

## GATE ACADEMY Presents

## Most Awaited Book For GATE - 2021

# Electrical Engineering 




## Volume - II

- Diligent Solution of GATE Previous Year Questions (1987-2020)
- Multi Method Approach for a Single Problem to Develop Crystal Clear Concepts
- Video Solution for Gonspicuous Questions to Enhance Problem Solving Skills
- This book is also a value addition for ESE/PSUs/ISRO/DRDO



## TOPIC WISE GATE SOLUTIONS 1987-2020

## IMPORTANCE of GATE

GATE examination has been emerging as one of the most prestigious competitive exam for engineers. Earlier it was considered to be an exam just for eligibility for pursuing PG courses, but now GATE exam has gained a lot of attention of students as this exam open an ocean of possibilities like:

1. Admission into IISc, IITs, IIITs, NITs

A good GATE score is helpful for getting admission into IISc, IITs, IIITs, NITs and many other renowned institutions for M.Tech./M.E./M.S. An M.Tech graduate has a number of career opportunities in research fields and education industries. Students get ₹ 12,400 per month as stipend during their course.
2. Selection in various Public Sector Undertakings (PSUs)

A good GATE score is helpful for getting job in government-owned corporations termed as Public Sector Undertakings (PSUs) in India like IOCL, BHEL, NTPC, BARC, ONGC, PGCIL, DVC, HPCL, GAIL, SAIL \& many more.
3. Direct recruitment to Group A level posts in Central government, i.e., Senior Field Officer (Tele), Senior Research Officer (Crypto) and Senior Research Officer (S\&T) in Cabinet Secretariat, Government of India, is now being carried out on the basis of GATE score.
4. Foreign universities through GATE

GATE has crossed the boundaries to become an international level test for entry into postgraduate engineering programmes in abroad. Some institutes in two countries Singapore and Germany are known to accept GATE score for admission to their PG engineering programmes.
5. National Institute of Industrial Engg. (NITIE)

- NITIE offers PGDIE / PGDMM / PGDPM on the basis of GATE scores. The shortlisted candidates are then called for group Discussion and Personal Interview rounds.
- NITIE offers a Doctoral Level Fellowship Programme recognized by Ministry of HRD (MHRD) as equivalent to PhD of any Indian University.
- Regular full time candidates those who will qualify for the financial assistance will receive $₹ 25,000$ during 1st and 2nd year of the Fellowship programme and ₹ 28,000 during 3rd, 4th and 5th year of the Fellowship programme as per MHRD guidelines.

6. Ph.D. in IISc/ IITs

- IISc and IITs take admissions for Ph.D. on the basis of GATE score.
- Earn a Ph.D. degree directly after Bachelor's degree through integrated programme.
- A fulltime residential researcher (RR) programme.

7. Fellowship Program in management (FPM)

- Enrolment through GATE score card
- Stipend of ₹ $22,000-30,000$ per month + HRA
- It is a fellowship program
- Application form is generally available in month of sept. and oct.

Note : In near future, hopefully GATE exam will become a mandatory exit test for all engineering students, so take this exam seriously. Best of LUCK !

## GATE Exam Pattern

| Section | Question No. | No. of Questions | Marks Per Question | Total Marks |
| :---: | :---: | :---: | :---: | :---: |
| General Aptitude | 1 to 5 | 5 | 1 | 5 |
|  | 6 to 10 | 5 | 2 | 10 |
| Technical <br> $+$ <br> Engineering <br> Mathematics | 1 to 25 | 25 | 1 | 25 |
|  | 26 to 55 | 30 | 2 | 60 |
| Total Duration : 3 hours |  | Total Questions : 65 | Total Mar | : 100 |
| Note : 40 to 45 marks will be allotted to Numerical Answer Type Questions |  |  |  |  |

## Pattern of Questions:

(i) Multiple Choice Questions (MCQ) carrying 1 or 2 marks each in all the papers and sections. These questions are objective in nature, and each will have a choice of four answers, out of which the candidate has to select (mark) the correct answer.
Negative Marking for Wrong Answers : For a wrong answer chosen in a MCQ, there will be negative marking. For 1-mark MCQ, 1/3 mark will be deducted for a wrong answer. Likewise for, 2-marks MCQ, 2/3 mark will be deducted for a wrong answer.
(ii) Numerical Answer Type (NAT) Questions carrying $\mathbf{1}$ or $\mathbf{2}$ marks each in all the papers and sections. For these questions, the answer is a signed real number, which needs to be entered by the candidate using the virtual numeric keypad on the monitor (Keyboard of the computer will be disabled). No choices will be shown for these type of questions. The answer can be a number such as $\mathbf{1 0}$ or $\mathbf{- 1 0}$ (an integer only). The answer may be in decimals as well, for example, $\mathbf{1 0 . 1}$ (one decimal) or $\mathbf{1 0 . 0 1}$ (two decimal) or $\mathbf{- 1 0 . 0 0 1}$ (three decimal). These questions will be mentioned with, up to which decimal places, the candidates need to make an answer. Also, an appropriate range will be considered while evaluating the numerical answer type questions so that the candidate is not penalized due to the usual round-off errors. Wherever required and possible, it is better to give NAT answer up to a maximum of three decimal places.

Note : There is NO negative marking for a wrong answer in NAT questions.

## What is special about this book ?

GATE ACADEMY Team took several years' to come up with the solutions of GATE examination. It is because we strongly believe in quality. We have significantly prepared each and every solution of the questions appeared in GATE, and many individuals from the community have taken time out to proof read and improve the quality of solutions, so that it becomes very lucid for the readers. Some of the key features of this book are as under :
This book gives complete analysis of questions chapter wise as well as year wise.
Video Solution of important conceptual questions has been given in the form of QR code and by scanning QR code one can see the video solution of the given question.

To Solutions has been presented in lucid and understandable language for an average student.
In In addition to the GATE syllabus, the book includes the nomenclature of chapters according to text books for easy reference.
Lo Last but not the least, author's 10 years experience and devotion in preparation of these solutions.
(b) Steps to Open Video solution through mobile.

(1) Search for QR Code scanner in Google Play / App Store.

(3) Scan the given QR Code for particular question.

(2) Download \& Install any QR Code Scanner App.

(4) Visit the link generated \& you'll be redirect to the video solution.

Note : For recent updates regarding minor changes in this book, visit www.gateacademy.co.in. We are always ready to appreciate and help you.

## PREFACE

It is our pleasure, that we insist on presenting "GATE 2021 Electrical Engineering (Volume - II)" authored for Electrical Engineering to all of the aspirants and career seekers. The prime objective of this book is to respond to tremendous amount of ever growing demand for error free, flawless and succinct but conceptually empowered solutions to all the question over the period 1987-2020.

This book serves to the best supplement the texts for Electronics \& Communication Engineering but shall be useful to a larger extent for Electronics \& Communication Engineering and Instrumentation Engineering as well. Simultaneously having its salient feature the book comprises:
$\stackrel{\Perp}{4}$ Step by step solution to all questions.
$\stackrel{4}{4}$ Complete analysis of questions chapter wise as well as year wise.
${ }^{4}$ Detailed explanation of all the questions.
${ }_{4}^{4}$ Solutions are presented in simple and easily understandable language.
${ }^{4}$ Video solutions for good questions.
4) It covers all GATE questions from 1987 to 2020 ( 34 years).

The authors do not sense any deficit in believing that this title will in many aspects, be different from the similar titles within the search of student.

In particular, we wish to thank GATE ACADEMY expert team members for their hard work and consistency while designing the script.

The final manuscript has been prepared with utmost care. However, going a line that, there is always room for improvement in anything done, we would welcome and greatly appreciate suggestion and correction for further improvement.

Sakshi Dhande<br>(Co-Director, GATE ACADEMY Learning Pvt. Ltd.)<br>Umesh Dhande<br>(Director, GATE ACADEMY Learning Pvt. Ltd.)

## 1. Electrical Machines

1. Single Phase Transformer
2. Three Phase Transformer
3. DC Machine
4. Synchronous Machine
5. Three Phase Induction Machine
6. Single Phase Induction Motor
7. Miscellaneous

## 2. Power Electronics

1. Power Semiconductor Devices
2. Single Phase Rectifier
3. Three Phase Rectifier
4. Choppers \& Commutation Techniques
5. Inverters
6. AC Voltage Controller
7. Miscellaneous

## 3. Power System Analysis

1. Parameters of Transmission Line
2. Performance of Transmission Lines
3. Voltage Control
4. Power Factor Improvement
5. Travelling Waves
6. Distribution Systems
7. Cables \& Insulators
8. Per Unit System
9. Symmetrical Faults
10. Symmetrical Components
11. Unsymmetrical Faults
12. Power System Stability
13. Load Flow Studies
14. Switch Gear \& Protection
15. Generating Power Stations
16. High Voltage DC Transmission

## 4. Electrical and Electronics Measurement

1. Error Analysis \& Measurement
2. Basic Instruments
3. Measurement of Resistance \& AC Bridges
4. Potentiometer
5. Measurement of Energy \& Power
6. CRO \& Electronic Measurement
7. Instrument Transformers

## 5. Electromagnetic Fields

1. Electrostatics
2. Magnetostatics
3. General Aptitude (GA)
4. Verbal Ability
5. Numerical Ability


## 1991 IIT Madras

1.1 The relative current directions through the primary $(P)$ and secondary $(S)$ of a single phase transformer connected to a resistive load on the secondary side, are indicated in the various cross-sectional views given in figure. Which of these are correct representations?
(A)

(B)

(C)

(D)

1.2 The hysteresis and eddy current losses of a single phase transformer working on $200 \mathrm{~V}, 50 \mathrm{~Hz}$ supply are $P_{h}$ and $P_{e}$ respectively. The percentage decrease in these, when operated at a $160 \mathrm{~V}, 40 \mathrm{~Hz}$ supply is
(A) 32,36
(B) 20, 36
(C) 25,20
(D) 40,80

## 1992 IIT Delhi

1.3 Two transformers of the same type, using the same grade of iron and conductor materials, are designed to work at the same flux and current densities, but the linear dimensions of one are two times those of the other in all respects. The ratio of kVA of the two transformers closely equals
(A) 16
(B) 8
(C) 4
(D) 2
1.4 Two transformers of different kVA ratings working in parallel share the load in proportion to their ratings when their
(A)per unit leakage impedances on the same kVA base are the same.
(B) per unit leakage impedances on their respective ratings are equal.
(C) ohmic values of the leakage impedance are inversely proportional to their ratings.
(D)ohmic values of the leakage magnetizing reactances are the same.
1.5 A Buchholz relay is used for
(A) protection of a transformer against all internal faults.
(B) protection of a transformer against external faults.
(C) protection of a transformer against both internal and external faults.
(D) protection of induction motors.

## 1993 IIT Bombay

1.6 A $50 \mathrm{~Hz}, 220 \mathrm{~V} / 440 \mathrm{~V}, 5 \mathrm{kVA}$ singlephase transformer operates on $220 \mathrm{~V}, 40$ Hz supply with secondary winding. Then
(A) the eddy current loss and hysteresis loss of the transformer decrease.
(B) the eddy current loss and hysteresis loss of the transformer increase.
(C) the hysteresis loss of the transformer increase while eddy current loss remains the same.
(D) the hysteresis loss remains the same whereas eddy current loss decreases.

## 1994 IIT Kharagpur

1.7 When a transformer winding suffers a short-circuit, the adjoining turns of the same winding experience
(A) an attractive force.
(B) a repulsive force.
(C) no force.
(D) none of the above.
1.8 Two transformers of identical voltage but of different capacities are operating in parallel.
For satisfactory load sharing
(A) impedances must be equal.
(B) per unit impedances must be equal.
(C) per unit impedances and $\frac{X}{R}$ ratios must be equal.
(D) impedances and $\frac{X}{R}$ ratios must be equal.

## 1995 IIT Kanpur

1.9 A $4 \mathrm{kVA}, 50 \mathrm{~Hz}$, single-phase transformer has a ratio $200 \mathrm{~V} / 400 \mathrm{~V}$. The data taken on the L.V. side at rated voltage show
that the open circuit input wattage is 80 W . The mutual inductance between the primary and the secondary windings is 1.91 H . (Neglecting the effect of winding resistance and leakage reactances). The value of current (in Amperes) taken by the transformer, if the no-load test is conducted on the H.V. side at rated voltage will be $\qquad$ .

## 1996 IISc Bangalore

1.10 The function of oil in a transformer is
(A) to provide insulation and cooling.
(B) to provide protection against lightning
(C) to provide protection against shortcircuit.
(D) to provide lubrication.
1.11 Auto transformer is used in transmission and distribution
(A) when operator is not available.
(B) when iron losses are to be reduced.
(C) when efficiency considerations can be ignored.
(D) when the transformation ratio is small.

## 1997 IIT Madras

1.12 The low voltage winding of a $400 \mathrm{~V} / 230$ V, single-phase, 50 Hz transformer is to be connected to a 25 Hz the supply voltage should be
(A) 230 V
(B) 460 V
(C) 115 V
(D) 65 V
1.13 A voltage $V=400 \sin 314.16 t$ is applied to a single-phase transformer on no-load. If the no-load current of the transformer is $2 \sin \left(314.16 t-85^{\circ}\right)$, the magnetization branch impedance will be approximately
(A) $141 \angle 90^{\circ}$
(B) $200 \angle-85^{\circ}$
(C) $200 \angle 85^{0}$
(D) $282 \angle-80^{\circ}$

## 1998 IIT Delhi

1.14 The laws of electromagnetic induction (Faraday's and Lenz's law) are summarized in the following equation
(A) $e=i R$
(B) $e=L \frac{d i}{d t}$
(C) $e=-\frac{d \psi}{d t}$
(D) None of these
1.15 The efficiency of a 100 kVA transformer is 0.98 at full as well as at half load. For this transformer at full load the copper loss
(A) is less than core loss.
(B) is equal to core loss.
(C) is more than core loss.
(D) none of the above.
1.16 The magnetizing current in a transformer is rich in
(A) $3^{\text {rd }}$ harmonic
(B) $5^{\text {th }}$ harmonic
(C) $7^{\text {th }}$ harmonic
(D) $13^{\text {th }}$ harmonic
1.17 A 50 Hz transformer having equal hysteresis and eddy current losses at rated excitation is operated at 45 Hz at $90 \%$ of its rated voltage. Compared to rated operating point, the core losses under this condition
(A) reduce by $10 \%$
(B) reduce by $19 \%$
(C) reduce by $14.5 \%$
(D) remain unchanged

## Common Data for Questions 1.18 \& 1.19

In a $50 \mathrm{kVA}, 11 \mathrm{KV} / 400 \mathrm{~V}$ transformers, the iron and copper losses are 500 W and 600 W respectively under rated conditions.
1.18 The efficiency on unity power factor at full load will be $\qquad$ .
1.19 The load at which maximum efficiency occurs and the iron and copper losses corresponding to this load respectively will be

## 1999 IIT Bombay

1.20 A $10 \mathrm{kVA}, 400 \mathrm{~V} / 200 \mathrm{~V}$ single-phase transformer with $10 \%$ impedance draws a steady short-circuit line current of
(A) 50 A
(B) 150 A
(C) 250 A
(D) 350 A
1.21 A $400 \mathrm{~V} / 100 \mathrm{~V}, 10 \mathrm{kVA}$ two winding transformer is reconnected as an auto transformer across a suitable voltage source. The maximum rating of such an arrangement could be
(A) 50 kVA
(B) 15 kVA
(C) 12.5 kVA
(D) 8.75 kVA
1.22 A $10 \mathrm{kVA}, 400 \mathrm{~V} / 200 \mathrm{~V}$, single-phase transformer with a percentage resistance of $3 \%$ and percentage reactance of $6 \%$ is supplying a current of 50 A to a resistive load. The value of the load voltage is
(A) 194 V
(B) 390 V
(C) 192 V
(D) 196 V
1.23 Two single-phase transformers $A$ and $B$ have the following parameters.
Transformer A: $400 \mathrm{~V} / 200 \mathrm{~V}, 10 \mathrm{kVA}$, percentage resistance and percentage reactance are $3 \%$ and $4 \%$ respectively.
Transformer B : $5 \mathrm{kVA}, 400 \mathrm{~V} / 200 \mathrm{~V}$, percentage resistance and percentage reactance are $4 \%$ and $3 \%$ respectively. These two transformers are connected in parallel and they share a common load of 12 kW at a power factor of 0.8 lag. Determine the active power ( $\mathrm{kW} \mathrm{)} \mathrm{and}$ reactive power (kVAR) delivered by transformer $A$.
(A) 2.5, 5
(B) $7.5,12.5$
(C) $7.5,6.78$
(D) $2.5,6.78$

## 2000 IIT Kharagpur

1.24 If an AC voltage wave is corrupted with an arbitrary number of harmonics, then
the overall frequency component is in terms of
(A) only the peak values.
(B) only the rms values.
(C) only the average values.
(D) all the three measures (peak, rms and average values).

## 2001 IIT Kanpur

1.25 A single-phase transformer is to be switched to the supply to have minimum inrush current. The switch should be closed at
(A) maximum supply voltage.
(B) zero supply voltage.
(C) $\frac{1}{\sqrt{2}}$ maximum supply voltage.
(D) $\frac{1}{2}$ maximum supply voltage.
1.26 The core flux of a practical transformer with a resistive load
(A) is strictly constant with load changes.
(B) increases linearly with load.
(C) increases as the square root of the load.
(D) decreases with increased load.
1.27 In the protection of transformers, harmonic restraint is used to guard against
(A) magnetizing inrush current.
(B) unbalanced operation.
(C) lightning.
(D) switching over-voltages.
1.28 The hysteresis loop of a magnetic material has an area of $5 \mathrm{~cm}^{2}$ with the scales given as $1 \mathrm{~cm}=2 \mathrm{AT}$ and $1 \mathrm{~cm}=$ 50 mWb . At 50 Hz , the total hysteresis loss is
(A) 15 W
(B) 20 W
(C) 25 W
(D) 50 W

## 2002 IISc Bangalore

1.29 In a single-phase 3-winding transformer, the turns ratio for primary : secondary : tertiary windings is $20: 4: 1$ with the lagging currents of 50 A at a power factor of 0.8 in the secondary winding and 60 A at power factor of 0.6 in the tertiary winding. Find the primary current and power factor.
(A) $2.5 \mathrm{~A}, 0.6$
(B) $12.5 \mathrm{~A}, 0.6$
(C) $200 \mathrm{~A}, 0.62$
(D) $2.5 \mathrm{~A}, 0.62$
1.30 A $1 \mathrm{kVA}, 230 \mathrm{~V} / 100 \mathrm{~V}$, single-phase, 50 Hz transformer having negligible winding resistance and leakage inductance is operating under saturation, while 250 $\mathrm{V}, 50 \mathrm{~Hz}$ sinusoidal supply is connected to the high voltage winding. A resistive load is connected to the low voltage winding which draws rated current. Which one of the following quantities will not be sinusoidal?
(A)Voltage induced across the low voltage winding
(B) Core flux
(C) Load current
(D) Current drawn from the source
1.31 A $400 \mathrm{~V} / 200 \mathrm{~V} / 200 \mathrm{~V}, 50 \mathrm{~Hz}$ three winding transformer is connected as shown in figure. The reading of the voltmeter ' $V$ ' will be

(A) 0 V
(B) 400 V
(C) 600 V
(D) 800 V

## Common Data for <br> Questions 1.32 \& 1.33

A single phase $6300 \mathrm{kVA}, 50 \mathrm{~Hz}, 3300 \mathrm{~V} / 400 \mathrm{~V}$ distribution transformer is connected between two 50 Hz supply systems, $A$ and $B$ as shown in figure, the transformer has 12 and 99 turns in the low and high voltage windings respectively. The magnetizing reactance of the transformer referred to the high voltage side is $500 \Omega$. The leakage reactance of the high and low voltage windings are $1.0 \Omega$ and $0.012 \Omega$ respectively. Neglect the winding resistance and core losses of the transformer. The Thevenin's voltage of system $A$ is 3300 V while that of system $B$ is 400 V . The short circuit reactance of system $A$ and $B$ are $0.5 \Omega$ and $0.010 \Omega$ respectively. If no power is transferred between $A$ and $B$, so that the two system voltage are in phase,

1.32 The magnetizing ampere turns of the transformer will be $\qquad$ .
1.33 The phase relation between the two system voltages will be
(A) $V_{A}$ leads $V_{B}$
(B) $V_{A}$ lags $V_{B}$
(C) Both $V_{A}$ and $V_{B}$ are in phase
(D) None of the above

## 2003 IIT Madras

1.34 A single phase transformer has a maximum efficiency of $90 \%$ at full load and unity power factor. Efficiency at half load, at the same power factor is
(A) $86.7 \%$
(B) $88.26 \%$
(C) $88.9 \%$
(D) $87.8 \%$
1.35 Figure shows an ideal single-phase transformer. The primary and secondary coils are wound on the core as shown. Turns ratio $\left(N_{1} / N_{2}\right)=2$. The correct phasors of voltages $E_{1}, E_{2}$ currents $I_{1}, I_{2}$ and core flux $\phi$ are as shown in

(A)

(B)

(C)

(D)

1.36 Figure shows an ideal three winding transformer. Windings are wound on the same core as shown. The turns ratio $N_{1}: N_{2}: N_{3}$ is $4: 2: 1$. A resistor of $10 \Omega$ is connected across winding-2. A capacitor of reactance $2.5 \Omega$ is connected across winding-3. Winding-1 is connected across a 400 V , ac supply. If the supply voltage phasor $V_{1}=400 \angle 0^{\circ} \mathrm{V}$, the supply current phasor $I_{1}$ is given by

(A) $(-10+j 10) \mathrm{A}$
(B) $(-10-j 10) \mathrm{A}$
(C) $(10+j 10) \mathrm{A}$
(D) $(10-j 10) \mathrm{A}$

## 2004 IIT Delhi

1.37 A $50 \mathrm{kVA}, 3300 \mathrm{~V} / 230 \mathrm{~V}$ single-phase transformer is connected as an autotransformer shown in figure. The nominal rating of the autotransformer will be

(A) 50.0 kVA
(B) 53.5 kVA
(C) 4717.4 kVA
(D) 767.4 kVA

## 2005 IIT Bombay

1.38 The equivalent circuit of a transformer has leakage reactances $X_{1}, X_{2}^{\prime}$ and magnetizing reactance $X_{m}$. Their magnitudes satisfy
(A) $X_{1} \gg X_{2}^{\prime} \gg X_{m}$
(B) $X_{1} \ll X_{2}^{\prime} \ll X_{m}$
(C) $X_{1} \approx X_{2}^{\prime} \gg X_{m}$
(D) $X_{1} \approx X_{2}^{\prime} \ll X_{m}$

## 2006 IIT Kharagpur

1.39 In a transformer, which of the following statements is valid?
(A) In an open circuit test, copper losses are obtained while in short circuit test core losses are obtained.
(B) In an open circuit test, current is drawn at high power factor.
(C) In a short circuit test, current is drawn at zero power factor.
(D)In an open circuit test, current is drawn at low power factor.
1.40 Two transformer are to be operated in parallel such that they share load in proportion to their kVA ratings. The rating of the first transformer is 500 kVA and its p.u. leakage impedance is 0.05 p.u. If the rating of second transformer is 250 kVA , its p.u. leakage impedance is
(A) 0.20
(B) 0.10
(C) 0.05
(D) 0.025

## Common Data for Questions 1.41 \& 1.42

A 300 kVA transformer has $95 \%$ efficiency at full load 0.8 pf lagging and $96 \%$ efficiency at half load, unity $p f$.
1.41 The iron loss $\left(P_{i}\right)$ and copper loss $\left(P_{c}\right)$ in kW , under full load operation are
(A) $P_{c}=4.12, P_{i}=8.51$
(B) $P_{c}=6.59, P_{i}=9.21$
(C) $P_{c}=8.51, P_{i}=4.12$
(D) $P_{c}=12.72, P_{i}=3.07$
1.42 What is the maximum efficiency (in \%) at unity power factor load?
(A) 95.1
(B) 96.2
(C) 96.4
(D) 98.1

## 2007 IIT Kanpur

1.43 In the figure, transformer $T_{1}$ has two secondaries, all three windings having the same number of turns and with polarities as indicated. One secondary is shorted by a $10 \Omega$ resistor $R$, and the other by a 15 mF capacitor. The switch $S$ is opened $(t=0)$ when the capacitor is charged to 5 V with the left plate as positive. At $\left(t=0^{+}\right)$the voltage $V_{p}$ and current $I_{R}$ are

(A) $-25 \mathrm{~V}, 0.0 \mathrm{~A}$
(B) very large voltage, very large current
(C) $5.0 \mathrm{~V}, 0.5 \mathrm{~A}$
(D) $-5.0 \mathrm{~V},-0.5 \mathrm{~A}$
1.44 In a transformer, zero voltage regulation at full load is
(A) not possible
(B) possible at unity power factor load
(C) possible at leading power factor load
(D) possible at lagging power factor load
1.45 A single-phase $50 \mathrm{kVA}, 250 \mathrm{~V} / 500 \mathrm{~V}$ two winding transformer has an efficiency of $95 \%$ at full load, unity power factor. If it is reconfigured as a $500 \mathrm{~V} / 750 \mathrm{~V}$ autotransformer, its efficiency at its new rated load at unity power factor will be
(A) $95.752 \%$
(B) $97.851 \%$
(C) $98.276 \%$
(D) $99.241 \%$

## 2008 IISc Bangalore

1.46 It is desired to measure parameters of 230 V/115 V, 2 kVA, single-phase transformer. The following wattmeters are available in a laboratory :
$W_{1}: 250 \mathrm{~V}, 10 \mathrm{~A}$, Low power factor
$W_{2}: 250 \mathrm{~V}, 5 \mathrm{~A}$, Low power factor
$W_{3}: 150 \mathrm{~V}, 10 \mathrm{~A}$, High power factor $W_{4}: 150 \mathrm{~V}, 5 \mathrm{~A}$, High power factor

The wattmeters used in open circuit test and short circuit test of the transformer will respectively be
(A) $W_{1}$ and $W_{2}$
(B) $W_{2}$ and $W_{4}$
(C) $W_{1}$ and $W_{4}$
(D) $W_{2}$ and $W_{3}$
1.47 The core of a two-winding transformer is subjected to a magnetic flux variation as indicated in the figure.



The induced emf $\left(e_{r s}\right)$ in the secondary winding as a function of time will be of the form
(A)

(B)

(C)

(D)


## 2009 IIT Roorkee

1.48 The single phase, 50 Hz iron core transformer in the circuit has both the vertical arms of cross sectional area $20 \mathrm{~cm}^{2}$ and both the horizontal arms of cross sectional area $10 \mathrm{~cm}^{2}$. If the two windings shown were wound instead on opposite horizontal arms, the mutual inductance will

(A) double
(B) remain same
(C) be halved
(D) become one quarter

## Common Data for Questions 1.49 \& 1.50

The circuit diagram shows a two-winding, lossless transformer with no leakage flux, excited from a current source $i(t)$, whose waveform is
also shown. The transformer has a magnetizing inductance of $400 / \pi \mathrm{mH}$.


1.49 The peak voltage across $A$ and $B$, with $S$ open is
(A) $\frac{400}{\pi} \mathrm{~V}$
(B) 800 V
(C) $\frac{4000}{\pi} \mathrm{~V}$
(D) $\frac{800}{\pi} \mathrm{~V}$
1.50 If the waveform of $i(t)$ is changed to $i(t)=10 \sin (100 \pi t) \mathrm{A}$, the peak voltage across $A$ and $B$ with $S$ closed is
(A) 400 V
(B) 240 V
(C) 320 V
(D) 160 V


The figure above shows coils 1 and 2, with dot markings as shown, having 4000 and 6000 turns respectively. Both the coils have a rated current of 25 A . Coil 1 is excited with single phase, 400 V, 50 Hz supply.
1.51 The coils are to be connected to obtain a $1-\phi, 400 / 1000 \mathrm{~V}$, auto-transformer to drive a load of 10 kVA . Which of the options given should be exercised to realize the required auto-transformer?
(A) Connected $A$ and $D$; Common $B$
(B) Connected $B$ and $D$; Common $C$
(C) Connected $A$ and $C$; Common $B$
(D) Connected $A$ and $C$; Common $D$
1.52 In the auto-transformer obtained in 1.51, the current in each coil is
(A)Coil-1 is 25 A and Coil-2 is 10 A
(B) Coil-1 is 10 A and Coil-2 is 25 A
(C) Coil-1 is 10 A and Coil-2 is 15 A
(D) Coil-1 is 15 A and Coil-2 is 10 A

## 2010 IIT Guwahati

1.53 A single-phase transformer has a turns ratio of $1: 2$, and is connected to a purely resistive load as shown in the figure. The magnetizing current drawn is 1 A , and the secondary current is 1 A . If core losses and leakage reactances are neglected, the primary current is

(A) 1.41 A
(B) 2 A
(C) 2.24 A
(D) 3 A

## 2011 IIT Madras

1.54 A single-phase air core transformer, fed from a rated sinusoidal supply, is operating at no load. The steady state magnetizing current drawn by the transformer from the supply will have the waveform
(A)

(B)

(C)

(D)


## 2012 IIT Delhi

1.55 A single phase $10 \mathrm{kVA}, 50 \mathrm{~Hz}$ transformer with 1 kV primary winding draws 0.5 A and 55 W , at rated voltage and frequency, on no load. A second transformer has a core with all its linear dimensions $\sqrt{2}$ times the corresponding dimensions of the first transformer. The core material and lamination thickness are the same in both transformers. The primary windings of both the transformers have the same number of turns. If a rated voltage of 2 kV at 50 Hz is applied to the primary of the second transformer, then the no load current and power respectively are
(A) $0.7 \mathrm{~A}, 77.8 \mathrm{~W}$
(B) $0.7 \mathrm{~A}, 155.6 \mathrm{~W}$
(C) $1 \mathrm{~A}, 110 \mathrm{~W}$
(D) $1 \mathrm{~A}, 220 \mathrm{~W}$

## 2013 IIT Bombay

1.56 A single-phase transformer has no-load loss of 64 W , as obtained from an opencircuit test. When a short-circuit test is performed on it with $90 \%$ of the rated currents flowing in its both LV and HV windings, the measured loss is 81 W . The transformer has maximum efficiency when operated at
(A) $50.0 \%$ of the rated current.
(B) $64.0 \%$ of the rated current.
(C) $80.0 \%$ of the rated current.
(D) $88.8 \%$ of the rated current.

## 2014 IIT Kharagpur

1.57 For a specified input voltage and frequency, if the equivalent radius of the core of a transformer is reduced by half, the factor by which the number of turns in the primary should change to maintain the same no load current is [Set - 01]
(A) $\frac{1}{4}$
(B) $\frac{1}{2}$
(C) 2
(D) 4
1.58 The core loss of a single-phase, 230/115 $\mathrm{V}, 50 \mathrm{~Hz}$ power transformer is measured from 230 V side by feeding the primary ( 230 V side) from a variable voltage variable frequency source while keeping the secondary open circuited. The core loss is measured to be 1050 W for 230 V , 50 Hz input. The core loss is again measured to be 500 W for $138 \mathrm{~V}, 30 \mathrm{~Hz}$ input. The hysteresis and eddy current losses of the transformer for $230 \mathrm{~V}, 50$ Hz input are respectively, [Set - 01]
(A) 508 W and 542 W
(B) 468 W and 582 W
(C) 498 W and 552 W
(D) 488 W and 562 W
1.59 For a single-phase, two winding transformer, the supply frequency and voltage are both increased by $10 \%$. The percentage changes in the hysteresis loss and eddy current loss, respectively, are
[Set - 02]
(A) 10 and 21
(B) -10 and 21
(C) 21 and 10
(D) -21 and 10
1.60 A single-phase, $50 \mathrm{kVA}, 1000 \mathrm{~V} / 100 \mathrm{~V}$ two winding transformer is connected as an autotransformer as shown in the figure.


The kVA rating of the auto transformer is
[Set - 02]
1.61 An open circuit test is performed on 50 Hz transformer, using variable frequency source and keeping $V / f$ ratio constant, to separate its eddy current and hysteresis losses. The variation of core loss/ frequency as function of frequency is shown in the figure.
[Set - 03]


The hysteresis and eddy current losses of the transformer at 25 Hz respectively are
(A) 250 W and 2.5 W
(B) 250 Wand 62.5 W
(C) 312.5 Wand 62.5 W
(D) 312.5 Wand 250 W
1.62 The load shown in the figure absorbs 4 kW at a power factor of 0.89 lagging.


Assuming the transformer to be ideal, the value of the reactance $X$ to improve the input power factor to unity is $\qquad$ .
[Set - 03]
1.63 The parameters measured for a 220 $\mathrm{V} / 110 \mathrm{~V}, 50 \mathrm{~Hz}, 1-\phi$ transformer are :
$\left[\begin{array}{l}\text { Self-inductance of } \\ \text { primary winding }\end{array}\right]=45 \mathrm{mH}$
$\left[\begin{array}{l}\text { Self-inductance of } \\ \text { secondary winding }\end{array}\right]=30 \mathrm{mH}$
Mutual inductance between primary and secondary windings $=20 \mathrm{mH}$.
Using the above parameters, the leakage ( $L_{l_{1}}, L_{l_{2}}$ ) and magnetizing ( $L_{m}$ ) inductances as referred to primary side in the equivalent circuit respectively are
[Set - 03]
(A) $5 \mathrm{mH}, 20 \mathrm{mH}$ and 40 mH
(B) $5 \mathrm{mH}, 80 \mathrm{mH}$ and 40 mH
(C) $25 \mathrm{mH}, 10 \mathrm{mH}$ and 20 mH
(D) $45 \mathrm{mH}, 30 \mathrm{mH}$ and 20 mH
1.64 The mean thickness and variance of silicon steel laminations are 0.2 mm and 0.02 mm respectively. The varnish insulation is applied on both the sides of the laminations. The mean thickness of one side insulation and its variance are 0.1 mm and 0.01 mm respectively. If the transformer core is made using 100 such varnish coated laminations, the mean thickness and variance of the core respectively are
[Set - 03]
(A) 30 mm and 0.22
(B) 30 mm and 2.44
(C) 40 mm and 2.44
(D) 40 mm and 0.24

## 2015 IIT Kanpur

1.65 The self-inductance of the primary winding of a single phase, 50 Hz , transformer is 800 mH , and that of the secondary winding is 600 mH . The mutual inductance between these two windings is 480 mH . The secondary winding of this transformer is short circuited and the primary winding is connected to a 50 Hz , single phase, sinusoidal voltage source. The current flowing in both the windings is less than their respective rated currents. The resistance of both windings can be neglected.
In this condition, what is the effective inductance (in mH ) seen by the source?
[Set - 01]
(A) 416
(B) 440
(C) 200
(D) 920
1.66 A $200 / 400 \mathrm{~V}$, 50 Hz , two - winding transformer is rated at 20 kVA . Its windings are connected as an autotransformer of rating $200 / 600 \mathrm{~V}$. A resistive load of $12 \Omega$ is connected to the high voltage ( 600 V ) side of the autotransformer. The value of equivalent load resistance (in Ohm) as seen from low voltage side is $\qquad$ .
[Set - 01]
1.67 Two single-phase transformers $T_{1}$ and $T_{2}$ each rated at 500 kVA are operated in parallel. Percentage impedances of $T_{1}$ and $T_{2}$ are $(1+j 6)$ and $(0.8+j 4.8)$ respectively. To share a load of 1000 kVA at 0.8 lagging power factor, the contribution of $T_{2}(\mathrm{in} \mathrm{kVA})$ is $\qquad$ .
[Set - 01]
1.68 A three-winding transformer is connected to an AC voltage source as shown in the figure. The number of turns are as follows : $N_{1}=100, N_{2}=50, N_{3}=50$. If the magnetizing current is neglected and the currents in two windings are $\bar{I}_{2}=2 \angle 30^{\circ} \mathrm{A}$ and $\bar{I}_{3}=2 \angle 150^{\circ} \mathrm{A}$, then what is the value of the current $\bar{I}_{1}$ in Ampere?
[Set - 02]

(A) $1 \angle 90^{\circ}$
(B) $1 \angle 270^{\circ}$
(C) $4 \angle 90^{\circ}$
(D) $4 \angle 270^{\circ}$
1.69 The primary mmf is least affected by the secondary terminal conditions in a
(A) Power transformer
(B) Potential transformer
(C) Current transformer
(D) Distribution transformer

## 2016 IISc Bangalore

1.70 If an ideal transformer has an inductive load element at port 2 as shown in the figure below, the equivalent inductance at port 1 is
[Set - 01]

(A) $n L$
(B) $n^{2} L$
(C) $\frac{n}{L}$
(D) $\frac{n^{2}}{L}$
1.71 A single-phase $400 \mathrm{~V}, 50 \mathrm{~Hz}$ transformer has an iron loss of 5000 W at the rated condition. When operated at $200 \mathrm{~V}, 25$ Hz , the iron loss is 2000 W . When operated at $416 \mathrm{~V}, 52 \mathrm{~Hz}$, the value of the hysteresis loss divided by the eddy current loss is $\qquad$ .
[Set-01]
1.72 A single-phase $22 \mathrm{kVA}, 2200 \mathrm{~V} / 220 \mathrm{~V}$, 50 Hz , distribution transformer is to be connected as an auto-transformer to get an output voltage of 2420 V . Its maximum kVA rating as an autotransformer is
[Set - 01]
(A) 22
(B) 24.2
(C) 242
(D) 2420
1.73 The following figure shows the connection of an ideal transformer with primary to secondary turns ratio of 1 : 100 . The applied primary voltage is 100 $\mathrm{V}(\mathrm{rms}), 50 \mathrm{~Hz}, \mathrm{AC}$. The rms value of the current $I$, in ampere, is $\qquad$ . [Set - 02]

1.74 A single phase, $2 \mathrm{kVA}, 100 / 200 \mathrm{~V}$ transformer is reconnected as an autotransformer such that its kVA rating is maximum. The new rating, in kVA , is
[Set - 02]

## 2018 IIT Guwahati

1.75 A single-phase $100 \mathrm{kVA}, 1000 \mathrm{~V} / 100 \mathrm{~V}$, 50 Hz transformer has a voltage drop of $5 \%$ across its series impedance at full load. Of this, $3 \%$ is due to resistance. The percentage regulation of the transformer at full load with 0.8 lagging power factor is
(A) 4.8
(B) 6.8
(C) 8.8
(D) 10.8
1.76 A transformer with toroidal core of permeability $\mu$ is shown in the figure. Assuming uniform flux density across the circular core cross - section of radius of $r \ll R$, and neglecting any leakage flux, the best estimate for the mean radius $R$ is

(A) $\frac{\mu V r^{2} N_{P}^{2} \omega}{I}$
(B) $\frac{\mu I r^{2} N_{P} N_{S} \omega}{V}$
(C) $\frac{\mu V r^{2} N_{P}^{2} \omega}{2 I}$
(D) $\frac{\mu I r^{2} N_{P}^{2} \omega}{2 V}$

## 2019 IIT Madras

1.77 A $5 \mathrm{kVA}, 50 \mathrm{~V} / 100 \mathrm{~V}$, single-phase transformer has a secondary terminal voltage of 95 V when loaded. The regulation of the transformer is
(A) $4.5 \%$
(B) $9 \%$
(C) $1 \%$
(D) $5 \%$
1.78 A single-phase transformer of rating 25 kVA, supplies a 12 kW load at power factor of 0.6 lagging. The additional load at unity power factor in kW (round off to two decimal places) that may be added before this transformer exceeds its rated kVA is $\qquad$ .
1.79 The magnetic circuit shown below has uniform cross-sectional area and air gap of 0.2 cm . The mean path length of the core is 40 cm . Assume that leakage and fringing fluxes are negligible. When the core relative permeability is assumed to be infinite, the magnetic flux density computed in the air gap is 1 tesla. With same Ampere-turns, if the core relative
permeability is assumed to be 1000 (linear), the flux density in tesla (round off to three decimal places) calculated in the air gap is $\qquad$ -


## 2020 IIT Delhi

1.80 A single-phase, $4 \mathrm{kVA}, 200 \mathrm{~V} / 100 \mathrm{~V}, 50$ Hz transformer with laminated CRGO steel core has rated no-load loss of 450 W . When the high-voltage winding is excited with $160 \mathrm{~V}, 40 \mathrm{~Hz}$ sinusoidal ac supply, the no-load losses are found to be 320 W . When the high-voltage winding of the same transformer is supplied from a $100 \mathrm{~V}, 25 \mathrm{~Hz}$ sinusoidal ac source, the no-load losses will be $\qquad$ W (rounded off to 2 decimal places).
1.81 Windings ' $A$ ', ' $B$ ' and ' $C$ ' have 20 turns each and are wound on the same iron core as shown, along with winding " X ' which has 2 turns. The figure shows the sense (clockwise/anti-clockwise) of each of the windings only and does not reflect the exact number of turns. If windings 'A', 'B' and ' C ' are supplied with balanced 3pbase voltages at 50 Hz and there is no core saturation, the no-load RMS voltage (in V , rounded off to 2 decimal places) across winding " X " is $\qquad$ .


## Answers Single Phase Transformer

| 1.1 | A, B | 1.2 | B | 1.3 | A | 1.4 | B, C | 1.5 | A |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.6 | C | 1.7 | A | 1.8 | C | 1.9 | 0.26 | 1.10 | A |
| 1.11 | D | 1.12 | C | 1.13 | C | 1.14 | C | 1.15 | C |
| 1.16 | A | 1.17 | C | 1.18 | 97.84 | 1.19 | $\begin{gathered} 45.64 \\ \mathrm{kVA}, \\ 500 \mathrm{~W}, \\ 500 \mathrm{~W} \end{gathered}$ | 1.20 | C |
| 1.21 | A | 1.22 | A | 1.23 | C | 1.24 | D | 1.25 | A |
| 1.26 | A | 1.27 | A | 1.28 | C | 1.29 | * | 1.30 | D |
| 1.31 | A | 1.32 | 652.806 | 1.33 | C | 1.34 | D | 1.35 | D |
| 1.36 | C | 1.37 | D | 1.38 | D | 1.39 | D | 1.40 | C |
| 1.41 | C | 1.42 | B | 1.43 | D | 1.44 | C | 1.45 | C |
| 1.46 | D | 1.47 | B | 1.48 | C | 1.49 | D | 1.50 | B |
| 1.51 | A | 1.52 | D | 1.53 | C | 1.54 | C | 1.55 | B |
| 1.56 | C | 1.57 | C | 1.58 | A | 1.59 | A | 1.60 | 550 |
| 1.61 | B | 1.62 | 23.62 | 1.63 | B | 1.64 | D | 1.65 | A |
| 1.66 | 1.33 | 1.67 | 555.55 | 1.68 | A | 1.69 | C | 1.70 | B |
| 1.71 | 1.44 | 1.72 | C | 1.73 | 10 | 1.74 | 6 | 1.75 | A |
| 1.76 | D | 1.77 | D | 1.78 | 7.21 | 1.79 | 0.835 | 1.80 | 162.5 |
| 1.81 | 46 |  |  |  |  |  |  |  |  |

## Explanations Single Phase Transformer

## $1.1 \quad$ (A) and (B)

From transformer action the operating flux should always be constant.
For this purpose (By Lenz's law) the flux produced by primary winding current opposes any change in secondary winding current to
maintain operating flux constant. (From no load to full load).


Option (A) :


Primary flux $\left(\phi_{P}\right)$ opposes secondary flux $\left(\phi_{S}\right)$.
Option (B) :


Primary flux $\left(\phi_{P}\right)$ opposes secondary flux $\left(\phi_{S}\right)$.
Option (C) :


Primary flux $\left(\phi_{P}\right)$ is assisting secondary flux $\left(\phi_{S}\right)$.
Option (D) :


Primary flux $\left(\phi_{P}\right)$ is assisting secondary flux $\left(\phi_{S}\right)$.
Thus, in options (A) and (B) the primary flux opposes the secondary flux.
Hence, the correct options are (A) and (B).


## 1.2 (B)

Given :
(i) Single phase transformer
(ii) Rated voltage, $V=200 \mathrm{~V}$
(iii) Frequency, $f=50 \mathrm{~Hz}$
(iv) Hysteresis losses is $P_{h}$
(v) Eddy current losses is $P_{e}$
(vi) New operating condition :

$$
160 \mathrm{~V}, 40 \mathrm{~Hz}
$$

Here, $\frac{V}{f}=\frac{200}{50}=\frac{160}{40}=4$ (constant)
Since, given $\frac{V}{f}$ ratio is constant.

$$
B_{\max } \propto \frac{V}{f}, \text { so } B_{\max }=\text { Constant }
$$

Case 1 : Hysteresis loss calculation

$$
\begin{aligned}
& P_{h}=K_{h} B_{\max }^{x} f v_{\text {core }} \\
& K_{h}=\text { Hysteresis constant } \\
& x=\text { Steinmetz coefficient } \\
& f=\text { Supply frequency } \\
& v_{\text {core }}=\text { Volume of core }
\end{aligned}
$$

As $K_{h}, B_{m}$, and $v_{\text {core }}$ are constant
Hence, $P_{h} \propto f$
\% change in hysteresis loss is given by,

$$
\begin{aligned}
& \% \Delta P_{h}=\frac{P_{h_{2}}-P_{h_{1}}}{P_{h_{1}}} \times 100 \\
& \% \Delta P_{h}=\frac{f_{2}-f_{1}}{f_{1}} \times 100=\frac{40-50}{50} \times 100 \\
& \% \Delta P_{h}=-20 \%
\end{aligned}
$$

Here, negative sign indicates decrement.
Case 2 : Eddy current loss calculation
$P_{e}=K_{e} B_{\max }^{2} f^{2} t^{2} v_{\text {core }}$
$K_{e}=$ Eddy current constant
$f=$ Supply frequency
$t=$ Thickness of lamination
$v_{\text {core }}=$ Volume of core
As $K_{e}, B_{m}, t$, and $v_{\text {core }}$ are constant
Hence, $P_{e} \propto f^{2}$

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$\%$ change in eddy current loss is given by,

$$
\begin{aligned}
& \% \Delta P_{e}=\frac{P_{e_{2}}-P_{e_{1}}}{P_{e_{1}}} \times 100 \\
& \% \Delta P_{e}=\frac{f_{2}^{2}-f_{1}^{2}}{f_{1}^{2}} \times 100=\frac{40^{2}-50^{2}}{50^{2}} \times 100 \\
& \% \Delta P_{e}=-36 \%
\end{aligned}
$$

Here, negative sign indicates decrement.
Hence, the correct option is (B).


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## Mey Point

For the problem based on core losses i.e. $P_{h}$ and $P_{e}$, always do check first that $(V / f)$ ratio is constant or not.
(i) If $\frac{V}{f}=$ constant or $B_{m} \propto\left(\frac{V}{f}\right)=$
constant $P_{h} \propto f$ and $P_{e} \propto f^{2}$
(ii) If $\frac{V}{f} \neq$ constant or $B_{m} \propto\left(\frac{V}{f}\right) \neq$
constant $P_{h} \propto\left(\frac{V}{f}\right)^{x} f \rightarrow P_{h} \propto V^{x} f^{1-x}$ and $P_{e} \propto V^{2}$

## $1.3 \quad$ (A)

## Given :

$\left[\begin{array}{l}\text { Linear dimensions } \\ \text { of } T_{2}\end{array}\right]=2 \times\left[\begin{array}{l}\text { Linear dimensions } \\ \text { of } T_{1}\end{array}\right]$
Where, $T_{1}=$ Transformer 1 and

$$
T_{2}=\text { Transformer } 2
$$

As the linear dimensions are doubled hence, the length and breadth of the core have been doubled. Hence, the area of $T_{2}$ transformer is now 4 times of the area of transformer $T_{1}$ as given below,
Area of $T_{2}=(2 \times 2)$ Area of $T_{1}$

$$
A_{2}=4 A_{1}
$$

Flux density, $B=$ Same
Emf, $\quad E=4.44 N_{f} B_{\text {max }} A$

$$
E \propto A
$$

Current density, $J=\frac{I}{A}=$ Same
Hence, current $I \propto A$
kVA rating $=E I \propto A A \propto A^{2}$
Ratio of kVA ratings,

$$
\frac{\mathrm{kVA}_{T_{2}}}{\mathrm{kVA}_{T_{1}}}=\left(\frac{A_{2}}{A_{1}}\right)^{2}=4^{2}=16
$$

Hence, the correct option is (A).

## Key Point

If linear dimensions of second transformer becomes $K$-times of first transformer then,
(i) $\mathrm{kVA} \propto k^{4}$
(ii) Losses $\propto k^{3}$
(iii) Surface area $\propto k^{2}$

## 1.4 (B) and (C)

Let, transformer $T_{1}$ and $T_{2}$ have the same voltage rating but different kVA rating $S_{A}$ and $S_{B}$ respectively.
And their leakage impedances in ohms be $Z_{A}$ and $Z_{B}$ respectively.

When these are operating in parallel, the circuit becomes


Load shared by transformer $A$ is given by,

$$
S_{A}=\frac{Z_{B}}{Z_{A}+Z_{B}} \times S_{L}
$$

Load shared by transformer $B$ is given by,

$$
\begin{aligned}
& S_{B}=\frac{Z_{A}}{Z_{A}+Z_{B}} \times S_{L} \\
& \frac{S_{A}}{S_{B}}=\frac{Z_{B}}{Z_{A}}
\end{aligned}
$$

So, ohmic values of the leakage impedances are inversely proportional to their ratings.
For proportional load sharing,

$$
\begin{aligned}
& \frac{Z_{B}}{Z_{A}}=\frac{S_{A}(\text { rated })}{S_{B}(\text { rated })} \\
& \frac{Z_{B}}{Z_{A}}=\frac{V_{L} \cdot I_{A}(\text { rated })}{V_{L} \cdot I_{B}(\text { rated })} \\
& \frac{Z_{B}}{\left\{\frac{V_{L}}{I_{B}(\text { rated })}\right\}}=\frac{Z_{A}}{\left\{\frac{V_{L}}{I_{A}(\text { rated })}\right\}} \\
& \left.Z_{B} \text { (p.u. }\right)=Z_{A} \text { (p.u.) }
\end{aligned}
$$

So, per unit leakage impedances on their respective rating should be equal.
Hence, the correct options are (B) and (C).

## 1.5 (A)

Gas operated relay or Buchholz relay : It is used to protect transformer against internal faults. It causes tripping for major fault and alarm for minor faults. The use of such a relay is possible only with transformers having conservators. Its malfunction may be caused by dropping of oil below relay.
Hence, the correct option is (A).

## $1.6 \quad$ (C)

Given :
(i) Single phase $220 \mathrm{~V} / 440 \mathrm{~V}$ transformer
(ii) Frequency, $f=50 \mathrm{~Hz}$,
(iii) kVA rating, $S=5 \mathrm{kVA}$
(iv) Supply: $220 \mathrm{~V}, 40 \mathrm{~Hz}$ at primary

Emf equation is given by,

$$
E=4.4 N f B_{m} A_{n}
$$

So, $\quad B_{m} \propto\left(\frac{V}{f}\right)$
Hysteresis loss is given by,

$$
P_{h}=K_{h} B_{\max }^{x} f v_{\text {core }}
$$

where, $x=1.6 \quad$ [Steinmetz constant]
Thus, $\quad P_{h} \propto B_{\max }^{1.6} f \propto \frac{V^{1.6}}{f^{1.6}} \times f \propto \frac{V^{1.6}}{f^{0.6}}$
Eddy current loss is given by,

$$
\begin{align*}
& P_{e}=K_{e} B_{\max }^{2} f^{2} t^{2} v_{\text {core }} \\
& P_{e} \propto B_{\max }^{2} f^{2} \propto \frac{V^{2}}{f^{2}} \times f^{2} \propto V^{2}  \tag{ii}\\
& V_{1}=V_{2}=220 \mathrm{~V}, f_{1}=50 \mathrm{~Hz}, f_{2}=40 \mathrm{~Hz}
\end{align*}
$$

From equation (i),

$$
\begin{align*}
& \frac{P_{h_{2}}}{P_{h_{1}}}=\left(\frac{f_{1}}{f_{2}}\right)^{0.6}=\left(\frac{50}{40}\right)^{0.6} \\
& P_{h_{2}}=1.1432 P_{h_{1}} \tag{iii}
\end{align*}
$$

Here, $\quad P_{h_{2}}>P_{h_{1}}$
From equation (ii),

$$
\begin{equation*}
P_{e_{2}}=P_{e_{1}}=\text { Constant } \tag{iv}
\end{equation*}
$$

From equations (iii) and (iv),

$$
P_{k_{2}}>P_{h_{1}} \& P_{e_{1}}=P_{e_{2}}
$$

Hence, the correct option is (C).

## 1.7 (A)

## Lorentz's principle :

Two conductors carrying current in the same direction will experience force of attraction and the conductors carrying current in opposite direction will experience force of repulsion.


Since, the direction of current in adjoining turns is same so, it will feel force of attraction.
Hence, the correct option is (A).

## 1.8 (C)

Given : The various conditions which must be fulfilled for the satisfactory parallel operation of two or more single phase transformers, are as follows :

1. The transformers must have the same voltage ratios, i.e. with the primaries connected to the same voltage source, the secondary voltages of all transformers should be equal in magnitude.
2. The equivalent leakage impedances in ohms should be inversely proportional to their respective kVA ratings. In other words, the per unit leakage impedances of the transformers based on their own kVA ratings must be equal.
3. The ratio of equivalent leakage reactance to equivalent resistance, i.e. $\frac{x_{e}}{r_{e}}$ should be same for all the transformers.
4. The transformers must be connected properly, so far as their polarities are concerned.
Hence, the correct option is (C).

## $1.9 \quad 0.26$

## Given :

(i) $1-\phi 200 \mathrm{~V} / 400 \mathrm{~V}$ transformer
(ii) kVA rating, $S=4 \mathrm{kVA}$
(iii) Supply frequency, $f=50 \mathrm{~Hz}$
(iv) Mutual inductance, $M=1.91 \mathrm{H}$
(v) O.C test at L.V. side, $P_{O . C}=80 \mathrm{~W}$
(vi) $\quad X_{m}=2 \pi f M$

Transformation ratio, $k=\frac{400}{200}=2$
The equivalent circuit referred to secondary side or HV side is shown below,


Magnetizing reactance (Referred to secondary) is given by,

$$
X_{m}^{\prime}=2 \pi f M K^{2}=2 \pi \times 50 \times 1.91 \times 2^{2}=2400 \Omega
$$

Magnetizing current is given by,

$$
I_{m}^{\prime}=\frac{V_{H . V .}}{X_{m}^{\prime}}=\frac{400}{2400}=0.167 \mathrm{~A}
$$

Losses must remain constant while measuring from both sides,

$$
P_{O . C(H V)}=P_{O . C(L V)}=80 \mathrm{~W}
$$

Core loss component of current,

$$
I_{C}^{\prime}=\frac{P_{O . C(H, V)}}{V_{H . V}}=\frac{80}{400}=0.2
$$

No load current is given by,

$$
\begin{aligned}
& I_{0}^{\prime}=I_{C}^{\prime}+j I_{m}^{\prime} \\
& \left|I_{0}^{\prime}\right|=\sqrt{I_{C}^{\prime 2}+I_{m}^{\prime 2}} \\
& \left|I_{0}^{\prime}\right|=\sqrt{0.2^{2}+0.167^{2}}=0.26 \mathrm{~A}
\end{aligned}
$$

Hence, the value of current on the H.V. side is 0.26 A.

### 1.10 (A)

Oil in a transformer acts as a coolant and insulant. It provides insulation to the windings and also works as a good coolant for cooling purposes.
Hence, the correct option is (A).

### 1.11 (D)

If $k$ is the transformation ratio of auto transformer (in step down format) then,

$$
\frac{S_{\text {auto }}}{S_{\text {two winding }}}=\frac{1}{1-k}
$$

Hence, auto transformer will be more advantageous when transformation ratio is nearer to unity (i.e. smaller). So, that maximum kVA rating can be achieved.
Hence, the correct option is (D).

### 1.12 (C)

Given :
(i) Single-phase $400 \mathrm{~V} / 230 \mathrm{~V}$ transformer,
(ii) Frequency :

$$
f_{\text {rated }}=50 \mathrm{~Hz}, f_{\text {supply }}=25 \mathrm{~Hz}
$$

For proper utilization of core, maximum flux density should be constant to avoid saturation of core (i.e. $B_{\text {max }}=$ constant).
Hence, we have to maintain $\frac{V}{f}$ ratio constant for proper utilization of core,

$$
\begin{aligned}
& \frac{V_{\text {rated }}}{f_{\text {rated }}}=\frac{V_{\text {supply }}}{f_{\text {supply }}} \\
& \frac{230 \mathrm{~V}}{50 \mathrm{~Hz}}=\frac{V_{\text {supply }}}{25 \mathrm{~Hz}} \\
& V_{\text {supply }}=115 \mathrm{~V}
\end{aligned}
$$

Hence, the correct option is (C).

### 1.13 (C)

Given :
(i) Instantaneous applied voltage,

$$
v(t)=400 \sin (314.16 t)
$$

(ii) Instantaneous no load current,

$$
i_{0}(t)=2 \sin \left(314.16 t-85^{0}\right)
$$

Therefore, $i_{0}(t)$ lags $v(t)$ by $85^{\circ}$.
At no load the equivalent circuit diagram of transformer (neglecting series impedance) is given by,


From diagram,

$$
v(t)=i_{0}(t) \times Z_{0}
$$

Phasor representation is shown below,


Shunt branch impedance is given by,

$$
Z_{0}=\frac{v(t)}{i_{0}(t)}=\frac{400 \sin \omega t}{2 \sin \left(\omega t-85^{0}\right)}
$$

Since, current is lagging voltage by $85^{\circ}$.
Hence, impedance angle is $85^{\circ}$

$$
Z_{0}=200 \angle 85^{\circ} \Omega
$$

Hence, the correct option is (C).


### 1.14 (C)

Faraday's $1^{\text {st }}$ law : Whenever the number of lines of flux linking with a coil changes an emf gets induced in that coil.
Faraday's $2^{\text {nd }}$ law : The magnitude of the induced emf is proportional to the rate of change of flux linkages.
Lenz's law : The effect (emf) opposes the cause (flux).


Hence, the correct option is (C).

### 1.15 (C)

Given :
(i) Transformer efficiency,

$$
\eta=0.98 \text { at full and half load }
$$

(ii) kVA rating, 100 kVA
(iii) $\cos \phi=$ power factor

The efficiency of the transformer is given by, Efficiency of transformer is given by,

$$
\begin{equation*}
\eta=\frac{m \mathrm{kVA} \cos \phi}{m \mathrm{kVA} \cos \phi+P_{i}+m^{2} P_{c u_{f}}} \tag{i}
\end{equation*}
$$

where, $m=$ Percentage of loading
$\cos \phi=$ Power factor

## Case I : At full load

Efficiency, $\eta=0.98$, $m=1$
Power factor is not given,
Assume $p f=1$

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Put the values in equation (i),

$$
\begin{align*}
& 0.98=\frac{(1 \times 100 \times 1)}{(1 \times 100 \times 1)+P_{i}+\left(1^{2} \times P_{c u_{n}}\right)} \\
& P_{i}+P_{c u_{n}}=2.04 \tag{ii}
\end{align*} .
$$

## Case II : At half load

Efficiency, $\eta=0.98, m=0.5$
Put the values in equation (i),

$$
\begin{align*}
& \frac{(0.5 \times 100 \times 1)}{(0.5 \times 100 \times 1)+P_{i}+\left(0.5^{2} \times P_{c u_{\Omega}}\right)}=0.98 \\
& P_{i}+0.25 P_{c u_{\Omega}}=1.02 \tag{iii}
\end{align*}
$$

Solving equations (ii) and (iii),

$$
P_{i}=0.68 \mathrm{~kW} \text { and } P_{c u_{n}}=1.36 \mathrm{~kW}
$$

Here, $\quad P_{c u_{\Omega}}>P_{i}$
Hence, the correct option is (C).

## $1.16 \quad$ (A)

The main objective of transformer is to get sinusoidal emf. For this the flux must be sinusoidal but because of hysteresis non linearity the relation between flux and magnetizing current is non-linear and it contain harmonics.

For higher order harmonics reactance ( $X=2 \pi n f L$ ) increases and amplitude of current decreases, making $3^{\text {rd }}$ harmonics as the dominant harmonic.

Waveform of magnetizing current is given by,


## $\square$ Key Point

The flux waveform under this conditions will be flat topped with $3^{\text {rd }}$ harmonics.


Hence, the correct option is (A).

### 1.17 (C)

Given :
(i) At $V_{1}$ (rated) Volts, $f_{1}=50 \mathrm{~Hz}$
(ii) At $V_{2}=0.9 V_{1}, f_{2}=45 \mathrm{~Hz}$
(iii) Hysteresis loss $=P_{h}$
(iv) Eddy current loss $=P_{e}$
(v) $\quad P_{h_{1}}=P_{e_{1}}=P$ (at rated condition)

Let, iron loss $=P_{i}$
As, $\frac{V_{2}}{f_{2}}=\frac{0.9 V_{1}}{45}=\frac{V_{1}}{50}=\frac{V_{1}}{f_{1}}=$ Constant
Hence, $\frac{V}{f}=$ Constant i.e. $B_{\max }=$ Constant
Hysteresis loss is given by,

$$
P_{h}=K_{h} B_{\max }^{1.6} f v_{\text {core }} \propto B_{\max }^{1.6} f \propto f
$$

$$
\frac{P_{h_{2}}}{P_{h_{1}}}=\frac{f_{2}}{f_{1}} \Rightarrow P_{h_{2}}=P_{h_{1}} \times \frac{45}{50}=0.9 P
$$

Eddy current loss is given by,

$$
\begin{aligned}
& P_{e}=K_{e} B_{\max }^{2} f^{2} t^{2} v_{\text {core }} \propto B_{\max }^{2} f^{2} \propto f^{2} \\
& \frac{P_{e_{2}}}{P_{e_{1}}}=\left(\frac{f_{2}}{f_{1}}\right)^{2} \\
& P_{e_{2}}=P_{e_{1}} \times\left(\frac{45}{50}\right)^{2}=0.81 P
\end{aligned}
$$

Old iron loss, $P_{i_{1}}=P_{h_{1}}+P_{e_{1}}=P+P=2 P$
New iron loss,

$$
P_{i_{2}}=P_{h_{2}}+P_{e_{2}}=0.9 P+0.81 P=1.71 P
$$

$\%$ change in core or iron loss is given by,

$$
\begin{aligned}
& \% \Delta P_{i}=\frac{P_{i_{2}}-P_{i_{1}}}{P_{i_{1}}} \times 100 \\
& \% \Delta P_{i}=\frac{1.71 P-2 P}{2 P} \times 100=-14.5 \%
\end{aligned}
$$

Core loss is reduced by $14.5 \%$.
Hence, the correct option is (C).


## $\begin{array}{ll}1.18 & \mathbf{9 7 . 8 4}\end{array}$

## Given :

(i) $11 \mathrm{kV} / 400 \mathrm{~V}$ transformer
(ii) kVA rating, $S=50 \mathrm{kVA}$
(iii) Iron losses, $P_{i}=500 \mathrm{~W}$
(iv) Full load copper loss, $P_{c u_{f}}=600 \mathrm{~W}$

Efficiency at any load is given by,

$$
\begin{align*}
& \eta=\frac{P_{\text {out }}}{P_{\text {in }}} \times 100=\frac{P_{\text {out }}}{P_{\text {out }}+\text { Losses }} \times 100 \\
& \eta=\frac{m \mathrm{kVA} \cos \phi}{m \mathrm{kVA} \cos \phi+P_{i}+m^{2} P_{\text {cu }}} . \tag{i}
\end{align*}
$$

where, $m=$ Percentage of loading
$\cos \phi=$ Power factor
Case I : At full load

$$
\begin{aligned}
& m=1, \cos \phi=1 \\
& \eta=\frac{50 \times 10^{3} \times 1}{50 \times 10^{3}+500+600} \times 100 \\
& \eta=97.84 \%
\end{aligned}
$$

Hence, the efficiency will be $\mathbf{9 7 . 8 4 \%}$.

## $1.19 \quad 45.64 \mathrm{kVA}, 500 \mathrm{~W}, 500 \mathrm{~W}$

Maximum efficiency occurs when,
Variable loss $=$ Constant loss
$m^{2} P_{c u_{n}}=P_{i}$

Where, $P_{c u m}=m^{2} P_{c u_{l}}=$ Copper loss at fraction of
load ( $m$ )
and $\quad P_{c u_{\Omega}}=$ Full load copper loss $(m=1)$

$$
\begin{aligned}
& m=\sqrt{\frac{P_{i}}{P_{u_{n}}}}=\sqrt{\frac{500}{600}} \\
& m=0.9128
\end{aligned}
$$

Loading at maximum efficiency condition is given by,

$$
\begin{aligned}
& \left.S\right|_{\eta_{\max }}=m \times S=0.9128 \times 50 \mathrm{kVA} \\
& \left.S\right|_{\eta_{\max }}=45.64 \mathrm{kVA}
\end{aligned}
$$

Since, iron losses are independent of loading. Therefore, iron and copper losses at this point are,

$$
P_{i}=P_{\text {cum }}=500 \mathrm{~W}
$$

Hence, the load at which maximum efficiency occurs is $\mathbf{4 5 . 6 4} \mathbf{~ k V A}$ and the iron and copper losses are $\mathbf{5 0 0} \mathbf{W}$ and $\mathbf{5 0 0} \mathbf{W}$ respectively.

## Key Point

$P_{c u_{g}}=P_{i}$ at full load $m=1$
At maximum efficiency,

$$
m^{2} P_{c u_{n}}=P_{i} \text { at any load } m
$$

### 1.20 (C)

## Given :

(i) Single-phase $400 \mathrm{~V} / 200 \mathrm{~V}$ transformer
(ii) kVA rating, $S=10 \mathrm{kVA}$
(iii) $\%$ Impedance, $Z_{(\%)}=10 \%$
i.e. $Z($ p.u. $)=0.1$

Rated primary current is given by,

$$
\begin{aligned}
& I_{P}=\frac{\mathrm{kVA}}{E_{1}}=\frac{10000}{400}=25 \mathrm{~A} \\
& \frac{I_{S C}(\text { steady })}{I_{f l}}=\frac{1}{Z_{p u}}
\end{aligned}
$$

$$
I_{S C}(\text { steady })=\frac{25}{0.1}=250 \mathrm{~A}
$$

Hence, the correct option is (C).

## $\square$ Key Point

Steady state short circuit line current is the current flowing through the transformer with secondary short circuited and rated voltage applied at primary.

Whereas, during short circuit as we calculate steady state short circuit line voltage at primary with secondary short circuited and rated current flowing through it.

### 1.21 (A)

## Given :

(i) Single-phase $400 \mathrm{~V} / 100 \mathrm{~V}$ transformer
(ii) kVA rating, $S=10 \mathrm{kVA}$
(iii) It is reconnected as auto transformer The equivalent circuit is shown below,


Primary current of transformer,

$$
I_{1}=\frac{10 \times 10^{3}}{400}=25 \mathrm{~A}
$$

Secondary current of transformer,

$$
I_{2}=\frac{10 \times 10^{3}}{100}=100 \mathrm{~A}
$$

## Method 1 : Additive polarity concept

For maximum kVA rating of auto transformer the transformation ratio to be near to unity.
Therefore, connecting $a$ to $d$ the auto transformer connection is shown below,


Primary current of auto-transformer,

$$
I_{P}=100+25=125 \mathrm{~A}
$$

Secondary current of auto-transformer,

$$
I_{S}=100 \mathrm{~A}
$$

Primary voltage, $V_{P}=400 \mathrm{~V}$
Secondary voltage, $V_{S}=500 \mathrm{~V}$
Maximum VA rating $=V_{P} I_{P}=V_{S} I_{S}$

$$
=500 \mathrm{~V} \times 100 \mathrm{~A}=50 \mathrm{kVA}
$$

Hence, the correct option is (A).

## Method 2 : Direct approach

If $k$ is the transformation ratio of auto transformer (in step down format) then,

$$
\frac{S_{\text {auto }}}{S_{\text {two winding }}}=\frac{1}{1-k}
$$

Here, $k=\frac{400}{500}=0.8$
Hence, $\frac{S_{\text {auto }}}{S_{\text {two winding }}}=\frac{1}{1-0.8}=5$

$$
\begin{aligned}
& S_{\text {auto }}=S_{\text {two winding }} \times 5 \\
& S_{\text {auto }}=10 \times 5=50 \mathrm{kVA}
\end{aligned}
$$

Hence, the correct option is (A).

### 1.22 (A)

Given :
(i) Single phase $400 / 200 \mathrm{~V}$ transformer
(ii) kVA rating, $S_{\text {rated }}=10 \mathrm{kVA}$
(iii) Resistance, $\% R=3 \%$
(iv) Reactance, $\% X=6 \%$
(v) Load current, $I=50 \mathrm{~A}$
(vi) As the load is resistive, $\cos \phi_{2}=1$ (upf)
\% Voltage regulation is given by,

$$
\% V R=m\left[\% R \cos \phi_{2}+\% X \sin \phi_{2}\right]
$$

## © Avoid this mistake

Never forget to calculate the value of $m$ (fraction of loading), if current is given.

$$
\begin{aligned}
& m=\frac{\mathrm{kVA}_{\text {operating }}}{\mathrm{kVA}_{\text {rating }}}=\frac{50 \times 200}{10 \times 10^{3}}=1 \\
& \% V R=\% R \times 1+\% X \times 0 \\
& \% V R=\% R=3 \%
\end{aligned}
$$

In per unit, $V R_{p u}=\frac{\% R}{100}=0.03$

$$
V R_{p u}=\frac{E_{2}-V_{2}}{E_{2}}=1-\frac{V_{2}}{E_{2}}
$$

Voltage across the load is given by,

$$
\begin{aligned}
& V_{2}=E_{2}\left[1-V R_{p u}\right] \\
& V_{2}=200[1-0.03]=194 \mathrm{~V}
\end{aligned}
$$

Hence, the correct option is (A).


### 1.23 (C)

Given :
For transformer A :
(i) Single-phase $400 \mathrm{~V} / 200 \mathrm{~V}$ transformer
(ii) kVA rating, $S_{A}=10 \mathrm{kVA}$
(iii) Resistance, $R_{p u}=0.03$
(iv) Reactance, $X_{p u}=0.04$

## For transformer B :

(i) Single-phase $400 \mathrm{~V} / 200 \mathrm{~V}$ transformer
(ii) kVA rating, $S_{B}=5 \mathrm{kVA}$
(iii) Resistance, $R_{p u}=0.04$
(iv) Reactance, $X_{p u}=0.03$

## For load :

(i) Power, $P_{L}=12 \mathrm{~kW}$
(ii) Power factor, $\cos \phi_{L}=0.8 \mathrm{lag}$

Load, $S_{L}($ in kVA$)=\frac{P_{L}}{\cos \phi_{L}}=\frac{12}{0.8}=15 \mathrm{kVA}$
Let base $\mathrm{kVA}=10 \mathrm{kVA}$

$$
\begin{aligned}
Z_{A(p u)}= & (0.03+j 0.04) \\
& \quad[\mathrm{At} \mathrm{common} \mathrm{base} \mathrm{of} 10 \mathrm{kVA}] \\
Z_{B\left(p u u_{\text {new }}\right.}= & \frac{\mathrm{MVA}_{\text {new }}}{\mathrm{MVA}_{\text {old }}} \times Z_{B(p u)_{\text {old }}} \\
Z_{B\left(p u u_{\text {new }}\right.}= & \frac{10}{5}(0.04+j 0.03)=0.08+j 0.06
\end{aligned}
$$

The equivalent circuit of parallel operation is shown below,


Load shared by transformer $A$ is given by,

$$
\begin{equation*}
S_{A}=\left(\frac{Z_{B}}{Z_{A}+Z_{B}}\right) \times S_{L} \tag{i}
\end{equation*}
$$

Load shared by transformer $B$ is given by,

$$
\begin{align*}
& S_{B}=\left(\frac{Z_{A}}{Z_{A}+Z_{B}}\right) \times S_{L}  \tag{ii}\\
& S_{L}=\text { Load kVA }
\end{align*}
$$

Put the values in equation (i),
$S_{A}=\frac{0.08+j 0.06}{0.03+j 0.04+0.08+j 0.06} \times 15 \angle-36.86^{\circ}$
$S_{A}=(7.46-j 6.78) \mathrm{kVA}$
Thus, Active power $=7.5 \mathrm{~kW}$
Reactive power $=6.78 \mathrm{kVAR}$
Hence, the correct option is (C).


Due to slip of tongue induced emf was pronounced as terminal voltage, actually it is induced emf (voltage).

### 1.24 (D)

If the waveform contains harmonics then its peak, rms and average values will be all the integral.
These are different from those of the fundamental. Hence, the correct option is (D).

### 1.25 (A)

From question,


When the value of input voltage is maximum, the rate of change of core flux is minimum, as both are $90^{\circ}$ out of phase in case of sinusoidal input.

So, if we close the switch at instant of maximum value of supply voltage flux demanded by core is zero, so that we get the minimum inrush current.


Hence, the correct option is (A).

## $1.26 \quad$ (A)

Flux, $\quad \phi_{m} \propto \frac{V}{f}$
The core flux $\phi_{m}$ (main flux) is produced by magnetizing current in the primary. On changing
load, secondary current changes, hence, secondary flux $\phi_{s}\left(\phi_{s}\right.$ opposes $\left.\phi_{m}\right)$ changes. Now to counter this primary current changes producing primary flux $\phi_{p}$ to compensate for $\phi_{s}$. Hence, core flux is maintained constant independent of load variations so, it is strictly constant with load changes.
Hence, the correct option is (A).

## @ Key Point

In a transformer, irrespective change in load,

$$
f=\text { Constant, and } \phi_{m}=\text { Constant }
$$

## $1.27 \quad$ (A)

In the protection of transformer, harmonic restraint is used to guard against magnetizing inrush current. It minimizes the inrush current. Hence, the correct option is (A).

### 1.28 (C)

## Given :

(i) Hysteresis loop area, $A=5 \mathrm{~cm}^{2}$
(ii) Abscissa $1 \mathrm{~cm}=2 \mathrm{AT}$
(iii) Ordinate $1 \mathrm{~cm}=50 \mathrm{mWb}$.

The B-H curve is shown below,


The hysteresis loss is calculated from the area of hysteresis loop.
Since, energy consumed due to hysteresis $\propto$ Area of loop.
The power loss due to hysteresis $=\frac{\text { Energy }}{\text { Time }}$

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Time period, $T=\frac{1}{f}$
The area of loop

$$
\begin{aligned}
& =5 \mathrm{~cm}^{2}=5(1 \mathrm{~cm} \text { of } y \times 1 \mathrm{~cm} \text { of } x) \\
& =5 \times 2 \times 0.05=0.5 \text { Joule }
\end{aligned}
$$

The hysteresis loss,

$$
P_{h}=\frac{0.5}{T}=0.5 f=25 \mathrm{~W}
$$

Hence, the correct option is (C).

### 1.29 (*)

Given :
(i) Single-phase, three-winding transformer
(ii) Turns ratio, $N_{1}: N_{2}: N_{3}=20: 4: 1$
(iii) Secondary winding, $I_{2}=50 \mathrm{~A}$ at 0.8 lag $p f$ i.e. $I_{2}=50 \angle-36.86^{\circ} \mathrm{A}$
(iv) Tertiary winding $I_{3}=60 \mathrm{~A}$ at 0.6 lag $p f$ i.e. $I_{3}=60 \angle-53.13^{\circ} \mathrm{A}$

Applying MMF balance equation,

$$
\begin{aligned}
& N_{1} I_{1}=N_{2} I_{2}+N_{3} I_{3} \\
& I_{1}=\frac{N_{2}}{N_{1}} I_{2}+\frac{N_{3}}{N_{1}} I_{3} \\
& I_{1}=\frac{4}{20} \times 50 \angle-36.86^{0}+\frac{1}{20} \times 60 \angle-53.13^{0} \\
& I_{1}=10 \angle-36.86^{0}+3 \angle-53.13^{0} \\
& I_{1}=8-j 6+1.8-j 2.4=9.8-j 8.4 \\
& I_{1}=12.9 \angle-40.59^{\circ} \mathrm{A}
\end{aligned}
$$

Hence, the primary current $I_{1}=12.9 \mathrm{~A}$ and power factor $\cos \phi_{1}=\cos (-40.59)=0.759$ lag. Hence, none of the options are correct.

### 1.30 (D)

## Given :

(i) Single phase $230 \mathrm{~V} / 100 \mathrm{~V}$ transformer
(ii) kVA rating, $S=1 \mathrm{kVA}$
(iii) Frequency, $f=50 \mathrm{~Hz}$

The equivalent circuit is shown below,


When the transformer feeds current to a linear load, the load current is sinusoidal. As the load current is sinusoidal the emf induced across the low voltage winding is also sinusoidal.

$$
e_{2}=i_{2} R \text { and } e_{2}=\frac{N_{2} d \phi}{d t}
$$

Since, core flux will also be sinusoidal but due to saturation, excitation current will not be sinusoidal but it would be peaky in nature due to enhancing $3^{\text {rd }}$ harmonic.
Because of this current, the current drawn from the source is not sinusoidal.

Hence, the correct option is (D).

## $1.31 \quad$ (A)

Given circuit is shown below,

[In the above figure polarity are marked by using dots.]

Applying KVL in the above loop,

$$
\begin{aligned}
& -E_{1}+E_{2}+E_{3}+V=0 \\
& -400+200+200+V=0
\end{aligned}
$$

$$
V=0 \mathrm{~V}
$$

Hence, the correct option is (A).


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## $1.32 \quad 652.806$

## Given :

(i) $3300 \mathrm{~V} / 400 \mathrm{~V}$ single phase transformer
(ii) Supply frequency, $f=50 \mathrm{~Hz}$
(iii) Magnetizing reactance, $X_{m}=500 \Omega$
(iv) Leakage reactances :

Primary, $X_{1}=1 \Omega$
Secondary, $X_{2}=0.012 \Omega$
(v) Turns:

Primary, $N_{1}=99$
Secondary, $N_{2}=12$
The system is shown below,

where, transformation ratio

$$
k=\frac{N_{2}}{N_{1}}=\frac{12}{99}=\frac{4}{33}=0.1212
$$

The secondary reactance refer to primary,

$$
X_{2}^{\prime}=\frac{X_{2}}{k^{2}}=\frac{0.012}{(4 / 33)^{2}}=0.82 \Omega
$$

The secondary reactance $X_{B}=0.01 \Omega$ referred to primary,

$$
X_{B}^{\prime}=\frac{X_{B}}{k^{2}}=\frac{0.01}{(4 / 33)^{2}}=0.681 \Omega
$$

The secondary voltage $V_{B}$ referred to primary,

$$
V_{B}^{\prime}=\frac{V_{B}}{k}=\frac{400}{(4 / 33)}=3300 \mathrm{~V}
$$

The equivalent circuit of above system referred to primary is given by,


To get the magnetizing MMF or ampere turn, first calculate the value of $I_{m}$. To get the value of $I_{m}$, we can opt different circuit analysis procedures as given below,
Using superposition theorem,

$$
I_{m}=I_{m_{1}}+I_{m_{2}}
$$

## Case I :

When $V_{A}$ is active. $V_{B}{ }^{\prime} \rightarrow S C$


## Case II :

When $V_{B}^{\prime}$ is active. $V_{A} \rightarrow S C$


$$
\begin{aligned}
I_{m_{2}} & =\frac{3300}{(j 500 \| j 0.6)+j 1.49} \times \frac{0.6}{500.6} \\
I_{m_{2}} & =1.893 \mathrm{~A}
\end{aligned}
$$

Hence, $I_{m}=I_{m_{1}}+I_{m_{2}}$

$$
I_{m}=4.701+1.893=6.594 \mathrm{~A}
$$

Hence, magnetizing Ampere turns,

$$
A T=N_{1} I_{m}=99 \times 6.594=652.806 \mathrm{AT}
$$

Hence, the magnetizing ampere turns is $\mathbf{6 5 2 . 8 0 6}$ AT.

### 1.33 (C)

Power transfer from system $A$ to $B$ is given by,

$$
P=\frac{V_{A} V_{B}}{X_{e q}} \sin \delta
$$

For $P=0$,

$$
\frac{V_{A} V_{B}}{X_{e q}} \sin \delta=0
$$

i.e. $\quad \sin \delta=0$

$$
\delta=0^{0}
$$

Thus, $V_{A}$ is in phase with $V_{B}$.
Hence, the correct option is (C).

### 1.34 (D)

Given :
(i) Single phase transformer
(ii) Maximum efficiency, $\eta_{\max }=90 \%$ at rated load and unity power factor.
Efficiency of single phase transformer is given by,

$$
\begin{equation*}
\eta=\frac{m \times(\mathrm{kVA}) \times \cos \phi}{m \times(\mathrm{kVA}) \cos \phi+P_{i}+m^{2} P_{c u_{f}}} \tag{i}
\end{equation*}
$$

where, $\mathrm{kVA}=$ Rating of transformer
$\cos \phi=$ Power factor
$P_{i}=$ Iron or core loss
$P_{c u_{f l}}=$ Full load copper loss
$m=$ Fraction of load

## Case I : At full load

## Given :

$$
\begin{aligned}
& \eta_{\max }=0.9 \\
& m=1.0 \\
& \cos \phi=1.0 \\
& P_{i}=P_{c u_{f}} \text { for } m=1.0
\end{aligned}
$$

From equation (i),

$$
\begin{aligned}
& \frac{1 \times(\mathrm{kVA}) \times 1}{1 \times(\mathrm{kVA}) \times 1+P_{i}+P_{c u_{n}}}=0.9 \\
& \frac{(\mathrm{kVA})}{(\mathrm{kVA})+2 P_{i}}=0.9
\end{aligned}
$$

$$
\begin{align*}
& P_{i}=0.055 \mathrm{kVA}  \tag{ii}\\
& P_{i}=P_{c u_{\ell}}=0.055 \mathrm{kVA}
\end{align*}
$$

## Case II : At half load

Given : $m=\frac{1}{2}, \cos \phi=1$
Efficiency is given by,

$$
\eta=\frac{\frac{1}{2} \times(\mathrm{kVA}) \times 1}{\frac{1}{2} \times(\mathrm{kVA}) \times 1+P_{i}+\left(\frac{1}{2}\right)^{2} P_{c u_{\Omega}}}
$$

[From equation (i)]

$$
\eta=\frac{0.5(\mathrm{kVA})}{0.5(\mathrm{kVA})+P_{i}+0.25 P_{c u_{f l}}}
$$

From equation (ii),

$$
\begin{aligned}
& \eta=\frac{0.5(\mathrm{kVA})}{\left[\begin{array}{r}
0.5(\mathrm{kVA})+0.055(\mathrm{kVA})+0.25 \\
\times 0.055(\mathrm{kVA})
\end{array}\right]} \\
& \eta=\frac{0.5}{0.5+0.055+0.25 \times 0.055}=0.878 \\
& \% \eta=87.8 \%
\end{aligned}
$$

Hence, the correct option is (D).

## $1.35 \quad$ (D)

## Given :

(i) Single phase transformer
(ii) Turns ratio, $N_{1} / N_{2}=2$

Magnetizing flux is given by,

$$
\phi_{m}=\phi_{\max } \sin \omega t
$$

Induced emf is given by,

$$
\begin{aligned}
& E_{2}=-N_{2} \frac{d \phi}{d t}=-N_{2} \frac{d}{d t}\left(\phi_{\max } \sin \omega t\right) \\
& E_{2}=-N_{2} \omega \phi_{\max } \cos \omega t \\
& E_{2}=-N_{2} \omega \phi_{\max } \sin (\omega t-90) \\
& E_{2}=E_{\max } \sin \left(\omega t-90^{\circ}\right) \\
& \left.\quad \quad \text { as, } E_{\max }=-N_{2} \omega \phi_{\max }\right)
\end{aligned}
$$

Therefore, $E_{2}$ lags $\phi_{m}$ by $90^{\circ}$.

For purely resistive load, $I_{2}=\frac{E_{2}}{R}$ is in phase with $E_{2}$.


From the equivalent circuit,


$$
\begin{aligned}
& \frac{E_{1}}{N_{1}}=\frac{E_{2}}{N_{2}} \Rightarrow E_{1}=2 E_{2} \\
& I_{1} N_{1}=I_{2} N_{2} \Rightarrow I_{1}=\frac{I_{2}}{2}
\end{aligned}
$$

$I_{1}$ is in phase opposition with $I_{2}$,

$$
E_{1}=I_{m} j X_{m}
$$



Phasor diagram is shown below,


Hence, the correct option is (D).

### 1.36 (C)

## Given :

(i) Three winding transformer
(i) Turns ratio, $N_{1}: N_{2}: N_{3}=4: 2: 1$
(iii) Resistance at secondary, $R=10 \Omega$
(iv) Capacitor at tertiary, $X_{C}=2.5 \Omega$
(v) Primary voltage, $V_{1}=400 \mathrm{~V}$

Given figure is shown below,


1. In primary, current flows from dot to coil.
2. In secondary and tertiary current flows from coil to dot.
3. Primary current produces main flux $\phi_{\text {main }}$ according to right hand thumb rule.
(Thumb gives

4. Secondary and tertiary currents produce flux $\phi_{S}$ and $\phi_{T}$ which oppose $\phi_{\text {main }}, \phi_{P}$ compensate $\phi_{S}$ and $\phi_{T}$.
In a transformer, emf per turn is constant.

$$
\begin{aligned}
& \frac{E_{1}}{N_{1}}=\frac{E_{2}}{N_{2}}=\frac{E_{3}}{N_{3}} \\
& \frac{400 \angle 0^{\circ}}{4}=\frac{E_{2}}{2}=\frac{E_{3}}{1} \\
& E_{2}=200 \angle 0^{\circ} \mathrm{V}, E_{3}=100 \angle 0^{\circ} \mathrm{V}
\end{aligned}
$$

Load current in secondary winding is given by,

$$
I_{2}=\frac{E_{2}}{R}=\frac{200 \angle 0^{0}}{10}=20 \angle 0^{\circ}=20 \mathrm{~A}
$$

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Load current in tertiary winding is given by,

$$
I_{3}=\frac{E_{3}}{-j X_{C}}=\frac{100 \angle 0^{0}}{-j 2.5}=40 \angle 90^{\circ}=j 40 \mathrm{~A}
$$

## Method 1 : MMF Balance Equation

MMF balance equation is given by,

$$
\begin{aligned}
& I_{1} N_{1}=I_{2} N_{2}+I_{3} N_{3} \\
& I_{1}=I_{2} \frac{N_{2}}{N_{1}}+I_{3} \frac{N_{3}}{N_{1}}=20 \times \frac{2}{4}+j 40 \times \frac{1}{4} \\
& I_{1}=(10+j 10) \mathrm{A}
\end{aligned}
$$

Hence, the correct option is (C).

## Method 2 : Power Balance Equation

Power balance equation is given by,

$$
\begin{aligned}
& S_{1}=S_{2}+S_{3} \\
& E_{1} I_{1}^{*}=E_{2} I_{2}^{*}+E_{3} I_{3}^{*} \\
& I_{1}^{*}=\frac{E_{2}}{E_{1}} I_{2}^{*}+\frac{E_{3}}{E_{1}} I_{3}^{*} \\
& I_{1}^{*}=\frac{200 \angle 0^{0}}{400 \angle 0^{0}} \times 20 \angle 0^{0}+\frac{100 \angle 0^{0}}{400 \angle 0^{0}} \times 40 \angle-90^{0} \\
& I_{1}^{*}=14.14 \angle-45^{0}
\end{aligned}
$$

Hence, $I_{1}=14.14 \angle 45^{0}=(10+j 10) \mathrm{A}$
Hence, the correct option is (C).

## Method 3 : Concept of Equivalent Circuit

Refer the secondary and tertiary circuit on primary side as shown below. While referring, it must be taken care of that the dotted terminal of secondary and tertiary must be connected to the dotted terminal of primary.

$I_{2}{ }^{\prime}=$ Secondary current referred to primary side.

$$
I_{2}^{\prime}=I_{2} \times \frac{N_{2}}{N_{1}}=20 \angle 0^{0} \times \frac{2}{4}=10 \angle 0^{0} \mathrm{~A}
$$

$$
I_{3}^{\prime}=I_{3} \times \frac{N_{3}}{N_{1}}=40 \angle 90^{\circ} \times \frac{1}{4}=10 \angle 90^{\circ} \mathrm{A}
$$

Hence, $I_{1}=I_{2}{ }^{\prime}+I_{3}{ }^{\prime}=\left(10 \angle 0^{0}\right)+\left(10 \angle 90^{\circ}\right)$

$$
I_{1}=14.14 \angle 45^{0}=(10+j 10) \mathrm{A}
$$

Hence, the correct option is (C).

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### 1.37 (D)

## Given :

(i) Single phase $3300 \mathrm{~V} / 230 \mathrm{~V}$ transformer
(ii) kVA rating, $\mathrm{kVA}_{\text {T.w. }}=50 \mathrm{kVA}$
(iii) It is connected as auto transformer

## Method 1 : Concept of Additive Polarity

The equivalent circuit of two winding transformer is shown below,


Primary current is,

$$
I_{1}=\frac{50 \times 1000}{3300}=15.15 \mathrm{~A}
$$

Secondary current is,

$$
I_{2}=\frac{50 \times 1000}{230}=217.39 \mathrm{~A}
$$

Two winding transformer can be connected as an autotransformer using additive polarity. In this we have two cases :

## Case I :

The turn ratio $(k)=\frac{V_{L}}{V_{H}}=\frac{3300}{3530}=0.9348$
The value of $k$ is close to unity. Hence it is a feasible connection.


From figure,
LV side current of auto-transformer is,

$$
I_{L}=15.15+217.39=232.54 \mathrm{~A}
$$

HV side current of auto-transformer is,

$$
I_{H}=217.39 \mathrm{~A}
$$

Hence, the nominal rating of autotransformer can be calculated as

$$
\begin{array}{ll}
S_{\text {auto }}=V_{H} I_{H} & \text { or } S_{\text {auto }}=V_{L} I_{L} \\
S_{\text {auto }}=3530 \times 217.39 & \text { or } S_{\text {auto }}=3300 \times 232.54 \\
S_{\text {auto }}=767.4 \mathrm{kVA} & \text { or } S_{\text {auto }}=767.4 \mathrm{kVA}
\end{array}
$$

## Case II :

The turn ratio $(k)=\frac{V_{L}}{V_{H}}=\frac{230}{3530}=0.0651$
The value of $k$ is very less. Hence it is not a feasible connection.


From figure,
LV side current of auto-transformer is,

$$
I_{L}=15.15+217.39=232.54 \mathrm{~A}
$$

HV side current of auto-transformer is,

$$
I_{H}=15.15 \mathrm{~A}
$$

Hence, the nominal rating of autotransformer can be calculated as

$$
\begin{array}{ll}
S_{\text {auto }}=V_{H} I_{H} & \text { or } S_{\text {auto }}=V_{L} I_{L} \\
S_{\text {auto }}=3530 \times 15.15 & \text { or } S_{\text {auto }}=230 \times 232.54 \\
S_{\text {auto }}=53.48 \mathrm{kVA} & \text { or } S_{\text {auto }}=53.48 \mathrm{kVA}
\end{array}
$$

Hence, this connection is not useful as the kVA rating of auto-transformer is very low.
Hence, maximum kVA rating of autotransformer will be 767.4 kVA .
Hence, the correct option is (D).

## Method 2

The kVA rating of auto-transformer is given by,

$$
(\mathrm{kVA})_{\text {auto-transformer }}=\frac{(\mathrm{kVA})_{\text {T.W. }}}{1-k}
$$

Where, $k$ is the turn ratio in step down format i.e.

$$
k=\frac{\text { Low voltage }}{\text { High voltage }}
$$

Here, we have two cases :
Case I :

$$
k=\frac{V_{L}}{V_{H}}=\frac{3300}{3530}=0.9348
$$

[Feasible as it is close to unity]

$$
S_{\text {auto }}=\frac{50 \times 10^{3}}{1-0.9348}=767.4 \mathrm{kVA}
$$

## Case II :

$$
\begin{aligned}
& k=\frac{V_{L}}{V_{H}}=\frac{230}{3530}=0.0651 \quad[\text { Not feasible }] \\
& S_{\text {auto }}=\frac{50 \times 10^{3}}{1-0.0651}=53.48 \mathrm{kVA}
\end{aligned}
$$

[Not useful]
Hence, maximum kVA rating of autotransformer will be 767.4 kVA .
Hence, the correct option is (D).

### 1.38 (D)

In a transformer, magnetizing flux $\phi_{m}$ linking with iron core, leakage flux $\phi_{l_{1}}$ and $\phi_{l_{2}}$ leaking through air.

As $\phi_{m}$ flows through low reluctance path so $\phi_{m} \gg\left(\phi_{l_{1}}\right.$ and $\left.\phi_{l_{2}}\right)$.
Hence, reactances $X_{m} \gg\left(X_{1} \simeq X_{2}{ }^{\prime}\right)$

$$
\underbrace{X_{1} \approx X_{2}^{\prime}}_{\text {Rangeinm } \Omega} \ll \underbrace{X_{m}}_{\text {Rangein } \Omega}
$$

Hence, the correct option is (D).

### 1.39 (D)

During open circuit, no load current $\left(I_{0}\right)$ is very small (mainly 2 to $5 \%$ of full load current), most of which is utilized for setting up the main flux and a small amount to supply the iron losses.


As seen from phasor diagram $I_{m}$ lags $V$ by $90^{\circ}$ and $I_{c} \ll I_{m}$.
$\phi_{0}=\tan ^{-1} \frac{I_{m}}{I_{c}}$ is very large and hence power factor $\left(\cos \phi_{0}\right)$ is low.
Hence, the correct option is (D).

## $1.40 \quad$ (C)

Transformer A :
(i) kVA rating, $S_{A}=500 \mathrm{kVA}$
(ii) Leakage impedance, $Z_{1(\mathrm{pu})}=0.05$ p.u.

## Transformer B :

(i) kVA rating, $S_{B}=250 \mathrm{kVA}$
(ii) Leakage impedance, $Z_{2(\mathrm{pu})}=$ ?

For parallel operation of two transformers if two transformers are sharing load in proportion to their rating (kVA), then their per unit impedance based on their respective ratings must be equal.
i.e. $Z_{1(p u)}=Z_{2(p u)}=0.05$ p.u.

Hence, the correct option is (C).

### 1.41 (C)

Given :
(i) kVA rating, $S=300 \mathrm{kVA}$
(ii) At full load
$\eta=95 \%, p f=0.8$, loading $(m)=1$
(iii) At half load
$\eta=96 \%, p f=1$ (unity), loading $(m)=0.5$
Efficiency of transformer is given by,

$$
\eta=\frac{m \times(\mathrm{kVA}) \times \cos \phi}{m \times(\mathrm{kVA}) \times \cos \phi+P_{i}+m^{2} P_{c u_{\Omega}}} \ldots \text { (i) }
$$

Where, $m=$ Fraction of load
$\mathrm{kVA}=$ Rating of transformer
$\cos \phi=$ Power factor
$P_{i}=$ Iron or core loss
$P_{c u_{n}}=$ Full load copper loss

## Case I : At full load

$$
m=1, \cos \phi=0.8 \mathrm{lag}, \eta=0.95
$$

Put the values in equation (i),

$$
\begin{align*}
& \frac{1 \times 300 \times 0.8}{1 \times 300 \times 0.8+P_{i}+(1)^{2} \times P_{c u_{\Omega}}}=0.95 \\
& P_{i}+P_{c u_{n}}=12.63 \mathrm{~kW} \tag{ii}
\end{align*}
$$

## Case II : At half load

$$
m=0.5, \quad \cos \phi=1.0, \quad \eta=0.96
$$

Put the values in equation (i),

$$
\begin{align*}
& \frac{0.5 \times 300 \times 1}{0.5 \times 300 \times 1+P_{i}+(0.5)^{2} \times P_{c u_{\Omega}}}=0.96 \\
& P_{i}+0.25 P_{c u_{\Omega}}=6.25 \mathrm{~kW} \tag{iii}
\end{align*}
$$

Solving equations (ii) and (iii),
Iron loss, $P_{i}=4.12 \mathrm{~kW}$
Full load copper loss, $P_{c u_{g}}=8.51 \mathrm{~kW}$
Hence, the correct option is (C).

### 1.42 (B)

For maximum efficiency at unity power factor load,

Loading $(m)=\sqrt{\frac{P_{i}}{P_{c u_{f l}}}}=\sqrt{\frac{4.12}{8.51}}=0.696$
Therefore, maximum efficiency is given by,

$$
\begin{aligned}
& \eta_{\max }=\frac{m \mathrm{kVA} \cos \phi}{m \mathrm{kVA} \cos \phi+2 P_{i}} \\
& \eta_{\max }=\frac{0.696 \times 300 \times 1}{0.696 \times 300 \times 1+4.12 \times 2}=0.962
\end{aligned}
$$

Therefore, $\% \eta_{\max }=96.2 \%$
Hence, the correct option is (B).

### 1.43 (D)

Given :
(i) Three winding transformer
(ii) Turns ratio, 1:1:1
(iii) Resistance, $R=10 \Omega$
(iv) Capacitance, $C=15 \mathrm{mF}$

As all the windings have same number of turns, so magnitude of induced emf in all windings will be same. Thus, turn ratio of the transformer will be $1: 1: 1$.


The negative terminal of capacitor is connected to dot.
If we refer both the secondaries to primary the equivalent circuit is shown below,
At $t=0^{+}$, the switch is opened.


Since, turns ratio $=1: 1$
Current and voltage remain same on both sides.
From the equivalent circuit,

$$
V_{P}=-V_{C}=-5 \mathrm{~V}
$$

And current $I_{R}$ is given by,

$$
I_{R}=\frac{V_{P}}{R}=\frac{-5}{10}=-0.5 \mathrm{~A}
$$

Hence, the correct option is (D).

## © Key Point

(i) Dotted ends represents positive terminals.
(ii) In primary, the current flows into the dotted terminal.
(iii) In secondary, current flows out of the dotted terminals.

### 1.44 (C)

In a transformer, zero voltage regulation at full load is possible only at leading $p f$ load.
At leading $p f$

$$
\begin{aligned}
& V R=R_{p u} \cos \phi_{2}-X_{p u} \sin \phi_{2}=0 \\
& R_{p u} \cos \phi_{2}=X_{p u} \sin \phi_{2} \\
& \frac{\sin \phi_{2}}{\cos \phi_{2}}=\frac{R_{p u}}{X_{p u}}=\frac{\frac{I_{2} R_{02}}{E_{2}}}{\frac{I_{2} X_{02}}{E_{2}}}
\end{aligned}
$$

Condition for zero voltage regulation :

$$
\tan \phi_{2}=\frac{R_{p u}}{X_{p u}}=\frac{R_{02}}{X_{02}}
$$

where, $\phi_{2}$ is load power factor angle.
Hence, the correct option is (C).

### 1.45 (C)

Given :
(i) Single phase $250 / 500 \mathrm{~V}$ transformer
(ii) kVA rating, $S=50 \mathrm{kVA}$,
(iii) Power factor, $\cos \phi=1.0$ (unity)
(iv) Full load efficiency, $\eta=0.95$


Primary current,

$$
I_{1}=\frac{50 \times 10^{3}}{500}=100 \mathrm{~A}
$$

Secondary current,

$$
I_{2}=\frac{50 \times 10^{3}}{250}=200 \mathrm{~A}
$$

Efficiency of transformer is given by,

$$
\eta=0.95=\frac{50 \times 10^{3} \times 1}{50 \times 10^{3} \times 1+P_{i}+P_{c u_{f}}}
$$

where, $P_{i}=$ Iron loss

$$
\begin{align*}
& P_{c u_{n}}=\text { Full load copper loss } \\
& P_{i}+P_{c u_{n}}=2631.57 \mathrm{~W} \tag{i}
\end{align*}
$$

Two winding transformer is reconfigured as auto transformer is shown below,


Primary current of auto-transformer,

$$
I_{L}=I_{1}+I_{2}=100+200=300 \mathrm{~A}
$$

Secondary current of auto-transformer,

$$
I_{H}=200 \mathrm{~A}
$$

kVA rating of auto transformer

$$
=500 \times 300=150 \mathrm{kVA}
$$

At full load unity power factor,
Efficiency,

$$
\begin{equation*}
\eta_{\text {auto }}=\frac{\left(150 \times 10^{3}\right) \times 1}{\left(150 \times 10^{3}\right) \times 1+P_{i}+P_{c u_{\Omega}}} \tag{ii}
\end{equation*}
$$

Since, the same core and same windings are used in both two winding and auto transformer. Hence, the losses remains same and also the voltage applied and current through the core is also same.

From equations (i) and (ii),
Auto-transformer efficiency is,

$$
\begin{aligned}
\% & \eta_{\text {auto }} \\
& =\frac{150 \times 1000}{150 \times 1000+2631.57} \times 100 \% \\
\% \eta_{\text {auto }} & =98.276 \%
\end{aligned}
$$

Hence, the correct option is (C).


### 1.46 (D)

## Given :

(i) Single phase $230 \mathrm{~V} / 115 \mathrm{~V}$ transformer
(ii) kVA rating, $S=2 \mathrm{kVA}$
(iii) Wattmeter ratings :
$W_{1}: 250 \mathrm{~V}, 10 \mathrm{~A}$, Low power factor
$W_{2}: 250 \mathrm{~V}, 5 \mathrm{~A}$, Low power factor
$W_{3}: 150 \mathrm{~V}, 10 \mathrm{~A}$, High power factor
$W_{4}: 150 \mathrm{~V}, 5 \mathrm{~A}$, High power factor

1. For open circuit test, the No-load current is negligibly small ( 2 to $5 \%$ of full load current) having low power factor (no load $p f$ is very small)
Therefore, wattmeter $W_{2}$ is suitable for open circuit test.
2. For short circuit test (on HV side) $V_{s c}$ needed for circulation of full load current is very low.
Full load current $=\frac{2000}{230}=8.7 \mathrm{~A}$ and
the power factor is high. It ranges from 0.5 to 0.6 .

Therefore, $W_{3}$ is suitable for short circuit test.
Hence, the correct option is (D).

### 1.47 (B)

From question,

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The magnetic flux waveform is shown below,


From equation of slope,

$$
\frac{y-y_{1}}{x-x_{1}}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}
$$

For slope $1(0<t<1)$,

$$
\begin{aligned}
& \frac{\phi(t)-0}{t-0}=\frac{0.12-0}{1-0} \\
& \phi(t)=0.12 t
\end{aligned}
$$

For slope $2(2<t<2.5)$,

$$
\begin{aligned}
& \frac{\phi(t)-0.12}{t-2}=\frac{0-0.12}{2.5-2} \\
& \phi(t)=-0.24(t-2)+0.12 \\
& \phi(t)=-0.24 t+0.6
\end{aligned}
$$

Equation of flux is shown below,

$$
\phi(t)=\left\{\begin{array}{llc}
0.12 t & ; & 0<t<1 \\
0.12 & ; & 1<t<2 \\
-0.24 t+0.6 & ; & 2<t<2.5
\end{array}\right.
$$

Emf induced in secondary is given by,

$$
e_{r s}=-\underbrace{\left[N_{2} \frac{d \phi(t)}{d t}\right]}_{\text {Faradays Law }}
$$

(Here negative sign is due to Lenz's law).

$$
\begin{aligned}
& e_{r s}=-200 \frac{d}{d t} \phi(t)
\end{aligned}
$$

$$
\begin{aligned}
& e_{r s}=\left\{\begin{array}{llc}
-200 \times 0.12=-24 \mathrm{~V} & ; & 0<t<1 \\
-200 \times 0=0 \mathrm{~V} & ; & 1<t<2 \\
-200 \times-0.24=48 \mathrm{~V} & ; & 2<t<2.5
\end{array}\right.
\end{aligned}
$$

Waveform of $e_{r s}$ is shown below,


Hence, the correct option is (B).


### 1.48 (C)

Given :
(i) Single phase iron core transformer
(ii) Frequency, $f=50 \mathrm{~Hz}$
(iii) Cross sectional area :

Vertical arms $=20 \mathrm{~cm}^{2}$
Horizontal arms $=10 \mathrm{~cm}^{2}$
Self-inductance is given by,
$L=\frac{N^{2} \mu A}{l}$
where, $N=$ Number of turns,
$A=$ Cross sectional area of core,
$l=$ Length of core,
$\mu=\mu_{0} \mu_{r}=$ Permeability of core material


Mutual inductance, $M=K \sqrt{L_{1} L_{2}}$
$L_{1}, L_{2}$ : Self-inductance of two windings
$K$ : Coefficient of coupling

$$
M \propto \sqrt{L_{1} L_{2}} \propto \sqrt{A_{1} A_{2}}
$$

$A_{1}, A_{2}$ : Cross section area of two windings.
If both windings are wound on vertical arms,


$$
A_{1}=A_{2}=A_{V}=20 \mathrm{~cm}^{2}
$$

Then, $\quad M_{V} \propto \sqrt{A_{1} A_{2}} \propto \sqrt{A_{V} A_{V}} \propto A_{V}=20 \mathrm{~cm}^{2}$
If both windings are wound on horizontal arms,


Then, $\quad M_{H} \propto \sqrt{A_{1} A_{2}} \propto \sqrt{A_{H} A_{H}} \propto A_{H}=10 \mathrm{~cm}^{2}$
So, $\quad \frac{M_{H}}{M_{V}}=\frac{A_{H}}{A_{V}}=\frac{10}{20}=\frac{1}{2}=$ Halved
Hence, the correct option is (C).

## LI Key Point



Mutual inductance $M=K \sqrt{L_{1} L_{2}}$
where, $K$ is coefficient of coupling
Range of $K \rightarrow 0<K<1$

Iron core : Tightly coupled $K>0.5$
(Perfect coupling $K=1$ )
Air core : Loosely coupled $K<0.5$
If $K$ is not given then it should be considered as 1.

### 1.49 (D)

## Given :

Magnetizing inductance, $L_{m}=\frac{400}{\pi} \mathrm{mH}$
Circuit is shown below,


Magnetizing inductance,

$$
L_{m}=\frac{400}{\pi} \mathrm{mH}=\frac{0.4}{\pi} \mathrm{H}
$$

Equivalent circuit referred to primary of $(1: 1)$ transformer


When switch is open, no current flows in $30 \Omega$ resistor and $i(t)$ flows through $L_{m}$.


$$
V_{A B}(t)=L_{m} \frac{d}{d t} i(t)=\frac{0.4}{\pi} \frac{d i(t)}{d t}
$$

Current waveform is shown below,


From equation of slope,

$$
\frac{Y-y_{1}}{X-x_{1}}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}
$$

For slope-1 $(0<t<5 \mathrm{~ms})$,

$$
\begin{aligned}
& \frac{i(t)-0}{t-0}=\frac{10-0}{(5-0) \times 10^{-3}} \\
& i(t)=2000 t
\end{aligned}
$$

For slope-2 ( $5 \mathrm{~ms}<t<15 \mathrm{~ms}$ ),

$$
\begin{aligned}
& \frac{i(t)-10}{t-\left(5 \times 10^{-3}\right)}=\frac{-10-10}{(15-5) \times 10^{-3}} \\
& i(t)=-2000 t+20
\end{aligned}
$$

For slope-3 ( $15 \mathrm{~ms}<t<20 \mathrm{~ms}$ ),

$$
\begin{aligned}
& \quad \frac{i(t)-(-10)}{t-\left(15 \times 10^{-3}\right)}=\frac{0-(-10)}{(20-15) \times 10^{-3}} \\
& i(t)=2000 t-40 \\
& i(t)=\left\{\begin{array}{l}
2000 t \quad 0 \quad 0<t<5 \mathrm{~ms} \\
-2000 t+20 ; \quad 5 \mathrm{~ms}<t<15 \mathrm{~ms} \\
2000 t-40 ; \quad 15 \mathrm{~ms}<t<20 \mathrm{~ms}
\end{array}\right. \\
& V_{A B}(t)=\frac{0.4}{\pi} \frac{d}{d t} i(t) \\
& V_{A B}(t)=\frac{0.4}{\pi}\left\{\begin{array}{l}
\frac{d}{d t}(2000 t) \quad ; \quad 0<t<5 \mathrm{~ms} \\
\frac{d}{d t}(-2000 t+20) ; \quad 5 \mathrm{~ms}<t>15 \mathrm{~ms} \\
\frac{d}{d t}(2000 t-40) \quad ; \quad 15 \mathrm{~ms}<t<20 \mathrm{~ms}
\end{array}\right.
\end{aligned}
$$

$$
V_{A B}(t)=\left\{\begin{array}{ccc}
\frac{800}{\pi} & ; & 0<t<5 \mathrm{~ms} \\
\frac{-800}{\pi} & ; & 5 \mathrm{~ms}<t<15 \mathrm{~ms} \\
\frac{800}{\pi} & ; & 15 \mathrm{~ms}<t<20 \mathrm{~ms}
\end{array}\right.
$$

Waveform of $V_{A B}$ is shown below,


Therefore, Peak value of $V_{A B}$ is $\frac{800}{\pi} \mathrm{~V}$.
Hence, the correct option is (D).


## $1.50 \quad$ (B)

Given : $i(t)=10 \sin (100 \pi t) \mathrm{A}$
When the switch is closed the circuit is shown below,


$$
\begin{aligned}
& j X_{m}=j \omega L_{m}=j 100 \pi \times \frac{0.4}{\pi}=j 40 \Omega \\
& V_{A B}(t)=i_{1}(t) \times j 40=i_{2}(t) \times 30 \\
& i_{1}(t)=\frac{30 \times i(t)}{30+j 40} \quad \quad \quad[\mathrm{By} \mathrm{CDR}] \\
& i_{2}(t)=\frac{j 40 \times i(t)}{30+j 40} \quad[\mathrm{By} \mathrm{CDR}]
\end{aligned}
$$

Voltage across $A$ and $B$ is given by,

$$
\begin{aligned}
& V_{A B}(t)=i_{2}(t) \times 30 \\
& V_{A B}(t)=\frac{j 40 \times 30 i(t)}{30+j 40}=\frac{j 1200 i(t)}{30+j 40} \\
& V_{A B}(t)=\frac{1200 \angle 90^{0}}{50 \angle 53.13^{0}} \times i(t)
\end{aligned}
$$

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$$
\begin{aligned}
& V_{A B}(t)=24 \angle 36.87^{0} \times i(t) \\
& V_{A B}(t)=\left(24 \angle 36.87^{0}\right) \times 10 \sin (100 \pi t) \\
& V_{A B}(t)=240 \sin \left(100 \pi t+36.87^{0}\right)
\end{aligned}
$$

### 1.52 (D)

Given : Auto-transformer is operating 10 kVA load.
The equivalent circuit is shown below,


From figure,
Load current, $I_{H}=\frac{10 \times 10^{3}}{1000}=10 \mathrm{~A}$
For transformer,
Output kVA $=$ Input kVA
$I_{L} \times 400=I_{H} \times 1000$
$I_{L}=\frac{10}{400} \times 1000=25 \mathrm{~A}$
Thus, current in coil- 1 is calculated by applying KCL at node $D$,

$$
I_{L}-I_{H}=25-10=15 \mathrm{~A} \text { and }
$$

Current in coil-2, $I_{H}=10 \mathrm{~A}$
Hence, the correct option is (D).

## $\square$ Key Point

(i) The current in any transformer is always decided by load only. Hence, if load kVA is mentioned, then the current will be decided by load only i.e. not by the rating of transformer unless it has been mentioned.
(ii) The comparison between two winding transformer and auto transformer is given below,
From above connection, we have
Connected $A$ and $D$, common $B$
Hence, the correct option is (A).

| Two-winding <br> Transformer | Auto <br> Transformer |
| :--- | :--- |
| Physically two windings <br> are accommodated so <br> volume of core is more. | Physically single winding <br> is accommodated so <br> volume of core is less. |
| Different high voltage <br> and low voltage so <br> insulation must be <br> provided individually. <br> Amount of insulation <br> required is more. | Part of high voltage <br> winding is low voltage so <br> insulation required is less. |
| More copper is required <br> as high voltage and low <br> voltage windings turns <br> are separate. | No separate windings so <br> amount of copper wire is <br> less. |

## $1.53 \quad$ (C)

Given :
(i) Single phase transformer
(ii) Magnetizing current $\left|I_{m}\right|=1 \mathrm{~A}$
(iii) Transformation ratio, $k=\frac{N_{2}}{N_{1}}=2$

Given figure is shown below,


Since load is purely resistive hence $I_{2}=1 \mathrm{~A}$ at upf.
Core loss and leakage reactance are neglected. Hence, magnetizing current $\left(I_{m}\right)$ will be in phase with flux ( $\phi$ ) i.e. $I_{m}=1 \angle-90^{\circ}=-j 1$.
Hence, equivalent circuit referred to primary side is shown below,

$\frac{I_{2}^{\prime}}{I_{2}}=\frac{2}{1}=k$

$$
I_{2}^{\prime}=2 \times I_{2}=2 \mathrm{~A}
$$

Primary current $I_{1}=I_{2}{ }^{\prime}+I_{m}=2-j 1$

$$
\left|I_{1}\right|=\sqrt{2^{2}+1^{2}}=2.24 \mathrm{~A}
$$

Hence, the correct option is (C).

### 1.54 (C)

Since, the transformer is an air cored.
So, there will be no saturation and hysteresis losses. Hence, $B-H$ curve will be linear.
The core flux $\left(\phi_{m}\right) \propto$ flux density $\left(B_{m}\right)$ and magnetizing current $\left(I_{m}\right) \propto$ field intensity $(H)$. The $B-H$ curve for air core material is linear and shown below,


From the $B-H$ curve it is clear that,

$$
I_{m}=k \phi_{m}
$$

where, $k=$ constant
As applied voltage is sinusoidal, hence the flux waveform will also be sinusoidal. Therefore, the shape of emf, flux ( $\phi$ ) and the magnetizing current $\left(I_{m}\right)$ drawn by the transformer from the supply is sinusoidal in nature.


Hence, the correct option is (C).

## Key Point

Air core :
(i) Poor magnetic coupling.
(ii) Low power electronic circuit.

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(iii) No saturation of magnetic material.
(iv) Linear $B$ - $H$ curve.

## Iron core :

(i) Ferromagnetic core.
(ii) Tight magnetic coupling.
(iii) High flux density and high power application.
(iv) Magnetic saturation will occur.
(v) Non-linear $B-H$ curve.

### 1.55 (B)

## Given :

(i) Transformer rating, $S=10 \mathrm{kVA}$
(ii) Frequency, $f=50 \mathrm{~Hz}$,
(iii) Primary supply, $E_{1}=1 \mathrm{kV}$,
(iv) No load loss, $P_{0_{1}}=55 \mathrm{~W}$
(v) No load current, $I_{0_{1}}=0.5 \mathrm{~A}$
(vi) Linear dimension of second transformer is $\sqrt{2}$ times of first transformer.
(vii) $2 \mathrm{kV}, 50 \mathrm{~Hz}$ is applied to primary of second transformer.


## Method 1

Emf equation is given by,

$$
E=4.44 f N \phi_{m}
$$

Since, $N$ and $f$ is constant

$$
\begin{align*}
& E \propto \phi_{m}  \tag{i}\\
& \phi_{m}=B_{m} \times A=\mu H \times A=\frac{\mu N I_{0} A}{l} \tag{ii}
\end{align*}
$$

Hence, $\phi_{m} \propto \frac{I_{0} A}{l}$

From equations (i) and (ii)

$$
E \propto \frac{I_{0} A}{l}
$$

where, $A=$ Cross sectional area of core,
$l=$ Length of core,
$I_{0}=$ No load current
$\frac{E_{2}}{E_{1}}=\frac{I_{0_{2}} A_{2}}{I_{0_{1}} A_{1}} \times \frac{l_{1}}{l_{2}}$
Since, linear dimensions of second transformer is $\sqrt{2}$ times of the first transformer (given).
So, $\quad A_{2}=(\sqrt{2})^{2} A_{1}$ and $l_{2}=\sqrt{2} l_{1}$
$E_{2}=2 \mathrm{kV}$
$E_{1}=1 \mathrm{kV} \quad$ (given)
Put the values in equation (iii),
$\frac{2}{1}=\frac{I_{0_{2}}}{I_{0_{1}}} \frac{(\sqrt{2})^{2} A_{1}}{A_{1}} \times \frac{l_{1}}{\sqrt{2} l_{1}}$
$I_{0_{2}}=\sqrt{2} I_{0_{1}} \quad$ (Given, $\left.I_{0_{1}}=0.5 \mathrm{~A}\right)$
$I_{0_{2}}=\sqrt{2} \times 0.5=0.707 \mathrm{~A}$
Maximum flux density of core is given by,

$$
\begin{aligned}
& B_{\max }=\frac{\phi_{\max }}{A} \\
& \phi_{\max }=\frac{E}{4.44 N \times f} \\
& \frac{B_{\max 2}}{B_{\max 1}}=\frac{\phi_{\max 2}}{A_{2}} \times \frac{A_{1}}{\phi_{\max 1}}
\end{aligned}
$$

$\frac{B_{\max 2}}{B_{\max 1}}=\frac{E_{2}}{4.44 N \times f \times A_{2}} \times \frac{4.44 N \times f \times A_{1}}{E_{1}}$
$\frac{B_{\max 2}}{B_{\max 1}}=\frac{2000}{2 A_{1}} \times \frac{A_{1}}{1000}$
$B_{\text {max } 2}=B_{\text {max } 1}$
Hence, core loss $\propto$ volume of core
$\frac{P_{o_{2}}}{P_{o_{1}}}=\frac{v_{2}}{v_{1}}=\frac{A_{2} l_{2}}{A_{1} l_{1}}=\frac{(\sqrt{2})^{2} \times \sqrt{2} A_{1} l_{1}}{A_{1} l_{1}}$
$\frac{P_{o_{2}}}{55}=\frac{(\sqrt{2})^{3}}{1}$

$$
P_{o_{2}}=\frac{(\sqrt{2})^{3}}{1} \times 55=155.6 \mathrm{~W}
$$

Hence, No load current is 0.7 A and no load power is 155.6 W
Hence, the correct option is (B).

## Method 2

No load current $\propto$ Linear dimension

$$
\begin{aligned}
& \frac{I_{1}}{I_{2}}=\frac{\text { dimension }_{1}}{\text { dimension }_{2}} \\
& \frac{0.5}{I_{2}}=\frac{1}{\sqrt{2}} \\
& I_{2}=\sqrt{2} \times 0.5=0.7 \mathrm{~A}
\end{aligned}
$$

Power $\propto(\text { Linear dimension })^{3}$

$$
\begin{aligned}
& \frac{P_{1}}{P_{2}}=\left(\frac{\text { dimension }_{1}}{\text { dimension }_{2}}\right)^{3} \\
& \frac{55}{P_{2}}=\left(\frac{1}{\sqrt{2}}\right)^{3} \\
& P_{2}=2 \sqrt{2} \times 55 \\
& P_{2}=1.414 \times 110=155.6 \mathrm{~W}
\end{aligned}
$$

Hence, the correct option is (B).

## $\square$ Key Point

Reluctance : Reluctance is basically inversely proportional to flux.
Reluctance, $S=\frac{l}{\mu A}=\frac{N I}{\mu H A}=\frac{N I}{B A}=\frac{N I}{\phi}$

$$
\phi=\frac{N I}{S} \quad\left[\text { Flux }=\frac{\text { MMF }}{\text { Reluctance }}\right]
$$

Ohm's law in magnetic field.

### 1.56 (C)

Given :
(i) Single phase transformer
(ii) No load loss, $P_{i}=64 \mathrm{~W}$
(iii) Short circuit test : $I_{s c}=0.9 I_{f l}$ ( $90 \%$ of rated current),

Copper loss, $P_{\text {cum }}=81 \mathrm{~W}$
where, $P_{\text {cum }}=$ Copper loss at $90 \%$ of rated current,

$$
\begin{equation*}
P_{c u m}=m^{2} P_{c u_{n}} \tag{i}
\end{equation*}
$$

where, $m=$ Friction of loading $=\frac{I_{s c}}{I_{\text {rated }}}=0.9$
Hence, from equation (i) the full load copper loss,

$$
P_{c u u_{f}}=\frac{P_{c u m}}{m^{2}}=\frac{81}{(0.9)^{2}}=100 \mathrm{~W}
$$

Now, for maximum efficiency of transformer,

$$
\begin{aligned}
& P_{i}=P_{c u m n}=m^{2} P_{c u_{n}} \\
& \left.m\right|_{\mathrm{at} \eta=\eta_{\max }}=\sqrt{\frac{P_{i}}{P_{c u_{n}}}}=\sqrt{\frac{64}{100}}=0.8
\end{aligned}
$$

So, $\%$ of loading $=0.8 \times 100=80 \%$
Thus, maximum efficiency will occur at $80 \%$ of rated current.
Hence, the correct option is (C).


### 1.57 (C)

Given :
(i) Voltage, frequency = constant
(ii) Equivalent radius of the core of a transformer is reduced by half.
(iii) No load current, $I_{0}=I_{m}=$ Constant

## Method 1

The induced emf in a transformer is given by,

$$
E=4.44 f N \phi
$$

Since, $\frac{E}{f}=$ Constant,$N \phi=$ Constant
i.e. $\quad \phi \propto \frac{1}{N}$

Flux in any magnetic circuit is given by,

$$
\begin{equation*}
\phi=\frac{\mathrm{MMF}}{\text { Reluctance }}=\frac{N I_{m}}{R}=\frac{N I_{m}}{l / \mu \mathrm{A}}=\frac{\mu N I_{m} A}{l} \tag{i}
\end{equation*}
$$

Where, $N=$ Number of turns
$I_{m}=$ Magnetizing current
$\mu=$ Permeability of core
$A=$ Cross sectional area of core
$l=$ Magnetic path length
Since, $\phi \propto \frac{1}{N}$
From equation (i)

$$
\frac{\mu N I_{m} A}{l} \propto \frac{1}{N}
$$

Hence, $\frac{\mu N^{2} I_{m} A}{l}=$ Constant
i.e. $\quad N^{2} I_{m} A=$ Constant

Since, No load current is constant.
Therefore, $N^{2} A=$ Constant
i.e. $\quad N^{2} \propto \frac{1}{A}$

$$
\begin{aligned}
& \left(\frac{N_{2}}{N_{1}}\right)^{2}=\frac{\pi r_{1}^{2}}{\pi r_{2}^{2}}=\left(\frac{r_{1}}{r_{2}}\right)^{2} \\
& \left(\frac{N_{2}}{N_{1}}\right)^{2}=\frac{\pi r_{1}^{2}}{\pi r_{2}^{2}}=\left(\frac{r_{1}}{r_{2}}\right)^{2}=\left(\frac{2 r_{2}}{r_{2}}\right)^{2}=4
\end{aligned}
$$

So, $\frac{N_{2}}{N_{1}}=2$ i.e. number of turns become double. Hence, the correct option is (C).

## Method 2

Considering, a square cored transformer with sides $=a$.
Then Area of core, $A=a^{2}$
When converted into equivalent circular core, with radius ' $r$ ' then the perimeter of both the cores will be same.
i.e., $\quad 4 a=2 \pi r$
$a \propto r$

Area of circular core, $A=\pi r^{2}$

$$
A \propto r^{2} \propto a^{2}
$$

If radius is halved, then $a$ is halved \& area becomes $\frac{A}{4}$
At no load, MMF $=\phi \times$ Reluctance

$$
\begin{array}{ll}
N I_{0}=\phi R & {[R=\text { Reluctance }]} \\
I_{0}=\frac{\phi R}{N}=\text { constant }
\end{array}
$$

Flux is constant as $V / f$ is constant

$$
\begin{aligned}
& I_{0} \propto \frac{R}{N}, R=\frac{l_{\text {core }}}{\mu_{\text {core } \cdot A_{\text {core }}}} \\
& I_{0} \propto \frac{l_{\text {core }}}{A_{\text {core }^{2}} \cdot N} \\
& \frac{I_{02}}{I_{01}}=\frac{l_{\text {core }_{2}}}{A_{\text {core }_{2}} \times N_{2}} \times \frac{A_{\text {core }_{1}}}{l_{\text {core }_{1}}} \times N_{1} \\
& \frac{I_{02}}{I_{01}}=\frac{l_{\text {core }_{1}} / 2}{A_{\text {core }_{1}} / 4 \times N_{2}} \times \frac{A_{\text {core }_{1}}}{l_{\text {core }_{1}}} N_{1} \\
& \frac{I_{02}}{I_{01}}=\frac{2 N_{1}}{N_{2}}=1 \quad \quad\left[\text { As } N_{2}=2 N_{1}\right] \\
& \frac{N_{2}}{N_{1}}=2
\end{aligned}
$$

Hence, the correct option is (C).

## $\square$ Key Point

## If the core is toroidal :



Mean radius of core, $x=\frac{R_{1}+R_{2}}{2}$
Equivalent radius of core, $r=\frac{R_{2}-R_{1}}{2}$
Mean circumference of core, $l=2 \pi x$
Cross sectional area of core, $A=\pi r^{2}$

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## $1.58 \quad$ (A)

Given :
(i) Single phase $230 / 115 \mathrm{~V}$ transformer
(ii) Supply voltage $=230 \mathrm{~V}, 50 \mathrm{~Hz}$,

$$
\left(\frac{V}{f}=\frac{230}{50}=\frac{23}{5}\right)
$$

Core loss, $P_{i_{1}}=1050 \mathrm{~W}$
(iii) Supply voltage $=138 \mathrm{~V}, 30 \mathrm{~Hz}$,

$$
\left(\frac{V}{f}=\frac{138}{30}=\frac{23}{5}\right)
$$

Core loss, $P_{i_{2}}=500 \mathrm{~W}$
In both above cases $\frac{V}{f}$ ratio is constant.
Core loss, $P_{i}=P_{h}+P_{e}$
where, $P_{h}=$ Hysteresis loss

$$
P_{e}=\text { Eddy current loss }
$$

For constant $\frac{V}{f}$ ratio,

$$
P_{e} \propto f \text { and } P_{h} \propto f^{2}
$$

Hence, core loss, $P_{i}=a f+b f^{2}$
where, $P_{i}=$ Core loss

$$
a \text { and } b=\text { Constant }
$$

(i) At $230 \mathrm{~V}, 50 \mathrm{~Hz}$ :

$$
\begin{equation*}
P_{i_{1}}=1050=a \times 50+b \times(50)^{2} \tag{i}
\end{equation*}
$$

(ii) At $\mathbf{1 3 8} \mathbf{V}, \mathbf{3 0 ~ H z}$ :

$$
\begin{equation*}
P_{i_{2}}=500=a \times 30+b \times(30)^{2} \tag{ii}
\end{equation*}
$$

From equation (i) and (ii),

$$
a=\frac{61}{6} \text { and } b=\frac{13}{60}
$$

Hence, at $230 \mathrm{~V}, 50 \mathrm{~Hz}$,
Hysteresis loss,

$$
P_{h}=a f=\frac{61}{6} \times 50=508.3 \mathrm{~W}
$$

Eddy current loss,

$$
P_{e}=b f^{2}=\frac{13}{60} \times(50)^{2}=541.67 \mathrm{~W}
$$

Hence, $P_{h}=508 \mathrm{~W}$ and $P_{e}=542 \mathrm{~W}$
Hence, the correct option is (A).

### 1.59 (A)

Given :
(i) Single-phase two winding transformer
(ii) Both supply frequency and voltage are increased by $10 \%$.
Since, both the supply frequency and voltage are increased by $10 \%$, hence $\frac{V}{f}$ ratio is constant.
Hence, $B_{\text {max }}=$ constant
Hysteresis loss is given by,

$$
\begin{aligned}
& P_{h}=K_{h} B_{\max }^{x} f v_{\text {core }} \\
& K_{h} B_{\max }^{x} v_{\text {core }}=\text { Constant }
\end{aligned}
$$

Hence, $P_{h} \propto f$
$\%$ change in hysteresis loss is given by,

$$
\begin{aligned}
& \% \Delta P_{h}=\frac{P_{h_{2}}-P_{h_{1}}}{P_{h_{1}}} \times 100 \\
& \% \Delta P_{h}=\frac{f_{2}-f_{1}}{f_{1}} \times 100=\frac{1.1 f_{1}-f_{1}}{f_{1}} \times 100 \\
& \% \Delta P_{h}=10 \%
\end{aligned}
$$

Eddy current loss is given by,

$$
\begin{aligned}
& P_{e}=K_{e} B_{\max }^{2} f^{2} t^{2} v_{\text {core }} \\
& K_{e} B_{\max }^{2} t^{2} v_{\text {core }}=\text { Constant }
\end{aligned}
$$

Hence, $P_{e} \propto f^{2}$
$\%$ change in eddy current loss is given by,

$$
\begin{aligned}
& \% \Delta P_{e}=\frac{P_{e_{2}}-P_{e_{1}}}{P_{e_{1}}} \times 100 \\
& \% \Delta P_{e}=\frac{f_{2}^{2}-f_{1}^{2}}{f_{1}^{2}} \times 100=\frac{\left(1.1 f_{1}\right)^{2}-f_{1}^{2}}{f_{1}^{2}} \times 100 \\
& \% \Delta P_{e}=\frac{1.21-1}{1} \times 100=21 \%
\end{aligned}
$$

Hence, the correct option is (A).

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### 1.60550

Given :
(i) Single phase $1000 \mathrm{~V} / 100 \mathrm{~V}$ transformer
(ii) kVA rating, $\mathrm{kVA}_{\text {T.w. }}=50 \mathrm{kVA}$
(iii) It is connected as auto transformer

## Method 1

The equivalent circuit can be represented as,


For two winding transformer :
Primary current is,

$$
I_{1}=\frac{50 \times 10^{3}}{1000}=50 \mathrm{~A}
$$

Secondary current is,

$$
I_{2}=\frac{50 \times 10^{3}}{100}=500 \mathrm{~A}
$$

Auto transformer rating 1000/1100 i.e. additive polarity and the equivalent circuit of autotransformer is shown below,


From figure,
Primary current of auto-transformer,

$$
I_{L}=50+500=550 \mathrm{~A} \text { and }
$$

Secondary current of auto-transformer,

$$
I_{H}=500 \mathrm{~A}
$$

The rating of auto transformer is given by,

$$
\begin{aligned}
& S_{\text {auto }}=V_{L} I_{L}=V_{H} I_{H} \\
& S_{\text {auto }}=1000 \mathrm{~V} \times 550 \mathrm{~A}=550 \mathrm{kVA}
\end{aligned}
$$

Hence, the kVA rating of the auto transformer is 550 kVA.

## Method 2

The kVA rating of auto transformer is given by,

$$
(\mathrm{kVA})_{\text {auto }}=\frac{(\mathrm{kVA})_{\mathrm{T} . \mathrm{W} .}}{1-k}
$$

Where, $K$ is transformation ratio in step down format i.e.

$$
k=\frac{\text { Low voltage }}{\text { High voltage }}=\frac{V_{L}}{V_{H}}
$$

Hence, $(\mathrm{kVA})_{\text {auto }}=\frac{50 \times 10^{3}}{1-1000 / 1100}$

$$
\begin{aligned}
& (\mathrm{kVA})_{\text {auto }}=\frac{50 \times 10^{3}}{11-10 / 11} \\
& (\mathrm{kVA})_{\text {auto }}=50 \times 10^{3} \times 11=550 \mathrm{kVA}
\end{aligned}
$$

Hence, the kVA rating of the auto-transformer is 550 kVA .

### 1.61 (B)

## Given :

(i) Supply frequency :

$$
f_{1}=50 \mathrm{~Hz}, \quad f_{2}=25 \mathrm{~Hz}
$$

(ii) $\frac{V}{f}$ is constant

Figure is shown below,


For constant $\frac{V}{f}$ ratio,

$$
P_{e} \propto f \text { and } P_{h} \propto f^{2}
$$

Hence, core loss, $P_{i}=a f+b f^{2}$
where, $P_{i}=$ Core loss

$$
\begin{equation*}
a \text { and } b=\text { Constant } \tag{i}
\end{equation*}
$$

Hence, $\left(\frac{P_{i}}{f}\right)=a+b f$
From figure,

## Case 1 :

At, $f=0 \mathrm{~Hz}, \frac{P_{i}}{f}=10$
Put the value in equation (i),

$$
\begin{aligned}
& 10=a+b \times 0 \\
& a=10
\end{aligned}
$$

## Case 2 :

At, $f=50 \mathrm{~Hz}, \frac{P_{i}}{f}=15$
Put the values in equation (i),

$$
\begin{aligned}
& 15=10+b \times 50 \\
& b=\frac{1}{10}
\end{aligned}
$$

Case 3 :
At, $f=25 \mathrm{~Hz}$
Hysteresis loss, $P_{h}=a f$

$$
P_{h}=10 \times 25=250 \mathrm{~W}
$$

Eddy current loss, $P_{e}=b f^{2}$

$$
P_{e}=\frac{1}{10} \times(25)^{2}=62.5 \mathrm{~W}
$$

Hence, $P_{h}=250 \mathrm{~W}$ and $P_{e}=62.5 \mathrm{~W}$ Hence, the correct option is (B).

\section*{| 1.62 | 23.62 |
| :--- | :--- |}

Given :
(i) Load : Power, $P=4 \mathrm{~kW}$,

Power factor $=\cos \phi_{1}=0.89$ lag
(ii) Transformation ratio, $k=\frac{1}{2}$

Circuit is shown below,


## Method 1

The equivalent circuit refer to primary is shown below,


Load current, $I_{L}^{\prime}=\frac{P}{V_{2}^{\prime} \cos \phi}=\frac{4 \times 10^{3}}{220 \times 0.89}$

$$
\begin{aligned}
& I_{L}^{\prime}=20.43 \angle-27.13^{0} \mathrm{~A} \\
&\left(\text { As } \phi=\cos ^{-1} 0.89=27.13^{0}\right)
\end{aligned}
$$

Case 1 : When reactance $X$ is not connected

$$
\begin{aligned}
I_{1} & =I_{L}^{\prime}=20.43 \angle-27.13^{0} \\
& =18.18-j 9.37 \mathrm{~A}
\end{aligned}
$$

Case 2 : When reactance $X$ is connected, Phasor diagram is given by,


Now, to make input power factor unity, the current through reactance is given by,

$$
I_{X}=I_{L_{p}}^{\prime}=I_{L}^{\prime} \sin (27.13)=9.37 \mathrm{~A}
$$

Also, $\quad I_{X}=\frac{V_{2}{ }^{\prime}}{X}$
Thus, $\quad X=\frac{220}{9.37}=23.62 \Omega$
Hence, the value of the reactance $X$ to improve the input power factor to unity is $23.62 \Omega$.

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## Method 2

The equivalent circuit refer to primary is shown below,


Now, the reactance power supplied by the capacitance is given by,

$$
\begin{aligned}
& Q_{C}=P\left[\tan \phi_{1}-\tan \phi_{2}\right] \\
& \phi_{1}=\cos ^{-1} 0.89=27.12^{0} \\
& \phi_{2}=\cos ^{-1} 1=0^{0}
\end{aligned}
$$

Hence, $Q_{C}=4 \times 10^{3}\left[\tan \left(27.12^{\circ}\right)-\tan \left(0^{\circ}\right)\right]$

$$
Q_{C}=2.05 \mathrm{kVAR}
$$

Also, $Q_{C}=\frac{\left(V_{2}^{\prime}\right)^{2}}{X}$

$$
X=\frac{220^{2}}{2.05 \times 10^{3}}=23.62 \Omega
$$

Hence, the value of the reactance $X$ to improve the input power factor to unity is $\mathbf{2 3 . 6 2} \boldsymbol{\Omega}$.

## Method 3


$X$ should be capacitive in order to compensate for reactive power and hence bring the $p f$ to unity. So, $I_{x}$ will lead $-E_{1}$ by $90^{\circ}$ and lag $+E_{1}$ by $90^{\circ}$. As per dot convention,

$$
I_{2}^{\prime}+I_{x}=I
$$

$I$ and $V_{1}$ are in phase (upf),

$$
\begin{gathered}
\phi_{1}=\cos ^{-1} 0.89=27.12^{0} \\
I_{2}=\frac{4000}{110 \times 0.89} \angle-27.12^{0}=40.85 \angle-27.12
\end{gathered}
$$

$$
I_{2}^{\prime}=k I_{2}=\frac{I_{2}}{2}=20.43 \angle-27.12
$$



So from $\triangle O A B$,

$$
\begin{aligned}
& \frac{A B}{O A}=\sin \left(27.12^{0}\right) \\
& \frac{I_{x}}{I_{2}^{\prime}}=\sin \left(27.12^{\circ}\right)
\end{aligned}
$$

Therefore, $I_{x}=I_{2}^{\prime} \sin 27.12$

$$
\begin{aligned}
& I_{x}=20.43 \times \sin 27.12=9.31 \mathrm{~A} \\
& \frac{E_{1}}{I_{x}}=\frac{220}{9.31}=23.62 \Omega
\end{aligned}
$$

Hence, the value of the reactance $X$ to improve the input power factor to unity is $\mathbf{2 3 . 6 2} \Omega$.

### 1.63 (B)

Given :
(i) Single-phase $220 / 110 \mathrm{~V}$ transformer
(ii) Supply frequency, $f=50 \mathrm{~Hz}$
(iii) $L_{l_{1}}=45 \mathrm{mH}, L_{l_{2}}=30 \mathrm{mH}$,

$$
M=20 \mathrm{mH}
$$

Transformation ratio is given by,

$$
k=\frac{N_{2}}{N_{1}}=\frac{E_{2}}{E_{1}}=\frac{110}{220}=\frac{1}{2}
$$

Secondary parameters are referred to primary side. The equivalent circuit is shown below,


From figure,

$$
\begin{aligned}
& L_{l_{1}}^{\prime}=L_{l_{1}}-\frac{M}{k}=45 \mathrm{mH}-\frac{20 \mathrm{mH}}{1 / 2} \\
& L_{l_{1}}^{\prime}=5 \mathrm{mH} \\
& L_{l_{2}}^{\prime}=\frac{L_{l_{2}}}{k^{2}}-\frac{M}{k}=\frac{30 \mathrm{mH}}{\left(\frac{1}{2}\right)^{2}}-\frac{20 \mathrm{mH}}{\frac{1}{2}} \\
& L_{l_{2}}^{\prime}=80 \mathrm{mH}
\end{aligned}
$$

$$
\bar{x}=\frac{\sum_{i=1}^{100} x_{i}}{100}=0.2 \mathrm{~mm}
$$

$$
\sum_{i=1}^{100} x_{i}=20 \mathrm{~mm}
$$

Magnetizing inductance is given by,

$$
L_{m}^{\prime}=\frac{M}{k}=\frac{20}{\left(\frac{1}{2}\right)}=40 \mathrm{mH}
$$

Thus, value of $L_{l_{1}}, L_{l_{2}}$ and $L_{m}$ refer to primary side is $5 \mathrm{mH}, 80 \mathrm{mH}$ and 40 mH .
Hence, the correct option is (B).


### 1.64 (D)

Given :
(i) Silicon steel lamination :

Mean thickness, $\bar{x}=0.2 \mathrm{~mm}$
Variance, $\sigma_{x}^{2}=0.02 \mathrm{~mm}$
(ii) Insulation:

Mean thickness, $\bar{y}=0.1 \mathrm{~mm}$
Variance, $\sigma_{y}^{2}=0.01 \mathrm{~mm}$
Let us consider a particular Si steel lamination i.e. $i^{\text {th }}$ lamination of thickness $x_{i}$.


For $n$ such laminations where, $n=100$
The mean value is given by,

$$
\bar{x}=\frac{x_{1}+x_{2}+\cdots \cdots+x_{n}}{n}=\frac{\sum_{i=1}^{n} x_{i}}{n}
$$

Variance $=\sigma_{x}^{2}=\frac{\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}}{n}$

$$
\begin{aligned}
& \sigma_{x}^{2}=\frac{\sum_{i=1}^{100}\left(x_{i}^{2}+(\bar{x})^{2}-2 x_{i} \bar{x}\right)}{100}=0.02 \\
& \sum_{i=1}^{100} x_{i}^{2}+(\bar{x})^{2} \times 100-2 \bar{x} \sum_{i=1}^{100} x_{i}=2 \\
& \sum_{i=1}^{100} x_{i}^{2}+100 \times(0.2)^{2}-2 \times 0.2 \times 20=2 \\
& \sum_{i=1}^{100} x_{i}^{2}=6
\end{aligned}
$$

The varnish insulation is applied on both the sides of the laminations.
The mean thickness of one side of insulation is $y_{i}$.
The equivalent circuit is shown below,


Mean $\bar{y}=0.1 \mathrm{~mm}$

$$
\bar{y}=\frac{\sum_{i=1}^{n} y_{i}}{n}
$$

$$
\begin{aligned}
& \frac{\sum_{i=1}^{100} y_{i}}{100}=0.1 \\
& \sum_{i=1}^{100} y_{i}=10
\end{aligned}
$$

Variance $\left(\sigma_{y}^{2}\right)=0.01$

$$
\begin{aligned}
& \sigma_{y}^{2}=\frac{\sum_{i=1}^{n}\left(y_{i}-\bar{y}\right)^{2}}{n} \\
& \sigma_{y}^{2}=\frac{\sum_{i=1}^{100}\left(y_{i}-\bar{y}\right)^{2}}{100}=0.01 \\
& \sum_{i=1}^{100}\left(y_{i}^{2}+\bar{y}^{2}-2 \bar{y} y_{i}\right)=1 \\
& \sum_{i=1}^{100} y_{i}^{2}+100 \times(\bar{y})^{2}-2 \bar{y} \sum_{i=1}^{100} y_{i}=1 \\
& \sum_{i=1}^{100} y_{i}^{2}+100 \times(0.1)^{2}-2 \times 0.1 \times 10=1 \\
& \sum_{i=1}^{100} y_{i}^{2}=2
\end{aligned}
$$

Hence, total thickness of one unit (both sides insulation)

$$
z_{i}=x_{i}+2 y_{i}
$$

If the transformer core is made using 100 such varnish coated laminations,
Then the equivalent circuit is shown below,


Thickness of core $=\sum_{i=1}^{n} z_{i}$

$$
\begin{aligned}
& =\sum_{i=1}^{100}\left(x_{i}+2 y_{i}\right)=\sum_{i=1}^{100} x_{i}+2 \sum_{i=1}^{100} y_{i} \\
& =20+2 \times 10=40 \mathrm{~mm}
\end{aligned}
$$

Mean $\bar{z}=\frac{\sum z_{i}}{n}=\frac{40}{100}=0.4 \mathrm{~mm}$
$\sum_{i=1}^{n} z_{i}^{2}=\sum_{i=1}^{n}\left(x_{i}+2 y_{i}\right)^{2}$
$\sum_{i=1}^{n} z_{i}^{2}=\sum_{i=1}^{100} x_{i}^{2}+4 y_{i}^{2}+4 x_{i} y_{i}$
$\sum_{i=1}^{n} z_{i}^{2}=\sum_{i=1}^{100} x_{i}^{2}+4 \sum_{i=1}^{100} y_{i}^{2}+4 \sum_{i=1}^{100}\left(x_{i} y_{1}\right)$
$\sum_{i=1}^{n} z_{i}^{2}=6+4 \times 2+4 \times \frac{1}{100}\left(\sum x_{i}\right)\left(\sum y_{i}\right)$
$\sum_{i=1}^{n} z_{i}^{2}=14+\frac{4}{100} \times 20 \times 10=22$
Variance of $z\left(\sigma_{z}^{2}\right)=\frac{\sum_{i=1}^{n}\left(z_{i}-\bar{z}\right)^{2}}{n}$

$$
\begin{aligned}
& \sigma_{z}^{2}=\frac{1}{100}\left[\sum_{i=1}^{100}\left(z_{i}^{2}+\bar{z}^{2}-2 \bar{z} z_{i}\right)\right] \\
& \left.\sigma_{z}^{2}=\frac{1}{100}\left[\sum_{i=1}^{100} z_{i}^{2}+(\bar{z})^{2} \times 100-2 \bar{z} \sum_{i=1}^{100} z_{i}\right)\right] \\
& \sigma_{z}^{2}=\frac{1}{100}\left[22+100 \times(0.4)^{2}-2 \times 0.4 \times 40\right] \\
& \sigma_{z}^{2}=0.06
\end{aligned}
$$

Standard deviation,

$$
\sigma_{z}=\sqrt{0.06}=0.245
$$

Hence, the correct option is (D).

## 10 Key Point

If $\quad z=\alpha x_{1}+\beta x_{2}+\gamma x_{3}$
$\operatorname{Var}(z)=\alpha^{2} \operatorname{Var}\left(x_{1}\right)+\beta^{2} \operatorname{Var}\left(x_{2}\right)+\gamma^{2} \operatorname{Var}\left(x_{3}\right)$
As, $\quad z=x+2 y$

$$
\begin{aligned}
& \operatorname{Var}(z)=\operatorname{Var}(x)+(2)^{2} \operatorname{Var}(y) \\
& \sigma_{z}^{2}=\sigma_{x}^{2}+4 \sigma_{y}^{2} \\
& \left(\sum_{i=1}^{n} x_{i}\right)\left(\sum_{i=1}^{n} y_{i}\right)=n \sum_{i=1}^{n} x_{i} y_{i}
\end{aligned}
$$

### 1.65 (A)

## Given :

(i) $1-\phi, 50 \mathrm{~Hz}$ transformer
(ii) Mutual inductance, $M=480 \mathrm{mH}$
(iii) Self-inductances :

Primary, $L_{1}=800 \mathrm{mH}$
Secondary, $L_{2}=600 \mathrm{mH}$
From the question, the equivalent circuit is shown below,


The equivalent circuit refer to primary is,


Leakage inductance of primary,

$$
L_{l_{1}}=L_{1}-\frac{M}{k}
$$

where, $k=$ transformation ratio $=1: 1$
Note : If value of $k$ is not specified, consider it as unity generally transformer with $k=1$ are used for electrical isolation.

$$
L_{l_{1}}=800-1 \times 480=320 \mathrm{mH}
$$

Secondary leakage inductance,

$$
\begin{aligned}
& L_{l_{2}}=\frac{L_{2}}{k^{2}}-\frac{M}{k} \\
& L_{l_{2}}=600-480=120 \mathrm{mH}
\end{aligned}
$$

Effective inductance seen from the source is given by,


$$
\begin{aligned}
& L_{A B}=L_{l_{1}}+\left(L_{l_{2}} \| M^{\prime}\right)=L_{l_{1}}+\frac{L_{l_{2}} \times M^{\prime}}{L_{l_{2}}+M^{\prime}} \\
& L_{A B}=320+\frac{120 \times 480}{120+480}=416 \mathrm{mH}
\end{aligned}
$$

Hence, the correct option is (A).

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## $1.66 \quad 1.33$

## Given :

(i) Two winding 200/400 V transformer,
(ii) Supply frequency, $f=50 \mathrm{~Hz}$
(iii) kVA rating $=20 \mathrm{kVA}$
(iv) $200 / 600 \mathrm{~V}$ auto transformer rating with $12 \Omega$ as load.
We form the autotransformer from the two winding transformer as,


Autotransformer transformation ratio,

$$
k=\frac{600}{200}=3
$$

## Method 1

$R$ is transformed to the low voltage side (i.e. primary side) its value becomes,

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$$
\frac{R}{k^{2}}=\frac{12}{9}=1.33
$$

Hence, the value of equivalent load resistance seen from L.V. side is $\mathbf{1 . 3 3} \Omega$.

## Method 2

Using power balance equation,

$$
\begin{align*}
& V_{L} I_{L}=V_{H} I_{H} \\
& 200 \times I_{L}=600 \times I_{H}  \tag{i}\\
& I_{H}=\frac{V_{H}}{R}=\frac{600}{12}=50 \mathrm{~A} \tag{ii}
\end{align*}
$$

From equations (i) and (ii),

$$
I_{L}=\frac{600 \times I_{H}}{200}=\frac{600 \times 50}{200}=150 \mathrm{~A}
$$

Hence, $R_{e q}=\frac{V_{L}}{I_{L}}=\frac{200}{150}=1.33 \Omega$
Hence, the value of equivalent load resistance seen from L.V. side is $\mathbf{1 . 3 3} \Omega$.

## $1.67 \quad 555.55$

## Given :

(i) Two single phase transformer $T_{1}$ and $T_{2}$
(ii) Percentage impedance :

$$
\begin{aligned}
& T_{1} \rightarrow z_{1}=1+j 6, \frac{X_{1}}{R_{1}}=6 \\
& T_{2} \rightarrow z_{2}=0.8+j 4.8, \frac{X_{2}}{R_{2}}=6
\end{aligned}
$$

(iii) Load: $1000 \mathrm{kVA}, 0.8 \mathrm{pf}$ lag

Here, $Z_{p u 2}<Z_{p u 1}$ and both the transformer have same $X / R$ ratio.
$\mathrm{So}, T_{2}$ share more load
Here, $S_{L}=$ load $=1000 \mathrm{kVA}$ at 0.8 lagging power factor.
For parallel operation of $T_{1}$ and $T_{2}$ load shared by transformer $T_{2}$ is given by,

$$
\begin{aligned}
& S_{2}=S_{L} \times \frac{Z_{1}}{Z_{1}+Z_{2}} \\
& S_{2}=\left(1000 \angle-36.87^{0}\right) \times \frac{1+j 6}{1+j 6+0.8+j 4.8}
\end{aligned}
$$

$$
\begin{aligned}
& S_{2}=\left(1000 \angle-36.87^{0}\right) \times \frac{1+j 6}{1.8+j 10.8} \\
& S_{2}=\left(1000 \angle-36.87^{0}\right) \times \frac{1+j 6}{1.8(1+j 6)} \\
& S_{2}=\frac{1000}{1.8} \angle-36.87^{0} \\
& S_{2}=555.55 \angle-36.87^{0}
\end{aligned}
$$

Thus, $S_{2}=555.55 \mathrm{kVA}$ at 0.8 pf lag Hence, the contribution of $T_{2}$ is $\mathbf{5 5 5 . 5 5} \mathbf{~ k V A}$.

### 1.68 (A)

Given :
(i) Three-winding transformer
(ii) Turns : $N_{1}=100, N_{2}=50, N_{3}=50$
(iii) Secondary current, $\bar{I}_{2}=2 \angle 30^{\circ} \mathrm{A}$,
(iv) Tertiary current, $\bar{I}_{3}=2 \angle 150^{\circ} \mathrm{A}$

Figure is shown below,


Since, given transformer is a three winding transformer.
mmf balance equation is given by,

$$
\begin{aligned}
& \bar{I}_{1} N_{1}=\bar{I}_{2} N_{2}+\bar{I}_{3} N_{3} \\
& \bar{I}_{1}=\bar{I}_{2} \frac{N_{2}}{N_{1}}+\bar{I}_{3} \frac{N_{3}}{N_{1}} \\
& \bar{I}_{1}=\frac{50}{100} \times 2 \angle 30+\frac{50}{100} \times 2 \angle 150 \\
& \bar{I}_{1}=1 \angle 90^{\circ} \mathrm{A}
\end{aligned}
$$

Hence, the correct option is (A).

### 1.69 (C)

## Potential transformer (PT) :

(i) The secondary side of a potential transformer is never kept short circuited

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because its secondary has to measure a low potential.
(ii) Load is always connected in parallel with PT.
(iii) It measures voltage hence parallel connection required.
(iv) Moving iron voltmeter commonly use to measure the voltage.

## Current transformer (CT) :



## Fig. Circuit diagram

(i) Secondary side of a current transformer is never kept open circuited because secondary flux reduces to zero and a huge voltage across the secondary winding of the current transformer (CT) but not in case of power transformer and potential transformer.
(ii) Load is always connected in series to CT.
(iii) High frequency transformer cores is generally made of ferrites.
(iv) Moving iron meter is commonly used to measure primary current of a transformer to mains.
(v) A current transformer passes always a constant value of current and will not respond to any changes on secondary side and will be unaffected by secondary side changes.

Hence, the correct option is (C).

## $1.70 \quad$ (B)

Given circuit is shown below,


From figure,
Transformation ratio, $k=\frac{N_{2}}{N_{1}}=\frac{1}{n}$
Port 2 inductance ' $L$ ' is referred to port 1 .
Then, $L$ becomes $\frac{L}{k^{2}}=n^{2} L \quad\left(\right.$ where, $\left.k=\frac{1}{n}\right)$
Hence, equivalent inductance at port 1 is $n^{2} L$.
Hence, the correct option is (B).

## $1.71 \quad 1.44$

Given :
(i) At $\mathbf{4 0 0 V}, 50 \mathrm{~Hz}$ :

$$
P_{i}=5000 \mathrm{~W}, \quad \frac{V}{f}=\frac{400}{50}=8
$$

(ii) At $200 \mathrm{~V}, 25 \mathrm{~Hz}$ :

$$
P_{i}=2000 \mathrm{~W}, \frac{V}{f}=\frac{200}{25}=8
$$

For constant $\frac{V}{f}$ ratio,

$$
P_{e} \propto f \text { and } P_{h} \propto f^{2}
$$

Hence, core loss, $P_{i}=a f+b f^{2}$

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where, $P_{i}=$ Core loss
$a$ and $b=$ Constant
(i) At $\mathbf{4 0 0} \mathbf{V}, 50 \mathrm{~Hz}$ :

$$
\begin{equation*}
P_{i_{1}}=5000=a \times 50+b \times(50)^{2} \tag{i}
\end{equation*}
$$

(ii) At $200 \mathrm{~V}, 25 \mathrm{~Hz}$ :

$$
\begin{equation*}
P_{i_{2}}=2000=a \times 25+b \times(25)^{2} \tag{ii}
\end{equation*}
$$

From equations (i) and (ii),

$$
a=60 \text { and } b=0.8
$$

Hence, at 416 V, 52 Hz ,
Hysteresis loss,

$$
\begin{aligned}
& \frac{V}{f}=\frac{416}{52}=8 \quad(\text { Also constant }) \\
& P_{h}=a f=60 \times 52=3120 \mathrm{~W}
\end{aligned}
$$

Eddy current loss,

$$
P_{e}=b f^{2}=0.8 \times(52)^{2}=2163.2 \mathrm{~W}
$$

Hence, the ratio of $P_{h}$ and $P_{e}$,

$$
\frac{P_{h}}{P_{e}}=\frac{3120}{2163.2}=1.44
$$

Hence, the ratio of hysteresis loss to eddy current loss is $\mathbf{1 . 4 4}$.

### 1.72 (C)

Given :
(i) Single phase $2200 \mathrm{~V} / 220 \mathrm{~V}$ transformer
(ii) kVA rating, $\mathrm{kVA}_{\text {T.w. }}=22 \mathrm{kVA}$
(iii) It is connected as auto transformer with output voltage of 2420 V .

## Method 1

Rated LV current $\left(I_{1}\right)=\frac{22 \times 10^{3}}{220}=100 \mathrm{~A}$
Rated HV current $\left(I_{2}\right)=\frac{22 \times 10^{3}}{2200}=10 \mathrm{~A}$
The equivalent two winding transformer is shown below,
$2200 / 220 \mathrm{~V}$


To get an output of 2420 V by an auto transformer, we have to use additive polarity $(2200+220) \mathrm{V}$.
The equivalent circuit is shown below,


Maximum kVA rating of auto transformer,

$$
V_{L} I_{L}=V_{H} I_{H}=2420 \times 100=242 \mathrm{kVA}
$$

Hence, the correct option is (C).

## Method 2

1. There are two possible connections available,
(i) $220 / 2420 \mathrm{~V}$ autotransformer

$$
(k=1 / 11)=0.09
$$

(ii) $2200 / 2420 \mathrm{~V}$ autotransformer $(k=10 / 11)=0.9$
2. The kVA rating of auto transformer is given by,

$$
(\mathrm{kVA})_{\text {auto }}=\frac{(\mathrm{kVA})_{\mathrm{T} . \mathrm{W} .}}{1-k}
$$

Where, $k$ is the transformation ratio of auto transformer in step down format i.e.

$$
k=\frac{\text { Low voltage }}{\text { High voltage }}
$$

Hence, for maximum rating of autotransformer ' $k$ ' should be close to unity i.e. 2200/2420 V
autotransformer connection will yield maximum kVA rating. Hence,

$$
\begin{aligned}
& (\mathrm{kVA})_{\text {auto }}=\frac{22 \times 10^{3}}{1-2200 / 2420} \\
& (\mathrm{kVA})_{\text {auto }}=\frac{22 \times 10^{3}}{1-10 / 11} \\
& (\mathrm{kVA})_{\text {auto }}=22 \times 11 \times 10^{3}=242 \mathrm{kVA}
\end{aligned}
$$

Hence, the correct option is (C).

## $1.73 \quad 10$

## Given :

(i) Transformation ratio, $K=100$
(ii) Supply frequency, $f=50 \mathrm{~Hz}$

Circuit is shown below,


For an ideal transformer, the equivalent circuit referred to primary side is shown below,


Transformation ratio,

$$
k=\frac{N_{2}}{N_{1}}=\frac{100}{1}=100
$$

From equivalent circuit,

$$
\begin{aligned}
& R^{\prime}=\frac{R}{k^{2}}=\frac{80 \times 10^{3}}{(100)^{2}}=8 \Omega \\
& X_{C}{ }^{\prime}=\frac{X_{C}}{k^{2}}=\frac{40 \times 10^{3}}{(100)^{2}}=4 \Omega
\end{aligned}
$$

Current, $I=\frac{V_{1}}{R^{\prime}+j\left(X_{L}-X_{C}{ }^{\prime}\right)}$

$$
\begin{aligned}
& I=\frac{100 \angle 0^{0}}{8+j(10-4)}=\frac{100 \angle 0^{0}}{8+j 6} \\
& |I|=\frac{100}{\sqrt{64+36}}=\frac{100}{10}=10 \mathrm{~A}
\end{aligned}
$$

Hence, the rms value of the current $I$ is $\mathbf{1 0} \mathbf{A}$.

## $1.74 \quad 6$

## Given :

(i) Single phase $100 \mathrm{~V} / 200 \mathrm{~V}$ transformer
(ii) kVA rating, $(\mathrm{kVA})_{\text {T.w. }}=2 \mathrm{kVA}$
(iii) It is connected as auto transformer such that its rating is maximum.

## Method 1

Rated LV current, $I_{1}=\frac{2 \times 10^{3}}{100}=20 \mathrm{~A}$
Rated HV currents, $I_{2}=\frac{2 \times 10^{3}}{200}=10 \mathrm{~A}$
The equivalent circuit of two winding transformer is shown below,


For getting maximum rating the equivalent auto transformer circuit is reconstructed using above circuit of two winding transformer is shown below,
In auto-transformer,
Primary current, $I_{L}=30 \mathrm{~A}$
Secondary current, $I_{H}=20 \mathrm{~A}$


So, kVA rating of the auto-transformer,
$V_{L} I_{L}=V_{H} I_{H}$
$300 \times 20=6 \mathrm{kVA}$
Hence, the new rating is $\mathbf{6 k V A}$.

## Method 2

1. Here two possible values of $k$ (in step down format).
(i) $k=1 / 3$
(ii) $k=2 / 3$
2. The kVA rating of auto transformer is given by,

$$
(\mathrm{kVA})_{\text {auto }}=\frac{(\mathrm{kVA})_{\text {T.w. }}}{1-k}
$$

Where, $k$ is the transformation ratio of auto transformer in step down format i.e.

$$
k=\frac{\text { Low voltage }}{\text { High voltage }}
$$

Hence, for maximum rating of autotransformer ' $k$ ' should be close to unity i.e. $k=2 / 3$ will yield maximum kVA rating. Hence,

$$
\begin{aligned}
& (\mathrm{kVA})_{\text {auto }}=\frac{(\mathrm{kVA})_{\text {T.w. }}}{1-k} \\
& (\mathrm{kVA})_{\text {auto }}=\frac{2 \times 10^{3}}{1-2 / 3} \\
& (\mathrm{kVA})_{\text {auto }}=2 \times 3=6 \mathrm{kVA}
\end{aligned}
$$

Hence, the new rating is $\mathbf{6 k V A}$.

### 1.75 (A)

Given :
(i) $1-\phi 1000 \mathrm{~V} / 100 \mathrm{~V}$ transformer
(ii) Voltage drop :

$$
\% R=3 \%, \% Z=5 \%
$$

(iii) Power factor, $p f=0.8$ lagging

$$
\% X=\sqrt{Z^{2}-R^{2}}=\sqrt{5^{2}-3^{2}}=4 \%
$$

Since, at full load, $x$ (Fraction of loading) $=1$.
Hence, voltage regulation (V.R.)

$$
\begin{aligned}
\quad & x \times(\% R \cos \phi+\% X \sin \phi) \\
\text { V.R. } & =1(3 \times 0.8+4 \times 0.6)=2.4+2.4=4.8 \%
\end{aligned}
$$

Hence, the correct option is (A).

### 1.76 (D)

Given :


Transformer with toroidal core of permeability $\mu$.
From the given figure,

$$
\begin{aligned}
& r=\text { equivalent radius of core } \\
& A=\pi r^{2}=\text { cross sectional area of core } \\
& R=\text { mean radius of core } \\
& l=2 \pi R=\text { mean length of core }
\end{aligned}
$$

The applied input voltage at primary is,

$$
v_{p}=N_{P} \frac{d \phi}{d t}=N_{P} \frac{d}{d t}\left(\frac{N_{P} i_{P}}{R_{L}}\right)
$$

where, $R_{L}=$ Reluctance

$$
\begin{aligned}
v_{P} & =\frac{N_{P}^{2}}{R_{L}} \frac{d}{d t}(I \sin \omega t) \\
& =\frac{N_{P}^{2}}{R_{L}} \times \omega I \cos \omega t=V \cos \omega t
\end{aligned}
$$

Where, $V=\frac{N_{P}^{2} \omega I}{R_{L}}=\frac{N_{P}^{2} \omega I}{\frac{l}{\mu A}}=\frac{N_{P}^{2} \omega I \mu A}{l}$

$$
=\frac{N_{P}^{2} \omega I \mu \pi r^{2}}{2 \pi R}
$$

Hence, $R=\frac{\mu I r^{2} N_{p}^{2} \omega}{2 V}$
Hence, the correct option is (D).

### 1.77 (D)

## Given :

(i) Rated $\mathrm{KVA}=5 \mathrm{kVA}$
(ii) Rated primary voltage, $E_{1}=50 \mathrm{~V}$
(iii) Rated secondary voltage, $E_{2}=100 \mathrm{~V}$
(iv) Terminal voltage, $\left(V_{t}\right)=95 \mathrm{~V}$

Voltage regulation $=\frac{E_{2}-V_{t}}{E_{2}}=\frac{100-95}{100}=\frac{5}{100}$
Voltage regulation $=0.05=5 \%$
Hence, the correct option is (D).

## $1.78 \quad \mathbf{7 . 2 1}$

Given :
(i) Load $1 \rightarrow 12 \mathrm{~kW}$ at 0.6 pf lag
(ii) Active power $P_{1}=12 \mathrm{~kW}$
(ii) Reactive power $Q_{1}=P_{1} \tan \phi_{1}$

$$
\begin{aligned}
& \phi_{1}=\cos ^{-1}(0.6)=53.13^{0} \\
& Q_{1}=12 \tan \left(53.13^{0}\right)=12 \times \frac{4}{3}=16 \mathrm{kVAR}
\end{aligned}
$$

(iv) Load 2 at unity $\operatorname{pf}\left(Q_{2}=0\right)$
(v) Transformer rating $S_{T}=25 \mathrm{KVA}$

Now $\quad \vec{S}_{T}=\vec{S}_{1}+\vec{S}_{2}$

$$
S_{T}=P_{1}+j Q_{1}+P_{2}+j Q_{2}
$$

$25=12+j 16+P_{2}+0$
$25^{2}=\left(12+P_{2}\right)^{2}+16^{2}$
$\left(12+P_{2}\right)^{2}=625-256$
$12+P_{2}=19.209$
$P_{2}=7.21 \mathrm{~kW}$
Hence, additional load at unity power factor in kW (round off to two decimal places) that may be added before this transformer exceeds its rated kVA is $7.21 \mathbf{k W}$.

## $1.79 \quad 0.835$

Given :
(i) Air gap length, $l_{\text {airgap }}=0.2 \mathrm{~cm}$
(ii) Mean path length, $l_{\text {core }}=40 \mathrm{~cm}$
(iii) Flux density computed in air gap, $B_{1}=1$ tesla
(iv) Core permeability, $\mu_{0}=1000$

Flux density in air gap, $B_{2}=$ ?


We know that,

$$
\begin{aligned}
& \mathrm{mmf}=\text { Ampere turns }=N I \\
& H l=N I=\mathrm{mmf}
\end{aligned}
$$

So, $\phi=\frac{\mathrm{mmf}}{\text { Reluctance }}$
Total reluctance,

$$
S_{T}=S_{\text {core }}+S_{\text {air gap }}
$$

$$
S_{T}=\frac{l_{\text {core }}}{\mu_{0} \cdot \mu_{r} \cdot A}+\frac{l_{\text {air } g a p}}{\mu_{0} \cdot 1 \cdot A}
$$

If each side of core is 10 cm , then total length of the core is 40 cm .
Therefore $l_{\text {core }}=40-0.2=39.8 \mathrm{~cm}$
Case 1: $\quad \operatorname{mmf}=\phi\left[S_{T}\right]$

$$
\begin{equation*}
N I=B_{1} \times A\left[\frac{l_{\text {air }}}{\mu_{0} \cdot A}\right] \tag{i}
\end{equation*}
$$

Case 2 : For same ampere turns

$$
\begin{equation*}
N \cdot I=B_{2} \cdot A\left[\frac{l_{\text {core }}}{\mu_{0} \mu_{r} A}+\frac{l_{\text {air }}}{\mu_{0} \cdot 1 \cdot A}\right] \ldots \tag{ii}
\end{equation*}
$$

From equation (i) and (ii),

$$
B_{1} \cdot A\left[\frac{l_{\text {air }}}{\mu_{0} \cdot A}\right]=B_{2} \cdot A\left[\frac{l_{\text {core }}}{\mu_{0} \mu_{r} A}+\frac{l_{\text {air }}}{\mu_{0} \cdot 1 \cdot A}\right]
$$

(Given $B_{1}=1$ Tesla, $l_{\text {air }}=0.2 \mathrm{~cm}$ )

$$
\begin{aligned}
& 1 \times 0.2=B_{2}\left[\frac{39.8}{1000}+\frac{0.2}{1}\right] \\
& 0.2=B_{2}[0.2+0.0398]
\end{aligned}
$$

$$
\begin{aligned}
& 0.2=B_{2}[0.2398] \\
& B_{2}=\frac{0.2}{0.2398}=0.8348 \mathrm{~T}
\end{aligned}
$$

Hence, the flux density calculated in the air gap is 0.834 T .

## $1.80 \quad 162.5$

Given : $4 \mathrm{kVA}, 200 \mathrm{~V} / 100 \mathrm{~V}$ single phase transformer
(i) $\quad V=200 \mathrm{~V}$
(ii) $f=50 \mathrm{~Hz}$
(iii) $P_{\text {(core) }}=450 \mathrm{~W}$

Case-I : $V_{1}=200 \mathrm{~V}, f_{1}=50 \mathrm{~Hz}$,

$$
\begin{align*}
P_{(\text {core }) 1} & =450 \mathrm{~W} \\
& \frac{V_{1}}{f_{1}}=\frac{200}{50}=4 \\
& P_{(\text {core })_{1}}=A f_{1}+B f_{1}^{2} \\
& 450=50 \mathrm{~A}+2500 \mathrm{~B} \tag{i}
\end{align*}
$$

Case-II : $V_{2}=160 \mathrm{~V}, f_{2}=40 \mathrm{~Hz}$,

$$
\begin{aligned}
P_{(\text {core })_{2}} & =320 \mathrm{~W} \\
\frac{V_{2}}{f_{2}} & =\frac{160}{40}=4
\end{aligned}
$$

$\frac{V}{f}$ is constant

$$
\begin{align*}
& P_{\text {(core }) 2}=A f_{2}+B f_{2}^{2} \\
& 320=40 A+1600 B \tag{ii}
\end{align*}
$$

Solving equation (i) and (ii)

$$
A=4, B=0.1
$$

Case-III : $V_{3}=100 \mathrm{~V}, f_{3}=25 \mathrm{~Hz}, P_{\text {(core) } 3}=$ ?

$$
\begin{aligned}
& P_{(\text {(core }) 3}=A f_{3}+B f_{3}^{2} \\
& P_{(\text {core }) 3}=162.5 \mathrm{~W}
\end{aligned}
$$

## $1.81 \quad 46$

## Given :

(i) $\quad N_{A}=20$
(ii) $\quad N_{B}=20$
(iii) $N_{C}=20$
(iv) $\quad N_{X}=2$

$$
\begin{aligned}
& V_{A}=230 \angle 0^{0} \\
& V_{B}=230 \angle-120^{\circ} \\
& V_{C}=230 \angle 120^{\circ}
\end{aligned}
$$

Voltage across

$$
\begin{aligned}
& =\frac{230}{10} \angle 180+\frac{230}{10} \angle-120+\frac{230}{10} \angle+120 \\
& =-46 \text { Volts }
\end{aligned}
$$

To obtain the no load rms voltage across winding " $X$ " we have to apply the superposition theorem. To obtain the voltage due to single source by short circuiting other two sources the entire circuit will be short circuited hence the voltage across winding " $X$ " will be zero.
By referring the primary side parameter to secondary side the transformer circuit will be modified as follow


Now considering the voltage induced in winding " $X$ " due to $A$ phase, the other two phases sources will be short circuited and the transformer circuit will be modified as follow


But here the instantaneous resultant flux due to all three phases are not considered at the same time which is wrong hence the given question is conceptually wrong.
Hence, IIT Delhi has declared it as MTA question.

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## Power Semiconductor Devices

## 1991 IIT Madras

1.1 The operating state that distinguish a silicon controlled rectifier (SCR) from a diode is
(A)forward conducting state.
(B) forwarding blocking state.
(C) reverse conducting state.
(D) reverse blocking state.
1.2 Match the functions of the following protective elements in SCR applications

| Protective element | SCR rating |
| :--- | :--- |
| (P) snubber | (a) $\frac{d i}{d t}$ limit |
| (Q) heat sink | (b) $\frac{d v}{d t}$ limit |
| (R) series reactor | (c) $i^{2} t$ limit |
| (S) to avoid runway <br> speeds on no-load | (d) junction |
| temperature limit |  |

Code :

|  | $\mathbf{P}$ | $\mathbf{Q}$ | $\mathbf{R}$ | $\mathbf{S}$ |
| :--- | :--- | :--- | :--- | :--- |
| (A) | b | d | a | c |
| (B) | a | c | b | d |
| (C) | a | b | c | d |
| (D) | d | c | b | a |

## 1993 IIT Bombay

1.3 The thermal resistance between the body of a power semiconductor device and the ambient is expressed as
(A) voltage across the device divided by current through the device.
(B) average power dissipated in the device divided by the square of the rms current in the device.
(C) average power dissipated in the device divided by the temperature difference from body to ambient.
(D) temperature difference from body to ambient divided by average power dissipated in the device.

## 1994 IIT Kharagpur

1.4 A triac can be triggered by a gate pulse of $\qquad$ polarity.
1.5 A switched mode power supply operating at 20 kHz to 100 kHz range uses as the main switching element,
(A) Thyristor
(B) MOSFET
(C) Triac
(D) UJT

## 1995 IIT Kanpur

1.6 Figure show two thyristor, each rated 500 A (continuous) sharing a load current. Current through thyristor $y$ is 120 A. The current through thyristor $x$ will be nearly $\qquad$ A.


## 1996 IISc Bangalore

1.7 The Triac can be used in
(A) inverter
(B) rectifier
(C) multi quadrant chopper
(D) ac voltage regulator
1.8 Which semiconductor power device out of the following is not a current triggered device?
(A) Thyristor
(B) G.T.O
(C) Triac
(D) MOSFET
1.9 Which of the following does not cause permanent damage to an SCR?
(A)High current
(B) High rate of rise of current
(C) High temperature rise
(D) High rate of rise of voltage

## 1998 IIT Delhi

1.10 The MOSFET switch in its ON state may be considered equivalent to
(A) resistor
(B) inductor
(C) capacitor
(D) battery
1.11 The uncontrolled electronic switch employed in power electronic converters is
(A) thyristor
(B) bipolar junction transistor
(C) diode
(D) MOSFET
1.12 In a commutation circuit employed to turn-off an SCR, satisfactory turn-off is obtained when
(A) Circuit turn-off time $<$ device turnoff time
(B) Circuit turn-off time $>$ device turnoff time
(C) Circuit time constant > device turnoff time
(D) Circuit time constant < device turnoff time

## 2001 IIT Kanpur

1.13 The main reason for connecting a pulse transformer at the output stage of a thyristor triggering circuit is to
(A)amplifying the power of the triggering pulse.
(B) provide electrical isolation.
(C) reduce the turn on time of the thyristor.
(D) avoid spurious triggering of the thyristor due to noise.

## 2003 IIT Madras

1.14 Figure shows a MOSFET with an integral body diode. It is employed as a power switching device in the ON and OFF states through appropriate control. The ON and OFF states of the switch are given on the $V_{D S}-I_{S}$ plane by

(A)

(B)

(C)

(D)

1.15 Figure shows a thyristor with the standard terminations of anode (A), cathode $(K)$, gate $(G)$ and the different junctions named $J_{1}, J_{2}$ and $J_{3}$. When the thyristor is turned on and conducting

(A) $J_{1}$ and $J_{2}$ are forward biased and $J_{3}$ is reverse biased
(B) $J_{1}$ and $J_{3}$ are forward biased and $J_{2}$ is reverse biased
(C) $J_{1}$ is forward biased and $J_{2}$ and $J_{3}$ are reverse biased.
(D) $J_{1}, J_{2}$ and $J_{3}$ are all forward biased

## 2004 IIT Delhi

1.16 A bipolar junction transistor (BJT) is used as a power control switch by biasing it in the cut-off region (OFF state) or in the saturation region (ON state). In the ON state, for the BJT
(A)both the base-emitter and basecollector junctions are reverse biased
(B) the base-emitter junction is reverse biased, and the base-collector junction is forward biased
(C) the base-emitter junction is forward biased, and the base-collector junction is reverse biased
(D)both the base-emitter and basecollector junctions are forward biased
1.17 A MOSFET rated for 15 A , carries a 10 A periodic current as shown in figure. The ON state resistance of the MOSFET is $0.15 \Omega$. The average ON state loss in the MOSFET is

(A) 33.8 W
(B) 15.0 W
(C) 7.5 W
(D) 3.8 W
1.18 The triggering circuit of a thyristor is shown in figure. The thyristor requires a gate current of 10 mA , for guaranteed turn-on. The value of R required for the thyristor to turn on reliably under all conditions of $V_{b}$ variation is

(A) $10000 \Omega$
(B) $1600 \Omega$
(C) $1200 \Omega$
(D) $800 \Omega$

## 2005 IIT Bombay

1.19 The conduction loss versus device current characteristic of a power MOSFET is best approximated by
(A)A parabola
(B) A straight line
(C) A rectangular hyperbola
(D) An exponentially decaying function
1.20 The figure shows the voltage across a power semiconductor device and the current through the device during a switching transitions. Is the transition a turn ON transition or a turn OFF transition? What is the energy lost during the transition?

(A)Turn ON, $\frac{V I}{2}\left(t_{1}+t_{2}\right)$
(B) Turn OFF, $V I\left(t_{1}+t_{2}\right)$
(C) Turn ON, $V I\left(t_{1}+t_{2}\right)$
(D) Turn OFF, $\frac{V I}{2}\left(t_{1}+t_{2}\right)$
1.21 An electronic switch $S$ is required to block voltages of either polarity during its OFF state as shown in the fig (a). This switch is required to conduct in only one direction during its ON state as shown in the figure (b).

(b)

Which of the following are valid realizations of the switch S ?
(P)

(Q)

(R)

(S)

(A) Only P
(B) $P$ and $Q$
(C) P and R
(D) R and S

## 2006 IIT Kharagpur

1.22 An SCR having a turn ON time of $5 \mu \mathrm{sec}$ latching current of 50 mA and holding current of 40 mA is triggered by a short duration pulse and is used in the circuit shown in fig. The minimum pulse width required to turn the SCR ON will be

(A) $251 \mu \mathrm{sec}$
(B) $150 \mu \mathrm{sec}$
(C) $100 \mu \mathrm{sec}$
(D) $5 \mu \mathrm{sec}$

## Common Data for <br> Questions 1.23 \& 1.24

A 1:1 pulse transformer (PT) is used to trigger the SCR in the adjacent figure. The SCR is rated at $1.5 \mathrm{kV}, 250$ A with $I_{L}=250 \mathrm{~mA}$, $I_{H}=150 \mathrm{~mA}$ and $I_{G \text { max }}=150 \mathrm{~mA}, I_{G \text { min }}=100 \mathrm{~mA}$ The SCR is connected to an inductive load, where $L=150 \mathrm{mH}$ in series with a small resistance and the supply voltage is 200 V dc. The forward drops of all transistors/diodes and gate-cathode junction during ON state are 1.0 V .

1.23 The value of resistance $R$ should be
(A) $4.7 \mathrm{k} \Omega$
(B) $470 \Omega$
(C) $47 \Omega$
(D) $4.7 \Omega$
1.24 The minimum approximate volt-second rating of the pulse transformer suitable for triggering the SCR should be : (volt - second rating is the maximum of product of the voltage and the width of the pulse that may be applied)
(A) $2000 \mu \mathrm{~V}$-s
(B) $200 \mu \mathrm{~V}$-s
(C) $20 \mu \mathrm{~V}$-s
(D) $2.0 \mu \mathrm{~V}-\mathrm{s}$

## 2009 IIT Roorkee

1.25 An SCR is considered to be a semicontrolled device because,
(A) it can be turned OFF but not ON with a gate pulse
(B) it conducts only during one halfcycle of an alternating current wave
(C) it can be turned ON but not OFF with a gate pulse
(D) it can be turned ON only during one half-cycle
1.26 Match the switch arrangements on the top row to the steady-state V-I characteristics on the lower row. The steady state operating points are shown by large black dots.
A.

(I)

B.

(II)

C.

(III)

D.

(IV)

(A)
(B)
(C)
(D)

| A-I | B-II | C-III | D-IV |
| :---: | :---: | :---: | :---: |
| A-II | B-IV | C-I | D-III |
| A-IV | B-III | C-I | D-II |
| A-IV | B-III | C-II | D-I |

## 2010 IIT Guwahati

1.27 Figure shows a composite switch consisting of a power transistor (BJT) in series with a diode. Assuming that the transistor switch and the diode are ideal, the $I-V$ characteristic of the composite switch is

(A)

(B)

(C)

(D)


## 2011 IIT Madras

1.28 Circuit turn-off time of an SCR is defined as the time
(A) taken by the SCR to turn off
(B) required for the SCR current to become zero
(C) for which the SCR is reverse biased by the commutation circuit
(D) for which the SCR is reverse biased to reduce its current below the holding current.

## 2012 IIT Delhi

1.29 The typical ratio of latching current to holding current in a 20 A thyristor is
(A) 5.0 A
(B) 2.0 A
(C) 1.0 A
(D) 0.5 A

## 2014 IIT Kharagpur

1.30 Figure shows four electronic switches (i), (ii), (iii) and (iv). Which of the switches can block voltages of either polarity (applied between terminals "a" and " $b$ ") when the active device is in the OFF state?
[Set - 01]

(i)

(ii)

(iii)

(iv)
(A) (i), (ii) and (iii)
(B) (ii), (iii) and (iv)
(C) (ii) and (iii)
(D) (i) and (iv)
1.31 The SCR in the circuit shown has a latching current of 40 mA . A gate pulse of $50 \mu \mathrm{~s}$ is applied to the SCR. The maximum value of $R$ in $\Omega$ to ensure successful firing of the SCR is
$\qquad$ .
[Set - 02]


## 2016 IISc Bangalore

1.32 The voltage $\left(v_{s}\right)$ across and the current ( $i_{s}$ ) through a semiconductor during a turn-ON transition are shown in figure. The energy dissipated during the turnON transition, in mJ , is $\qquad$ .

1.33 A steady dc current of 100 A is flowing through a power module ( $\mathrm{S}, \mathrm{D}$ ) as shown in Figure (a). The V-I characteristics of the IGBT (S) and the diode (D) are shown in Figures (b) and (c), respectively. The conduction power loss in the power module (S, D), in watts, is $\qquad$ -.
[Set - 01]



V-I characteristic of diode
(c)
[Set - 01]

## 2017 IIT Roorkee

1.34 For the power semiconductor devices IGBT, MOSFET, Diode and Thyristor, which one of the following statements is TRUE?
[Set-01]
(A) All the four are majority carrier devices.
(B) All the four are minority carrier devices.
(C) IGBT and MOSFET are majority carrier devices, whereas Diode and Thyristor are minority carrier devices.
(D) MOSFET is majority carrier device, whereas IGBT, Diode Thyristor are minority carrier devices.

## 2018 IIT Guwahati

1.35 Four power semiconductor devices are shown in the figure along with their relevant terminals. The devices that can carry dc current continuously in the direction shown when gated appropriately is (are)


Thyristor


Triac




MOSFET
(A) Triac only
(B) Triac and MOSFET
(C) Triac and GTO
(D) Thyristor and triac

## 2020 IIT Delhi

1.36 A double pulse measurement for an inductively loaded circuit controlled by

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## Answers Power Semiconductor Devices

| 1.1 | B | 1.2 | A | 1.3 | D | 1.4 | +ve and <br> -ve | 1.5 | B |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.6 | 100 | 1.7 | D | 1.8 | D | 1.9 | D | 1.10 | C |
| 1.11 | C | 1.12 | B | 1.13 | B | 1.14 | B | 1.15 | B |
| 1.16 | D | 1.17 | C | 1.18 | D | 1.19 | A | 1.20 | A |
| 1.21 | C | 1.22 | B | 1.23 | C | 1.24 | $*$ | 1.25 | C |
| 1.26 | C | 1.27 | D | 1.28 | C | 1.29 | B | 1.30 | C |
| 1.31 | 6060 | 1.32 | 75 | 1.33 | 170 | 1.34 | D | 1.35 | B |
| 1.36 | 3 |  |  |  |  |  |  |  |  |

## Explanations Power Semiconductor Devices

## Application of Switches



## 1.1 (B)

Operating states of diode : Forward conduction, backward (reverse) blocking

Operating states of SCR : Forward blocking, forward conduction and reverse blocking
So, "forward blocking state" distinguishes a SCR from a diode.
Hence, the correct option is (B).

## $1.2 \quad$ (A)

(a) Reactor is a inductor that does not allow sudden change in current so for $d i / d t$ protection, series reactor is used.
(b) Switching problem arises due to $d v / d t$ limit and whenever there is a switching problem, a snubber circuit is used.
(c) $i^{2} t$ limit is limited/fixed to avoid runway speed on no load.
(d) Thyristors are mounted on heat sink for thermal protection.

Hence, the correct option is (A).

## 1.3 (D)

The thermal resistance between the body of power semiconductor device and the ambient is expressed as temperature difference from body to ambient divided by average power dissipated in the device.
Hence, the correct option is (D).

### 1.4 Positive and Negative

TRIAC is a three terminal bi-directional semiconductor device and can be triggered by a gate pulse of both positive and negative polarity.
For positive gate pulse triac works in forward conduction mode.
For negative gate pulse triac works in reverse conduction mode.

## Key point



Fig. TRIAC symbol

|  |  |
| :--- | :--- |
| 1.5 | (B) |

Power MOSFET is used in switched mode power supply operating at 20 kHz to 100 kHz range.
Hence, the correct option is (B).

## $1.6 \quad 100$

The thyristor $x$ and thyristor $y$ are connected in parallel. Hence, the voltage across both the thyristors are same.
Let, the current through thyristor $x$ and $y$ are $I_{x}$ and $I_{y}$ respectively, when they are ON .

Thus, $I_{x} \times 0.06=I_{y} \times 0.05$

$$
I_{x}=\frac{I_{y} \times 0.05}{0.06}=\frac{120 \times 0.05}{0.06}=100 \mathrm{~A}
$$

Hence, the current through thyristor $x$ will be nearly $\mathbf{1 0 0 ~ A . ~}$

## 1.7 (D)

Triac is a bidirectional thyristor, which is used for the control of power in ac circuits. i.e. it works as a AC voltage regulator.
Hence, the correct option is (D).

## 1.8 (D)

MOSFET is voltage control device. The gate to source voltage is used to trigger the MOSFET. Hence, the correct option is (D).

## 1.9 (D)

High current : $I^{2} R$ Losses of thyristor are high for high current which can damage the SCR.
High rate of rise of current : It can damage junction $J_{2}$ (inner junction of SCR).
High temperature rise : It can damage whole of SCR.
High rate of rise of voltage : If rate of rise of voltage is high, it can result in false triggering
as $\frac{d V}{d t}$ triggering and would turn ON SCR without gate pulse. But it cannot cause permanent damage of an SCR.
Hence, the correct option is (D).

### 1.10 (C)

MOSFET switch in its ON state is equivalent to capacitor and the value of this capacitance is a function of bias voltage.
Hence, the correct option is (C).

### 1.11 (C)

Diode is an uncontrolled electronic switch because turn ON and turn OFF characteristics of diode depends on the circuit in which it is connected. However, SCR's BJT's and MOSFET's possess controlled characteristics. These are turned ON, when a current or voltage signal is given to their controlled terminal.
Hence, the correct option is (C).

### 1.12 (B)

For satisfactory turn off of an SCR.
Circuit turn off time $>$ device turn off time.
Hence, the correct option is (B).

### 1.13 (B)

Pulse transformer is used for electrical isolation at the output stage of a thyristor triggering circuit.
Hence, the correct option is (B).

### 1.14 (B)



Case I : The antiparallel diode conducts in reverse direction when reverse current flows through diode $D$. Diode is short circuited.
$I_{S}<0$ and $V_{D S}=0$
Case II : When MOSFET is in ON state,

$$
I_{S}>0 \text { and } V_{D S}=0
$$

Case III : When MOSFET is in OFF state, $I_{S}=0$ and $V_{D S}>0$.
Thus, the characteristics of the device are shown below,


Hence, the correct option is (B).

### 1.15 (B)

Given : Forward conduction mode
Anode is positive with respect to cathode. Under forward conduction mode $J_{1}$ and $J_{3}$ are in forward bias and $J_{2}$ will be in reverse bias always. In FCM $J_{2}$ junction breaks down due to gate current.
Hence, the correct option is (B).

### 1.16 (D)

For Bipolar junction transistor (BJT)

| S. <br> No | Base- <br> emitter <br> junction | Base- <br> Collector <br> junction | State |
| :---: | :--- | :--- | :--- |
| 1. | Forward <br> Biased | Forward <br> Biased | ON <br> (Saturation) |
| 2. | Forward <br> Biased | Reversed <br> Biased | Active region <br> (Amplification) |
| 3. | Reversed <br> Biased | Reversed <br> Biased | OFF (cut off) |
| 4. | Reversed <br> Biased | Forward <br> Biased | Inverse Active <br> (Attenuation) |

Hence, the correct option is (D).

### 1.17 (C)

Given :


MOSFET is ON, for $0<\omega t<\pi$
MOSFET is OFF, for $\pi<\omega t<2 \pi$

$$
R_{O N}=0.15 \Omega, I=10 \mathrm{~A}
$$

The rms current, $I_{r m s}$ through the MOSFET is given by,

$$
\begin{aligned}
& I_{r m s}=\sqrt{\frac{1}{2 \pi} \int_{0}^{\pi}(10)^{2} d(\omega t)} \\
& I_{r m s}=\sqrt{\frac{1}{2 \pi} \times 10^{2}[\omega t]_{0}^{\pi}}=5 \sqrt{2} \mathrm{~A}
\end{aligned}
$$

Average ON state power loss in the MOSFET,

$$
\begin{aligned}
& P_{L}=I_{r m s}^{2} \times R=(5 \sqrt{2})^{2} \times 0.15 \\
& P_{L}=7.5 \mathrm{~W}
\end{aligned}
$$

Hence, the correct option is (C).

### 1.18 (D)

Given : $I_{g \text { min }}=10 \mathrm{~mA}$,
Given triggering circuit is shown below,


The variation in triggering voltage is,

$$
V_{b}=12 \pm 4 \mathrm{~V}
$$

Thus the minimum and maximum triggering voltage is, $\left(V_{b}\right)_{\min }=8 \mathrm{~V}$ and $\left(V_{b}\right)_{\text {max }}=16 \mathrm{~V}$.

The thyristor must turn ON even for minimum value of $V_{b}$ then it can turn ON for higher values.
Gate current required by thyristor to turn ON,

$$
\left(I_{g}\right)_{\min }=10 \mathrm{~mA}
$$

Neglecting the gate to source voltage drop

$$
\begin{aligned}
& \left(I_{g}\right)_{\min }=\frac{\left(V_{b}\right)_{\min }}{R} \\
& 10 \times 10^{-3}=\frac{12-4}{R}
\end{aligned}
$$

$$
R=800 \Omega
$$

Hence, the correct option is (D).

### 1.19 (A)

For a power MOSFET,
Let, $\quad I=$ Device current
$R_{O N}=\mathrm{ON}$ state resistance
Conduction loss $P=I^{2} R_{O N}$
Therefore, conduction loss versus device current characteristic can be best approximated by a parabola.
Hence, the correct option is (A).

## $1.20 \quad$ (A)

Given figure is shown below,


From the given figure, voltage $v$ starts decreasing during interval $t_{2}$ and becomes zero and during interval $t_{1}$ current $i$ starts increasing and becomes constant, so power semiconductor device is turning ON.
During $t_{1}$ time interval, $v$ is constant.
Thus, energy loss $E_{1}=\int v i d t$

$$
E_{1}=v \int i d t
$$

Where, $\int i d t=$ Area under $i-t$ curve.

$$
E_{1}=V \times \frac{1}{2} \times I \times t_{1}
$$

During $t_{2}$ time interval, $i$ is constant.
Thus, energy loss $E_{2}=\int v i d t$

$$
E_{2}=i \int v d t
$$

where, $\int v . d t=$ Area under curve $v-t$.

$$
E_{2}=I \times \frac{V}{2} \times t_{2}
$$

Total Energy loss during the transition,

$$
\begin{aligned}
& E=E_{1}+E_{2} \\
& E=\frac{V I}{2} t_{1}+\frac{V I}{2} t_{2} \\
& E=\frac{V I}{2}\left(t_{1}+t_{2}\right)
\end{aligned}
$$

Hence, the correct option is (A).

### 1.21 (C)



1. $\quad$ Switch $S$ blocks voltage of both polarity therefore it can block forward as well as reverse voltage

2. Current through $S$ flows in forward direction only.
From the given realization,
(P)


Thyristor blocks voltage in either polarity until gate is triggered and once it is triggered current flows in forward direction.

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(Q)


When reverse voltage is applied across device, current flows through diode $D$. So, these devices do not satisfy the requirement.
(R)


Transistor in series with diode blocks voltage in either polarity until base signal is applied. Once the base signal is applied, the device conducts in forward direction.
(S)


Transistor in series with a diode blocks voltage in either polarity until base signal is applied and current flows in only forward direction but an antiparallel diode is connected across the series combination of BJT and diode which allows current to flow in reverse direction.
Hence, the correct option is (C).


### 1.22 (B)

Given :


Current through inductor is given by,

$$
\begin{aligned}
& I_{L}=\frac{V}{R_{1}}\left(1-e^{-t \frac{R_{1}}{L}}\right) \\
& I_{L}=\frac{100}{20}\left(1-e^{-\frac{20 t}{0.5}}\right)=5\left(1-e^{-40 t}\right)
\end{aligned}
$$

Current through $5 \mathrm{k} \Omega$ resistor,

$$
\begin{aligned}
& I_{R}=\frac{V}{R_{2}}=\frac{100}{5 \times 10^{3}} \\
& I_{R}=20 \mathrm{~mA}=0.02 \mathrm{~A}
\end{aligned}
$$

Anode current,

$$
I_{A}=I_{L}+I_{R}
$$

The minimum pulse width required to turn ON the SCR will be equal to time required by anode current to reach magnitude of latching current that is,

$$
\begin{aligned}
& I_{A} \geq \text { Latching current } \\
& 0.02+5\left(1-e^{-40 t}\right)=50 \times 10^{-3} \\
& e^{-40 t}=0.994 \\
& -40 t=\ln 0.994 \\
& t=1.50 \times 10^{-4} \mathrm{sec} \\
& t=150 \mu \mathrm{sec}
\end{aligned}
$$

Hence, the correct option is (B).


### 1.23 (C)

Given configuration is shown below,


When switch is ON.


Hence, the correct option is (C).

### 1.24

Given :
(i) $\quad I_{L}=250 \mathrm{~mA}$
(ii) $L=150 \mathrm{mH}$


For turning ON the SCR,

$$
I_{L}=I_{A}
$$

Applying KVL, thyristor current is,

$$
\begin{aligned}
& I_{A}=\left(\frac{199}{R_{L}}\right)\left(1-e^{\frac{-R t}{L}}\right) \\
& 250 \times 10^{-3}=\left(\frac{199}{1}\right)\left(1-e^{\frac{-1 \times t}{150 \times 10^{-3}}}\right) \\
& 1.25628 \times 10^{-3}=1-e^{\frac{-1000 t}{150}} \\
& e^{-6.667 t}=1-1.25628 \times 10^{-3} \\
& e^{-6.667 t}=0.9987 \\
& e^{6.667 t}=\frac{1}{0.9987} \\
& 6.667 t=\ln 1.00125 \\
& t_{p}=188.55 \mu \mathrm{sec}
\end{aligned}
$$

Maximum pulse width [ $\mathrm{V}-\mu \mathrm{sec}$ ] of transformer

$$
=10 \times 188.55=1885.5 \mathrm{~V}-\mu \mathrm{sec}
$$

Which is very close to option (A).

### 1.25 (C)

SCR can be turn ON by applying gate pulse. When SCR is turned ON gate losses control over SCR and it cannot be turned OFF by gate circuit. Hence, SCR is semi controlled device.
Hence, the correct option is (C).

### 1.26 (C)

Device (A) :


When diode is Forward Biased i.e ON,

$$
V_{s}=0, i_{s}>0
$$

When diode is reversed biased i.e OFF,

$$
V_{s}<0, i_{s}=0
$$



Device (B) :


When thyristor is in reverse blocking mode,

$$
V_{s}<0, i_{s}=0
$$

When thyristor is in forward blocking,

$$
V_{s}>0, i_{s}=0
$$

When thyristor is in forward conduction mode,

$$
V_{s}=0, i_{s}>0
$$

## Device (C) :



When device is ON, $V_{s}=0, i_{s}>0$
When device is OFF, $V_{s}>0, i_{s}=0$


Device (D) :


Reverse current can flow through the diode,
So $i_{s}<0$ and $V_{s}=0$
During ON state, $i_{s}>0, V_{s}=0$

During OFF state, $i_{s}=0, V_{s}=0$


Hence, the correct option is (C).


### 1.27 (D)

Given composite switch is shown below,


Case I : When $V>0$, both transistor and diode will be ON
Hence, voltage across device will be zero and current ( $I$ ) will flow through device.
Case II : When $V<0$, both transistor and diode will be OFF. Hence the device will offer infinite resistance so current $I$ will be zero.
Thus $I-V$ characteristic


Hence the correct option is (D).

### 1.28 (C)

The turn OFF time provided to the thyristor by a circuit is called circuit turn OFF time. It is defined as the time for which the SCR is reverse biased by the commutation circuit.
Hence, the correct option is (C).

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### 1.29 (B)

For medium power thyristor, the ratio of latching current to holding current is between 1.5 to 2 A .

Hence, the correct option is (B).

## $1.30 \quad$ (C)

(ii)


Transistor in series with diode blocks voltages of either polarity until base signal is applied. Once the base signal is applied, the device conducts in forward direction.

(iii)


When thyristor is in reverse blocking mode,

$$
V_{s}<0, i_{s}=0
$$

Thyristor is in forward blocking,

$$
V_{s}>0, i_{s}=0
$$

Thyristor is in forward conduction mode,

$$
V_{s}=0, i_{s}>0
$$



Hence, the correct option is (C).

## $1.31 \quad \mathbf{6 0 6 0}$

Given :


Current through inductor is given by,

$$
I_{L}=\frac{V}{R_{1}}\left(1-e^{-t / \tau}\right)
$$

Where $\tau=\frac{L}{R_{1}}=\frac{200 \mathrm{mH}}{500}=400 \mu \mathrm{sec}$

$$
I_{L}=\frac{100}{500}\left(1-e^{-t / 400 \mu \mathrm{sec}}\right)
$$

Gate pulse $t=50 \mu \mathrm{sec}$

$$
\begin{equation*}
I_{L}=\frac{100}{500}\left(1-e^{-50 / 400}\right)=23.5 \mathrm{~mA} \tag{i}
\end{equation*}
$$

Current through $R, I_{R}=\frac{100}{R}$
Latching current, $I_{\text {Latch }}=40 \mathrm{~mA}$
From equation (i), (ii) and (iii),
For successful firing of SCR,

$$
\begin{aligned}
& I_{\text {Latch }}=I_{L}+I_{R} \\
& 40 \times 10^{-3}=23.5 \times 10^{-3}+\frac{100}{R_{\max }} \\
& R_{\max }=6060 \Omega
\end{aligned}
$$

Hence, the maximum value of $R$ is $\mathbf{6 0 6 0} \Omega$.


## M Key point

Latching current : It is the minimum amount of current required to maintain the thyristor in on-state immediately after a thyristor is turned ON.
Holding current : It is a minimum current that is required to maintain the thyristor in onstate not allowing it to turn OFF.

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## $1.32 \quad 75$

Power loss $P_{s}=v_{s} i_{s}$ is


Energy loss during time $T_{1}$,

$$
\begin{aligned}
& E_{1}=\int_{0}^{T_{1}} v i d t=600 \times \int_{0}^{T_{1}} i d t \\
& E_{1}=600 \times \text { Area under } T_{1} \\
& E_{1}=600 \times \frac{1}{2} \times 150 \times 1 \times 10^{-6}=45 \mathrm{~mJ}
\end{aligned}
$$

Energy loss during time $T_{2}$,

$$
\begin{aligned}
& E_{2}=\int_{0}^{T_{2}} v i d t=100 \times \int_{0}^{T_{2}} v d t \\
& E_{2}=100 \times \text { Area under } T_{2} \\
& E_{2}=100 \times \frac{1}{2} \times 600 \times 10^{-6} \times 1=30 \mathrm{~mJ}
\end{aligned}
$$

Total energy loss

$$
E_{1}+E_{2}=45+30=75 \mathrm{~mJ}
$$

Hence, the energy dissipated during the turnON transition is $\mathbf{7 5} \mathbf{~ m J}$.

## $1.33 \quad 170$

Given : V - I characteristic of IGBT and diode is shown below,

V-I characteristic of IGBT
(b)

(c)

Since, both the diode and IGBT are unidirectional devices. As per the direction of
current given in fig (a), diode should be in turn ON to provide path for current to flow.
Since only diode is ON, therefore conduction power loss in power module is,

$$
\begin{aligned}
& P_{L}=V_{o d} \times I+I^{2} \times\left(\frac{d V}{d I}\right) \\
& P_{L}=100 \times 0.7+100^{2} \times 0.01=170 \mathrm{~W}
\end{aligned}
$$

Hence, the conduction power loss in the power module (S, D) is $\mathbf{1 7 0} \mathbf{W}$.

| D | Scan for Video Solution |  |
| :---: | :---: | :---: |
| $\square \square$ Key point |  |  |

IGBT : It is an insulated-gate bipolar transistor (IGBT) is a three terminal power semiconductor device primarily used as an electronic switch which, as it was developed, come to combine high efficiency and fast switching.


### 1.34 (D)

MOSFET is majority carrier device.
IGBT, diode, thyristor are minority carrier device.
Hence, the correct option is (D).

### 1.35 (B)

Given : Power Semiconductor device
(i)


Thyristor

An SCR allows only anode to cathode current. Hence, the given current can not flow through SCR.
(ii)


Triac
A triac is a bidirectional current flow device. Hence, the given current can flow from terminal $M T_{2}$ to terminal $M T_{1}$.
(iii)


GTO
A GTO is a gate turn off thyristor. It is a unidirectional current flow device. It allows only anode to cathode current. Hence, the given current can not flow through GTO.
(iv) Here we can discuss two cases :

Case I : MOSFET without body diode


The given MOSFET is a D-MOSFET and body diode is not mentioned here. Hence, it allows only drain to source current. Hence the given current can not flow through MOSFET.

Case II : A MOSFET with body diode


A practical MOSFET have inherent body diode and a MOSFET with body diode is a bi-directional current conduction device. Hence, the given current can flow through the body diode. GATE 2018 organizing institute in considering this as correct.
Hence, the correct option is (B).

## $1.36 \quad 3$

Given plot of diode current versus time and inductively loaded circuit is shown below,


Considering the load to be highly inductive
$\therefore \quad$ Load current will be constant
Load current $=$ Switch current + Diode current (diode current is considered to evaluate the reverse recovery characteristics of the diode)
So, $\quad I_{\text {IGBT }}=I_{\text {Load }}-I_{\text {Diode }}$
$I_{\text {IGBT }}$ will experience the highest current stress when the switch current will be maximum
Switch current $=($ Load current - Diode current $)$ to be maximum diode current has to be minimum.
Diode current is minimum at point 3 .
Hence, the correct point is (3).


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## Formulas \& Logics

Inductance of Transmission Line :

1. Internal inductance of an isolated current carrying conductor :

Internal inductance for magnetic material,

$$
\left(\mu_{r} \neq 1\right) \Rightarrow L_{\mathrm{int}}=\frac{\lambda_{\mathrm{int}}}{I}=\frac{1}{2} \times 10^{-7} \mu_{r} \mathrm{H} / \mathrm{m}
$$

Internal inductance for non-magnetic material

$$
\left(\mu_{r}=1\right) \Rightarrow L_{\mathrm{int}}=\frac{1}{2} \times 10^{-7} \mathrm{H} / \mathrm{m}
$$

2. External inductance of isolated conductor upto an external point :

$$
L_{\mathrm{ext}}=2 \times 10^{-7} \ln \left(\frac{D}{r}\right) \mathrm{H} / \mathrm{m}=0.2 \ln \left(\frac{D}{r}\right) \mathrm{mH} / \mathrm{km}
$$

where, $r$ : Radius of core conductor, $D$ : Distance upto an external point from centre of core
3. Total inductance due to internal flux and external flux :

For magnetic material $\left(\mu_{r} \neq 1\right)$,

$$
L_{\mathrm{tot}}=2 \times 10^{-7}\left[\frac{\mu_{r}}{4}+\ln \left(\frac{D}{r}\right)\right] \mathrm{H} / \mathrm{m}
$$

For non-magnetic material ( $\mu_{r}=1$ ),

$$
L_{\mathrm{tot}}=2 \times 10^{-7} \ln \left(\frac{D}{r^{\prime}}\right) \mathrm{H} / \mathrm{m}=0.2 \ln \left(\frac{D}{r^{\prime}}\right) \mathrm{mH} / \mathrm{km}
$$

where, $r^{\prime}=r e^{-1 / 4}=0.7788 r, r^{\prime}:$ Geometric Mean Radius (GMR)
Inductive reactance, $X_{L}=\omega L=2 \pi f L \Omega / \mathrm{m}$

## 4. Inductance of single phase two wire line :

Consider a single-phase line consisting of two conductors $a$ and $b$, of equal radius $r$. They are situated at distance $D$ meters.

These conductors carry the same current in opposite directions so that one serves return path for the other.

Total inductance of two wire line,
Loop inductance $=$ Sum of inductance of both conductors

$$
\begin{aligned}
& L_{\text {total }}=L_{\text {interral }}+L_{\text {external }}=L_{\text {Loop }} \\
& L_{\text {Total }}=4 \times 10^{-7} \ln \left(\frac{D}{0.7788 r}\right) \mathrm{H} / \mathrm{m} \\
& L_{\text {Total }}=4 \times 10^{-7} \ln \left(\frac{D}{r^{\prime}}\right) \mathrm{H} / \mathrm{m}
\end{aligned}
$$


where, $r^{\prime}=r e^{-1 / 4}=0.7788 r$
$r^{\prime}$ : Geometric mean radius (GMR)
The inductance of individual conductor is one-half the total inductance of a two-wire line.
5. Inductance of symmetrical three phase line :

A three-phase line is said to be symmetrical when its conductors are situated at the corners of an equilateral triangle.

Let, the spacing between the conductors is $D$, and the radius of each conductor is $r$.

The inductance per conductor of a three phase symmetrically spaced line is,

$$
\begin{aligned}
& L=2 \times 10^{-7} \ln \left(\frac{D}{r^{\prime}}\right) \mathrm{H} / \mathrm{m} \\
& L=0.2 \ln \left(\frac{D}{r^{\prime}}\right) \mathrm{mH} / \mathrm{km}
\end{aligned}
$$



## 6. Inductance of Un-symmetrical :

Transposed Three Phase Line :
The average inductance per phase of a transposed line is,

$$
\begin{aligned}
& L=2 \times 10^{-7} \times \ln \left[\frac{\left(D_{12} D_{23} D_{31}\right)^{1 / 3}}{r^{\prime}}\right] \\
& L=2 \times 10^{-7} \times \ln \left[\frac{D_{e q}}{r^{\prime}}\right] \mathrm{H} / \mathrm{m}
\end{aligned}
$$



The quantity $D_{e q}=D_{m}=\left(D_{12} D_{23} D_{31}\right)^{1 / 3}$ is the geometric mean of the three unequal spacing and is called the equivalent delta spacing or the equivalent equilateral spacing between the conductors of the line.

$$
r^{\prime}=r e^{-1 / 4}=0.7788 r
$$

where, $r^{\prime}$ : Geometric Mean Radius (GMR)
If the conductors are in the same horizontal or vertical plane, the arrangement is known as flat spacing.

From figure, $\quad D_{a b}=D_{b c}=D, D_{c a}=2 D$

$$
D_{a b}=D_{b c}=\frac{1}{2} D_{c a}=D
$$

The equivalent spacing,

$$
D_{e q}=(D \times D \times 2 D)^{1 / 3}=1.26 D
$$




Vertical and horizontal flat spacing of three-phase lines

## Capacitance (Shunt Parameter) of Transmission Line:

## 1. Capacitance of a two wire line :

Consider a single phase line consisting of two conductors $a$ and $b$, of equal radius $r$. They are situated at distance $D$ meters. These conductors carry the same charge in opposite directions.
The line to line capacitance between the conductors is given by,

$$
C_{a b}=\frac{q_{a}}{V_{a b}}=\frac{\pi \varepsilon}{\ln (D / r)} \mathrm{F} / \mathrm{m}
$$

The capacitance to neutral or capacitance to ground is,

$$
C_{n}=\frac{2 \pi \varepsilon}{\ln (D / r)} \mathrm{F} / \mathrm{m}
$$



Two wire line

Capacitive reactance between one conductor and neutral,

$$
X_{c}=\frac{1}{\omega C_{n}}=\frac{1}{2 \pi f C_{n}} \Omega-\mathrm{m}
$$

2. Capacitance of a symmetrical three phase line :

A three phase line is said to be symmetrical when its conductors are situated at the corners of an equilateral triangle.
The line-to-neutral capacitance,

$$
C_{n}=\frac{q_{a}}{V_{a n}}=\frac{2 \pi \varepsilon}{\ln (D / r)} \mathrm{F} / \mathrm{m}
$$


3. Capacitance of un-symmetrical transposed three phase line : The average line to neutral capacitance of a transposed line is,

$$
\begin{aligned}
& C_{n}=\frac{q_{a}}{V_{a n}}=\frac{2 \pi \varepsilon}{\ln \left(\frac{D_{e q}}{r}\right)} \mathrm{F} / \mathrm{m} \\
& C=\frac{2 \pi \varepsilon}{\ln \left(\frac{\sqrt[3]{D_{12} D_{23} D_{31}}}{r}\right)} \mathrm{F} / \mathrm{m}
\end{aligned}
$$



The charging current per phase is, $I_{C}=j \omega C_{N} V_{P}$ Amp
Total reactive volt amperes, $Q_{C}=3 V_{P} I_{C}=\frac{3 V_{P}^{2}}{X_{C}}=\frac{V_{L}^{2}}{X_{C}}$ VAR .

## 1992 IIT Delhi

1.1 The inductance of a power transmission line increases with
(A) decrease in line length.
(B) increase in diameter of conductor.
(C) increase in spacing between the phase conductors.
(D) increase in load current carried by the conductors.

## 1993 IIT Bombay

1.2 A three phase overhead transmission line has its conductors horizontally spaced with spacing between adjacent conductors equal to ' $d$ '. If now the conductors of the line are rearranged to form an equilateral triangle of sides equal to ' $d$ 'then
(A) average capacitance and inductance will increase.
(B) average capacitance will increases and inductance will increase.
(C) average capacitance will increase and inductance will decrease.
(D) surge impedance loading of the line increases.

## 1994 IIT Kharagpur

1.3 The increase in resistance due to nonuniform distribution current in a conductor is known as
(A) Corona effect.
(B) Skin effect.
(C) Both (A) and (B)
(D) None of these

## 1996 IISc Bangalore

1.4 For equilateral spacing of conductors of an un-transposed 3-phase line, we have
(A) balanced receiving end voltage and no communication interference.
(B) unbalance receiving end voltage and no communication interference.
(C) balance receiving end voltage and communication interference.
(D) unbalance receiving end voltage and communication interference.

## 1997 IIT Madras

1.5 Bundling of conductors $\qquad$ .
(A)improves the appearance of the transmission line
(B) improves mechanical stability of the line
(C) decreases system stability
(D) increases the short circuit current
1.6 The velocity of propagation of electromagnetic wave on an underground cable with relative permittivity of 3 will be ___ $\times 10^{8} \mathrm{~m} / \mathrm{sec}$.
1.7 Two power systems $A$ and $B$ each having a regulation $(R)$ of 0.05 p.u. on their respective bases and a stiffness (damping coefficient) of 0.75 p.u. are connected through a tie-line, initially carrying no power. The capacity of system $A$ is 2000 MW and that of system $B$ is 3000 MW . If there is an increase in load of 200 MW in system $A$, the change in the steady state power transfer (in MW) will be $\qquad$ .

## 1999 IIT Bombay

1.8 For a single-phase overhead line having solid copper conductors of diameter 1 cm , spaced 60 cm between centers, the inductance in $\mathrm{mH} / \mathrm{km}$ is
(A) $0.05+0.2 \ln 60$
(B) $0.2 \ln 60$
(C) $0.05+0.2 \ln \left(\frac{60}{0.5}\right)$
(D) $0.2 \ln \left(\frac{60}{0.5}\right)$

## 2001 IIT Kanpur

1.9 The conductors of a 10 km long, single phase, two wire line are separated by a distance of 1.5 m . The diameter of each conductor is 1 cm . If the conductors are of copper, the inductance of the circuit is
(A) 50.0 mH
(B) 45.3 mH
(C) 23.8 mH
(D) 19.6 mH

## 2002 IISc Bangalore

1.10 Consider a long, two-wire line composed of solid round conductors. The radius of both conductors is 0.25 cm and the distance between their centers is $l$ meters. If this distance is doubled, then the inductance per unit length
(A) doubles.
(B) halves.
(C) increase but does not double.
(D) decrease but does not halve.
1.11 A long wire composed of a smooth round conductor runs above parallel to the ground (Assumed to be large conductive plane). A high voltage exists between the conductor and the ground. The maximum electric stress occurs at
(A) the upper surface of the conductor.
(B) the lower surface of the conductor.
(C) the ground surface.
(D) midway between the conductor and the ground.

## 2006 IIT Kharagpur

1.12 A single phase transmission line and a telephone line are both symmetrically strung one below the other, in horizontal configurations, on a common tower. The shortest and longest distance between the phase and telephone conductors are 2.5 m and 3 m respectively. The voltage (volt/km) induced in the telephone circuit, due to 50 Hz current of 100 amps in the power circuit is
(A) 4.81
(B) 3.56
(C) 2.29
(D) 1.27

## 2007 IIT Kanpur

1.13 Consider a bundled conductor of an overhead line consisting of three identical sub-conductors placed at the corners of an equilateral triangle as shown in the figure. If we neglect the charges on the other phase conductors and ground and assume that spacing between sub-conductors is much large than their radius, the maximum electric field intensity is experienced at

(A) point $X$
(B) point $Y$
(C) point $Z$
(D) point $W$

## 2011 IIT Madras

1.14 A nuclear power station of 500 MW capacity is located at 300 km away from a load center. Select the most suitable power evacuation transmission configuration among the following options
(A)


$$
132 \mathrm{kV}, 300 \mathrm{~km} \text { double circuit }
$$

(B)

$132 \mathrm{kV}, 300 \mathrm{~km}$ single circuit with $40 \%$ series capacitor compensation
(C)

(D)

$400 \mathrm{kV}, 300 \mathrm{~km}$ double circuit

## 2014 IIT Kharagpur

1.15 The horizontally placed conductors of a single-phase line operating at 50 Hz are having outside diameter of 1.6 cm , and the spacing between centers of the conductors is 6 m . The permittivity of free space is $8.854 \times 10^{-12} \mathrm{~F} / \mathrm{m}$. The capacitance to ground per kilometer of each line is
[Set-02]
(A) $4.2 \times 10^{-9} \mathrm{~F}$
(B) $8.4 \times 10^{-9} \mathrm{~F}$
(C) $4.2 \times 10^{-12} \mathrm{~F}$
(D) $8.4 \times 10^{-12} \mathrm{~F}$

## 2015 IIT Kanpur

1.16 A composite conductor consists of three conductors of radius $R$ each. The conductors are arranged as shown below. The geometric mean radius (GMR) (in cm ) of the composite conductor is $k R$. The value of $k$ is
[Set - 02]


## 2016 IISc Bangalore

1.17 A single-phase transmission line has two conductors each of 10 mm radius. These are fixed at a center-to-center distance of 1 m in a horizontal plane. This is now converted to a three-phase transmission line by introducing a third conductor of the same radius. This conductor is fixed at an equal distance $D$ from the two single-phase conductors. The threephase line is fully transposed. The positive sequence inductance per phase of the three-phase system is to be $5 \%$ more than that of the inductance per conductor of the single-phase system. The distance D, in meters, is $\qquad$ .
[Set - 01]

## 2017 IIT Roorkee

1.18 Consider an overhead transmission line with 3-phase, 50 Hz balanced system with conductors located at the vertices of an equilateral triangle of length $D_{a b}=D_{b c}=D_{c a}=1 \mathrm{~m}$ as shown in figure below. The resistances of the conductors are neglected. The geometric mean radius (GMR) of each conductor is 0.01 m . Neglecting the effect of ground, the magnitude of positive sequence reactance in $\Omega / \mathrm{km}$ (rounded off to three decimal places) is $\qquad$ .
[Set - 02]


## Answers Parameters of Transmission Line

| 1.1 | C | 1.2 | C \& D | 1.3 | B | 1.4 | A | 1.5 | D |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.6 | 1.732 | 1.7 | 80 | 1.8 | C | 1.9 | C | 1.10 | C |
| 1.11 | B | 1.12 | C | 1.13 | B | 1.14 | D | 1.15 | B |
| 1.16 | 1.913 | 1.17 | 1.439 | 1.18 | 0.289 |  |  |  |  |

## Explanations Parameters of Transmission Line

## $1.1 \quad$ (C)

Inductance per phase of a transmission line is given by,

$$
L_{p h}=0.2 \ln \left(\frac{D}{r^{\prime}}\right) \mathrm{mH} / \mathrm{km}
$$

Total inductance of line,

$$
\begin{align*}
& L_{T}=L_{p h} \times l \\
& L_{T}=0.2 \ln \left(\frac{D}{r^{\prime}}\right) \times l \mathrm{mH} \tag{i}
\end{align*}
$$

where, $D=$ Space between two conductors,

$$
r^{\prime}=0.7788 r
$$

= Effective radius of conductor
$r=$ Radius of conductor
$l=$ Length of line in km
From equation (i), it is clear that the inductance of the power transmission line increases with
(i) Increase in spacing between the phase conductors.
(ii) Decrease in radius of the conductors.
(iii) Increase in length of line.
(iv) Inductance is independent of load current.
Hence, the correct option is (C).

## 1.2 (C) \& (D)

Case 1 : Horizontal spacing
Three conductors of radius $r$ are arranged in flat spacing


Geometric mean distance,

$$
\begin{align*}
& D_{m}=\sqrt[3]{D_{12} \cdot D_{23} \cdot D_{13}} \\
& D_{m}=\sqrt[3]{d \times d \times 2 d}=1.26 d \tag{i}
\end{align*}
$$

## Case 2 : Equilateral spacing

Three conductors of radius $r$ are arranged in equilateral triangle.


New geometric mean distance,

$$
\begin{align*}
& D_{m}^{\prime}=\sqrt[3]{D_{12} \times D_{23} \times D_{13}} \\
& D_{m}^{\prime}=\sqrt[3]{d \times d \times d}=d \tag{ii}
\end{align*}
$$

From equation (i) and (ii), $D_{m}^{\prime}<D_{m}$
For 3- $\phi$ overhead transmission line, average capacitance per phase and average inductance per phase are given by,

$$
\begin{aligned}
C_{p h} & =\frac{2 \pi \varepsilon_{0}}{\ln \left(\frac{D_{m}}{r}\right)} \mathrm{F} / \mathrm{m} \\
L_{p h} & =2 \times 10^{-7} \ln \left(\frac{D_{m}}{r}\right) \mathrm{H} / \mathrm{m} \\
C_{p h} & \propto \frac{1}{\ln \left(\frac{D_{m}}{r}\right)} \text { and } L_{p h} \propto \ln \left(\frac{D_{m}}{r}\right)
\end{aligned}
$$

From flat horizontal spacing ( $D_{m}, C_{p h}, L_{p h}$ ) to equilateral spacing ( $D_{m}{ }^{\prime}, C_{p h}{ }^{\prime}, L_{p h}{ }^{\prime}$ )

$$
\begin{aligned}
& D_{m}^{\prime}<D_{m} \quad \Rightarrow \quad C_{p h}{ }^{\prime}>C_{p h} \\
& L_{p h}^{\prime}<L_{p h}
\end{aligned}
$$

Hence, average capacitance increases and average inductance decreases.
Surge impedance is given by,

$$
Z_{C}=\sqrt{\frac{L_{p h}}{C_{p h}}}
$$

As, average capacitance increases and average inductance decreases, surge impedance decreases an hence surge impedance loading increases.
Hence, the correct option are (C) \& (D).

## $1.3 \quad$ (B)

When a conductor carries a steady current or d.c. current, this current is uniformly distributed over the whole cross-section of the conductor.
When a conductor carries alternating current, the current distance distribution is non-uniform.


The non-uniform distribution of current through the given cross sectional area of the conductor when it is operated on alternating current system is called skin effect. The main reason for skin effect is non-uniform distribution of current and flux linkages.
Due to skin effect the effective cross sectional area offered to the flow of current decreases which increases effective resistance.

$$
R_{A C}=(1.2 \text { to } 1.6) R_{d c}
$$

The electric current flows mainly at the skin of the conductor between the outer surface and a level called the skin depth.
The skin depth $\delta$ depends on following factor :

$$
\delta=\frac{1}{\sqrt{\pi f \mu_{r} \mu_{0} \sigma}}
$$

The skin effect is inversely proportional to skin depth.
Skin effect increases due to increase in frequency, permeability, conductivity, diameter and cross section area of the conductor.

Hence, the correct option is (B).

## 1.4 (A)

|  |  | Line configuration |  |  |
| :---: | :--- | :---: | :---: | :---: |
| S. | Parameters | Symmetrical line <br> (equilateral spacing) <br> Transposed or <br> untransposed | Untransposed <br> Unsymmetrical <br> line | Transposed <br> Unsymmetrica <br> 1 line |
| $\mathbf{1 .}$ | Flux linkage/phase | Same | Unequal | Equal |
| $\mathbf{2 .}$ | Inductance/phase | Same | Unequal | Equal |
| $\mathbf{3 .}$ | Voltage drop/phase | Same | Unequal | Equal |
| $\mathbf{4 .}$ | Receiving end voltage/phase <br> (for balanced sending end <br> voltage and line current) | Same | Unequal | Equal |
| $\mathbf{5 .}$ | Communication interference <br> (in a telephone line running <br> close to power line) | Does not exit due to <br> equilateral spacing | Exists | Eliminated <br> because of <br> transposition |

Hence, the correct option is (A).

## 1.5 (D)

Bundling of conductors increases the Geometric Mean Radius (GMR) keeping Geometric Mean Distance (GMD) constant.
Option (B) : Due to bundling of conductor the overall cross section area of single wire increases and hence, the mechanical strength of the line reduces.

Option (C) : System stability is defined as the maximum power which can be transferred without loss of synchronism $\left(P_{\max }=\frac{V_{S} V_{R}}{X}\right)$. Due to bundling of conductors line inductance and reactance ( $X=2 \pi f L$ ) decreases. As a result $P_{\text {max }}$ increases and hence, the system stability increases.

Option (D) : Short circuit current ( $I_{S C}$ ) is inversely proportional to reactance. As a result of bundling $I_{S C}$ increases.

Hence, the correct option is (D).

## $1.6 \quad 1.732$

Given :
(i) Underground cable
(ii) Relative permittivity, $\varepsilon_{r}=3$

Velocity of wave propagation is given by,

$$
\begin{aligned}
& V_{p}=\frac{1}{\sqrt{\mu_{0} \varepsilon_{0} \varepsilon_{r}}}=\frac{1}{\sqrt{\varepsilon_{r}}} \times \frac{1}{\sqrt{\mu_{0} \varepsilon_{0}}} \\
& V_{p}=\frac{1}{\sqrt{\varepsilon_{r}}} \times c=\frac{3 \times 10^{8}}{\sqrt{\varepsilon_{r}}}=\frac{3 \times 10^{8}}{\sqrt{3}} \\
& V_{p}=\sqrt{3} \times 10^{8}=1.732 \times 10^{8} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Hence, the velocity of propagation of electromagnetic wave is $\mathbf{1 . 7 3 2} \times \mathbf{1 0}^{\mathbf{8}} \mathbf{~ m} / \mathbf{s}$.

## $1.7 \quad 80$

Given :
(i) Power systems $A$ and $B$
(ii) Regulation, $R=0.5 \mathrm{pu}$
(iii) Stiffness, $D=0.75$ pu
(iv) System capacity :
$P_{A}=2000 \mathrm{MW}, P_{B}=3000 \mathrm{MW}$
(v) Change in load, $\Delta P=200 \mathrm{MW}$

Since, regulation and damping coefficient of both the systems are same and hence, change in frequency of system A and system B will be same.
Thus, for same change in frequency the extra power shared will be proportional to the rating i.e.,

$$
\begin{align*}
& \frac{\Delta P_{A}}{\Delta P_{B}}=\frac{P_{A}}{P_{B}}=\frac{2000}{3000} \\
& \Delta P_{A}=\frac{2}{3} \Delta P_{B} \tag{i}
\end{align*}
$$

Also, $\Delta P=\Delta P_{A}+\Delta P_{B}$

$$
\begin{aligned}
& \Delta P=\left(1+\frac{2}{3}\right) \Delta P_{B}=\frac{5}{3} \Delta P_{B} \\
& \Delta P=\frac{5}{3} \Delta P_{B}=200 \\
& \Delta P_{B}=120 \mathrm{MW}
\end{aligned}
$$

From equation (i),

$$
\Delta P_{A}=\frac{2}{3} \times 120=80 \mathrm{MW}
$$

Hence, the change in the steady state power transfer in system $A$ is $\mathbf{8 0} \mathbf{~ M W}$.

## 1.8 (C)

Given :
(i) Single-phase overhead line
(ii) Diameter, $d=1 \mathrm{~cm}$

$$
\text { Radius, } r=\frac{d}{2}=0.5 \mathrm{~cm}
$$

(iii) Spacing between conductor, $D=60 \mathrm{~cm}$ The total inductance of a single conductor is given by,

$$
\begin{aligned}
& L_{\text {tot }}=L_{\text {int }}+L_{\text {ext }} \\
& L_{\text {tot }}=\frac{1}{2} \times 10^{-7}+2 \times 10^{-7} \ln \left(\frac{D}{r}\right) \mathrm{H} / \mathrm{m}
\end{aligned}
$$

Distance of separation, $D=60 \mathrm{~cm}$,

Radius $r=\frac{1}{2} \mathrm{~cm}=0.5 \mathrm{~cm}$

$$
\begin{aligned}
& L=0.5 \times 10^{-7}+2 \times 10^{-7} \ln \left(\frac{60}{0.5}\right) \mathrm{H} / \mathrm{m} \\
& L=0.05+0.2 \ln \left(\frac{60}{0.5}\right) \mathrm{mH} / \mathrm{km}
\end{aligned}
$$

Hence, the correct option is (C).

## $1.9 \quad$ (C)

Given :
(i) Single phase two wire line
(ii) Radius, $r=0.5 \mathrm{~cm}=0.005 \mathrm{~m}$
(iii) Length, $l=10 \mathrm{~km}$,
(iv) Distance between conductors,

$$
D=1.5 \mathrm{~m}=150 \mathrm{~cm}
$$

Radius of each conductor, $r=\frac{d}{2}=\frac{1}{2}=0.5 \mathrm{~cm}$
Loop inductance, $L=4 \times 10^{-7} \ln \left(\frac{D}{r^{\prime}}\right)$

$$
\begin{aligned}
& L=4 \times 10^{-7} \ln \left(\frac{150}{0.7788 \times 0.5}\right) \\
& L=23.81 \times 10^{-7} \mathrm{H} / \mathrm{m}=2.381 \mathrm{mH} / \mathrm{km}
\end{aligned}
$$

Length of line, $l=10 \mathrm{~km}$
Hence, inductance, $L=2.381 \times 10=23.81 \mathrm{mH}$ Hence, the correct option is (C).

## $1.10 \quad$ (C)

## Given :

(i) Single phase two wire line
(ii) Radius, $r=0.25 \mathrm{~cm}$
(iii) Distance between conductors, $D=l$


Loop inductance is given by,

$$
L=4 \times 10^{-7} \ln \left(\frac{D}{0.7788 r}\right) \mathrm{H} / \mathrm{m}
$$

When $D=1 \mathrm{~m}=100 \mathrm{~cm}$,

$$
\begin{aligned}
& L_{1}=4 \times 10^{-7} \ln \left(\frac{100}{0.7788 \times 0.25}\right) \\
& L_{1}=2.5 \mathrm{mH} / \mathrm{km}
\end{aligned}
$$

If distance is doubled, $D=2 \mathrm{~m}=200 \mathrm{~cm}$, then

$$
\begin{aligned}
& L_{2}=4 \times 10^{-7} \ln \left(\frac{200}{0.7788 \times 0.25}\right) \\
& L_{2}=2.77 \mathrm{mH} / \mathrm{km}
\end{aligned}
$$

Thus, the inductance increases does not get doubled.
Hence, the correct option is (C).

### 1.11 (B)

The ground is assumed as large conducting plane.
The electric flux lines on the surface of conductor are radial (i.e. perpendicular to the plane). The electric flux density $(\vec{D})$ is maximum at the lower surface of the conductor and $\vec{E}=\frac{\vec{D}}{\varepsilon_{0}}$.
where, vector field $\vec{D}$ is called the electric flux density and $\varepsilon_{0}=$ Permittivity of air.
Hence, the correct option is (B).

### 1.12 (C)

Given :
(i) Single phase transmission line placed symmetrically near telephone line.
(ii) Short distance, $D_{1}=2.5 \mathrm{~m}$
(iii) Long distance, $D_{2}=3 \mathrm{~m}$
(iv) Current in power circuit, $I=100 \mathrm{~A}$
(v) Operating frequency, $f=50 \mathrm{~Hz}$

Interference with communication line (Power line and telephone line) :
Single phase power line : Consider a single phase overhead transmission system consisting
of two conductors $P_{1}$ and $P_{2}$ and two telephone conductors $T_{1}$ and $T_{2}$ below the power line conductors running on the same supports as shown in figure. The radius of each conductor is $r$.


Currents in power lines are balanced i.e.,

$$
I_{P_{1}}+I_{P_{2}}=0 \text { or } I_{P_{1}}=+I, I_{P_{2}}=-I
$$

Mutual inductance between power line and telephone line is given by,

$$
\begin{aligned}
& M_{P T}=\frac{\lambda_{T}}{I} \\
& M_{P T}=2 \times 10^{-7} \ln \left[\frac{D_{P_{1} T_{2}} D_{P_{2} T_{1}}}{D_{P_{1} T_{1}} D_{P_{2} T_{2}}}\right] \mathrm{H} / \mathrm{m}
\end{aligned}
$$

Voltage induced in the telephone circuit due to current $I$ in power line,

$$
V_{T}=\omega M_{P T} I=\omega \lambda_{T} \mathrm{~V} / \mathrm{m}
$$

If $D_{P_{1} T_{1}}=D_{P_{2} T_{2}}=D_{1}=$ shortest distance between power line and telephone line conductors, and $D_{P_{1} T_{2}}=D_{P_{2} T_{1}}=D_{2}=$ longest distance between power line and telephone line conductors.
Mutual inductance between power line and telephone line is given by,

$$
\begin{aligned}
& M_{P T}=\frac{\lambda_{T}}{I}=4 \times 10^{-7} \ln \left[\frac{D_{2}}{D_{1}}\right] \mathrm{H} / \mathrm{m} \\
& M_{P T}=4 \times 10^{-7} \ln \left(\frac{D_{2}}{D_{1}}\right)=0.4 \ln \left(\frac{3}{2.5}\right) \\
& M_{P T}=0.073 \mathrm{mH} / \mathrm{km} \\
& V_{T}=\omega M_{P T} I \\
& V_{T}=2 \pi \times 50 \times 100 \times 73 \times 10^{-6} \\
& V_{T}=2.29 \mathrm{~V} / \mathrm{km}
\end{aligned}
$$

Hence, the correct option is (C).

### 1.13 (B)

Given circuit is shown below,


$\bigcirc$
The electric field intensity at different points are shown below,
(i) At point $X$ :

(ii) At point $Y$ :

(iii) At point $Z$ :

(iv) At point $W$ :


From observation (i), (ii), (iii) and (iv), it is observed that maximum field intensity is experienced at point $Y$ at the outer surface of a bundled conductor consisting of three identical sub-conductors placed at the corners of an equilateral triangle.
Hence, the correct option is (B).

### 1.14 (D)

Given : Nuclear power station of 500 MW capacity is located at 300 km away from a load center.
A transmission line of length more than 250 km is called long transmission line. The power transmission capability of a line can be enhanced by decreasing the inductance of line and enhancing the voltage level.
For the given system the line is required to transmit a power of 500 MW which is very large. So the voltage level should be high and inductance should be low.
A double circuit line reduces the inductance and also improves the reliability of the line.
This can be achieved by using a 400 kV and double circuit line as shown in figure,

$400 \mathrm{kV}, 300 \mathrm{~km}$ double circuit
Hence, the correct option is (D).

### 1.15 (B)

## Given :

(i) Single-phase line
(ii) Horizontal spacing
(iii) Diameter, $d=1.6 \mathrm{~cm}$

Radius, $r=\frac{d}{2}=0.8 \mathrm{~cm}$
(iii) Spacing between conductor, $D=6 \mathrm{~m}$

Horizontally placed conductors of a singlephase line operating at 50 Hz .


$$
\begin{aligned}
& r=0.8 \mathrm{~cm}, D=6 \mathrm{~m} \\
& \epsilon_{0}=8.854 \times 10^{-12} \mathrm{~F} / \mathrm{m}
\end{aligned}
$$

For single phase line the capacitance to ground $C_{a n} /$ phase is given by,

$$
\begin{aligned}
& C_{a n}=\frac{2 \pi \epsilon_{0} \epsilon_{r}}{\ln \frac{D}{r}}=\frac{2 \pi \times \epsilon_{0} \times 1}{\ln \left(\frac{6}{0.8 \times 10^{-2}}\right)} \\
& C_{a n}=8.404 \times 10^{-12} \mathrm{~F} / \mathrm{m} \\
& C_{a n}=8.404 \times 10^{-9} \mathrm{~F} / \mathrm{km}
\end{aligned}
$$

Hence, the correct option is (B).

\section*{| 1.16 | 1.913 |
| :--- | :--- |}

Given : The geometric mean radius (GMR) (in cm ) of the composite conductor is $k R$.


The geometric mean radius (GMR) of composite conductor is given by,
where, $D_{a a}=D_{b b}=D_{c c}=0.7788 R$

$$
\begin{aligned}
& D_{a b}=D_{a c}=D_{b a}=D_{b c}=D_{c a}=D_{a c}=3 R \\
& D_{s}=\left[(3 R)^{6}(0.7788 R)^{3}\right]^{\frac{1}{9}}=1.913 R
\end{aligned}
$$

Given : GMR $=k R$
Therefore, $k R=1.913 R$

$$
k=1.913
$$

Hence, the value of $k$ is $\mathbf{1 . 9 1 3}$.

## $\begin{array}{ll}1.17 & 1.439\end{array}$

## Given :

(i) Single-phase two wire line
(ii) Radius, $r=10 \mathrm{~mm}$
(iii) Spacing between conductor, $D=1 \mathrm{~m}$
(iv) $\left(\begin{array}{c}\text { Per phase } \\ \text { Inductance } \\ \text { for 3-фline }\end{array}\right)=1.05 \times\left(\begin{array}{c}\text { Per phase } \\ \text { Inductance } \\ \text { for 1- } \phi \text { line }\end{array}\right)$

## Single phase line :



Inductance per conductor of single phase system,

$$
\begin{aligned}
& L_{1}=2 \times 10^{-7} \ln \left(\frac{D}{r^{\prime}}\right) \\
& L_{1}=2 \times 10^{-7} \ln \left(\frac{1000}{0.7788 \times 10}\right) \mathrm{H} / \mathrm{m} \\
& L_{1}=0.971 \mathrm{mH} / \mathrm{km}
\end{aligned}
$$

## Three phase line :



Inductance per conductor,

$$
\begin{aligned}
& L_{2}=5 \% \text { more than } L_{1} \\
& L_{1}=1.05 L_{1}=1.05 \times 0.971 \\
& L_{2}=1.01955 \mathrm{mH} / \mathrm{km} \\
& L_{2}=2 \times 10^{-7} \ln \left(\frac{D_{e q}}{r^{\prime}}\right) \\
& L_{2}=2 \times 10^{-7} \ln \left(\frac{\sqrt[3]{D \times D \times 1000}}{0.7788 \times 10}\right) \mathrm{H} / \mathrm{m}
\end{aligned}
$$

$$
\begin{aligned}
& 1.01955=0.2 \ln \left(\frac{\sqrt[3]{1000 \times D^{2}}}{7.788}\right) \mathrm{mH} / \mathrm{km} \\
& 5.0925=\ln \left(\frac{\sqrt[3]{1000 \times D^{2}}}{7.788}\right) \mathrm{mH} / \mathrm{km} \\
& \frac{\sqrt[3]{1000 \times D^{2}}}{7.788}=e^{5.0925}=162.796
\end{aligned}
$$

On solving, $D=1427.59 \mathrm{~mm}=1.4276 \mathrm{~m}$
Hence, the distance $D$ is $\mathbf{1 . 4 3 9} \mathbf{~ m}$.

## $1.18 \quad 0.289$

Given :
(i) 3-phase overhead transmission line
(ii) Arrangement :


$$
D_{a b}=D_{b c}=D_{c a}=1 \mathrm{~m}
$$

(iii) Operating frequency, $f=50 \mathrm{~Hz}$
(iv) Geometric mean radius (GMR),

$$
D_{s}=0.01 \mathrm{~m}
$$

The inductance per phase is given by,

$$
\begin{aligned}
& L=2 \times 10^{-7} \ln \left(\frac{D_{\text {mutaal }}}{D_{\text {self }}}\right) \mathrm{H} / \mathrm{m} \\
& L=2 \times 10^{-7} \ln \left(\frac{1}{0.01}\right) \\
& L=9.21 \times 10^{-7} \mathrm{H} / \mathrm{m} \\
& L=0.921 \times 10^{-3} \mathrm{H} / \mathrm{km} \\
& X_{L}=2 \pi f L \\
& X_{L}=2 \times 3.14 \times 50 \times 0.921 \times 10^{-3} \Omega / \mathrm{km} \\
& X_{L}=0.289 \Omega / \mathrm{km}
\end{aligned}
$$

Hence, the positive sequence reactance is $\mathbf{0 . 2 8 9}$
$\boldsymbol{\Omega} / \mathbf{k m}$.



## Error Analysis

## \& Measurement

## 1992 IIT Delhi

1.1 A resistance is measured by the voltmeter ammeter method employing d.c. excitation and a voltmeter of very high resistance connected directly across the unknown resistance. If the voltmeter and ammeter readings are subjected to maximum possible errors of $\pm 2.4 \%$ and $\pm 1.0 \%$ respectively, then the magnitude of the maximum possible percentage error in the value of resistance deduced from the measurement is nearly
(A) $1.4 \%$
(B) $1.7 \%$
(C) $2.4 \%$
(D) $3.4 \%$

## 1994 IIT Kharagpur

1.2 A precise measurement guarantees accuracy of the measured quantity. (True/False).

## 1995 IIT Kanpur

1.3 Four ammeters $M_{1}, M_{2}, M_{3}, M_{4}$ with the following specifications are available.

| Instrument | Type | Full <br> scale <br> value <br> (A) | Accuracy <br> \% of FS |
| :---: | :---: | :---: | :---: |
| $M_{1}$ | $31 / 2$ digit dual <br> slope | 20 | $\pm 0.10$ |


| $M_{2}$ | PMMC | 10 | $\pm 0.20$ |
| :---: | :---: | :---: | :---: |
| $M_{3}$ | Electrodynamic | 5 | $\pm 0.50$ |
| $M_{4}$ | Moving iron | 1 | $\pm 1.00$ |

A current of 1 A is to be measured. To obtain minimum error in the reading, one should select meter
(A) $M_{1}$
(B) $M_{2}$
(C) $M_{3}$
(D) $M_{4}$

## 1997 IIT Madras

1.4 In the circuit shown in figure, for measuring resistance ' $R$ ' if the ammeter indicates 1 A and the voltmeter indicates 100 V , then the value of $R$ is $\qquad$ Ohms and the error in measurement using the ratio $V / I$ is $\qquad$ \%.


## 1999 IIT Bombay

1.5 A current of $\left[2+\sqrt{2} \sin \left(314 t+30^{\circ}\right)\right.$ $\left.+2 \sqrt{2} \cos \left(952 t+45^{\circ}\right)\right]$ is measured with a thermocouple type, 5 A full-scale, class- 1 meter. The meter reading would lie in the range
(A) $5 \mathrm{~A} \pm 1 \%$
(B) $(2+3 \sqrt{2}) \mathrm{A} \pm 1 \%$
(C) $3 \mathrm{~A} \pm 1.7 \%$
(D) $2 \mathrm{~A} \pm 2.5 \%$

## 2001 IIT Kanpur

1.6 Resistance $R_{1}$ and $R_{2}$ have, respectively, nominal value of $10 \Omega$ and $5 \Omega$ and tolerances of $\pm 5 \%$ and $10 \%$. The range of values for the parallel combination of $R_{1}$ and $R_{2}$ is
(A) $3.077 \Omega$ to $3.636 \Omega$
(B) $2.805 \Omega$ to $3.371 \Omega$
(C) $3.237 \Omega$ to $3.678 \Omega$
(D) $3.192 \Omega$ to $3.435 \Omega$

## 2006 IIT Kharagpur

1.7 A variable $w$ is related to three other variables $x, y, z$ as $w=\frac{x y}{z}$. The variables are measured with meters of accuracy $\pm 0.5 \%$ reading, $\pm 1 \%$ of full scale value and $\pm 1.5 \%$ reading respectively. The actual readings of the three meters are 80, 20 and 50 with 100 being the full scale value for all three. The maximum uncertainty in the measurement of $w$ will be
(A) $\pm 0.5 \% \mathrm{rdg}$
(B) $\pm 5.5 \% \mathrm{rdg}$
(C) $\pm 6.7 \% \mathrm{rdg}$
(D) $\pm 7.0 \% \mathrm{rdg}$

## 2009 IIT Roorkee

1.8 The measurement system shown in the figure uses three sub-systems in cascade whose gains are specified as $G_{1}, G_{2}$ and $\frac{1}{G_{3}}$. The relative small errors associated
with each respective sub-system as $G_{1}, G_{2}$ and $G_{3}$ are $\varepsilon_{1}, \varepsilon_{2}$ and $\varepsilon_{3}$. The error associated with the output is :

(A) $\varepsilon_{1}+\varepsilon_{2}+\frac{1}{\varepsilon_{3}}$
(B) $\frac{\varepsilon_{1} \cdot \varepsilon_{2}}{\varepsilon_{3}}$
(C) $\varepsilon_{1}+\varepsilon_{2}-\varepsilon_{3}$
(D) $\varepsilon_{1}+\varepsilon_{2}+\varepsilon_{3}$

## 2014 IIT Kharagpur

1.9 Suppose the resistors $R_{1}$ and $R_{2}$ are connected in parallel to give an equivalent resistor $R$. If resistors $R_{1}$ and $R_{2}$ have tolerance of $1 \%$ each, the equivalent resistor $R$ for resistors $R_{1}=300 \Omega$ and $R_{2}=200 \Omega$ will have tolerance of
[Set - 02]
(A) $0.5 \%$
(B) $1 \%$
(C) $1.2 \%$
(D) $2 \%$

## 2015 IIT Kanpur

1.10 A capacitive voltage divider is used to measure the bus voltage $V_{\text {bus }}$ in a highvoltage $50 \mathrm{~Hz}, \mathrm{AC}$ system as shown in the figure. The measurement capacitors $C_{1}$ and $C_{2}$ have tolerances of $\pm 10 \%$ on their nominal capacitance values. If the bus voltage $V_{\text {bus }}$ is $100 \mathrm{kV} \mathrm{rms}$, maximum rms output voltage $V_{\text {out }}$ (in kV ), considering the capacitor tolerance, is $\qquad$ .
[Set - 02]

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## 2017 IIT Roorkee

1.11 Two resistors with nominal resistance values $R_{1}$ and $R_{2}$ have additive uncertainties $\Delta R_{1}$ and $\Delta R_{2}$ respectively. When these resistances are connected in parallel, the standard deviation of the error in the equivalent resistance $R$ is
(A) $\pm \sqrt{\left(\frac{\partial R}{\partial R_{1}} \Delta R_{1}\right)^{2}+\left(\frac{\partial R}{\partial R_{2}} \Delta R_{2}\right)^{2}}$
(B) $\pm \sqrt{\left(\frac{\partial R}{\partial R_{2}} \Delta R_{1}\right)^{2}+\left(\frac{\partial R}{\partial R_{1}} \Delta R_{2}\right)^{2}}$
(C) $\pm \sqrt{\left(\frac{\partial R}{\partial R_{1}}\right)^{2} \Delta R_{2}+\left(\frac{\partial R}{\partial R_{2}}\right)^{2} \Delta R_{1}}$
(D) $\pm \sqrt{\left(\frac{\partial R}{\partial R_{1}}\right)^{2} \Delta R_{1}+\left(\frac{\partial R}{\partial R_{2}}\right)^{2} \Delta R_{2}}$
1.12 The following measurements are obtained on a single phase load : $V=220 \mathrm{~V} \pm 1 \%, \quad I=5.0 \mathrm{~A} \pm 1 \% \quad$ and $W=555 \mathrm{~W} \pm 2 \%$. If the power factor is calculated using these measurements, the worst case error in the calculated power factor in percent is $\qquad$ .
(Give answer up to one decimal place.)

## 2020 IIT Delhi

1.13 A non-ideal Si-based pn junction diode is tested by sweeping the bias applied
across its terminals from -5 V to +5 V . The effective thermal voltage $V_{T}$, for the diode is measured to be $(29 \pm 2) \mathrm{mV}$. The resolution of voltage source in the measurement range is 1 mV . The percentage uncertainty (rounded off to 2 decimal places) in the measured current at a bias voltage of 0.02 V is $\qquad$ .
-

## Answers Error Analysis \& Measurement

| 1.1 | D | 1.2 | False | 1.3 | D | 1.4 | 111.11, <br> $-9.99 \%$ | 1.5 | C |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.6 | A | 1.7 | D | 1.8 | C | 1.9 | B | 1.10 | 12.22 |
| 1.11 | A | 1.12 | 4 | 1.13 | 4.75 |  |  |  |  |

## Explanations Error Analysis \& Measurement

## 1.1 (D)

## Given :

(i) Maximum possible error in voltmeter

$$
= \pm 2.4 \%
$$

(ii) Maximum possible error in ammeter $= \pm 1.0 \%$
In voltmeter ammeter method, the measured value of resistance is given by,

$$
R=\frac{\text { Voltmeter reading }}{\text { Ammeter reading }}=\frac{V}{I}
$$

So, $\quad \% \varepsilon_{R}= \pm\left(\varepsilon_{v}+\varepsilon_{I}\right) \%$
The maximum possible percentage error in resistance is given by,

$$
\begin{aligned}
& \% \varepsilon_{R}(\max )= \pm\left[\varepsilon_{v}(\max )+\varepsilon_{I}(\max )\right] \% \\
& \% \varepsilon_{R}(\max )= \pm(2.4+1.0) \%= \pm 3.4 \%
\end{aligned}
$$

Hence, the correct option is (D).

## $@$ Key Point

If $X_{1}=a+\varepsilon_{r_{1}}, X_{2}=b+\varepsilon_{r_{2}}, X_{3}=c+\varepsilon_{r_{3}}$ where, $\varepsilon_{r_{1}}, \varepsilon_{r_{2}}$ and $\varepsilon_{r_{3}}$ are the error in $X_{1}, X_{2}$ and $X_{3}$
(1) For summation or difference :

$$
X_{e q}=X_{1} \pm X_{2} \pm X_{3}
$$

$\% \varepsilon_{r}\left(\operatorname{in} X_{e q}\right)= \pm\left[\frac{a}{a+b+c} \varepsilon_{r_{1}}+\frac{b}{a+b+c} \varepsilon_{r_{2}}+\right]$

## (2) Multiplication of quantities :

$$
\begin{aligned}
& X_{e q}=X_{1} \times X_{2} \times X_{3} \\
& \% \varepsilon_{r}\left(\text { in } X_{e q}\right)= \pm\left(\varepsilon_{r_{1}}+\varepsilon_{r_{2}}+\varepsilon_{r_{3}}\right) \%
\end{aligned}
$$

(3) Division of quantities :

$$
\begin{aligned}
& X_{e q}=\frac{X_{1} X_{2}}{X_{3}} \\
& \% \varepsilon_{r}\left(\text { in } X_{e q}\right)= \pm\left(\varepsilon_{r_{1}}+\varepsilon_{r_{2}}+\varepsilon_{r_{3}}\right) \%
\end{aligned}
$$

## (4) Error due to composite factor :

$$
\begin{aligned}
& X_{e q}=\frac{X_{1}^{m} X_{2}^{n}}{X_{3}^{p}} \text { or } \frac{X_{1}^{m} X_{3}^{p}}{X_{2}^{n}} \\
& \% \varepsilon_{r}\left(\text { in } X_{e q}\right)= \pm\left(m \varepsilon_{r_{1}}+n \varepsilon_{r_{2}}+p \varepsilon_{r_{3}}\right) \%
\end{aligned}
$$

## $1.2 \quad$ False

Preciseness is a measure of reproducibility of the measurement. It does not guarantee the accuracy.
Hence, the correct answer is false.

## $\square$ Key Point

(1) Accuracy : It indicates the degree of closeness of measured quantity to true value.
(2) Precision : The most repeatable value or reproducible value out of set of record is called precision.

## Note :

(i) Accurate instrument may be précised instrument but precision does not give the guarantee of the accuracy.
(ii) We prefer always accurate as well as precise instrument. Precision characteristic should be supporting characteristic to accuracy.

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## 1.3 (D)

Given :

| Instrument | Type | Full <br> scale <br> value <br> (A) | Accuracy <br> \% of FS |
| :---: | :---: | :---: | :---: |
| $M_{1}$ | $31 / 2$ digit dual <br> slope | 20 | $\pm 0.10$ |
| $M_{2}$ | PMMC | 10 | $\pm 0.20$ |
| $M_{3}$ | Electrodynamic | 5 | $\pm 0.50$ |
| $M_{4}$ | Moving iron | 1 | $\pm 1.00$ |

Since, error in reading $=F S D \times$ Accuracy
Error in reading of meter $M_{1}$ is given by,

$$
\begin{equation*}
=20 \times \frac{ \pm 0.1}{100}= \pm 0.02 \mathrm{~A} \tag{i}
\end{equation*}
$$

Error in reading of meter $M_{2}$ is given by,

$$
\begin{equation*}
=10 \times \frac{ \pm 0.2}{100}= \pm 0.02 \mathrm{~A} \tag{ii}
\end{equation*}
$$

Error in reading of meter $M_{3}$ is given by,

$$
\begin{equation*}
=5 \times \frac{ \pm 0.5}{100}= \pm 0.025 \mathrm{~A} \tag{iii}
\end{equation*}
$$

Error in reading of meter $M_{4}$ is given by,

$$
\begin{equation*}
=1 \times \frac{ \pm 1.00}{100}= \pm 0.01 \mathrm{~A} \tag{iv}
\end{equation*}
$$

From equation (i), (ii), (iii) and (iv), it is clear that meter $M_{4}$ has least error.

Hence, the correct option is (D)


## $\square$ Key Point

(i) $\%$ error at any value

$$
=\frac{\% \text { error at full value }}{\text { Reading (at any value) }} \times \text { Full value }
$$

(ii) If accuracy are given in terms of full scale then,
Error in reading $=F S D \times$ Accuracy (at full scale)

## $1.4 \quad$ 111.11, -9.99\%

(i) Voltmeter reading $=100 \mathrm{~V}$
(ii) Ammeter reading $=1 \mathrm{~A}$

Thus, the measured value of resistance is given by,

$$
=\frac{100 \mathrm{~V}}{1 \mathrm{~A}}=100 \Omega
$$

Current through voltmeter is given by,

$$
=\frac{100 \mathrm{~V}}{1000 \Omega}=0.1 \mathrm{~A}
$$

Given figure is shown below,


Current through resistor, $R=0.9 \mathrm{~A}$
So, true value of resistance, $R$ (actual)

$$
R=\frac{\text { Voltage across } R}{\text { Current through } R}=\frac{100}{0.9}=111.11 \Omega
$$

Thus, \% error $=\frac{\text { Measured } R-\operatorname{True} R}{\operatorname{True} R} \times 100 \%$

$$
=\frac{100-111.11}{111.11} \times 100=-9.99 \%
$$

Hence, the value of $R$ is $\mathbf{1 1 1 . 1 1 ~ \mathbf { o h m s } \text { and the }}$ error in measurement using the ratio $V / I$ is - 9.99\%.

## 1.5 (C)

Given :
(i) $\quad I=\left[2+\sqrt{2} \sin \left(314 t+30^{\circ}\right)\right.$

$$
\left.+2 \sqrt{2} \cos \left(952 t+45^{\circ}\right)\right]
$$

(ii) Full scale reading of thermocouple meter $=5 \mathrm{~A}$
(iii) The meter is the class -1 meter.

Since, thermocouple type instrument measures the r.m.s. value.
Calculation of $I_{r m s}$ :

$$
\begin{aligned}
& I_{r m s}=\sqrt{2^{2}+\left(\frac{\sqrt{2}}{\sqrt{2}}\right)^{2}+\left(\frac{2 \sqrt{2}}{\sqrt{2}}\right)^{2}} \\
& I_{r m s}=\sqrt{4+1+4}=3 \mathrm{~A}
\end{aligned}
$$

## $\square$ Key Point

If any signal in the form of,
$x(t)=A \pm B \sin (\omega t+\phi) \pm C \sin 3(\omega t+\phi) \pm \ldots .$. or
$x(t)=A \pm B \cos \omega t \pm C \cos 3 \omega t \pm \ldots$.
Then,
(i) The average value of $x(t)$ is given by,

$$
x_{\text {avg }}=\text { constant term }=\mathrm{DC} \text { value }=A
$$

(ii) The rms value of $x(t)$ is given by,

$$
x_{r n s}=\sqrt{A^{2}+\left(\frac{B}{\sqrt{2}}\right)^{2}+\left(\frac{C}{\sqrt{2}}\right)^{2}+\ldots \ldots .}
$$

For class 1 meter, error is the $1 \%$ of full scale.
So, for 3 A error is given by,

$$
\% \in_{I}=1 \times \frac{5}{3}=1.67 \%
$$

Thus, the meter reading would lie in the range of $3 \mathrm{~A} \pm 1.67 \%$

Hence, the correct option is (C).

| D | Scan for Video Solution |  |
| :---: | :---: | :---: |

## 1.6 (A)

Given :
(i) Nominal value of resistance
$R_{1}=10 \Omega$ with tolerances of $\pm 5 \%$ i.e.
$R_{1}=10 \pm 5 \%$

So, range of $R_{1}=10 \pm 10 \times \frac{5}{100}$
Thus, range of $R_{1}=9.5 \Omega$ to $10.5 \Omega$.
(ii) Nominal value of resistance
$R_{2}=5 \Omega$ with tolerances of $\pm 10 \%$ i.e.
$R_{2}=5 \pm 10 \%$
So, range of $R_{2}=5 \pm 5 \times \frac{10}{100}$
Thus, range of $R_{2}=4.5 \Omega$ to $5.5 \Omega$.
For parallel combination of $R_{1}$ and $R_{2}$, the equivalent figure is represented as,


The value of $R_{e q}$ when $R_{1}=9.5 \Omega$ and $R_{2}=4.5 \Omega$ is given by,

$$
R_{e q}=\frac{9.5 \times 4.5}{9.5+4.5}=3.05 \Omega
$$

The value of $R_{e q}$ when $R_{1}=10.5 \Omega$ and $R_{2}=5.5 \Omega$ is given by,

$$
R_{e q}=\frac{10.5 \times 5.5}{10.5+5.5}=3.61 \Omega
$$

Thus, the range of values for the parallel combination of $R_{1}$ and $R_{2}$ is $3.05 \Omega$ to $3.61 \Omega$

Hence, the correct option is (A).

## 1.7 (D)

Given :
(i) Four variables $w, x, y$ and $z$ are related as, $w=\frac{x y}{z}$
(ii) Accuracy of meter $x= \pm 0.5 \%$, Accuracy of meter $y= \pm 1 \%$ of full scale,
Accuracy of meter $z= \pm 1.5 \%$
(iii) Actual readings of meter $x, y$ and $z$ are 80, 20 and 50 respectively.
(iv) Full scale value $=100$

## Method 1

Since, uncertainty is given in reading values in options.
Converting all the uncertainties in reading value,
Error at reading value

$$
=\frac{\text { Error at full scale } \times \text { Fullscale value }}{\text { Reading value }}
$$

For $\boldsymbol{x}, \quad \delta x= \pm 0.5 \%$
For $y, \delta y= \pm 1 \% \times \frac{100}{20}= \pm 5 \%$
For $z, \delta z= \pm 1.5 \%$
Therefore,

$$
\begin{aligned}
& \delta w=\delta x+\delta y+\delta z \\
& \delta w= \pm 0.5 \%+ \pm 5 \%+ \pm 1.5 \% \\
& \delta w= \pm 7 \%
\end{aligned}
$$

Hence, the correct option is (D).

$$
\begin{aligned}
& \text { Method } 2 \\
& w=\frac{x y}{z} \\
& \delta x= \pm 0.5 \% \text { of } 80 \text { (reading) } \\
& \delta x= \pm \frac{0.5}{100} \times 80= \pm 0.4 \\
& \delta y= \pm 1 \% \text { of } 100 \text { (Full scale) } \\
& \delta y=\frac{ \pm 1}{100} \times 100=1 \\
& \delta z= \pm 1.5 \% \text { of } 50 \text { (reading) } \\
& \delta z=\frac{ \pm 1.5 \times 50}{100}= \pm 0.75
\end{aligned}
$$

Taking $\log$ on both sides of equation (i),

$$
\begin{aligned}
& \log w=\log x+\log y-\log z \\
& \frac{\delta w}{w}=\frac{\delta x}{x}+\frac{\delta y}{y}-\frac{\delta z}{z}
\end{aligned}
$$

For maximum uncertainty

$$
\begin{aligned}
& \frac{\delta w}{w}= \pm\left(\frac{0.4}{80}+\frac{1}{20}+\frac{0.75}{50}\right) \times 100 \\
& \frac{\delta w}{w}= \pm 7 \%
\end{aligned}
$$

Hence, the correct option is (D).

## 1.8 (C)

Given :
(i) The gains of the given sub-systems are $G_{1}, G_{2}$ and $\frac{1}{G_{3}}$.
(ii) The relative small errors associated with each respective sub-system are $\varepsilon_{1}, \varepsilon_{2}$ and $\varepsilon_{3}$.

Overall gain is given by,

$$
\frac{\text { Output }}{\text { Input }}=\frac{G_{1} G_{2}}{G_{3}}
$$

Thus, relative error is given by,

$$
\frac{\partial G_{1}}{G_{1}}+\frac{\partial G_{2}}{G_{2}}-\frac{\partial G_{3}}{G_{3}}=\varepsilon_{1}+\varepsilon_{2}-\varepsilon_{3}
$$

Hence, the correct option is (C).

## 1.9 (B)

Given :
(i) $R_{1}=300 \Omega, R_{2}=200 \Omega$
(ii) Each have tolerance of $1 \%$

For parallel combination equivalent resistance is given by,

$$
\begin{aligned}
& R_{e q}=\frac{R_{1} R_{2}}{R_{1}+R_{2}} \\
& R_{1} R_{2}=200 \times 300=6 \times 10^{4} \Omega \\
& R_{1}+R_{2}=500 \Omega
\end{aligned}
$$

$$
R_{e q}=120 \Omega
$$

$$
R_{1}=200 \pm 1 \%=(200 \pm 2) \Omega
$$

$$
R_{2}=300 \pm 1 \%=(300 \pm 3) \Omega
$$

Considering case of positive error,

$$
R_{e q}=\frac{202 \times 303}{202+303}=121.2 \Omega
$$

Therefore, percentage error

$$
\% \in_{r}=\frac{121.2-120}{120} \times 100=1 \%
$$

Hence, the correct option is (B).

## $1.10 \quad 12.22$

Given :


Since, capacitors $C_{1}$ and $C_{2}$ have the tolerance of $\pm 10 \%$ i.e.

$$
C_{1}=1 \mu \mathrm{~F} \pm 10 \%, C_{2}=9 \mu \mathrm{~F} \pm 10 \%
$$

So, $\quad C_{1 \max }=1.1 \mu \mathrm{~F}$ and $C_{1 \text { min }}=0.9 \mu \mathrm{~F}$
Similarly,

$$
C_{2 \max }=9.9 \mu \mathrm{~F} \text { and } C_{2 \text { min }}=8.1 \mu \mathrm{~F}
$$

Applying the voltage division rule in the above figure, the output voltage $\left(V_{\text {out }}\right)$ is given by,

$$
V_{\text {out }}=V_{\text {bus }} \times \frac{C_{1}}{C_{1}+C_{2}}
$$

For $V_{\text {out }}$ to be maximum, $C_{1}$ must be maximum and $\left(C_{1}+C_{2}\right)$ must be minimum.
So, $\quad V_{\text {out (max) }}=V_{\text {bus }} \times \frac{C_{1 \max }}{C_{1 \text { min }}+C_{2 \min }}$

$$
V_{\text {out }(\max )}=100 \times \frac{1.1}{0.9+8.1}=12.22 \mathrm{kV}
$$

Hence, the maximum rms output voltage $V_{\text {out }}$, considering the capacitor tolerance is $\mathbf{1 2 . 2 2} \mathbf{k V}$. Note : If in the question asked for the minimum input voltage then,
For $V_{\text {out }}$ to be minimum, $C_{1}$ must be minimum and $\left(C_{1}+C_{2}\right)$ must be maximum

$$
V_{\text {out }(\min )}=100 \times \frac{0.9}{1.1+9.9}=8.18 \mathrm{kV}
$$

### 1.11 (A)

Given : Two resistances $R_{1}$ and $R_{2}$ are connected in parallel. Their equivalent resistance will be $\frac{1}{R}=\frac{1}{R_{1}}+\frac{1}{R_{2}}$. The standard deviation of the error in the equivalent resistance $R$ is,

$$
\pm \sqrt{\left(\frac{\partial R}{\partial R_{1}} \Delta R_{1}\right)^{2}+\left(\frac{\partial R}{\partial R_{2}} \Delta R_{2}\right)^{2}}
$$

Hence, the correct option is (A).

## $1.12 \quad 4$

Given : $V=220 \pm 1 \%, I=5 \pm 1 \%$,

$$
W=555 \pm 2 \%
$$

Since, Power, $P=V I \cos \phi$,
where, power factor $=\cos \phi$,

$$
\begin{aligned}
& W=V I \cos \phi \\
& \text { p.f. }=\frac{W}{V I}=\frac{555 \pm 2 \%}{(220 \pm 1 \%)(5 \pm 1 \%)} \\
& \text { p.f. }=\frac{555}{220 \times 5} \pm 4 \% \\
& \text { p.f. }=0.5 \pm 4 \%
\end{aligned}
$$

Hence, the worst case error in the calculated power factor is $\mathbf{4 \%}$.


## $1.13 \quad 4.75$

Given : $V_{D}=0.02 \mathrm{~V}$

$$
\begin{aligned}
& V_{T}=(29 \pm 2) \mathrm{mV}=(0.029 \pm 0.002) \\
& I_{D}=I \cong I_{0} e^{V_{D} / \mathfrak{n} V_{T}}
\end{aligned}
$$

Non ideal silicon diode is as shown below


Applying $\log$ on both sides on diode current equation

$$
\ln (I)=\ln \left(I_{0}\right)+\frac{V_{D}}{\eta V_{T}}
$$

Differentiating partially with respect to $V_{T}$,

$$
\begin{aligned}
& \frac{\partial I}{I}=0+\frac{V_{D}}{\eta} \times\left(-\frac{1}{V_{T}^{2}}\right) \partial V_{T} \\
& \frac{\partial I}{I}=-\frac{I V_{D}}{\eta V_{T}^{2}} \times \partial V_{T}
\end{aligned}
$$

For $[\eta=1]$,

$$
\frac{\partial I}{\partial V_{T}}=-\frac{I \cdot V_{D}}{V_{T}^{2}}
$$

The expression of uncertainty for diode current equation is given below,

$$
\begin{aligned}
W_{\text {res }} & =W_{I}= \pm \sqrt{\left(\frac{\partial I}{\partial V_{T}}\right)^{2} \cdot W_{v}^{2}}= \pm \frac{\partial I}{\partial V_{T}} \times W_{V} \\
W_{\text {res }}=W_{I}= \pm \frac{I \cdot V_{D}}{V_{T}^{2}} \cdot W_{V} & = \pm \frac{I \times 0.02}{(0.029)^{2}} \times 0.002 \\
& = \pm 0.0475 I
\end{aligned}
$$

$\% \frac{W_{I}}{I}= \pm 0.0475 \times 100= \pm 4.75$
Percentage uncertainty $= \pm 4.75 \%$


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## Electrostatics

## 1992 IIT Delhi

1.1 Which of the following equations represents the Gauss' law in a homogeneous isotropic medium
(A) $\oiint \vec{D} \cdot d \vec{S}=\iiint \rho d v$
(B) $\nabla \times \vec{H}=\vec{D}$
(C) $\nabla \cdot \vec{J}+\rho=0$
(D) $\nabla \cdot \vec{E}=\frac{\rho}{\varepsilon}$
1.2 An electrostatic potential is given by $\phi=2 x \sqrt{y}$ volts in the rectangular coordinate system. The magnitude of the electric field at $x=1 \mathrm{~m}, y=1 \mathrm{~m}$ is
$\qquad$ V/m.

## 1994 IIT Kharagpur

1.3 In electrostatic field $\nabla \times \vec{E} \equiv 0$ [TRUE/FALSE]
1.4 When a charge is given to a conductor
(A) it distributes uniformly all over the surface.
(B) it distributes uniformly all over the volume.
(C) it distributes on the surface, inversely proportional to the radius of curvature.
(D) it stays where it was placed.
1.5 A hollow conductor is at a potential $V$. The potential at any point inside the hollow is $\qquad$
(A) 0
(B) $V$
(C) $>V$
(D) $<V$

## 1995 IIT Kanpur

1.6 A spherical conductor of radius ' $a$ ' with charge ' $q$ ' is placed concentrically inside an uncharged and unearthed spherical conducting shell of inner and outer radii $r_{1}$ and $r_{2}$ respectively. Taking potential to be zero at infinity, the potential at any point $P$ within the shell ( $r_{1}<r<r_{2}$ ) will be

(A) $\frac{q}{4 \pi \varepsilon_{0} r}$
(B) $\frac{q}{4 \pi \varepsilon_{0} a}$
(C) $\frac{q}{4 \pi \varepsilon_{0} r_{2}}$
(D) $\frac{q}{4 \pi \varepsilon_{0} r_{1}}$

## 1996 IISc Bangalore

1.7 If $v, w, q$ stand for voltage, energy and charge, then $v$ can be expressed as
(A) $v=\frac{d q}{d w}$
(B) $v=\frac{d w}{d q}$
(C) $d v=\frac{d w}{d q}$
(D) $d v=\frac{d q}{d w}$
1.8 Inside a hollow conducting sphere
(A) electric field is zero.
(B) electric field is a non-zero constant.
(C) electric field changes with the magnitude of the charge given to the conductor.
(D) electric field changes with distance from the centre of the sphere.

## 1997 IIT Madras

1.9 The capacitance of the arrangement shown in figure is $\qquad$ pF . 20 cm

1.10 The capacitance of an isolated sphere of radius 10 cm in air is equal to $\qquad$ pF .
1.11 The velocity of propagation of electromagnetic wave in an underground cable with relative permittivity of 3 will be $\qquad$ $\times 10^{8} \mathrm{~m} / \mathrm{sec}$.
1.12 In a uniform electric field, field lines and equipotentials
(A) are parallel to one another
(B) intersect at $45^{\circ}$
(C) intersect at $30^{\circ}$
(D) are orthogonal

## 1999 IIT Bombay

1.13 When the plate area of a parallel plate capacitor is increased keeping the capacitor voltage constant, the force between the plates
(A) increase.
(B) decrease.
(C) remains constant.
(D) may increase or decrease depending on the metal making up the plates.

## 2001 IIT Kanpur

1.14 Given the potential function in free space to be $V(x, y, z)=\left(50 x^{2}+50 y^{2}\right.$ $+50 z^{2}$ ) volts, the magnitude (in volts/ metre) and the direction of the electric field at a point $(1,-1,1)$, where the dimensions are in meters are
(A) $100 ;(\hat{i}+\hat{j}+\hat{k})$
(B) $\frac{100}{\sqrt{3}} ;(\hat{i}-\hat{j}+\hat{k})$
(C) $100 \sqrt{3} ;[(-\hat{i}+\hat{j}-\hat{k}) / \sqrt{3}]$
(D) $100 \sqrt{3} ;[(-\hat{i}-\hat{j}-\hat{k}) / \sqrt{3}]$
1.15 The electric field $\vec{E}$ (in volts / metre) at the point $(1,1,0)$ due to a point charge of $+1 \mu \mathrm{C}$ located at $(-1,1,1)$ (coordinates in metres) is
(A) $\frac{10^{-6}}{20 \sqrt{5} \pi \varepsilon_{0}}(2 \hat{i}-\hat{k})$
(B) $\frac{10^{-6}}{20 \pi \varepsilon_{0}}(2 \hat{i}-\hat{k})$
(C) $\frac{-10^{-6}}{20 \sqrt{5} \pi \varepsilon_{0}}(2 \hat{i}-\hat{k})$
(D) $\frac{-10^{-6}}{20 \pi \varepsilon_{0}}(2 \hat{i}-\hat{k})$

## 2003 IIT Madras

1.16 If the electric field intensity is given by $\vec{E}=\left(x \vec{u}_{x}+y \vec{u}_{y}+z \vec{u}_{z}\right) \mathrm{V} / \mathrm{m}$ the potential difference between $x(2,0,0)$ and $y(1$, $2,3)$ is
(A) +1 V
(B) -1 V
$(\mathrm{C})+5 \mathrm{~V}$
$(\mathrm{D})+6 \mathrm{~V}$
1.17 A point of +1 nC is placed in a space with permittivity of $8.85 \times 10^{-12} \mathrm{~F} / \mathrm{m}$ as shown in figure. The potential difference $V_{P Q}$ between two points $P$ and $Q$ at distance of 40 mm and 20 mm respectively from the point charge is

(A) 0.22 kV
(B) -225 V
(C) -2.24 kV
(D) 15 V
1.18 A parallel plate capacitor has an electrode area of $100 \mathrm{~mm}^{2}$, width spacing of 0.1 mm between the electrodes. The dielectric between the plates is air with a permittivity of $8.85 \times 10^{-12} \mathrm{~F} / \mathrm{m}$. The charge on the capacitor is 100 V . The stored energy in the capacitor is
(A) 8.85 pJ
(B) 440 pJ
(C) 22.1 nJ
(D) 44.3 nJ
1.19 A composite parallel plate capacitor is made up of two different dielectric materials with different thicknesses $\left(t_{1}\right.$ and $t_{2}$ ) as shown in figure. The two different dielectric materials are separated by a conducting foil $F$. The voltage of the conducting foil is

(A) 52 V
(B) 60 V
(C) 67 V
(D) 33 V

## 2004 IIT Delhi

1.20 A parallel plate capacitor is shown in figure. It is made of two square metal plates of 400 mm side. The 14 mm space between the plates is filled with two layers of dielectrics of $\varepsilon_{r}=4$, $d=6 \mathrm{~mm}$ thick and $\varepsilon_{r}=2, d=8 \mathrm{~mm}$ thick. Neglecting fringing of field at the edges the capacitance is

(A) 1298 pF
(B) 944 pF
(C) 354 pF
(D) 257 pF

## 2005 IIT Bombay

1.21 If $\vec{E}$ is the electric field intensity, $\nabla \cdot(\nabla \times \vec{E})$ is equal to
(A) $\vec{E}$
(B) $|\vec{E}|$
(C) Null vector
(D) Zero
1.22 The charge distribution in a metal-dielectric-semiconductor specimen is shown in the figure. The negative charge density decreases linearly in the semiconductor as shown. The electric field distribution is shown in below figure.

(A)

(B)

(C)

(D)


## 2006 IIT Kharagpur

1.23 Which of the following statements holds for the divergence of electric and magnetic flux densities?
(A) Both are zero
(B) These are zero for static densities but non zero for time varying densities
(C) It is zero for the electric flux density
(D) It is zero for the magnetic flux density
1.24 Consider the following statements with reference to the equation

$$
\nabla \cdot \bar{J}=-\frac{\partial \rho_{v}}{\partial t}
$$

1. This is a point form of the continuity equation.
2. Divergence of current density is equal to the decrease of volume charge density per unit time at every point.
3. This is Maxwell's divergence equation.
4. This represents the conservation of charge.

Select the correct answer
(A) Only 2 and 4 are true
(B) 1, 2 and 3 are true
(C) 2, 3 and 4 are true
(D) 1, 2 and 4 are true

## 2007 IIT Kanpur

1.25 A solid sphere made of insulating material has a radius $R$ and has total charge $Q$ distributed uniformly in its volume. What is the magnitude of the electric field intensity, $E$, at a distance $r(0<r<R)$ inside the sphere?
(A) $\frac{1}{4 \pi \varepsilon_{0}} \frac{Q r}{R^{3}}$
(B) $\frac{3}{4 \pi \varepsilon_{0}} \frac{Q r}{R^{3}}$
(C) $\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r^{3}}$
(D) $\frac{1}{4 \pi \varepsilon_{0}} \frac{Q R}{r^{3}}$

## Statement for Linked Answer Questions 1.26 \& 1.27

An inductor designed with 400 turns coil wound on an iron core of $16 \mathrm{~cm}^{2}$ cross sectional area and with a cut of an air gap length of 1 mm . The coil is connected to a $230 \mathrm{~V}, 50 \mathrm{~Hz}$ ac supply. Neglect coil resistance, core loss, iron reluctance and leakage inductance.
$\left(\mu_{0}=4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m}\right)$
1.26 The current in the inductor is
(A) 18.08 A
(B) 9.04 A
(C) 4.56 A
(D) 2.28 A
1.27 The average force on the core to reduce the air gap will be
(A) 832.29 N
(B) 1666.22 N
(C) 3332.47 N
(D) 6664.84 N

## 2008 IISc Bangalore

1.28 Two point charges $Q_{1}=10 \mu \mathrm{C}$ and $Q_{2}=20 \mu \mathrm{C}$ are placed at coordinates $(1,1,0)$ and $(-1,-1,0)$ respectively. The total electric flux passing through a plane $z=20$ will be
(A) $7.5 \mu \mathrm{C}$
(B) $13.5 \mu \mathrm{C}$
(C) $15.0 \mu \mathrm{C}$
(D) $22.5 \mu \mathrm{C}$
1.29 A capacitor consists of two metal plates each $500 \times 500 \mathrm{~mm}^{2}$ and spaced 6 mm apart. The space between the metal plates is filled with a glass plate of 4 mm thickness and a layer of paper of 2 mm thickness. The relative permittivity of the glass and paper are 8 and 2 respectively. Neglecting the fringing effect, the capacitance will be (Given that $\varepsilon_{0}=8.85 \times 10^{-12} \mathrm{~F} / \mathrm{m}$ )
(A) 983.3 pF
(B) 1475 pF
(C) 637.5 pF
(D) 9956.25 pF
1.30 The coil of 300 turns is wound on a nonmagnetic core having a mean circumference of 300 mm and a crosssection area of $300 \mathrm{~mm}^{2}$. The inductance of the coil corresponding to a magnetizing current of 3 A will be (Given that $\mu_{0}=4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m}$ )
(A) $37.68 \mu \mathrm{H}$
(B) $113.04 \mu \mathrm{H}$
(C) 37.68 mH
(D) 113.04 mH

## 2011 IIT Madras

1.31 A capacitor is made with a polymeric dielectric having an $\varepsilon_{r}$ of 2.26 and a dielectric breakdown strength of 50 $\mathrm{kV} / \mathrm{cm}$. The permittivity of free space is $8.85 \mathrm{pF} / \mathrm{m}$. If the rectangular plate of the capacitor have a width of 20 cm and a length of 40 cm , then the maximum electric charge in the capacitor is
(A) $2 \mu \mathrm{C}$
(B) $4 \mu \mathrm{C}$
(C) $8 \mu \mathrm{C}$
(D) $10 \mu \mathrm{C}$

## 2012 IIT Delhi

1.32 The direction of vector $A$ is radially outward from the origin, with $|A|=k r^{n}$ where $r^{2}=x^{2}+y^{2}+z^{2}$ and $k$ is a constant. The value of $n$ for which $\nabla \cdot \vec{A}=0$ is
(A) -2
(B) 2
(C) 1
(D) 0

## 2013 IIT Bombay

1.33 A dielectric slab with $500 \mathrm{~mm} \times 500$ mm cross-section is 0.4 m long. The slab is subjected to a uniform electric field of $\vec{E}=6 \vec{a}_{x}+8 \vec{a}_{y} \mathrm{kV} / \mathrm{mm}$. The relative permittivity of the dielectric material is equal to 2 . The value of constant $\quad \varepsilon=2 \times 8.85 \times 10^{-12} \mathrm{~F} / \mathrm{m}$. The energy stored in the dielectric in Joules is
(A) $8.85 \times 10^{-11}$
(B) $8.85 \times 10^{-5}$
(C) 88.5
(D) 885

## 2014 IIT Kharagpur

1.34 $C_{0}$ is the capacitance of a parallel plate capacitor with air as dielectric (as shown in figure (a)). If, half of the entire gap as shown in figure (b) is filled with a dielectric of permittivity $\varepsilon_{r}$, the expression for the modified capacitance is
[Set - 01]

(A) $\frac{C_{0}}{2}\left(1+\varepsilon_{r}\right)$
(B) $\left(C_{0}+\varepsilon_{r}\right)$
(C) $\frac{C_{0}}{2} \varepsilon_{r}$
(D) $C_{0}\left(1+\varepsilon_{r}\right)$
1.35 A parallel plate capacitor consisting two dielectric materials is shown in the figure. The middle dielectric slab is placed symmetrically with respect to the plates.


If the potential difference between one of the plates and the nearest surface of dielectric interface is 2 Volts, then the ratio $\varepsilon_{1}: \varepsilon_{2}$ is
[Set - 02]
(A) $1: 4$
(B) $2: 3$
(C) $3: 2$
(D) $4: 1$
1.36 A perfectly conducting metal plate is placed in $x-y$ plane in a right handed coordinate system.
A charge of $+32 \pi \varepsilon_{0} \sqrt{2}$ columbs is placed at coordinate $(0,0,2) . \varepsilon_{0}$ is the
permittivity of free space. Assume $\hat{i}, \hat{j}, \hat{k}$ to be unit vectors along $x, y$ and $z$ axis respectively. At the coordinate $(\sqrt{2}, \sqrt{2}, 0)$, the electric field vector $\vec{E}$ (Newtons/Columb) will be [Set - 03]

(A) $2 \sqrt{2} \hat{k}$
(B) $-2 \hat{k}$
(C) $2 \hat{k}$
(D) $-2 \sqrt{2} \hat{k}$
1.37 A hollow metallic sphere of radius $r$ is kept at potential of 1 volt. The total electric flux coming out of the concentric spherical surface of radius $R(>r)$ is
[Set - 03]
(A) $4 \pi \varepsilon_{0} r$
(B) $4 \pi \varepsilon_{0} r^{2}$
(C) $4 \pi \varepsilon_{0} R$
(D) $4 \pi \varepsilon_{0} R^{2}$

## 2015 IIT Kanpur

1.38 Consider a function $\vec{f}=\frac{1}{r^{2}} \vec{r}$, where $r$ is the distance from the origin and $\vec{r}$ is the unit vector in the radial direction. The divergence of this function over a sphere of radius $R$, which includes the origin, is
[Set - 01]
(A) 0
(B) $2 \pi$
(C) $4 \pi$
(D) $R \pi$
1.39 A parallel plate capacitor is partially filled with glass of dielectric constant 4.0 as shown below. The dielectric strengths of air and glass are $30 \mathrm{kV} / \mathrm{cm}$

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and $300 \mathrm{kV} / \mathrm{cm}$, respectively. The maximum voltage (in kilovolts), which can be applied across the capacitor without any breakdown, is $\qquad$ .

1.40 Two semi-infinite dielectric regions are separated by a plane boundary at $y=0$. The dielectric constants of region $1(y<$ $0)$ and region $2(y>0)$ are 2 and 5 , respectively. Region 1 has uniform electric field $\vec{E}=3 \vec{a}_{x}+4 \vec{a}_{y}+2 \vec{a}_{z}$, where $\vec{a}_{x}, \vec{a}_{y}$ and $\vec{a}_{z}$ are unit vectors along the $x, y$ and $z$ axes, respectively. The electric field in region 2 is
[Set - 02]
(A) $3 \vec{a}_{x}+1.6 \vec{a}_{y}+2 \vec{a}_{z}$
(B) $1.2 \vec{a}_{x}+4 \vec{a}_{y}+2 \vec{a}_{z}$
(C) $1.2 \vec{a}_{x}+4 \vec{a}_{y}+0.8 \vec{a}_{z}$
(D) $3 \vec{a}_{x}+10 \vec{a}_{y}+0.8 \vec{a}_{z}$
1.41 Two semi-infinite conducting sheets are placed at right angles to each other as shown in the figure. A point charge of + $Q$ is placed at a distance of $d$ from both sheets. The net force on the charge is $\frac{Q^{2}}{4 \pi \varepsilon_{0}} \frac{K}{d^{2}}$, where $K$ is given by [Set - 02]

(A) 0
(B) $-\frac{1}{4} \hat{i}-\frac{1}{4} \hat{j}$
(C) $-\frac{1}{8} \hat{i}-\frac{1}{8} \hat{j}$
(D) $\frac{1-2 \sqrt{2}}{8 \sqrt{2}} \hat{i}+\frac{1-2 \sqrt{2}}{8 \sqrt{2}} \hat{j}$
1.42 Match the following :
[Set - 02]
P. Stoke's Theorem
Q. Gauss's Theorem
R. Divergence Theorem
S. Cauchy's Integral Theorem

1. $\oiint \vec{D} \cdot d \vec{S}=Q$
2. $\oint f(z) d z=0$
3. $\iiint(\nabla \cdot \vec{A}) d v=\oiint A \cdot d \vec{S}$
4. $\iint(\nabla \times \vec{A}) \cdot d \vec{S}=\oint \vec{A} \cdot d \vec{l}$

P $\mathbf{Q} \mathbf{R} \mathbf{S}$
(A) $2 \quad 1 \quad 4 \quad 3$
(B) $4 \quad 1 \quad 3 \quad 2$
(C) $4 \begin{array}{lll}3 & 1 & 2\end{array}$
(D) $3 \quad 4 \quad 2 \quad 1$

## 2016 IISc Bangalore

1.43 In cylindrical coordinate system, the potential produced by a uniform ring charge is given by $\phi=f(r, z)$, where $f$ is a continuous function of $r$ and $z$. Let $\vec{E}$ be the resulting electric field. Then the magnitude of $\nabla \times \vec{E}$
[Set - 01]
(A) increases with $r$
(B) is 0
(C) is 3
(D) decreases with $z$
1.44 Two electric charges $q$ and $-2 q$ are placed at $(0,0)$ and $(6,0)$ on the $x-y$ plane. The equation of the zero equipotential curve in the $x-y$ plane is
[Set - 01]
(A) $x=-2$
(B) $y=2$
(C) $x^{2}+y^{2}=2$
(D) $(x+2)^{2}+y^{2}=16$
1.45 A parallel plate capacitor filled with two dielectrics is shown in the figure below. If the electric field in the region A is 4 $\mathrm{kV} / \mathrm{cm}$, the electric field in the region B , in $\mathrm{kV} / \mathrm{cm}$, is
[Set - 02]

(A) 1
(B) 2
(C) 4
(D) 16
1.46 Two electrodes, whose cross-sectional view is shown in the figure below, are at the same potential. The maximum electric field will be at the point

(A) A
(C) C

(B) B
(D) D

## 2017 IIT Roorkee

1.47 Consider an electron, a neutron and a proton initially at rest and placed along a straight line such that the neutron is exactly at the center of the line joining the electron and proton. At $t=0$, the particles are released but are constrained to move along the same straight line. Which of these will collide first?
(A)the particles will never collide
(B) all will collide together
(C) proton and neutron
(D) electron and neutron
1.48 A thin soap bubble of radius, $R=1 \mathrm{~cm}$ and thickness $a=3.3 \mu \mathrm{~m}(a \ll R)$, is at a potential of 1 V with respect to a reference point at infinity. The bubble bursts and becomes a single spherical drop of soap (assuming all the soap is contained in the drop) of radius $r$. The volume of the soap in the thin bubble is $4 \pi R^{2} a$ and that of the drop is $\frac{4}{3} \pi r^{3}$. The potential in volts, of the resulting single spherical drop with respect to the same reference point at infinity is
$\qquad$ . (Give the answer up to two
decimal places).
[Set - 02]

1.49 The figures show diagrammatic representations of vector fields $\vec{X}, \vec{Y}$ and $\vec{Z}$ respectively. Which one of the following choices is true?

(A) $\nabla \cdot \vec{X}=0, \nabla \times \vec{Y} \neq 0, \nabla \times \vec{Z}=0$
(B) $\nabla \cdot \vec{X} \neq 0, \nabla \times \vec{Y}=0, \nabla \times \vec{Z} \neq 0$
(C) $\nabla \cdot \vec{X} \neq 0, \nabla \times \vec{Y} \neq 0, \nabla \times \vec{Z} \neq 0$
(D) $\nabla \cdot \vec{X}=0, \nabla \times \vec{Y}=0, \nabla \times \vec{Z}=0$
1.50 Consider a solid sphere of radius 5 cm made of a perfect electric conductor. If one million electrons are added to this sphere, these electrons will be distributed
(A) Uniformly over the entire volume of the sphere
(B) Uniformly over the outer surface of the sphere
(C) Concentrated around the centre of the sphere
(D) Along a straight line passing through the centre of the sphere

## 2018 IIT Guwahati

1.51 A positive charge of 1 nC is placed at $(0,0,0.2)$ where all dimensions are in metres. Consider the $x-y$ plane to be a conducting ground plane.
Take $\varepsilon_{0}=8.85 \times 10^{-12} \mathrm{~F} / \mathrm{m}$.
The $Z$ component of the $E$ field at $(0,0$, 0.1 ) is closest to
(A) $899.18 \mathrm{~V} / \mathrm{m}$
(B) $-899.18 \mathrm{~V} / \mathrm{m}$
(C) $999.09 \mathrm{~V} / \mathrm{m}$
(D) - $999.09 \mathrm{~V} / \mathrm{m}$
1.52 The capacitance of an air-filled parallelplate capacitor is 60 pF . When a dielectric slab whose thickness is half the distance between the plates, is placed on one of the plates covering it entirely, the capacitance becomes 86 pF . Neglecting the fringing effects, the relative permittivity of the dielectric is
$\qquad$ (up to 2 decimal places).

## 2019 IIT Madras

1.53 A co-axial cylindrical capacitor shown in Figure (i) has dielectric with relative permittivity $\varepsilon_{r 1}=2$. When one-fourth portion of the dielectric is replaced with
another dielectric of relative permittivity $\varepsilon_{r 2}$, as shown in Figure (ii), the capacitance is doubled. The value of $\varepsilon_{r 2}$
is $\qquad$ -.


Fig. (i)


Fig. (ii)

## 2020 IIT Delhi

1.54 Let $\hat{a}_{r}, \hat{a}_{\phi}, \hat{a}_{z}$ be unit vector along $r, \phi$ and $z$ direction respectively in the cylindrical coordinate system. For the electric flux density given by $\vec{D}=\left(15 \hat{a}_{r}+2 r \hat{a}_{\phi}-3 r z \hat{a}_{z}\right) \mathrm{C} / \mathrm{m}^{2}$, the total electric flux, in Coulomb emanating from the volume enclosed by solid cylinder of radius 3 m and height 5 m oriented along the $z$-axis with its base at the origin is
(A) $54 \pi$
(B) $180 \pi$
(C) $90 \pi$
(D) $108 \pi$
1.55 The static electric field inside a dielectric medium with relative permittivity, $\varepsilon_{r}=2.25$, expressed in cylindrical coordinate system is given by the following expression

$$
\mathbf{E}=\mathbf{a}_{r} 2 r+\mathbf{a}_{\phi}\left(\frac{3}{r}\right)+\mathbf{a}_{z} 6
$$

where $\mathbf{a}_{r}, \mathbf{a}_{\phi}, \mathbf{a}_{z}$ are unit vectors along $r, \phi$ and $z$ directions, respectively. If the above expression represents a valid electrostatic field inside the medium, then the volume charge density
associated with this field in terms of free space permittivity $\varepsilon_{0}$ ，in SI units is given by
（A） $3 \varepsilon_{0}$
（B） $5 \varepsilon_{0}$
（C） $4 \varepsilon_{0}$
（D） $9 \varepsilon_{0}$

## ※ぬが

## Answers Electrostatics

| 1.1 | A，D | 1.2 | 2.236 | 1.3 | True | 1.4 | A | 1.5 | B |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.6 | C | 1.7 | B | 1.8 | A | 1.9 | 4.72 | 1.10 | 11.111 |
| 1.11 | 1.732 | 1.12 | D | 1.13 | A | 1.14 | C | 1.15 | A |
| 1.16 | C | 1.17 | B | 1.18 | D | 1.19 | B | 1.20 | D |
| 1.21 | D | 1.22 | A | 1.23 | D | 1.24 | D | 1.25 | A |
| 1.26 | D | 1.27 | A | 1.28 | C | 1.29 | B | 1.30 | B |
| 1.31 | C | 1.32 | A | 1.33 | C | 1.34 | A | 1.35 | C |
| 1.36 | B | 1.37 | A | 1.38 | C | 1.39 | 18.75 | 1.40 | A |
| 1.41 | D | 1.42 | B | 1.43 | B | 1.44 | D | 1.45 | C |
| 1.46 | A | 1.47 | D | 1.48 | 10.04 | 1.49 | C | 1.50 | B |
| 1.51 | D | 1.52 | 2.53 | 1.53 | 10 | 1.54 | B | 1.55 | C |

## Explanations Electrostatics

## $1.1 \quad$（A）and（D）

Gauss law ：The electric flux passing through any closed surface is equal to the total charge in the volume enclosed by that surface．

$$
\begin{align*}
\psi & =Q_{e n c} \\
\psi & =\oint_{S} \vec{D} \cdot d \vec{S} \tag{i}
\end{align*}
$$

where，$\vec{D}=$ Electric flux density， $\mathrm{C} / \mathrm{m}^{2}$
Total charge，$Q=\int_{v} \rho d v$
where，$\rho=$ volume charge density， $\mathrm{C} / \mathrm{m}^{3}$
From equation（i）and（ii），we get

$$
\oint \vec{D} \cdot d \vec{S}=\int_{v} \rho d v
$$

According to Maxwell＇s $1^{\text {st }}$ equation

$$
\begin{array}{ll}
\nabla \cdot \vec{D}=\rho_{v} & \\
\nabla \cdot \vec{E}=\frac{\rho}{\varepsilon} & {[\vec{D}=\varepsilon \vec{E}]}
\end{array}
$$

Hence，the correct options are（A）and（D）．

## $1.2 \quad 2.236$

Given :
Electrostatic potential, $\phi=2 x \sqrt{y}$ Volts
Electric field intensity is given by,

$$
\begin{align*}
& \vec{E}=-\nabla \phi=-\left[\frac{\partial \phi}{\partial x} \vec{a}_{x}+\frac{\partial \phi}{\partial y} \vec{a}_{y}\right] \\
& \vec{E}=-\left[\frac{\partial(2 x \sqrt{y})}{\partial x} \vec{a}_{x}+\frac{\partial(2 x \sqrt{y})}{\partial y} \vec{a}_{y}\right] \\
& \vec{E}=-\left[2 \sqrt{y} \vec{a}_{x}+\frac{x}{\sqrt{y}} \vec{a}_{y}\right] \quad \ldots(i) \tag{i}
\end{align*}
$$

At the point $x=1 \mathrm{~m}, y=1 \mathrm{~m}$,

$$
\begin{aligned}
& E_{x}=-2, E_{y}=-1 \\
& \vec{E}=-\left[2 \vec{a}_{x}+1 \vec{a}_{y}+0 \vec{a}_{z}\right]
\end{aligned}
$$

Magnitude of electric field intensity is given by,

$$
|\vec{E}|=E=\sqrt{2^{2}+1^{2}}=2.236 \mathrm{~V} / \mathrm{m}
$$

Hence, the magnitude of the electric field is $\mathbf{2 . 2 3 6}$ V/m.

### 1.3 True

$\oint \vec{E} \cdot d \vec{l}$ gives the potential difference around a closed path $C$, which is zero in electrostatic field.

By Stoke's theorem,

$$
\oint_{C} \vec{E} \cdot d \vec{l}=\int_{S}(\nabla \times \vec{E}) \cdot d \vec{S}
$$

$\nabla \times \vec{E}=0$ in electrostatic field i.e. electric field is conservative in nature.

Hence, the given statement is true.

## $1.4 \quad$ (A)

Given : A charge is given to a conductor.

When a charge is given to a conductor, it distributes uniformly all over the surface as the charge cannot reside inside the conductor for electrostatic condition.

Hence, the correct option is (A).

## 1.5 (B)

Given : A hollow conductor is at a potential $V$.
Conductor surface is an equipotential surface, hence everywhere on the surface potential will be $V$. Charge inside a hollow conductor is zero and by Gauss's Law the electric field inside the hollow conductor would also be zero. The electric field is related to electric potential as,

$$
E=-\frac{d V}{d r}
$$

Due to zero electric field, the potential gradient is zero and hence the potential remains same at all points inside the conductor.

For electric field to be zero inside hollow conductor sphere potential difference between a point on surface and inside conductor must be same i.e. $V$.

Hence, the correct option is (B).

## 1.6 (C)

## Given :

(i) A spherical conductor of radius ' $a$ ' with charge ' $q$ ' is placed concentrically inside an uncharged and unearthed spherical conducting shell of inner and outer radii $r_{1}$ and $r_{2}$ respectively,

(ii) The potential is zero at infinity

When charge on inner conductor is $+q$. The inner surface of conducting shell has induced charge equal to $-q$ and outer surface equal to $+q$. The total charge enclosed in a spherical Gaussian surface outside the shell is $+q$. The potential at distance ' $r$ ' from centre, outside the shell, can be given by,

$$
V=-\int_{\infty}^{r} \bar{E} \cdot \bar{d} r
$$



The electric field at distance ' $r$ ' is obtained by applying Gauss's Law as,

$$
\begin{aligned}
& \bar{E}=\frac{1}{4 \pi \epsilon_{0}} \times \frac{q}{r^{2}} \vec{a}_{r} \\
& V=-\int_{\infty}^{r} \frac{1}{4 \pi \varepsilon_{0}} \times \frac{q}{r^{2}} \vec{a}_{r} d r \vec{a}_{r} \\
& V=-\int_{\infty}^{r} \frac{1}{4 \pi \varepsilon_{0}} \times \frac{q}{r^{2}} d r=-\int_{\infty}^{r} \frac{1}{4 \pi \varepsilon_{0}} \times \frac{q}{r^{2}} d r \\
& V=\frac{q}{4 \pi \varepsilon_{0}} \times \frac{1}{r}
\end{aligned}
$$

At $r=r_{2}$, the value of $V$ is given by,

$$
V=\frac{q}{4 \pi \varepsilon_{0}} \times \frac{1}{r_{2}}
$$

Electric field inside the conducting shell for $r_{1}<r<r_{2}$, is zero because the charge enclosed by a Gaussian surface with $r_{1}<r<r_{2}$ is zero. So the electric potential is constant.

$$
V=\frac{q}{4 \pi \varepsilon_{0}} \times \frac{1}{r_{2}}
$$

Hence, the correct option is (C).

## $1.7 \quad$ (B)

Given : $v, w, q$ stand for voltage, energy and charge respectively.
The above quantities are related as, $v=\frac{d w}{d q}$
i.e. electric potential difference is defined as work done per unit test charge in bringing a charge from one point to another point.
Hence, the correct option is (B).

## $1.8 \quad$ (A)

The potential of a shell is same in its interior and surface. For a shell of radius $R$ with charge $Q$ potential on the surface or any point inside is given by

$$
\begin{aligned}
& V \propto \frac{Q}{R} \\
& E=-\nabla V
\end{aligned}
$$

But $V$ is constant. So, $E=0$ inside hollow conducting sphere.
Hence, the correct option is (A).

\section*{| 1.9 | 4.72 |
| :--- | :--- |}

Given :


From figure, $d_{1}=d_{2}=10 \mathrm{~cm}, d_{3}=20 \mathrm{~cm}$,

$$
\begin{aligned}
& \varepsilon_{1}=1, \varepsilon_{2}=2 \text { and } \varepsilon_{3}=4 \\
& A=(20 \times 20) \mathrm{cm}^{2}, \varepsilon_{0}=\frac{1}{36 \pi \times 10^{9}} \mathrm{~F} / \mathrm{m}
\end{aligned}
$$

Let, $C_{1}, C_{2}$ and $C_{3}$ be the capacitances due to dielectrics with $\varepsilon_{1}, \varepsilon_{2}$ and $\varepsilon_{3}$ respectively.
Voltage between the plates is divided between $C_{1}$ and $C_{2}$ and voltage across $C_{3}$ is same as the voltage between the plates.
Therefore, the given capacitance structure may be identified as series combination of $C_{1}$ and $C_{2}$, in parallel with $C_{3}$.
For parallel plate capacitor, capacitance is given by,

$$
\begin{aligned}
& C=\frac{\varepsilon_{0} A}{d} \\
& C_{1}=\frac{\varepsilon_{0} \varepsilon_{1}\left(\frac{A}{2}\right)}{d_{1}}, C_{2}=\frac{\varepsilon_{0} \varepsilon_{2}\left(\frac{A}{2}\right)}{d_{2}} \text { and } \\
& C_{3}=\frac{\varepsilon_{0} \varepsilon_{3}\left(\frac{A}{2}\right)}{d_{3}}
\end{aligned}
$$

The given dielectric arrangement can be treated as a combination of three capacitors.

$$
\begin{aligned}
& C_{e q}=\frac{\frac{1}{C_{1}}=\frac{C_{2}}{\frac{1}{C_{1}}+\frac{1}{C_{2}}}+C_{3}}{C_{e q}}=\frac{\frac{\varepsilon_{0} \varepsilon_{1}\left(\frac{A}{2}\right)}{d_{1}} \times \frac{\varepsilon_{0} \varepsilon_{2}\left(\frac{A}{2}\right)}{\varepsilon_{0} \varepsilon_{1}\left(\frac{A}{2}\right)}}{\frac{\varepsilon_{0} \varepsilon_{2}\left(\frac{A}{2}\right)}{d_{1}}+\frac{\varepsilon_{0} \varepsilon_{3}\left(\frac{A}{2}\right)}{d_{2}}}+\frac{d_{3}}{C_{0} \varepsilon_{1} \varepsilon_{2}\left(\frac{A}{2}\right)} \\
& C_{e q}= \\
& \frac{d_{1} d_{2}}{\frac{\varepsilon_{1}}{d_{1}}+\frac{\varepsilon_{2}}{d_{2}}}+\frac{A}{d_{3}}
\end{aligned}
$$

$$
\begin{aligned}
& C_{e q}= \frac{\frac{10^{-9}}{36 \pi} \times 1 \times 2\left(\frac{20 \times 20 \times 10^{-4}}{2}\right)}{10 \times 10^{-2} \times 10 \times 10^{-2}} \\
& \frac{1}{10 \times 10^{-2}}+\frac{2}{10 \times 10^{-2}} \\
&+\frac{\frac{10^{-9}}{36 \pi} \times 4\left(\frac{20 \times 20 \times 10^{-4}}{2}\right)}{20 \times 10^{-2}}
\end{aligned} \quad \begin{aligned}
& C_{e q}=4.72 \mathrm{pF}
\end{aligned}
$$

Hence, the capacitance of the arrangement shown in figure is 4.72 pF .


## $1.10 \quad 11.11$

Given : An isolated sphere of radius 10 cm ,
Capacitance of isolated sphere of radius ( $r=10$ cm ) is given by,

$$
\begin{aligned}
& C=4 \pi \varepsilon a=\frac{4 \pi}{36 \pi \times 10^{9}} \times \frac{10}{100} \mathrm{~F} \\
& C=\frac{1}{9 \times 10^{10}} \mathrm{~F}=\frac{100}{9} \mathrm{pF}=11.11 \mathrm{pF}
\end{aligned}
$$

Hence, the capacitance of an isolated sphere of


## $\begin{array}{ll}1.11 & 1.732\end{array}$

Given : $\varepsilon_{r}=3$
Velocity of propagation of electromagnetic wave in an underground cable is given by,

$$
\begin{aligned}
& v=\frac{1}{\sqrt{\mu \varepsilon}}=\frac{1}{\sqrt{\mu_{0} \varepsilon_{0} \varepsilon_{r}}} \\
& v=\frac{3 \times 10^{8}}{\sqrt{3}} \mathrm{~m} / \mathrm{s}=\sqrt{3} \times 10^{8} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Hence, the velocity of propagation of electromagnetic wave in an underground cable is $\mathbf{1 . 7 3 2} \mathbf{~ m} / \mathrm{s}$.

### 1.12 (D)

Given : Uniform electric field,
Field lines i.e., direction of $\vec{E}$ fields at any point is orthogonal or normal to the equipotential surface in the direction of decreasing potentials.
Hence, the correct option is (D).

### 1.13 (A)

Given : The plate area of a parallel plate capacitor is increased keeping the capacitor voltage constant.
Capacitance of parallel plate capacitor is given by,

$$
C=\frac{\varepsilon A}{d}
$$

Force between plates of parallel plate capacitor is given by,

$$
\begin{aligned}
& F=E \cdot Q=\frac{V}{d} \cdot Q \\
& F=\frac{V}{d} \cdot(C V) \\
& F=\frac{V^{2}}{d} \cdot \frac{\varepsilon \cdot A}{d} \\
& F=\frac{V^{2}}{d^{2}} \varepsilon \cdot A \\
& F \propto A
\end{aligned}
$$

Thus, force between the plates increases as area increases.
Hence, the correct option is (A).

## $1.14 \quad$ (C)

Given: $V(x, y, z)=50 x^{2}+50 y^{2}+50 z^{2}$
Electric field is given by,

$$
\begin{aligned}
\vec{E} & =-\nabla V=-\frac{\partial V}{\partial x} \hat{i}-\frac{\partial V}{\partial y} \hat{j}-\frac{\partial V}{\partial z} \hat{k} \\
\vec{E} & =-100 x \hat{i}-100 y \hat{j}-100 z \hat{k}
\end{aligned}
$$

At point ( $1,-1,1$ ),

$$
\vec{E}=-100 \hat{i}+100 \hat{j}-100 \hat{k}
$$

Magnitude of electric field is given by,

$$
|\vec{E}|=\sqrt{100^{2}+100^{2}+100^{2}}=100 \sqrt{3}
$$

Direction of electric field is given by,

$$
\hat{E}=\frac{\vec{E}}{|\vec{E}|}=\frac{1}{\sqrt{3}}(-\hat{i}+\hat{j}-\hat{k})
$$

Hence, the correct option is (C).

### 1.15 (A)

Given : $B\left(x_{2}, y_{2}, z_{2}\right)=B(1,1,0)$ and

$$
A\left(x_{1}, y_{1}, z_{1}\right)=A(-1,1,1)
$$



Displacement vector is given by,

$$
\begin{aligned}
\vec{r} & =\overrightarrow{A B}=[1-(-1)] \vec{a}_{x}+(1-1) \vec{a}_{y}+(0-1) \vec{a}_{z} \\
\vec{r} & =2 \vec{a}_{x}+0 \vec{a}_{y}-1 \vec{a}_{z}
\end{aligned}
$$

Magnitude is given by,

$$
r=|\vec{r}|=A B=\sqrt{2^{2}+0^{2}+1^{2}}=\sqrt{5}
$$

Unit vector is given by,

$$
\hat{a}_{r}=\frac{a_{r}}{\left|\vec{a}_{r}\right|}=\frac{2 \vec{a}_{x}-\vec{a}_{z}}{\sqrt{4+1}}=\frac{1}{\sqrt{5}}\left(2 \vec{a}_{x}-\vec{a}_{z}\right)
$$

Electric field is given by,

$$
\begin{aligned}
\vec{E} & =\frac{Q}{4 \pi \varepsilon r^{2}} \hat{a}_{r} \\
\vec{E} & =\frac{10^{-6}}{4 \pi \varepsilon_{0} \times 5} \frac{1}{\sqrt{5}}\left(2 \vec{a}_{x}-\vec{a}_{z}\right) \mathrm{V} / \mathrm{m} \\
\vec{E} & =\frac{10^{-6}}{20 \sqrt{5} \pi \varepsilon_{0}}(2 \hat{i}-\hat{k})
\end{aligned}
$$

where, $\vec{a}_{x}=\hat{i}, \vec{a}_{y}=\hat{j}$ and $\vec{a}_{z}=\hat{k}$
Hence, the correct option is (A).

## $1.16 \quad$ (C)

Given : $\vec{E}=\left(x \vec{u}_{x}+y \vec{u}_{y}+z \vec{u}_{z}\right)$
Potential difference is given by,

$$
V_{x y}=V_{x}-V_{y}=-\int_{y}^{x} \vec{E} \cdot d \vec{l}
$$

Here, $\quad d \vec{l}=d x \vec{x}_{x}+d y \vec{u}_{y}+d z \vec{u}_{z}$

$$
\begin{aligned}
& \vec{E} \cdot d \vec{l}=x d x+y d y+z d z \\
& V_{x y}=-\int_{y}^{x} \vec{E} \cdot d \vec{l}=-\int_{1}^{2} x d x-\int_{2}^{0} y d y-\int_{3}^{0} z d z \\
& V_{x y}=-\left[\frac{x^{2}}{2}\right]_{1}^{2}-\left[\frac{y^{2}}{2}\right]_{2}^{0}-\left[\frac{z^{2}}{2}\right]_{3}^{0} \\
& V_{x y}=-\left[2-\frac{1}{2}\right]-[-2]-\left[-\frac{9}{2}\right] \\
& V_{x y}=-\frac{3}{2}+2+\frac{9}{2}=3+2=5 \mathrm{~V}
\end{aligned}
$$

Hence, the correct option is (C).

### 1.17 (B)

Given : A point of +1 nC is placed in a space with permittivity of $8.85 \times 10^{-12} \mathrm{~F} / \mathrm{m}$ as shown in figure,


Potential due to a point charge is given by,

$$
V=\frac{q}{4 \pi \varepsilon r}
$$

where, $r$ is the distance of point from charge $q$. Potential difference between $P$ and $Q$ is given by,

$$
\begin{aligned}
& V_{P}-V_{Q}=\frac{q}{4 \pi \varepsilon_{0}(O P)}-\frac{q}{4 \pi \varepsilon_{0}(O Q)} \\
& V_{P}-V_{Q}=\frac{q}{4 \pi \varepsilon_{0} r_{1}}-\frac{q}{4 \pi \varepsilon_{0} r_{2}}
\end{aligned}
$$

$$
V_{P}-V_{Q}=\frac{q}{4 \pi \varepsilon_{0}}\left(\frac{1}{r_{1}}-\frac{1}{r_{2}}\right)
$$

$$
V_{P}-V_{Q}=10^{-9} \times 9 \times 10^{9} \times\left[\frac{1}{40 \times 10^{-3}}-\frac{1}{20 \times 10^{-3}}\right]
$$

$$
V_{P}-V_{Q}=9 \times(-25) \text { Volts }=-225 \text { Volts }
$$

Hence, the correct option is (B).


### 1.18 (D)

Given : $A=100 \mathrm{~mm}^{2}=100 \times 10^{-6} \mathrm{~m}^{2}$,
$d=0.1 \mathrm{~mm}=0.1 \times 10^{-3} \mathrm{~m}, \varepsilon_{0}=8.85 \times 10^{-12} \mathrm{~F} / \mathrm{m}$,
$V=100 \mathrm{~V}$
For parallel plate capacitor, capacitance is given by,

$$
C=\frac{\varepsilon_{0} A}{d}=\frac{8.85 \times 10^{-12} \times 100 \times 10^{-6}}{0.1 \times 10^{-3}} \mathrm{~F}
$$

$$
C=8.85 \times 10^{-12} \mathrm{~F}
$$

Energy stored in the capacitor is given by,

$$
\begin{aligned}
& W=\frac{1}{2} C V^{2} \\
& W=\frac{1}{2} \times 8.85 \times 10^{-12} \times(100)^{2} \\
& W=44.25 \times 10^{-9} \text { Joules } \simeq 44.3 \mathrm{~nJ}
\end{aligned}
$$

Hence, the correct option is (D).

### 1.19 (B)

Given : A composite parallel plate capacitor is made up of two different dielectric materials with different thicknesses ( $t_{1}$ and $t_{2}$ ) as shown in figure,


## Method 1

The given dielectric arrangement can be treated as a combination of two capacitors $C_{1}$ and $C_{2}$ in series.


$$
\begin{aligned}
& C_{1}=\frac{\varepsilon_{0} \varepsilon_{r_{1}} A}{t_{1}}=\frac{3 \varepsilon_{0} A}{0.5}=6 \varepsilon_{0} A \\
& C_{2}=\frac{\varepsilon_{0} \varepsilon_{r_{2}} A}{t_{2}}=\frac{4 \varepsilon_{0} A}{1}=4 \varepsilon_{0} A
\end{aligned}
$$

Here, the two capacitors are in series.
Equivalent capacitance is given by,

$$
\begin{aligned}
& \frac{1}{C_{e q}}=\frac{1}{C_{1}}+\frac{1}{C_{2}} \\
& C_{e q}=\frac{C_{1} C_{2}}{C_{1}+C_{2}}=\frac{4 \times 6}{4+6} \varepsilon_{0} A=\frac{24}{10} \varepsilon_{0} A \\
& Q=C_{e q} V=\left(\frac{24}{10} \varepsilon_{0} A\right)(100)=240 \varepsilon_{0} A
\end{aligned}
$$

Since, the two capacitors are in series.
Thus, $Q_{1}=Q_{2}=Q$

$$
Q_{2}=C_{2} \cdot V_{F}
$$

where, $V_{F}$ is the voltage of the conducting foil and also voltage across $C_{2}$.

$$
V_{F}=\frac{Q}{C_{2}}=\frac{240 \varepsilon_{0} A}{4 \varepsilon_{0} A}=60 \mathrm{Volts}
$$

Hence, the correct option is (B).

## Method 2

(i) Voltage across $C_{2}$ is $V_{F}$.
(ii) Voltage across $C_{1}$ is $100-V_{F}$.

Then, $C_{1}\left(100-V_{F}\right)=C_{2} V_{F}$

$$
\begin{aligned}
& 6 \varepsilon_{0} A\left(100-V_{F}\right)=4 \varepsilon_{0} A V_{F} \\
& 6\left(100-V_{F}\right)=4 V_{F} \\
& V_{F}=\frac{600}{10}=60 \mathrm{~V}
\end{aligned}
$$

Hence, the correct option is (B).

### 1.20 (D)

Given : A parallel plate capacitor is made of two square metal plates of 400 mm side,


Area of cross section is given by,

$$
\begin{aligned}
& A=400 \times 400 \mathrm{~mm}^{2}=160000 \mathrm{~mm}^{2} \\
& A=0.16 \mathrm{~m}^{2}
\end{aligned}
$$

The above figure is represented by,


For parallel plate capacitor, capacitance is given by,

$$
\begin{aligned}
& C=\frac{\varepsilon_{0} \varepsilon_{r} A}{d} \\
& C_{1}=\frac{\varepsilon_{0} \varepsilon_{r_{1}} A}{d_{1}}, \quad C_{2}=\frac{\varepsilon_{0} \varepsilon_{r_{2}} A}{d_{2}}
\end{aligned}
$$

Here, the two capacitors are in series.
Equivalent capacitance is given by,

$$
\frac{1}{C_{e q}}=\frac{1}{C_{1}}+\frac{1}{C_{2}}
$$

$$
\begin{gathered}
C_{e q}=\frac{C_{1} C_{2}}{C_{1}+C_{2}}=\frac{\frac{\varepsilon_{0} \varepsilon_{r_{1}} A}{d_{1}} \cdot \frac{\varepsilon_{0} \varepsilon_{r_{2}} A}{d_{2}}}{\frac{\varepsilon_{0} \varepsilon_{r_{1}} A}{d_{1}}+\frac{\varepsilon_{0} \varepsilon_{r_{2}} A}{d_{2}}} \\
C_{e q}=\varepsilon_{0} A\left[\frac{\varepsilon_{r_{1}} \cdot \varepsilon_{r_{2}}}{\varepsilon_{r_{1}} \cdot d_{2}+\varepsilon_{r_{2}} \cdot d_{1}}\right] \\
C_{e q}=8.85 \times 10^{-12} \times 0.16\left[\frac{4 \times 2}{(4 \times 8+2 \times 6) \times 10^{-3}}\right] \\
C_{e q}=\frac{8.85 \times 0.16 \times 8 \times 10^{-12}}{44 \times 10^{-3}} \mathrm{~F} \\
C_{e q}= \\
2.57 \times 10^{-10} \mathrm{~F}=257 \mathrm{pF}
\end{gathered}
$$

Hence, the correct option is (D).


### 1.21 (D)

Given : $\vec{E}$ is the electric field intensity,
So, $\quad \nabla \cdot(\nabla \times \vec{E})=0$
This is because divergence of a curl is always zero by vector identity.
Hence, the correct option is (D).

### 1.22 (A)

Given : The charge distribution in a metal-dielectric-semiconductor specimen is shown in the figure.


In metals there is no electric field.

In dielectric, there will be a constant electric field.

$$
\left(\frac{\rho}{\varepsilon_{0} \varepsilon_{r}}, \text { where } \rho \text { is the surface charge density }\right)
$$

In semiconductor, the electric field will vary according to the charge distribution.
Hence, the correct option is (A).

### 1.23 (D)

From Maxwell's equation,

$$
\nabla \cdot \vec{B}=0
$$

This simply suggests that there can't be magnetic monopoles.
Hence, the correct option is (D).

### 1.24 (D)

Given : $\nabla \cdot \bar{J}=-\frac{\partial \rho_{v}}{\partial t}$
where $J$ is the current density and $\rho_{v}$ is the volume charge density.
The divergence of current density is equal to the decrease of volume charge density per unit time at every point. The above equation is called current continuity equation or continuity equation.
The continuity equation is derived from the principle of conservation of charge and essentially states that there can be no accumulation of charge at any point.
$\frac{\partial \rho_{v}}{\partial t}$ also represents conservation of charges.
Hence, the correct option is (D).

## $1.25 \quad$ (A)

Given : A solid sphere made of insulating material has a radius $R$ and has total charge $Q$ distributed uniformly in its volume.
Charge inside the smaller sphere of radius ' $r$ ' is given by,


$$
q=\frac{Q}{\frac{4}{3} \pi R^{3}} \cdot \frac{4}{3} \pi r^{3}=\frac{Q}{R^{3}} \cdot r^{3}
$$

Electric field intensity is given by,

$$
\begin{aligned}
& E=\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{q}{r^{2}} \\
& E=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{R^{3}} \cdot \frac{r^{3}}{r^{2}}=\frac{Q \cdot r}{4 \pi \varepsilon_{0} R^{3}}
\end{aligned}
$$

Hence, the correct option is (A).

### 1.26 (D)

Given :

$$
\begin{equation*}
N=400, A=16 \mathrm{~cm}^{2} \tag{i}
\end{equation*}
$$

(ii) $\quad V=230 \mathrm{~V}, l=1 \mathrm{~mm}$

Inductance of solenoid is given by,

$$
\begin{aligned}
& L=\frac{\mu_{0} N^{2} A}{l} \\
& L=\frac{4 \pi \times 10^{-7} \times(400)^{2} \times 16 \times 10^{-4}}{\left(1 \times 10^{-3}\right)} \\
& L=0.3217 \mathrm{H}
\end{aligned}
$$

Inductive reactance is given by,

$$
X_{L}=2 \pi f L
$$

Current, $I=\frac{V}{X_{L}}=\frac{V}{2 \pi f L}=\frac{230}{2 \pi \times 50 \times 0.3217}$

$$
I=2.275 \mathrm{~A}
$$

Hence, the correct option is (D).

## $1.27 \quad$ (A)

Energy stored in inductor is given by,

$$
W_{E}=\frac{1}{2} L I^{2}
$$

$$
\begin{aligned}
& W_{E}=\frac{1}{2} \times 0.3217 \times(2.275)^{2} \\
& W_{E}=0.83249 \mathrm{~J}
\end{aligned}
$$

Force to reduce 1 mm air gap $=\frac{W_{E}}{d}$.

$$
F=\frac{0.83249}{1 \times 10^{-3}}=832.49 \mathrm{~N}
$$

Hence, the correct option is (A).

### 1.28 (C)

Given : Two point charges $Q_{1}=10 \mu \mathrm{C}$ and $Q_{2}=20 \mu \mathrm{C}$ are placed at coordinates $(1,1,0)$ and $(-1,-1,0)$ respectively,


The charges are lying on the $x y$ plane $(z=20)$. Hence half of the flux will pass through above the plane (i.e. $z>0$ ) and half will pass through below the plane (i.e. $z<0$ ).
Now, total flux $=$ Total charge enclosed

$$
=10 \mu \mathrm{C}+20 \mu \mathrm{C}=30 \mu \mathrm{C}
$$

Total flux passing through infinite plane $z=20$ will be $\frac{30}{2} \mu \mathrm{C}=15 \mu \mathrm{C}$

Hence, the correct option is (C).

### 1.29 (B)

## Given :

(i) A capacitor consists of two metal plates each $500 \times 500 \mathrm{~mm}^{2}$ and spaced 6 mm apart.
(ii) The space between the metal plates is filled with a glass plate of 4 mm thickness and a layer of paper of 2 mm thickness.
(iii) The relative permittivity of the glass and paper are 8 and 2 respectively.
The equivalent figure is represented as,


Area of cross section $=500 \times 500 \mathrm{~mm}^{2}$

$$
\begin{aligned}
& A=500 \times 500 \times 10^{-6} \mathrm{~m}^{2}=0.25 \mathrm{~m}^{2} \\
& d_{g}=4 \mathrm{~mm}=4 \times 10^{-3} \mathrm{~m} \\
& d_{p}=2 \mathrm{~mm}=2 \times 10^{-3} \mathrm{~m}
\end{aligned}
$$

The given dielectric arrangement can be treated as a combination of two capacitors in series.


Equivalent capacitance is given by,

$$
\begin{aligned}
& \frac{1}{C_{e q}}=\frac{1}{C_{g}}+\frac{1}{C_{p}}=\frac{d_{g}}{\varepsilon_{0} \varepsilon_{r g} A}+\frac{d_{p}}{\varepsilon_{0} \varepsilon_{r p} A} \\
& \frac{1}{C_{e q}}=\frac{d_{g} \cdot \varepsilon_{r p}+d_{p} \cdot \varepsilon_{r g}}{\varepsilon_{0} A \varepsilon_{r g} \cdot \varepsilon_{r p}} \\
& C_{e q}=\frac{\left(\varepsilon_{0} A\right) \cdot \varepsilon_{r g} \cdot \varepsilon_{r p}}{d_{g} \cdot \varepsilon_{r p}+d_{p} \cdot \varepsilon_{r g}} \\
& C_{e q}=\frac{8.85 \times 10^{-12} \times 0.25 \times 8 \times 2}{4 \times 10^{-3} \times 2+2 \times 10^{-3} \times 8} \\
& C_{e q}=\frac{8.85 \times 8 \times 2 \times 10^{-12} \times 0.25}{24 \times 10^{-3}} \\
& C_{e q}=1475 \mathrm{pF}
\end{aligned}
$$

## $1.30 \quad$ (B)

Given :
(i) $N=300$, Circumference $=2 \pi r=300 \mathrm{~mm}$
(ii) $A=300 \mathrm{~mm}^{2}=300 \times 10^{-6} \mathrm{~m}^{2}, I=3 \mathrm{~A}$

Inductance of solenoid is given by,

$$
\begin{aligned}
& L=\frac{\mu_{0} N^{2} A}{l} \\
& L=\frac{4 \pi \times 10^{-7} \times 300 \times 300 \times 300 \times 10^{-6}}{300 \times 10^{-3}} \\
& L=113.04 \mu \mathrm{H}
\end{aligned}
$$

Hence, the correct option is (B).

### 1.31 (C)

## Given :

(i) A capacitor is made with a polymeric dielectric having an $\varepsilon_{r}$ of 2.26 and a dielectric breakdown strength of 50 $\mathrm{kV} / \mathrm{cm}$.
(ii) The permittivity of free space is 8.85 $\mathrm{pF} / \mathrm{m}$.
(iii) The rectangular plate of the capacitor have a width of 20 cm and a length of 40 cm .

$$
\begin{aligned}
& A=20 \times 40 \mathrm{~cm}^{2}=8 \times 10^{-2} \mathrm{~m}^{2} \\
& \varepsilon=\varepsilon_{0} \varepsilon_{r}=2.26 \times 8.85 \times 10^{-12} \mathrm{~F} / \mathrm{m}
\end{aligned}
$$

Breakdown strength of the dielectric

$$
=50 \mathrm{kV} / \mathrm{cm}=5 \times 10^{6} \text { Volts } / \mathrm{m}
$$

For a distance of separation, $d$ meter; maximum voltage that can be applied,

$$
V_{m}=5 \times 10^{6} d \text { Volts }
$$

Maximum charge in the capacitor is given by,

$$
\begin{aligned}
& Q_{m}=C V_{m}, \text { where, } C=\frac{\varepsilon A}{d} \\
& Q_{m}=\frac{\varepsilon A}{d} \times 5 \times 10^{6} d=5 \times 10^{6} \varepsilon A
\end{aligned}
$$

Hence, the correct option is (B).

$$
\begin{aligned}
& Q_{m}=5 \times 10^{6} \times 2.26 \times 8.85 \times 10^{-12} \times 8 \times 10^{-2} \mathrm{C} \\
& Q_{m}=8 \mu \mathrm{C}
\end{aligned}
$$

Hence, the correct option is (C).

### 1.32 (A)

Given : $\vec{A}=k r^{n} \hat{i}_{r}$
In spherical coordinate divergence is given by,

$$
\begin{aligned}
& \nabla \cdot \vec{A}=\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(r^{2} \cdot k r^{n}\right)=\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(k r^{n+2}\right) \\
& \nabla \cdot \vec{A}=\frac{k}{r^{2}}(n+2) r^{n+1}=(n+2) k r^{n-1}
\end{aligned}
$$

For $\nabla \cdot \vec{A}=0$,
$n+2=0$
$n=-2$
Hence, the correct option is (A).

## $1.33 \quad$ (C)

Given :
(i) A dielectric slab with $500 \mathrm{~mm} \times 500$ mm cross-section is 0.4 m long.
(ii) The slab is subjected to a uniform electric field of $\vec{E}=6 \vec{a}_{x}+8 \vec{a}_{y} \mathrm{kV} / \mathrm{mm}$.
(iii) The relative permittivity of the dielectric material is equal to 2 .
(iv) $\varepsilon=2 \times 8.85 \times 10^{-12} \mathrm{~F} / \mathrm{m}$

Magnitude of electric field can be written as,

$$
|\vec{E}|=\sqrt{6^{2}+8^{2}}=10 \mathrm{kV} / \mathrm{mm}
$$

Energy stored in dielectric is given by,

$$
\begin{aligned}
& W_{E}=\int_{v} \frac{1}{2} \varepsilon E^{2} d v=\frac{1}{2} \varepsilon E^{2} \int_{v} d v \\
& W_{E}=\frac{1}{2} \varepsilon E^{2} \times \text { Volume }=\frac{1}{2} \varepsilon E^{2} \times A \times l \\
& W_{E}=\frac{1}{2} \times 2 \times\left(8.85 \times 10^{-12}\right) \times\left(\frac{10 \times 10^{3}}{10^{-3}}\right)^{2} \\
& \\
& \quad \times\left(500 \times 10^{-3}\right)^{2} \times 0.4 \\
& W_{E}=88.5 \text { Joule }
\end{aligned}
$$

Hence, the correct option is (C).

## $1.34 \quad$ (A)

Given :
(i) $\quad C_{0}$ is the capacitance of a parallel plate capacitor with air as dielectric as shown in figure,

$$
\varepsilon_{r}=1 \quad \text { Air }
$$

(ii) In the new arrangement half of the entire gap is filled with a dielectric of permittivity $\varepsilon_{r}$ as shown in figure,


Let, $A$ be the area of the parallel plate capacitor and $d$ be the distance between the plates.
With air dielectric :


Capacitance, $C_{0}=\frac{\varepsilon_{0} A}{d}$
With new arrangement,


Let, $C_{1}$ be the capacitance of half portion with air as dielectric medium and $C_{2}$ be capacitance with a dielectric of permittivity $\varepsilon_{r}$.

Then, $C_{1}=\frac{\varepsilon_{0}\left(\frac{A}{2}\right)}{d}$ (As area becomes half) and $C_{2}=\frac{\varepsilon_{0} \varepsilon_{r}\left(\frac{A}{2}\right)}{d}$ (As area becomes half)

These two capacitances are in parallel if voltage is applied between the plates as same potential
difference will be there between both the capacitances.


Equivalent capacitance is given by,

$$
\begin{aligned}
& C_{e q}=C_{1}+C_{2} \\
& C_{e q}=\frac{\varepsilon_{0} A}{2 d}+\frac{\varepsilon_{0} \varepsilon_{r} A}{2 d} \\
& C_{e q}=\frac{\varepsilon_{0} A}{2 d}\left(1+\varepsilon_{r}\right)
\end{aligned}
$$

From equation (i),

$$
C_{e q}=\frac{C_{0}}{2}\left(1+\varepsilon_{r}\right)
$$

Hence, the correct option is (A).


## $1.35 \quad$ (C)

Given : A parallel plate capacitor consisting two dielectric materials is shown in the figure,

10 Volt


Let, $A=$ Area of plates,
$C_{1}=C_{3}$ be capacitance formed with dielectric having dielectric constant $\varepsilon_{1}$.
$C_{e q}$ be the equivalent capacitance.
$C_{2}$ be the capacitance formed with dielectric having dielectric constant $\varepsilon_{2}$.

Then, $C_{1}=C_{3}=\frac{\varepsilon_{0} \varepsilon_{1} A}{\frac{d}{4}}=\frac{4 \varepsilon_{0} \varepsilon_{1} A}{d}$
and

$$
C_{2}=\frac{\varepsilon_{0} \varepsilon_{2} A}{\frac{d}{2}}=\frac{2 \varepsilon_{0} \varepsilon_{2} A}{d}
$$

The given dielectric arrangement can be treated as a combination of three capacitors in series.


Equivalent capacitance $=C_{e q}$

$$
\frac{1}{C_{e q}}=\frac{1}{C_{1}}+\frac{1}{C_{2}}+\frac{1}{C_{3}}=\frac{2}{C_{1}}+\frac{1}{C_{2}}
$$

$\left(C_{1}=C_{3}, C_{1}, C_{2}, C_{3}\right.$ all are in series)
$\frac{1}{C_{e q}}=\frac{2 d}{4 \varepsilon_{0} \varepsilon_{1} A}+\frac{d}{2 \varepsilon_{0} \varepsilon_{2} A}$
$C_{e q}=\frac{2 \varepsilon_{1} \varepsilon_{2} \varepsilon_{0} A}{d\left(\varepsilon_{1}+\varepsilon_{2}\right)}$
$V_{e q}=$ Total voltage $=10 \mathrm{~V}$
$V_{1}=V_{3}=2 \mathrm{~V}$.
Since, $C \propto \frac{1}{V}$
$\frac{C_{e q}}{C_{1}}=\frac{V_{1}}{V_{e q}}$
$C_{e q}=\frac{C_{1}}{5}$
$\frac{2 \varepsilon_{0} \varepsilon_{1} \varepsilon_{2} A}{d\left(\varepsilon_{1}+\varepsilon_{2}\right)} \cdot \frac{d}{4 \varepsilon_{0} \varepsilon_{1} A}=\frac{1}{5}$
$\frac{\varepsilon_{2}}{2\left(\varepsilon_{1}+\varepsilon_{2}\right)}=\frac{1}{5}$
$5 \varepsilon_{2}=2 \varepsilon_{1}+2 \varepsilon_{2}$
$2 \varepsilon_{1}=3 \varepsilon_{2}$

$$
\varepsilon_{1}: \varepsilon_{2}=3: 2
$$

Hence, the correct option is (C).


## $1.36 \quad$ (B)

Given : $Q=32 \pi \varepsilon_{0} \sqrt{2} \mathrm{C}$ at coordinate $(0,0,2)$.


Due to charge at $(0,0,2)$, and conductor plane there is an image at $(0,0,-2)$.
Electric field intensity at any point is given by,

$$
\vec{E}=\frac{Q}{4 \pi \varepsilon_{0} R^{3 / 2}} \vec{R}
$$

Electric field intensity due to charge $Q$ at $(0,0$, 2) is given by,

$$
\begin{aligned}
\vec{E} & =\frac{Q}{4 \pi \varepsilon_{0} R_{1}^{3 / 2}} \vec{R}_{1} \\
\vec{E}_{1} & =\frac{32 \pi \varepsilon_{0} \sqrt{2}}{4 \pi \varepsilon_{0}(2+2+4)^{\frac{3}{2}}}\left(\sqrt{2} \vec{a}_{x}+\sqrt{2} \vec{a}_{y}-2 \vec{a}_{z}\right)
\end{aligned}
$$

Electric field intensity due to charge $Q$ at $(0,0$, -2 ) is given by,

$$
\begin{aligned}
& \vec{E}=\frac{Q}{4 \pi \varepsilon_{0} R_{2}^{3 / 2}} \vec{R}_{2} \\
& \vec{E}_{2}=\frac{-32 \pi \varepsilon_{0} \sqrt{2}}{4 \pi \varepsilon_{0}(2+2+4)^{\frac{3}{2}}}\left(\sqrt{2} \vec{a}_{x}+\sqrt{2} \vec{a}_{y}+2 \vec{a}_{z}\right) \\
& \vec{E}=\vec{E}_{1}+\vec{E}_{2}=0 \vec{a}_{x}+0 \vec{a}_{y}-2 \vec{a}_{z}=-2 \vec{a}_{z}
\end{aligned}
$$

Hence, the correct option is (B).


### 1.37 (A)

Given : A hollow metallic sphere of radius $r$ is kept at potential of 1 volt.


Hollow metallic sphere of radius ' $r$ ' i.e. equipotential surface or uniformly charged ' $q$ '. Potential on the surface is given by,

$$
V_{0}=\frac{q}{4 \pi \varepsilon_{0} r}=1
$$

Charge, $q=4 \pi \varepsilon_{0} r$
For spherical surfaces of radius $R>r$, from Gauss's law,
Total flux leaving the surface $\psi=$ Total charge enclosed $=q=4 \pi \varepsilon_{0} r$

Hence, the correct option is (A).

## $1.38 \quad$ (C)

Given : $\vec{f}=\frac{1}{r^{2}} \vec{r}$
From divergence theorem,

$$
\int_{v}(\nabla \cdot \vec{f}) d v=\oiint_{s} \vec{f} \cdot d \vec{S}
$$

So, $\quad \oiint_{s} \vec{f} \cdot d \vec{S}=\oiint_{s}\left(\frac{1}{r^{2}} \vec{r}\right) \cdot\left(r^{2} \sin \theta d \theta d \phi \vec{r}\right)$

$$
\oiint_{s} \vec{f} \cdot d \vec{S}=\oiint_{s} \sin \theta d \theta d \phi=4 \pi
$$

Hence, the correct option is (C).

## $\begin{array}{ll}1.39 & 18.75\end{array}$

Given :
(i) A parallel plate capacitor is partially filled with glass of dielectric constant 4.0 as shown below,

(ii) Dielectric strength of air,

$$
E_{10}=30 \mathrm{kV} / \mathrm{cm}=3 \mathrm{kV} / \mathrm{mm}
$$

(iii) Dielectric strength of glass,
$E_{20}=300 \mathrm{kV} / \mathrm{cm}=30 \mathrm{kV} / \mathrm{mm}$
It can be assumed as two capacitors are connected in series (as they carry same current and they are having same charge).


$$
\begin{aligned}
& C_{1}=\frac{\varepsilon_{0} \varepsilon_{r_{1}} A}{d}=\frac{\varepsilon_{0} A}{d}=C_{0} \text { let } \\
& C_{2}=\frac{\varepsilon_{0} \varepsilon_{r_{1}} A}{d}=\frac{4 \varepsilon_{0} A}{d}=4 C_{0}
\end{aligned}
$$

The equivalent capacitance is given by,

$$
\begin{aligned}
& C=\frac{C_{1} C_{2}}{C_{1}+C_{2}}=\frac{C_{0} \times 4 C_{0}}{C_{0}+4 C_{0}}=\frac{4 C_{0}}{5} \\
& Q=C_{1} V_{1}=C_{2} V_{2}=C V
\end{aligned}
$$

where $V=V_{1}+V_{2}$

$$
\begin{aligned}
& C_{0} V_{1}=4 C_{0} V_{2} \\
& V_{1}=4 V_{2}
\end{aligned}
$$



Dielectric strength of air $E_{10}=3 \mathrm{kV} / \mathrm{mm}$
For a distance $d=5 \mathrm{~mm}$
So, maximum voltage allowed across $C_{1}$ is given by,

$$
V_{10}=E_{10} \times d=15 \mathrm{kV}
$$

For $V_{10}=15 \mathrm{kV}$,

$$
V_{2}=\frac{V_{10}}{4}=\frac{15}{4}=3.75 \mathrm{kV}
$$

Electric field across $C_{2}$ is given by,

$$
\begin{aligned}
& E_{2}=\frac{V_{2}}{d}=\frac{3.75}{5}=0.75 \mathrm{kV} / \mathrm{mm} \\
& E_{2}=7.5 \mathrm{kV} / \mathrm{cm}
\end{aligned}
$$

Dielectric strength of glass,

$$
E_{20}=300 \mathrm{kV} / \mathrm{cm}=30 \mathrm{kV} / \mathrm{mm}
$$

As, $E_{2}<E_{20}$
So, it is allowed.
Total voltage across (equivalent) capacitor is given by,

$$
V=V_{1}+V_{2}=15+3.75=18.75 \mathrm{kV}
$$

For $E_{2}=E_{20}=30 \mathrm{kV} / \mathrm{mm}$ (Maximum allowed)

$$
\begin{aligned}
& V_{20}=E_{20} \times d=30 \times 5=150 \mathrm{kV} \\
& V_{1}=4 V_{20}=4 \times 150=600 \mathrm{kV}
\end{aligned}
$$

Electric field across $C_{1}$ is given by,

$$
E_{1}=\frac{V_{1}}{d}=\frac{600}{5}=120 \mathrm{kV} / \mathrm{mm}
$$

As $\quad E_{1}>E_{10}(=3 \mathrm{kV} / \mathrm{mm})$
In this condition, there will be breakdown in $C_{1}$ so it is not allowed.

Hence, the maximum voltage is $\mathbf{1 8 . 7 5} \mathbf{~ k V}$.


## $1.40 \quad$ (A)

Given :
(i) Two semi-infinite dielectric regions are separated by a plane boundary at $y=0$.
(ii) The dielectric constants of region $1(y<$ $0)$ and region $2(y>0)$ are 2 and 5 , respectively.


Normal component of electric field is $4 \vec{a}_{y}$, and tangential component is $3 \vec{a}_{x}+2 \vec{a}_{z}$.

$$
E_{1}=E_{t_{1}}+E_{n_{1}}
$$

From boundary condition,

$$
\begin{aligned}
& E_{t_{1}}=E_{t_{2}} \\
& E_{t_{2}}=3 \vec{a}_{x}+2 \vec{a}_{z}
\end{aligned}
$$

And $\quad D_{n_{1}}=D_{n_{2}}$

$$
\varepsilon_{1} E_{n_{1}}=\varepsilon_{2} E_{n_{2}}
$$

$$
E_{n_{2}}=\frac{\varepsilon_{1} \times 4 \vec{a}_{y}}{\varepsilon_{2}}=\frac{2 \times 4 \vec{a}_{y}}{5}=1.6 \vec{a}_{y}
$$

The electric field in region 2 will be

$$
\vec{E}_{2}=E_{t_{2}}+E_{n_{2}}=3 \vec{a}_{x}+1.6 \vec{a}_{y}+2 \vec{a}_{z}
$$

Hence, the correct option is (A).

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### 1.41 (D)

Given :
(i) Two semi-infinite conducting sheets are placed at right angles to each other as shown in the figure,

(ii) Net force on the charge is

$$
\begin{equation*}
\frac{Q^{2}}{4 \pi \varepsilon_{0}} \frac{K}{d^{2}} \tag{i}
\end{equation*}
$$

Since, there are two semi-infinite conducting sheets and these are connected to ground we can apply method of images. Due to image of charge, the new charges will be created as shown in the figure.


Force on (4) due to (1), (2), (3) will be

$$
\vec{F}_{\text {total }}=\vec{F}_{41}+\vec{F}_{42}+\vec{F}_{43}
$$

Force on (4) due to (1) is given by,

$$
\vec{F}_{41}=\frac{-Q^{2}}{4 \pi \varepsilon_{0}} \cdot \frac{1}{4 d^{2}} \vec{a}_{y}
$$

Force on (4) due to (2) is given by,

$$
\vec{F}_{42}=\frac{+Q^{2}}{4 \pi \varepsilon_{0}} \cdot \frac{1}{8 d^{2}}\left(\frac{\vec{a}_{y}}{\sqrt{2}}+\frac{\vec{a}_{x}}{\sqrt{2}}\right)
$$

Force on (4) due to (3) is given by,

$$
\vec{F}_{43}=\frac{-Q^{2}}{4 \pi \varepsilon_{0}} \cdot \frac{1}{4 d^{2}} \vec{a}_{x}
$$

The net force on charge $Q$ is given by,

$$
\begin{equation*}
F_{t o t a l}=\frac{Q^{2}}{4 \pi \varepsilon_{0} d^{2}}\left(\frac{1-2 \sqrt{2}}{8 \sqrt{2}} \vec{a}_{x}+\frac{1-2 \sqrt{2}}{8 \sqrt{2}} \vec{a}_{y}\right) \tag{ii}
\end{equation*}
$$

On comparing equation (i) and (ii),

$$
K=\frac{1-2 \sqrt{2}}{8 \sqrt{2}} \vec{a}_{x}+\frac{1-2 \sqrt{2}}{8 \sqrt{2}} \vec{a}_{y}
$$

Hence, the correct option is (D).


### 1.42 (B)

$P \rightarrow 4$ Stokes theorem

$$
\oint_{C} \vec{A} \cdot d \vec{l}=\iint_{S}(\nabla \times \vec{A}) \cdot d \vec{S}
$$

where, $C$ is the closed path and $S$ is any open surface opened at boundary ' $C$ '.
$Q \rightarrow 1$ Gauss's theorem

$$
\oiint_{S} \vec{D} \cdot d \vec{S}=Q
$$

$R \rightarrow 3$ Divergence theorem

$$
\oiint_{S} \vec{A} \cdot d \vec{S}=\iiint_{v}(\nabla \cdot \vec{A}) d v
$$

where, $S$ is any closed surface and $v$ is the volume bounded by ' $S$ '.
$S \rightarrow 2$ Cauchy's integral theorem

$$
\oint_{C} f(z) d z=0
$$

where, $C$ is any closed curve and $f(z)$ has no pole either inside the curve ' $C$ ' or on the boundary of curve $C$.
Hence, the correct option is (B).

## $1.43 \quad$ (B)

Given : In cylindrical coordinate system, the potential produced by a uniform ring charge is given by $\phi=f(r, z)$, where $f$ is a continuous function of $r$ and $z$.

Since, uniform charged ring is given, it can be considered as static. A static electric charge produces an electric field for which $\nabla \times \vec{E}=0$
i.e. conservation theorem states that equipotenttial surfaces have zero work done on them, so whenever we apply Stoke's theorem on it, it clears that $\nabla \times \vec{E}$ becomes zero proving that $\vec{E}$ is static and irrotational.

Hence, the correct option is (B).

### 1.44 (D)

Given : Charge $q$ is located at $(0,0)$ and $-2 q$ is located at $(6,0)$.


Potential due to charge $Q$ is given by,

$$
V=\frac{Q}{4 \pi \varepsilon r}
$$

Potential at point $P$ due to charge $q$ is given by,

$$
V_{q}=\frac{q}{4 \pi \varepsilon \sqrt{x^{2}+y^{2}}}
$$

Potential at point $P$ due to charge $-2 q$ is given by,

$$
V_{-2 q}=\frac{-2 q}{4 \pi \varepsilon \sqrt{(x-6)^{2}+y^{2}}}
$$

To get zero equipotential, sum of potential at point $P$ due to charge $q$ and $-2 q$ will be zero.

Thus, $\quad V_{\text {total }}=0$

$$
\begin{aligned}
& \frac{q}{4 \pi \varepsilon \sqrt{x^{2}+y^{2}}}+\frac{-2 q}{4 \pi \varepsilon \sqrt{(x-6)^{2}+y^{2}}}=0 \\
& \sqrt{(x-6)^{2}+y^{2}}=2\left(\sqrt{x^{2}+y^{2}}\right)
\end{aligned}
$$

$$
\begin{aligned}
& x^{2}+36-12 x+y^{2}=4 x^{2}+4 y^{2} \\
& 3 x^{2}+3 y^{2}+12 x=36 \\
& x^{2}+y^{2}+4 x=12 \\
& x^{2}+4 x+4+y^{2}=16 \\
& (x+2)^{2}+y^{2}=16
\end{aligned}
$$

Hence, the correct option is (D).

### 1.45 (C)

## Given :

(i) A parallel plate capacitor filled with two dielectrics is shown in the figure below,
$\left.\begin{array}{c|c}\hline A \\ \varepsilon_{r}=1\end{array} \quad \begin{array}{c}B \\ \varepsilon_{r}=4\end{array}\right] 2 \mathrm{~cm}$
(ii) Electric field in the region $A$ is $4 \mathrm{kV} / \mathrm{cm}$. From above figure it is clear that two capacitors are connected in parallel, so the voltage across them is same and electric field is also same.


$$
E=\frac{V}{d} \quad \text { i.e. potential gradient }
$$

So electric field in the region $B$ is $4 \mathrm{kV} / \mathrm{cm}$ Hence, the correct option is (C).


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## $1.46 \quad$ (A)

Given : Two electrodes, whose cross-sectional view is shown in the figure below, are at the same potential,


Since, both the electrodes are at the same potential. We can say that both have same nature of charge either positive or negative.

In such a case, if we draw electric field, the field due to both will be in same direction at $A$ and hence get added up. So, electric field at $A$ will be maximum.


Hence, the correct option is (A).

### 1.47 (D)

Given :
(i) An electron, a neutron and a proton initially at rest and placed along a straight line such that the neutron is exactly at the center of the line joining the electron and proton.
(ii) At $t=0$, the particles are released but are constrained to move along the same straight line.
From given data, position of electron, neutron and proton are shown in below figure.


Charge on electron, $e^{-}=-q=-1.6 \times 10^{-19}$
Charge on proton, $p^{+}=q=1.6 \times 10^{-19}$
Mass of electron, $m_{e}=9.1 \times 10^{-31} \mathrm{~kg}$
Mass of proton, $m_{p}=1.6 \times 10^{-27} \mathrm{~kg}$
Force on electron $F_{e}$ and proton $F_{p}$ and their direction are shown in above figure. Force between electron and proton is given by,

$$
F_{e}=F_{p}=\frac{q^{2}}{4 \pi \varepsilon_{0} x^{2}}
$$

Acceleration of electron, $a_{e}=\frac{F}{m_{e}}$

Acceleration of proton, $a_{p}=\frac{F}{m_{p}}$
Since, $m_{e}<m_{p}$

$$
\begin{aligned}
& \frac{F}{m_{e}}>\frac{F}{m_{p}} \\
& a_{e}>a_{p}
\end{aligned}
$$

Hence, electron will move towards neutron faster than proton. So, electron and neutron will collide first.
Hence, the correct option is (D).

## $\begin{array}{ll}1.48 & \mathbf{1 0 . 0 4}\end{array}$

Given :
(i) A thin soap bubble of radius, $R=1 \mathrm{~cm}$ and thickness $a=3.3 \mu \mathrm{~m}(a \ll R)$, is at a potential of 1 V with respect to a reference point at infinity.

(ii) Potential of soap bubble $=1 \mathrm{~V}$
(iii) Volume of soap bubble $=4 \pi R^{2} a$
(iv) Volume of soap drop $=\frac{4}{3} \pi r^{3}$


## Method 1

When soap bubble burst, then it becomes single spherical drop. So, volume will not change.
Volume of soap bubble = Volume of soap drop

$$
\begin{aligned}
& 4 \pi R^{2} a=\frac{4}{3} \pi r^{3} \\
& r^{3}=3 R^{2} a \\
& r^{3}=3 \times 1 \times 3.3 \times 10^{-4} \\
& r=(0.99)^{1 / 3} \times 10^{-1} \mathrm{~cm}
\end{aligned}
$$

At distance $x$ from center we take strip of width of $d x$. The differential volume of that strip is given by,

$$
V=4 \pi x^{2} d x
$$

Let, total charge on soap bubble $=Q$
Charge present in $\frac{4}{3} \pi\left[(a+R)^{3}-R^{3}\right]$ is $Q$.
Then charge present in differential volume $4 \pi x^{2} d x$ is

$$
\begin{aligned}
& d q=\frac{Q}{\frac{4}{3} \pi\left[(a+R)^{3}-R^{3}\right]} \times 4 \pi x^{2} d x \\
& d q=\frac{3 Q}{(a+R)^{3}-R^{3}} \times x^{2} d x
\end{aligned}
$$

Potential of soap bubble

$$
\begin{aligned}
& V=\int_{x=R}^{a+R} \frac{k d q}{x}=\int_{R}^{a+R} \frac{k \cdot 3 Q \cdot x^{2} d x}{\left[(a+R)^{3}-R^{3}\right] x} \\
& V=\frac{3 k Q}{\left[(a+R)^{3}-R^{3}\right]}\left[\frac{x^{2}}{2}\right]_{R}^{a+R} \\
& V=\frac{3 k Q}{2\left[(a+R)^{3}-R^{3}\right]}\left[(a+R)^{2}-R^{2}\right]
\end{aligned}
$$

From, $a^{2}-b^{2}=(a+b)(a-b)$ and

$$
\begin{gathered}
a^{3}-b^{3}=(a-b)\left(a^{2}+a b+b^{2}\right) \\
V=\frac{3 k Q}{2} \frac{(a+R-R)(a+R+R)}{(a+R-R)\left[(a+R)^{2}+R^{2}+(a+R) R\right]} \\
V=\frac{3 k Q}{2} \frac{(a+2 R)}{\left[(a+R)^{2}+R^{2}+(a+R) R\right]}
\end{gathered}
$$

[Since $a \ll R$ ]

$$
V=\frac{3}{2} \frac{k Q \times 2 R}{3 R^{2}}=\frac{k Q}{R}
$$

For $V=1 \mathrm{~V}, R=1 \mathrm{~cm}$

$$
k Q=1 \mathrm{~V} \times 1 \mathrm{~cm}=1 \mathrm{~V}-\mathrm{cm}
$$

Electric potential of solid sphere is given by,

$$
\begin{aligned}
& V=\frac{k Q}{r}=\frac{1}{(0.99)^{1 / 3} \times 10^{-1}} \\
& V=\frac{10}{(0.99)^{1 / 3}}=10.04 \mathrm{~V}
\end{aligned}
$$

Hence, the potential in volts, of the resulting single spherical drop with respect to the same reference point at infinity is $\mathbf{1 0 . 0 4} \mathbf{V}$.

## Method 2

Since, $Q_{1}=Q_{2}$ and charge density ' $\rho_{v}$ ' must be same in both cases.
Volume occupied in the two cases must be same.

$$
\begin{aligned}
& \frac{4}{3} \pi r^{3}=4 \pi R^{2} \cdot a \\
& r=\left(3 R^{2} a\right)^{1 / 3} \\
& r=\left[3 \times\left(10^{-2}\right)^{2} \times\left(3.3 \times 10^{-6}\right)\right]^{1 / 3} \\
& r=0.996 \times 10^{-3} \mathrm{~m}=0.996 \mathrm{~mm}
\end{aligned}
$$

Let us consider the bigger bubble,

$$
\begin{aligned}
& R \gg a \\
& V_{1}=\frac{Q_{1}}{4 \pi \varepsilon R} \\
& Q_{1}=4 \pi \varepsilon R V_{1}
\end{aligned}
$$

For smaller bubble,

$$
\begin{aligned}
& V_{2}=\frac{Q_{2}}{4 \pi \varepsilon r} \\
& Q_{2}=4 \pi \varepsilon r V_{2} \\
& Q_{1}=Q_{2} \\
& 4 \pi \varepsilon r V_{2}=4 \pi \varepsilon R V_{1} \\
& V_{2}=\frac{R}{r} V_{1}
\end{aligned}
$$

$$
V_{2}=\frac{10^{-2}}{0.996 \times 10^{-3}} \times 1=10.04 \mathrm{~V}
$$

Hence, the potential in volts, of the resulting single spherical drop with respect to the same reference point at infinity is $\mathbf{1 0 . 0 4} \mathbf{V}$.

### 1.49 (C)

Given : The diagrammatic representations of vector fields $\vec{X}, \vec{Y}$ and $\vec{Z}$ respectively,


(i) Divergence of any vector $(\nabla \cdot \vec{A})$ is zero if total outward flow $=$ Total inward flow.
(ii) Curl of any vector $(\nabla \times \vec{A})$ is zero if there is no rotation in space.
From fig (i), $\nabla \cdot \vec{X} \neq 0$,
$\nabla \times \vec{X}=0 \quad$ [ $X$ is going away]
From fig (ii), $\nabla \cdot \vec{Y}=0$,
$\nabla \times \vec{Y} \neq 0 \quad$ [ $Y$ has circular rotation]
From fig (iii), $\nabla \cdot \vec{Z} \neq 0$,

$$
\nabla \times \vec{Z} \neq 0 \quad[Z \text { has circular rotation }]
$$

Hence, the correct option is (C).

### 1.50 (B)

## Given :

(i) Consider a solid sphere of radius 5 cm made of a perfect electric conductor
(ii) One million electrons are added to this sphere.
The electric field inside a perfect electric conductor is zero, hence all the charge added to the sphere will be distributed uniformly over the
surface of the sphere, so that at every point on the sphere, the electric field from the surface of the sphere is radial in direction.
In other words, for ideal conductor, no charge can be inside the surface of conductor and charge will be uniformly distributed over the outer surface of sphere.
Hence, the correct option is (B).

### 1.51 (D)

Given : Charge $=1 \mathrm{nC}$ at $(0,0,0.2)$. Since the charge is placed above conducting grounded plane there will be an image charge below the grounded conducting plane as per method of image concept.


$$
\begin{gathered}
z=0 \\
\text { plane } \\
P\left\{\begin{array}{c}
\uparrow^{z} \\
(0,0,0.2) \\
(0,0,0.1)
\end{array}\right. \\
\begin{array}{c}
V=0 \\
\text { Equipotential } \\
\text { surface }
\end{array} \\
\hline(0,0,-0.2)
\end{gathered}
$$

Electric field at any point is given by,

$$
\vec{E}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q \vec{R}}{|\vec{R}|^{3}}
$$

$\vec{R}=$ displacement vector between charge to point of interest.
Electric field at point $P$ due to point charge +1 nC ,
$\vec{E}_{1}=\frac{1}{4 \pi \varepsilon_{0}} \frac{1 \times 10^{-9}\left[(0-0) \vec{a}_{x}+(0-0) \vec{a}_{y}+(0.1-0.2) \vec{a}_{z}\right]}{(0.1)^{3}}$
$\vec{E}_{1}=-898.755 \vec{a}_{z} \mathrm{~V} / \mathrm{m}$
Electric field point $P$ due to point charge -1 nC
$\vec{E}_{2}=\frac{1}{4 \pi \varepsilon_{0}} \frac{-1 \times 10^{-9}\left[(0-0) \vec{a}_{x}+(0-0) \vec{a}_{y}+(0.1+0.2) \vec{a}_{z}\right]}{(0.3)^{3}}$
$\vec{E}_{2}=-99.86 \vec{a}_{z} \mathrm{~V} / \mathrm{m}$
Total electric field at point $P$ due to both charges,

$$
\begin{aligned}
\vec{E} & =\vec{E}_{1}+\vec{E}_{2} \\
\vec{E} & =-898.755-99.86=-998.61 \vec{a}_{z} \mathrm{~V} / \mathrm{m}
\end{aligned}
$$

Hence, the correct option is (D).

\section*{| 1.52 | 2.53 |
| :--- | :--- |}

Given : Capacitance of an air-filled parallel plate capacitor,

$$
\begin{equation*}
C_{0}=\frac{\varepsilon_{0} A}{d}=60 \mathrm{pF} \tag{i}
\end{equation*}
$$

Overall capacitance, $C_{e q}=86 \mathrm{pF}$


Plate area ' $A$ '
Fig. 1


Fig. 2
From fig. 2, after filling with dielectric it has become series combination of two capacitances overall capacitance for series combination is given by,

$$
\begin{align*}
& C_{e q}=\frac{C_{1} C_{2}}{C_{1}+C_{2}}=86 \mathrm{pF}  \tag{ii}\\
& C_{1}=\frac{\varepsilon_{0} A}{d / 2} \text { and } C_{2}=\frac{\varepsilon_{0} \varepsilon_{r} A}{d / 2}
\end{align*}
$$

From equation (ii),

$$
C_{e q}=\frac{\frac{\varepsilon_{0} A}{d / 2} \times \frac{\varepsilon_{0} \varepsilon_{r} A}{d / 2}}{\frac{\varepsilon_{0} A}{d / 2}+\frac{\varepsilon_{0} \varepsilon_{r} A}{d / 2}}=\frac{\varepsilon_{0} A}{d / 2} \frac{\left(\varepsilon_{r}\right)}{\left(1+\varepsilon_{r}\right)}
$$

$$
\begin{aligned}
& 86=120\left(\frac{\varepsilon_{r}}{1+\varepsilon_{r}}\right) \\
& 86\left(1+\varepsilon_{r}\right)=120 \varepsilon_{r} \\
& 86=34 \varepsilon_{r} \\
& \varepsilon_{r}=2.529
\end{aligned}
$$

Hence, the relative permittivity of the dielectric is $\mathbf{2 . 5 3}$.

## $1.53 \quad 10$

Given : $c_{1}=$ Capacitance of figure 1.
$c_{2}=$ Capacitance of figure 2.
$c_{2}=2 c_{1}$


Fig. (1)


Fig. (2)

Figure 1 has dielectric with $\varepsilon_{r_{1}}=2$ has capacitance $c_{1}$.
When one fourth portion of figure 1 is replaced with another dielectric of $\varepsilon_{r_{2}}$ then capacitance.
For co-axial cylindrical capacitor

$$
\begin{aligned}
& c_{1}=\frac{2 \pi \varepsilon_{0} l_{1}}{\ln \left(\frac{R}{r}\right)} \text { as } l_{1}, \ln \left(\frac{R}{r}\right) \text { is constant. } \\
& c_{2}=\frac{2 \pi \varepsilon_{0} l_{2}}{\ln \left(\frac{R}{r}\right)} \text { as } l_{2}, \ln \left(\frac{R}{r}\right) \text { is constant. } \\
& c_{1} \propto \varepsilon_{r}
\end{aligned}
$$

As $3 / 4^{\text {th }}$ of $c_{1}$ is going to be same and $1 / 4^{\text {th }}$ is going to be replaced with $\varepsilon_{r_{2}}$.

$$
\begin{aligned}
& \therefore \quad \frac{3}{4} c_{1}+\frac{1}{4} c_{2}=c_{2}=2 c_{1} \\
& \therefore \quad \frac{1}{4} c_{2}=\frac{5}{4} c_{1}
\end{aligned}
$$

$$
\begin{aligned}
& c_{2}=5 c_{1} \\
& \frac{c_{1}}{c_{2}}=\frac{\varepsilon_{r_{1}}}{\varepsilon_{r_{2}}} \\
& \frac{c_{1}}{5 c_{1}}=\frac{\varepsilon_{r_{1}}}{\varepsilon_{r_{2}}}
\end{aligned}
$$

$$
\therefore \quad \frac{1}{5}=\frac{2}{\varepsilon_{r_{2}}}
$$

$$
\varepsilon_{r_{2}}=10
$$

Hence, the value of $\varepsilon_{r 2}$ is $\mathbf{1 0}$.

## $1.54 \quad$ (B)

Given : $\vec{D}=15 \hat{a}_{r}+2 r \hat{a}_{\phi}-3 r z \hat{a}_{z} \mathrm{C} / \mathrm{m}^{2}$


Flux passing through closed surface is given by,

$$
\phi=\oint_{s} \vec{D} \cdot d \vec{s}
$$

From divergence theorem,

$$
\oint_{s} \vec{D} \cdot d \vec{s}=\int_{v}(\nabla \cdot \vec{D}) d v
$$

In cylindrical co-ordinate system,

$$
\begin{aligned}
& \nabla \cdot \vec{D}=\frac{1}{r} \frac{\partial}{\partial r}\left(r D_{r}\right)+\frac{1}{r} \frac{\partial D_{\phi}}{\partial \phi}+\frac{\partial D_{z}}{\partial z} \\
& \nabla \cdot \vec{D}=\frac{1}{r} \frac{\partial}{\partial r}(15 r)+\frac{1}{r} \frac{\partial}{\partial \phi}(2 r)+\frac{\partial}{\partial z}(-3 r z) \\
& \nabla \cdot \vec{D}=\frac{15}{r}+0-3 r=\frac{15}{r}-3 r \\
& \oint \vec{D} \cdot d \vec{s}=\int_{z=0}^{5} \int_{\phi=0}^{2 \pi} \int_{r=0}^{3}\left(\frac{15}{r}-3 r\right) r d r d \phi d z
\end{aligned}
$$

$$
\begin{aligned}
& \oint_{s} \vec{D} \cdot d \vec{s}=\int_{r=0}^{3}\left(15-3 r^{2}\right) d r \int_{\phi=0}^{2 \pi} d \phi \int_{z=0}^{5} d z \\
& \oint_{s} \vec{D} \cdot d \vec{s}=\left[15 r-r^{3}\right]_{0}^{3}(2 \pi)(5) \\
& \oint_{s} \vec{D} \cdot d \vec{s}=(45-27) 10 \pi \\
& \oint_{s} \vec{D} \cdot d \vec{s}=180 \pi
\end{aligned}
$$

Hence, the correct option is (B).

### 1.55 (C)

Given : E = $\mathbf{a}_{r} 2 r+\mathbf{a}_{\phi}\left(\frac{3}{r}\right)+\mathbf{a}_{z} 6$
From Maxwell's equation,

$$
\begin{aligned}
P_{V} & =\nabla \cdot \vec{D} \\
P_{V} & =\epsilon_{0}(\nabla \cdot \vec{E}) \\
P & =\nabla \cdot \vec{E} \\
P & =\frac{1}{r} \frac{\partial}{\partial r}\left(r E_{r}\right)+\frac{1}{r} \frac{\partial}{\partial \phi} E \phi+\frac{\partial E_{z}}{\partial z} \\
P & =\frac{1}{r} \frac{\partial}{\partial r}\left(2 r^{2}\right)+0+0 \\
P & =\frac{4 r}{r}=4 \\
\Rightarrow \quad P_{V} & =\varepsilon_{0} P=4 \varepsilon_{0}
\end{aligned}
$$

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## 2010 IIT Guwahati

## One Mark

1.1 Which of the following options is the closest in meaning to the word below:

## Circuitous

(A) cyclic
(B) indirect
(C) confusing
(D) crooked
1.2 The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair.

## Unemployed : Worker

(A) fallow : land
(B) unaware : sleeper
(C) wit : jester
(D) renovated : house
1.3 Choose the most appropriate word from the options given below to complete the following sentence :
If we manage to $\qquad$ our natural resources, we would leave a better planet for our children.
(A) uphold
(B) restrain
(C) cherish
(D) conserve
1.4 Choose the most appropriate word from the options given below to complete the following sentence :
His rather casual remarks on politics
$\qquad$ his lack of seriousness about
the subject.
(A) masked
(B) belied
(C) betrayed
(D) suppressed

## Two Marks

1.5 Modern warfare has changed from large scale clashes of armies to suppression of civilian populations. Chemical agents that do their work silently appear to be suited to such warfare; and regretfully, there exist people in military establishments who think that chemical agents are useful tools for their cause.
Which of the following statements best sums up the meaning of the above passage :
(A) Modern warfare has resulted in civil strife.
(B) Chemical agents are useful in modern warfare.
(C) Use of chemical agents in warfare would be undesirable.
(D)People in military establishments like to use chemical agents in war.

## 2011 IIT Madras

## One Mark

1.6 Choose the word from the options given below that is most nearly opposite in meaning to the given word:

## Frequency

(A) Periodicity
(B) Rarity
(C) Gradualness
(D) Persistency
1.7 Choose the most appropriate word from the options given below to complete the following sentence :
Under ethical guidelines recently adopted by the Indian Medical Association, human genes are to be manipulated only to correct diseases for which $\qquad$ treatments are unsatisfactory.
(A) Similar
(B) Most
(C) Uncommon
(D) Available
1.8 The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair :

## Gladiator : Arena

(A) Dancer : Stage
(B) Commuter : Train
(C) Teacher: Classroom
(D) Lawyer : Courtroom
1.9 Choose the most appropriate word from the options given below to complete the following sentence :
It was her view that the country's problem had been
by foreign technocrats, so that to invite them to come back would be counter-productive.
(A) Identified
(B) Ascertained
(C) Exacerbated
(D) Analysed

## Two Marks

1.10 The horse has played a little known but very important role in the field of medicine. Horses were injected with toxins of diseases until their blood built up immunities. Then a serum was made from their blood. Serums to fight with diphtheria and tetanus were developed this way.

It can be inferred from the passage that horses were
(A) given immunity to diseases.
(B) generally quite immune to diseases.
(C) given medicines to fight toxins.
(D) given diphtheria and tetanus serums.

## 2012 IIT Delhi

## One Mark

1.11 Choose the most appropriate alternative from the options given below to complete the following sentence :
If the tired soldier wanted to lie down, he $\qquad$ the mattress out on the balcony.
(A) should take
(B) shall take
(C) should have taken
(D) will have taken
1.12 Choose the most appropriate word from the options given below to complete the following sentence:
Given the seriousness of the situation that he had to face, his ___was impressive.
(A) beggary
(B) nomenclature
(C) jealousy
(D) nonchalance
1.13 Which one of the following options is the closest in meaning to the word given below?

## Latitude

(A) Eligibility
(B) Freedom
(C) Coercion
(D) Meticulousness
1.14 One of the parts ( $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ ) in the sentence given below contains an ERROR. Which one of the following is INCORRECT?

## I requested that he should be given the driving test today instead of tomorrow

(A) requested that
(B) should be given
(C) the driving test
(D) instead of tomorrow

## Two Marks

1.15 One of the legacies of the Roman legions was discipline. In the legions, military law prevailed and discipline was brutal. Discipline on the battlefield kept units obedient, intact and fighting, even when the odds and conditions were against them.
Which one of the following statements best sums up the meaning of the above passage?
(A) Thorough regimentation was the main reason for the efficiency of the Roman legions even in adverse circumstances.
(B) The legions were treated inhumanly as if the men were animals.
(C) Discipline was the armies' inheritance from their seniors.
(D) The harsh discipline to which the legions were subjected to led to the odds and conditions being against them.

## 2013 IIT Bombay

## One Mark

1.16 Choose the grammatically CORRECT sentence
(A) Two and two add four.
(B) Two and two become four.
(C) Two and two are four.
(D) Two and two make four.
1.17 Statement : You can always give me a ring whenever you need. Which one of the following is the best inference from the above statement?
(A) Because I have a nice caller tune.
(B) Because I have a better telephone facility.
(C) Because a friend in need is a friend indeed.
(D) Because you need not pay towards the telephone bills when you give me a ring.
1.18 Complete the sentence

Dare $\qquad$ mistakes
(A) Commit
(B) To commit
(C) Committed
(D) Committing
1.19 They were requested not to quarrel with others.

Which one of the following options is the closest in meaning to the word quarrel?
(A)Make out
(B) Call out
(C) Dig out
(D) Fall out

## Two Marks

1.20 Statement : There were different streams of freedom movements in colonial India carried out by the moderates, liberals, radicals, socialists, and so on.
Which one of the following is the best inference from the above statement?
(A) The emergence of nationalism in colonial India led to our Independence
(B) Nationalism in India emerged in the context of colonialism
(C) Nationalism in India is homogeneous
(D) Nationalism in India is heterogeneous

## 6.4 <br> 2014 IIT Kharagpur

## One Mark : Set - 01

1.21 Which of the following options is the closest in meaning to the phrase underlined in the sentence below? It is fascinating to see life forms cope with varied environmental conditions.
(A) adopt to
(B) adapt to
(C) adept in
(D) accept with
1.22 Choose the most appropriate word from the options given below to complete the following sentence.
He could not understand the judges awarding her the first prize, because he thought that her performance was quite $\qquad$ -
(A) superb
(B) medium
(C) mediocre
(D) exhilarating
1.23 In a press meet on the recent scam, the minister said, "The buck stops here" what did the minister convey by the statement?
(A)He wants all the money
(B) He will return the money
(C) He will assume final responsibility
(D) He will resist all enquiries

## Two Marks : Set - 01

1.24 The Palghat Gap (or Palakkad Gap), a region about 30 km wide in the southern part of the western Ghats in India, is lower than the hilly terrain to its north and south. The exact reasons for the formation of this gap are not clear. It results in the neighbouring regions of Tamil Nadu getting more rainfall from the south west monsoon and the neighbouring regions of Kerala having higher summer temperatures.

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What can be inferred from this passage?
(A)The Palghat gap is caused by high rainfall and high temperatures in southern Tamil Nadu and Kerala.
(B) The regions in Tamil Nadu and Kerala that are near the Palghat Gap are low lying.
(C) The low terrain of the Palghat Gap has a significant impact on weather patterns in neighbouring parts of Tamil Nadu and Kerala.
(D) Higher summer temperatures result in higher rainfall near the Palghat Gap area.
1.25 Geneticists say that they are very close to confirming the generating roots of psychiatric illnesses such as depression and schizophrenia, and consequently, that doctors will be able to eradicate these diseases through early identification and gene therapy.
On which of the following assumptions does the statement above rely?
(A)Strategies are now available for eliminating psychiatric illnesses.
(B) Certain psychiatric illnesses have a genetic basis.
(C) All human diseases can be traced back to genes and how they are expressed.
(D)In the future, genetics will become the only relevant field for identifying psychiatric illnesses.

## One Mark : Set - 02

1.26 Choose the most appropriate phrase from the options given below to complete the following sentence:
India is post-colonial country because

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(A) It was a former British colony.
(B) Indian information Technology professionals have colonized the world.
(C) India does not follow any colonial practices.
(D) India has helped other countries gain freedom.
1.27 Who $\qquad$ was coming to see us this evening?
(A) you said
(B) did you say
(C) did you say that
(D) Had you said
1.28 Match the columns.

## Column 1

1. Eradicate
2. Distort
3. Saturate
4. Utilize

## Column 2

P. Misrepresent
Q. Soak Completely
R. Use
S. Destroy utterly
(A) 1:S, 2:P, 3:Q, 4:R
(B) 1:P, 2:Q, 3:R, $4: S$
(C) $1: Q, 2: R, 3: S, 4: P$
(D) 1:S, 2:P, 3;R, 4:Q

## Two Marks : Set - 02

1.29 The old city of Koenigsberg, which had a German majority population before World War 2, is now called Kaliningrad. After the events of the war, Kaliningrad is now a Russian territory and has a predominantly Russian population. It is bordered by the Baltic sea on the north and the countries of Poland to the south and west and Lithuania to the east respectively. Which of the statements below can be inferred from this passage?
(A) Kaliningrad was historically Russian in its ethnic make up.
(B) Kaliningrad is a part of Russia despite it not being contiguous with the rest of Russia.
(C) Koenigsberg was renamed Kaliningrad, as that was its original Russian name.
(D)Poland and Lithuania are on the route from Kaliningrad to the rest of Russia.
1.30 The number of people diagnosed with dengue fever (contracted from the bite of a mosquito) in north India is twice the number diagnosed last year. Municipal authorities have concluded that measures to control the mosquito population have failed in this region.
Which one of the following statements, if true, does not contradict this conclusion?
(A) A high proportion of the affected population has returned from neighbouring countries where dengue is prevalent.
(B) More cases of dengue are now reported because of an increase in the municipal office's administrative efficiency.
(C) Many more cases of dengue are being diagnosed this year since the introduction of a new and effective diagnostic test.
(D) The number of people with malarial fever (also contracted from mosquito bites) has increased this year.

## One Mark : Set - 03

1.31 While trying to collect an envelope I from under the table, Mr . X fell down II III and was losing consciousness.

IV

Which one of the above underlined parts of the sentence is NOT appropriate?
(A)I
(B) II
(C) III
(D)IV
$\mathbf{1 . 3 2}$ If she $\qquad$ how to calibrate the instrument, she $\qquad$ done the experiment.
(A) knows, will have
(B) knew, had
(C) had known, could have
(D) should have known, would have
1.33 Choose the word that is opposite in meaning to the word "coherent".
(A) sticky
(B) well-connected
(C) rambling
(D) friendly

## Two Marks : Set - 03

1.34 A dance programme is scheduled for 10:00 am, some students are participating in the programme and they need to come an hour earlier than the start of the event. These students should be accompanied by a parent. Other students and parents should come in time for the programme. The instruction you think that is appropriate for this is
(A) Students should come at 9:00 am and parents should come at 10:00 am.
(B) Participating students should come at 9:00 am accompanied by a parent, and other parents and students should come by 10:00 am.
(C) Students who are not participating should come by 10:00 am and they should not bring their parents. Participating students should come at 9:00 am.
(D)Participating students should come before 9:00 am parents who accompany them should come at 9:00 am. All others should come at 10:00.
1.35 By the beginning of the $20^{\text {th }}$ century, several hypotheses were being proposed, suggesting a paradigm shift in our understanding of the universe. However, the clinching evidence was provided by experimental measurements of the position of a star which was directly behind our sun.
Which of the following inference (s) may be drawn from the above passage?
(i) Our understanding of the universe changes based on the positions of stars.
(ii) Paradigm shifts usually occur at the beginning of centuries.
(iii)Stars are important objects in the universe.
(iv)Experimental evidence was important in confirming this paradigm shift.
(A)(i), (ii) and (iv)
(B) (iii) only
(C) (i) and (iv)
(D) (iv) only

## 2015 IIT Kanpur

## One Mark : Set - 01

1.36 Which of the following options is the closest in meaning to the sentence below?
She enjoyed herself immensely at the party.
(A) She had a terrible time at the party.
(B) She had a horrible time at the party.
(C) She had a terrific time at the party.
(D) She had a terrifying time at the party.
1.37 Didn't you buy $\qquad$ when you went shopping?
(A) any paper
(B) much paper
(C) no paper
(D) a few paper
1.38 Which one of the following combinations is incorrect?
(A) Acquiescence - Submission
(B) Wheedle - Roundabout
(C) Flippancy - Lightness
(D) Profligate - Extravagant

## Two Marks : Set - 01

1.39 Select the alternative meaning of the underlined part of the sentence.
The chain snatchers took to their heels when the police party arrived.
(A) took shelter in a thick jungle.
(B) open indiscriminate fire.
(C) took to flight.
(D) unconditionally surrendered.
1.40 The given statement is followed by some courses of action. Assuming the statement to be true decide the correct option.

## Statement :

There has been a significant drop in the water level in the lakes supplying water to the city.
I. The water supply authority should impose a partial cut in supply to tackle the situation.
II. The government should appeal to all the residents through mass media for minimal use of water.
III. The government should ban the water supply in lower areas.
(A) Statements I and II follow.
(B) Statements I and III follow.
(C) Statements II and III follow.
(D) All statements follow.

## One Mark : Set - 02

1.41 Choose the statement where underlined word is used correctly.
(A) The industrialist had a personnel jet.
(B) I write my experience in my personnel diary.
(C) All personnel are being given the day off.
(D)Being religious is a personnel aspect.
1.42 A generic term that includes various items of clothing such as a skirt, a pair of trousers and a shirt is
(A) fabric
(B) textile
(C) fibre
(D) apparel
1.43 We $\qquad$ our friend's birthday and we $\qquad$ how to make it up to him.
(A) completely forgot---don't just know
(B) forgot completely---don't just know
(C) completely forgot---just don't know
(D) forgot completely---just don't know

## Two Marks : Set - 02

1.44 Out of the following four sentences, select the most suitable sentence with respect to grammar and usage :
(A) Since the report lacked needed information, it was of no use to them.
(B) The report was useless to them because there were no needed information in it.
(C) Since the report did not contain the needed information, it was not real useful to them.
(D) Since the report lacked needed information, it would not had been useful to them.

## 2016 IISc Bangalore

## One Mark : Set - 01

1.45 The man who is now Municipal Commissioner worked as $\qquad$ .
(A) the security guard at a university.
(B) a security guard at the university.
(C) a security guard at university.
(D) the security guard at the university.
1.46 Nobody knows how the Indian cricket team is going to cope with the difficult and seamer-friendly wickets in Australia.
Choose the option which is closest in meaning to the underlined phrase in the above sentence.
(A) put up with
(B) put in with
(C) put down to
(D) put up against
1.47 Find the odd one in the following group of words.
mock, deride, praise, jeer
(A) mock
(B) deride
(C) praise
(D) jeer

## Two Marks : Set - 01

1.48 Computers were invented for performing only high-end useful computations. However, it is no understatement that they have taken over our world today. The internet, for example, is ubiquitous. Many believe that the internet itself is an unintended consequence of the original invention. With the advent of mobile computing on our phones, a whole new dimension is now enabled. One is left wondering if all these developments are good or, more importantly, required.
Which of the statement(s) below is/are logically valid and can be inferred from the above paragraph?
(i) The author believes that computers are not good for us.
(ii) Mobile computers and the internet are both intended inventions.
(A) (i) only
(B) (ii) only
(C) both (i) and (ii)
(D) neither (i) nor (ii)
1.49 All hill-stations have a lake. Ooty has two lakes.
Which of the statement(s) below is/are logically valid and can be inferred from the above sentences?
(i) Ooty is not a hill-station.
(ii) No hill-station can have more than one lake.
(A) (i) only
(B) (ii) only
(C) both (i) and (ii)
(D) neither (i) nor (ii)

## One Mark : Set - 02

1.50 The chairman requested the aggrieved shareholders to $\qquad$ him.
(A) bare with
(B) bore with
(C) bear with
(D) bare
1.51 Identify the correct spelling out of the given options :
(A)Managable
(B) Manageable
(C) Mangaeble
(D) Managible
1.52 R2D2 is a robot. R2D2 can repair aeroplanes. No other robot can repair aeroplanes.
Which of the following can be logically inferred from the above statements?
(A)R2D2 is a robot which can only repair aeroplanes.
(B) R2D2 is the only robot which can repair aeroplanes.
(C) R2D2 is a robot which can repair only aeroplanes.
(D) Only R2D2 is a robot.

## Two Marks : Set - 02

1.53 A poll of students appearing for masters in engineering indicated that $60 \%$ of the students believed that mechanical engineering is a profession unsuitable for women. A research study on women with masters or higher degrees in mechanical engineering found that $99 \%$ of such women were successful in their professions.
Which of the following can be logically inferred from the above paragraph?
(A) Many students have misconceptions regarding various engineering disciplines.
(B) Men with advanced degrees in mechanical engineering believe women are well suited to be mechanical engineers.
(C) Mechanical engineering is a profession well suited for women with masters or higher degrees in mechanical engineering.
(D) The number of women pursing higher degrees in mechanical engineering is small.
1.54 Sourya committee had proposed the establishment of Sourya Institute of Technology (SITs) in line with Indian Institutes of Technology (IITs) to cater to the technology and industrial needs of a developing country.
Which of the following can be logical inferred from the above sentence?
Based on the proposal,
(i) In the initial years, SIT students will get degrees from IIT.
(ii) SITs will have a distinct national objective.
(iii)SIT like institutions can only be established in consultation with IIT.
(iv)SITs will serve technological needs of a developing country.
(A) (iii) and (iv) only
(B) (i) and (iv) only
(C) (ii) and (iv) only
(D) (ii) and (iii) only

## 2017 IIT Roorkee

## One Mark : Set - 01

1.55 Research in the workplace reveals that people work for many reasons $\qquad$ .
(A)money beside.
(B) beside money.
(C) money besides.
(D) besides money.
1.56 After Rajendra Chola returned from his voyage to Indonesia, he $\qquad$ to visit the temple in Thanjavur.
(A) was wishing
(B) is wishing
(C) wished
(D) had wished
1.57 Rahul, Murali, Srinivas and Arul are seated around a square table. Rahul is sitting to the left of Murali, Srinivas is sitting to the right of Arul. Which of the following pairs are seated opposite each other?
(A)Rahul and Murali
(B) Srinivas and Arul
(C) Srinivas and Murali
(D) Srinivas and Rahul

## Two Marks : Set - 01

1.58 "The hold of the nationalist imagination on our colonial past is such that anything inadequately or improperly nationalist is just not history".

Which of the following statements best reflects the author's opinion?
(A) Nationalist are highly imaginative.
(B) History is viewed through the filter of nationalism.
(C) Our colonial past never happened.
(D) Nationalism has to be adequately and properly imagined.

## One Mark : Set - 02

1.59 There are five buildings called $V, W, X$, $Y$ and $Z$ in a row (not necessarily in that order). $V$ is to the West of $W, Z$ is to the East of $X$ and the West of $V, W$ is to the West of $Y$. Which is the building in the middle?
(A) $V$
(B) $W$
(C) $X$
(D) $Y$
1.60 Saturn is $\qquad$ to be seen on a clear night with the naked eye.
(A) Enough bright
(B) Bright enough
(C) As enough bright
(D) Bright as enough
1.61 Choose the option with words that are not synonyms.
(A) Aversion, dislike
(B) Luminous, radiant
(C) Plunder, loot
(D) Yielding, resistant

## Two Marks : Set - 02

1.62 "We lived in a culture that denied any merit to literary works, considering them important only when they were handmaidens to something seemingly more urgent-namely ideology. This was a country where all gestures, even the most private were interpreted in political terms".

The author's belief that ideology is not as important as literature is revealed by the word :
(A) 'Culture'
(B) 'Seemingly'
(C) 'Urgent'
(D) 'Political'
1.63 There are three boxes. One contains apples, another contains oranges and the last one contains both apples and oranges. All three are known to be incorrectly labelled. If you are permitted to open just one box and then pull out and inspect only one fruit, which box would you open to determine the contents of all three boxes?
(A) The box labelled 'Apples'
(B) The box labelled 'Apples and Oranges'
(C) The box labelled 'Oranges'
(D) Cannot be determined

## 2018 IIT Guwahati

## One Mark

1.64 "Since you have gone off the $\qquad$ the
$\qquad$ sand is likely to damage the car". The words that best fill the blanks in the above sentence are
(A) course, coarse
(B) course, course
(C) coarse, course
(D) coarse, coarse
1.65 "A common misconception among writers is that sentence structure mirrors thought, the more $\qquad$ the structure, the more complicated the ideas."
The words that best fill the blanks in the above sentence are
(A) detailed
(B) simple
(C) clear
(D) convoluted

## 2019 IIT Madras

## One Mark

1.66 The passengers were angry $\qquad$ the airline staff about the delay.
(A) about
(B) towards
(C) with
(D) on
1.67 I am not sure if the bus that has been booked will be able to $\qquad$ all the students.
(A) accommodate
(B) sit
(C) deteriorate
(D) fill
1.68 Newspapers are a constant source of delight and recreation for me. The
$\qquad$ trouble is that I read $\qquad$ many of them.
(A) only, quite
(B) even, too
(C) only, too
(D) even, quite

## Two Marks

1.69 Consider five people - Mita, Ganga, Rekha, Lakshmi and Sana. Ganga is taller than both Rekha and Lakshmi. Lakshmi is taller than Sana. Mita is taller than Ganga.

Which of the following conclusions are true?

1. Lakshmi is taller than Rekha
2. Rekha is shorter than Mita
3. Rekha is taller than Sana
4. Sana is shorter than Ganga
(A) 2 and 4
(B) 1 only
(C) 1 and 3
(D) 3 only
1.70 An award-winning study by a group of researchers suggests that men are as prone to buying on impulse as women but women feel more guilty about shopping.

Which one of the following statements can be inferred from the given text?
(A) All men and women indulge in buying on impulse
(B) Some men and women indulge in buying on impulse
(C) Few men and women indulge in buying on impulse
(D) Many men and women indulge $m$ buying on impulse

## 2020 IIT Delhi

1.71 This book, including all its chapters,
$\qquad$ interesting. The students as well as instructor $\qquad$ in agreement about it.
(A)is, was
(B) is, are
(C) were, was
(D) are, are
1.72 Stock markets $\qquad$ at the news of the coup.
(A) plugged
(B) plunged
(C) poised
(D) probed
1.73 People were prohibited $\qquad$ their vehicles near the entrance of the main administrative building.
(A) to park
(B) to have parked
(C) from parking
(D) parking
1.74 Select the word that fits the analogy

Do : Undo : : Trust : $\qquad$
(A) Intrust
(B) Untrust
(C) Entrust
(D) Distrust

## Answers Verbal Ability

| 1.1 | B | 1.2 | A | 1.3 | D | 1.4 | C | 1.5 | D |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.6 | B | 1.7 | D | 1.8 | A | 1.9 | C | 1.10 | B |
| 1.11 | C | 1.12 | D | 1.13 | B | 1.14 | B | 1.15 | A |
| 1.16 | D | 1.17 | C | 1.18 | A | 1.19 | D | 1.20 | D |
| 1.21 | B | 1.22 | C | 1.23 | C | 1.24 | C | 1.25 | B |
| 1.26 | A | 1.27 | B | 1.28 | A | 1.29 | B | 1.30 | D |
| 1.31 | D | 1.32 | C | 1.33 | C | 1.34 | B | 1.35 | D |
| 1.36 | C | 1.37 | A | 1.38 | B | 1.39 | C | 1.40 | A |
| 1.41 | C | 1.42 | D | 1.43 | C | 1.44 | A | 1.45 | B |
| 1.46 | A | 1.47 | C | 1.48 | D | 1.49 | D | 1.50 | C |
| 1.51 | B | 1.52 | B | 1.53 | C | 1.54 | C | 1.55 | D |
| 1.56 | C | 1.57 | C | 1.58 | B | 1.59 | A | 1.60 | B |
| 1.61 | D | 1.62 | B | 1.63 | B | 1.64 | A | 1.65 | D |
| 1.66 | C | 1.67 | A | 1.68 | C | 1.69 | A | 1.70 | B |
| 1.71 | B | 1.72 | B | 1.73 | C | 1.74 | D |  |  |

## Explanations Verbal Ability

## 1.1 (B)

Circuitous : Deviating from a straight course; indirect
Cyclic : Recurring in cycle.
Indirect : Not leading by straight line.
Confusing : Lacking clarity.
Crooked : For shapes (irregular in shape).
E.g. a crooked path.

Hence, the correct option is (B).

## 1.2 (A)

A worker who is unemployed is unproductive just as a land which is fallow is unproductive.
Hence, the correct option is (A).

## 1.3 <br> (D)

Uphold : Cause to remain (not appropriate).
Restrain : Keep under control (not appropriate).
Cherish : Be fond of (not related).

Conserve : Keep in safety and protect from harm, decay, loss or destruction (most appropriate).
Hence, the correct option is (D).

## 1.4 (C)

Masked : Hide under a false appearance (opposite).
Belied : Be in contradiction with (not appropriate).
Betrayed : Reveal unintentionally (most appropriate).
Suppressed : To put down by force or authority (irrelevant).
Hence, the correct option is (C).

## 1.5 (D)

(A) Modern warfare has resulted in civil strife : There is no direct consequence of warfare given, so it is not appropriate.
(B) Chemical agents are useful in modern warfare : Passage does not say whether chemical agents are useful or not, so it is not appropriate.
(C) Use of chemical agents in warfare would be undesirable : Given that people in military think these are useful, undesirable is wrong.
(D) People in military establishment like to use chemical agents in war : Correct choice as last statement tells that military people think that chemical agents are useful tools for their cause (work silently in warfare).
Hence, the correct option is (D).

## 1.6 (B)

Frequency : Means occurring at frequent intervals. So opposite of it is rarity which means infrequency of occurrence.
Hence, (B) is the most appropriate answer.
Periodicity : The equality or state of being periodic; recurrence at regular intervals.
Rarity : The quality or state of being rare; infrequency of occurrence.
Gradualness : The quality of change in ascending or descending in moderate rate.
Persistency : Persistent determination; the quality of being determined to do or achieve something.
Hence, the correct option is (B).

## 1.7 (D)

Similar : Showing resemblance in qualities, characteristics, or appearance; alike but not identical.
Most : Superlative of many, much.
(i) Greatest in number.
(ii) Greatest in amount, extent, or degree.

## Uncommon :

(i) Not common; rare.
(ii) Wonderful; remarkable.

Available : Present and ready for use; at hand; accessible. (most appropriate)
Hence, the correct option is (D).

## $1.8 \quad$ (A)

Gladiator performs his action in arena, similarly dancer's performance required stage. A person, usually a professional combatant, a captive, or a slave, trained to entertain the public by engaging in mortal combat with another person or a wild animal in the ancient Roman arena.
Arena : An enclosed area for the presentation of sports events and spectacles.
Dancer : A performer who dances professionally.
Stage : A raised platform on which theatrical performances are presented.
Commuter : One that travels regularly from one place to another, as from suburb to city and back.
Train : A series of connected railroad cars pulled or pushed by one or more locomotives.
Teacher : One who teaches, especially one hired to teach.
Classroom : A room of place specially in a school in which classes are conducted.
Lawyer : One whose profession is to give legal advice and assistance to clients and represent them in court or in other legal matters.
Courtroom : A room in which the proceedings of a court are held.
Hence, the correct option is (A).

## 1.9 (C)

Identified : To establish the identity of, to ascertain the origin, nature, of definitive characteristics.

Ascertained : To discover with certainty, as through examination or experimentation.
Exacerbated : To increase the severity, violence, or bitterness of ; aggravate.
Analyzed : To examine in detail in order to discover meaning, essential features, etc.
Hence, the correct option is (C).

## $1.10 \quad$ (B)

The author in the paragraph states that horses built up immunities when they are injected with toxins of diseases i.e. horses are generally immune to diseases.
Hence, the correct option is (B).

### 1.11 (C)

There are a number of different ways to use 'should have been' or 'should have done' (i.e. should have + past participle).

For example :
You may also hear 'should have been/done' used when you expect a confirmation that something had a particular outcome in the past. This usage would be similar to 'must have been/done'
Hence, the correct option is (C).

### 1.12 (D)

Considering the first half of the statement we can depict that "the situation was serious / intense" but the second half suggests that "his efforts / reactions were commendable", thus it will be a "positive or a neutral word" but definetly not a "negative word."
Beggary : Extreme poverty, penury, the state of condition of being a beggar.
Nomenclature : The terminology used in particular science, art, activity, etc.
Jealousy : The state or quality of being jealous.
Nonchalance : The trait of remaining calm and seeming not to care, a causal lack of concern.
Hence, the correct option is (D).

### 1.13 (B)

Latitude : Freedom from normal restraints, limitations or regulations.

Eligibility : The quality or state of being eligible.
Freedom : The condition of being free of restraints.

Coercion : Refer to force.
Meticulousness : Extremely careful and precise.
Hence, the correct option is (B).

### 1.14 (B)

The error is 'he should be given'. The correct way of writing the sentence is 'he should give'.
Hence, the correct option is (B).

### 1.15 (A)

Here, there are three key words "legacy", "legions" and "Discipline". Even though the paragraph starts with "legacy" but the basic idea revolves around "Strict Discipline", the author has mentioned that the main reason because of which the legion is "obedient, intact and fighting" is "Discipline". In option (A), we can see that "Strict Discipline" has been featured as "Regimentation."
Hence, correct option is (A).

## $\square$ Key Point

Legion = Noun
(i) A division of the Roman army, usually comprising 3000 to 6000 soldiers.
(ii) A military or semi-military unit.

Regimentation $=$ Noun
(i) The act of regimenting or the state of being regimented.
(ii) The strict discipline and enforced uniformity characteristic of military groups.

### 1.16 (D)

"Two and two make four". The word "make" means "amount to". Two and two amount to (or make) four.
"Two and two are four" suggests that they are separate numbers.
Grammatically both options (C) and (D) are used in different sense. Hence, the general usage is two and two make four.
Hence, the correct option is (D).

### 1.17 (C)

"You can always give me a ring whenever you need."
This statement, means that whenever there is any problem or need, feel free to call me. It simply means "a friend is need is a friend indeed."
Hence, the correct option is (C).

### 1.18 (A)

'Dare to commit mistakes'
This is a continuous sentence, not a complete one. Hence, option (B) is incorrect. The sentence is in simple present tense. Hence, options (C) and (D) are also incorrect.
'Dare commit mistakes'
It is a complete sentence on its own.
Hence, the correct option is (A).

### 1.19 (D)

Make out : To cause to exist or happen; bring about; to understand.
Call out : To cause to assemble ; summon.
Dig out : To remove; unearth.
Fall out : To quarrel.
The closest option is fall out.
Hence, the correct option is (D).

### 1.20 (D)

In the above statement, the movement in colonial India is carried out by different persons of different philosophies.

For example, some are moderates, some are liberals, etc. Therefore, different philosophies shows heterogeneous character. Thus, Nationalism in India is heterogeneous. Hence, the correct option is (D).

### 1.21 (B)

'Cope with' means to 'adapt to' which means adjust to something.
Adopt to : Take legally.
Adept in : Skillful or Skilful.
Accept with : Agree.
Hence, the correct option is (B).

### 1.22 (C)

There is a contrast in the sentence, 'he could not understand why the judges were awarding her the first prize because he thought that her performance was' not up to the mark OR not very good.
Superb : Excellent.
Medium : A way of doing something.
Mediocre : Not very good.
Exhilarating : Feeling happy or thrilling.
As her performance was not so good, the word 'Mediocre' suffices the requirement.
Hence, the correct option is (C).

### 1.23 (C)

'The buck stops here' means that responsibility for something should not be passed to someone else i.e. assume the final responsibility.
Hence, the correct option is (C).

### 1.24 (C)

Option (A) can not be inferred from the passage because the palghat gap is causing the high rainfall and high temperatures, not the other way around. Option (B) and (D) are nowhere nearly mentioned in the passage.
Therefore, it can be inferred from the pasage that the low terrain of the palghat gap has a significant impact on weather patterns in neighbouring parts of Tamil Nadu and Kerala.
Hence, the correct option is (C).

### 1.25 (B)

As it has been indicated that doctors will be able to eradicate psychiatric diseases such as depression and schizophrenia through gene therapy. So, these have a genetic basis.
Hence, the correct option is (B).

### 1.26 (A)

India is a post colonical country because it was a former british colony.
Hence, the correct option is (A).

### 1.27 (B)

When a simple sentence is converted to question form and the sentence is in past tense then in the question form, verb is changed to present tense with a 'did' added to the sentence. Therefore, only option (B) or (C) can be correct. Since, 'that' refers to non-living thus option (C) is incorrect.
Hence, the correct option is (B).

### 1.28 (A)

Eradicate : Destroy utterly.
Distorts : Misrepresent.
Saturate : Soak completely.
Utilize : Use, consume.
This indicates that the correct sequence must be $\mathrm{S}, \mathrm{P}, \mathrm{Q}$ and R .
Hence, the correct option is (A).

### 1.29 (B)

The author states in the passage that after World War-II Kaliningrad became Russian territory but it is not contiguous with Russia because it used to have a German majority population.
Hence, the correct option is (B).

### 1.30 (D)

Municipal authorities admitted that it has been unable to control the mosquito population which is why the number of people with malarial fever has increased this year.
Hence, the correct option is (D).

### 1.31 (D)

The statement clearly says that 'while trying to collect an envelope from under the table, Mr. X fell down' which is in past tense. The statement continues with 'and' so the rest of the part of the sentence should also be in past tense (and lost consciousness). Therefore, part (IV) of the sentence is incorrect.
Hence, the correct option is (D).

### 1.32 (C)

Option (A) : Knows, will have
Present tense, future tense
Option (B) : Knew, had
Past tense, past tense
Option (C) : Had known, could have
Past perfect tense, perfect conditional
Option (D) : Should have known, would have Present perfect tense, future tense
In a type three conditional sentence, the tense in 'If' clause is past prefect and the tense in the main clause is prefect continuous conditional.
Hence, the correct option is (C).

### 1.33 (C)

Coherent : Logically consistent i.e. unified whole.
Rambling : Lengthy and confused.
The words sticky, well-connected and friendly are self-explanatory.
Hence, the correct option is (C).

### 1.34 (B)

The passage states that students participating in the programme, should come with a parent one hour earlier ( 9 am ) and other students and parents should come in time for the programme (10 am) which clearly directs to option (B).
Hence, the correct option is (B).

### 1.35 (D)

The experimental measurements of the position of a star directly behind our sun gives proof for the hypothesis.
Hence, the correct option is (D).

## $1.36 \quad$ (C)

The phrase "enjoyed herself immensely" means she had a fantastic or terrific time.
Hence, the correct option is (C).

### 1.37 (A)

For the given sentence, "any paper" is the best and only suitable option.
Hence, the correct option is (A).

### 1.38 (B)

Acquiescence : The reluctant acceptance of something without protest.
Submission : The action of accepting or yielding to a superior force.
Wheedle : Get to do something by gentle urging special attention or flattery.
Roundabout : Not straight forward or direct.
Flippancy : Lack of respect or seriousness.
Lightness : Lack of depth or seriousness.
Profligate : Recklessly extravagant or wasteful in the use of resources.
Extravagant : Lack of restraint or exceeding what is reasonable.
Only the combination in option (B) have different meanings.
Hence, the correct option is (B).

### 1.39 (C)

"Took to their heels" is an idiom which means running away or "took to flight". Hence, the correct option is (C).

### 1.40 (A)

The first two statements are appropriate to tackle the situation.
The third statement implies to put a ban on water supply in lower areas which is morally wrong and should not be done.
Hence, the correct option is (A).

### 1.41 (C)

Personnel : A person or a group of people employed in an organization or place of work.
Personal : Relating to a particular person or individual.

Only in the statement in option (C) word personnel is used appropriately.
Hence, the correct option is (C).

### 1.42 (D)

Fabric: A cloth made by weaving or knitting fibres.
Textile : Any cloth or goods produced by weaving or knitting.
Fibre : A fine, threadlike piece, as of cotton, jute or asbestos.
Apparel : A garment, clothing especially outerwear or attire.
Hence, the correct option is (D).

### 1.43 (C)

'Completely' is an adverb which should be used before the verb 'forgot'. And 'just' should be used before "don't know".
The words in option (C) are arranged in right sequence.
Hence, the correct option is (C).

### 1.44 (A)

Correct versions :
Option (B) : The report was useless to them because there was no needed information in it.
Option (C) : Since the report did not content the needed information, it was not really useful to them.
Option (D) : Since the report lacked needed information, it would not have been useful to them.
Therefore only option (A) is correct with respect to grammar and usage.
Hence, the correct option is (A).

### 1.45 (B)

Options (A) and (D) are incorrect because the post of security guard is a general post. So 'the security' is not correct. The word 'university' is always prefixed with 'the' being important. So, option (C) is also incorrect.
Hence, the correct option is (B).

### 1.46 (A)

"cope with" means to put up with or tolerate.
Hence, the correct option is (A).

### 1.47 (C)

Mock, deride and jeer are synonyms which means mockery. Therefore, the odd one is 'praise'.
Hence, the correct option is (C).

### 1.48 (D)

Statement 1 in the question is nowhere nearly mentioned in the passage.
It is clearly said in the passage that internet is an unintended consequence which makes statement II wrong.
Hence, the correct option is (D).

### 1.49 (D)

Statement (i) is not true because Ooty is a hill station and it has two lakes. Statement (ii) is also not true, because in given sentences, for hill stations at least one lake is compulsory but nothing is mentioned about number of lakes.
Hence, the correct option is (D).

## $1.50 \quad$ (C)

Bare with : invitation to undress.
Bore with : past tense of bear with.
Bear with : tolerance or patience
Bare : it is an adjective and it means lacking the appropriate covering.
Hence, the correct option is (C).

### 1.51 (B)

'Manageable' is the correct spelling. Hence, the correct option is (B).

## $1.52 \quad$ (B)

The statement 'No other robot can repair aeroplanes' means R2D2 is the only robot which can repair aeroplanes and is the best inference drawn.
Hence, the correct option is (B).

### 1.53 (C)

A poll says that women with masters or higher degrees in mechanical engineering are successful in their professions. This statement leads to the option (C), which is the best inference drawn.
Hence, the correct option is (C).

## $1.54 \quad$ (C)

In the passage it is said that SIT will cater to the technology industrial needs of a developing country. Statement (i) and (iii) state phrases like 'in the initial years' and 'SIT like institutions can only be established in consultation with IIT' cannot be logically inferred, so (ii) and (iv) are the best inferences.
Hence, the correct option is (C).

### 1.55 (D)

Besides means in addition to or apart from. The statement says that 'people work for many reasons' which will continue with apart from money. So, the correct assertion is,
"Research in the workplace reveals that people work for many reasons besides money".
Hence, the correct option is (D).

### 1.56 (C)

When the main clause is in the past tense or past perfect tense, the subordinate clause must be in the past or past perfect tense. So, the correct assertion is,
"After Rajendra Chola returned from his voyage to Indonesia, he wished to visit the temple in Thanjavur".
Hence, the correct option is (C).

### 1.57 (C)

## Given :

(i) Rahul, Murali, Srinivas and Arul are sitting around a square table.
(ii) Rahul is just left to Murali.
(iii) Srinivas is just right to Arul.


It can be concluded that Rahul is opposite to Arul and Srinivas is opposite to Murali.
Hence, the correct option is (C).

### 1.58 (B)

The author said in the passage that everything related to colonial past has a hold of nationalism in people's imagination. Nationalism that is improperly represented is not history that is History is viewed through the filter of nationalism.

This statement best reflects the author's opinion based on the given paragraph.
Hence, the correct option is (B).

### 1.59 (A)

Given : There are 5 buildings $V, W, X, Y$ and $Z$.
$V$ is to the West of $W$ : (V)-W
$Z$ is to the East of $X: \quad X-(Z)$
$Z$ is also to the West of $V:(\boldsymbol{X}-\boldsymbol{Z})-\boldsymbol{V}-\boldsymbol{W}$
$W$ is to the West of $Y: \quad(\boldsymbol{X}-\boldsymbol{Z}-\boldsymbol{V}-\boldsymbol{W})-\boldsymbol{Y}$
The building in the middle is $\boldsymbol{V}$.
Hence, the correct option is (A).

### 1.60 (B)

With adjectives and adverb, enough comes after adjectives and adverbs.
With nouns, enough comes before noun.
In the given question, enough is used with bright which is an adjective, so enough will come after the adjective.
So bright enough is the correct option (B).
Hence, the correct option is (B).

### 1.61 (D)

Yielding : Tending to give up under pressure.
Resistant : Offer resistance or opposing.

Luminous : Bright or shining.
Radiant : Glowing brightly
Aversion : Strong dislike
Plunder : Rob or steal goods
Loot : Rob, sack
Hence, the correct option is (D).

### 1.62 (B)

Seemingly means external appearance as disguised from true character.
'Considering them important only when they were handmaidens (means helping) to something seemingly more urgent' but in reality it is not so urgent.
So, ideology is not as important as literature is revealed by the word "seemingly" only.
Hence, the correct option is (B).

### 1.63 (B)

Given : All boxes have been labelled incorrectly.

So,
Statement 1 : Box labelled "Apples" is either "Oranges" or "Apples and Oranges".

Statement 2 : Box labelled "Oranges" is either "Apples" or "Apples and Oranges".
Statement 3 : Box labelled "Apples and Oranges" is either "Apples" or "Oranges".
After opening the box labelled "Apples and Oranges", if we get Apple in it then it is sure that the box labelled "Apples and Oranges" is actually Apple.
So, Box labelled "Oranges" can have either only 'oranges' or 'apples and oranges'. Since, all boxes are incorrectly labelled, box labelled as oranges can not have only oranges. Therefore, it must be having oranges and apple and box labelled "Apples" has to be "Oranges".
Hence, the correct option is (B).

### 1.64 (A)

course : The route or direction followed by a ship, air craft or road.
coarse : Rough or harsh in texture.
Hence, the correct option is (A).

### 1.65 (D)

The misconception is that ideas or thoughts mirror the sentence structure i.e. thoughts and sentence structure are similar. If one of them is complicated then the other will be as well.
Convoluted : Extremely complex and difficult to follow.
From all the options, convoluted is the best suitable answer.
Hence, the correct option is (D).

### 1.66 (C)

While expressing certain feelings towards someone, we use the word 'with' or 'of'. For example:
(i) I am disappointed with you.
(ii) I like being with you.
(iii) I am so proud of you.

Therefore, the correct flow of the sentence will be,
"The passengers were angry with the airline staff about the delay."
Hence, the correct option is (C).

### 1.67 (A)

Accommodate : Fit in according to requirement or providing sufficient space. In the given options, only accommodate is the appropriate word.
Hence, the correct option is (A).

### 1.68 (C)

In the given statement, use of 'even' is inappropriate because it makes the sentence meaningless. Therefore, option (B) and (D) are
incorrect. 'Quite' cannot be used before many (grammatically incorrect).
Hence, the correct option is (C).

### 1.69 (A)

From the given information, we can draw as follows


There is no clear information about height of Rekha except that Rekha is shorter than Mita and Ganga. Sana is shorter than Mita, Ganga and Lakshmi.
Hence, the correct option is (A).

## $1.70 \quad$ (B)

In the given passage, it is not mention whether all or a few or a lot of men and women are prone to buying on impulse. So the use of 'some' is appropriate.
Hence, the correct option is (B).

### 1.71 (B)

'Book' is a singular noun, so it will take singular verb 'is'
When 'as well as' is part of the sentence then the verb must agree with the noun before 'as well as'.
So "the students as well as" will take the verb 'are'
Hence the correct option is (B).

### 1.72 (B)

The word 'Coup' means a sudden, illegal and often violent change of government and this situation will lead to sudden 'jump'; drop or fall
of shares for which the appropriate word is ＇plunged＇
Hence，the correct option is（B）．

### 1.73 （C）

The verb＇prohibit＇means to forbid or prevent， it is followed by the preposition＇form＇．
Hence，the correct option is（C）．

### 1.74 （D）

Given words in the first pair are antonyms， correct antonym for Trust is distrust．

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