Electric Current & Ohm's Law

Unit: Electricity & Magnetism

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 5.2, 5.5

AP Physics 1 Learning Objectives: 1.B.1.1, 1.B.1.2, 1.E.2.1

Knowledge/Understanding Goals:

- electric potential (voltage)
- electric current
- resistance

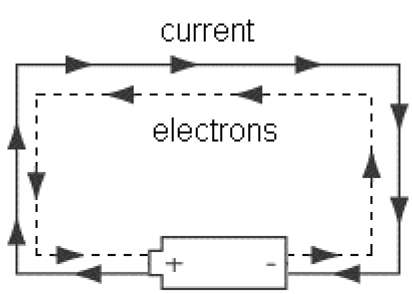
Skills:

- perform calculations involving voltage, current, resistance and power

Language Objectives:

- Understand and correctly use the terms "current," "direct current," "alternating current," "potential difference," "voltage," "resistance," "power," and "work."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to Ohm's Law.

Notes:



electric current: the flow of charged particles from one place to another, caused by a difference in electric potential. The direction of the electric current is defined as the direction that a positively-charged particle would move. Note, however, that the particles that are actually moving are electrons, which are negatively charged. This means that electric current "travels" in the *opposite* direction from the electrons.

Use this space for summary and/or additional notes.

Electric current ($\vec{I}$) is a vector quantity and is measured in amperes (A), often abbreviated as "amps". One ampere is one coulomb per second.

$$I = \frac{\Delta q}{t}$$

voltage (potential difference): the difference in electric potential energy between two locations, per unit of charge.

Potential difference is the work (W) done on a charge per unit of charge (q). Potential difference (V) is a scalar quantity (in DC circuits) and is measured in volts (V), which are equal to joules per coulomb.

$$V = \frac{W}{q}$$

The total voltage in a circuit is usually determined by the power supply that is used for the circuit (usually a battery in DC circuits).

resistance: the amount of electromotive force (electric potential) needed to force a given amount of current through an object.

Resistance (R) is a scalar quantity and is measured in ohms (Ω). One ohm is one volt per ampere.

$$R = \frac{V}{I}$$

This relationship is Ohm's Law, named for the German physicist Georg Ohm. Ohm's Law is more commonly written:

$$I = \frac{V}{R} \qquad \text{or} \qquad V = IR$$

Simply put, Ohm's Law states that an object has an ability to resist electric current flowing through it. The more resistance an object has, the more voltage you need to force electric current through it. Or, for a given voltage, the more resistance an object has, the less current will flow through it.

Resistance is an intrinsic property of a substance. In this course, we will limit problems that involve calculations to ohmic resistors, which means their resistance does not change with temperature.

Choosing the voltage and the arrangement of objects in the circuit (which determines the resistance) is what determines how much current will flow.

Electrical engineers use arrangements of resistors in circuits in order to adjust the amount of current that flows through the components.

Use this space for summary and/or additional notes.

resistivity: the innate ability of a substance to offer electrical resistance. The resistance of an object is therefore a function of the resistivity of the substance (ρ), and of the length (L) and cross-sectional area (A) of the object. In MKS units, resistivity is measured in ohm-meters (Ω·m).

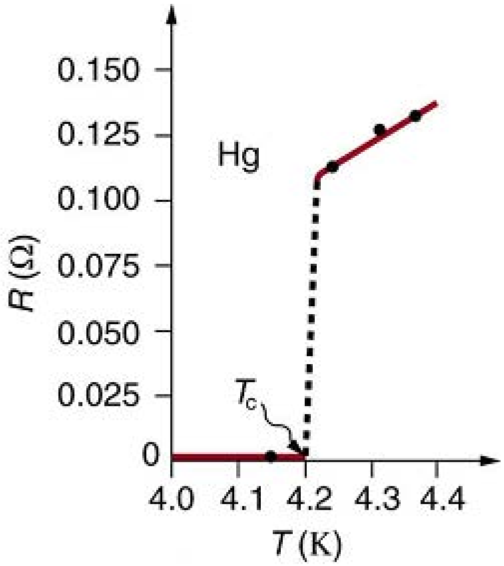
$$R = \frac{\rho L}{A}$$

Resistivity changes with temperature. For small temperature differences (less than 100°C), resistivity is given by:

$$\rho = \rho_o (1 + \alpha \Delta T)$$

where ρ_o is the resistivity at some reference temperature and α is the temperature coëfficient of resistivity for that substance. For conductors, α is positive (which means their resistivity increases with temperature). For metals at room temperature, resistivity typically varies from +0.003 to +0.006 K^{-1}.

Some materials become superconductors (essentially zero resistance) at very low temperatures. The temperature below which a material becomes a superconductor is called the critical temperature (T_c). For example, the critical temperature for mercury is 4.2 K, as shown in the graph to the right.



conductivity: the innate ability of a substance to conduct electricity. Conductivity (σ) is the inverse of resistivity, and is measured in siemens (S). Siemens used to be called mhos (symbol ℧). ("Mho" is "ohm" spelled backwards.)
Note: conductivity is not tested on the AP Physics 1 exam.

$$\sigma = \frac{1}{\rho}$$

Use this space for summary and/or additional notes.

ohmic resistor: a resistor whose resistance is the same regardless of voltage and current. The filament of an incandescent light bulb is an example of a non-ohmic resistor, because the current heats up the filament, which increases its resistance. (This is necessary in order for the filament to also produce light.)

capacitance: the ability of an object to hold an electric charge. Capacitance (C) is a scalar quantity and is measured in farads (F). One farad equals one coulomb per volt. Note: capacitance is covered in AP Physics 2, and is not tested on the AP Physics 1 exam.

$$C = \frac{q}{V}$$

power: as discussed in the mechanics section of this course, power (P) is the work done per unit of time and is measured in watts (W).

In electric circuits:

$$P = \frac{W}{t} = VI = I^2R = \frac{V^2}{R}$$

work: recall from mechanics that work (W) equals power times time, and is measured in either newton-meters (N·m) or joules (J):

$$W = Pt = VIt = I^2Rt = \frac{V^2t}{R} = Vq$$

Electrical work or energy is often measured in kilowatt-hours (kW·h).

$$1\,\text{kW·h} \equiv 3.6\times10^6\ \text{J} \equiv 3.6\,\text{MJ}$$

Summary of Terms and Variables

Term	Variable	Unit	Term	Variable	Unit
charge	q or Q	coulomb (C)	resistance	R	ohm (Ω)
current	I	ampere (A)	power	P	watt (W)
voltage	V	volt (V)	work	W	joule (J)

Use this space for summary and/or additional notes.

Alternating Current vs. Direct Current

Electric current can move in two ways.

direct current: electric current flows through the circuit, starting at the positive terminal of the battery or power supply, and ending at the negative terminal. Batteries supply direct current. A typical AAA, AA, C, or D battery supplies 1.5 volts DC.

alternating current: electric current flows back and forth in one direction and then the other, like a sine wave. The current alternates at a particular frequency. In the U.S., household current is 110 volts AC with a frequency of 60 Hz.

Alternating current requires higher voltages in order to operate devices, but has the advantage that the voltage drop is much less over a length of wire than with direct current.

Note that alternating current is not tested on the AP Physics 1 exam.

Sample Problems:

Q: A simple electrical device uses 1.5 A of current when plugged into a 110 V household electrical outlet. How much current would the same device draw if it were plugged into a 12 V outlet in a car?

A: Resistance is a property of a specific object. Because we are not told otherwise, we assume the device is ohmic and the resistance is the same regardless of the current.

To solve this problem, we need to find the resistance from the numbers when the device is plugged into the household outlet, and use that resistance along with the car's voltage to find the current it draws when it is plugged into the car's 12 V plug.

In the household outlet:

$$R = \frac{V}{I} = \frac{110}{1.5} = 73.\overline{3}\ \Omega$$

In the car:

$$I = \frac{V}{R} = \frac{12}{73.\overline{3}} = 0.163\ \text{A}$$

Use this space for summary and/or additional notes.

Q: A laptop computer uses 10 W of power. The laptop's power supply adjusts the current so that the power is the same regardless of the voltage supplied. How much current would the computer draw from a 110 V household outlet? How much current would the same laptop computer need to draw from a 12 V car outlet?

A: Household outlet:

$$P = VI$$

$$I = \frac{P}{V} = \frac{10}{110} = 0.091\text{ A}$$

Car outlet:

$$I = \frac{P}{V} = \frac{10}{12} = 0.8\overline{3}\text{ A}$$

Q: A 100 Ω resistor is 0.70 mm in diameter and 6.0 mm long. Suppose you wanted to make a 470 Ω resistor out of the same material. If you wanted to use a piece of the same diameter material, what would the length need to be? If, instead, you wanted to make a resistor the same length, what would the new diameter need to be?

A: In both cases, $R = \frac{\rho \ell}{A}$.

For a resistor of the same diameter (same cross-sectional area), ρ and A are the same, which gives:

$$\frac{R'}{R} = \frac{\ell'}{\ell}$$

$$\ell' = \frac{R'\ell}{R} = \frac{(470)(6.0)}{100} = 28.2\text{ mm}$$

For a resistor of the same length:

$$\frac{R'}{R} = \frac{A}{A'} = \frac{\pi r^2}{\pi (r')^2} = \frac{\pi (d/2)^2}{\pi (d'/2)^2} = \frac{d^2}{(d')^2}$$

$$d' = \sqrt{\frac{Rd^2}{R'}} = d\sqrt{\frac{R}{R'}} = 0.70\sqrt{\frac{100}{470}} = 0.70\sqrt{0.213} = 0.323\text{ mm}$$

Use this space for summary and/or additional notes.

Homework Problems

1. An MP3 player uses a standard 1.5 V battery. How much resistance is in the circuit if it uses a current of 0.010 A?

   Answer: 150 Ω

2. How much current flows through a hair dryer plugged into a 110 V circuit if it has a resistance of 25 Ω?

   Answer: 4.4 A

3. A battery pushes 1.2 A of charge through the headlights in a car, which has a resistance of 10 Ω. What is the potential difference across the headlights?

   Answer: 12 V

4. A 0.7 mm diameter by 60 mm long pencil "lead" is made of graphite, which has a resistivity of approximately $1.0\times10^{-4}\ \Omega\cdot\text{m}$. What is its resistance?

   Answer: 15.6 Ω

Use this space for summary and/or additional notes.

5. A cylindrical object has radius r and length ℓ and is made from a substance with resistivity ρ. A potential difference of V is applied to the object. Derive an expression for the current that flows through it.

   Answer: $I = \frac{VA}{\rho\ell}$

6. A circuit used for electroplating copper applies a current of 3.0 A for 16 hours. How much charge is transferred?

   Answer: 172 800 C

7. What is the power when a voltage of 120 V drives a 2.0 A current through a device?

   Answer: 240W

8. What is the resistance of a 40. W light bulb connected to a 120 V circuit?

   Answer: 360 Ω

9. If a component in an electric circuit dissipates 6.0 W of power when it draws a current of 3.0 A, what is the resistance of the component?

   Answer: 0.67 Ω

Use this space for summary and/or additional notes.

10. Some children are afraid of the dark and ask their parents to leave the hall light on all night. Suppose the hall light in a child's house has two 60. W incandescent light bulbs (120 W total), the voltage is 120 V, and the light is left on for 8.0 hours.

    a. How much current flows through the light fixture?

    Answer: 1.0 A

    b. How many kilowatt-hours of energy would be used in one night?

    Answer: 0.96 kW·h

    c. If the power company charges 22 ¢ per kilowatt-hour, how much does it cost to leave the light on overnight?

    Answer: 21.1 ¢

    d. If the two 60 W incandescent bulbs are replaced by LED bulbs that use 8.5 W each, how much money would the family save each night?

    Answer: 18.1 ¢

Use this space for summary and/or additional notes.

Electrical Components

Unit: Electricity & Magnetism

NGSS Standards: HS-PS2-6

MA Curriculum Frameworks (2006): 5.3

AP Physics 1 Learning Objectives: N/A

Knowledge/Understanding Goals:

- recognize common components of electrical circuits

Language Objectives:

- Recognize and be able to name and draw symbols for each of the electrical components described in this section.

Labs, Activities & Demonstrations:

- Show & tell with actual components.

Notes:

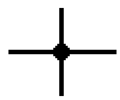
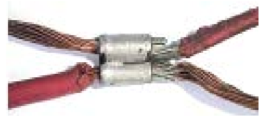
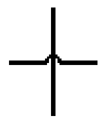
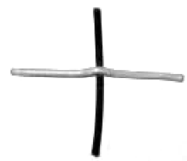
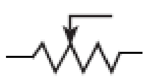
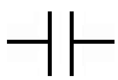
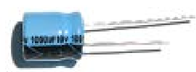
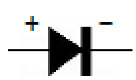
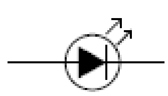
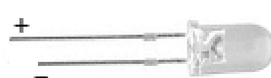
electrical component: an object that performs a specific task in an electric circuit. A circuit is a collection of components connected together so that the tasks performed by the individual components combine in some useful way.

circuit diagram: a picture that represents a circuit, with different symbols representing the different components.

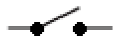
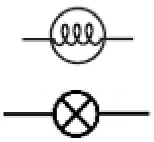
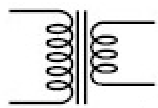
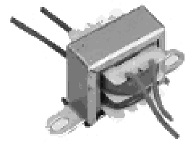
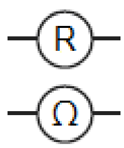
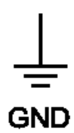
Use this space for summary and/or additional notes.

Electrical Components

The following table describes some of the common components of electrical circuits, what they do, and the symbols that are used to represent them in circuit diagrams.

Component	Symbol	Picture	Description
wire			Carries current in a circuit.
junction			Connection between two or more wires.
unconnected wires			Wires cross but are not connected.
battery			Supplies current at a fixed voltage.
resistor			Resists flow of current.
potentiometer (rheostat, dimmer)			Provides variable (adjustable) resistance.
capacitor			Stores charge.
diode			Allows current to flow in only one direction (from + to −).
light-emitting diode (LED)			Diode that gives off light.

Use this space for summary and/or additional notes.

Component	**Symbol**	**Picture**	**Description**
switch			Opens / closes circuit.
incandescent lamp (light)			Provides light (and resistance).
transformer		 	Increases or decreases voltage.
voltmeter			Measures voltage (volts).
ammeter			Measures current (amperes).
ohmmeter			Measures resistance (ohms).
fuse			Opens circuit if too much current flows through it.
ground		 (clamps to water pipe)	Neutralizes charge.

Use this space for summary and/or additional notes.

Circuits

Unit: Electricity & Magnetism

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 5.3

AP Physics 1 Learning Objectives: 5.C.3.2, 5.C.3.3

Knowledge/Understanding Goals:

- how resistance limits current in a circuit
- the difference between series and parallel circuits

Language Objectives:

- Explain why resistors are necessary in electric circuits.
- Understand and correctly use the terms "series" and "parallel" as applied to electric circuits.

Labs, Activities & Demonstrations:

- Example circuit with light bulbs & switches.
- Fuse demo using a single strand from a multi-strand wire.

Notes:

<u>circuit</u>: an arrangement of electrical components that allows electric current to pass through them so that the tasks performed by the individual components combine in some useful way.

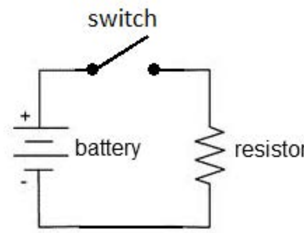
<u>closed circuit</u>: a circuit that has a complete path for current to flow from the positive terminal of the battery or power supply through the components and back to the negative terminal.

<u>open circuit</u>: a circuit that has a gap such that current cannot flow from the positive terminal to the negative terminal.

<u>short circuit</u>: a circuit in which the positive terminal is connected directly to the negative terminal with no load (resistance) in between.

Use this space for summary and/or additional notes.

A diagram of a simple electric circuit might look like the diagram to the right.



When the switch is closed, the electric current flows from the positive terminal of the battery through the switch, through the resistor, and back to the negative terminal of the battery.

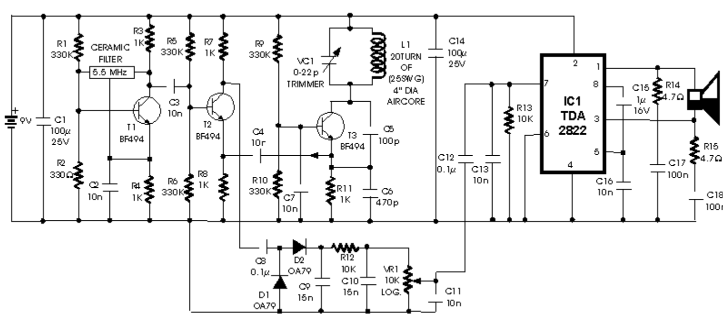
An electric circuit needs a power supply (often a battery) that provides current at a specific difference in electric potential (voltage), and one or more components that use the energy provided by the battery.

The battery continues to supply current, provided that:

1. There is a path for the current to flow from the positive terminal to the negative terminal, and
2. The total resistance of the circuit is small enough to allow the current to flow.

If the circuit is broken, current cannot flow and the chemical reactions inside the battery stop.

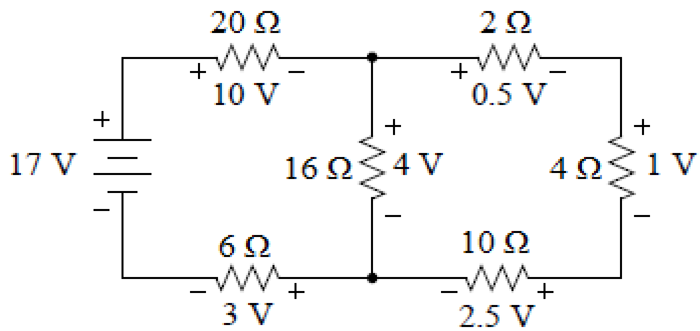
Of course, as circuits become more complex, the diagrams reflect this increasing complexity. The following is a circuit diagram for a metal detector:



Analyzing an electrical circuit means figuring out the potential difference (voltage), current, and/or resistance contributed by each component of a circuit.

Use this space for summary and/or additional notes.

The following is an example of a circuit with one 17 V battery and six resistors. Notice that the voltages and resistances are all labeled.

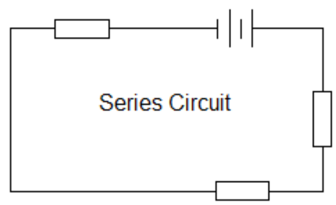


Use this space for summary and/or additional notes.

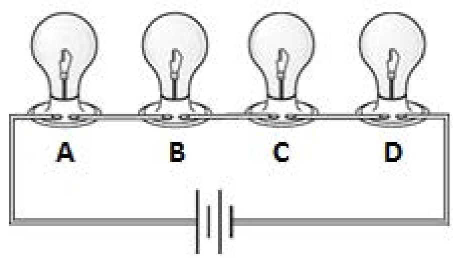
Series vs. Parallel Circuits

If a circuit has multiple components, they can be arranged in series or parallel.

series: Components in **series** lie along the same path, one after the other.



In a series circuit, all of the current flows through every component, one after another. If the current is interrupted anywhere in the circuit, no current will flow. For example, in the following series circuit, if any of light bulbs A, B, C, or D is removed, no current can flow and none of the light bulbs will be illuminated.

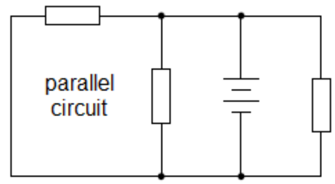


Because some voltage is "used up" by each bulb in the circuit, each additional bulb means the voltage is divided among more bulbs and is therefore less for each bulb. This is why light bulbs get dimmer as you add more bulbs in series.

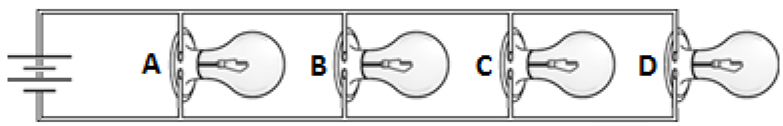
Christmas tree lights used to be wired in series. This caused a lot of frustration, because if one bulb burned out, the entire string went out, and it could take several tries to find which bulb was burned out.

Use this space for summary and/or additional notes.

parallel: Components in **parallel** lie in separate paths.

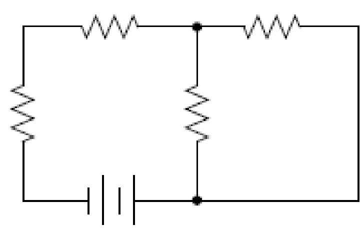


In a parallel circuit, the current divides at each junction, with some of the current flowing through each path. If the current is interrupted in one path, current can still flow through the other paths. For example, in the following parallel circuit, if any of light bulbs A, B, C, or D is removed, current still flows through the remaining bulbs.



Because the voltage across each branch is equal to the total voltage, all of the bulbs will light up with full brightness, regardless of how many bulbs are in the circuit. (However, each separate light bulb draws the same amount of current as if it were the only thing in the circuit, so the total current in the circuit increases with each new branch. This is why you trip a circuit breaker or blow a fuse if you have too many high-power components plugged into the same circuit.)

Note that complex circuits may have some components that are in series with each other and other components that are in parallel.

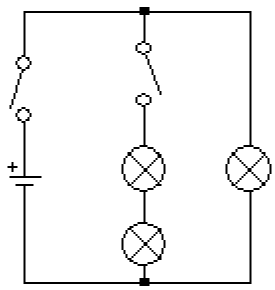


Use this space for summary and/or additional notes.

Sample Problem:

Q: A circuit consists of a battery, two switches, and three light bulbs. Two of the bulbs are in series with each other, and the third bulb is in parallel with the others. One of the switches turns off the two light bulbs that are in series with each other, and the other switch turns off the entire circuit. Draw a schematic diagram of the circuit, using the correct symbol for each component.

A:

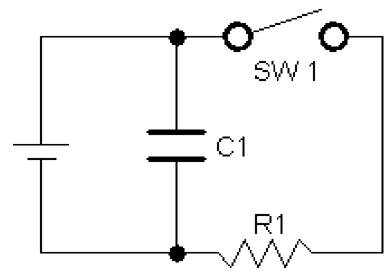


Note that no sensible person would intentionally wire a circuit this way. It would make much more sense to have the second switch on the branch with the one light bulb, so you could turn off either branch separately or both branches by opening both switches. This is an example of a strange circuit that a physics teacher would use to make sure you really can follow exactly what the question is asking!

Use this space for summary and/or additional notes.

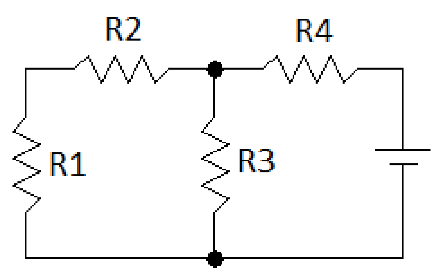
Homework Problems

1. The following circuit contains a battery, switch (SW1), capacitor (C1), and resistor (R1).



Which of components C1 and SW1 are in series with R1? Which are in parallel with R1?

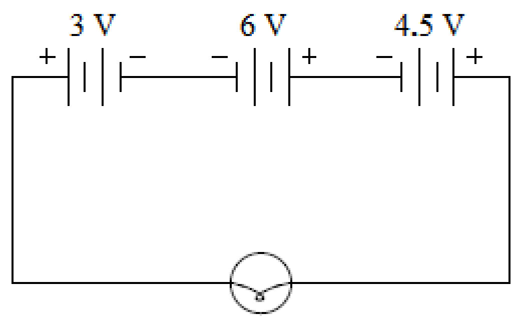
2. The following circuit contains a battery and four resistors (R1, R2, R3, and R4).



Which resistors are in series with R1? Which are in parallel with R1?

Use this space for summary and/or additional notes.

3. The following bizarre circuit contains three batteries and a light bulb. What is the potential difference across the light bulb?
(*Hint: remember to check the +/− orientation of the batteries.*)



4. A circuit is powered by a 9 V battery. The circuit has a 100 Ω resistor in series with the battery, and then the circuit splits. One branch contains only a switch. The second branch contains a 100 μF capacitor in series with another switch. Draw a diagram for this circuit.

Use this space for summary and/or additional notes.

Kirchhoff's Rules

Unit: Electricity & Magnetism

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 5.3

AP Physics 1 Learning Objectives: 5.B.9.1, 5.B.9.2, 5.B.9.3, 5.C.3.1

Knowledge & Understanding:

- Understand Kirchhoff's junction rule and Kirchhoff's loop rule.

Skills:

- Use Kirchhoff's rules to determine voltage, current and resistance in complex circuits.

Language Objectives:

- Accurately describe how to measure voltage, current and resistance in an electric circuit, using appropriate academic language.

Notes:

In 1845, the German physicist Gustav Kirchhoff came up with two simple rules that describe the behavior of current in complex circuits. Those rules are:

<u>Kirchhoff's junction rule</u>: the total current coming into any junction must equal the total current coming out of the junction.

The junction rule is based on the concept that electric charge cannot be created or destroyed. Current is simply the flow of electric charge, so any charges that come into a junction must also come out of it.

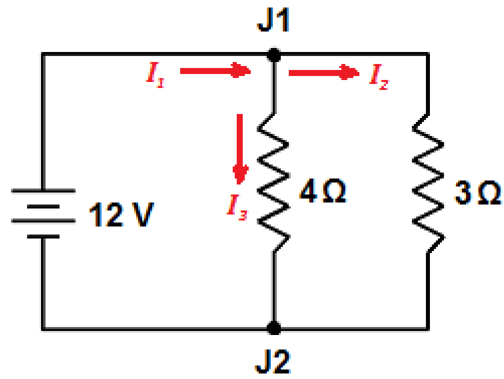
<u>Kirchhoff's loop rule</u>: the sum of the voltages around any closed loop must add up to zero.

The loop rule is based on the concept that voltage is the difference in electric potential between one location in the circuit and another. If you come back to the same point in the circuit, the difference in electric potential between where you started and where you ended (the same place) must be zero. Therefore, any increases and decreases in voltage around the loop must cancel.

Use this space for summary and/or additional notes.

Junction Rule Example:

As an example of the junction rule, consider the following circuit:



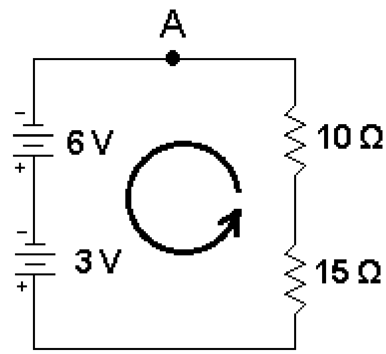
The junction rule tells us that the current flowing into junction J1 must equal the current flowing out. If we assume current I_1 flows into the junction, and currents I_2 and I_3 flow out of it, then $I_1 = I_2 + I_3$.

We know that the voltage across both resistors is 12 V. From Ohm's Law we can determine that the current through the 3 Ω resistor is $I_2 = 4\text{ A}$, and the current through the 4 Ω resistor is $I_3 = 3\text{ A}$. The junction rule tells us that the total current must therefore be $I_1 = I_2 + I_3 = 4\text{ A} + 3\text{ A} = 7\text{ A}$.

Use this space for summary and/or additional notes.

Loop Rule Example:

For the loop rule, consider the following circuit:



If we start at point A and move counterclockwise around the loop (in the direction of the arrow), the voltage should be zero when we get back to point A.

For this example, we are moving around the circuit in the same direction that the current flows, because that makes the most intuitive sense. However, it wouldn't matter if we moved clockwise instead—just as with vector quantities, we choose a positive direction and assign each quantity to a positive or negative number accordingly, and the math tells us what is actually happening.

Starting from point A, we first move through the 6 V battery. We are moving from the negative pole to the positive pole of the battery, so the voltage increases by +6 V. When we move through the second battery, the voltage increases by +3 V.

Next, we move through the 15 Ω resistor. When we move through a resistor in the positive direction (of current flow), the voltage drops, so we assign the resistor a voltage of $-15I$ (based on $V = IR$, where I is the current through the resistor). Similarly, the voltage across the 10 Ω resistor is $-10I$. Applying the loop rule gives:

$$6 + 3 + (-15I) + (-10I) = 0$$
$$9 - 25I = 0$$
$$9 = 25I$$
$$I = \frac{9}{25} = 0.36\text{ A}$$

Use this space for summary and/or additional notes.

Series Circuits

Unit: Electricity & Magnetism

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 5.3

AP Physics 1 Learning Objectives: 5.B.9.1, 5.B.9.2, 5.B.9.3, 5.C.3.1, 5.C.3.2, 5.C.3.3

Knowledge/Understanding Goals:

- the difference between series and parallel circuits

Skills:

- calculate voltage, current, resistance, and power in series circuits.

Language Objectives:

- Understand and correctly use the term “series circuit.”
- Set up and solve word problems relating to electrical circuits with components in series.

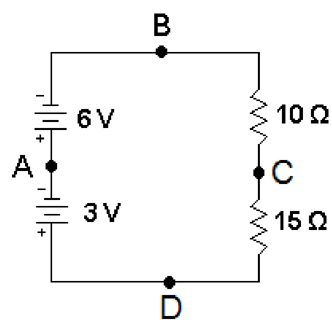
Labs, Activities & Demonstrations:

- Circuit with light bulbs wired in series.

Notes:

Analyzing Series Circuits

The following circuit shows two batteries and two resistors in series:



Use this space for summary and/or additional notes.

Current

Because there is only one path, all of the current flows through every component. This means the current is the same through every component in the circuit:

$$I_{total} = I_1 = I_2 = I_3 = \ldots$$

Voltage

In a series circuit, if there are multiple voltage sources (*e.g.,* batteries), the voltages add:

$$V_{total} = V_1 + V_2 + V_3 + \ldots$$

In the above circuit, there are two batteries, one that supplies 6 V and one that supplies 3 V. The voltage from A to B is +6 V, the voltage from A to D is −3 V (note that A to D means measuring from negative to positive), and the voltage from D to B is (+3 V) + (+6 V) = +9 V.

Resistance

If there are multiple resistors, each one contributes to the total resistance and the resistances add:

$$R_{total} = R_1 + R_2 + R_3 + \ldots$$

In the above circuit, the resistance between points B and D is 10Ω + 15Ω = 25Ω.

Power

In all circuits (series and parallel), any component that has resistance dissipates power whenever current passes through it. The total power consumed by the circuit is the sum of the power dissipated by each component:

$$P_{total} = P_1 + P_2 + P_3 + \ldots$$

Use this space for summary and/or additional notes.

Calculations

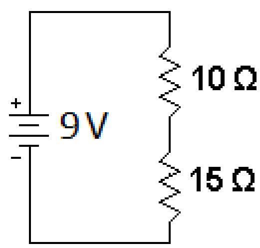
You can calculate the voltage, current, resistance, and power of each component separately and also the entire circuit using the equations:

$$V = IR \qquad P = VI = I^2R = \frac{V^2}{R}$$

"Solving" the circuit for these quantities is much like solving a Sudoku puzzle. You systematically decide which variables (for each component and/or the entire circuit) you have enough information to solve for. Each result enables you to determine more and more of the, until you have found all of the quantities you need.

Sample Problem:

Suppose we are given the following circuit:



and we are asked to fill in the table:

	R_1	R_2	Total
Voltage (V)			9 V
Current (I)			
Resistance (R)	10 Ω	15 Ω	
Power (P)			

First, we recognize that resistances in series add, which gives us:

	R_1	R_2	Total
Voltage (V)			9 V
Current (I)			
Resistance (R)	10 Ω	15 Ω	***25 Ω***
Power (P)			

Use this space for summary and/or additional notes.

Now, we know two variables in the "Total" column, so we use $V = IR$ to find the current. Because this is a series circuit, the total current is also the current through R_1 and R_2.

$$V = IR$$
$$9 = (I)(25)$$
$$I = \frac{9}{25} = 0.36\text{ A}$$

	R_1	R_2	Total
Voltage (V)			9 V
Current (I)	***0.36 A***	***0.36 A***	***0.36 A***
Resistance (R)	10 Ω	15 Ω	25 Ω
Power (P)			

As soon as we know the current, we can find the voltage across R_1 and R_2, again using $V = IR$.

	R_1	R_2	Total
Voltage (V)	***3.6 V***	***5.4 V***	9 V
Current (I)	0.36 A	0.36 A	0.36 A
Resistance (R)	10 Ω	15 Ω	25 Ω
Power (P)			

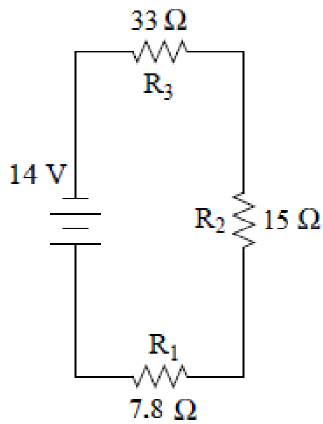
Finally, we can fill in the power, using $P = VI$, $P = I^2R$, and/or $P = \frac{V^2}{R}$:

	R_1	R_2	Total
Voltage (V)	3.6 V	5.4 V	9 V
Current (I)	0.36 A	0.36 A	0.36 A
Resistance (R)	10 Ω	15 Ω	25 Ω
Power (P)	***1.30 W***	***1.94 W***	***3.24 W***

Use this space for summary and/or additional notes.

Homework Problems

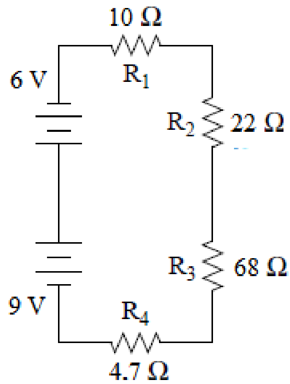
1. Fill in the table for the following circuit:



	R_1	R_2	R_3	Total
Voltage (*V*)				14 V
Current (*I*)				
Resist. (*R*)	7.8 Ω	15 Ω	33 Ω	
Power (*P*)				

Use this space for summary and/or additional notes.

2. Fill in the table for the following circuit:



	R_1	R_2	R_3	R_4	Total
Voltage (*V*)					
Current (*I*)					
Resist. (*R*)	10 Ω	22 Ω	68 Ω	4.7 Ω	
Power (*P*)					

Use this space for summary and/or additional notes:

Parallel Circuits

Unit: Electricity & Magnetism

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 5.3

AP Physics 1 Learning Objectives: 5.B.9.1, 5.B.9.2, 5.B.9.3, 5.C.3.1, 5.C.3.2, 5.C.3.3

Skills:

- calculate voltage, current, resistance, and power in parallel circuits.

Language Objectives:

- Understand and correctly use the term "parallel circuit."
- Set up and solve word problems relating to electrical circuits with components in parallel.

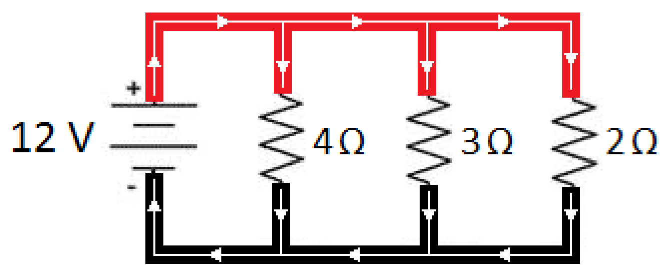
Labs, Activities & Demonstrations:

- Circuit with light bulbs wired in parallel.

Notes:

Parallel Circuits

The following circuit shows a battery and three resistors in parallel:



Current

The current divides at each junction (as indicated by the arrows). This means the current through each path must add up to the total current:

$$I_{total} = I_1 + I_2 + I_3 + \ldots$$

Use this space for summary and/or additional notes.

Voltage

In a parallel circuit, the potential difference (voltage) across the battery is always the same (12 V in the above example). Therefore, the potential difference between *any point* on the top wire and *any point* on the bottom wire must be the same. This means the voltage is the same across each path:

$$V_{total} = V_1 = V_2 = V_3 = \ldots$$

Power

Just as with series circuits, in a parallel circuit, any component that has resistance dissipates power whenever current passes through it. The total power consumed by the circuit is the sum of the power dissipated by each component:

$$P_{total} = P_1 + P_2 + P_3 + \ldots$$

Resistance

If there are multiple resistors, the effective resistance of each path becomes less as there are more paths for the current to flow through. The total resistance is given by the formula:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

Some students find it confusing that the combined resistance of a group of resistors in series is always less than any single resistor by itself.

Use this space for summary and/or additional notes.

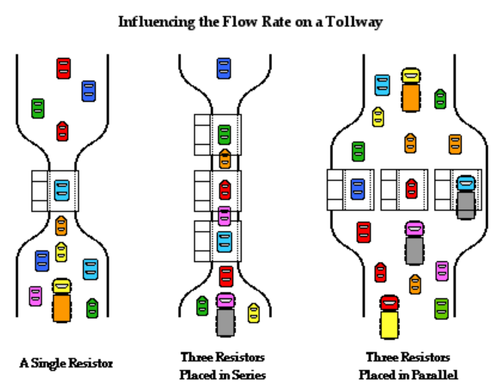
Electric current is analogous to water in a pipe:

- The current corresponds to the flow rate.
- The voltage corresponds to the pressure between one side and the other.
- The resistance would correspond to how small the pipe is (i.e., how hard it is to push water through the pipes). A smaller pipe has more resistance; a larger pipe will let water flow through more easily than a smaller pipe.

The voltage (pressure) drop is the same between one side and the other because less water flows through the smaller pipes and more water flows through the larger ones until the pressure is completely balanced. The same is true for electrons in a parallel circuit.

The water will flow through the *set* of pipes more easily than it would through any one pipe by itself. The same is true for resistors. As you add more resistors, you add more pathways for the current, which means less total resistance.

Another common analogy is to compare resistors with toll booths on a highway.



One toll booth slows cars down while the drivers pay the toll.

Multiple toll booths in series would slow traffic down more.

Multiple toll booths in parallel make traffic flow faster because there are more paths for the cars to follow. Each additional toll booth further reduces the resistance to the flow of traffic.

Use this space for summary and/or additional notes.

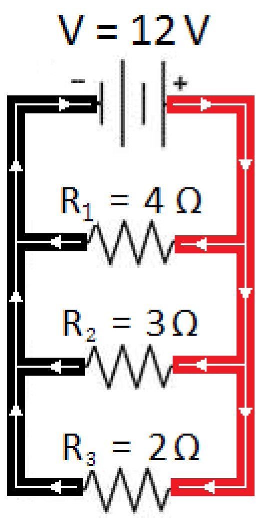
Calculations

Just as with series circuits, you can calculate the voltage, current, resistance, and power of each component and the entire circuit using the equations:

$$V = IR \qquad P = VI = I^2R = \frac{V^2}{R}$$

Sample Problem

Suppose we are given the following circuit:



and we are asked to fill in the table:

	R_1	R_2	R_3	Total
Voltage (V)				12 V
Current (I)				
Resistance (R)	4 Ω	3 Ω	2 Ω	
Power (P)				

Because this is a parallel circuit, the total voltage equals the voltage across all three branches, so we can fill in 12 V for each resistor.

The next thing we can do is use $V = IR$ to find the current through each resistor:

	R_1	R_2	R_3	Total
Voltage (V)	12 V	12 V	12 V	12 V
Current (I)	***3 A***	***4 A***	***6 A***	***13 A***
Resistance (R)	4 Ω	3 Ω	2 Ω	
Power (P)				

In a parallel circuit, the current adds, so the total current is 3 + 4 + 6 = 13 A.

Use this space for summary and/or additional notes.

Now, we have two ways of finding the total resistance.

We can use $V = IR$ for the voltage and total current:

$$V = IR$$
$$12 = 13R$$
$$R = \frac{12}{13} = 0.923\,\Omega$$

Or we can use the formula for resistances in parallel:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$\frac{1}{R_{total}} = \frac{1}{4} + \frac{1}{3} + \frac{1}{2}$$
$$\frac{1}{R_{total}} = \frac{3}{12} + \frac{4}{12} + \frac{6}{12} = \frac{13}{12}$$
$$R_{total} = \frac{12}{13} = 0.923\,\Omega$$

Now we have:

	R_1	R_2	R_3	Total
Voltage (V)	12 V	12 V	12 V	12 V
Current (I)	3 A	4 A	6 A	13 A
Resistance (R)	4 Ω	3 Ω	2 Ω	***0.923 Ω***
Power (P)				

Use this space for summary and/or additional notes.

As we did with series circuits, we can calculate the power, using $P = VI$, $P = I^2R$, and/or $P = \frac{V^2}{R}$:

	R_1	R_2	R_3	Total
Voltage (V)	12 V	12 V	12 V	12 V
Current (I)	3 A	4 A	6 A	13 A
Resistance (R)	4 Ω	3 Ω	2 Ω	0.923 Ω
Power (P)	***36 W***	***48 W***	***72 W***	***156 W***

Batteries in Parallel

One question that has not been answered yet is what happens when batteries are connected in parallel.

If the batteries have the same voltage, the potential difference (voltage) remains the same, but the total current is the combined current from the two batteries.

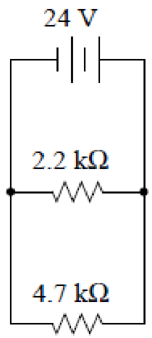
However, if the batteries have different voltages there is a problem, because each battery attempts to maintain a constant potential difference (voltage) between its terminals. This results in the higher voltage battery overcharging the lower voltage battery.

Remember that physically, batteries are electrochemical cells—small solid-state chemical reactors with redox reactions taking place in each cell. If one battery overcharges the other, material is deposited on the cathode (positive terminal) until the cathode becomes physically too large for its compartment, at which point the battery bursts and the chemicals leak out.

Use this space for summary and/or additional notes.

Homework Problems

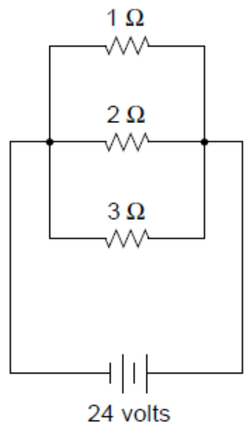
1. Fill in the table for the following circuit:



	R_1	R_2	Total
Voltage (V)			24 V
Current (I)			
Resist. (R)	2.2 kΩ	4.7 kΩ	
Power (P)			

Use this space for summary and/or additional notes.

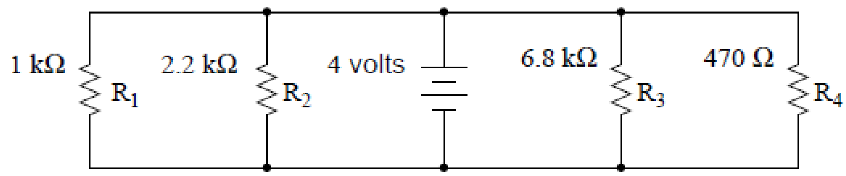
2. Fill in the table for the following circuit:



	R_1	R_2	R_3	Total
Voltage (V)				24 V
Current (I)				
Resist. (R)	1 Ω	2 Ω	3 Ω	
Power (P)				

Use this space for summary and/or additional notes:

3. Fill in the table for the following circuit:



	R_1	R_2	R_3	R_4	Total
Voltage (V)					4 V
Current (I)					
Resistance (R)	1 kΩ	2.2 kΩ	6.8 kΩ	470 Ω	
Power (P)					

Use this space for summary and/or additional notes:

Mixed Series & Parallel Circuits

Unit: Electricity & Magnetism

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 5.3

AP Physics 1 Learning Objectives: 5.B.9.1, 5.B.9.2, 5.B.9.3, 5.C.3.1, 5.C.3.2, 5.C.3.3

Skills:

- analyze circuits by replacing networks of resistors with a single resistor of equivalent resistance.

Language Objectives:

- Set up and solve word problems involving electrical circuits with some components in series and others in parallel with each other.

Labs, Activities & Demonstrations:

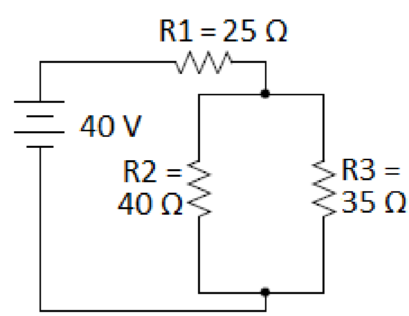
- Light bulb mystery circuits.

Notes:

Mixed Series and Parallel Circuits (Resistors Only)

If a circuit has mixed series and parallel sections, you can determine the various voltages, currents and resistances by applying Kirkhoff's Rules and/or by "simplifying the circuit." Simplifying the circuit, in this case, means replacing resistors in series or parallel with a single resistor of equivalent resistance.

For example, suppose we need to solve the following mixed series & parallel circuit for voltage, current, resistance and power for each resistor:



Use this space for summary and/or additional notes.

Because the circuit has series and parallel sections, we cannot simply use the series and parallel rules across the entire table.

	R_1	R_2	R_3	Total
Voltage (V)				40 V
Current (I)				
Resistance (R)	25 Ω	40 Ω	35 Ω	
Power (P)				

We can use Ohm's Law $(V = IR)$ and the power equation $(P = VI)$ on each individual resistor and the totals for the circuit (columns), but we need two pieces of information for each resistor in order to do this.

Our strategy will be:

1. Simplify the resistor network until all resistances are combined into one equivalent resistor to find the total resistance.
2. Use $V = IR$ to find the total current.
3. Work backwards through your simplification, using the equations for series and parallel circuits in the appropriate sections of the circuit until you have all of the information.

Step 1: If we follow the current through the circuit, we see that it goes through resistor R1 first. Then it splits into two parallel pathways. One path goes through R2 and the other goes through R3.

There is no universal shorthand for representing series and parallel components, so let's define the symbols "—" to show resistors in series, and "||" to show resistors in parallel. The above network of resistors could be represented as:

R1 — (R2 || R3)

Now, we simplify the network just like a math problem—start inside the parentheses and work your way out.

Use this space for summary and/or additional notes.

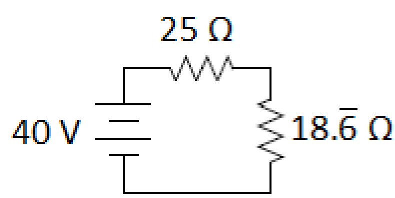
Step 2: Combine the parallel 40Ω and 35Ω resistors into a single equivalent resistance:

$$\frac{1}{R_{total}} = \frac{1}{40} + \frac{1}{35}$$

$$\frac{1}{R_{total}} = 0.0250 + 0.0286 = 0.0536$$

$$R_{total} = \frac{1}{0.0536} = 18.\overline{6}\,\Omega$$

Now, our circuit is equivalent to:



Step 3: Add the two resistances in series to get the total combined resistance of the circuit:

$$18.\overline{6} + 25 = 43.\overline{6}\,\Omega$$

Step 4: Now that we know the total voltage and resistance, we can use Ohm's Law to find the total current:

$$V = IR$$

$$40 = I(43.\overline{6})$$

$$I = \frac{40}{43.\overline{6}} = 0.916\text{ A}$$

While we're at it, let's use $P = VI = (40)(0.916) = 36.6\text{ W}$ to find the total power.

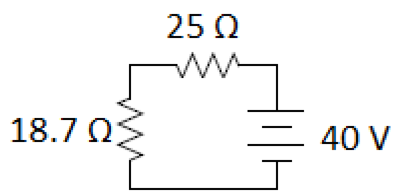
Use this space for summary and/or additional notes.

Now we have:

	R_1	R_2	R_3	Total
Voltage (V)				40 V
Current (I)				***0.916 A***
Resistance (R)	25 Ω	40 Ω	35 Ω	***$43.\bar{6}$ Ω***
Power (P)				***36.6 W***

Now we work backwards.

The next-to-last simplification step was:



The 25 Ω resistor is R1. All of the current goes through it, so the current through R1 must be 0.916 A. Using Ohm's Law, this means the voltage drop across R1 must be:

$$V = IR$$
$$V = (0.916)(25) = 22.9\text{ V}$$

and the power must be:

$$P = VI$$
$$P = (22.9)(0.916) = 21.0\text{ W}$$

This means that the voltage across the parallel portion of the circuit (R2 || R3) must be 40 – 22.9 = 17.1 V.

Use this space for summary and/or additional notes.

	R_1	R_2	R_3	Total
Voltage (V)	***22.9 V***	***17.1 V***	***17.1 V***	40 V
Current (I)	***0.916 A***			0.916 A
Resistance (R)	25 Ω	40 Ω	35 Ω	$43.\overline{6}$ Ω
Power (P)	***21.0 W***			36.6 W

We can use this and Ohm's Law to find the current through one branch:

$$V_{40} = V_{35} = 40 - V_1 = 40 - 22.9 = 17.1V$$

$$V_{40} = I_{40} R_{40}$$

$$I_{40} = \frac{V_{40}}{R_{40}} = \frac{17.1}{40} = 0.428\ A$$

We can use Kirkhoff's Junction Rule to find the current through the other branch:

$$I_{total} = I_{40} + I_{35}$$

$$0.916 = 0.428 + I_{35}$$

$$I_{35} = 0.488\text{ A}$$

This gives us:

	R_1	R_2	R_4	Total
Voltage (V)	22.9 V	17.1 V	17.1 V	40 V
Current (I)	0.916 A	***0.428 A***	***0.488 A***	0.916 A
Resistance (R)	25 Ω	40 Ω	35 Ω	$43.\overline{6}$ Ω
Power (P)	21.0 W			36.6 W

Use this space for summary and/or additional notes.

Finally, because we now have voltage, current and resistance for each of the resistors R_2 and R_3, we can use $P = VI$ to find the power:

$V_2 = I_2 R_2$	$V_3 = I_3 R_3$
$V_2 = (0.428)(10)$	$V_3 = (0.428)(30)$
$V_2 = 4.28\text{ V}$	$V_3 = 12.84\text{ V}$
$P_2 = V_2 I_2$	$P_3 = V_3 I_3$
$P_2 = (4.28)(0.428)$	$P_3 = (12.84)(0.428)$
$P_2 = 1.83\text{ W}$	$P_3 = 5.50\text{ W}$

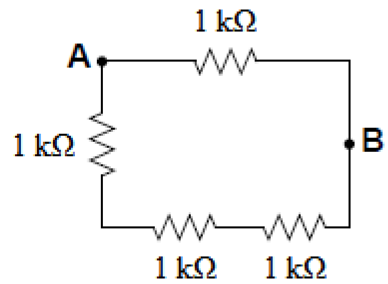
	R_1	R_2	R_4	Total
Voltage (V)	22.9 V	17.1 V	17.1 V	40 V
Current (I)	0.916 A	0.428 A	0.488 A	0.916 A
Resistance (R)	25 Ω	40 Ω	35 Ω	$43.\overline{6}$ Ω
Power (P)	21.0 W	***7.32 W***	***8.34 W***	36.6 W

Alternately, because the total power is the sum of the power of each component, once we had the power in all but one resistor, we could have subtracted from the total to find the last one.

Use this space for summary and/or additional notes.

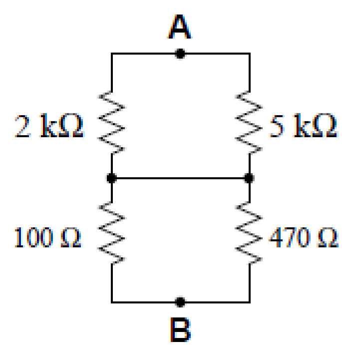
Homework Problems

1. What is the equivalent resistance between points **A** and **B**?



Answer: 750 Ω

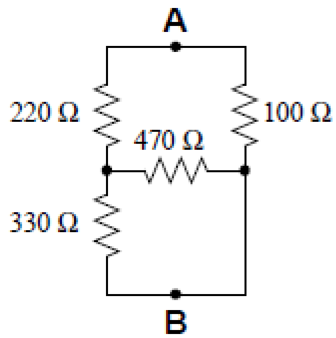
2. What is the equivalent resistance between points **A** and **B**?



Answer: 1511 Ω or 1.511 kΩ

Use this space for summary and/or additional notes.

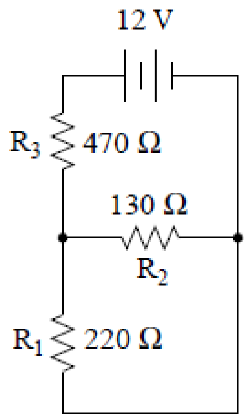
3. What is the equivalent resistance between points **A** and **B**?



Answer: 80.5 Ω

Use this space for summary and/or additional notes.

4. Fill in the table for the circuit below:



	R_1	R_2	R_3	Total
Voltage (V)				12 V
Current (I)				
Resistance (R)	220 Ω	130 Ω	470 Ω	
Power (P)				

Use this space for summary and/or additional notes.

Measuring Voltage, Current & Resistance

Unit: Electricity & Magnetism

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 5.3

AP Physics 1 Learning Objectives: N/A

Knowledge & Understanding:

- how voltage, current and resistance are measured.
- how positive and negative numbers for voltage and current correspond with the direction of current flow.

Skills:

- measure voltage, current and resistance.

Language Objectives:

- Accurately describe how to measure voltage, current and resistance in an electric circuit, using appropriate academic language.

Labs, Activities & Demonstrations:

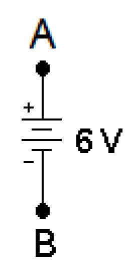
- Show & tell with digital multi-meter.

Notes:

Analyzing an electrical circuit means figuring out the potential difference (voltage), current, and/or resistance in each component of a circuit. In order to analyze actual circuits, it is necessary to be able to measure these quantities.

Measuring Voltage

Suppose we want to measure the electric potential (voltage) across the terminals of a 6 V battery. The diagram would look like this:

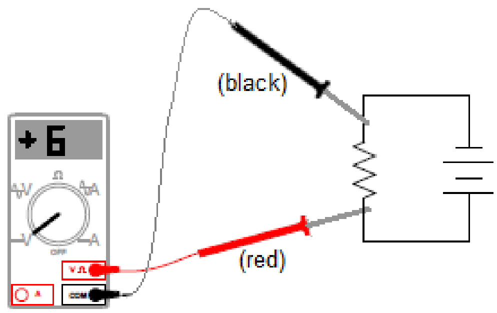


The voltage between points A and B is either +6 V or −6 V, depending on the direction. The voltage from A to B (positive to negative) is +6 V, and the voltage from B to A (negative to positive) is −6 V.

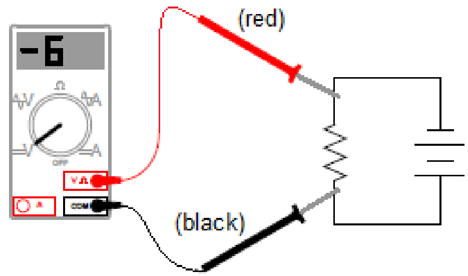
Use this space for summary and/or additional notes:

When measuring voltage, the circuit needs to be powered up with current flowing through it. Make sure that the voltmeter is set for volts (DC or AC, as appropriate) and that the red lead is plugged into the V Ω socket (for measuring volts or ohms). Then touch the two leads *in parallel* with the two points you want to measure the voltage across. (*Remember that voltage is the same across all branches of a **parallel** circuit. You want the voltmeter in parallel so the voltmeter reads the same voltage as the voltage across the component that you are measuring.*)

On a voltmeter (a meter that measures volts or voltage), the voltage is measured assuming the current is going from the red (+) lead to the black (−) lead. In the following circuit, if you put the red (+) lead on the more positive end of a resistor and the black (−) lead on the more negative end, the voltage reading would be positive. In this circuit, the voltmeter reads a potential difference of +6 V:



If you switch the leads, so the black (−) lead is on the more positive end and the red (+) lead is on the more positive end, the voltage reading would be negative. In this circuit, the voltmeter reads −6 V:



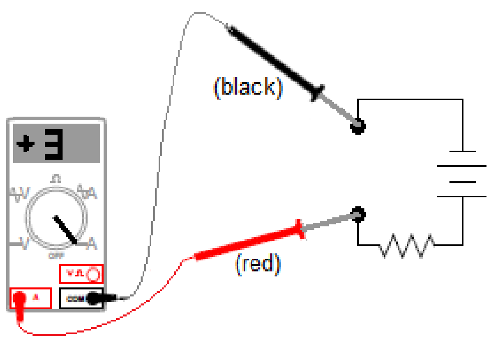
The reading of −6 V indicates that the potential difference is 6 V, but the current is actually flowing in the opposite direction from the way the voltmeter is measuring—from the black (−) lead to the red (+) lead.

Use this space for summary and/or additional notes.

Measuring Current

When measuring current, the circuit needs to be open between two points. Make sure the ammeter is set for amperes (A), milliamperes (mA) or microamperes (μA) AC or DC, depending on what you expect the current in the circuit to be. Make sure the red lead is plugged into appropriate socket (A if the current is expected to be 0.5 A or greater, or mA/μA if the current is expected to be less than 0.5 A). Then touch one lead to each of the two contact points, so that the ammeter is *in series* with the rest of the circuit. (*Remember that current is the same through all components in a* ***series*** *circuit. You want the ammeter in series so that all of the current flows through it.*)

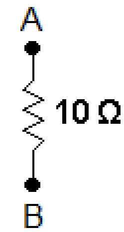
On an ammeter (a meter that measures current), the current is measured assuming that it is flowing from the red (+) lead to the black (−) lead. In the following circuit, if you put the red (+) lead on the side that is connected to the positive terminal and the black (−) lead on the end that is connected to the negative terminal, the current reading would be positive. In this circuit, the current is +3 A:



As with the voltage example above, if you switched the leads, the reading would be −3 A instead of +3 A.

Use this space for summary and/or additional notes.

Measuring Resistance



Resistance does not have a direction. If you placed an ohmmeter (a meter that measures resistance) across points A and B, it would read 10 Ω regardless of which lead is on which point.

However, because an ohmmeter needs to supply a small amount of current across the component and measure the resistance, the reading is more susceptible to measurement problems, such as the resistance of the wire itself, how well the probes are making contact with the circuit, *etc.* It is often more reliable to measure the voltage and current and calculate resistance using Ohm's Law ($V = IR$).

Use this space for summary and/or additional notes.