Chapter 20 Electric Potential and Electric Potential Energy

Electric Potential and Electric Potential Energy (EPE)

Energy conservation: Very important in mechanics. Also very useful in electrostatics

Objects in gravitational fields have potential energy (PE).
Similarly, Charges in electric fields have ELECTRIC POTENTIAL ENERGY (**U**)
To move an object in a gravitational field requires work. There is a change in the object's PE.
Similarly, To move a charge in an electric field requires work. There is a change in the charge's **EPE**.

In gravitational fields a force is required to move masses apart.
That is, positive work is done (by an external agent-you).
Similarly, In electric fields a force is required to push like charges together.
That is positive work is done (by an external agent-you).

Electric Potential and Electric Potential Energy (EPE)
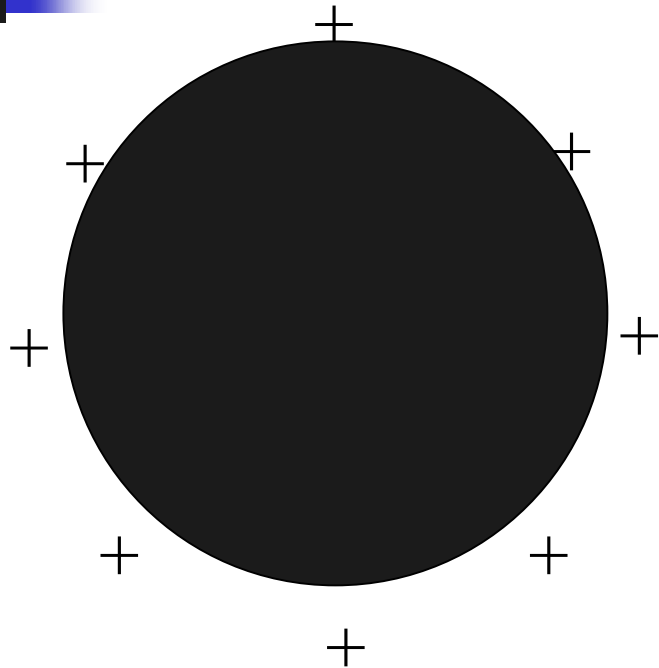
Force Required

Force Not Required.
Energy from field given to charge

Force required to push charges together. That is, positive work needs to be done (by you).
When charges pushed together there is an increase in **U.**
If charges released they will accelerate apart and the EPE is turned into KE.
Same thing in gravitational fields.

Figure 20–1 Change in electric potential energy

a) A positive test charge q_0 experiences a downward force due to the electric field E. If the charge is moved upward a distance d the work done by the electric field is $-(q_0)Ed$. At the same time, the electric potential energy of the system increases by $(q_0)Ed$. The situation is analogous to that of an object in a gravitational field. (b) If a ball is lifted against the force exerted by gravity, the gravitational potential energy of the system increases.

$$\Delta U = q_0 E d$$

$$\Delta U = mgd$$

E

d

q_0

$$F = q_0 E$$

m

d

$$F = mg$$
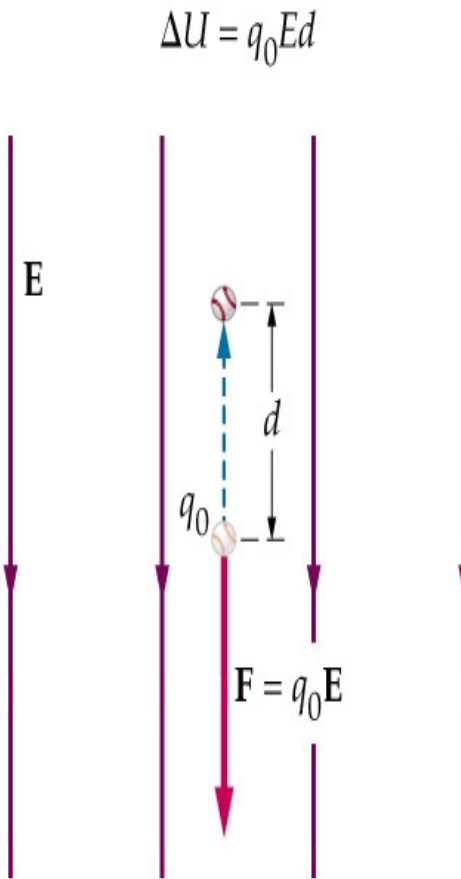
(a)

(b)
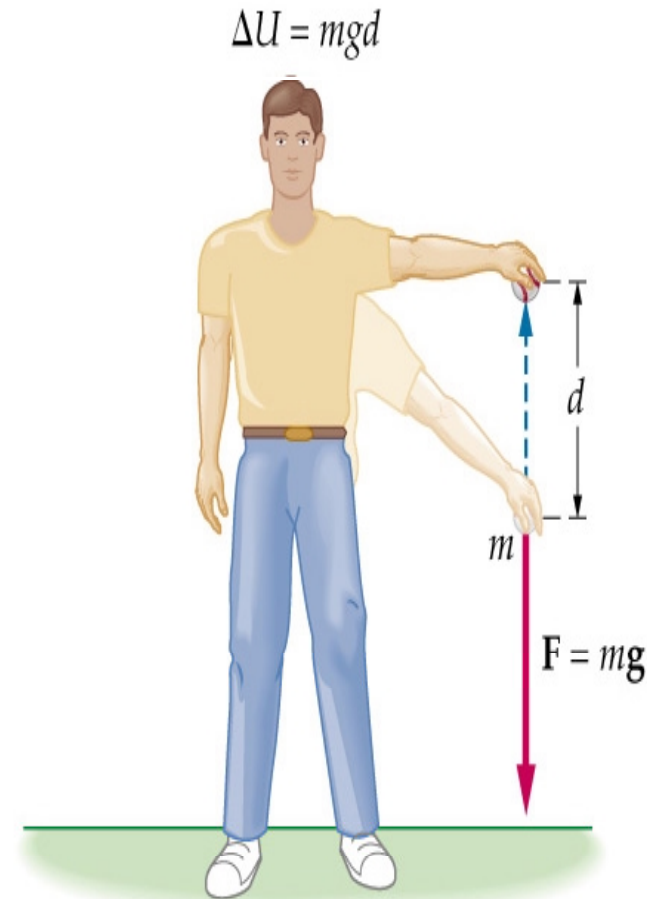
Change in electric potential energy

EPE Increases   PE Increases

$\Delta U = q_0 Ed$

E

d

q_0
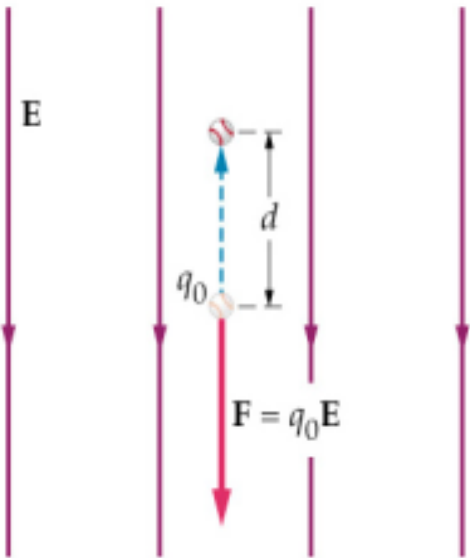
$F = q_0 E$

Work Done by an **Electric Force, $W=-q_o Ed$**
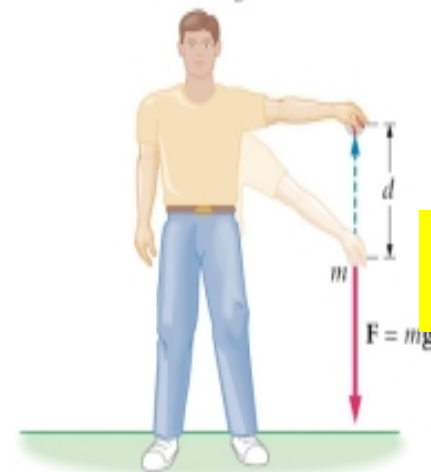Work Done by an **YOU, $-W=q_o Ed$**

$$\Delta U = U_B - U_A = -W = q_0 Ed$$

Change in electric potential energy
of a charge on moving from A to B

(a)

In the above: You do positive work,
the field does negative work.
When things happen spontaneously:
The field does positive work YOU do negative work

$\Delta U = mgd$

d

m

$F = mg$

(b)

Change in electric potential

The change in potential energy or the work done depends on the size of the charge being moved.

$$\Delta V = V_B - V_A = \Delta U / q_0 = -W / q_0$$

Potential difference
between points A to B
Measured in Volts

1 Volt = 1 J/C

Work Done By an **Electric FORCE**
It's important to know who does the work!

Electrical potential difference more important than the actual value of electric potential

For a uniform field between two plates
$$\Delta V = \Delta U / q_0 = q_0 Ed / q_0 = Ed \qquad \text{or} \qquad E = -\Delta V / \Delta s$$

Potential difference depends only on the plates and NOT on any charge being moved. The work done depends on the size of the charge being moved.

Figure 20–2 Electric field and electric potential

As a charge q_0 moves in the direction of the electric field, E, the electric potential, V, decreases. In particular, if the charge moves a distance $\triangle s$, the electric potential decreases by the amount

$$\triangle V = -E \triangle s.$$
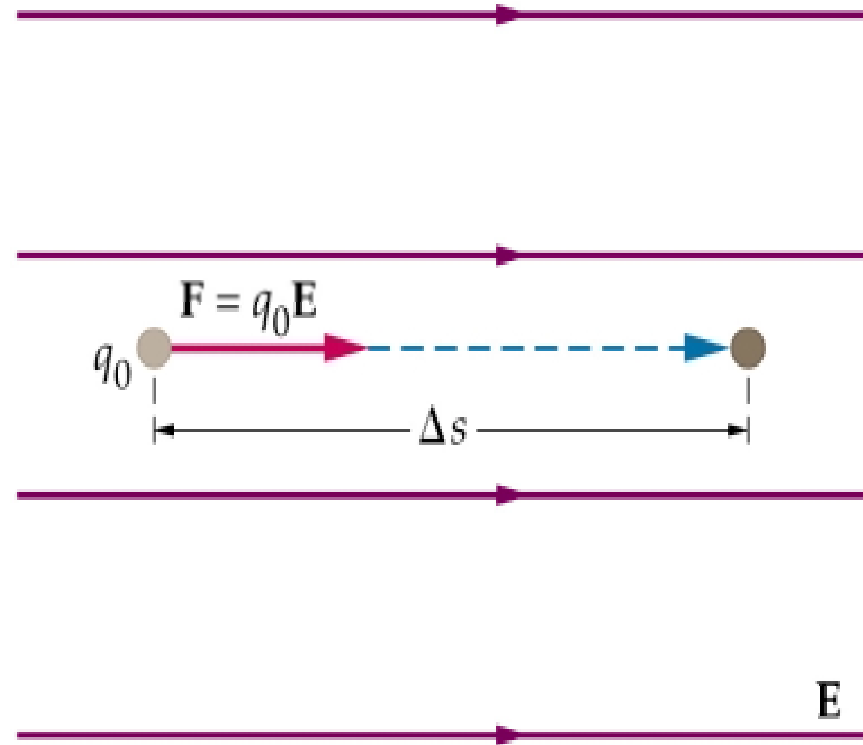
How is electric field related to electric potential?

Use Dimensional or unit analysis
Electric field is measured in units of N/C

$F=ma$

$$\frac{N}{C} = \frac{(kgms^{-2})}{C}$$

$WD=Fd$

$$(kgms^{-2})m = J$$

$$\frac{N}{C} = \frac{J}{mC} = \frac{V}{m}$$

So

$$E = \frac{V}{d}$$

Figure 20–3 The electric potential for a constant electric field

The electric potential, V, decreases as one moves in the direction of the electric field. In the case shown here, the electric field is constant; as a result, the electric potential decreases uniformly with distance. We have arbitrarily set the potential equal to zero at the right-hand plate.
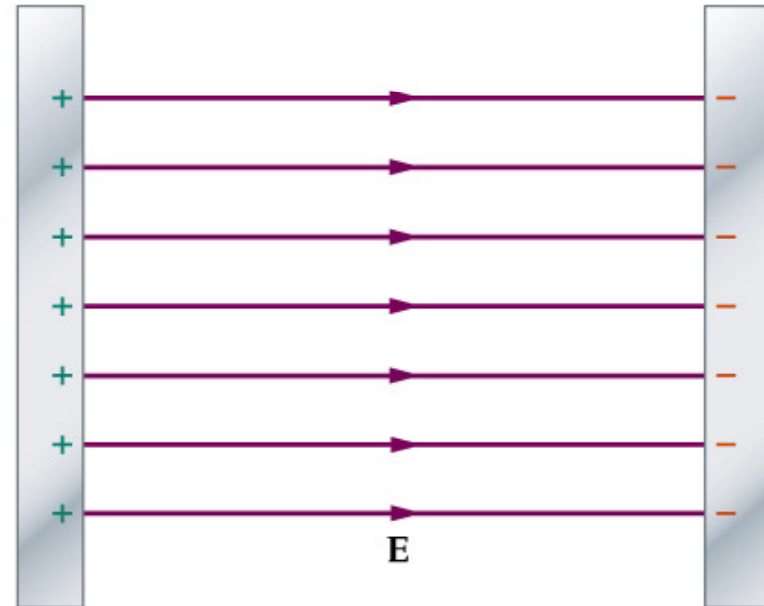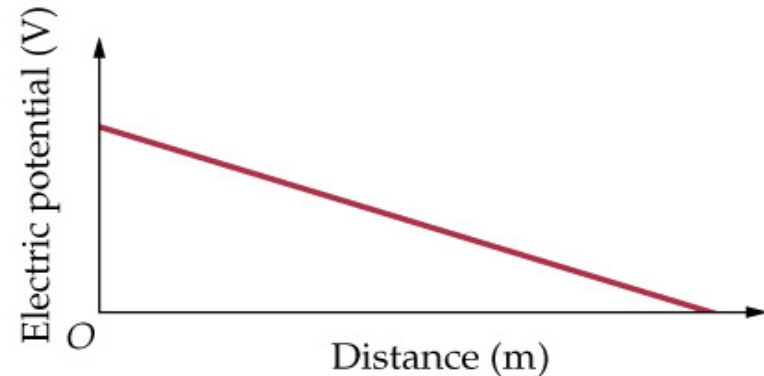
Problem 10: A pair of parallel plates is charged by a 24 V battery. How much work is required to move a charge of –4 µC from the positive to the negative plate?

B   **A**

24V
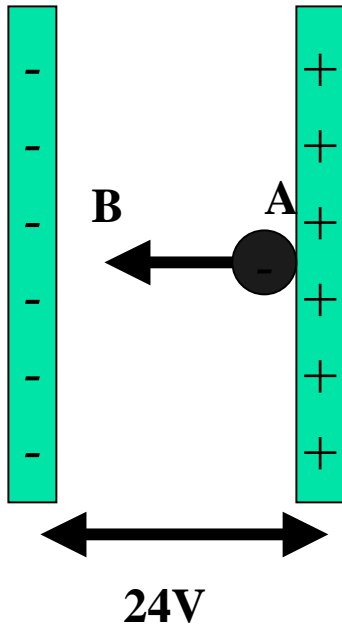
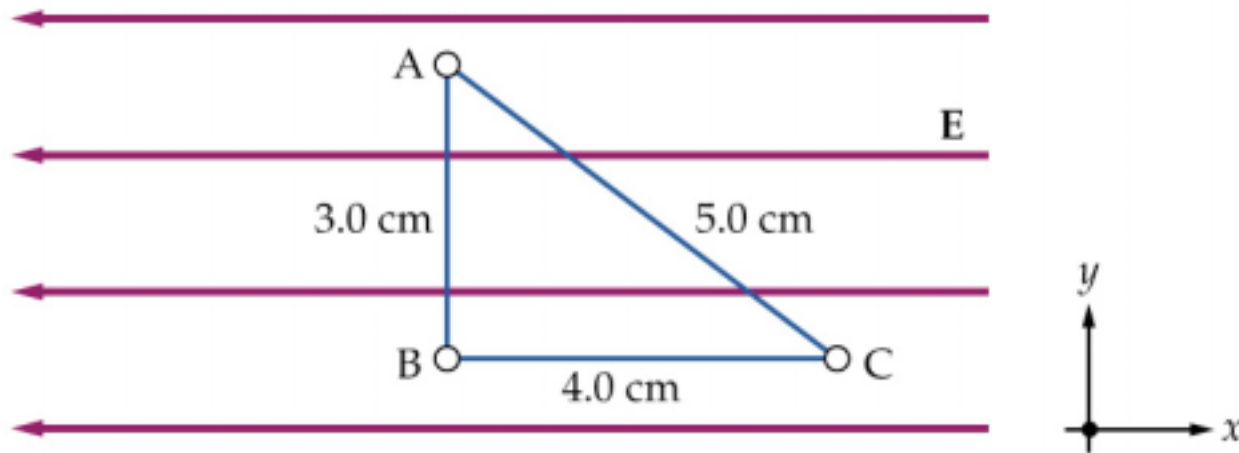NOTICE: Which plate is at higher potential and WHY?

You will have to do positive work to move the charge as directed.

Problem 13: A proton is accelerated by a potential difference of 10 kV. How fast is the proton moving if it started from rest (**7 marks**)?

Problem 11

$$11.a \quad \Delta V = V_B - V_A = 0 \text{ because } V_B = V_A$$

$$b \quad \Delta V = V_B - V_C = -Ed = -(1200\frac{N}{C})(0.040 \text{ m}) = -48V$$

$$c \quad \Delta V = V_C - V_A = Ed = (1200\frac{N}{C})(0.040 \text{ m}) = +48V$$

Problem 14

The electric potential along the x-axis is shown. Find the electric field at positions 1-4.
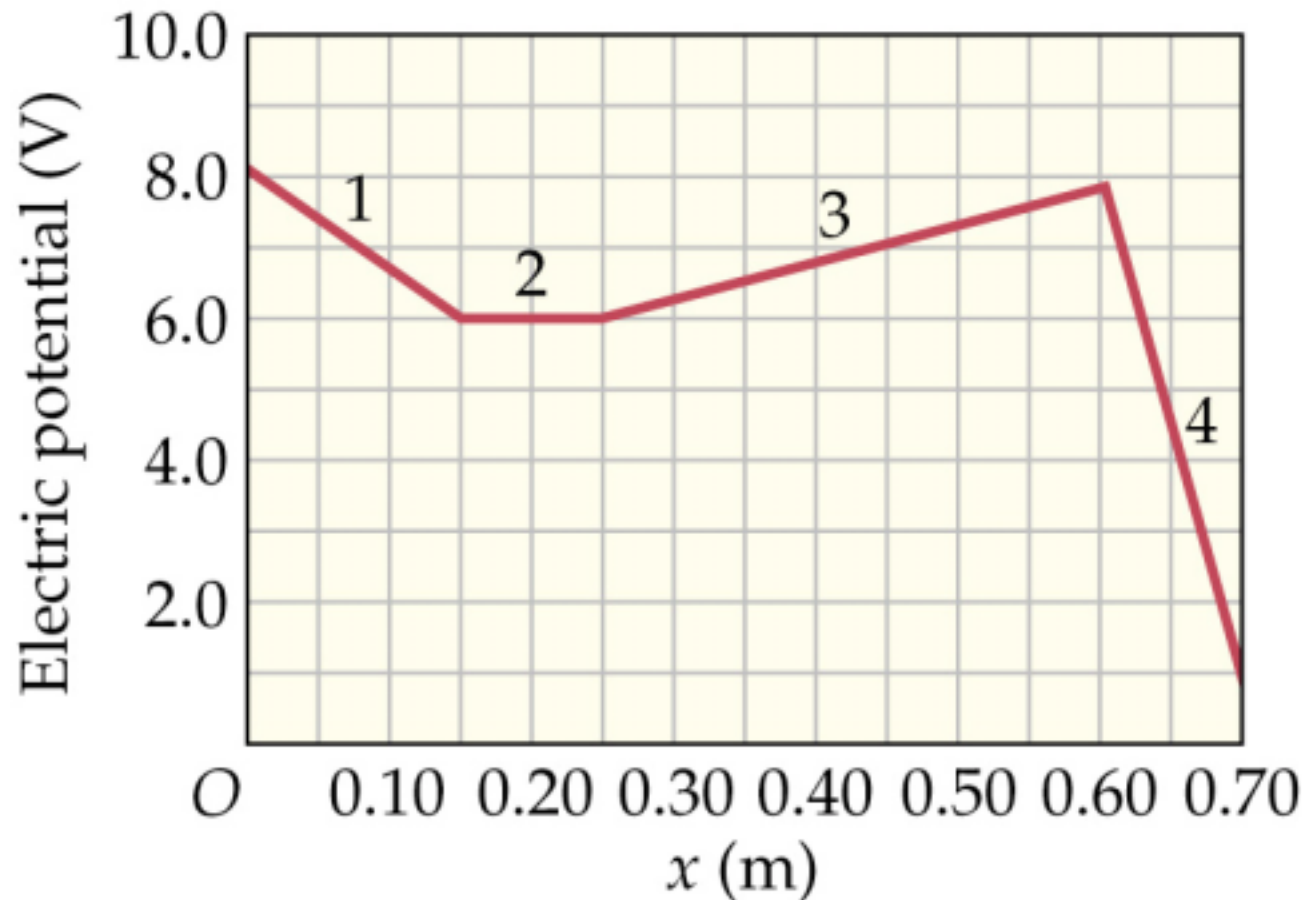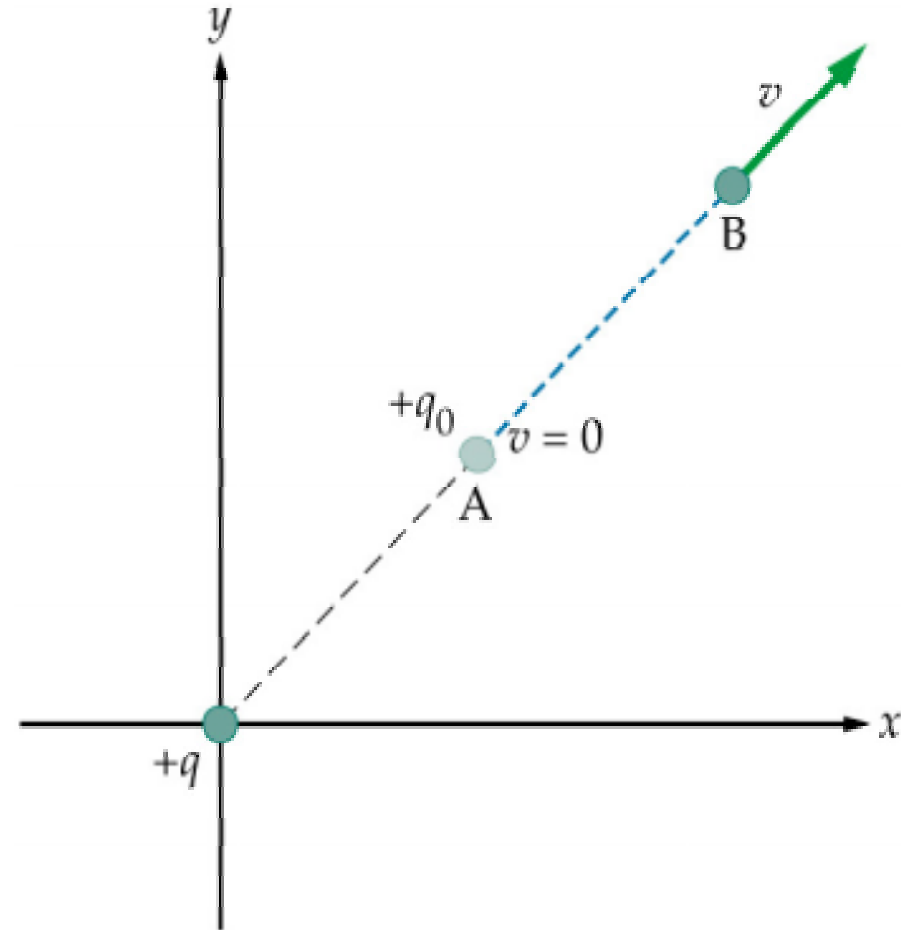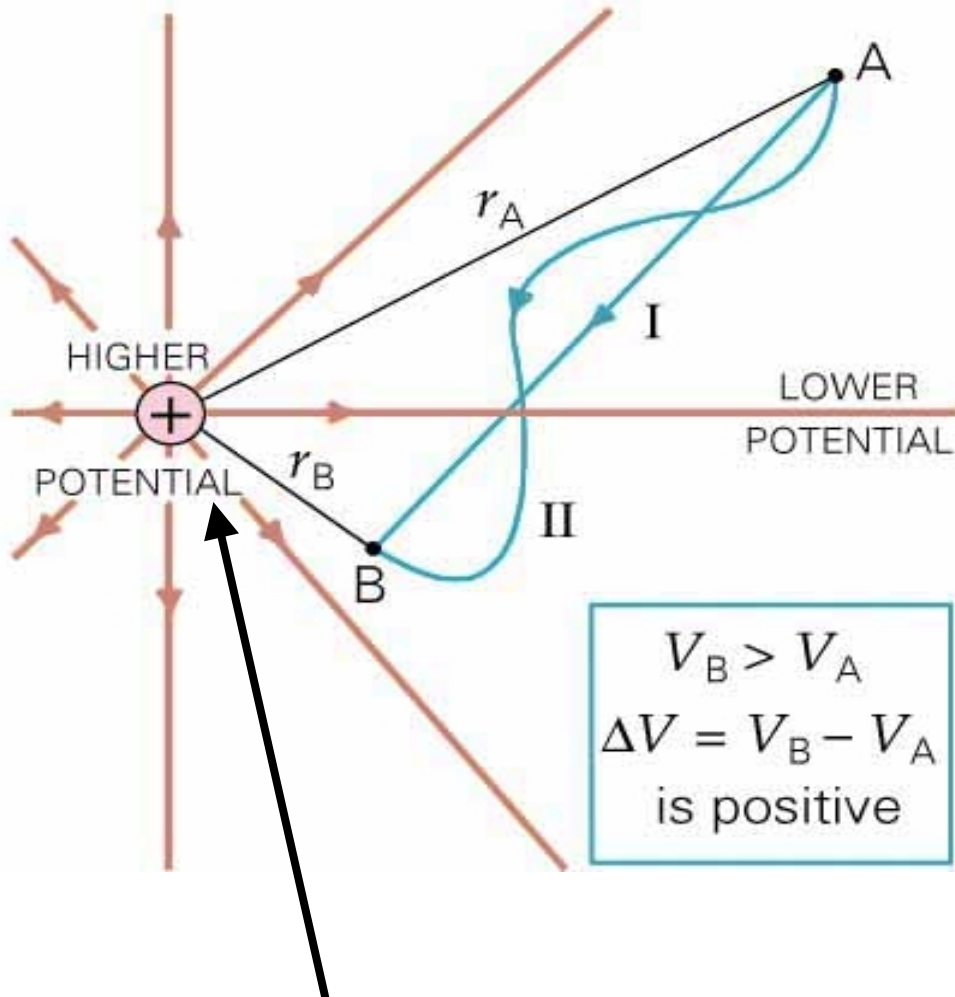
Figure 20–4 Energy conservation in an electrical system

A test charge, $+q_0$, is released from rest at point A. When it reaches point B its kinetic energy will have increased by the same amount that its electric potential energy has decreased.

The electric potential of a point charge

HIGHER POTENTIAL

LOWER POTENTIAL

r_A

r_B

I

II

A

B

$V_B > V_A$
$\Delta V = V_B - V_A$
is positive

charge Q

$$\Delta V = V_B - V_A = \frac{kQ}{r_B} - \frac{kQ}{r_A}$$

(need calculus to prove this)
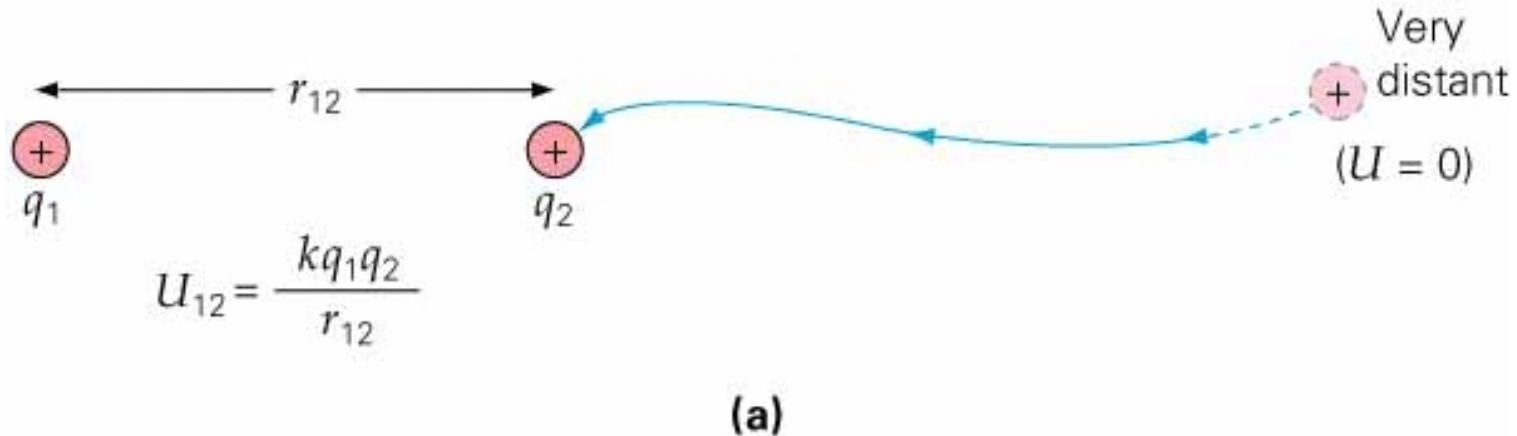
At r_A= infinity, the potential is defined to be ZERO

$$\Delta V = V_B = \frac{kQ}{r_B}$$

or $$V = \frac{kQ}{r}$$

Equation is valid ONLY for a point charge

Mutual electric potential energy of point charges

$$U_{12} = \frac{kq_1q_2}{r_{12}}$$

(a)

If we move a positive charge from a very great distance to a distance r_{12} from another charge, there is an increase in potential energy because positive work must be done to bring the mutually repelling charges closer together.

See active example 20.3
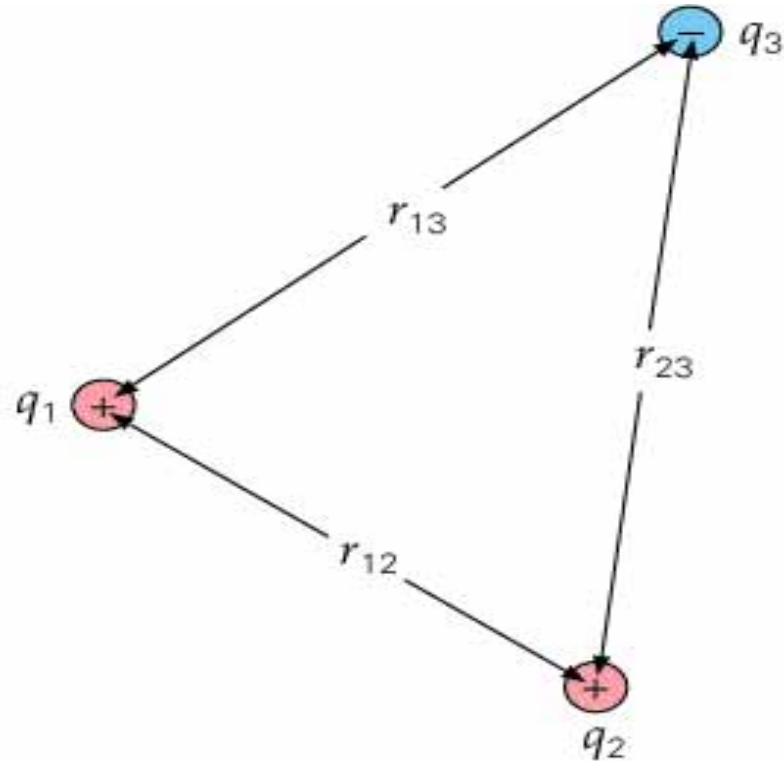
$$U_{total} = U_{12} + U_{23} + U_{13}$$

(b)

For more than two charges, the system's electric potential energy is the sum of the energies of each pair.

Figure 20–6 Equipotentials for a point charge

Equipotential surfaces for a positive point charge located at the origin. Near the origin, where the equipotentials are closely spaced, the potential varies rapidly with distance and the electric field is large.
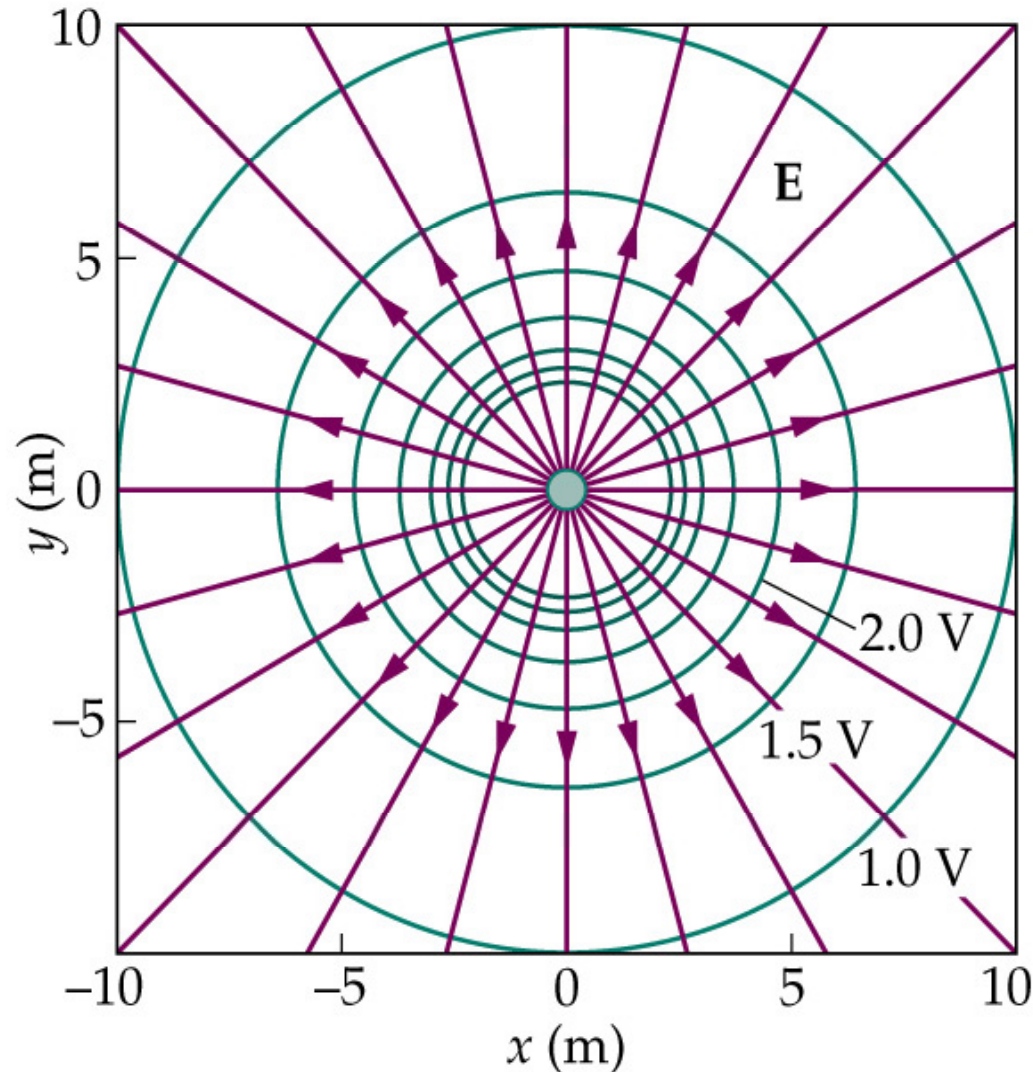
Figure 20–7 Equipotential surfaces for a uniform electric field

Electric field lines are perpendicular to the equipotential. surfaces.

Electric field lines point in the direction of decreasing potential.
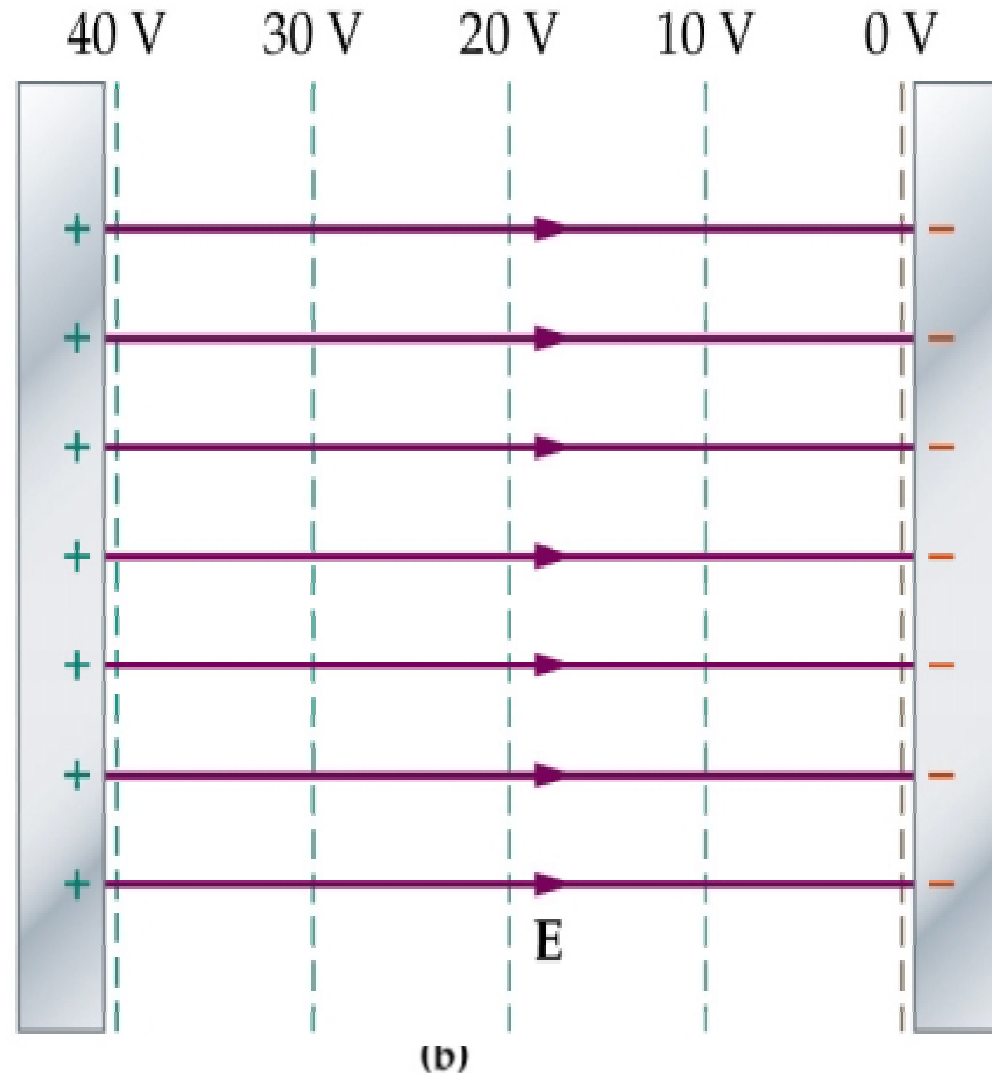

(b)

Figure 20–9 Electric charges on the surface of ideal conductors

a) On a spherical conductor the charge is distributed uniformly over the surface. (b) On a conductor of arbitrary shape the charge is more concentrated, and the electric field is more intense, where the conductor is more sharply curved. Note that in all cases the electric field is perpendicular to the surface of the conductor.
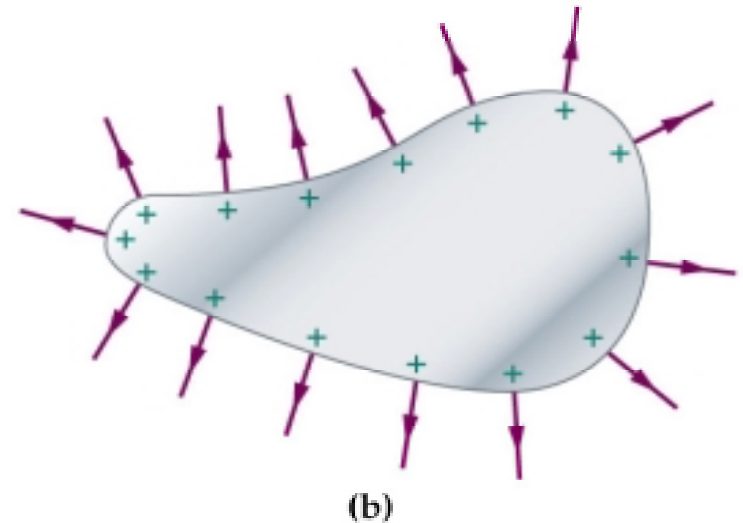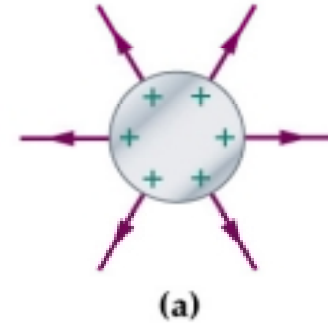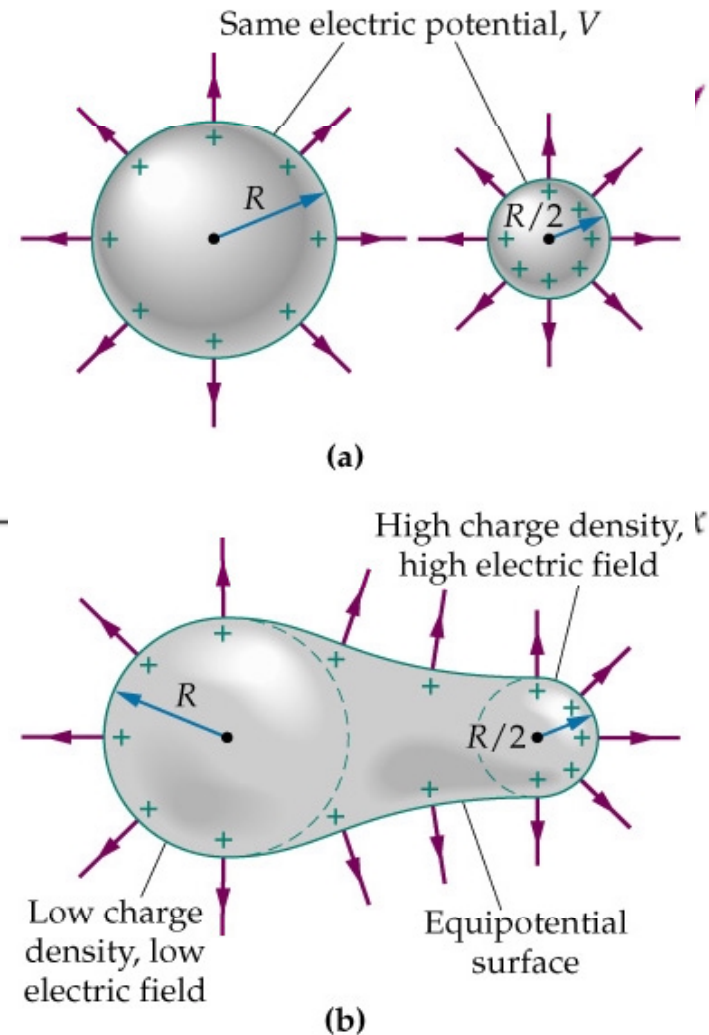
(a)

(b)

Figure 20–10 Charge concentration near points

(a) If two spheres of different radii have the same electric potential at their surfaces, the sphere with the smaller radius of curvature has the greater charge density and the greater electric field. (b) An arbitrarily shaped conductor can be approximated by spheres with the same potential at the surface and varying radii of curvature. It follows that the more sharply curved end of a conductor has a greater charge density and a more intense field.

Same electric potential, V

R

$R/2$

(a)

High charge density, high electric field

R

$R/2$

Low charge density, low electric field

Equipotential surface

(b)

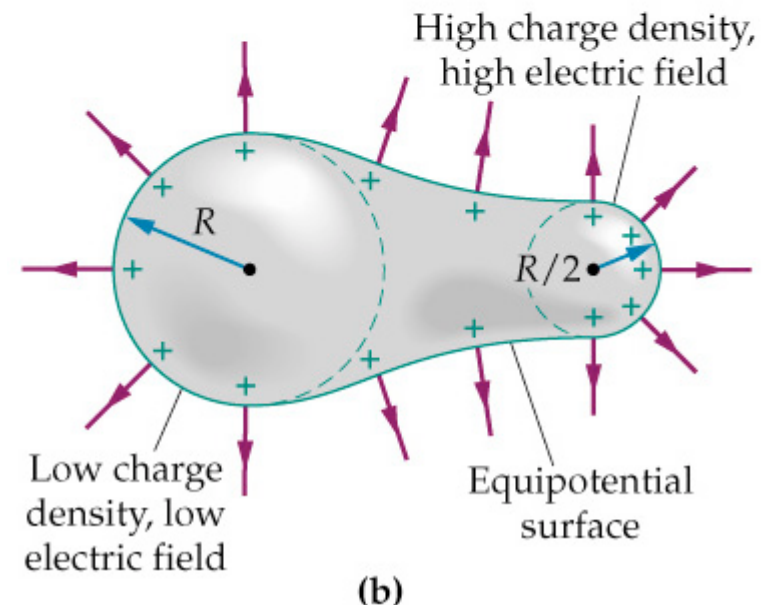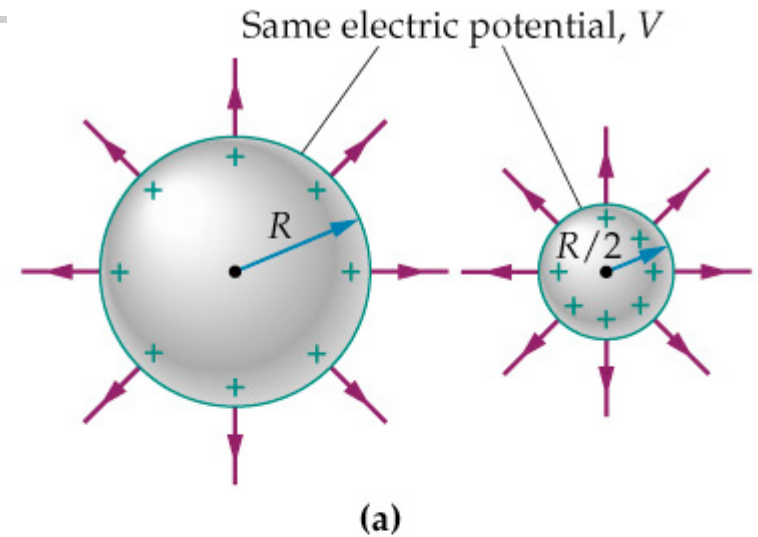# Electric charges on the surface of ideal conductors

Conducting sphere radius R,

$$\sigma = \frac{Q}{4\pi R^2}, \qquad V = \frac{kQ}{R} = \frac{k\sigma 4\pi R^2}{R} = 4\pi k\sigma R$$

Conducting sphere radius $\frac{R}{2}$,

$$2\sigma = \frac{2Q}{4\pi R^2}, \qquad V = \frac{kQ}{R} = 4\pi k\sigma R$$

Smaller spheres have higher charge densities to be at the same potential. Also electric field is larger for the small sphere and at right angles to the surface.

Same electric potential, V

R

$R/2$

(a)

High charge density, high electric field

R

$R/2$

Low charge density, low electric field

Equipotential surface

(b)

The electron-volt as an energy unit

Explain why an electron-volt (eV) is a unit of energy, not a voltage. Which is larger, a gigaelectron-volt or a mega-electron volt?

The electron volt unit comes from

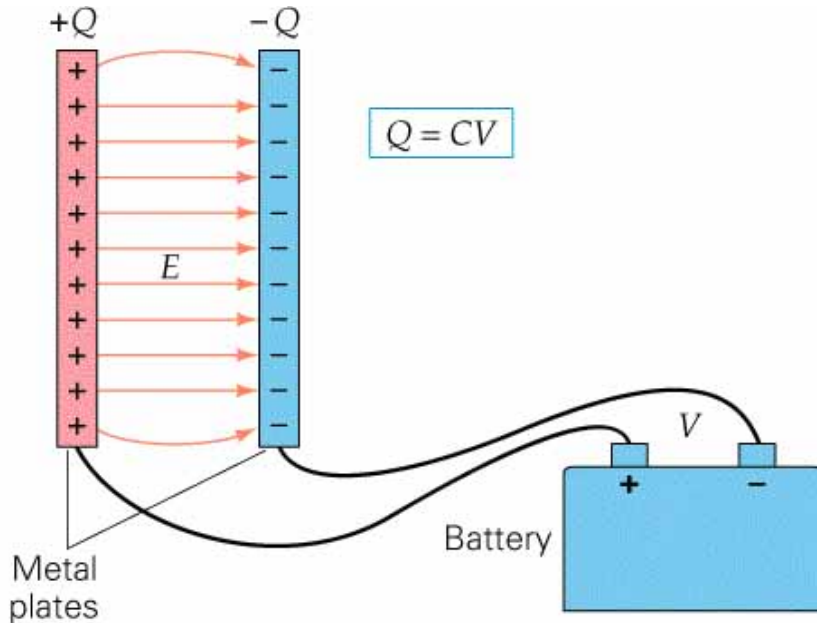$$e\Delta V = q\Delta V = \Delta U_e.$$

It is also the kinetic energy gained by an electron when it goes through a potential difference of 1 V. So it is a unit of energy.

A GeV is larger than a MeV by 1000 times.

Parallel plate capacitor

Q is a measure of the charge on either plate.

$+Q$ $-Q$

E

Metal plates

$Q = CV$

V

Battery

(a) Parallel-plate capacitor

C

V
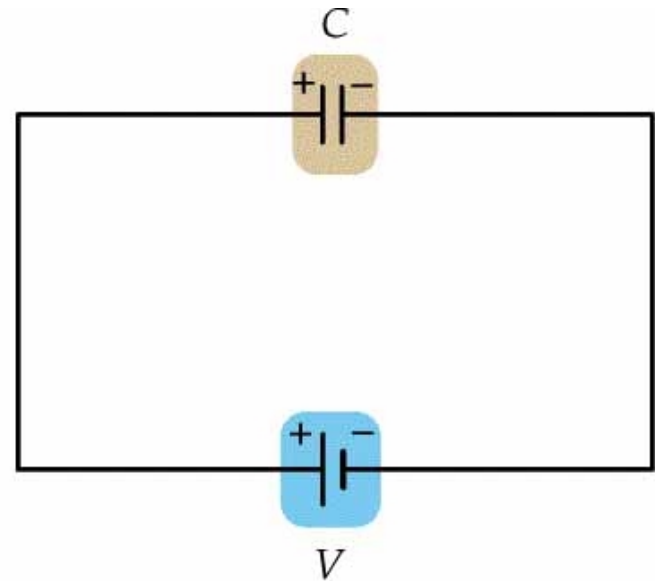
(b) Schematic circuit diagram

Capacitance: A measure of the capacity that the plates have for charge.

From experiment $Q \propto V$

Parallel plate capacitor

$Q \; \alpha \; V$ or $Q = CV$

$$C = \frac{Q}{V}$$

Capacitance measured in FARADS, F

1 F is a large unit. Use pF=10^{-12} F or μF =10^{-6} F

Can a capacitor with large plates hold more charge?
That is, will it have a larger capacity for charge?

Re*member*

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} = \frac{4\pi kQ}{A} \qquad and \qquad E = Vd \quad So \quad Vd = \frac{4\pi kQ}{A} \Rightarrow \frac{Q}{V} = \frac{A}{4\pi kd}$$

BUT

$$C = \frac{Q}{V} = \frac{A}{4\pi kd} = \frac{\varepsilon_0 A}{d}$$

$$C = \frac{\varepsilon_0 A}{d}$$

Figure 20–13 A parallel-plate capacitor

A parallel-plate capacitor, with plates of area A, separation d, and charges of magnitude Q. The capacitance of such a capacitor is
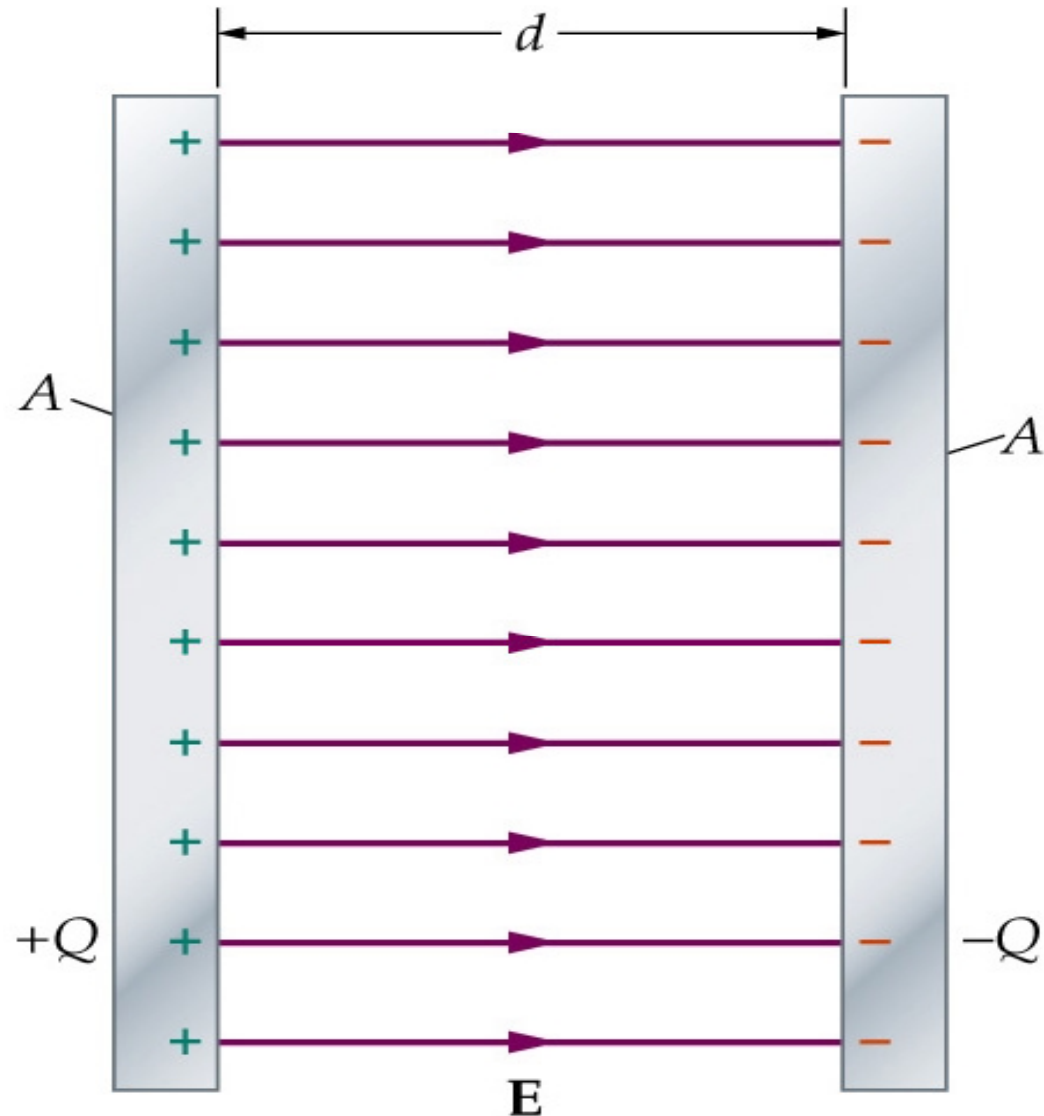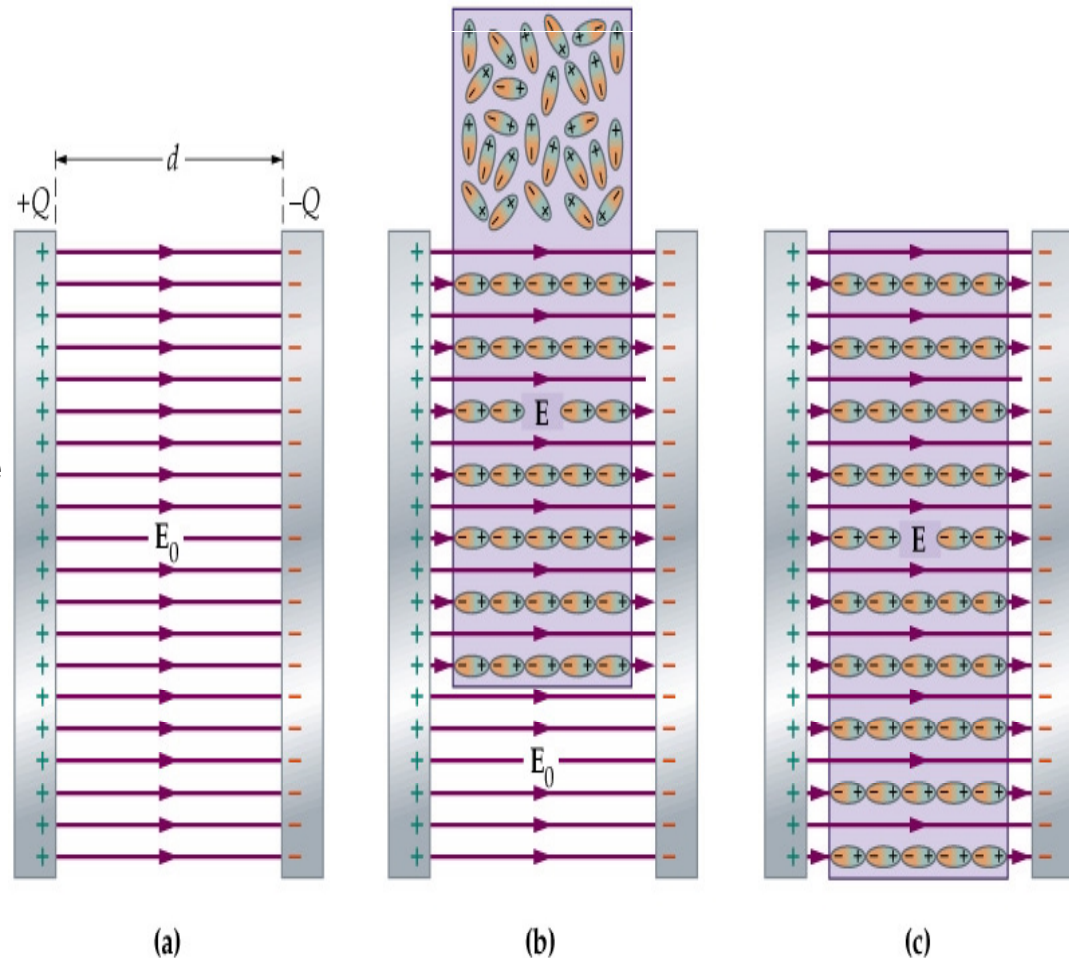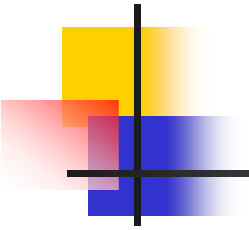
$C = e_0 A/d.$

Figure 20–15 The effect of a dielectric on the electric field of a capacitor

When a dielectric is placed in the electric field between the plates of a capacitor, the molecules of the dielectric tend to become oriented with their positive ends pointing toward the negatively-charged plate and their negative ends pointing toward the positively-charged plate. The result is a buildup of positive charge on one surface of the dielectric and of negative charge on the other. Since field lines start on positive charges and end on negative charges, we see that the number of field lines within the dielectric is reduced. Thus, within the dielectric the applied electric field, E_0, is partially cancelled. Because the strength of the electric field is less, the voltage between the plates is less as well. Since V is smaller while Q remains the same, the capacitance, C = Q/V, is increased by the dielectric.

The effect of a dielectric on the electric field of a capacitor
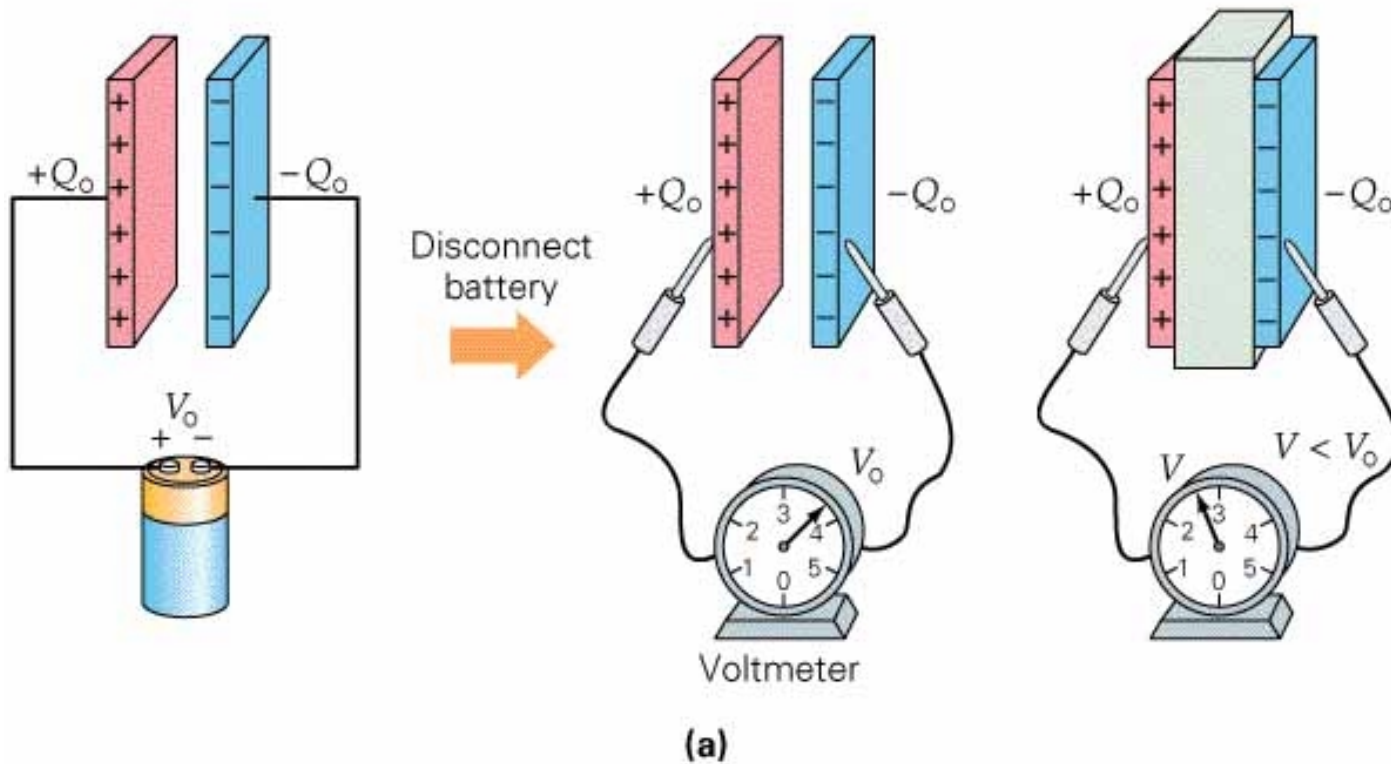
Botttom Line: Dielectrics reduce the effective electric field between the plates.

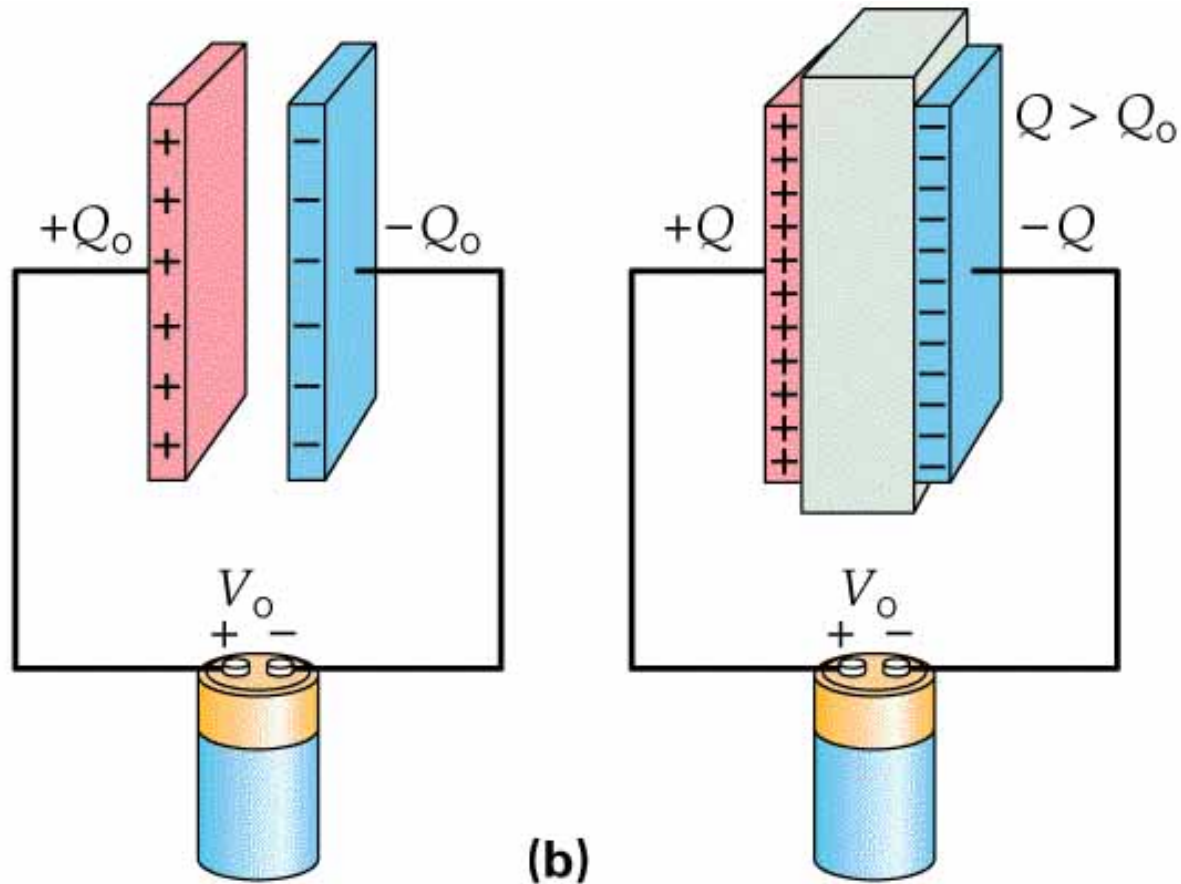*Definition: $\kappa = E_0 / E = V_0 / V$.   In addition*

$$C = \kappa C_0 = \kappa \varepsilon_0 A/d$$

↑

***NOT** the k in Coulombs Law Equation*

(a)

If battery disconnected then the total charge on the plates cannot change when dielectric inserted.

(b)

*If battery is connected when dielectric inserted
then the total voltage between the plates cannot change.
The charge on the plates can change, however.*

An empty capacitor is connected to a 12 V battery and charged up.
The capacitor is then disconnected and a slab of dielectric material (κ=2.8)
is inserted between the plates. Find the amount by which the potential
difference across the plates changes. Specify if this is an increase or decrease.

7.7 V decrease

Capacitors in the real world

Table 20.1: Dielectrics

Substance	Dielectric Constant, κ
Vacuum	1
Air	1.00059
Teflon	2.1
Mylar	3.1
Paper	3.7
Mica	5.4
Pyrex glass	5.6
Neoprene	6.7
Water	80.4

Figure 20–17 The energy required to charge a capacitor

A capacitor has a charge of magnitude Q on its plates and a potential difference V between the plates. Transferring a small charge increment, +△Q, from the negative plate to the positive plate increases the electric potential energy of the capacitor by the amount △U = (△Q)V...
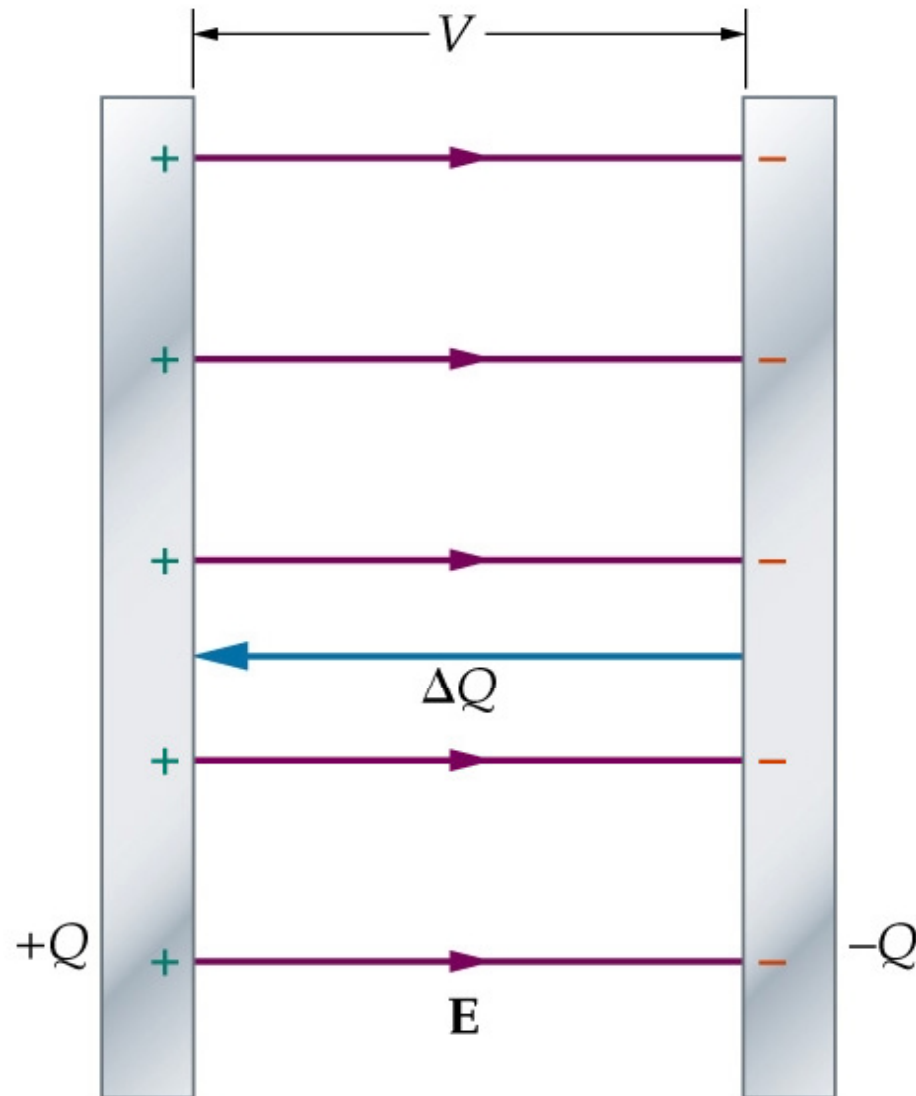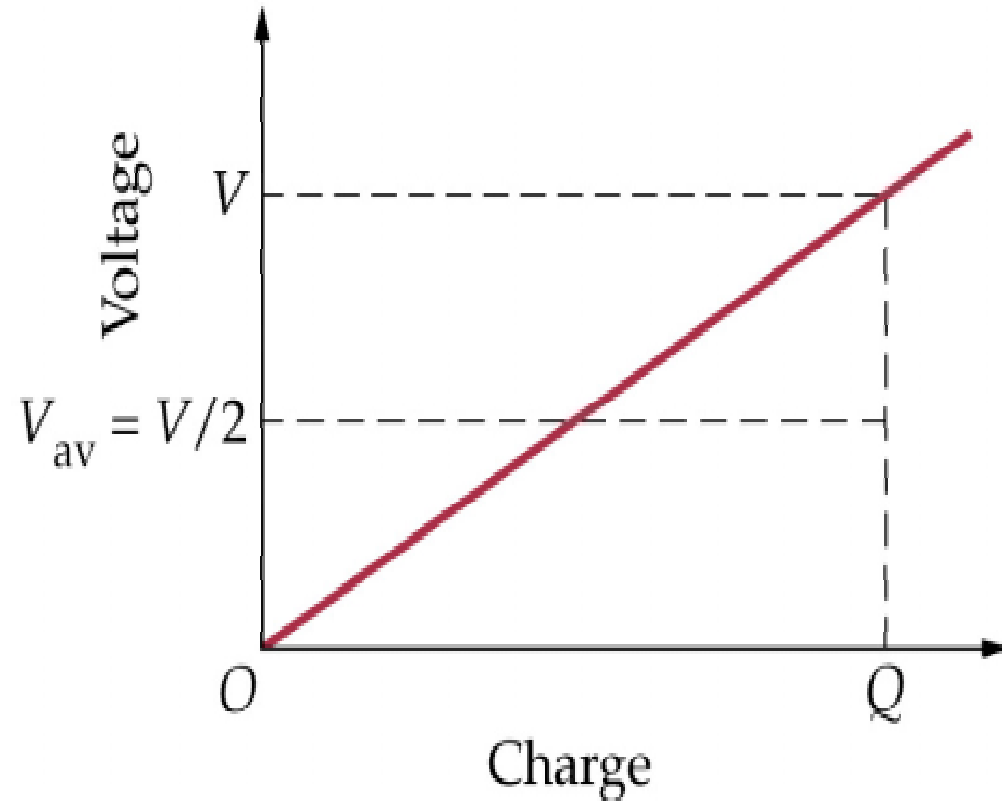
Figure 20–18 The voltage of a capacitor being charged

The voltage V between the plates of a capacitor increases linearly with the charge Q on the plates, $V = Q/C$. Therefore, if a capacitor is charged to a final voltage of V, the average voltage during charging was $V_{av} = 1/2 \, V$.

The energy of a capacitor being charged

When you have charged plates there is a lot of electrical potential energy stored that can be used to do work. For example, accelerate charges.
BUT How much energy is stored?

$$U_C = Q\bar{V}$$

$$= \frac{1}{2}QV$$

$$= \frac{Q^2}{2C}$$

$$= \frac{1}{2}CV^2$$

If you know any two of Q, V or C it is possible to calculate the energy stored.

$$\bar{V} = \frac{V+0}{2} = \frac{V}{2}$$

Voltage (V)

Slope = 1/C

Area = 1/2QV

Charge (C)