ELECTRICAL MACHINES II

Lecturer: Dr. Suad Ibrahim Shahl

Syllabus

I. Introduction to AC Machine
II. Synchronous Generators
III. Synchronous Motors
IV. Three-Phase Induction Machines
V. Three-Phase Induction Motors
VI. Induction Generators
VII. Induction Regulators

Recommended Textbook :

1) M.G.Say
   Alternating Current Machines
   Pitman Pub.
2) A.S. Langsdorf
   Theory of AC Machinery
   McGRAW-HILL Pub.

I. Introduction to AC Machines

Classification of AC Rotating Machines

•Synchronous Machines:

•Synchronous Generators: A primary source of electrical energy.

•**Synchronous Motors:** Used as motors as well as power factor compensators (synchronous condensers).

•Asynchronous (Induction) Machines:

•**Induction Motors:** Most widely used electrical motors in both domestic and industrial applications.

•**Induction Generators:** Due to lack of a separate field excitation, these machines are rarely used as generators.

Energy Conversion

- Generators convert mechanical energy to electric energy.
- Motors convert electric energy to mechanical energy.
- The construction of motors and generators are similar.
- Every generator can operate as a motor and vice versa.
- The energy or power balance is :
  - Generator: Mechanical power = electric power + losses
  - Motor: Electric Power = Mechanical Power + losses.

AC winding design

The windings used in rotating electrical machines can be classified as

- ❖ **Concentrated Windings**
  - All the winding turns are wound together in series to form one multi-turn coil
  - All the turns have the same magnetic axis
  - Examples of concentrated winding are
    - field windings for salient-pole synchronous machines
    - D.C. machines
    - Primary and secondary windings of a transformer

- ❖ **Distributed Windings**
  - All the winding turns are arranged in several full-pitch or fractional-pitch coils
  - These coils are then housed in the slots spread around the air-gap periphery to form phase or commutator winding
  - Examples of distributed winding are
    - Stator and rotor of induction machines
    - The armatures of both synchronous and D.C. machines

Armature windings, in general, are classified under two main heads, namely,

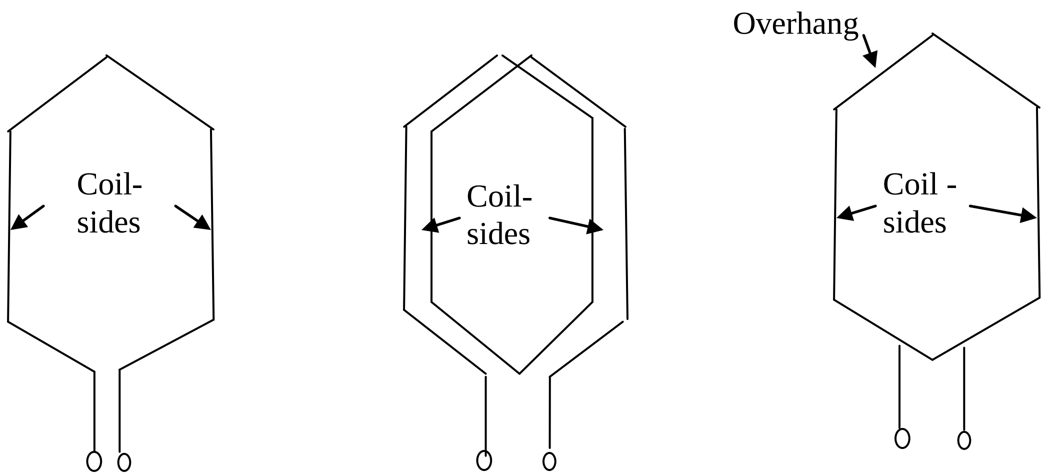
- ❖ **Closed Windings**
  - There is a closed path in the sense that if one starts from any point on the winding and traverses it, one again reaches the starting point from where one had started
  - Used only for D.C. machines and A.C. commutator machines

- ❖ **Open Windings**
  - Open windings terminate at suitable number of slip-rings or terminals
  - Used only for A.C. machines, like synchronous machines, induction machines, etc

Some of the terms common to armature windings are described below:

1. **Conductor.** A length of wire which takes active part in the energy-conversion process is a called a conductor.
2. **Turn.** One turn consists of two conductors.
3. **Coil.** One coil may consist of any number of turns.
4. **Coil –side.** One coil with any number of turns has two coil-sides.

The number of conductors ***(C)*** in any coil-side is **equal** to the number of turns ***(N)*** in that coil.

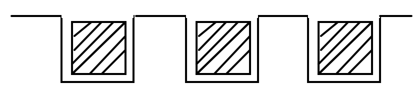


One-turn coil **two-turn coil** **multi-turn coil**

5. Single- layer and double layer windings.

❖ ***Single- layer winding***

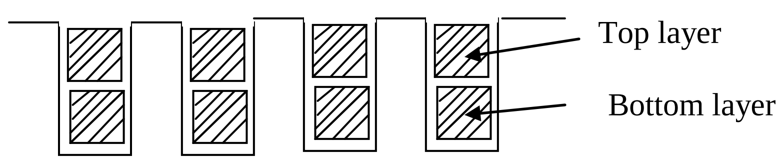
- One coil-side occupies the total slot area
- Used only in small ac machines



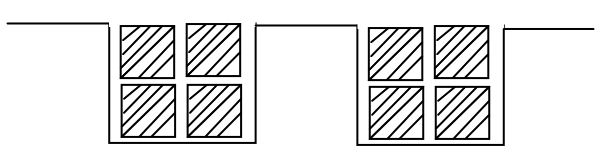
one coil-side per slot

❖ ***Double- layer winding***

- Slot contains even number (may be 2,4,6 etc.) of coil-sides in two layers
- Double-layer winding is more common above about 5kW machines



Two coil –sides per slot

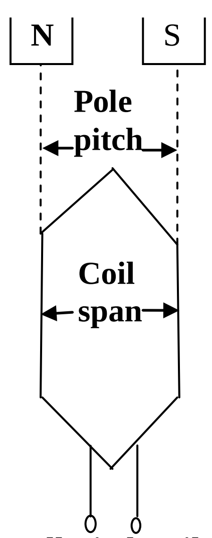


4-coil-sides per slot

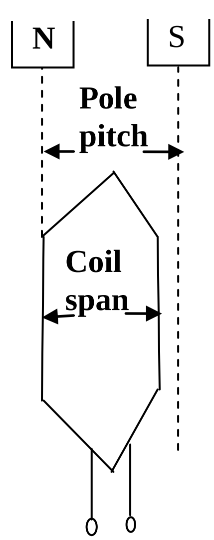
The advantages of double-layer winding <u>over</u> single layer winding are as follows:

a. Easier to manufacture and lower cost of the coils
b. Fractional-slot winding can be used
c. Chorded-winding is possible
d. Lower-leakage reactance and therefore , better performance of the machine
e. Better emf waveform in case of generators

6. **Pole – pitch.** A pole pitch is defined as the peripheral distance between identical points on two adjacent poles. Pole pitch is always equal to 180^o electrical.
7. **Coil–span or coil-pitch.** The distance between the two coil-sides of a coil is called coil-span or coil-pitch. It is usually measured in terms of teeth, slots or electrical degrees.
8. **Chorded-coil.**
   - ❖ If the coil-span (or coil-pitch) is **<u>equal</u>** to the pole-pitch, then the coil is termed a ***full-pitch coil***.
   - ❖ in case the coil-pitch is **<u>less</u>** than pole-pitch, then it is called ***chorded***, ***short-pitch*** or ***fractional-pitch coil***
     - ➢ if there are ***S*** slots and ***P*** poles, then pole pitch $\boldsymbol{Q = \frac{S}{P}}$ slots per pole
     - ➢ if coil-pitch $\boldsymbol{y = \frac{S}{P}}$, it results in ***full-pitch winding***
     - ➢ in case coil-pitch $\boldsymbol{y < \frac{S}{P}}$, it results in ***chorded***, ***short-pitched*** or ***fractional-pitch***



Full-pitch coil



Short-pitched or chorded coil

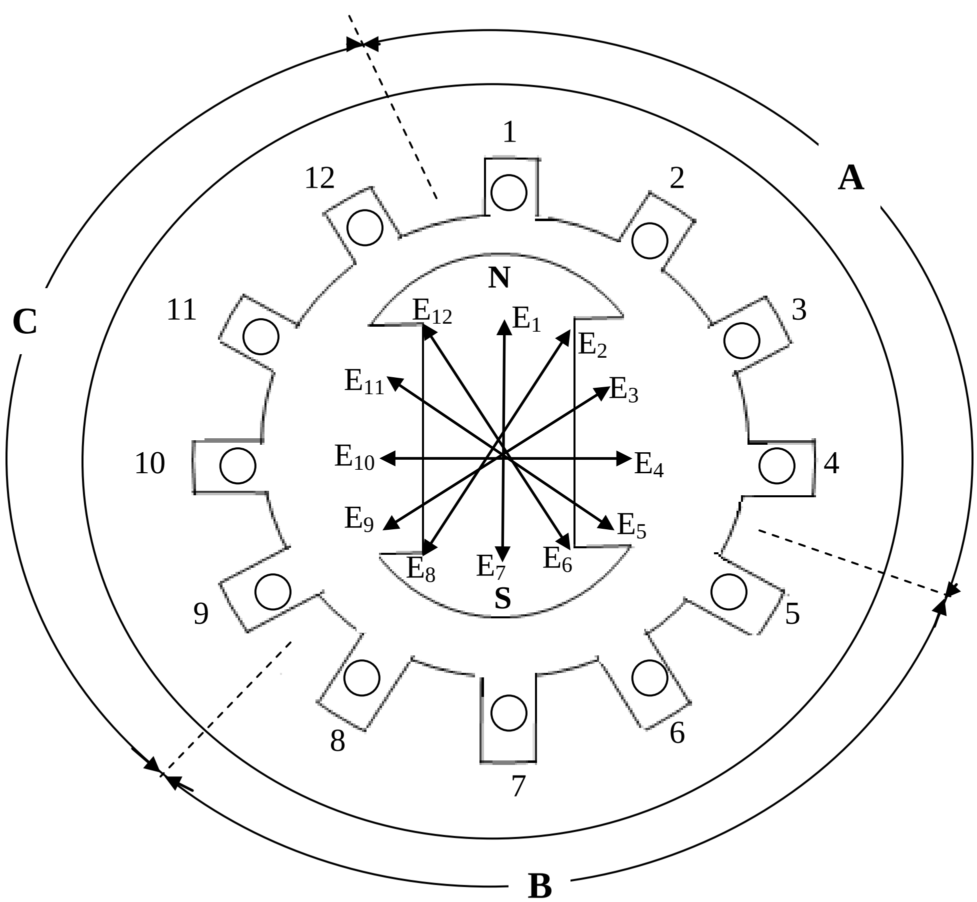
In ***AC armature windings***, the separate coils may be connected in several different manners, but the two most common methods are ***lap*** and ***wave***

In polyphase windings it is essential that

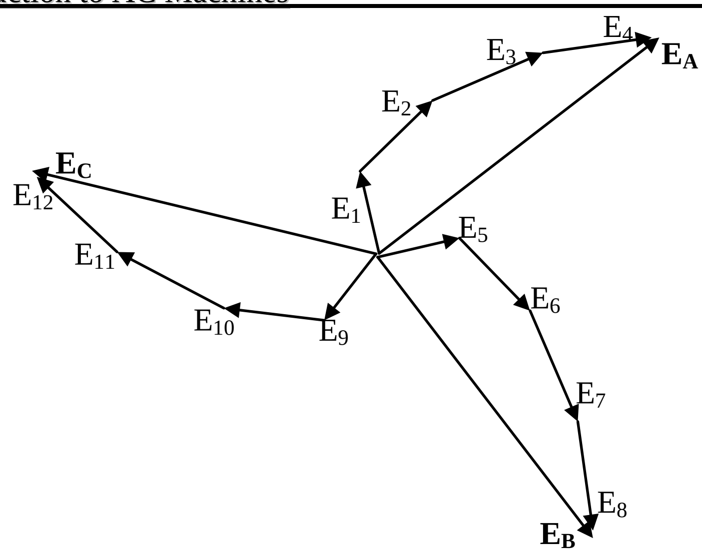
- The generated ***emfs*** of all the phases are of ***equal magnitude***
- The ***waveforms*** of the ***phase emfs*** are ***identical***
- The ***frequency*** of the ***phase emfs*** are ***equal***
- The ***phase emfs*** have mutual time-phase displacement of $\boldsymbol{\beta = \frac{2\pi}{m}}$ electrical radians. Here ***m*** is the number of phases of the a.c. machine.

Phase spread

Where field winding on the rotor to produce 2 poles and the stator carries 12 conductors housed in 12 slots.



3-phase winding - phase spread is 120°



Time phase angle is 120° between E_A, E_B and E_C

- Maximum emf $\mathbf{E_m}$ induced in **conductor 1** $\left(E_1 = \frac{E_m}{\sqrt{2}}\right)$
- **Zero** emf induced in **conductor 4** (conductor 4 is cutting zero lines of flux)
- the emf generated in **conductor 7** is maximum (conductor 7 is cutting maximum lines of flux from ***S*** pole)
- the polarity of emf in ***conductor 7*** will be opposite to that in ***conductor 1***, $\boldsymbol{E_7 = \frac{E_m}{\sqrt{2}}}$, opposite to $\boldsymbol{E_1}$
- similarly the emfs generated in conductors 2, 3, 5, 6 and in conductor 8 to 12 can be represented by phasors E_2, E_3 , E_5 , E_6 and E_8 to E_{12}
- the **slot angle pitch** is given by $\gamma = \frac{180^o}{Slots\ per\ pole} = \frac{180^o}{6} = 30^o$
- if

$$\left.\begin{array}{r} back\ end\ of\ conductor\ \mathbf{1}\ is\ connected\ to\ back\ end\ of\ conductor\ \mathbf{2} \\ front\ end\ of\ conductor\ \mathbf{2}\ is\ connected\ to\ front\ end\ of\ conductor\ \mathbf{3} \\ back\ end\ of\ conductor\ \mathbf{3}\ is\ connected\ to\ back\ end\ of\ conductor\ \mathbf{3} \end{array}\right\}\ E_A = E_1 + E_2 + E_3 + E_4$$

Similarly, $E_B = E_5 + E_6 + E_7 + E_8$ & $E_C = E_9 + E_{10} + E_{11} + E_{12}$

- the ***phase belt*** or ***phase band*** may be defined as the ***group*** of adjacent slots belonging to one phase under one pole-pair
  Conductors 1, 2, 3 and 4 constitute ***first phase group***
  Conductors 5, 6, 7 and 8 constitute ***second phase group***
  Conductors 9, 10, 11 and 12 constitute ***third phase group***

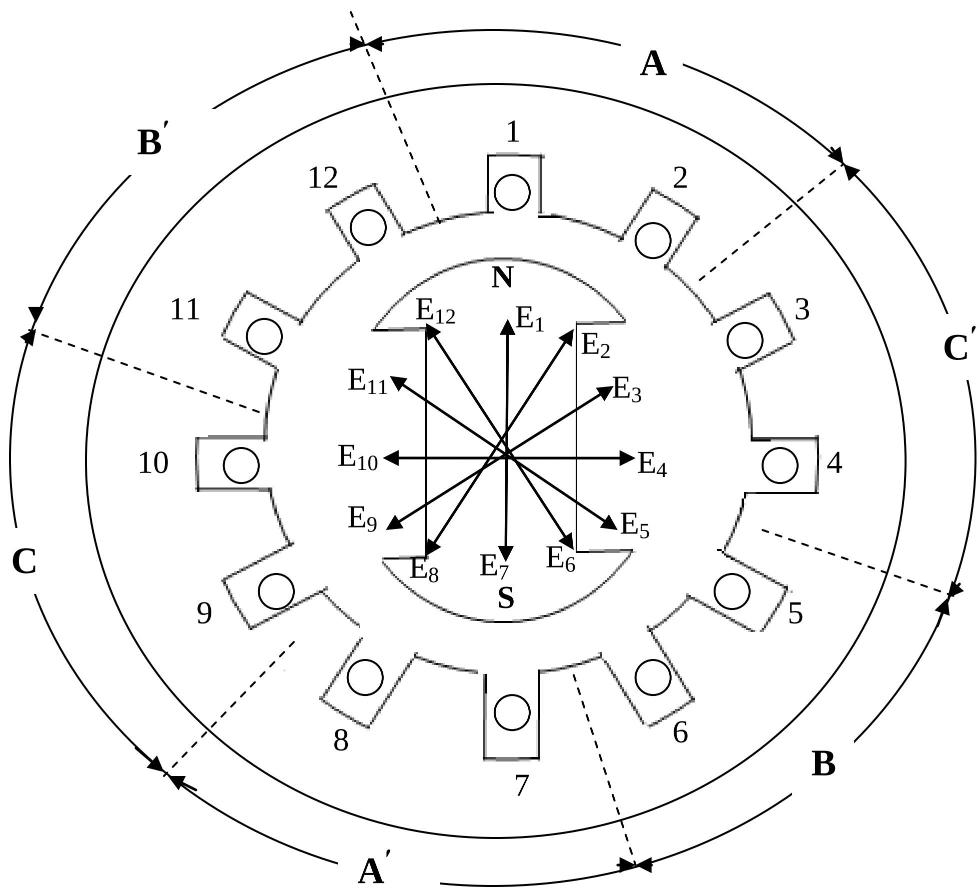
- the angle subtended by one phase group is called phase spread, symbol σ
  $\sigma = q\gamma\ = 4 \times 30^o$ where
  $q = number\ of\ slots\ per\ pole\ per\ phse = \frac{S}{Pm}$

Sequence of phase-belts (groups)

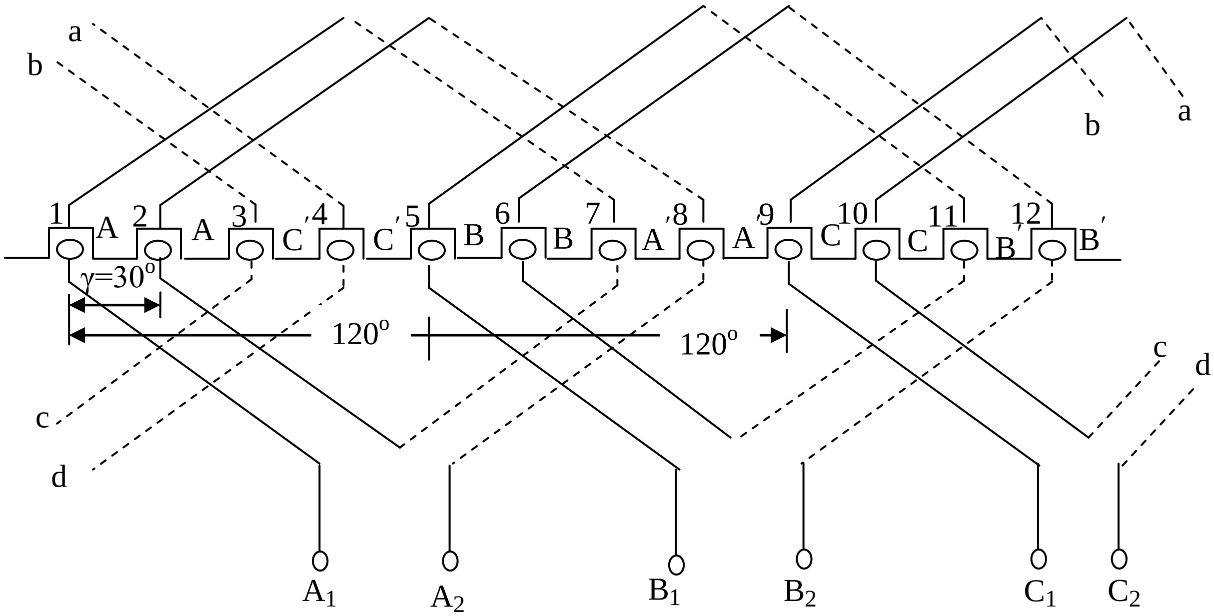
Let

12-conductors can be used to obtain three-phase single – layer winding having a phase spread of 60° ($\sigma = 60^o$)

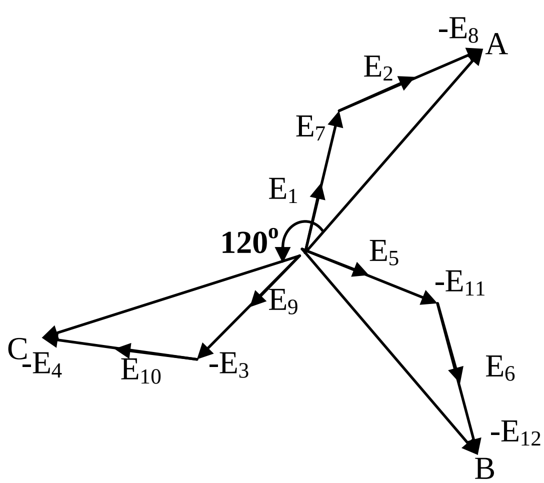
- coil pitch or coil span $\boldsymbol{y}$ = pole pitch $\boldsymbol{\tau} = \frac{S}{P} = \frac{12}{2} = 6$
- for 12 slots and 2 poles, slot angular pitch **γ =30°**
- for $\sigma = 60^o$, two adjacent slots must belong to the same phase



3-phase winding, phase spread is 60°



(a)



(b)

Phase spread of 60°, 12 slots,2 pole winding arrangement
(b) Time-phase diagram for the emfs generated in (a)

Double Layer Winding

- synchronous machine armatures and induction –motor stators above a few kW, are wound with double layer windings
- if the number of slots per pole per phase $\boldsymbol{q} = \frac{\boldsymbol{S}}{\boldsymbol{mP}}$ is an integer, then the winding is called an ***integral-slot winding***
- in case the number of slots per pole per phase, q is not an integer, the winding is called ***fractional-slot winding.*** For example
  - *a 3-phase winding with 36 slots and 4 poles is an integral slot winding, because* $q = \frac{36}{3\times4} = 3 \; is \; an \; integer$
  - *a 3-phase winding with 30 slots and 4 poles is a fractional slot winding, because* $q = \frac{30}{3\times4} = \frac{5}{2} \; is \; not \; an \; integer$
- the number of coils ***C*** is always equal to the number of slots ***S***, ***C=S***

1- Integral Slot Winding

Example: make a winding table for the armature of a 3-phase machine with the following specifications:

Total number of slots = 24 Double – layer winding
Number of poles = 4 Phase spread=60°
Coil-span = full-pitch

(a) Draw the detailed winding diagram for one phase only
(b) Show the star of coil-emfs. Draw phasor diagram for narrow-spread(σ=60°) connections of the 3-phase winding showing coil-emfs for phases A and B only.

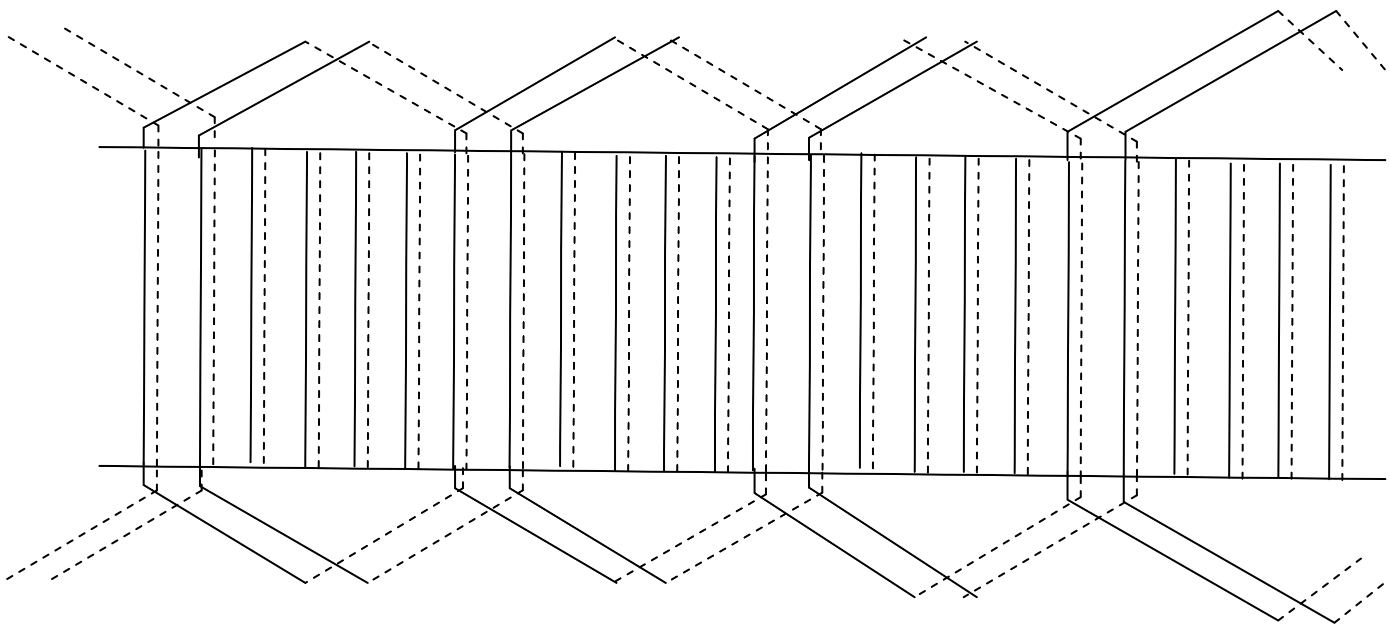
Solution: slot angular pitch, $\gamma = \frac{4\times180^o}{24} = 30^o$

Phase spread, $\sigma = 60^o$

Number of slots per pole per phase, $q = \frac{24}{3\times4} = 2$

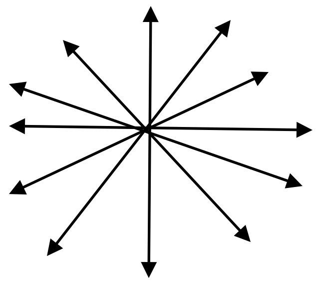
Coil span = full pitch = $\frac{24}{4} = 6$

(a)

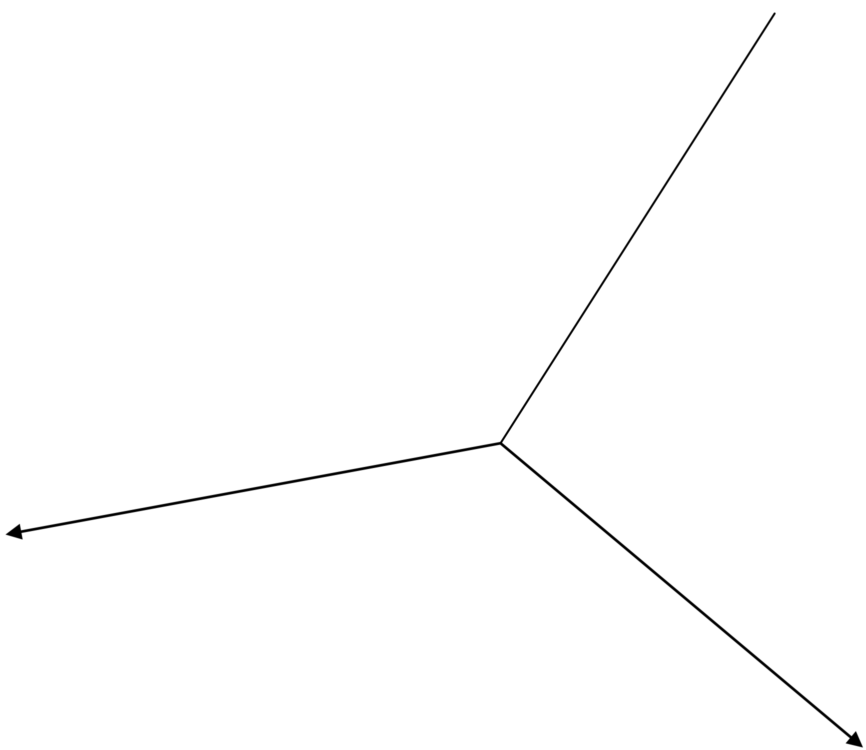


Detailed double layer winding diagram for phase A for 3-phase armature having 24 slots, 4 poles, phase spread 60^o

(c) The ***star*** of ***coil emfs*** can be drawn similar to the star of ***slot emfs*** or star of ***conductor emfs***



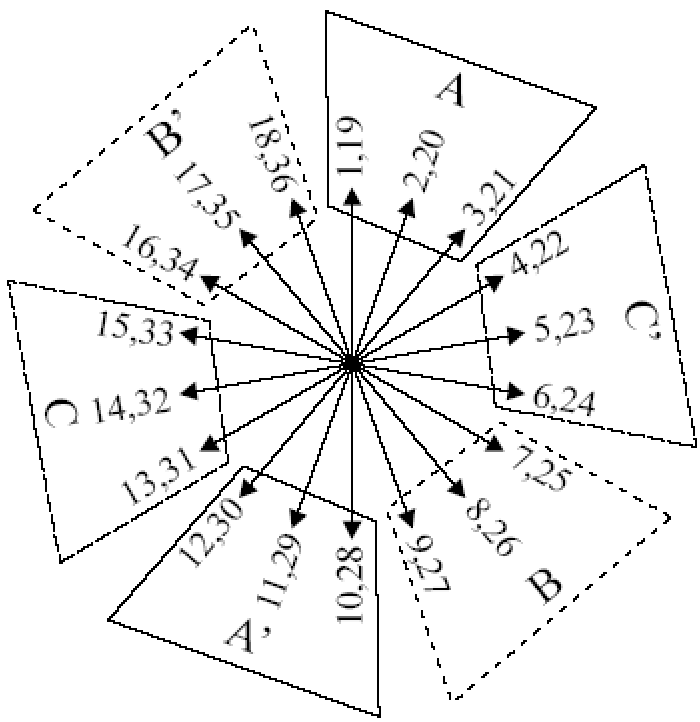
Phasor diagram showing the phasor sum of coil-emfs to obtain phase voltages A and B



2. integral slot chorded winding

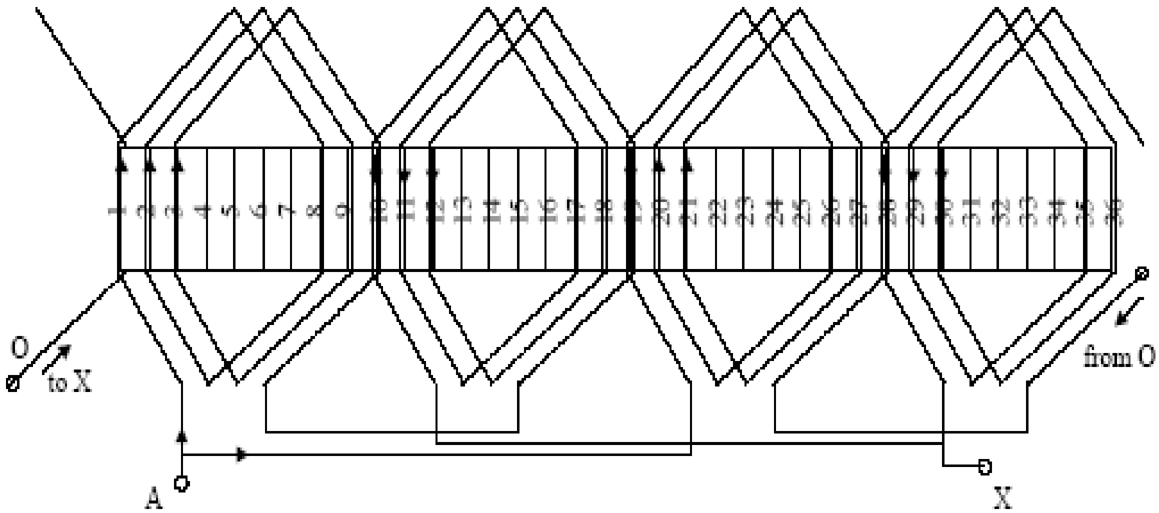
- ❖ Coil span (coil pitch) < pole pitch ($y < \tau$)
- ❖ The advantages of using chorded coils are:
  - ➢ To reduce the **amount of copper** required for the end-connections (or over hang)
  - ➢ To reduce the **magnitude of certain harmonics** in the waveform of phase emfs and mmfs
- ❖ The coil span generally varies from **2/3 pole pitch** to **full pole pitch**

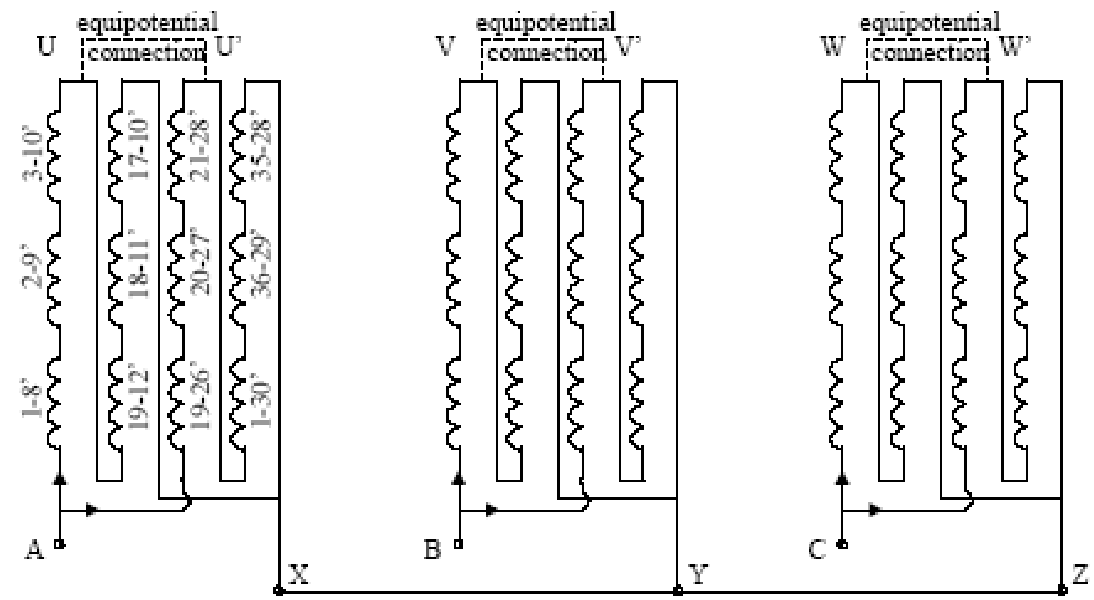
Example. Let us consider a double-layer three-phase winding with q = 3, p = 4, (S = pqm = 36 slots), chorded coils $y/\tau = 7/9$



The star of slot emf phasors for a double-layer winding p = 4 poles, q = 3 slots/pole/phase, m = 3, S = 36

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
layer1	A	A	A	C'	C'	C'	B	B	B	A'	A'	A'	C	C	C	B'	B'	B'	A	A	A	C'	C'	C'	B	B	B	A'	A'	A'	C	C	C	B'	B'	B'
layer2	A	C'	C'	C'	B	B	B	A'	A'	A'	C	C	C	B'	B'	B'	A	A	A	C'	C'	C'	B	B	B	A'	A'	A'	C	C	C	B'	B'	B'	A	A





Double-layer winding: p = 4 poles, q = 3, y/τ = 7/9, S = 36 slots.

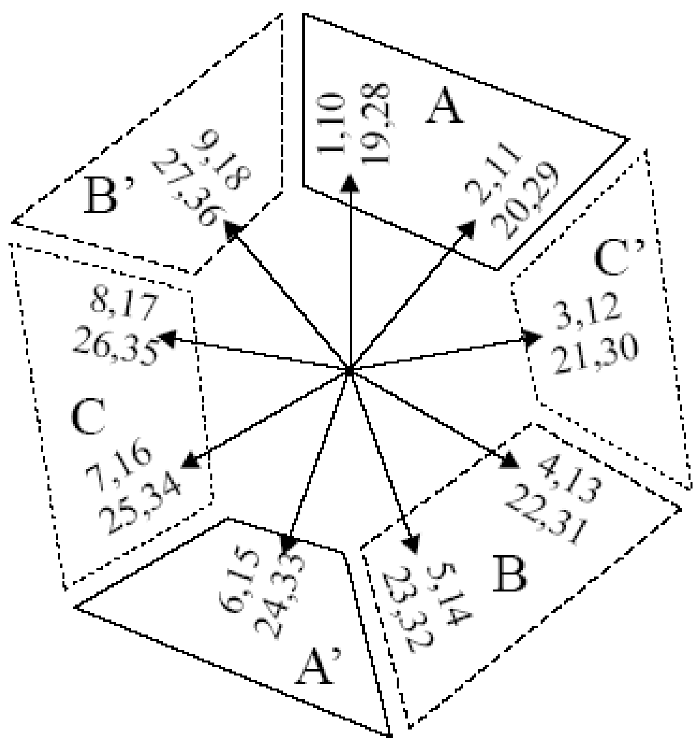
3. Fractional Slot Windings

If the number of slots ***q*** of a winding is a fraction, the winding is called a ***fractional slot winding.***

Advantages of fractional slot windings when compared with integral slot windings are:

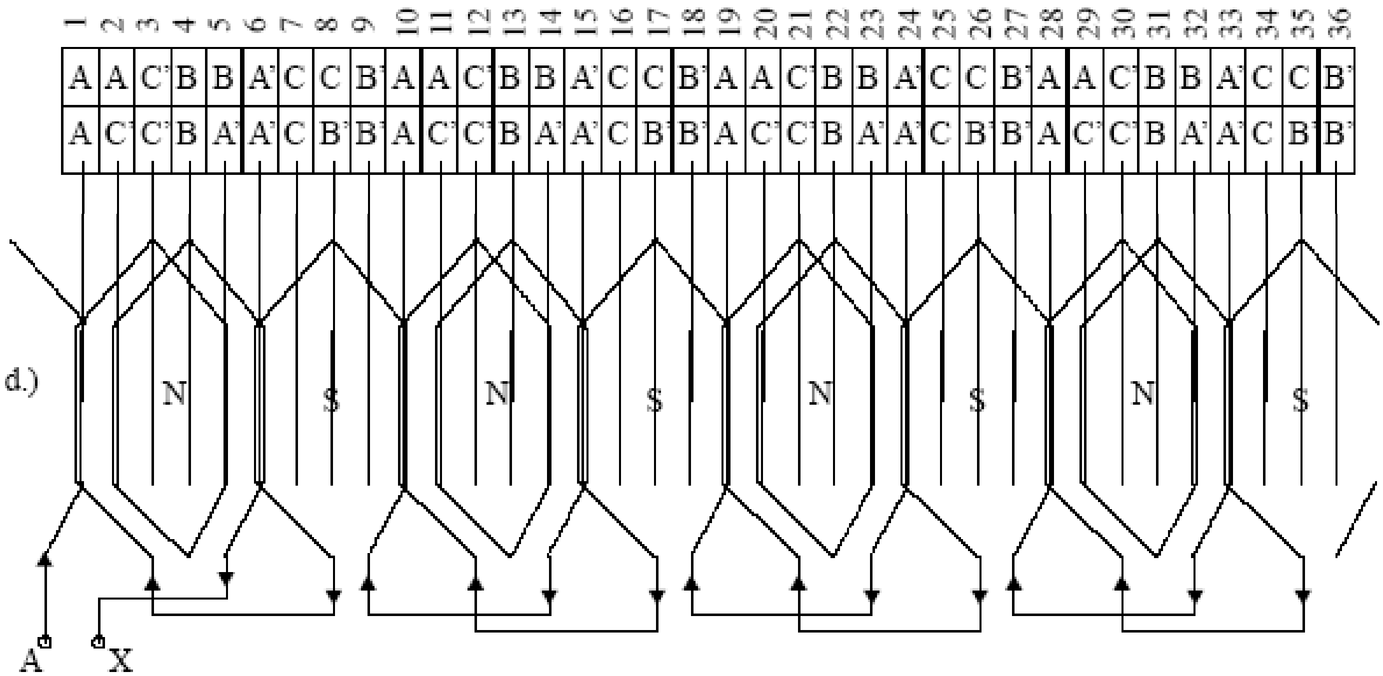
1. a great freedom of choice with respect to the number of slot a possibility to reach a suitable magnetic flux density
2. this winding allows more freedom in the choice of coil span
3. if the number of slots is predetermined, the fractional slot winding can be applied to a wider range of numbers of poles than the integral slot winding the segment structures of large machines are better controlled by using fractional slot windings
4. this winding reduces the high-frequency harmonics in the emf and mmf waveforms

Let us consider a small induction motor with $p = 8$ and $q = 3/2$, $m = 3$.
The total number of slots S = pqm = 8*3*3/2= 36 slots. The coil span y is
$y = (S/p) = (36/8) = 4$ slot pitches



Fractionary q (q = 3/2, p = 8, m = 3,S = 36) winding- emf star,

- The actual value of q for each phase under neighboring poles is 2 and 1, respectively, to give an average of 3/2



Fractionary q (q = 3/2, p = 8, m = 3, S = 36) winding slot/phase allocation & coils of phase A

Single – Layer Winding

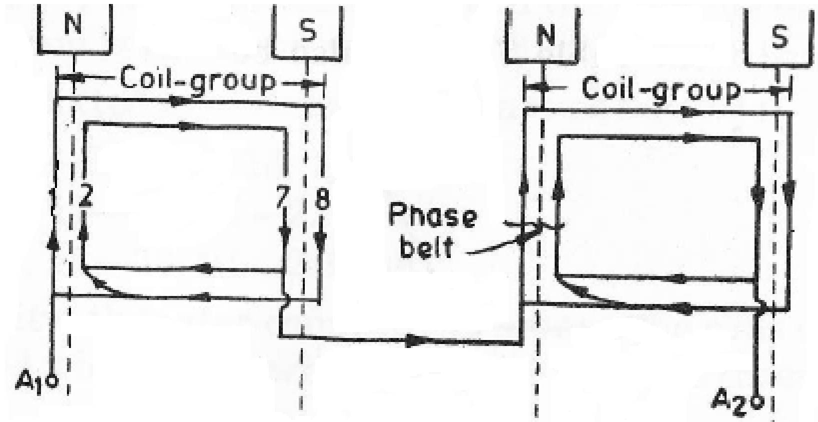
- One coil side occupies one slot completely, in view of this, number of coils $\boldsymbol{C}$ is equal to half the number of slots $\boldsymbol{S}$, $\boldsymbol{C} = \frac{1}{2}\boldsymbol{S}$
- The 3-phase single –layer windings are of two types
    1. Concentric windings
    2. Mush windings

Concentric Windings

- The coils under one pole pair are wound in such a manner as if these have one center
- the concentric winding can further be sub-divided into
    1. half coil winding or unbifurcated winding
    2. Whole coil winding or bifurcated winding

Half coil winding

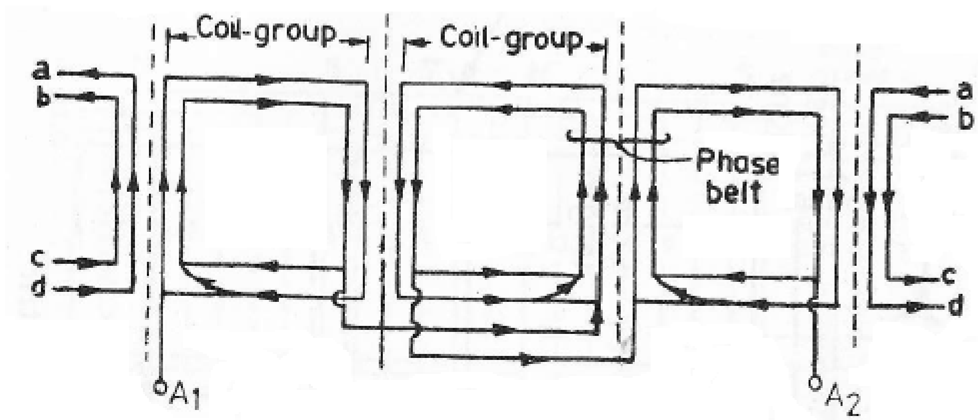
➔For phase **A** only



- The half coil winding arrangement with 2-slots per pole per phase and for σ=60^o
- A coil group may be defined as the group of coils having the same center
- The number of coils in each coil group = the number of coil sides in each phase belt (phase group)
- The carry current in the same direction in all the coil groups

whole coil winding

➔For phase **A** only



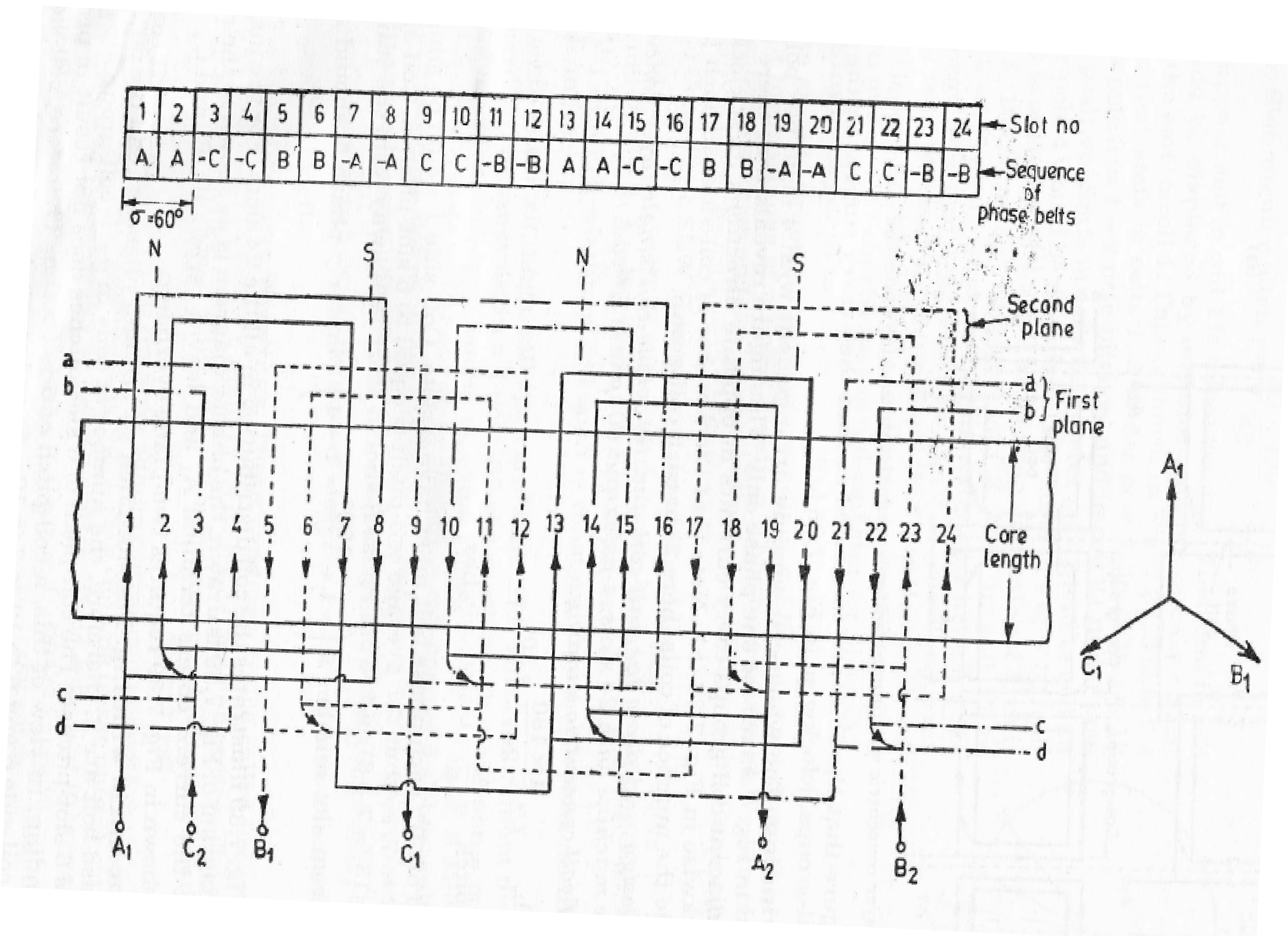
- The whole coil winding arrangement with 2-slots per pole per phase
- The number of coil sides in each phase belt (here 4) are double the number of coils (here 2) in each coil group
- There are ***P*** coil groups and the adjacent coil groups carry currents in opposite directions

Example. Design and draw ***(a)*** half coil and ***(b)*** whole coil single layer concentric windings for a 3-phase machine with 24-slots, 4-poles and 60^o phase spread.

Solution: *(a) half coil concentric winding*

$$Slots\ angular\ pitch, \gamma = \frac{4\times180^o}{24} = 30^o$$

$$Full\ pitch\ or\ pole\ pitch = \frac{24}{4} = 6\ slots\ pitches$$

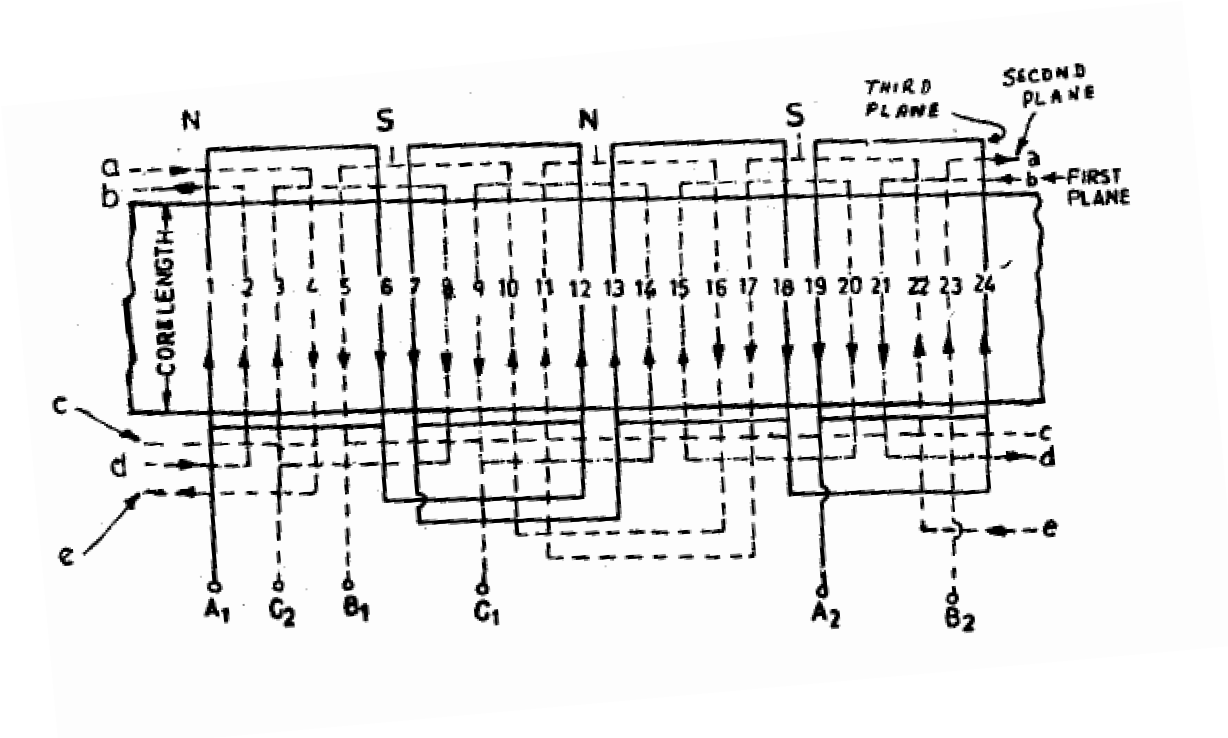


Half-coil winding diagram for 24 slots, 4 poles, 60° phase spread single layer concentric winding (two – plane overhang)

(b) Whole-coil concentric winding

For slot pitch $\gamma = 30^o$ & phase spread $\sigma = 60^o$,

- The number of coils per phase belt = 2
- The number of coils in each coil group = 1
- The pole pitch=6
- The coil pitch of 6 slot pitches does not result in proper arrangement of the winding
- In view of this, a coil pitch of 5 is chosen

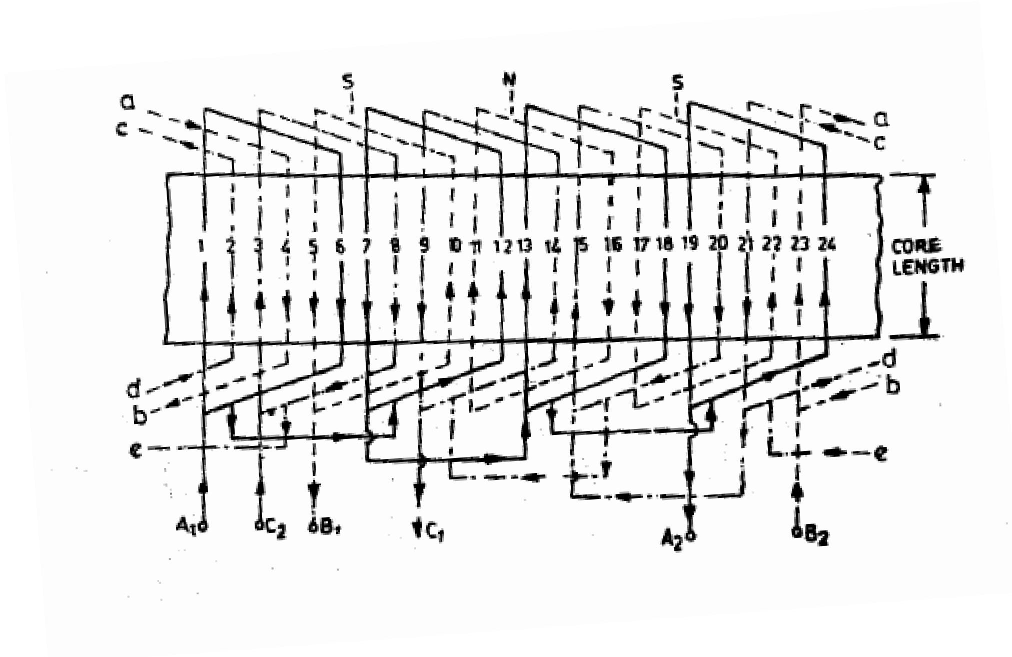


Whole-coil winding arrangement of 24 slots, 4 poles, 60^o phase spread, single layer concentric winding (three-plane overhang)

Mush Winding

- The coil pitch is the same for all the coils
- Each coil is first wound on a trapezoidal shaped former. Then the short coil sides are first fitted in alternate slots and the long coil sides are inserted in the remaining slots
- The number of slots per pole per phase must be a whole number
- The coil pitch is always odd

For example, for 24 slots, 4 poles, single-layer mush winding, the pole pitch is 6 slots pitches. Since the coil pitch must be odd, it can be taken as 5 or 7. Choosing here a coil pitch of 5 slot pitches.



Single – layer mush winding diagram for 24 slots, 4 poles and 60^o phase spread

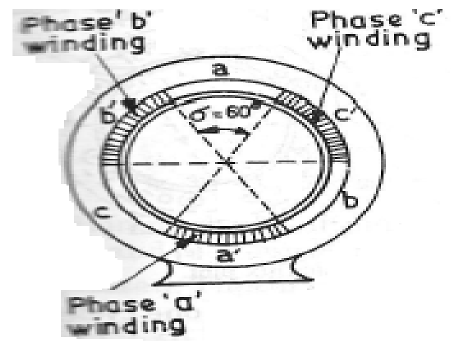
H.W: Design and draw

1. 3-phase, 24-slots, 2-poles single-layer winding (half coil winding)
2. a.c. winding: 3-phase, 4 -pole, 24- slots, double layer winding with full pitch coils (phase B& phase C)
3. a.c. winding: 3-phase, 4 -pole, 24- slots, double layer winding with chorded coils $y/\tau = 5/6$
4. 10 -pole, 48- slots, fractional 3-phase double layer winding

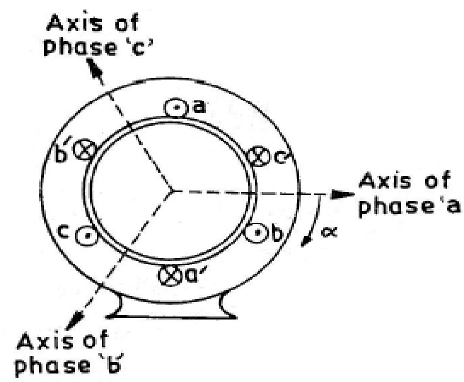
Rotating Magnetic Field

- ❖ When balanced 3-phase currents flow in balanced 3-phase windings, a rotating magnetic field is produced.
- ❖ All 3-phase ac machines are associated with rotating magnetic fields in their air-gaps.

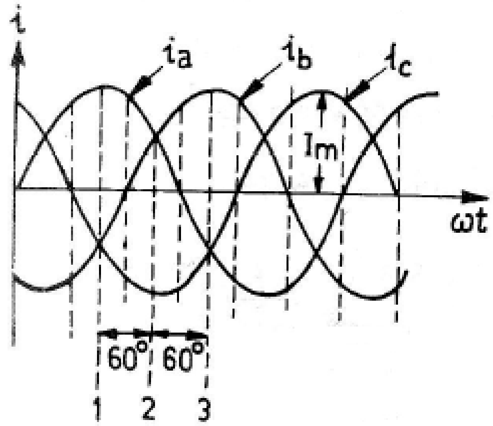
For example, a 2-pole 3-phase stator winding



- ➢ The three windings are displaced from each other by 120° along the air-gap periphery.
- ➢ Each phase is distributed or spread over 60° (called phase-spread σ=60°)

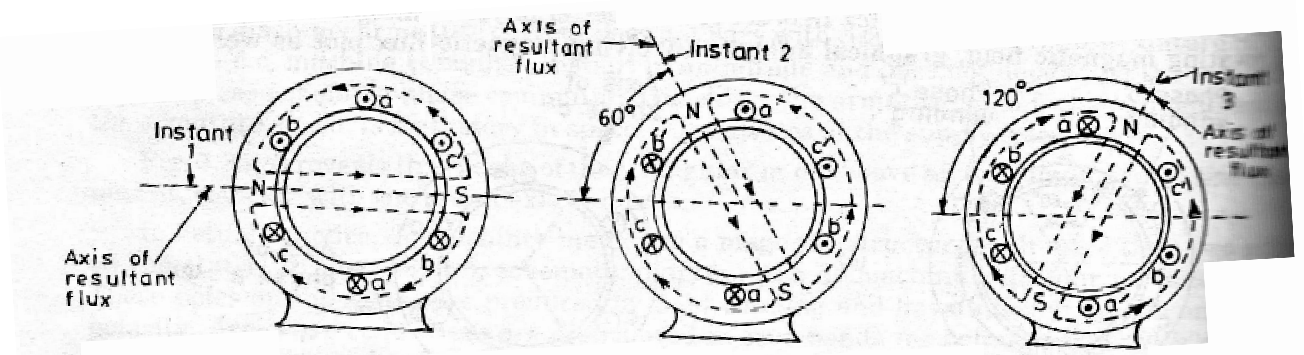


- ➢ The 3-phase winding ***a, b, c*** is represented by three full pitched coils, ***aa′ , bb′ , cc′***
- ➢ For instance, the concentrated full-pitched coil ***aa′*** represents phase a winding in all respects
- ➢ A current in ***phase a*** winding establishes magnetic flux directed along the magnetic axis of coil ***aa′***
- ➢ Positive currents are assumed to be flowing as indicated by crosses in coil-sides ***a′ , b′ , c′***



Magnetic flux plot

- At the ***instant 1***, the current in **phase a** is ***positive*** and ***maximum*** $\boldsymbol{I_m}$ $\left(\boldsymbol{i_b = i_c = -\frac{I_m}{2}}\right)$
- At the ***instant 2***, $\boldsymbol{i_a = \frac{I_m}{2}}$ **,** $\boldsymbol{i_b = \frac{I_m}{2}}$ and $\boldsymbol{i_c = -I_m}$
- At the ***instant 3***, $\boldsymbol{i_a = -\frac{I_m}{2}}$ **,** $\boldsymbol{i_b = I_m}$ and $\boldsymbol{i_c = -\frac{I_m}{2}}$



Production of rotating magnetic field illustrated by magnetic flux plot

- The 2 poles produced by the resultant flux are seen to have turned through further 60°
- The space angle traversed by rotating flux is equal to the time angle traversed by currents

❖ The rotating field speed, for a ***P***-pole machine, is

➔ $\frac{\boldsymbol{1}}{\boldsymbol{P/2}}$ revolution in one cycle

➔ $\frac{\boldsymbol{f}}{\boldsymbol{P/2}}$ revolutions in ***f*** cycles

➔ $\frac{f}{P/2}$ revolutions in one second [because f cycles are completed in one second]

Here f is the frequency of the phase currents. If n_s denotes the rotating field speed in revolutions per sec, then

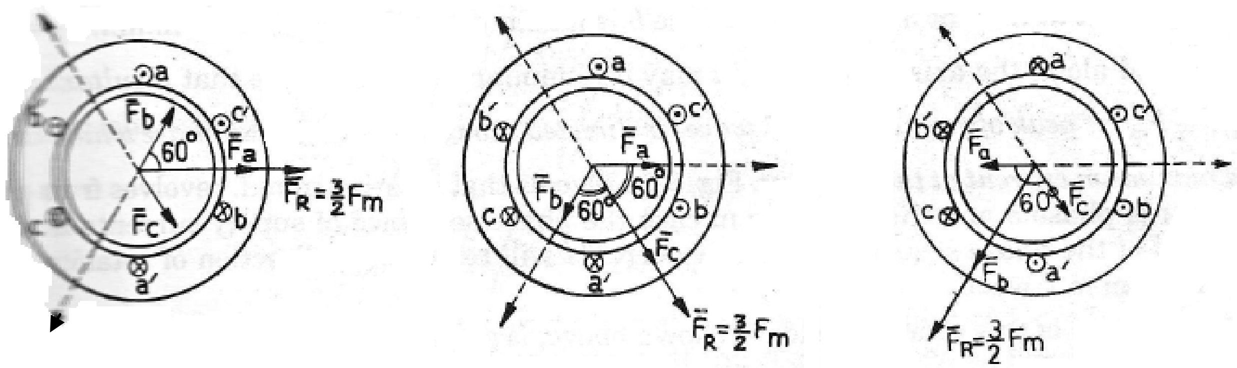
$$n_s = \frac{f}{P/2} = \frac{2f}{P}$$

Or

$$N_s = \frac{120f}{p}\ r.p.m$$ [***The speed at which rotating magnetic field revolves is called the Synchronous speed]***

Space phasor representation

- When currents $\boldsymbol{i_a}$, $\boldsymbol{i_b}$, $\boldsymbol{i_c}$ flow in their respective phase windings, then the three stationary pulsation m.m.fs $\overline{F_a}$,$\overline{F_b}$, $\overline{F_c}$ combine to give the resultant m.m.f. $\overline{F_R}$ which is rotating at synchronous speed.



Production of rotating magnetic field illustrated by space phasor m.m.fs.

- ***At the instant 1,***

$$i_a = I_m \quad \text{➔}\ space\ phasor\ \bar{F}_a = maximum\ m.m.f.F_m$$

$$i_b = i_c = -\frac{I_m}{2} \quad \text{➔}\ the\ m.m.f.phasors\ \bar{F}_b = \bar{F}_c = \frac{F_m}{2}$$

The resultant of m.m.fs. $\boldsymbol{\bar{F}_a}$, $\boldsymbol{\bar{F}_b}$, $\boldsymbol{\bar{F}_c}$ is $\boldsymbol{\bar{F}_R}$ and its magnitude is given by

$$\boxed{F_R = F_m + \frac{2F_m}{2}\cos 60^o = \frac{3}{2}F_m}$$

The vertical component of $\boldsymbol{\bar{F}_b}$ **&** $\boldsymbol{\bar{F}_c}$ cancel each other.

- ***At the instant 2,***

$i_a = i_b = \frac{I_m}{2}$ & $i_c = -I_m$

$the\ m.m.f.phasors\ \bar{F}_a = \bar{F}_b = \frac{F_m}{2}$ & $space\ phasor\ \bar{F}_c = maximum\ m.m.f.F_m$

The resultant m.m.f. $F_R = \frac{3}{2} F_m$ [it rotate by a space angle of 60^o clockwise]

- ***At the instant 3,***

$i_a = i_c = -\frac{I_m}{2}$ & $i_b = I_m$

The resultant m.m.f. $F_R = \frac{3}{2} F_m$ [The resultant m.m.f. has turned through a further space angle of 60^o from its position occupied at instant 2]

A constant-amplitude rotating m.m.f. or rotating field is produced in the air-gap of a three-phase machines at synchronous speed

Sinusoidal rotating mmf wave creates in phase sinusoidal rotating flux density wave in the air gap; the peak value of ***B- wave*** is given by

$$B_p = \frac{\mu_0 F_R}{g} = \frac{3}{2}\frac{\mu_0 F_m}{g}$$ Where ***g*** is air-gap length

Example*: Prove that a rotating magnetic field of constant amplitude is produced when 3-phase balanced winding is excited by three-phase balanced currents.*

Solution: three – phase balanced currents given by

$$\left.\begin{aligned} i_a &= I_{max}\cos\omega t \\ i_b &= I_{max}\cos(\omega t - 120°) \\ i_c &= I_{max}\cos(\omega t - 240°) \end{aligned}\right\} \qquad \text{------ (1)}$$

The three mmfs $\boldsymbol{F_a}$, $\boldsymbol{F_b}$ and $\boldsymbol{F_c}$ can be expressed mathematically as

$$F_a = F_m \cos\alpha \cos\omega t$$
$$F_b = F_m \cos(\alpha - 120°) \cos(\omega t - 120°)$$
$$F_c = F_m \cos(\alpha - 240°), \cos(\omega t - 240°)$$

Angle $\boldsymbol{\alpha}$ is measured from the axis of ***phase a***

The mmf of phase *a* can be expressed as

$$F_a = \frac{1}{2} F_m \cos(\alpha - \omega t) + \frac{1}{2} F_m \cos(\alpha + \omega t) \qquad \text{------ (2)}$$

Similarly, for phases ***b*** & ***c,***

$$F_b = \frac{1}{2} F_m \cos(\alpha - 120° - \omega t + 120°) + \frac{1}{2} F_m \cos(\alpha - 120° + \omega t - 120°)$$
$$= \frac{1}{2} F_m \cos(\alpha - \omega t) + \frac{1}{2} F_m \cos(\alpha + \omega t - 240°) \qquad \text{------ (3)}$$
$$F_c = \frac{1}{2} F_m \cos(\alpha - 240° - \omega t + 240°) + \frac{1}{2} F_m \cos(\alpha - 240° + \omega t - 240°)$$
$$= \frac{1}{2} F_m \cos(\alpha - \omega t) + \frac{1}{2} F_m \cos(\alpha + \omega t - 120°) \qquad \text{------ (4)}$$

The resultant mmf $F_R(\alpha, t)$ can be obtained by adding the three mmfs given by Eqs. (1), (2) and (3).

$$\therefore \quad F_R(\alpha, t) = F_a + F_b + F_c$$
$$= \frac{1}{2} F_m [\cos(\alpha - \omega t) + \cos(\alpha + \omega t) + \cos(\alpha - \omega t) + \cos(\alpha + \omega t - 240°)$$
$$+ \cos(\alpha - \omega t) + \cos(\alpha + \omega t - 120°)]$$
$$= \frac{3}{2} F_m \cos(\alpha - \omega t) + \frac{1}{2} F_m [\cos(\alpha + \omega t) + \cos(\alpha + \omega t - 120°)$$
$$+ \cos(\alpha + \omega t - 240°)] \qquad \text{------ (5)}$$

But $\frac{1}{2}F_m\cos(\alpha+\omega t)$, $\frac{1}{2}F_m\cos(\alpha+\omega t-120°)$, $\frac{1}{2}F_m\cos(\alpha+\omega t-240°)$ represent three **mmf** sinusoids displaced in phase by 120°. Therefore, there sum must be zero, *i.e.*,

$$\frac{1}{2}F_m\,[\cos(\alpha+\omega t)+\cos(\alpha+\omega t-120)+\cos(\alpha+\omega t-240)]=0$$

Eq.(5), therefore, reduces to

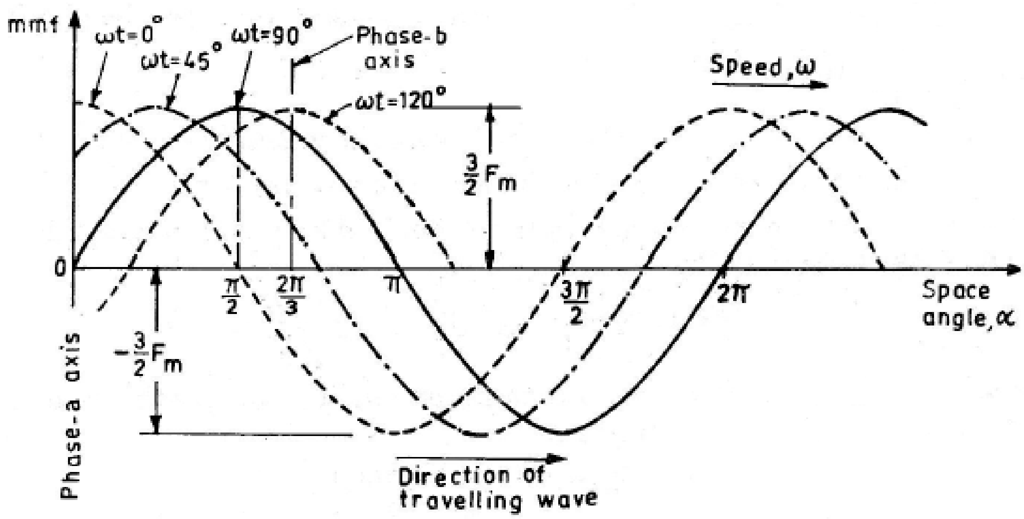
$$F_r(\alpha,t)=\frac{3}{2}F_m\cdot\cos(\alpha-\omega t) \qquad \text{------ (6)}$$

It can be shown that Eq.(6) represents a travelling mmf wave of constant amplitude $\frac{3}{2}\boldsymbol{F_m}$

At $\omega t=0°$, $F_R(\alpha,t)=\frac{3}{2}F_m\cos\alpha$

At $\omega t=45°$, $F_R(\alpha,t)=\frac{3}{2}F_m\cos(\alpha-45°)$

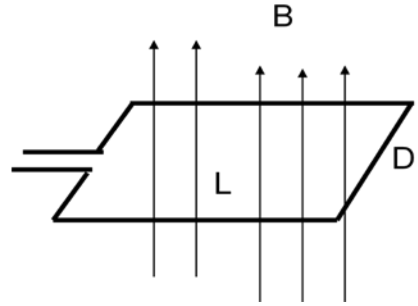
At $\omega t=90°$, $F_R(\alpha,t)=\frac{3}{2}F_m\cos(\alpha-90°)=\frac{3}{2}F_m\sin\alpha$



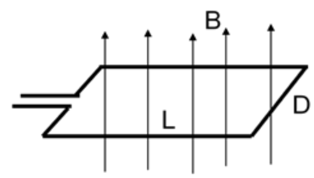
A graphical plot indicating that $\frac{3}{2}F_m\cos(\alpha-\omega t)$ is a constant amplitude m.m.f. wave travelling in the positive α direction.

H.W: ***A three-phase, Y-connected winding is fed from 3-phase balanced supply, with their neutrals connected together. If one of the three supply leads gets disconnected, find what happens to the m.m.f. wave .***

Electromotive Force (EMF) Equation



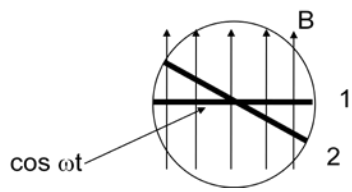
- A wire loop is rotated in a magnetic field.
  - N is the number of turns in the loop
  - L is the length of the loop
  - D is the width of the loop
  - B is the magnetic flux density
  - n is the number of revolutions per seconds

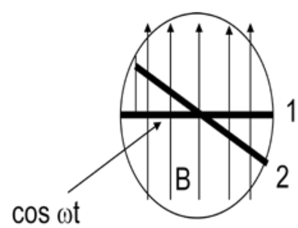


- A wire loop is rotated in a magnetic field.
- The magnetic flux through the loop changes by the position

$$\Phi(t) = B\,D\,L\cos(\omega\,t)$$

$$\omega = 2\pi\,n$$





- Position 1 all flux links with the loop
- Position 2 the flux linkage reduced
- The change of flux linkage induces a voltage in the loop

$$E\ (t) = N\frac{d\Phi(t)}{dt} = N\,B\,D\,L\frac{d[\cos(\omega\,t)]}{dt} = N\,B\,D\,L\,\omega\,\sin(\omega\,t)$$

- The induced voltage is an ac voltage
- The voltage is sinusoidal
- The rms value of the induced voltage loop is:

$$E_{rms} = \frac{N\,B\,D\,L\,\omega}{\sqrt{2}}$$

The r.m.s value of the generated emf in a full pitched coil is

$$E = \frac{E_{max}}{\sqrt{2}}\ , \quad \text{where } E_{max} = \omega_r N\emptyset = 2\pi f N\emptyset \quad [\emptyset = BDL]$$

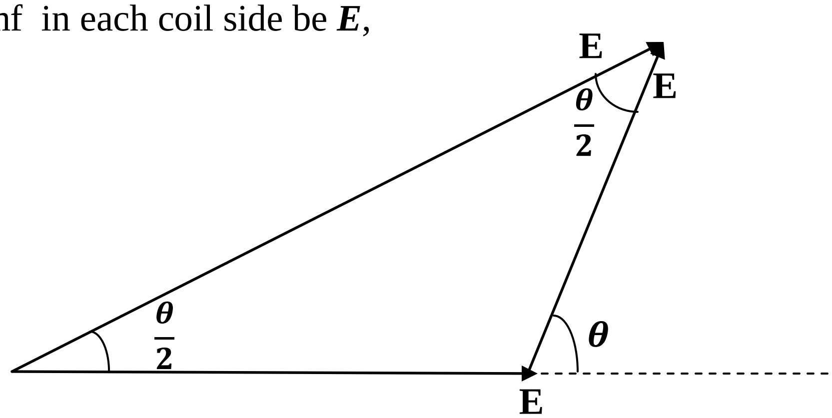
$$\therefore \quad E = \frac{E_{max}}{\sqrt{2}} = \sqrt{2}\ \pi\ fN\emptyset = 4.44fN\emptyset$$

Winding Factor (Coil Pitch and Distributed Windings)

Pitch Factor or Coil Pitch

The ratio of phasor (vector) sum of induced emfs per coil to the arithmetic sum of induced emfs per coil is known as ***pitch factor (K$_p$)*** or ***coil span factor (K$_c$)*** which is always less than unity.

Let the coil have a pitch short by angle $\boldsymbol{\theta}$ electrical space degrees from full pitch and induced emf in each coil side be $\boldsymbol{E}$,



- If the coil would have been full pitched, then total induced emf in the coil would have been $\boldsymbol{2E}$.
- when the coil is short pitched by $\boldsymbol{\theta}$ electrical space degrees the resultant induced emf, $\boldsymbol{E_R}$ in the coil is phasor sum of two voltages, $\boldsymbol{\theta}$ apart

$$E_R = 2E\cos\frac{\theta}{2}$$

Pitch factor, $$\boldsymbol{K_p = \frac{Phasor\ sum\ of\ coil\ side\ emfs}{Arithmetic\ sum\ of\ coil\ side\ emfs} = \frac{2E\cos\frac{\theta}{2}}{2E} = \cos\frac{\theta}{2}}$$

Example. The coil span for the stator winding of an alternator is 120°. Find the chording factor of the winding.

Solution: Chording angle, $\theta = 180^o - coil\ span = 180^o - 120^o = 60^o$

Chording factor, $K_p = \cos\frac{\theta}{2} = \cos\frac{60^o}{2} = 0.866$

Distribution Factor

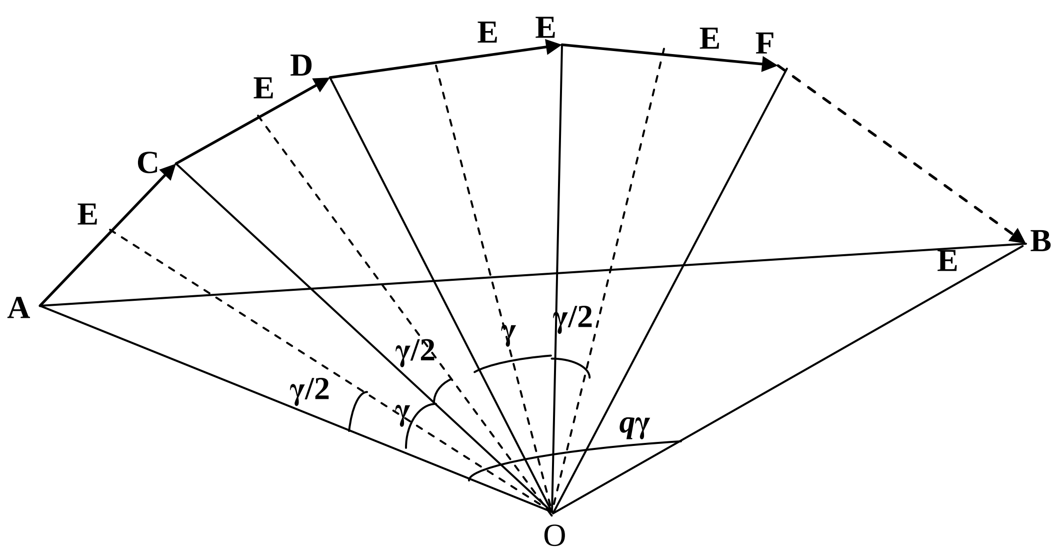
The ratio of the phasor sum of the emfs induced in all the coils distributed in a number of slots under one pole to the arithmetic sum of the emfs induced(or to the resultant of emfs induced in all coils concentrated in one slot under one pole) is known as ***breadth factor (K_b)*** or ***distribution factor (K_d)***

$$K_d = \frac{EMF\ induced\ in\ distributed\ winding}{EMF\ induced\ if\ the\ winding\ would\ have\ been\ concentrated}$$

$$= \frac{Phasor\ \ sum\ \ of\ \ component\ \ \ emfs}{Arithmetic\ \ sum\ \ of\ \ component\ \ \ emfs}$$

- The distribution factor is always less than unity.

- Let no. of slots per pole = Q and no. of slots per pole per phase = q
  Induced emf in each coil side = E_c
  Angular displacement between the slots, $\gamma = \frac{180^o}{Q}$
- The emf induced in different coils of one phase under one pole are represented by side AC, CD, DE, EF… Which are equal in magnitude (say each equal E_c) and differ in phase (say by γ^o) from each other.



If bisectors are drawn on AC, CD, DE, EF… they would meet at common point *(O)*. The point *O* would be the circum center of the circle having AC, CD, DE, EF…as the chords and representing the emfs induced in the coils in different slots.

EMF induced in each coil side, $E_c = AC = 2OA \sin\frac{\gamma}{2}$

$Arithmetic\ sum = q \times 2 \times OA \sin\frac{\gamma}{2}$

$\therefore$ The resultant emf, $E_R = AB = 2 \times OA \sin\frac{AOB}{2} = 2 \times OA \sin\frac{q\gamma}{2}$

& distribution factor, $k_d = \frac{Phasor\ sum\ of\ components\ emfs}{Arithmetic\ sum\ of\ components\ emfs}$

$$= \frac{2 \times OA \sin\frac{q\gamma}{2}}{q \times 2 \times OA \sin\frac{\gamma}{2}} = \boldsymbol{\frac{\sin\frac{q\gamma}{2}}{q \sin\frac{\gamma}{2}}}$$

Example. Calculate the distribution factor for a 36-slots, 4-pole, single layer 3-phase winding.

Solution: No. of slots per pole, $Q = \frac{36}{4} = 9$

No. of slots per pole per phase, $q = \frac{Q}{Number\ of\ phases} = \frac{9}{3} = 3$

Angular displacement between the slots, $\gamma = \frac{180^o}{Q} = \frac{180^o}{9} = 20^o$

Distribution factor, $K_d = \frac{\sin\frac{q\gamma}{2}}{q \sin\frac{\gamma}{2}} = \frac{\sin\frac{3\times 20^o}{2}}{3 \sin\frac{20^o}{2}} = \frac{1}{3}\frac{\sin\ 30^o}{\sin\ 10^o} = 0.96$

Example1. A 3-phase, 8-pole, 750 r.p.m. star-connected alternator has 72 slots on the armature. Each slot has 12 conductors and winding is short chorded by 2 slots. Find the induced emf between lines, given the flux per pole is 0.06 Wb.

Solution:

Flux per pole, $\emptyset = 0.06\ Wb$

$$f = \frac{pn}{60} = \frac{4 \times 750}{60} = 50\ Hz$$

Number of conductors connected in series per phase,

$$Z_s = \frac{Number\ \ of\ \ conductors\ \ per\ \ slot\ \times number\ \ of\ \ slots}{Number\ \ of\ \ phases}$$

$$= \frac{12 \times 72}{3} = 288$$

Number of turns per phase, $T = \frac{Z_s}{2} = \frac{288}{2} = 144$

Number of slots per pole, $Q = \frac{72}{8} = 9$

Number of slots per pole per phase, $q = \frac{Q}{3} = \frac{9}{3} = 3$

Angular displacement between the slots, $\gamma = \frac{180^o}{Q} = \frac{180^o}{9} = 20^o$

Distribution factor, $K_d = \frac{\sin\frac{q\gamma}{2}}{q\sin\frac{\gamma}{2}} = \frac{\sin\frac{3 \times 20^o}{2}}{3\sin\frac{20^o}{2}} = \frac{1}{3}\frac{\sin\ 30^o}{\sin\ 10^o} = 0.96$

Chording angle, $\theta = 180^o \times \frac{2}{9} = 40^o$

Pitch factor, $K_p = \cos\frac{\theta}{2} = \cos\frac{40^o}{2} = \cos 20^o = 0.94$

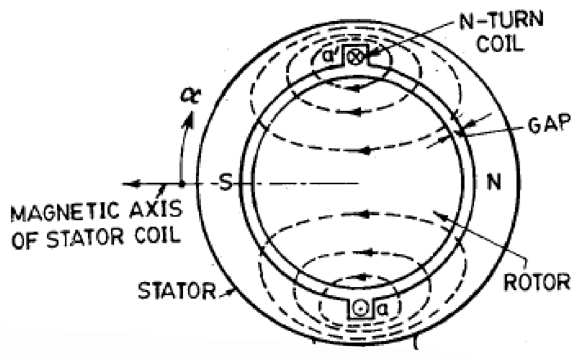
Induced emf between lines, $E_L = \sqrt{3} \times 4.44 \times K_d \times K_p \times \emptyset \times f \times T$

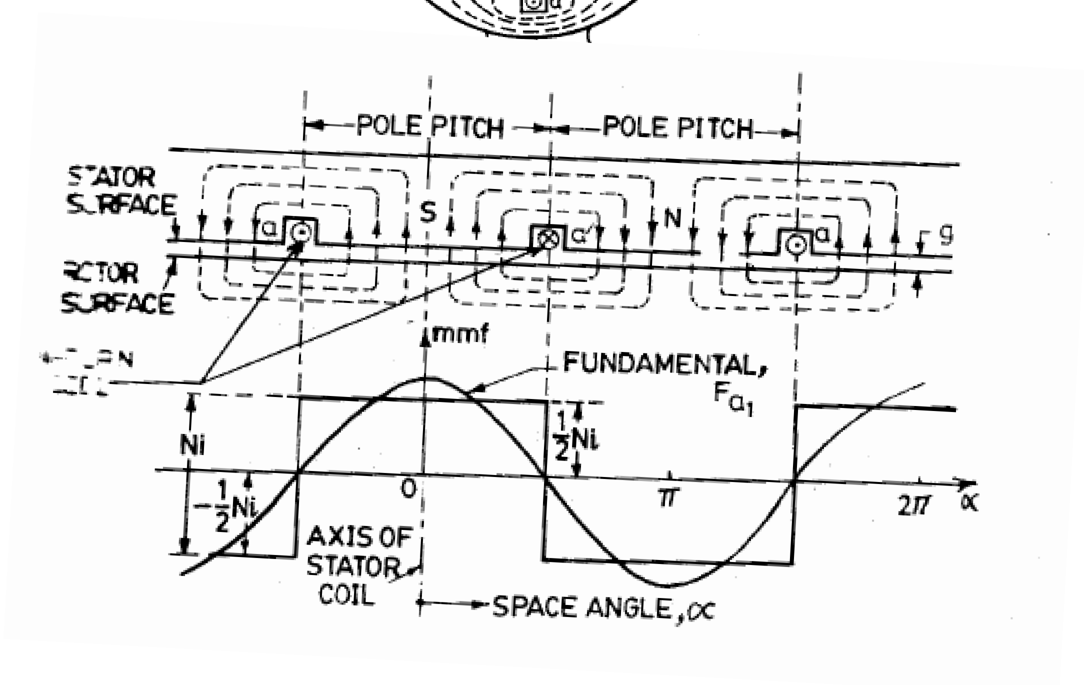
$$= \sqrt{3} \times 4.44 \times 0.96 \times 0.94 \times 0.06 \times 50 \times 144 = 2998\ V$$

Magnetomotive Force (mmf) of AC Windings

M.m.f. of a coil

- the variation of magnetic potential difference along the air –gap periphery is of rectangular waveform and of magnitude $\frac{1}{2}Ni$
- The amplitude of mmf wave varies with time, but not with space
- The air –gap mmf wave is time-variant but space invariant
- The air –gap mmf wave at any instant is rectangular





Mmf distribution along air-gap periphery

The fundamental component of rectangular wave is found to be

$$F_{a1} = \frac{4}{\pi} \cdot \frac{Ni}{2} \cos\alpha = F_{1p} \cos\alpha$$

Where

$\boldsymbol{\alpha}$ = electrical space angle measured from the magnetic axis of the stator coil

Here $\boldsymbol{F_{1p}}$, the peak value of the sine mmf wave for a 2-pole machine is given by

$$F_{1p} = \frac{4}{\pi} \cdot \frac{Ni}{2} \; AT\ per\ pole$$

When $\boldsymbol{i=0} \Rightarrow \boldsymbol{F_{1p}=0}$

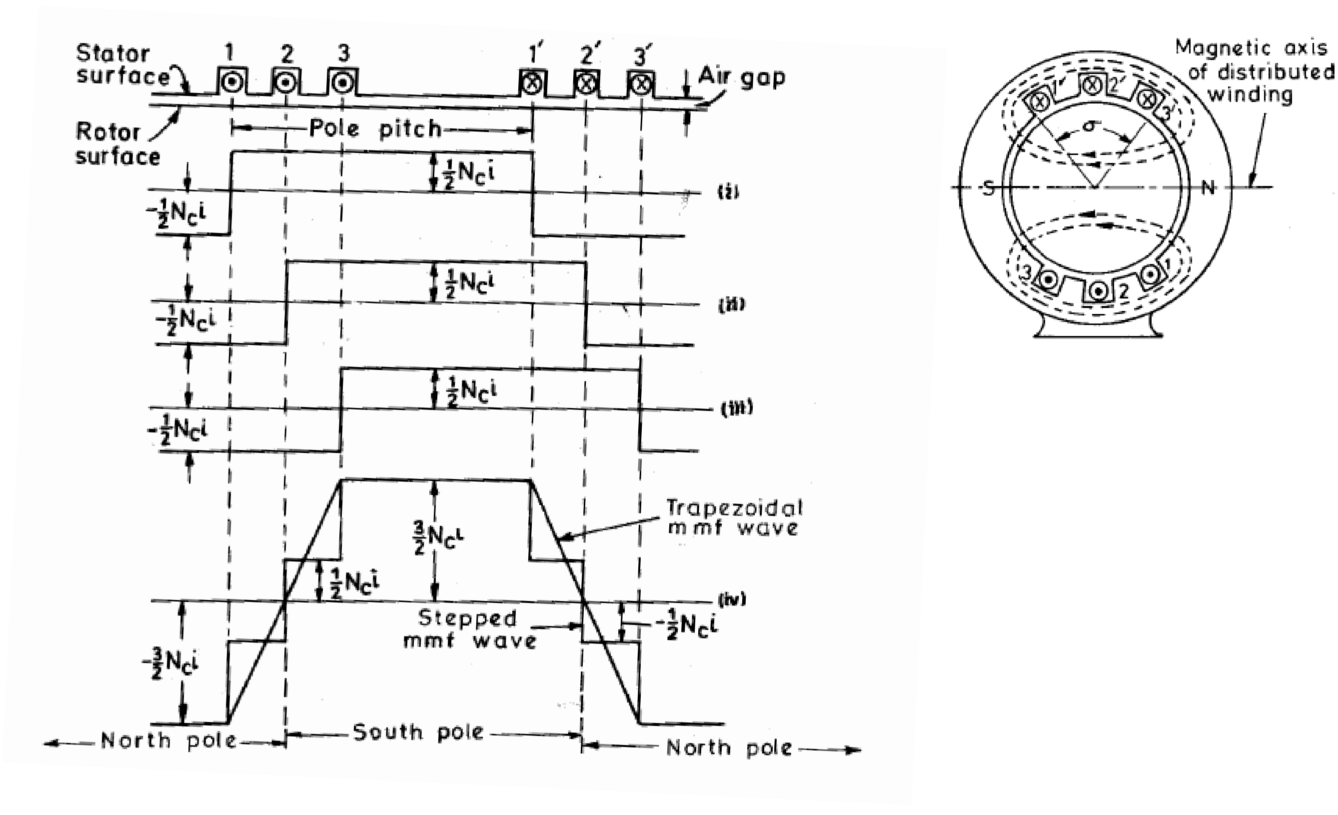
$\boldsymbol{i=I_{max}=\sqrt{2}I}$

For 2-pole machine $$F_{1pm} = \frac{4}{\pi} \cdot \frac{N\sqrt{2}I}{2} \; AT\ per\ pole$$

For p-pole machine $$F_{1pm} = \frac{4}{\pi} \cdot \frac{N\sqrt{2}I}{P} \; AT\ per\ pole$$

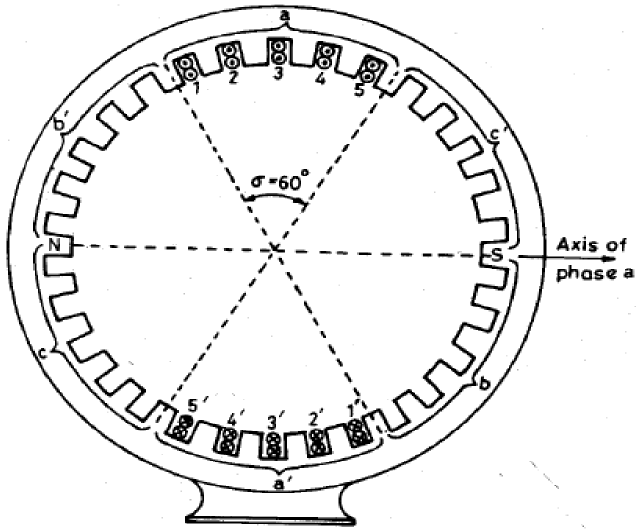
M.m.f of distributed windings

- The mmf distribution along the air gap periphery depends on the nature of slots, winding and the exciting current
- The effect of winding distribution has changed the shape of the mmf wave, from rectangular to stepped



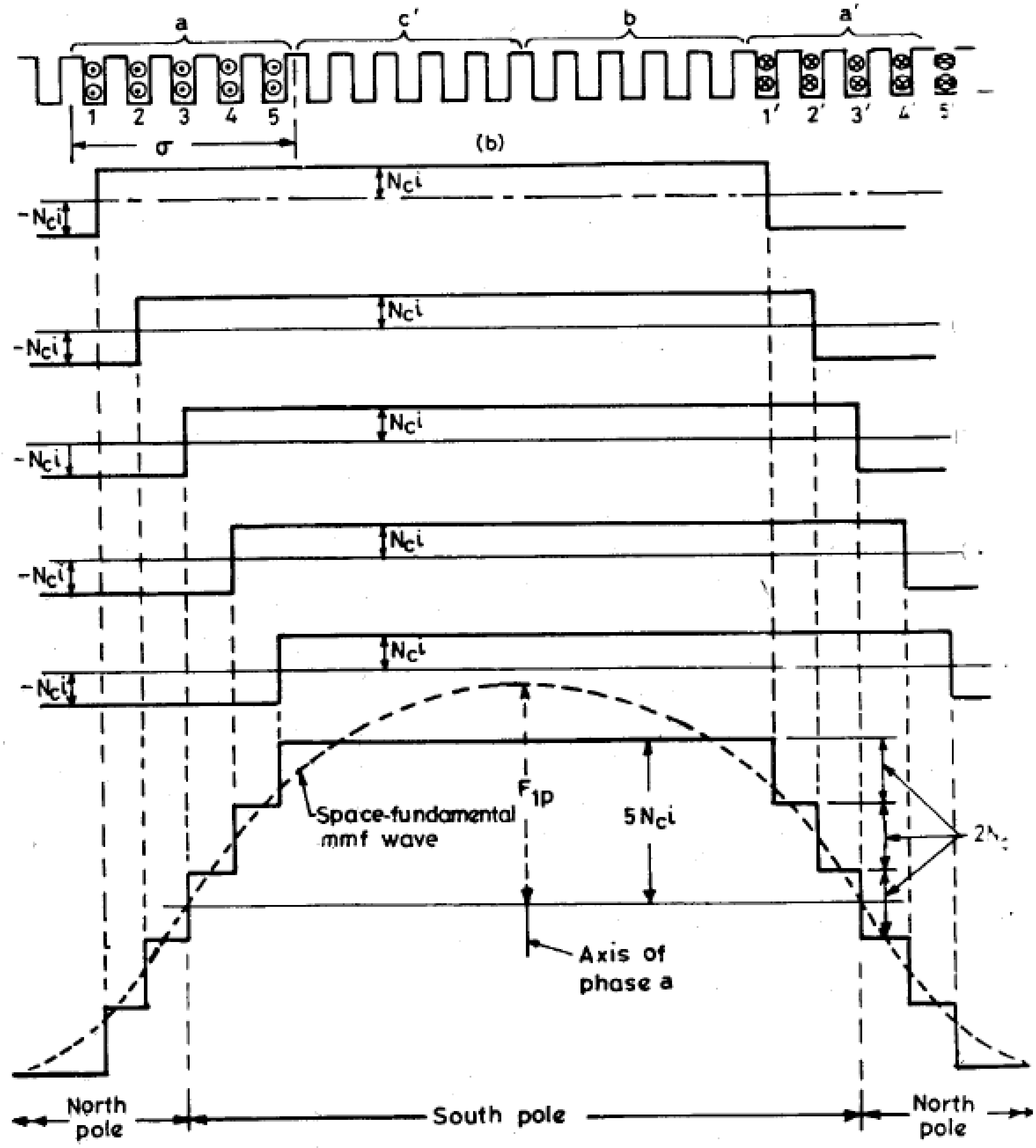
Developed diagram and mmf wave of the machine
(each coil has N_c turns and each turn carries i amperes)

Example: a 3-phase, 2-pole stator has double-layer full pitched winding with 5 slots per pole per phase. If each coil has N_c turns and i is the conductor current, then sketch the mmf wave form produced by phase A alone.



A 3-phase, 2-pole stator with double-layer winding having 5 slots per pole per phase

- For any closed path around slot 1, the total current enclosed is $\boldsymbol{2N_c i}$ ampere
- Magnetic potential difference across each gap is $\frac{1}{2}[2N_c i] = N_c i$
- The mmf variation from $-N_c i$ to $+N_c i$ at the middle of slot 1
- The mmf variation for slot $1'$ is from $+N_c i$ to $-N_c i$
- The mmf variation for coil $11'$ is of rectangular waveform with amplitude $\pm N_c i$. similarly, the rectangular mmf waveforms of amplitude $\pm N_c i$ are sketched for the coils $22'$, …, $55'$
- The combined mmf produced by 5 coils is obtained by adding the ordinates of the individual coil mmfs.
- The resultant mmf waveform consists of a series of steps each of height $2N_c i$ = (conductors per slot) (conductor current)
- The amplitude of the resultant mmf wave is $5N_c i$.



Mmf waveforms

Harmonic Effect

- The flux distribution along the air gaps of alternators usually is non-sinusoidal so that the emf in the individual armature conductor likewise is non-sinusoidal
- The sources of harmonics in the output voltage waveform are the non-sinusoidal waveform of the field flux.
- Fourier showed that any periodic wave may be expressed as the sum of a d-c component (zero frequency) and sine (or cosine) waves having fundamental and multiple or higher frequencies, the higher frequencies being called harmonics.

The emf of a phase due to the ***fundamental component*** of the flux per pole is:

$$E_{ph1} = 4.44 f K_{w1} T_{ph} \emptyset_1$$

Where $K_{w1} = k_{d1}.K_{p1}$ is the winding factor. For the ***nth harmonic***

$$E_{phn} = 4.44 n f K_{wn} T_{ph} \emptyset_n$$

The **nth harmonic** and ***fundamental*** emf components are related by

$$\frac{E_{phn}}{E_{ph1}} = \frac{B_n K_{wn}}{B_1 K_{w1}}$$

The r.m.s. phase emf is:

$$E_{ph} = \sqrt{\left(E_{ph1}{}^2 + E_{ph3}{}^2 + \cdots + E_{phn}{}^2\right)}$$

- All the ***odd harmonics*** (third, fifth, seventh, ninth, etc.) are present in the phase voltage to some extent and need to be dealt with in the design of ac machines.
- Because the resulting voltage waveform is symmetric about the center of the rotor flux, no ***even harmonics*** are present in the phase voltage.
- In ***Y- connected***, the *third-harmonic* voltage between any two terminals will be zero. This result applies not only to third-harmonic components but also to any multiple of a third-harmonic component (such as the ninth harmonic). Such special harmonic frequencies are called ***triplen harmonics***.

❖ The pitch factor of the coil at the harmonic frequency can be expressed as

$$\boxed{K_{pn} = \cos\frac{n\theta}{2}}$$ where *n* is the number of the harmonic

Elimination or Suppressed of Harmonics

Field flux waveform can be made as much sinusoidal as possible by the following methods:

1. Small air gap at the pole centre and large air gap towards the pole ends
2. ***Skewing:*** skew the pole faces if possible
3. ***Distribution*:** distribution of the armature winding along the air-gap periphery
4. ***Chording:*** with coil-span less than pole pitch
5. ***Fractional slot winding***
6. ***Alternator connections:*** star or delta connections of alternators suppress triplen harmonics from appearing across the lines

For example, for a coil-span of two-thirds $\left(\frac{2}{3} rd\right)$ of a pole pitch

$$Coil-span, \propto = \frac{2}{3} \times 180^o = 120^o \ (in\ electrical\ degrees)$$

$$Chording\ angle, \theta = 180^o - \alpha = 180^o - 120^o = 60^o$$

$$K_{p1} = \cos\frac{n\theta}{2} = \cos\frac{60^o}{2} = \cos 30^o = 0.866$$

For the 3rd harmonic: $K_{p3} = \cos\frac{3\times 60^o}{2} = \cos 90^o = 0;$

Thus all 3rd (and triplen) harmonics are eliminated from the coil and phase emf . The triplen harmonics in a 3-phase machine are normally eliminated by the phase connection.

Example: *An 8-pole, 3-phase, 60^o spread, double layer winding has 72 coils in 72 slots. The coils are short-pitched by two slots. Calculate the winding factor for the fundamental and third harmonic.*

Solution: No. of slots per pole, $Q = \frac{72}{8} = 9$

No. of slots per pole per phase, $q = \frac{Q}{m} = \frac{9}{3} = 3$

Angular displacement between the slots, $\gamma = \frac{180^o}{Q} = \frac{180^o}{9} = 20^o$

$$Coil\ span, \propto = \frac{180^o \times coil\ span\ in\ terms\ of\ slots}{No.of\ slots\ per\ pole}$$
$$= \frac{180^o(9-2)}{9} = 140^o$$

$$Chording\ angle, \theta = 180^o - coil\ span = 180^o - 140^o = 40^o$$

For the fundamental component

$$Distribution\ factor, K_d = \frac{\sin q\gamma/2}{q \sin \gamma/2} = \frac{\sin 3 \times \frac{20^o}{2}}{3 \sin \frac{20^o}{2}} = 0.96$$
$$Pitch\ factor, K_p = \cos\frac{\theta}{2} = \cos\frac{40^o}{2} = 0.94$$
$$Winding\ factor, K_w = K_d \times K_p = 0.96 \times 0.94 = 0.9$$

For the third harmonic component (n=3)

$$Distribution\ factor, K_{d3} = \frac{\sin nq\gamma/2}{q \sin n\gamma/2} = \frac{\sin\frac{3 \times 3 \times 20^o}{2}}{3 \sin\frac{3 \times 20^o}{2}} = 0.666$$
$$Pitch\ factor, K_{p3} = \cos\frac{3\theta}{2} = \cos\frac{3 \times 40^o}{2} = 0.5$$
$$Winding\ factor, K_{w3} = K_{d3} \times K_{p3} = 0.666 \times 0.5 = 0.333$$

Example3: *Calculate the r.m.s. value of the induced e.m.f. per phase of a 10-pole, 3-phase, 50Hz alternator with 2 slots per pole per phase and 4 conductors per slot in two layers. The coil span is 150^o .the flux per pole has a fundamental component of 0.12Wb and a third harmonic component.*

Solution: No. of slots/pole/phase, $q = 2$

No. of slots/pole, $Q = qm = 2 \times 3 = 6$

No. of slots/phase =$2pq = 10 \times 2 = 20$

No. of conductors connected in series, $Z_s = 20 \times 4 = 80$

No. of series turns/phase, $T = \frac{Z_s}{2} = \frac{80}{2} = 40$

Angular displacement between adjacent slots, $\gamma = \frac{180^o}{Q} = \frac{180^o}{6} = 30^o$

$$Distribution\ factor, K_d = \frac{\sin q\gamma/2}{q \sin \gamma/2} = \frac{\sin \frac{2 \times 30^o}{2}}{2 \sin \frac{30^o}{2}} = 0.966$$

$$Coil\ span\ factor, K_p = \cos\frac{\theta}{2} = \cos\frac{(180^o - 150^o)}{2} = \cos 15^o = 0.966$$

Induced emf per phase (fundamental component),

$$E_{ph1} = 4.44\ K_d K_p \emptyset f T$$
$$= 4.44 \times 0.966 \times 0.966 \times 0.12 \times 50 \times 40 = 994.4\ V$$

For third harmonic component of flux

$$Distribution\ factor, K_{d3} = \frac{\sin qn\gamma/2}{q \sin n\gamma/2} = \frac{\sin \frac{2 \times 3 \times 30^o}{2}}{3 \sin \frac{3 \times 30^o}{2}} = 0.707$$

$$Coil\ span\ factor, K_{p3} = \cos 3\frac{(180^o - 150^o)}{2} = \cos 45^o = 0.707$$

$$Frequency, f_3 = 3 \times f = 3 \times 50 = 150$$

Flux per pole, $\emptyset_3 = \frac{1}{3} \times 0.12 \times \frac{20}{100} = 0.008\ Wb$

Induced emf per phase (third harmonic component)

$$E_{ph3} = 4.44\ K_{d3} K_{p3} \emptyset_3 f_3 T$$
$$= 4.44 \times 0.707 \times 0.707 \times 0.008 \times 150 \times 40 = 106.56\ V$$

Induced emf per phase,

$$E_{ph} = \sqrt{{E_{ph1}}^2 + {E_{ph3}}^2} = \sqrt{(994.4)^2 + (106.56)^2} = 1000\ V$$

H.W

1. Three-phase voltages are applied to the three windings of an electrical machine. If any two supply terminals are interchanged, show that the direction of rotating mmf wave is reversed,through its amplitude remains unaltered.

2. A 3-phase 4-pole alternator has a winding with 8 conductors per slot. The armature has a total of 36 slots. Calculate the distribution factor. What is the induced voltage per phase when the alternator is driven at 1800 RPM, with flux of 0.041 Wb in each pole? **(Answer. 0.96, 503.197 Volts/phase)**

3. A 10 MVA ,11 KV,50 Hz ,3-phase star-connected alternator s driven at 300 RPM. The winding is housed in 360 slots and has 6 conductors per slot, the coils spanning (5/6) of a pole pitch. Calculate the sinusoidally distributed flux per pole required to give a line voltage of 11 kV on open circuit, and the full-load current per conductor. **(Answer. 0.086 weber , 524.864 Amps)**

4. A three phase ***four*** pole winding is excited by balanced three phase 50 Hz currents. Although the winding distribution has been designed to minimize the harmonics, there remains some third and fifth spatial harmonics. Thus the phase *A mmf* can be written as

$$F_a = \left(F_1 \cos\theta + F_3 \cos 3\theta + F_5 \cos 5\theta\right)\cos\omega t$$

Similar expressions can be written for phase *B* (replacing θ by $\theta - 120^o$ and ωt by $\omega t - 120^o$) and phase *C* (replacing θ by $\theta + 120^o$ and ωt by $\omega t + 120^o$).
Derive the expression for the total three phase *mmf*, and show that the fundamental and the 5th harmonic components are rotating