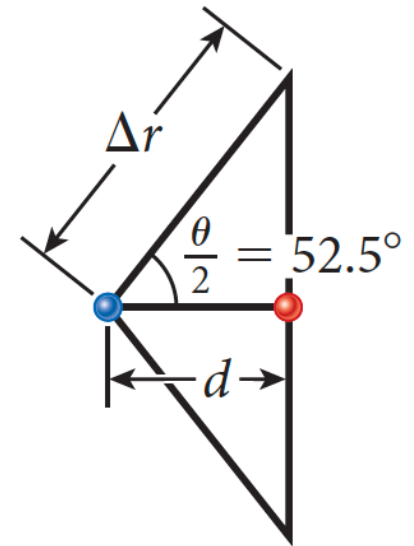
PROBLEM

- Suppose we approximate the water molecules as two positive charges located at the center of the hydrogen atoms and two negative charges located at the center of the oxygen atom
- What is the electric dipole moment of a water molecule?

SOLUTION

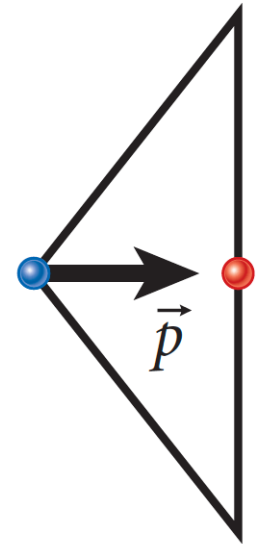
- The center of charge of the two positive charges is located exactly halfway between the centers of the hydrogen atoms
- The distance between the positive and negative charge centers is

$$d = \Delta r \cos\left(\frac{\theta}{2}\right) = (10^{-10} \text{ m}) \cos(52.5^\circ) = 0.6 \cdot 10^{-10} \text{ m}$$



Electric Dipole Moment of Water

- This distance times the transferred charge, $q = 2e$, is the magnitude of the dipole moment of water
$$p = 2ed = 2(1.6 \cdot 10^{-19} \text{ C})(0.6 \cdot 10^{-10} \text{ m}) = 2 \cdot 10^{-29} \text{ C m}$$
- This oversimplified calculation comes fairly close to the measured value of the electric dipole moment of water of $6.2 \cdot 10^{-30} \text{ C m}$
- The fact that the real dipole moment of water is less than our calculated result indicates that the two electrons of the hydrogen atoms are not pulled all the way to the oxygen, but only one-third of the way



Demo: electric force on dipole

- Demo: electric force on conducting end of stick balanced at mid-point
- Demo: electric force on water
 - Force due to dipole moment of water
- Demo: electric force on wooden end of stick balanced at mid-point

Explanation of the Demo

- The presence of the charged stick polarizes the atoms even on the wooden side of the stick

Polarization of an Atom

