Chapter 24

Electric Potential

Electric Potential

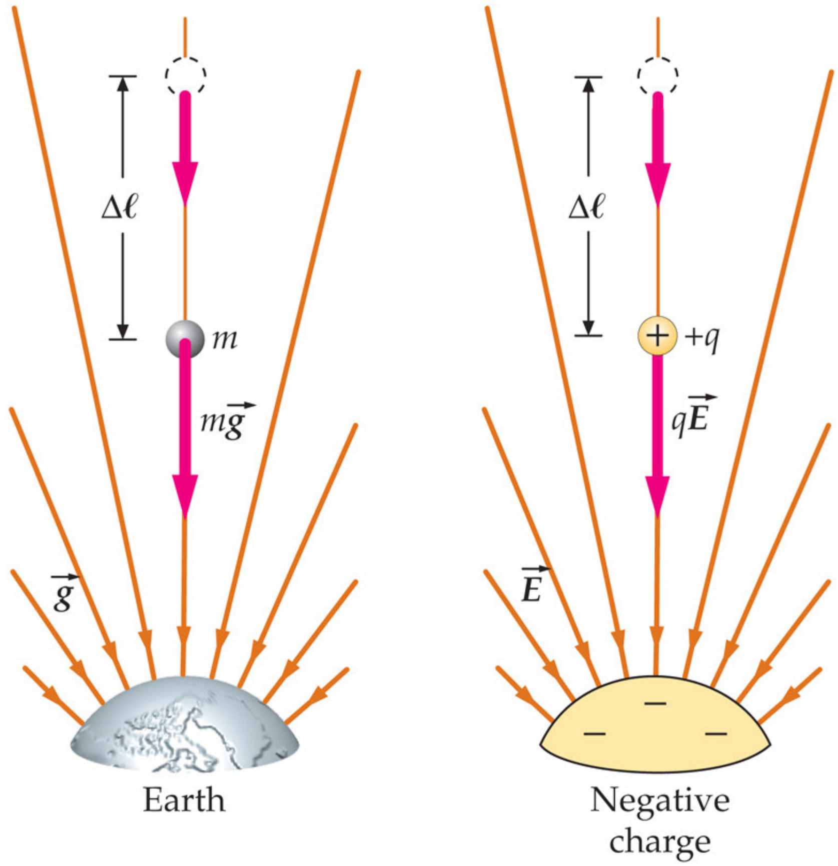
1. Potential Difference
2. Potential Due to a System of Point Charges
3. Computing the Electric Field from the Potential
4. V for Continuous Charge Distributions
5. Equipotential Surfaces
6. Electrostatic Potential Energy

Conservative Force

For conservative forces we can introduce the concept of a potential energy.

Potential energy is the energy of position whereas *kinetic energy* is the energy of motion.

Gravitational Analogy



The positive charge is used to make the analogy more transparent.

An Example from Last Semester

If you throw a ball straight up in the air its initial kinetic energy is soon reduced to zero at the top of its motion.

The "loss of kinetic energy" is only apparent. The kinetic energy is restored to its original value when the ball returns to its starting position.

The use of the concept of potential energy allows us to maintain the <u>conservation of total mechanical energy</u>.

Total Mechanical Energy = KE + PE

Examples of Forces

<u>Conservative Forces</u>

Gravitation

Electric force

Magnetic force

Spring force

<u>Non-Conservative Forces</u>

Friction

Air resistance

The kinetic energy is transformed into heat and is then outside the mechanical system.

The magnetic force has some restrictions on it since in general a velocity dependent force is not conservative.

Work and the Conservative Force

The characteristic of a conservative force is that the work done on an object moving it, in a force field, from point A to point B is *independent of the path* taken between A and B.

If the path forms a closed loop then the work done is zero.

$$W = \oint_C \vec{F} \cdot d\vec{L} = 0$$

Potential Energy - Potential Function

There is a *potential energy* (U) and a *potential function* (V) and they are closely related.

Potential Energy

$$dU = -\vec{F} \bullet d\vec{L}$$

$$\vec{F} = q\vec{E}$$

$$dU = -q\vec{E} \bullet d\vec{L}$$

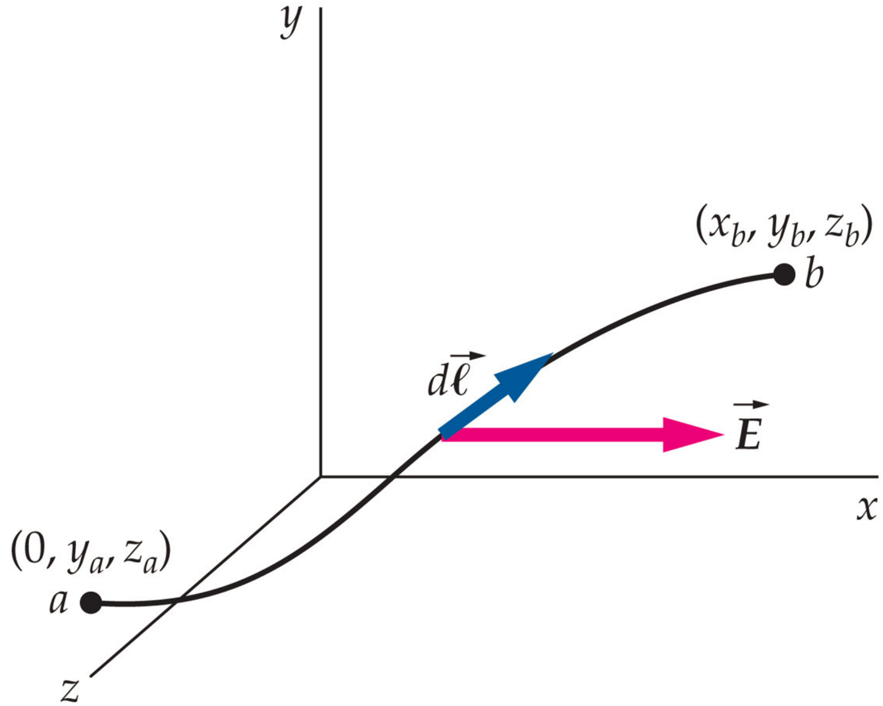
Potential Function

$$dU = -q_0 \vec{E} \bullet d\vec{L}$$

$$dV \equiv \frac{dU}{q_0} = -\vec{E} \bullet d\vec{L}$$

$$\Delta V = V_b - V_a = \frac{\Delta U}{q_0} = -\int_a^b \vec{E} \bullet d\vec{L}$$

The Change in Potential a$\longrightarrow$ b



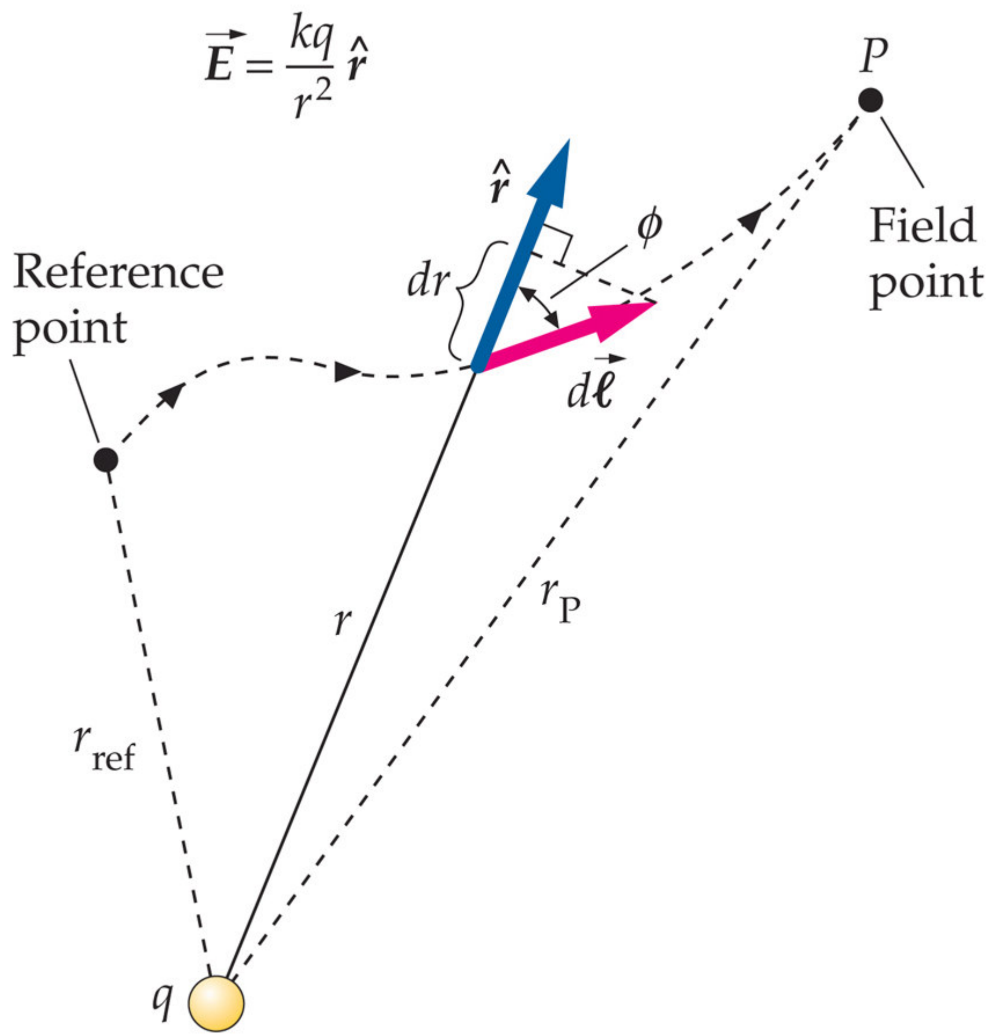
The *integral* is equal to the *difference* between the *potential function* evaluated at the end points.

$$\Delta V = V_b - V_a = \frac{\Delta U}{q_0} = -\int_a^b \vec{E} \bullet d\vec{L}$$

Potentials for Discrete Charges

Potential and the Reference Point

The potential function needs a common reference point so that the calculated potential differences will have physical meaning.



Potential and the Point Charge

$$\vec{E} = \frac{kq}{r^2}\hat{r}$$

$$V_P - V_{ref} = -\int_{ref}^{P} \vec{E}\bullet d\vec{L} = -\int_{ref}^{P} \frac{kq}{r^2}\hat{r}\bullet d\vec{L} = -\int_{ref}^{P} \frac{kq}{r^2}\,dr$$

$$V_P - V_{ref} = V_P - 0 = -kq\int_{ref}^{P} \frac{dr}{r^2} = \frac{kq}{r_P} - \frac{kq}{r_{ref}}$$

$$V = \frac{kq}{r};\quad as\ r_{ref}\ goes\ to\ \infty$$ ⟵ Final form

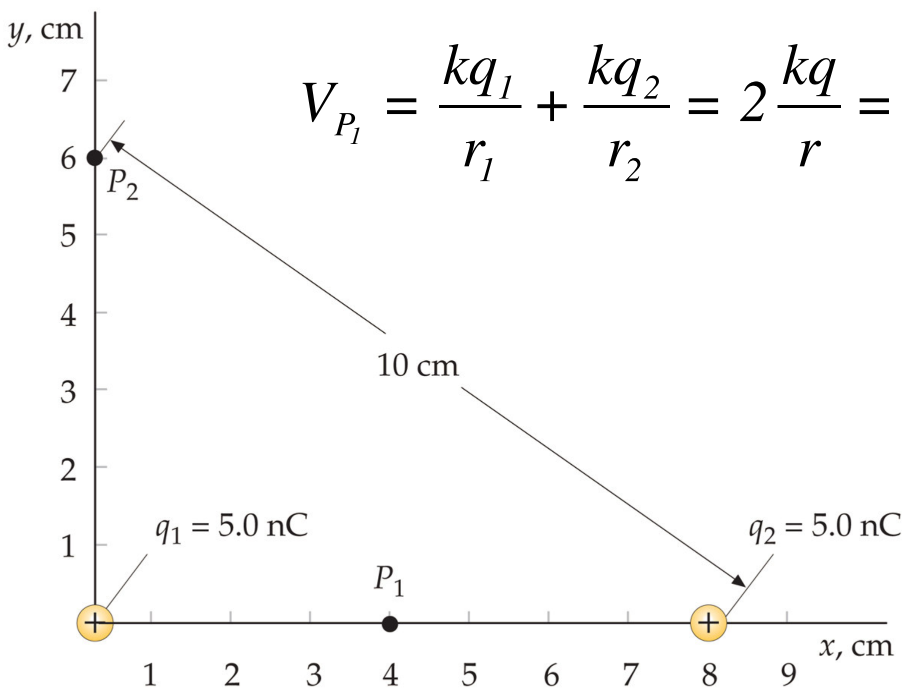
Potential Due to Two Point Charges

Find V at points P_1 and P_2

$$V = \sum_i \frac{kq_i}{r_i} = \frac{kq_1}{r_1} + \frac{kq_2}{r_2}$$

$$V_{P_2} = \frac{kq_1}{r_1^{'}} + \frac{kq_2}{r_2^{'}} = \frac{9x10^9 * 5.0x10^{-9}}{.060} + \frac{9x10^9 * 5.0x10^{-9}}{.010} = 1200\,V$$

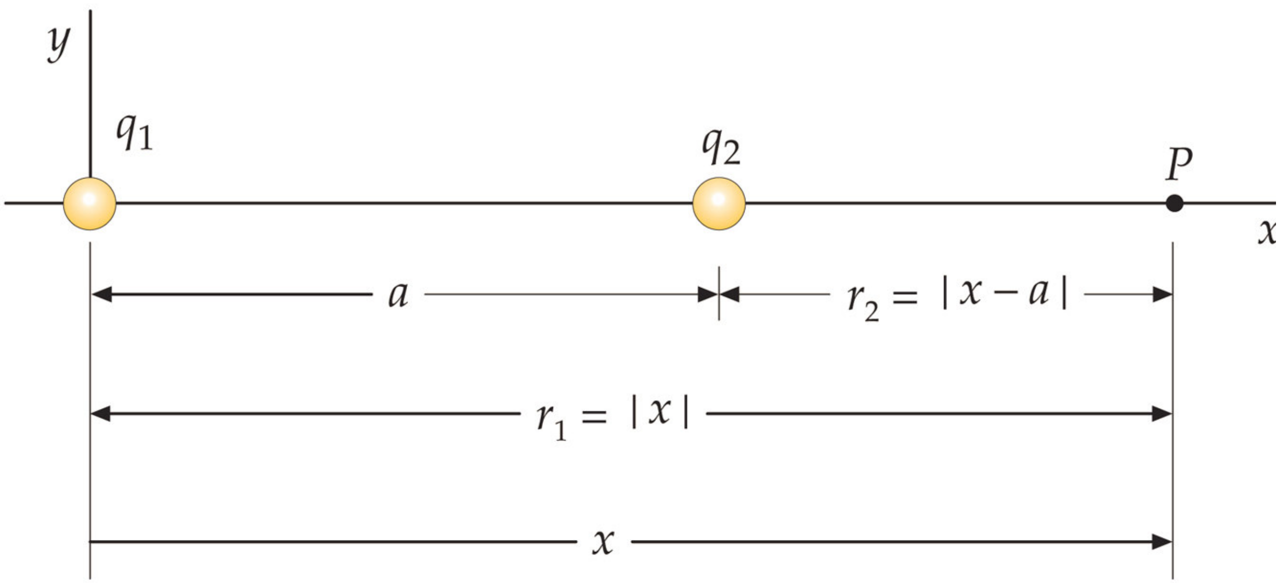
$$V_{P_1} = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} = 2\frac{kq}{r} = 2\frac{9x10^9 * 5.0x10^{-9}}{.040} = 2247\,V$$



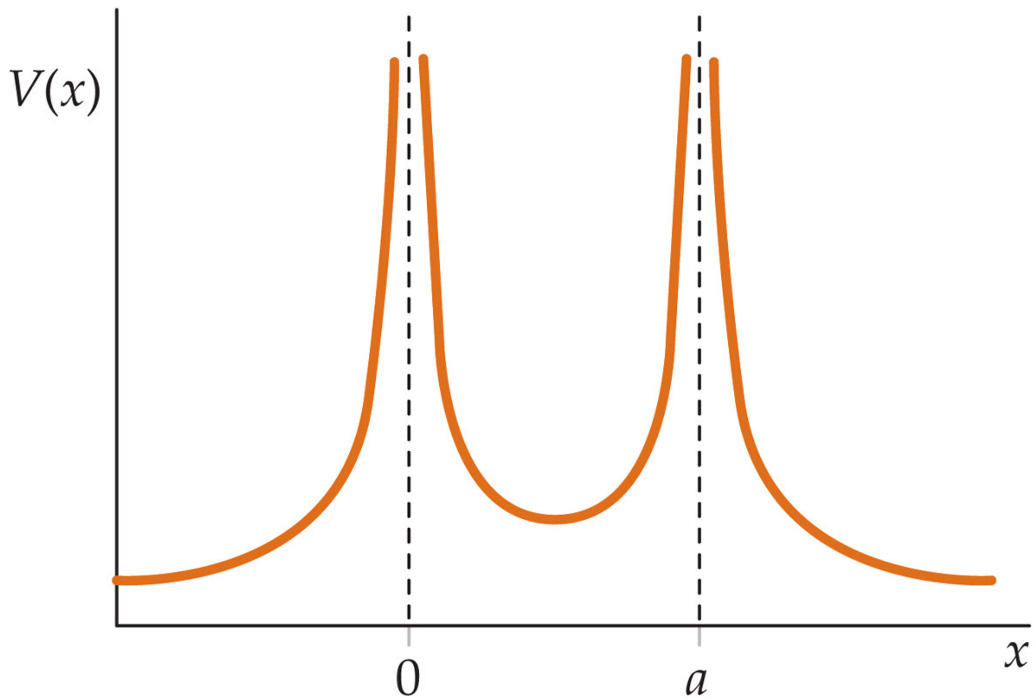
Potential Calculation for 2 Point Charges

$$V = \sum_i \frac{kq_i}{r_i} = \frac{kq_1}{r_1} + \frac{kq_2}{r_2}$$

$$V = \frac{kq_1}{|x|} + \frac{kq_2}{|x - a|}$$



Potential for 2 Point Charges



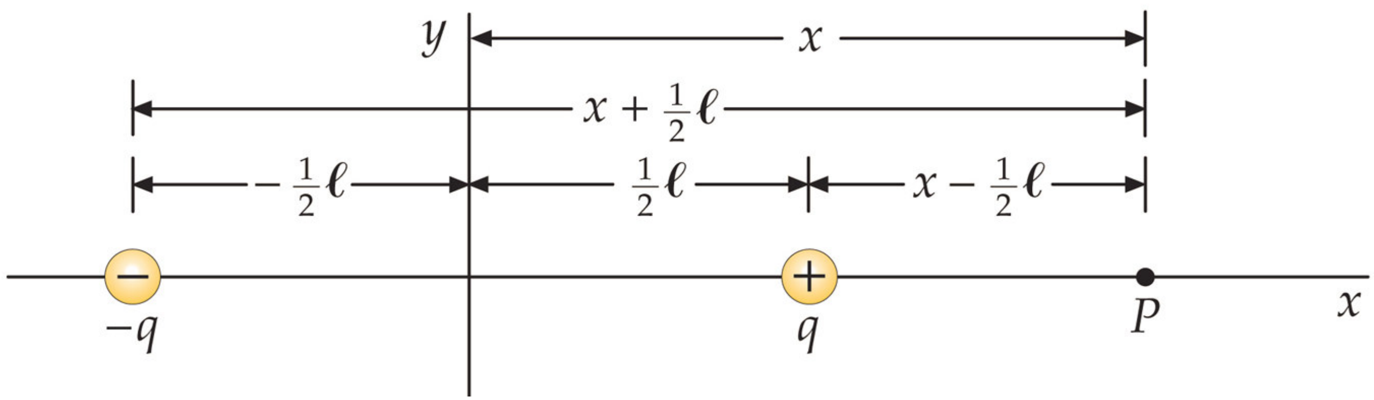
In contrast to the electric field component graphs the potential graphs, being scalar quantities, depend on the sign of the charge, there is no direction.

Electric Dipole Geometry for Potential Calculation

$$V = \frac{kql}{x^2 - \left(l^2/4\right)};\ x > \frac{l}{2}$$

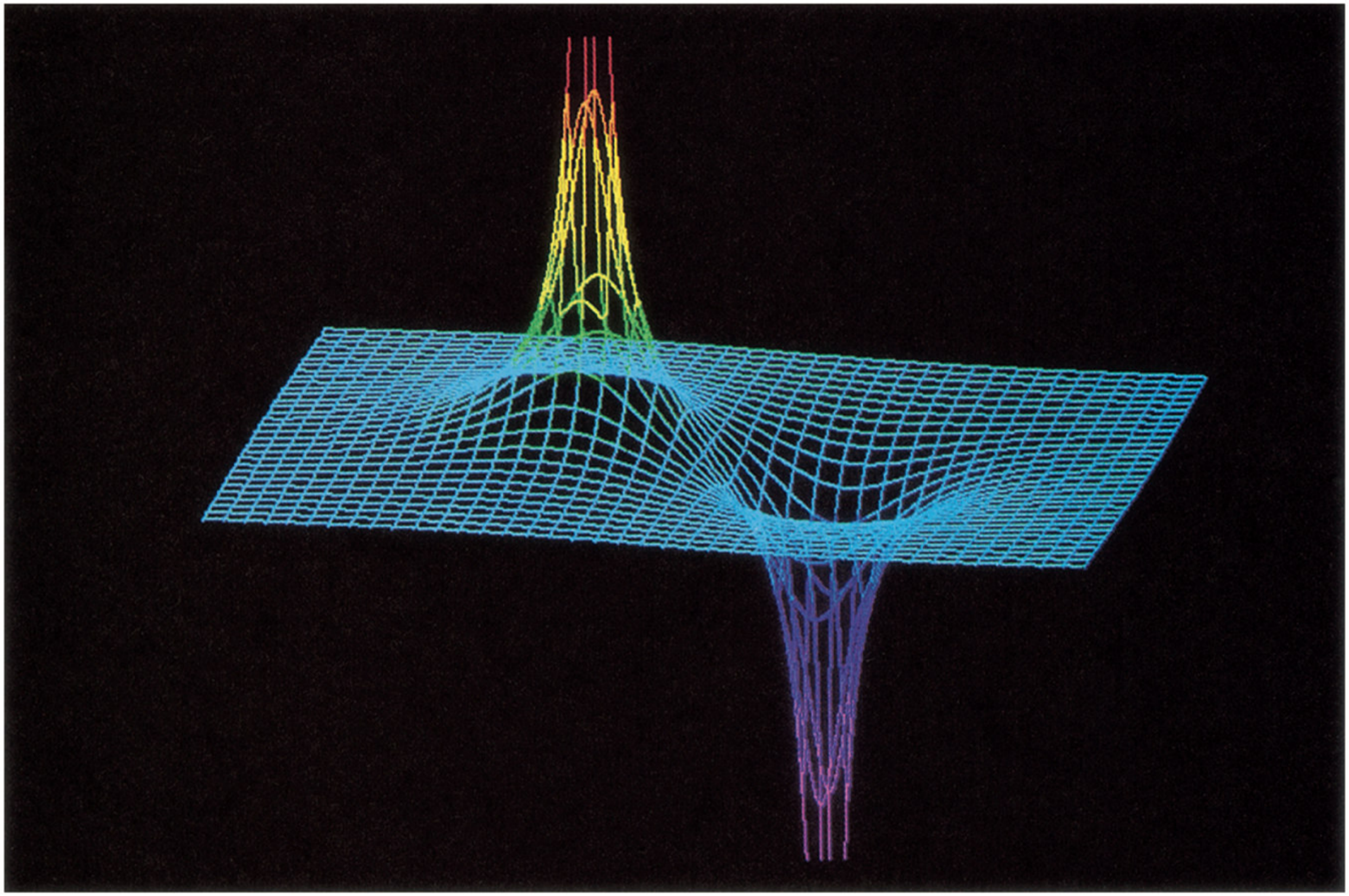
$$V \approx \frac{kql}{x^2} = \frac{kp}{x^2};\ x \gg l$$

p = ql is the dipole moment



The dipole configuration requires equal and opposite charges.

3D Image of the Electric Dipole Potential



Electric Field from the Electric Potential

For one dimension

$$dV(x) = -\vec{E} \bullet d\vec{l} = -\vec{E} \bullet dx\hat{i} = -(\vec{E} \bullet \hat{i})dx = -E_x dx$$

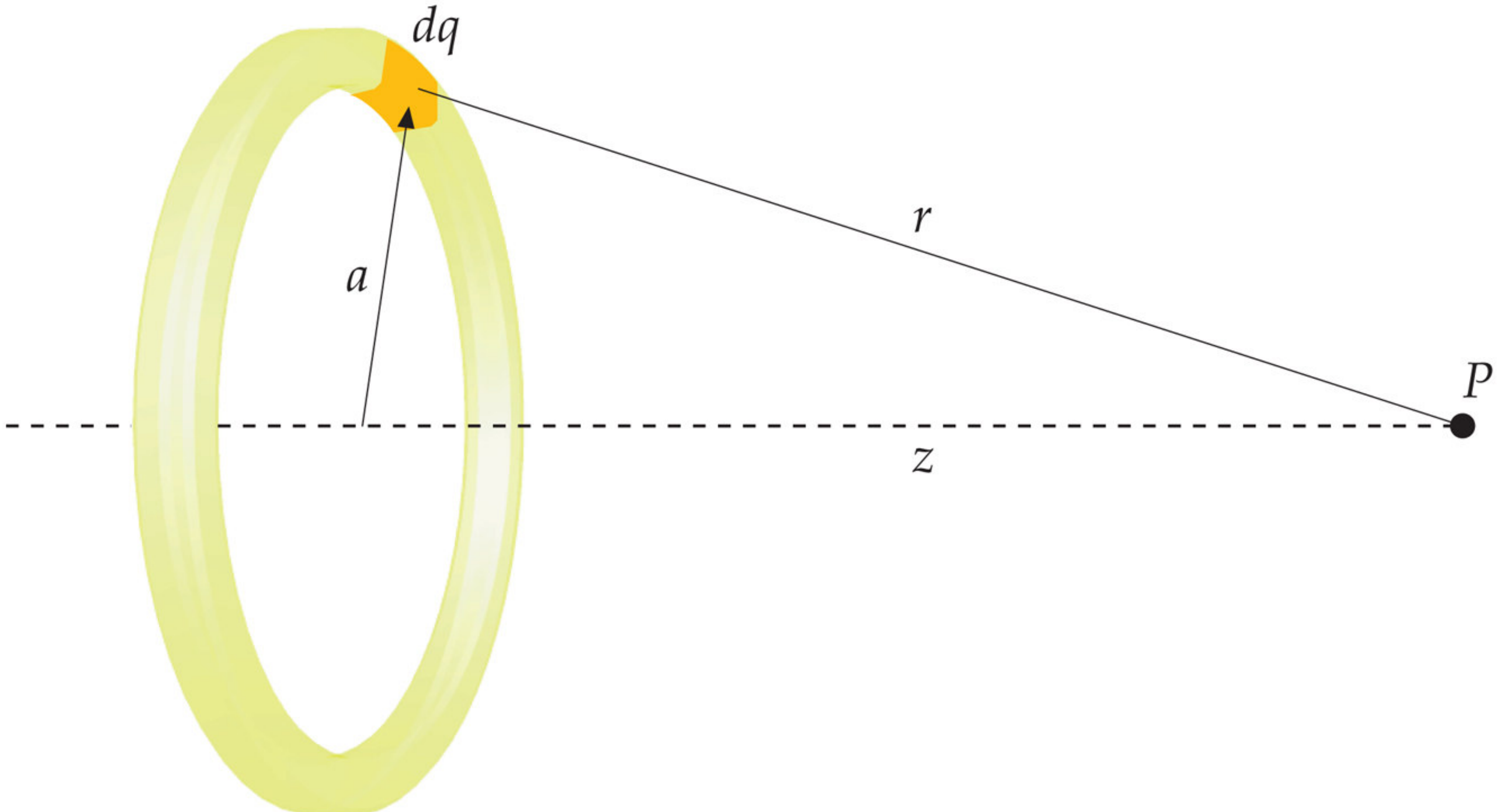
$$E_x = -\frac{dV(x)}{dx}$$

In general

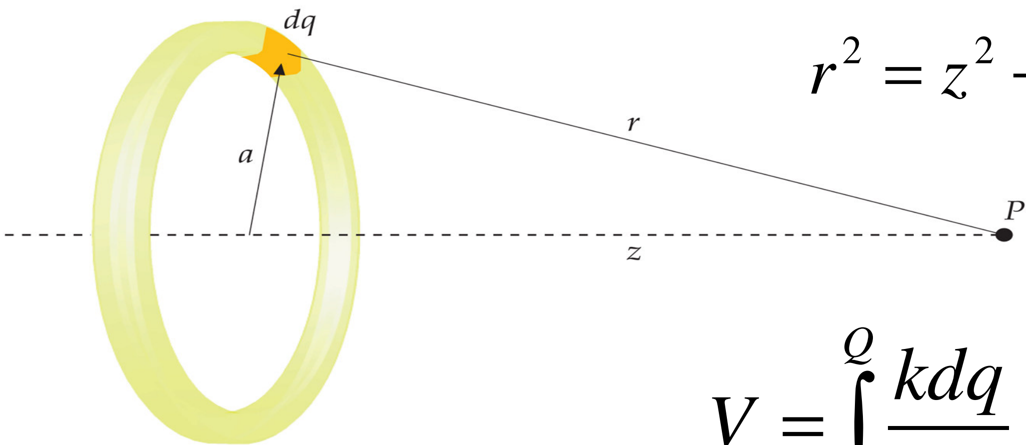
$$\vec{E} = -\nabla V = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k} \right) = -gradV$$

Potentials for Continuous Charge Distributions

Potential Due to a Ring Charge



Potential Due to a Ring Charge



$$r^2 = z^2 + a^2$$

$$V = \int_0^Q \frac{kdq}{r} = \frac{k}{r}\int_0^Q dq = \frac{kQ}{r}$$

$$V = \frac{kQ}{\left[z^2 + a^2\right]^{1/2}} = \frac{kQ}{\left[z^2\right]^{1/2}\left[1 + \frac{a^2}{z^2}\right]^{1/2}} = \frac{kQ}{|z|\left[1 + \frac{a^2}{z^2}\right]^{1/2}}$$

Electric Field Due to a Ring Charge

$$E_z = -\frac{\partial V}{\partial z} = -kQ\frac{\partial}{\partial z}\left[z^2 + a^2\right]^{-1/2} = \frac{1}{2}kQ\left[z^2 + a^2\right]^{-3/2} 2z$$

$$\vec{E} = E_z\hat{k} = \frac{kQz}{\left[z^2 + a^2\right]^{3/2}}\hat{k}$$

$$\vec{E} = E_z\hat{k} = \frac{kQz}{\left[z^2 + a^2\right]^{3/2}}\hat{k} = \frac{kQz}{\left[z^2\right]^{3/2}\left[1 + \frac{a^2}{z^2}\right]^{3/2}}\hat{k} = \frac{kQz}{|z|^3}\left[1 + \frac{a^2}{z^2}\right]^{-3/2}\hat{k}$$

Electric Field Due to a Ring Charge

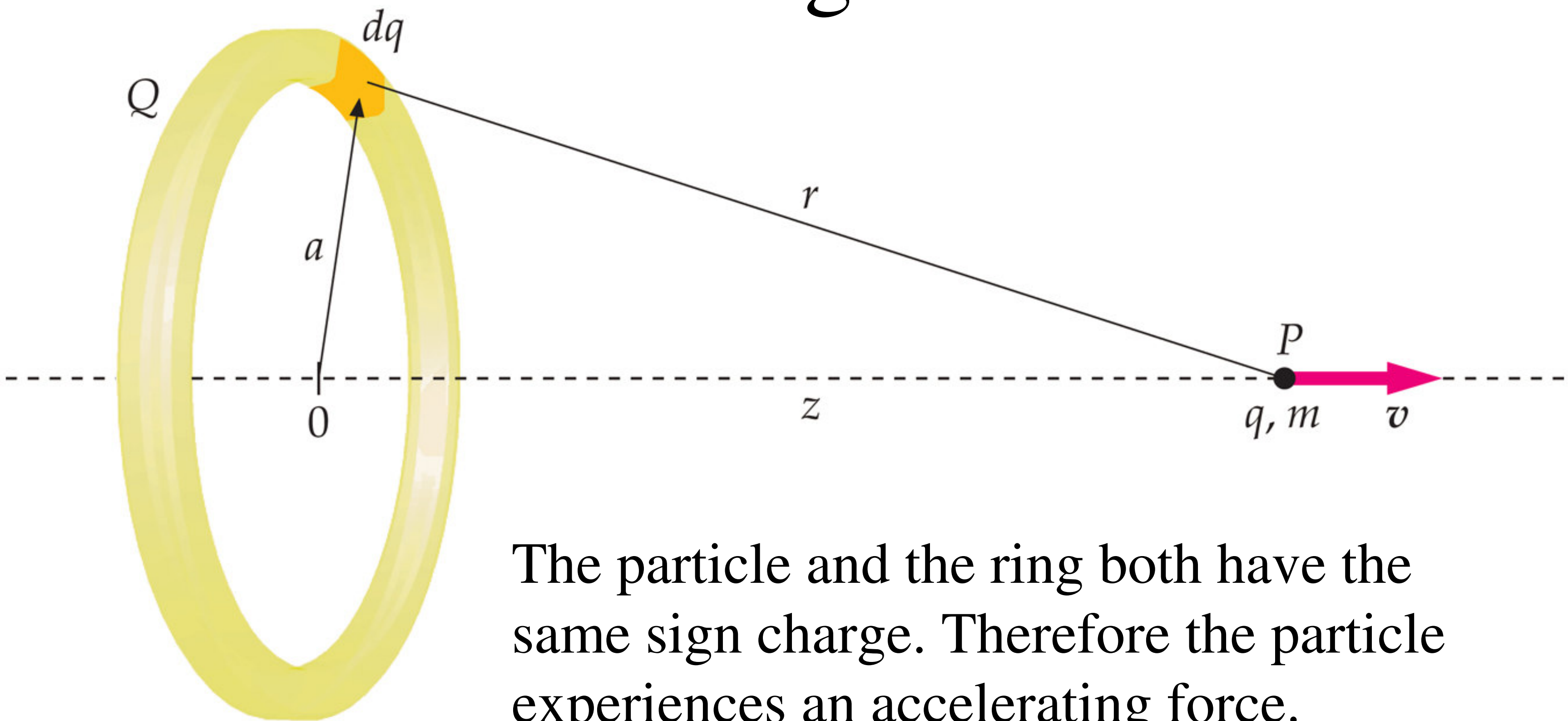
Far Away from the Ring

$$\vec{E} = E_z\hat{k} = \frac{kQz}{|z|^3}\left[1+\frac{a^2}{z^2}\right]^{-3/2}\hat{k} = \frac{kQz}{|z|^3}\left[1-\frac{3}{2}\frac{a^2}{z^2}\right]\hat{k}$$

$$\lim_{z\to\infty}\frac{kQz}{|z|^3}\left[1-\frac{3}{2}\frac{a^2}{z^2}\right]\hat{k} = \frac{kQ}{z^2}\hat{k}$$

This the electric field of a point charge of value Q

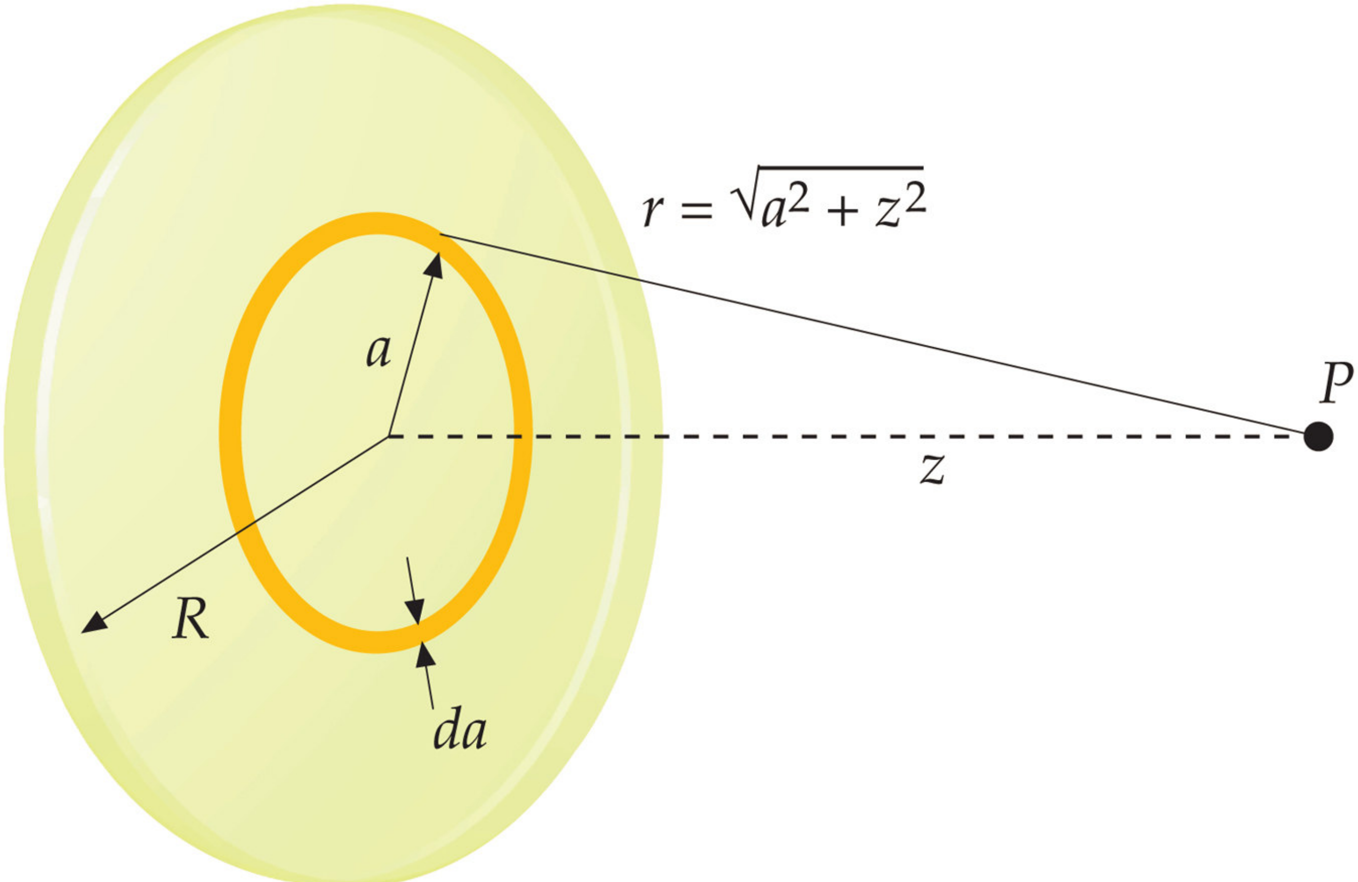
Particle Acceleration Due to a Ring Charge



The particle and the ring both have the same sign charge. Therefore the particle experiences an accelerating force.

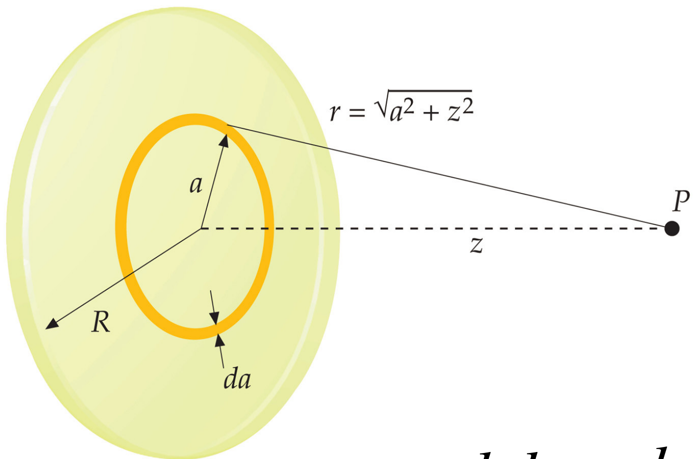
Do calculation

Potential Due to a Disk Charge



For simplicity we only consider potentials along the z-axis

Potential Due to a Disk Charge



$$dV = \frac{kdq}{r}$$

$$dq = \sigma dA = \sigma 2\pi a da$$

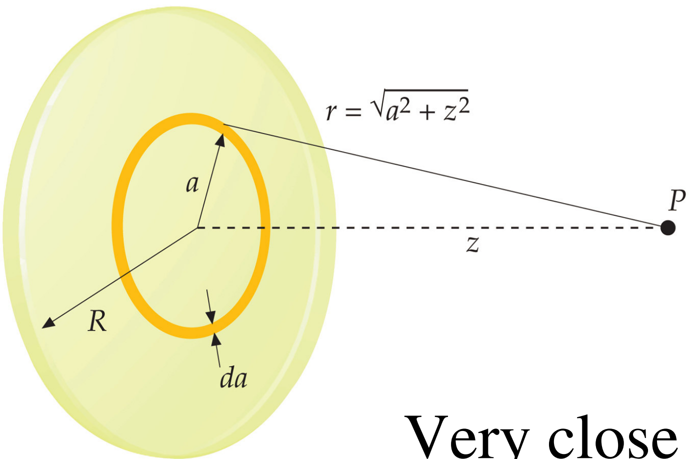
$$\sigma = Q/_{\pi R^2}$$

$$dV = \frac{kdq}{r} = \frac{k\left(2\pi\sigma a da\right)}{\left(z^2 + a^2\right)^{1/2}}$$

$$\int_0^V dV = V = 2k\sigma\pi\int_0^R \frac{a da}{\left(z^2 + a^2\right)^{1/2}}$$

$$V(z) = 2\pi k\sigma\left|z\right|\left(\sqrt{1 + \frac{R^2}{z^2}} - 1\right)$$

Potential Close to a Disk Charge



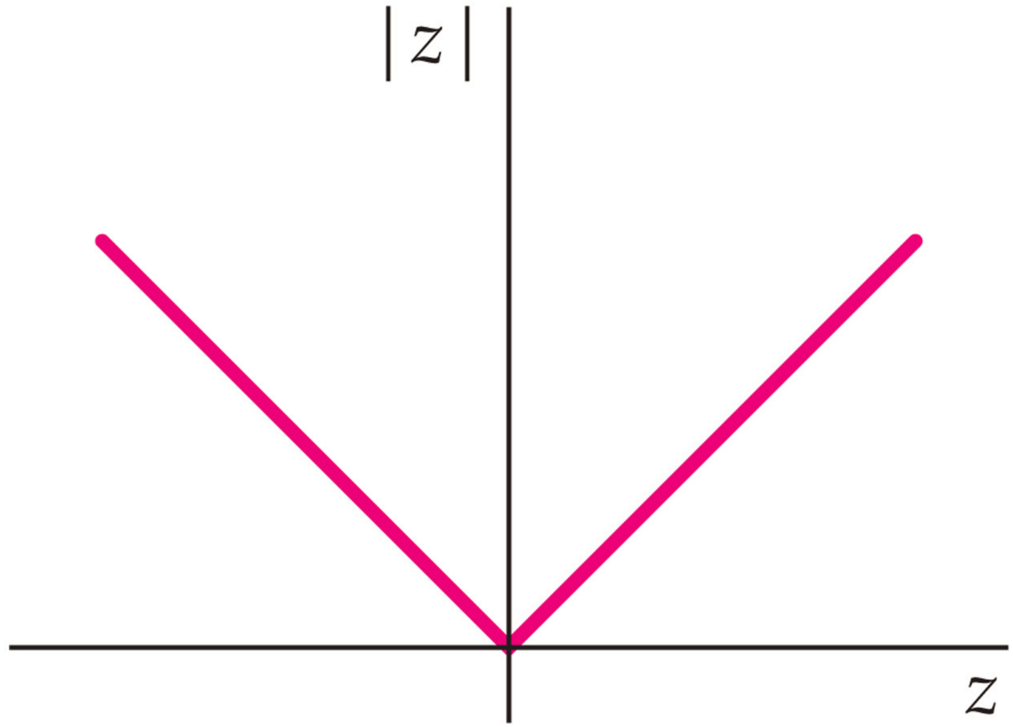
$$V(z) = 2\pi k\sigma |z| \left(\sqrt{1 + \frac{R^2}{z^2}} - 1 \right)$$

Very close to the disk z<<R. The disk appears to effectively be an infinite plane.

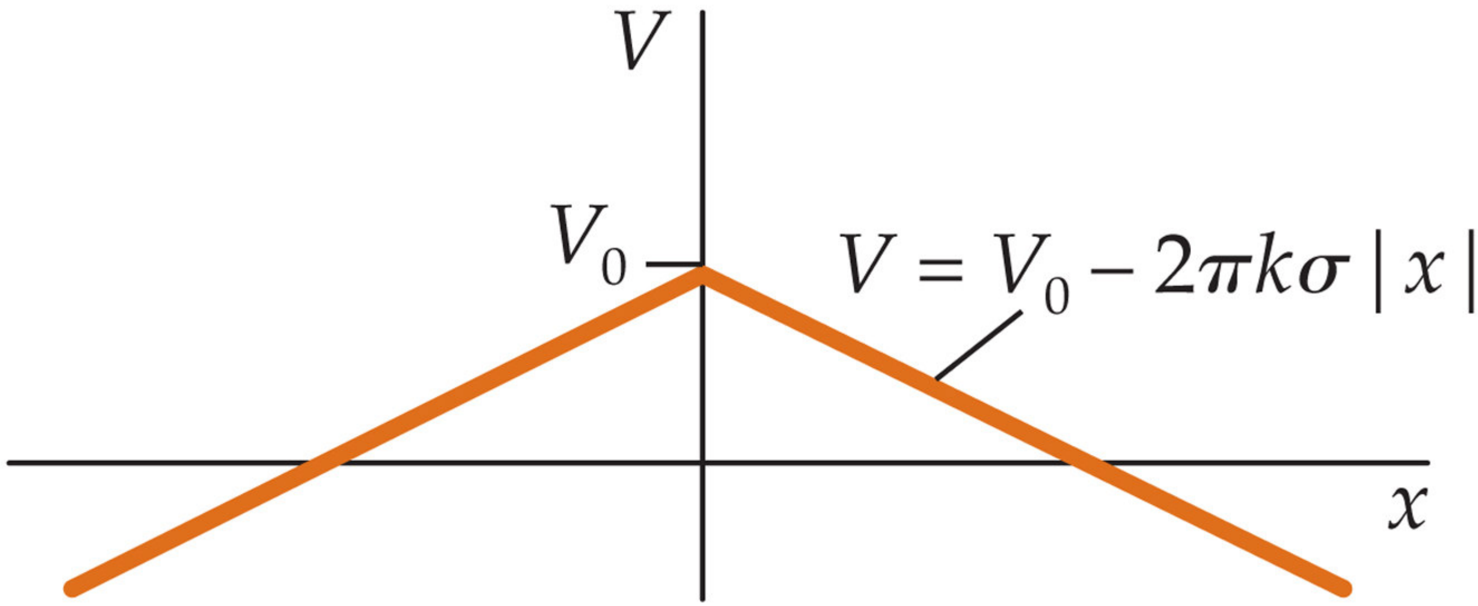
$$V(z) = 2\pi k\sigma |z| \left(\sqrt{1 + \frac{R^2}{z^2}} - 1 \right) \approx 2\pi k\sigma |z| \left(\frac{R}{|z|} - 1 \right)$$

$$V(z) = 2\pi k\sigma R - 2\pi k\sigma |z| = V_0 - 2\pi k\sigma |z|$$

The Absolute Value Operation



Potential Due to an Infinite Plane of Charge



The potential function is required to be continuous but its derivatives need not be continuous.

The Electric Field Due to an Infinite Plane of Charge (z>0)

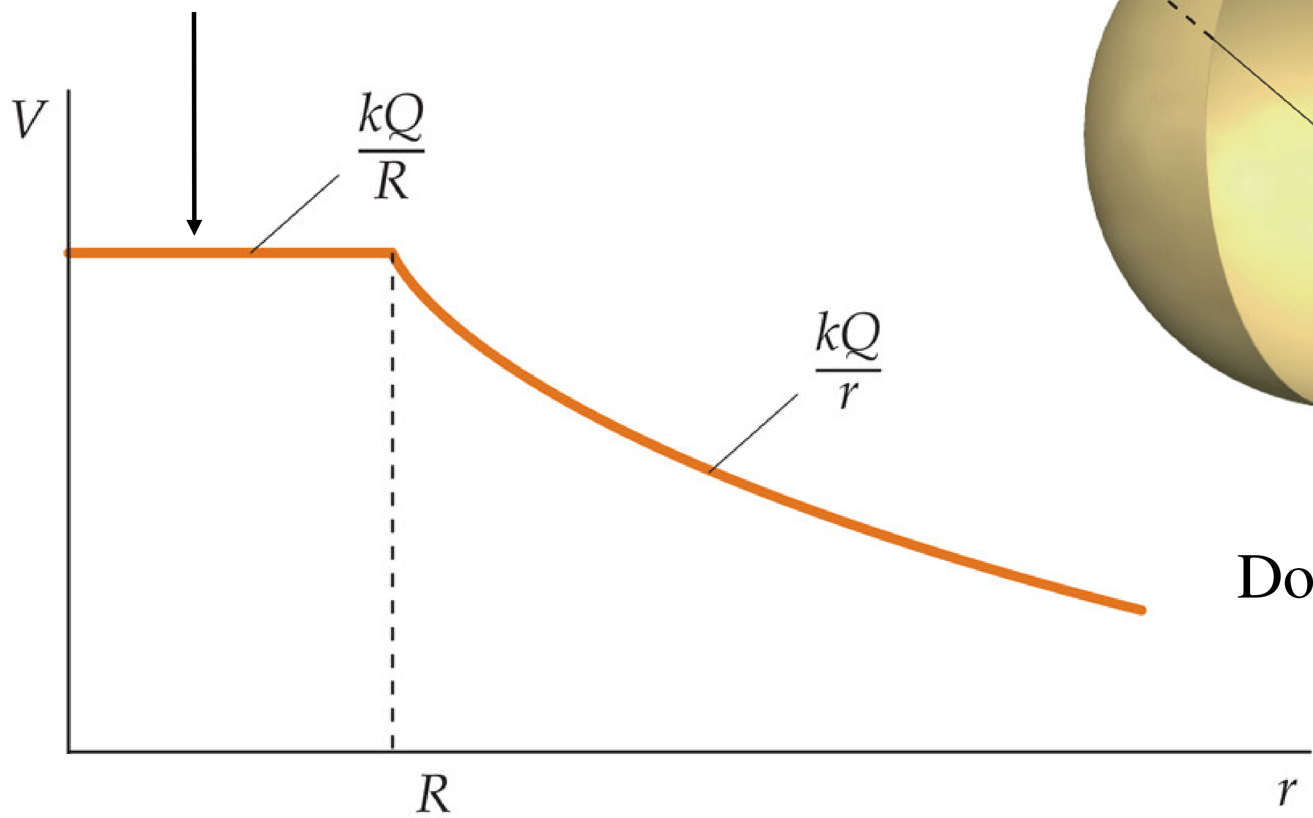
$$V(z) = V_0 - 2\pi k\sigma|z|$$

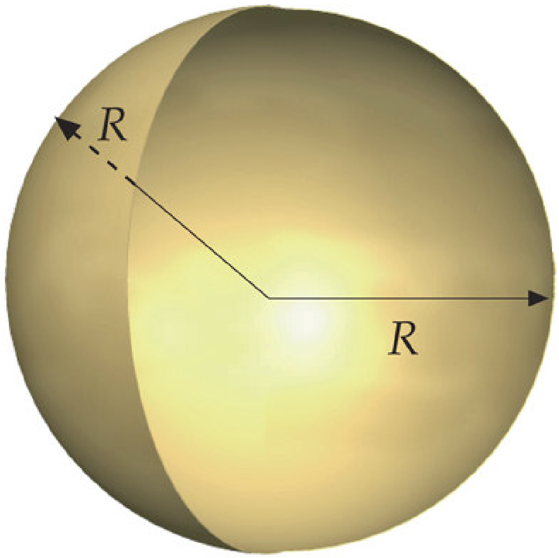
$$E_n(z) = -\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}\left(2\pi k\sigma|z|\right) = 2\pi k\sigma$$

$$E_n(z) = 2\pi\left(\frac{1}{4\pi\varepsilon_0}\right)\sigma = \frac{\sigma}{2\varepsilon_0}$$

Potential Due to a Thin Spherical Shell

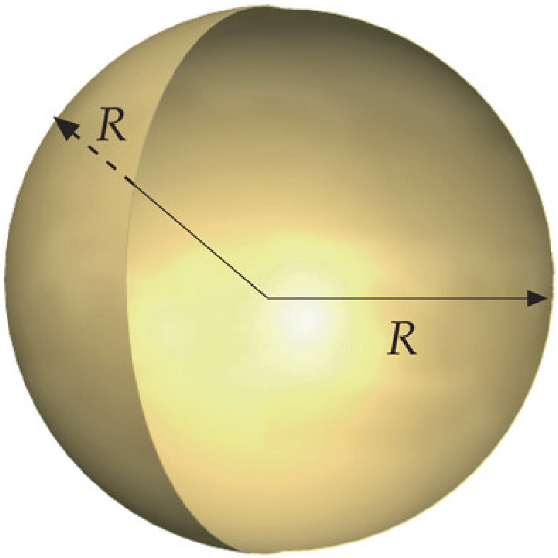
No electric field inside, therefore
V is constant inside the sphere.





Do calculation

Potential Due to a Thin Spherical Shell



$$dV = -\vec{E} \bullet d\vec{l} \quad \text{For r > R, } \vec{E} = \frac{kQ}{r^2}\hat{r}$$

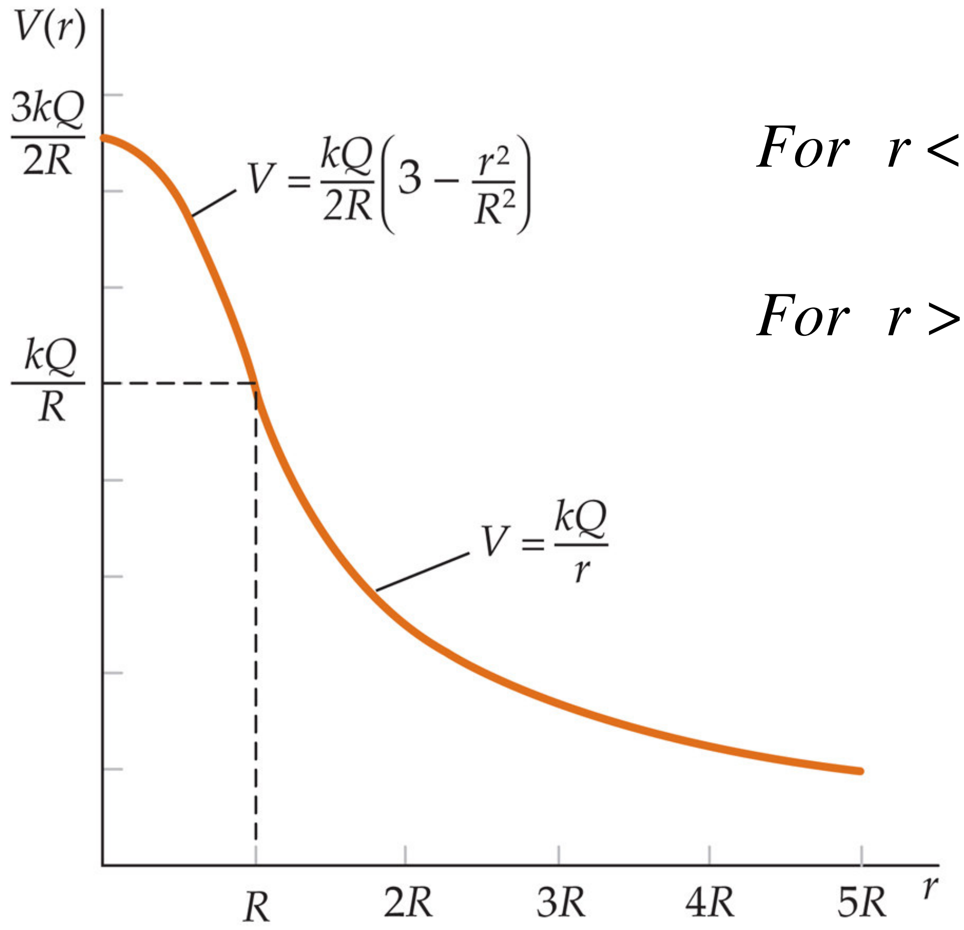
$$dV = -\frac{kQ}{r^2}\hat{r} \bullet d\vec{l}$$

$$V_p = -\int_{+\infty}^{r_p} \frac{kQ}{r^2} dr = -kQ \int_{+\infty}^{r_p} r^{-2} dr = \frac{kQ}{r_p}$$

For r < R

$$V_p = -\int_{+\infty}^{r_p} \frac{kQ}{r^2} dr = -kQ \int_{+\infty}^{R} r^{-2} dr - \int_{R}^{r_p} (0)\ dr = \frac{kQ}{R}$$

Potential Due to a Uniformly Charged Solid Sphere



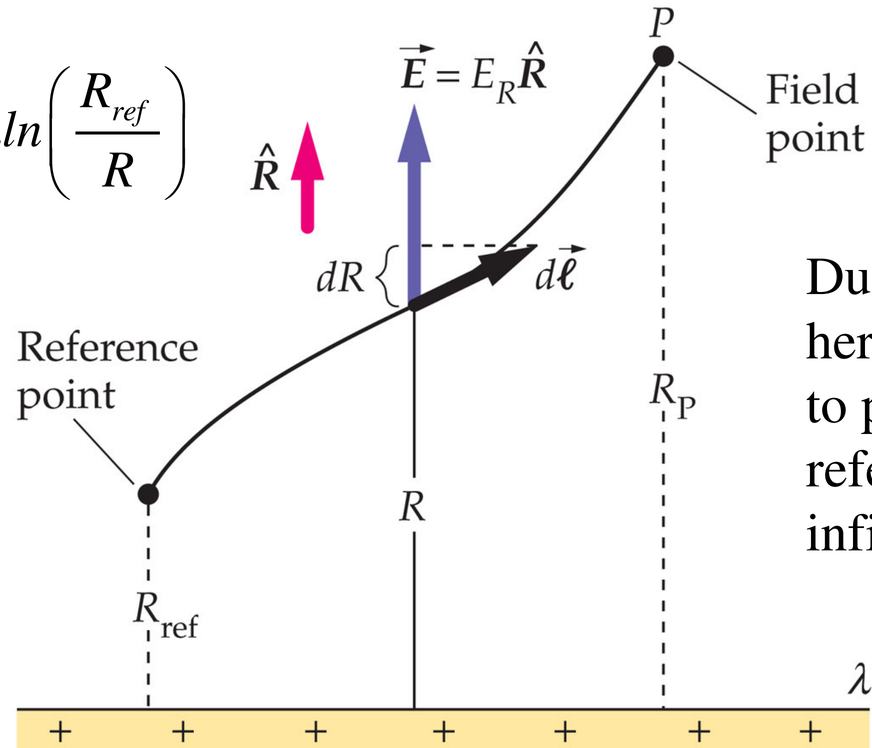
$$For \quad r < R, \quad E_r = \frac{kQ}{R^3} r$$

$$For \quad r > R, \quad E_r = \frac{kQ}{R^2}$$

Potential For an Infinite Line Charge

$$\Delta V = V_P - V_{Ref} = -\int_{Ref}^{P} \vec{E} \cdot d\vec{L}$$

$$V_P - V_{ref} = V_P - 0 = 2k\lambda \ln\left(\frac{R_{ref}}{R}\right)$$

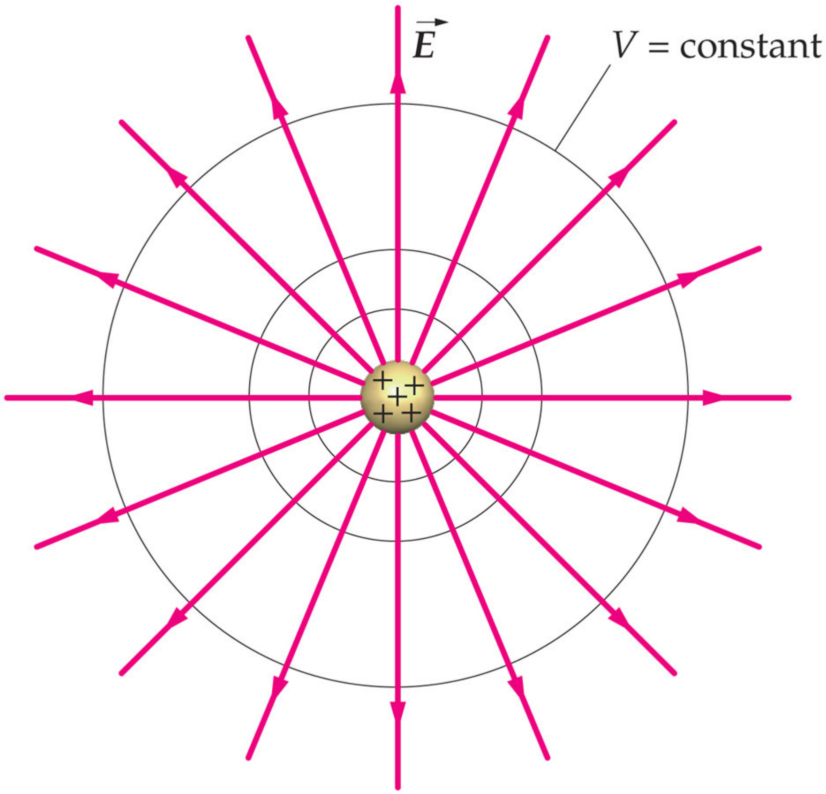


Here we are going to use our knowledge of the field to determine the potential.

Due to the geometry here we are unable to place the reference point at infinity.

Since it is the "difference in potential" that has a physical meaning, whatever reference point we pick will drop out upon taking the difference of potential.

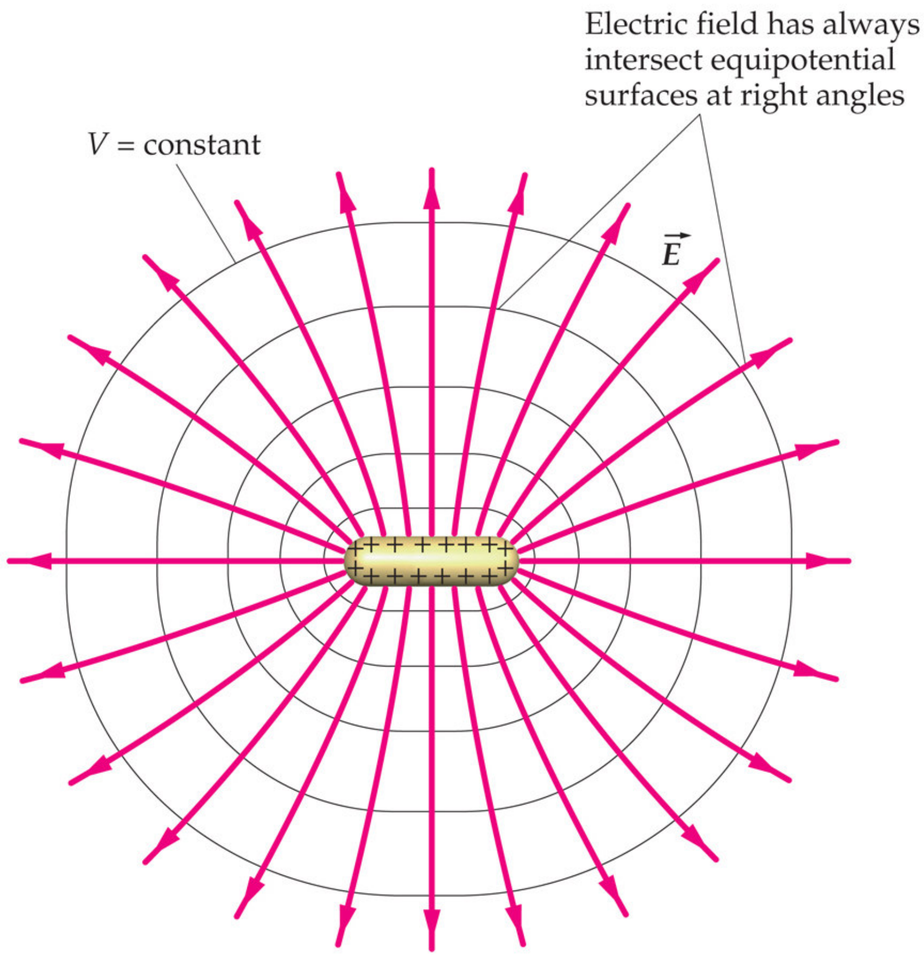
Equipotential Surfaces



The equipotentials (gray lines) are represented by contours along which the value of the potential function is constant.

Potential lines and electric field lines are everywhere perpendicular.

Equipotential Surfaces

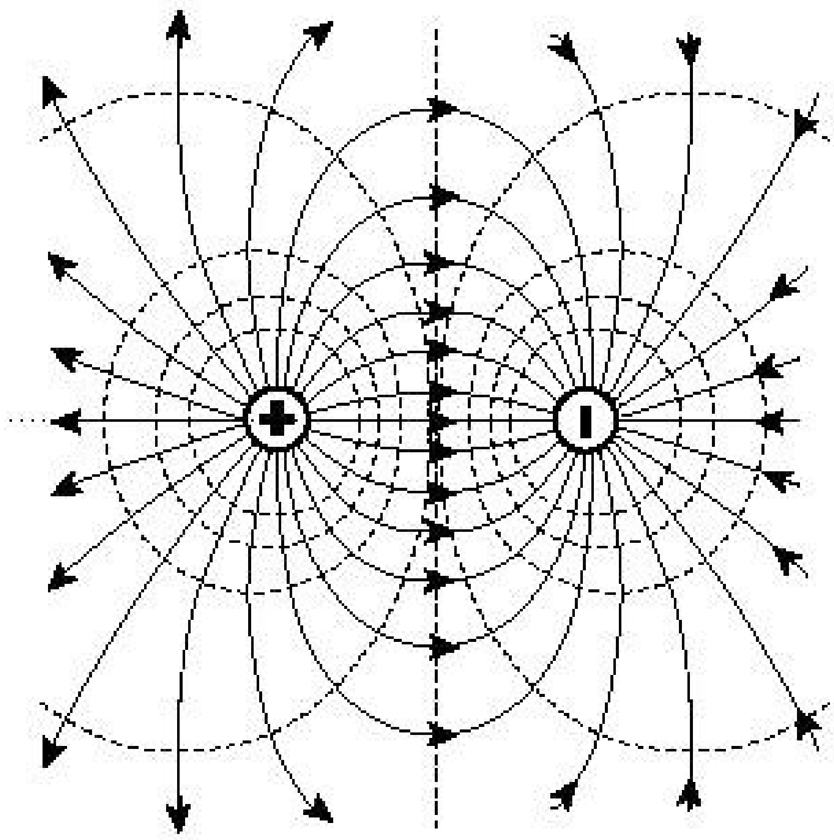


Close to the surface of the conductor the equipotential surfaces follow the shape of the conductor.

The surface of a conductor is an equipotential surface.

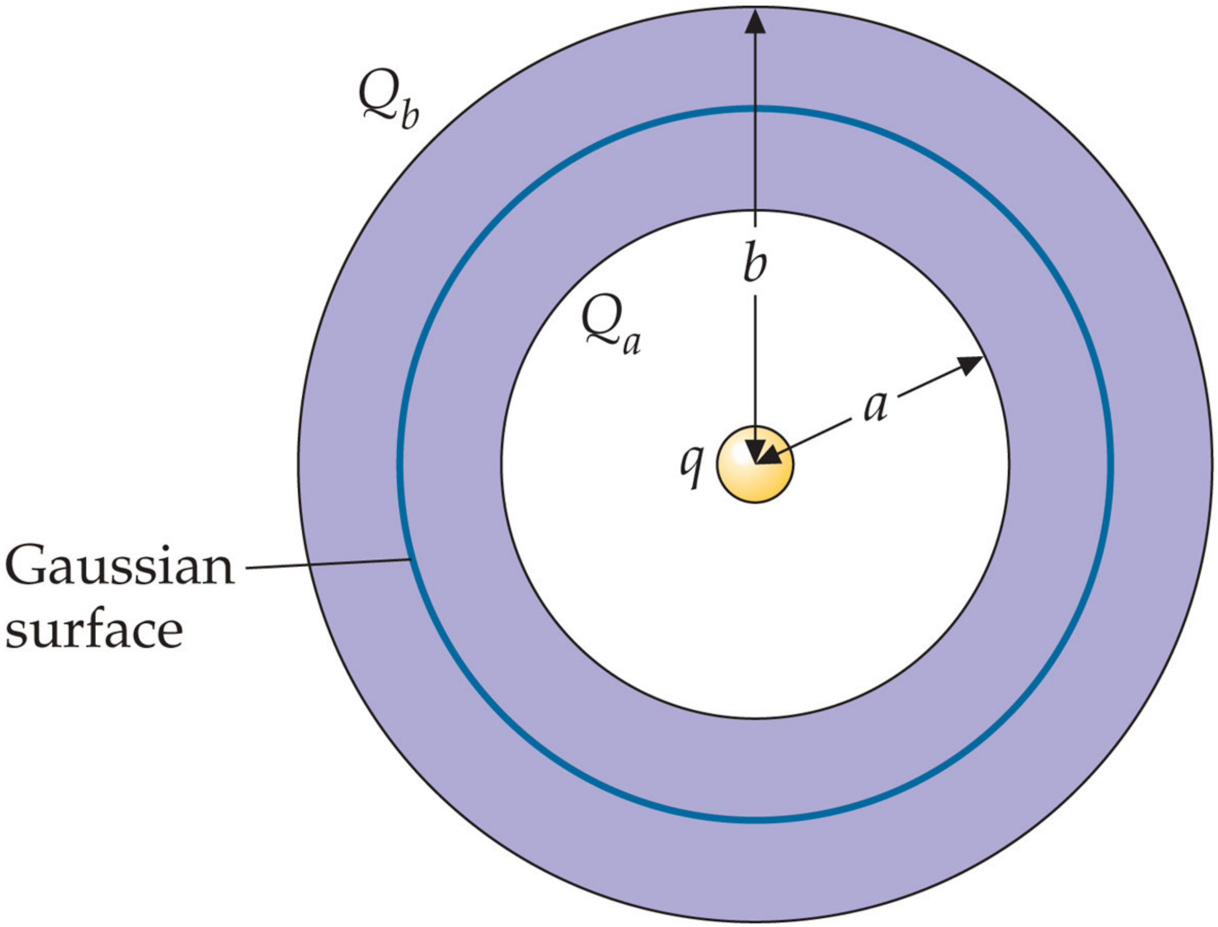
Equipotential Surfaces-Point Charges

Field Lines

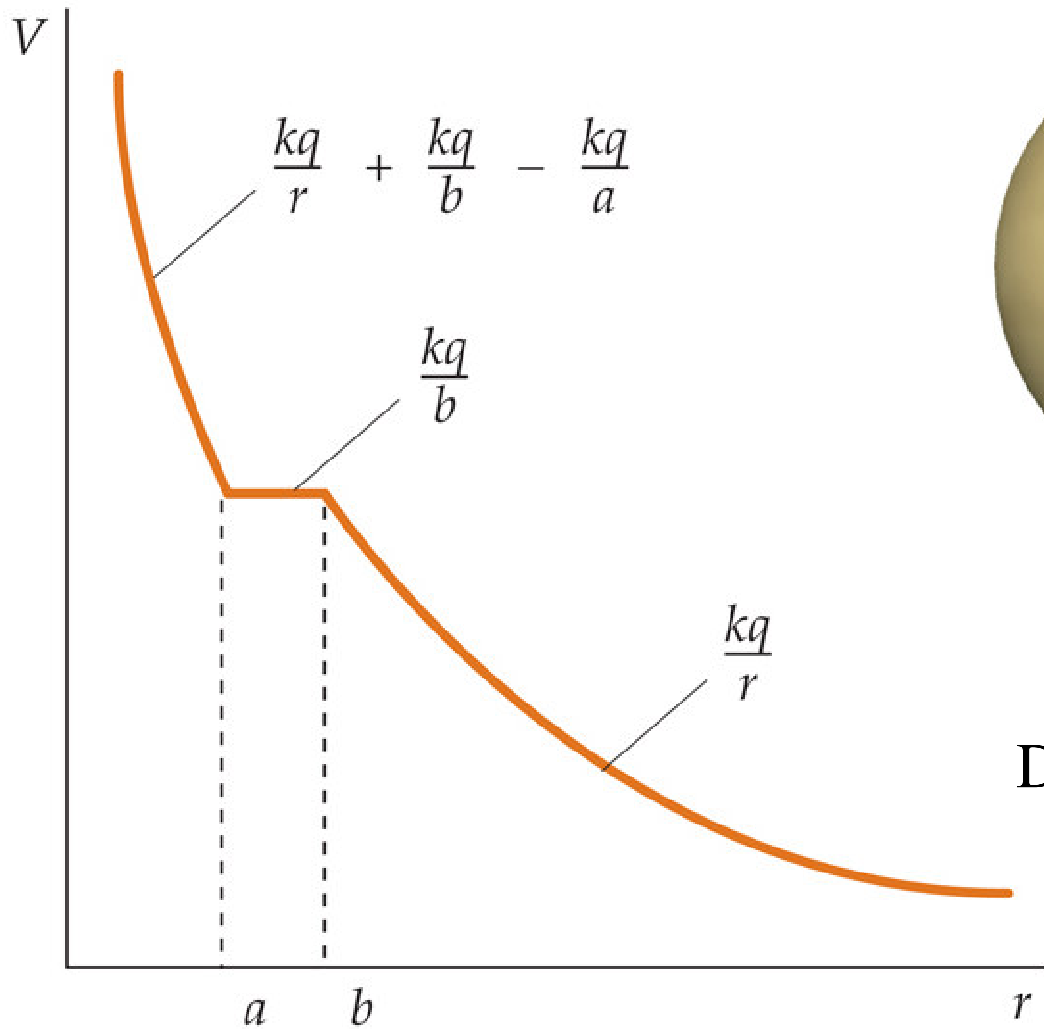


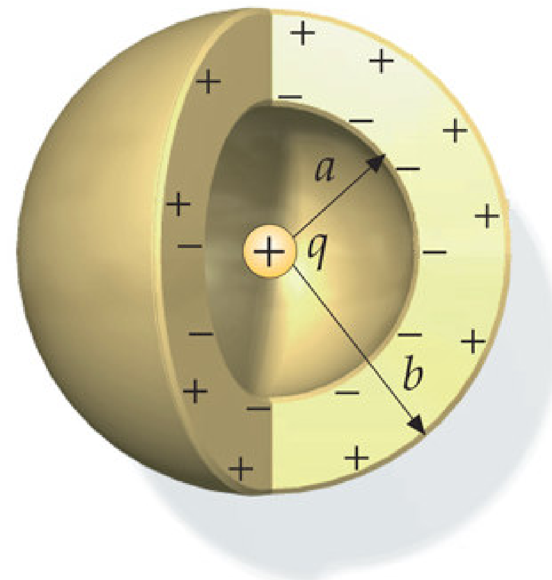
Potential Lines

Potentials Due to Spherical Shells



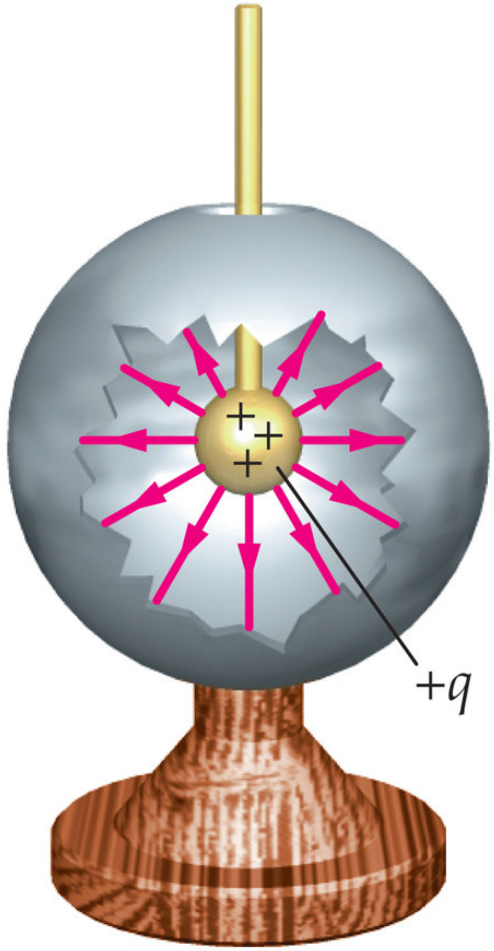
Potentials Due to Spherical Shells





Do calculation

Primitive Method of Storing a Large Charge or Creating a High Potential



Spherical conductor on the end of an insulating rod

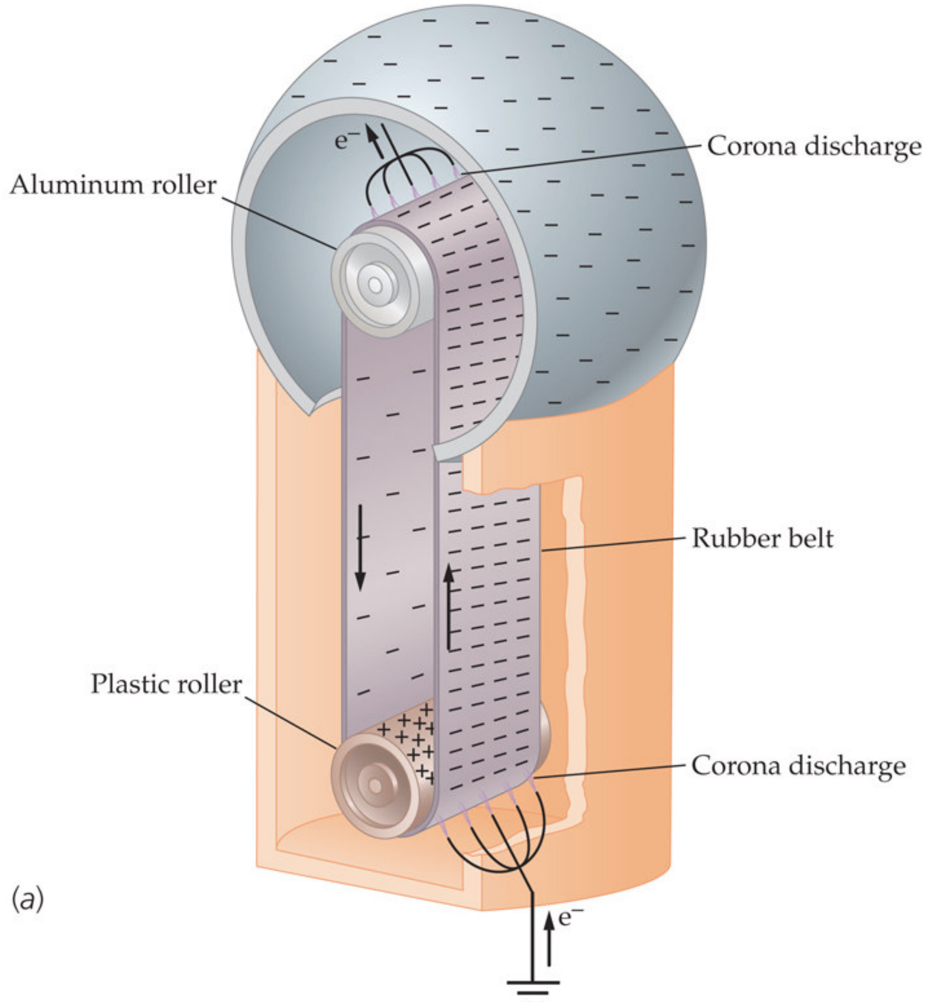
Spherical conductor is charged positively by removing a quantity of electrons.

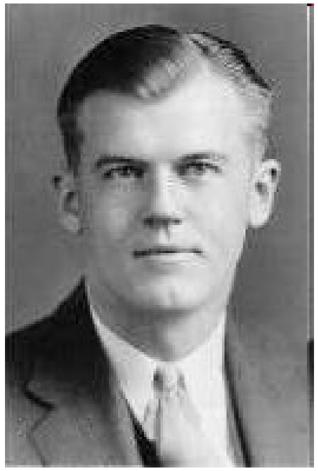
Touching the charged sphere to the inner surface of the large hollow conducting sphere causes electrons to transfer to the small spherical conductor - the large sphere is left with a net positive charge.

Repeat the process to build up the charge.

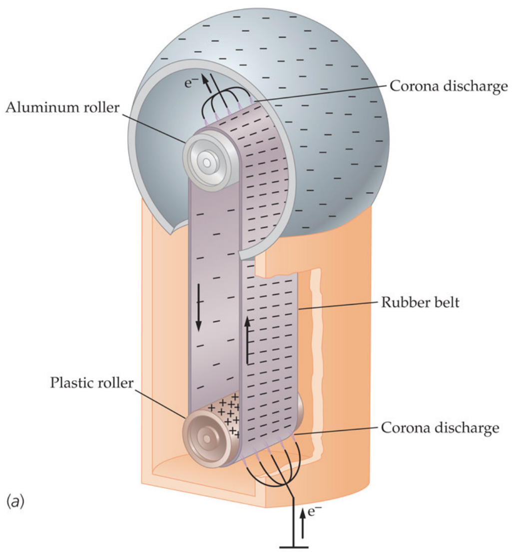
Eventually the process saturates.

The Van de Graff Generator





Robert J. Van de Graaff
1901-1967



The energies produced by Van de Graaff atomic particle accelerators are limited to about 30 MeV, even with tandem generators accelerating doubly charged (for example alpha) particles.

More modern particle accelerators using different technology produce much higher energies, thus *Van de Graaff particle accelerators have become largely obsolete*.

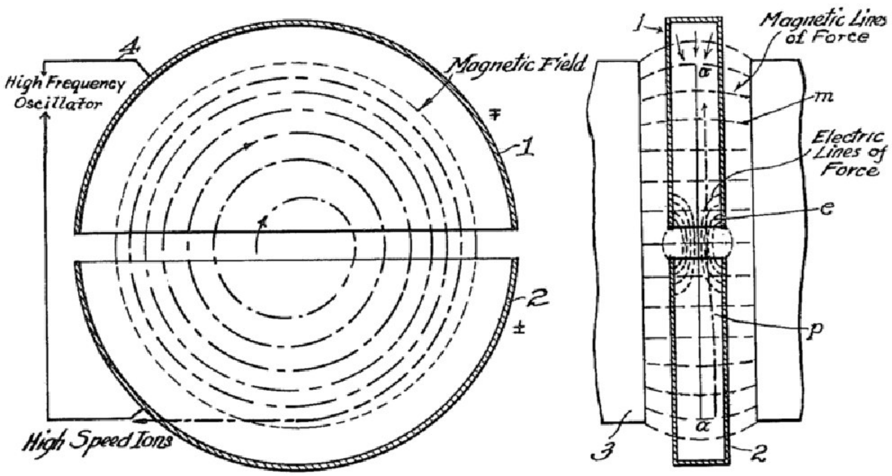
They are still used to some extent for graduate student research at colleges and universities and as ion sources for high energy bursts.

Ernest O. Lawrence
1901- 1958

Ernest Orlando Lawrence was an American physicist and Nobel Laureate, known for his invention, utilization, and improvement of the cyclotron atom-smasher beginning in 1929, based on his studies of the works of Rolf Widerøe, and his later work in uranium-isotope separation for the Manhattan Project.

Lawrence had a long career at the University of California, where he became a Professor of Physics. In 1939, Lawrence was awarded the **Nobel Prize** in Physics for his work in inventing the cyclotron and developing its applications.

Chemical element number 103 is named "lawrencium" in Lawrence's honor.



Source: http://en.wikipedia.org/wiki/E._O._Lawrence

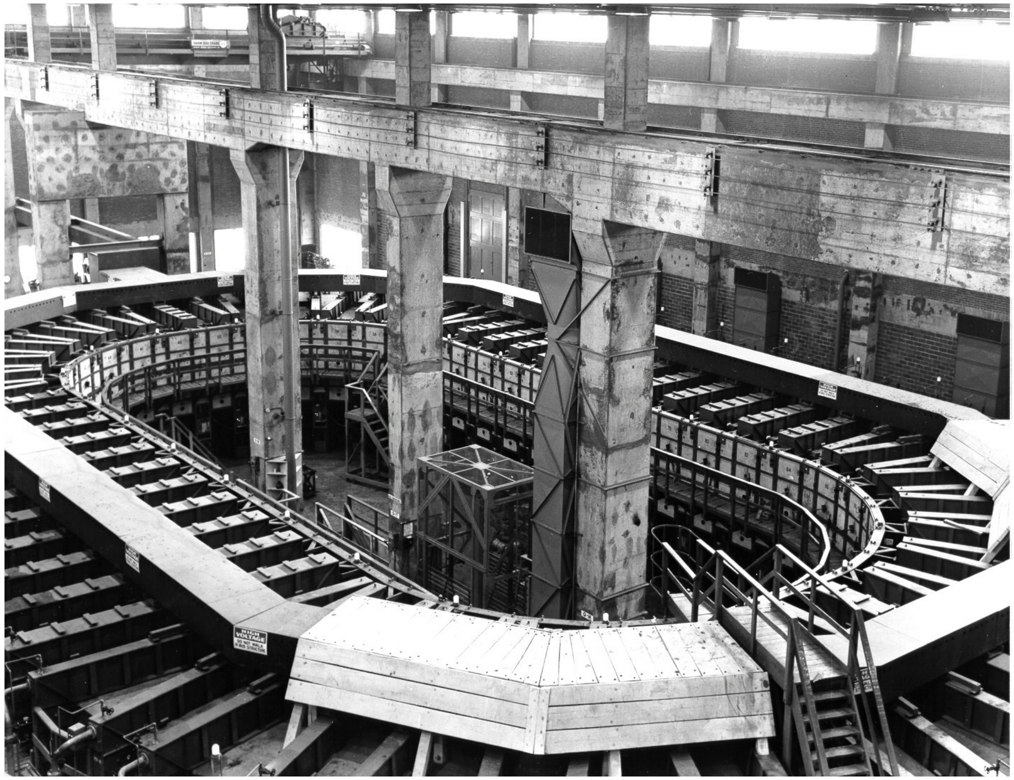
The Beginning of the Cyclotron

Lawrence read a journal article about producing very high-energy particles required for atomic disintegration by means of a succession of very small "pushes."

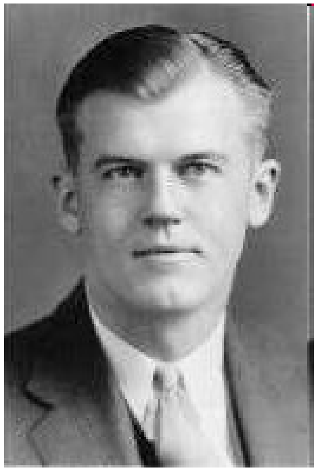
In an attempt to make the accelerator more compact, Lawrence decided to use a circular accelerating chamber situated between the poles of a large electromagnet. The magnetic field would hold the charged particles in a planar spiral path as they were accelerated between the two semicircular electrodes. These electrodes would be sandwiched between the magnetic poles and connected to an alternating potential. After about a hundred circular orbits, the particles would impact the target as a beam of high-energy particles. Lawrence had discovered a method for obtaining particles of very high energy *without the use of any high voltage*.

Source: http://en.wikipedia.org/wiki/E._O._Lawrence

Alpha Track Calutron at the Y-12 Plant (Site X) at Oak Ridge, Tennessee



Giant calutron plants developed at Lawrence's laboratory were used at Site X during World War II to purify uranium for use in the first atomic bomb. Alpha Track Calutron at the Y-12 Plant (Site X) at Oak Ridge, Tennessee from the Manhattan Project, used for uranium enrichment.

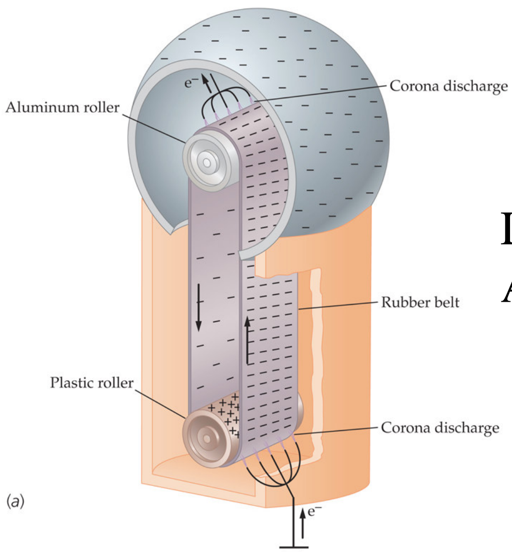


Van de Graaff was jealous of his more successful rival - Lawrence. In later years at HVE his engineers could not even discuss a design that involved particles traveling in a circular motion.

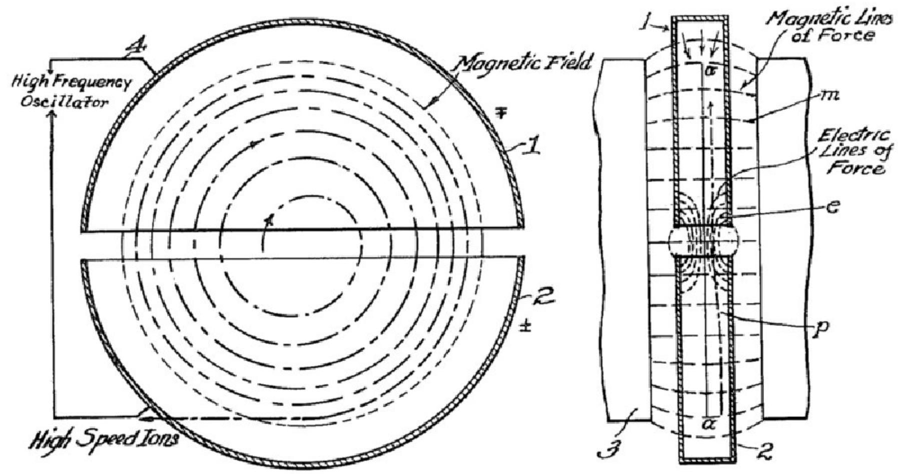
Robert J. Van de Graaff
1901-1967

Circular Acceleration

Ernest O. Lawrence
1901- 1958

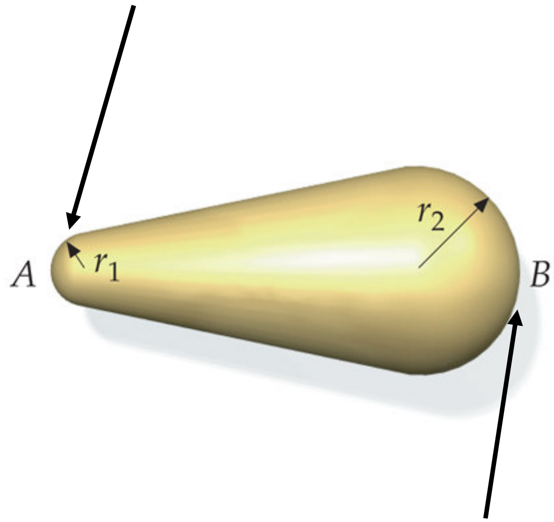


Linear Acceleration

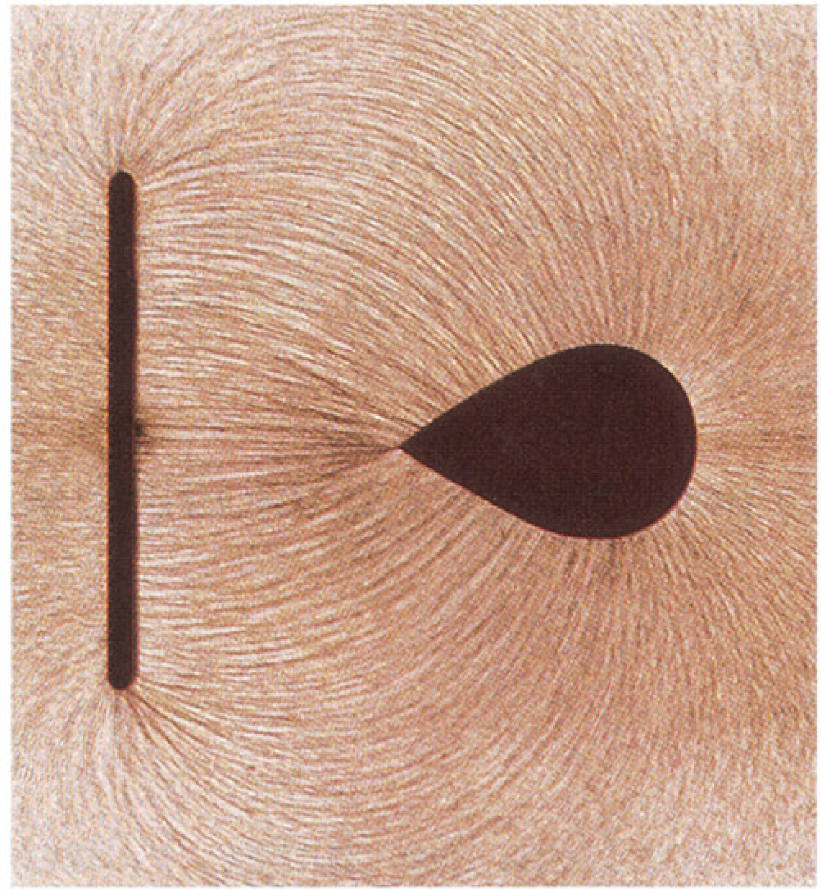


Dielectric Breakdown

High E-Field



Low E-Field



Jacob's Ladder serves no scientific purpose but every mad scientist in the movies had to have at least one in his lab.

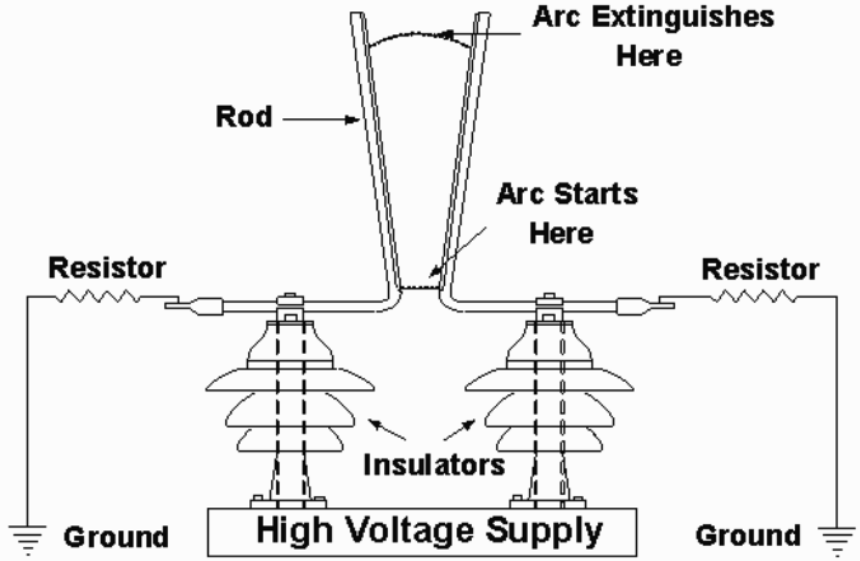


Figure 1: Basic structure of Jacob's ladder

"Air by nature is an Insulator. For it to become a Conductor, it must experience Dielectric Breakdown. This occurs if a voltage of 30,000V per centimeter is applied to the air. The air is then Ionized. In a Jacobs Ladder, Dielectric breakdown occurs when a voltage of 30,000 V is supplied to the bottom of the Rods."

"An Electric Arc is formed between the Rods and rises along with the hot air. As the arc length increases, its power consumption increases and there is insufficient Voltage to maintain the Electric Arc. The arc extinguishes and the cycle repeats with a new arc forming at the bottom of the rods."

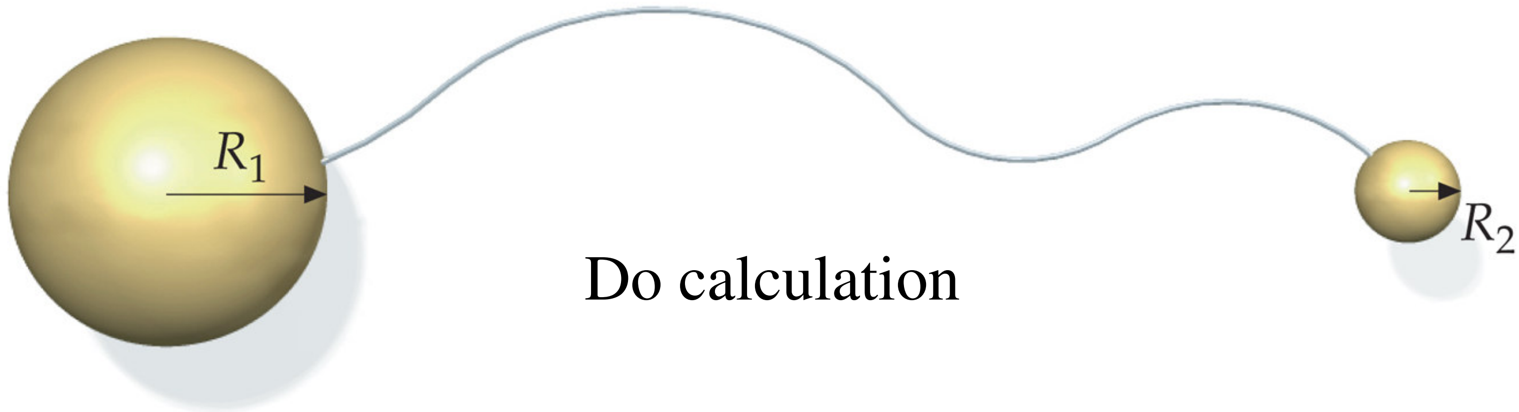
Source: http://macao.communications.museum/eng/Exhibition/secondfloor/moreinfo/2_1_1_JacobsLadder.html

Dielectric Strengths

Substance	Dielectric Strength (MV/m)
Helium	0.15
Air	3.0 (depends on pressure)
Alumina	13.4
Window glass	9.8 - 13.8
Silicone oil, Mineral oil	15-Oct
Benzene	16
Polystyrene	19.7
Polyethylene	18.9 - 21.7
Neoprene rubber	15.7 - 27.6
Ultra pure Water	30
High Vacuum (field emission limited)	20 - 40 (depends on electrode shape)
Fused silica	25 - 40
Waxed paper	40 - 60
PTFE (Teflon)	60
Mica	20 - 70
Thin films of SiO_2 in ICs	> 1000

Source: http://en.wikipedia.org/wiki/Dielectric_strength

Two Charged Spherical Conductors

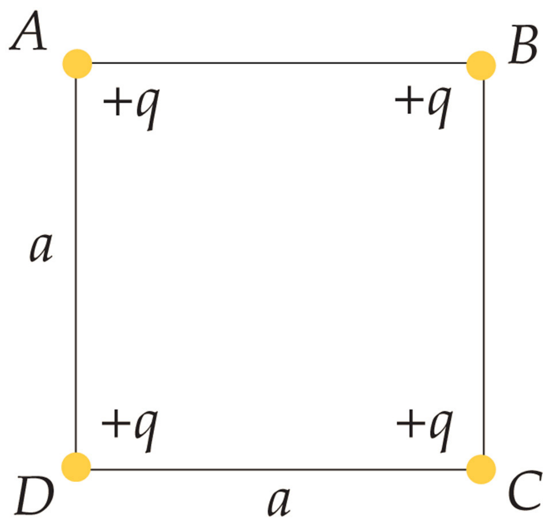


Do calculation

The thin conducting wire connecting the two spheres serves the same purpose as allowing the two spheres to touch.

This contact allows charge to transfer (remember only the electrons move) between the spheres and to bring the conductor surfaces to the same potential.

U - Electrostatic Potential Energy



- The electrostatic energy of a system of charges is equal to the Work required to assemble the charges from infinity.
- The Work is the difference in potential energy between the charge at infinity and at its final position.
- Since U at infinity is zero, the work is just U evaluated at the final charge positions.

U - Electrostatic Potential Energy

$$U = \frac{1}{2}\sum_{i=1}^{n} q_i V_i$$

For point charges

$$U = \frac{1}{2} Q V$$

A conductor with a charge Q at potential V

$$U = \frac{1}{2}\sum_{i=1}^{n} Q_i V_i$$

For a system of conductors

Extra Slides

Conservative Forces and Potentials from Vector Analysis

$$W = \oint_C \vec{F} \cdot d\vec{l} = 0$$

Work around a closed loop = 0

$$\oint_C \vec{F} \cdot d\vec{l} = \int_S \left(\vec{\nabla} \times \vec{F} \right) \cdot d\vec{a}$$

Stokes Theorem

$$\int_S \left(\vec{\nabla} \times \vec{F} \right) \cdot d\vec{a} = 0 \quad \Rightarrow \vec{\nabla} \times \vec{F} = 0$$

$$\vec{F} = -\vec{\nabla} V \quad since \quad \vec{\nabla} \times \left(\vec{\nabla} V \right) = 0$$

Therefore a potential function V exists for a conservative force.

Vector Analysis

$\varphi \; and \; \psi \; are \; scalar \; functions$

$\vec{F} \; and \; \vec{G} \; are \; vector \; functions$

$$\vec{\nabla}\varphi = grad \;\; \varphi = gradient \; of \;\; \varphi$$

$$\vec{\nabla} \cdot \vec{F} = div \;\; \vec{F} = divergence \; of \;\; \vec{F}$$

$$\vec{\nabla} \times \vec{F} = curl \;\; \vec{F} = curl \; of \;\; \vec{F}$$

Vector Analysis

Gradient $$\vec{\nabla}\varphi = \frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}$$

Divergence $$\vec{\nabla} \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

Curl $$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

Formulas from Vector Analysis Involving Differential Operators

(I–1)	$\nabla(\varphi + \psi) = \nabla\varphi + \nabla\psi$
(I–2)	$\nabla\varphi\psi = \varphi\nabla\psi + \psi\nabla\varphi$
(I–3)	$\mathrm{div}\,(\mathbf{F} + \mathbf{G}) = \mathrm{div}\,\mathbf{F} + \mathrm{div}\,\mathbf{G}$
(I–4)	$\mathbf{curl}\,(\mathbf{F} + \mathbf{G}) = \mathbf{curl}\,\mathbf{F} + \mathbf{curl}\,\mathbf{G}$
(I–5)	$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times \mathbf{curl}\,\mathbf{G} + \mathbf{G} \times \mathbf{curl}\,\mathbf{F}$
(I–6)	$\mathrm{div}\,\varphi\mathbf{F} = \varphi\,\mathrm{div}\,\mathbf{F} + \mathbf{F} \cdot \nabla\varphi$
(I–7)	$\mathrm{div}\,(\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \mathbf{curl}\,\mathbf{F} - \mathbf{F} \cdot \mathbf{curl}\,\mathbf{G}$
(I–8)	$\mathrm{div}\,\mathbf{curl}\,\mathbf{F} = 0$
(I–9)	$\mathbf{curl}\,\varphi\mathbf{F} = \varphi\,\mathbf{curl}\,\mathbf{F} + \nabla\varphi \times \mathbf{F}$
(I–10)	$\mathbf{curl}\,(\mathbf{F} \times \mathbf{G}) = \mathbf{F}\,\mathrm{div}\,\mathbf{G} - \mathbf{G}\,\mathrm{div}\,\mathbf{F} + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$
(I–11)	$\mathbf{curl}\,\mathbf{curl}\,\mathbf{F} = \mathbf{grad}\,\mathrm{div}\,\mathbf{F} - \nabla^2\mathbf{F}$
(I–12)	$\mathbf{curl}\,\nabla\varphi = 0$
(I–13)	$\oint_S \mathbf{F} \cdot \mathbf{n}\, da = \int_V \mathrm{div}\,\mathbf{F}\, dv$
(I–14)	$\oint_C \mathbf{F} \cdot d\mathbf{l} = \int_S \mathbf{curl}\,\mathbf{F} \cdot \mathbf{n}\, da$
(I–15)	$\oint_S \varphi\mathbf{n}\, da = \int_V \nabla\varphi\, dv$
(I–16)	$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_V \mathbf{F}\,\mathrm{div}\,\mathbf{G}\, dv + \int_V (\mathbf{G} \cdot \nabla)\mathbf{F}\, dv$
(I–17)	$\oint_S \mathbf{n} \times \mathbf{F}\, da = \int_V \mathbf{curl}\,\mathbf{F}\, dv$
(I–18)	$\oint_C \varphi\, d\mathbf{l} = \int_S \mathbf{n} \times \nabla\varphi\, da$