Introduction to Electrodynamics

Fourth Edition

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Introduction to Electrodynamics

Fourth Edition

David J Griffiths
Reed College, Oregon

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Contents

Preface **xii**

Advertisement **xiv**

1 ■ Vector Analysis **1**

1.1 Vector Algebra 1
1.1.1 Vector Operations 1
1.1.2 Vector Algebra: Component Form 4
1.1.3 Triple Products 7
1.1.4 Position, Displacement, and Separation Vectors 8
1.1.5 How Vectors Transform 10
1.2 Differential Calculus 13
1.2.1 "Ordinary" Derivatives 13
1.2.2 Gradient 13
1.2.3 The Del Operator 16
1.2.4 The Divergence 17
1.2.5 The Curl 18
1.2.6 Product Rules 20
1.2.7 Second Derivatives 22
1.3 Integral Calculus 24
1.3.1 Line, Surface, and Volume Integrals 24
1.3.2 The Fundamental Theorem of Calculus 29
1.3.3 The Fundamental Theorem for Gradients 29
1.3.4 The Fundamental Theorem for Divergences 31
1.3.5 The Fundamental Theorem for Curls 34
1.3.6 Integration by Parts 36
1.4 Curvilinear Coordinates 38
1.4.1 Spherical Coordinates 38
1.4.2 Cylindrical Coordinates 43
1.5 The Dirac Delta Function 45
1.5.1 The Divergence of $\hat{\mathbf{r}}/r^2$ 45
1.5.2 The One-Dimensional Dirac Delta Function 46
1.5.3 The Three-Dimensional Delta Function 50

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CAMBRIDGE
Cambridge University Press
978-1-108-42041-9 — Introduction to Electrodynamics
David J. Griffiths
Frontmatter
More Information

1.6 The Theory of Vector Fields 52
1.6.1 The Helmholtz Theorem 52
1.6.2 Potentials 53

2 ■ Electrostatics 59
2.1 The Electric Field 59
2.1.1 Introduction 59
2.1.2 Coulomb's Law 60
2.1.3 The Electric Field 61
2.1.4 Continuous Charge Distributions 63
2.2 Divergence and Curl of Electrostatic Fields 66
2.2.1 Field Lines, Flux, and Gauss's Law 66
2.2.2 The Divergence of E 71
2.2.3 Applications of Gauss's Law 71
2.2.4 The Curl of E 77
2.3 Electric Potential 78
2.3.1 Introduction to Potential 78
2.3.2 Comments on Potential 80
2.3.3 Poisson's Equation and Laplace's Equation 83
2.3.4 The Potential of a Localized Charge Distribution 84
2.3.5 Boundary Conditions 88
2.4 Work and Energy in Electrostatics 91
2.4.1 The Work It Takes to Move a Charge 91
2.4.2 The Energy of a Point Charge Distribution 92
2.4.3 The Energy of a Continuous Charge Distribution 94
2.4.4 Comments on Electrostatic Energy 96
2.5 Conductors 97
2.5.1 Basic Properties 97
2.5.2 Induced Charges 99
2.5.3 Surface Charge and the Force on a Conductor 103
2.5.4 Capacitors 105

3 ■ Potentials 113
3.1 Laplace's Equation 113
3.1.1 Introduction 113
3.1.2 Laplace's Equation in One Dimension 114
3.1.3 Laplace's Equation in Two Dimensions 115
3.1.4 Laplace's Equation in Three Dimensions 117
3.1.5 Boundary Conditions and Uniqueness Theorems 119
3.1.6 Conductors and the Second Uniqueness Theorem 121

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CAMBRIDGE
Cambridge University Press
978-1-108-42041-9 — Introduction to Electrodynamics
David J. Griffiths
Frontmatter
More Information

3.2 The Method of Images 124
3.2.1 The Classic Image Problem 124
3.2.2 Induced Surface Charge 125
3.2.3 Force and Energy 126
3.2.4 Other Image Problems 127
3.3 Separation of Variables 130
3.3.1 Cartesian Coordinates 131
3.3.2 Spherical Coordinates 141
3.4 Multipole Expansion 151
3.4.1 Approximate Potentials at Large Distances 151
3.4.2 The Monopole and Dipole Terms 154
3.4.3 Origin of Coordinates in Multipole Expansions 157
3.4.4 The Electric Field of a Dipole 158

4 ■ Electric Fields in Matter 167
4.1 Polarization 167
4.1.1 Dielectrics 167
4.1.2 Induced Dipoles 167
4.1.3 Alignment of Polar Molecules 170
4.1.4 Polarization 172
4.2 The Field of a Polarized Object 173
4.2.1 Bound Charges 173
4.2.2 Physical Interpretation of Bound Charges 176
4.2.3 The Field Inside a Dielectric 179
4.3 The Electric Displacement 181
4.3.1 Gauss's Law in the Presence of Dielectrics 181
4.3.2 A Deceptive Parallel 184
4.3.3 Boundary Conditions 185
4.4 Linear Dielectrics 185
4.4.1 Susceptibility, Permittivity, Dielectric Constant 185
4.4.2 Boundary Value Problems with Linear Dielectrics 192
4.4.3 Energy in Dielectric Systems 197
4.4.4 Forces on Dielectrics 202

5 ■ Magnetostatics 210
5.1 The Lorentz Force Law 210
5.1.1 Magnetic Fields 210
5.1.2 Magnetic Forces 212
5.1.3 Currents 216
5.2 The Biot-Savart Law 223
5.2.1 Steady Currents 223
5.2.2 The Magnetic Field of a Steady Current 224

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CAMBRIDGE

Cambridge University Press
978-1-108-42041-9 — Introduction to Electrodynamics
David J. Griffiths
Frontmatter
More Information

5.3 The Divergence and Curl of B 229
5.3.1 Straight-Line Currents 229
5.3.2 The Divergence and Curl of B 231
5.3.3 Ampère's Law 233
5.3.4 Comparison of Magnetostatics and Electrostatics 241
5.4 Magnetic Vector Potential 243
5.4.1 The Vector Potential 243
5.4.2 Boundary Conditions 249
5.4.3 Multipole Expansion of the Vector Potential 252

6 ■ Magnetic Fields in Matter 266

6.1 Magnetization 266
6.1.1 Diamagnets, Paramagnets, Ferromagnets 266
6.1.2 Torques and Forces on Magnetic Dipoles 266
6.1.3 Effect of a Magnetic Field on Atomic Orbits 271
6.1.4 Magnetization 273
6.2 The Field of a Magnetized Object 274
6.2.1 Bound Currents 274
6.2.2 Physical Interpretation of Bound Currents 277
6.2.3 The Magnetic Field Inside Matter 279
6.3 The Auxiliary Field H 279
6.3.1 Ampère's Law in Magnetized Materials 279
6.3.2 A Deceptive Parallel 283
6.3.3 Boundary Conditions 284
6.4 Linear and Nonlinear Media 284
6.4.1 Magnetic Susceptibility and Permeability 284
6.4.2 Ferromagnetism 288

7 ■ Electrodynamics 296

7.1 Electromotive Force 296
7.1.1 Ohm's Law 296
7.1.2 Electromotive Force 303
7.1.3 Motional emf 305
7.2 Electromagnetic Induction 312
7.2.1 Faraday's Law 312
7.2.2 The Induced Electric Field 317
7.2.3 Inductance 321
7.2.4 Energy in Magnetic Fields 328
7.3 Maxwell's Equations 332
7.3.1 Electrodynamics Before Maxwell 332
7.3.2 How Maxwell Fixed Ampère's Law 334
7.3.3 Maxwell's Equations 337

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CAMBRIDGE

Cambridge University Press
978-1-108-42041-9 — Introduction to Electrodynamics
David J. Griffiths
Frontmatter
More Information

7.3.4 Magnetic Charge 338
7.3.5 Maxwell's Equations in Matter 340
7.3.6 Boundary Conditions 342

8 ■ Conservation Laws 356

8.1 Charge and Energy 356
8.1.1 The Continuity Equation 356
8.1.2 Poynting's Theorem 357
8.2 Momentum 360
8.2.1 Newton's Third Law in Electrodynamics 360
8.2.2 Maxwell's Stress Tensor 362
8.2.3 Conservation of Momentum 366
8.2.4 Angular Momentum 370
8.3 Magnetic Forces Do No Work 373

9 ■ Electromagnetic Waves 382

9.1 Waves in One Dimension 382
9.1.1 The Wave Equation 382
9.1.2 Sinusoidal Waves 385
9.1.3 Boundary Conditions: Reflection and Transmission 388
9.1.4 Polarization 391
9.2 Electromagnetic Waves in Vacuum 393
9.2.1 The Wave Equation for E and B 393
9.2.2 Monochromatic Plane Waves 394
9.2.3 Energy and Momentum in Electromagnetic Waves 398
9.3 Electromagnetic Waves in Matter 401
9.3.1 Propagation in Linear Media 401
9.3.2 Reflection and Transmission at Normal Incidence 403
9.3.3 Reflection and Transmission at Oblique Incidence 405
9.4 Absorption and Dispersion 412
9.4.1 Electromagnetic Waves in Conductors 412
9.4.2 Reflection at a Conducting Surface 416
9.4.3 The Frequency Dependence of Permittivity 417
9.5 Guided Waves 425
9.5.1 Wave Guides 425
9.5.2 TE Waves in a Rectangular Wave Guide 428
9.5.3 The Coaxial Transmission Line 431

10 ■ Potentials and Fields 436

10.1 The Potential Formulation 436
10.1.1 Scalar and Vector Potentials 436
10.1.2 Gauge Transformations 439

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CAMBRIDGE
Cambridge University Press
978-1-108-42041-9 — Introduction to Electrodynamics
David J. Griffiths
Frontmatter
More Information

10.1.3 Coulomb Gauge and Lorenz Gauge 440
10.1.4 Lorentz Force Law in Potential Form 442
10.2 Continuous Distributions 444
10.2.1 Retarded Potentials 444
10.2.2 Jefimenko's Equations 449
10.3 Point Charges 451
10.3.1 Liénard-Wiechert Potentials 451
10.3.2 The Fields of a Moving Point Charge 456

11 ■ Radiation 466
11.1 Dipole Radiation 466
11.1.1 What is Radiation? 466
11.1.2 Electric Dipole Radiation 467
11.1.3 Magnetic Dipole Radiation 473
11.1.4 Radiation from an Arbitrary Source 477
11.2 Point Charges 482
11.2.1 Power Radiated by a Point Charge 482
11.2.2 Radiation Reaction 488
11.2.3 The Mechanism Responsible for the Radiation Reaction 492

12 ■ Electrodynamics and Relativity 502
12.1 The Special Theory of Relativity 502
12.1.1 Einstein's Postulates 502
12.1.2 The Geometry of Relativity 508
12.1.3 The Lorentz Transformations 519
12.1.4 The Structure of Spacetime 525
12.2 Relativistic Mechanics 532
12.2.1 Proper Time and Proper Velocity 532
12.2.2 Relativistic Energy and Momentum 535
12.2.3 Relativistic Kinematics 537
12.2.4 Relativistic Dynamics 542
12.3 Relativistic Electrodynamics 550
12.3.1 Magnetism as a Relativistic Phenomenon 550
12.3.2 How the Fields Transform 553
12.3.3 The Field Tensor 562
12.3.4 Electrodynamics in Tensor Notation 565
12.3.5 Relativistic Potentials 569

A ■ Vector Calculus in Curvilinear Coordinates 575
A.1 Introduction 575
A.2 Notation 575

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CAMBRIDGE

Cambridge University Press
978-1-108-42041-9 — Introduction to Electrodynamics
David J. Griffiths
Frontmatter
More Information

A.3 Gradient 576
A.4 Divergence 577
A.5 Curl 579
A.6 Laplacian 581

B ■ The Helmholtz Theorem **582**

C ■ Units **585**

Index **589**

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Preface

This is a textbook on electricity and magnetism, designed for an undergraduate course at the junior or senior level. It can be covered comfortably in two semesters, maybe even with room to spare for special topics (AC circuits, numerical methods, plasma physics, transmission lines, antenna theory, etc.) A one-semester course could reasonably stop after Chapter 7. Unlike quantum mechanics or thermal physics (for example), there is a fairly general consensus with respect to the teaching of electrodynamics; the subjects to be included, and even their order of presentation, are not particularly controversial, and textbooks differ mainly in style and tone. My approach is perhaps less formal than most; I think this makes difficult ideas more interesting and accessible.

For this new edition I have made a large number of small changes, in the interests of clarity and grace. In a few places I have corrected serious errors. I have added some problems and examples (and removed a few that were not effective). And I have included more references to the accessible literature (particularly the *American Journal of Physics*). I realize, of course, that most readers will not have the time or inclination to consult these resources, but I think it is worthwhile anyway, if only to emphasize that electrodynamics, notwithstanding its venerable age, is very much alive, and intriguing new discoveries are being made all the time. I hope that occasionally a problem will pique your curiosity, and you will be inspired to look up the reference—some of them are real gems.

I have maintained three items of unorthodox notation:

- The Cartesian unit vectors are written $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ (and, in general, all unit vectors inherit the letter of the corresponding coordinate).

- The distance from the z axis in cylindrical coordinates is designated by s, to avoid confusion with r (the distance from the *origin*, and the radial coordinate in spherical coordinates).

- The script letter $\boldsymbol{\mathscr{r}}$ denotes the vector from a source point $\mathbf{r}'$ to the field point $\mathbf{r}$ (see Figure). Some authors prefer the more explicit $(\mathbf{r} - \mathbf{r}')$. But this makes many equations distractingly cumbersome, especially when the unit vector $\hat{\boldsymbol{\mathscr{r}}}$ is involved. I realize that unwary readers are tempted to interpret $\boldsymbol{\mathscr{r}}$ as $\mathbf{r}$—it certainly makes the integrals easier! *Please take note:* $\boldsymbol{\mathscr{r}} \equiv (\mathbf{r} - \mathbf{r}')$, which is *not* the same as $\mathbf{r}$. I think it's good notation, but it does have to be handled with care.[1]

[1] In MS Word, $\boldsymbol{\mathscr{r}}$ is "Kaufmann font," but this is very difficult to install in TeX. TeX users can download a pretty good facsimile from my web site.

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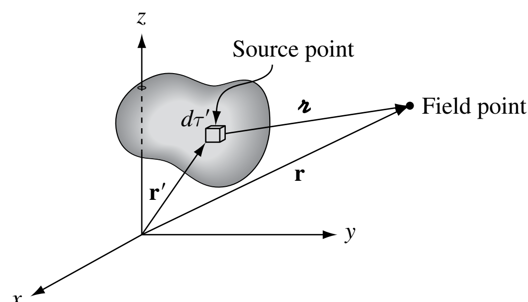

As in previous editions, I distinguish two kinds of problems. Some have a specific pedagogical purpose, and should be worked immediately after reading the section to which they pertain; these I have placed at the pertinent point within the chapter. (In a few cases the solution to a problem is used later in the text; these are indicated by a bullet (•) in the left margin.) Longer problems, or those of a more general nature, will be found at the end of each chapter. When I teach the subject, I assign some of these, and work a few of them in class. Unusually challenging problems are flagged by an exclamation point (**!**) in the margin. Many readers have asked that the answers to problems be provided at the back of the book; unfortunately, just as many are strenuously opposed. I have compromised, supplying answers when this seems particularly appropriate. A complete solution manual is available (to instructors) from the publisher; go to the Cambridge University Press web site to order a copy

I have benefitted from the comments of many colleagues. I cannot list them all here, but I would like to thank the following people for especially useful contributions to this edition: Burton Brody (Bard), Catherine Crouch (Swarthmore), Joel Franklin (Reed), Ted Jacobson (Maryland), Don Koks (Adelaide), Charles Lane (Berry), Kirk McDonald[2] (Princeton), Jim McTavish (Liverpool), Rich Saenz (Cal Poly), Darrell Schroeter (Reed), Herschel Snodgrass (Lewis and Clark), and Larry Tankersley (Naval Academy). Practically everything I know about electrodynamics—certainly about teaching electrodynamics—I owe to Edward Purcell.

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[2]Kirk's web site, http://www.hep.princeton.edu/∼mcdonald/examples/, is a fantastic resource, with clever explanations, nifty problems, and useful references.

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WHAT IS ELECTRODYNAMICS, AND HOW DOES IT FIT INTO THE GENERAL SCHEME OF PHYSICS?

Four Realms of Mechanics

In the diagram below, I have sketched out the four great realms of mechanics:

Classical Mechanics (Newton)	**Quantum Mechanics** (Bohr, Heisenberg, Schrödinger, et al.)
Special Relativity (Einstein)	**Quantum Field Theory** (Dirac, Pauli, Feynman, Schwinger, et al.)

Newtonian mechanics is adequate for most purposes in "everyday life," but for objects moving at high speeds (near the speed of light) it is incorrect, and must be replaced by special relativity (introduced by Einstein in 1905); for objects that are extremely small (near the size of atoms) it fails for different reasons, and is superseded by quantum mechanics (developed by Bohr, Schrödinger, Heisenberg, and many others, in the 1920's, mostly). For objects that are both very fast *and* very small (as is common in modern particle physics), a mechanics that combines relativity and quantum principles is in order; this relativistic quantum mechanics is known as quantum field theory—it was worked out in the thirties and forties, but even today it cannot claim to be a completely satisfactory system. In this book, save for the last chapter, we shall work exclusively in the domain of classical mechanics, although electrodynamics extends with unique simplicity to the other three realms. (In fact, the theory is in most respects *automatically* consistent with special relativity, for which it was, historically, the main stimulus.)

Four Kinds of Forces

Mechanics tells us how a system will behave when subjected to a given *force*. There are just *four* basic forces known (presently) to physics: I list them in the order of decreasing strength:

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1. Strong
2. Electromagnetic
3. Weak
4. Gravitational

The brevity of this list may surprise you. Where is friction? Where is the "normal" force that keeps you from falling through the floor? Where are the chemical forces that bind molecules together? Where is the force of impact between two colliding billiard balls? The answer is that *all* these forces are *electromagnetic.* Indeed, it is scarcely an exaggeration to say that we live in an electromagnetic world—virtually every force we experience in everyday life, with the exception of gravity, is electromagnetic in origin.

The **strong forces**, which hold protons and neutrons together in the atomic nucleus, have extremely short range, so we do not "feel" them, in spite of the fact that they are a hundred times more powerful than electrical forces. The **weak forces**, which account for certain kinds of radioactive decay, are also of short range, and they are far weaker than electromagnetic forces. As for gravity, it is so pitifully feeble (compared to all of the others) that it is only by virtue of huge mass concentrations (like the earth and the sun) that we ever notice it at all. The electrical repulsion between two electrons is 10^{42} times as large as their gravitational attraction, and if atoms were held together by gravitational (instead of electrical) forces, a single hydrogen atom would be much larger than the known universe.

Not only are electromagnetic forces overwhelmingly dominant in everyday life, they are also, at present, the *only* ones that are completely understood. There is, of course, a classical theory of gravity (Newton's law of universal gravitation) and a relativistic one (Einstein's general relativity), but no entirely satisfactory quantum mechanical theory of gravity has been constructed (though many people are working on it). At the present time there is a very successful (if cumbersome) theory for the weak interactions, and a strikingly attractive candidate (called **chromodynamics**) for the strong interactions. All these theories draw their inspiration from electrodynamics; none can claim conclusive experimental verification at this stage. So electrodynamics, a beautifully complete and successful theory, has become a kind of paradigm for physicists: an ideal model that other theories emulate.

The laws of classical electrodynamics were discovered in bits and pieces by Franklin, Coulomb, Ampère, Faraday, and others, but the person who completed the job, and packaged it all in the compact and consistent form it has today, was James Clerk Maxwell. The theory is now about 150 years old.

The Unification of Physical Theories

In the beginning, **electricity** and **magnetism** were entirely separate subjects. The one dealt with glass rods and cat's fur, pith balls, batteries, currents, electrolysis, and lightning; the other with bar magnets, iron filings, compass needles, and the North Pole. But in 1820 Oersted noticed that an *electric* current could deflect

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a *magnetic* compass needle. Soon afterward, Ampère correctly postulated that *all* magnetic phenomena are due to electric charges in motion. Then, in 1831, Faraday discovered that a moving *magnet* generates an *electric* current. By the time Maxwell and Lorentz put the finishing touches on the theory, electricity and magnetism were inextricably intertwined. They could no longer be regarded as separate subjects, but rather as two *aspects* of a *single* subject: **electromagnetism**.

Faraday speculated that light, too, is electrical in nature. Maxwell's theory provided spectacular justification for this hypothesis, and soon **optics**—the study of lenses, mirrors, prisms, interference, and diffraction—was incorporated into electromagnetism. Hertz, who presented the decisive experimental confirmation for Maxwell's theory in 1888, put it this way: "The connection between light and electricity is now established . . . In every flame, in every luminous particle, we see an electrical process . . . Thus, the domain of electricity extends over the whole of nature. It even affects ourselves intimately: we perceive that we possess . . . an electrical organ—the eye." By 1900, then, three great branches of physics–electricity, magnetism, and optics–had merged into a single unified theory. (And it was soon apparent that visible light represents only a tiny "window" in the vast spectrum of electromagnetic radiation, from radio through microwaves, infrared and ultraviolet, to x-rays and gamma rays.)

Einstein dreamed of a further unification, which would combine gravity and electrodynamics, in much the same way as electricity and magnetism had been combined a century earlier. His **unified field theory** was not particularly successful, but in recent years the same impulse has spawned a hierarchy of increasingly ambitious (and speculative) unification schemes, beginning in the 1960s with the **electroweak** theory of Glashow, Weinberg, and Salam (which joins the weak and electromagnetic forces), and culminating in the 1980s with the **superstring** theory (which, according to its proponents, incorporates all four forces in a single "theory of everything"). At each step in this hierarchy, the mathematical difficulties mount, and the gap between inspired conjecture and experimental test widens; nevertheless, it is clear that the unification of forces initiated by electrodynamics has become a major theme in the progress of physics.

The Field Formulation of Electrodynamics

The fundamental problem a theory of electromagnetism hopes to solve is this: I hold up a bunch of electric charges *here* (and maybe shake them around); what happens to some *other* charge, over *there?* The classical solution takes the form of a **field theory**: We say that the space around an electric charge is permeated by electric and magnetic **fields** (the electromagnetic "odor," as it were, of the charge). A second charge, in the presence of these fields, experiences a force; the fields, then, transmit the influence from one charge to the other—they "mediate" the interaction.

When a charge undergoes *acceleration,* a portion of the field "detaches" itself, in a sense, and travels off at the speed of light, carrying with it energy, momentum, and angular momentum. We call this **electromagnetic radiation**. Its exis-

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tence invites (if not *compels*) us to regard the fields as independent dynamical entities in their own right, every bit as "real" as atoms or baseballs. Our interest accordingly shifts from the study of forces between charges to the theory of the fields themselves. But it takes a charge to *produce* an electromagnetic field, and it takes another charge to *detect* one, so we had best begin by reviewing the essential properties of electric charge.

Electric Charge

1. *Charge comes in two varieties,* which we call "plus" and "minus," because their effects tend to *cancel* (if you have $+q$ and $-q$ at the same point, electrically it is the same as having no charge there at all). This may seem too obvious to warrant comment, but I encourage you to contemplate other possibilities: what if there were 8 or 10 different species of charge? (In chromodynamics there are, in fact, *three* quantities analogous to electric charge, each of which may be positive or negative.) Or what if the two kinds did not tend to cancel? The extraordinary fact is that plus and minus charges occur in *exactly* equal amounts, to fantastic precision, in bulk matter, so that their effects are almost completely neutralized. Were it not for this, we would be subjected to enormous forces: a potato would explode violently if the cancellation were imperfect by as little as one part in 10^{10}.

2. *Charge is conserved:* it cannot be created or destroyed—what there is now has always been. (A plus charge can "annihilate" an equal minus charge, but a plus charge cannot simply disappear by itself—*something* must pick up that electric charge.) So the total charge of the universe is fixed for all time. This is called **global** conservation of charge. Actually, I can say something much stronger: Global conservation would allow for a charge to disappear in New York and instantly reappear in San Francisco (that wouldn't affect the *total*), and yet we know this doesn't happen. If the charge *was* in New York and it *went* to San Francisco, then it must have passed along some continuous path from one to the other. This is called **local** conservation of charge. Later on we'll see how to formulate a precise mathematical law expressing local conservation of charge—it's called the **continuity equation**.

3. *Charge is quantized.* Although nothing in classical electrodynamics requires that it be so, the *fact* is that electric charge comes only in discrete lumps—integer multiples of the basic unit of charge. If we call the charge on the proton $+e$, then the electron carries charge $-e$; the neutron charge zero; the pi mesons $+e$, 0, and $-e$; the carbon nucleus $+6e$; and so on (never $7.392e$, or even $1/2e$).[3] This fundamental unit of charge is extremely small, so for practical purposes it is usually appropriate to ignore quantization altogether. Water, too, "really" consists of discrete lumps (molecules); yet, if we are dealing with reasonably large

[3] Actually, protons and neutrons are composed of three **quarks,** which carry fractional charges ($\pm\frac{2}{3}e$ and $\pm\frac{1}{3}e$). However, *free* quarks do not appear to exist in nature, and in any event, this does not alter the fact that charge is quantized; it merely reduces the size of the basic unit.

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quantities of it we can treat it as a continuous fluid. This is in fact much closer to Maxwell's own view; he knew nothing of electrons and protons—he must have pictured charge as a kind of "jelly" that could be divided up into portions of any size and smeared out at will.

Units

The subject of electrodynamics is plagued by competing systems of units, which sometimes render it difficult for physicists to communicate with one another. The problem is far worse than in mechanics, where Neanderthals still speak of pounds and feet; in mechanics, at least all equations *look* the same, regardless of the units used to measure quantities. Newton's second law remains $\mathbf{F} = m\mathbf{a}$, whether it is feet-pounds-seconds, kilograms-meters-seconds, or whatever. But this is not so in electromagnetism, where Coulomb's law may appear variously as

$$\mathbf{F} = \frac{q_1 q_2}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}} \text{ (Gaussian)}, \quad \text{or} \quad \mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}} \text{ (SI)}, \quad \text{or} \quad \mathbf{F} = \frac{1}{4\pi}\frac{q_1 q_2}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}} \text{ (HL)}.$$

Of the systems in common use, the two most popular are **Gaussian** (cgs) and **SI** (mks). Elementary particle theorists favor yet a third system: **Heaviside-Lorentz**. Although Gaussian units offer distinct theoretical advantages, most undergraduate instructors seem to prefer SI, I suppose because they incorporate the familiar household units (volts, amperes, and watts). In this book, therefore, I have used SI units. Appendix C provides a "dictionary" for converting the main results into Gaussian units.

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