Electrodynamics of Continuous Media in the Extreme Relativistic Regime

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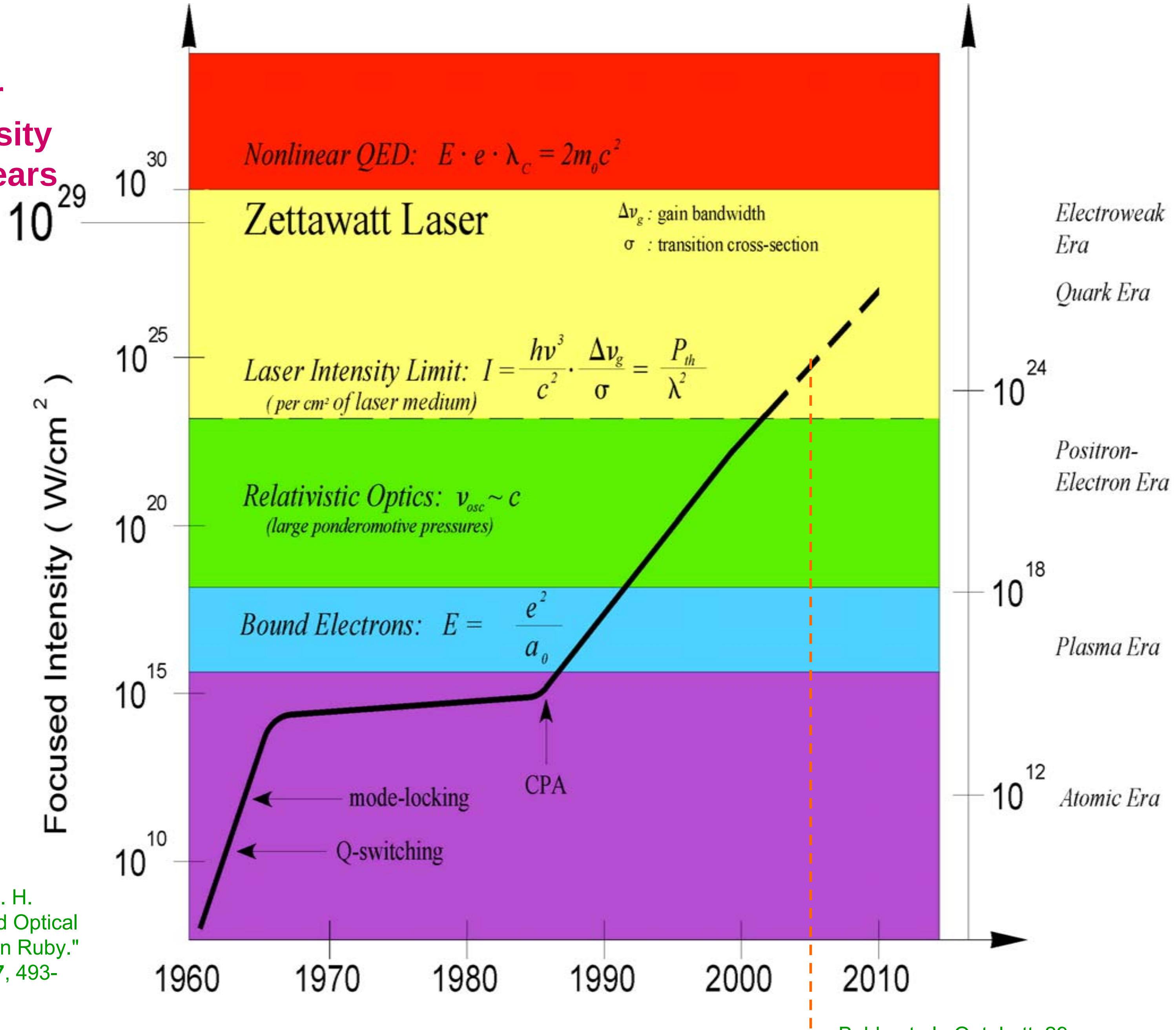
Laser Intensity vs. years



Maiman, T. H. "Stimulated Optical Radiation in Ruby." *Nature* **187**, 493-494, 1960

Mourou, G. A., Barty, C. P. J., and Perry, M. D., 1998, Phys. Today 51, 22

Bahk, et al., Opt. Lett. 29, 2837 (2004)

Ultra-Relativistic Effects in Laser – Plasma Interaction

Quiver electron energy becomes larger than $m_e c^2$ when the dimensionless amplitude of the laser pulse is greater than unity: $a_0 = eE_0/m_e \omega c > 1$. The electron energy scales as $\mathcal{E} = m_e c^2 a_0^2/2$.

This corresponds for 1μm laser wavelength to the intensity above 1.35×10^{18}W/cm^2. Recently Bahk, et al., Opt. Lett. 29, 2837 (2004), reported the experimental demonstration of $I=10^{22}$W/cm^2.

The ELI will achieve even high intensity.

For such intense laser the nonlinear plasma electrodynamics becomes of the key importance with charged particle (electron and ion) acceleration, laser pulse shortening and its frequency upshifting.

Laser Accelerators of Charged Particles

A. Electron acceleration in the wake wave left in a tenuous plasma behind the ultra-short laser pulse, by the Laser Wake Field Acceleration **(LWFA)** mechanism or/and direct electron acceleration by the laser field.

B. Ion acceleration in the regimes of strong electric charge separation (when the electrons accelerated by the laser radiation leave the irradiated by the laser pulse region) and by the Radiation Pressure Dominated Acceleration **(RPDA)** mechanism when the ions are trapped inside the plasma cloud, which is accelerated by the light pressure.

Laser Pulse Shortening and Inensification during Nonlinear Laser-Plasma Interaction

A. Laser pulse shortening with its intensification and the frequency upshifting during interaction with nonlinear Langmuir waves in the Flying Mirror Light Intensification **(FMLI)** process.

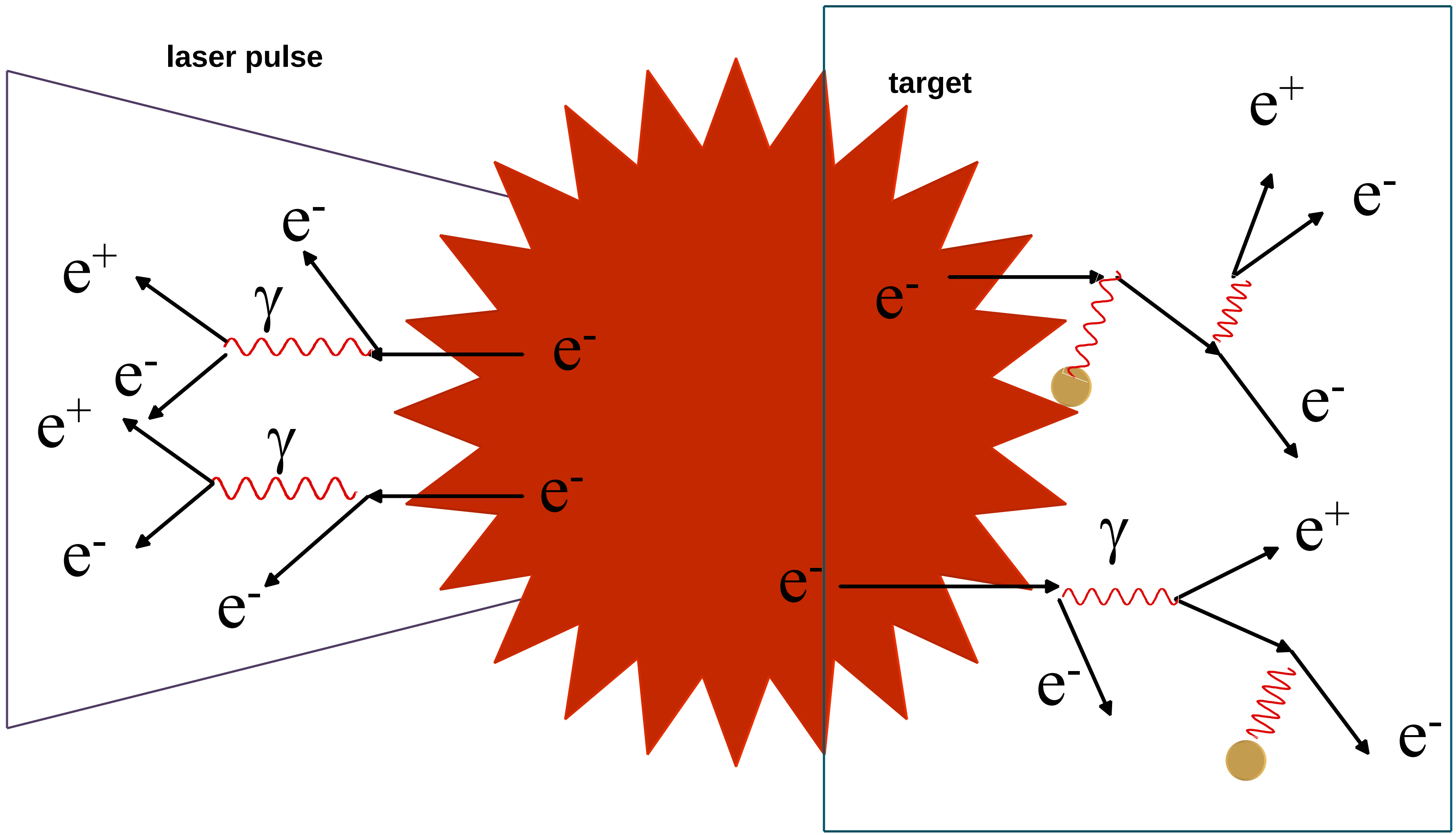
Extreme Intensity and Power Laser Radiation for Nonlinear Vacuum Probing

A. Electron-positron pair creation in vacuum.

B. Nonlinear refraction index due to vacuum polarization.

QED processes

Electron-positron pair generation; Bremsstrahlung; Inverse Compton Scattering; Trident process; Bethe-Heitler process



Upper Limit on the Electric Field Amplitude

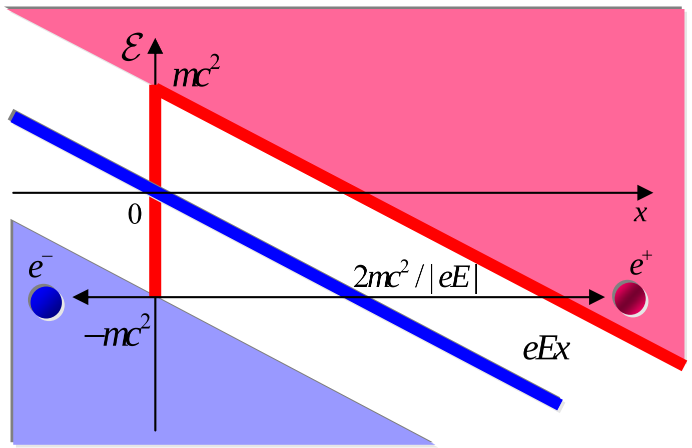
We reach a limit when the nonlinear QED with the electron-positron pair creation in the vacuum comes into play, at the critical QED electric field, which corresponds to so strong electric field that produces a work on the Compton length equal to $m_e c^2$, i.e.

$$E_{QED} = m_e^2 c^3 / e\hbar$$

It corresponds to the intensity $\approx 10^{29} W/cm^2$

W.Heisenberg, H.Euler (1936)
J. Schwinger (1951)
E. Brezin & C. Itzykson (1970)
V. S. Popov (1971)

Sub-barrier tunneling



$$w = \frac{1}{4\pi^2}\frac{c}{\lambda\!\!\!^{-}{}_C^4}\left(\frac{E}{E_{QED}}\right)^2 \exp\left(-\frac{\pi E_{QED}}{E}\right)$$

ELECTRON-POSITRON PAIR PRODUCTION BY FOCUSED ELECTROMAGNETIC PULSES

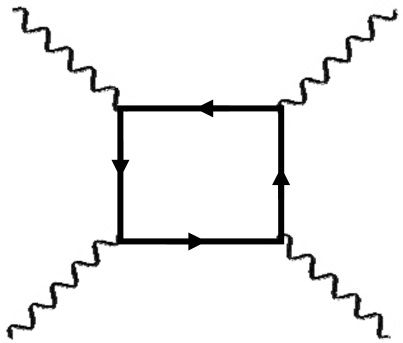
Pair production by single focused pulse: N. B. Narozhny, et al., Phys. Lett. A 330, 1 (2004). Electron-positron pairs produced by focused laser pulse intensity two orders of magnitude less than critical ~10^{27} W/cm^2

Pair production by oppositely directed focused laser pulses: N. B. Narozhny, et al., JETP Lett., 80, 434 (2004). Pair production at intensities one-two orders of magnitude less than single focused pulse ~10^{26} W/cm^2

Nonlinear QED Vacuum

In a strong EM field vacuum behaves similarly to a birefracting, i.e. anisotropic medium. This fact is known since papers published by Halpern (1933), and by Heisenberg & Euler (1936). After discovering the pulsars and with the emerging of the lasers able to generate relativistically strong EM fields, it becomes clear that the effects of vacuum polarization can be observed in cosmos and under laboratory conditions .

One of the most beautiful effects predicted by QED is photon-photon scattering due to vacuum polarization. This process is described by the diagram:



The cross section of photon-photon scattering in the limit

$\hbar\omega \ll m_e c^2$ is equal to

$$\sigma = \frac{973}{10125\pi}\alpha^2 r_e^{\,2}\left(\frac{\hbar\omega}{m_e c^2}\right)^6$$

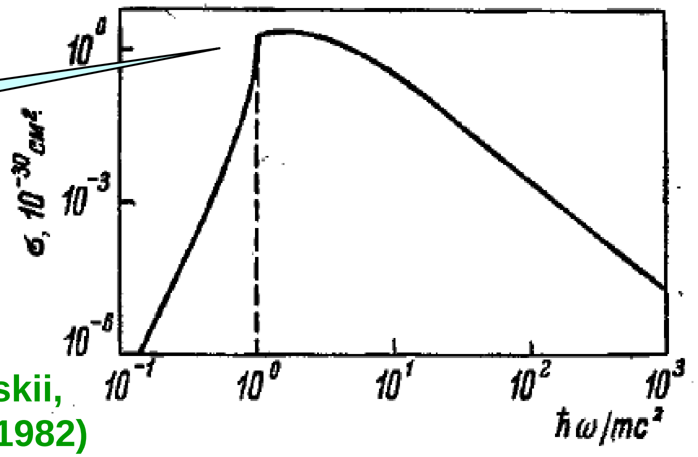
for $\hbar\omega \gg m_e c^2$ we have

$$\sigma = 4.7\alpha^4\left(\frac{c}{\omega}\right)^2$$

it reaches its maximum $\sigma_{\max} \approx 10^{-30}\,cm^{-2}$

at $\hbar\omega \approx 1.5\, m_e c^2$

e^+e^- pair creation



Berestetskii, Lifshitz, Pitaevskii,
Quantum Electrodynamics **(1982)**

Heisenberg-Euler Lagrangian

The intense laser light utilization for studies of nonlinear QED vacuum were discussed by Aleksandrov, et al., (1985), Rozanov (1993,1998), Marklund and Shukla (2005) who considered theoretically a number of nonlinear processes: 4-wave interactions, induced focusing, etc.

Theoretical description of nonlinear QED vacuum in the limit

$$E \ll E_{QED} \quad \text{and} \quad \hbar\omega \ll m_e c^2$$

is based on the Heisenberg-Euler Lagrangian

$$\mathcal{L} = \frac{1}{16\pi} F_{\alpha\beta} F^{\alpha\beta} - \frac{\kappa}{64\pi} \left[5\left(F_{\alpha\beta} F^{\alpha\beta} \right)^2 - 14 F_{\alpha\beta} F^{\beta\gamma} F_{\gamma\delta} F^{\delta\mu} \right]$$

with $\kappa = e^4 \hbar / 45\pi m_e^{\,4} c^7$ and $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$

Interaction of two counter-propagating pulses is described by

$$\partial_t E_1 + \partial_z E_1 = i n_2 \omega_1 |E_2|^2 E_1; \quad \partial_t E_2 - \partial_z E_2 = i n_2 \omega_2 |E_1|^2 E_2$$

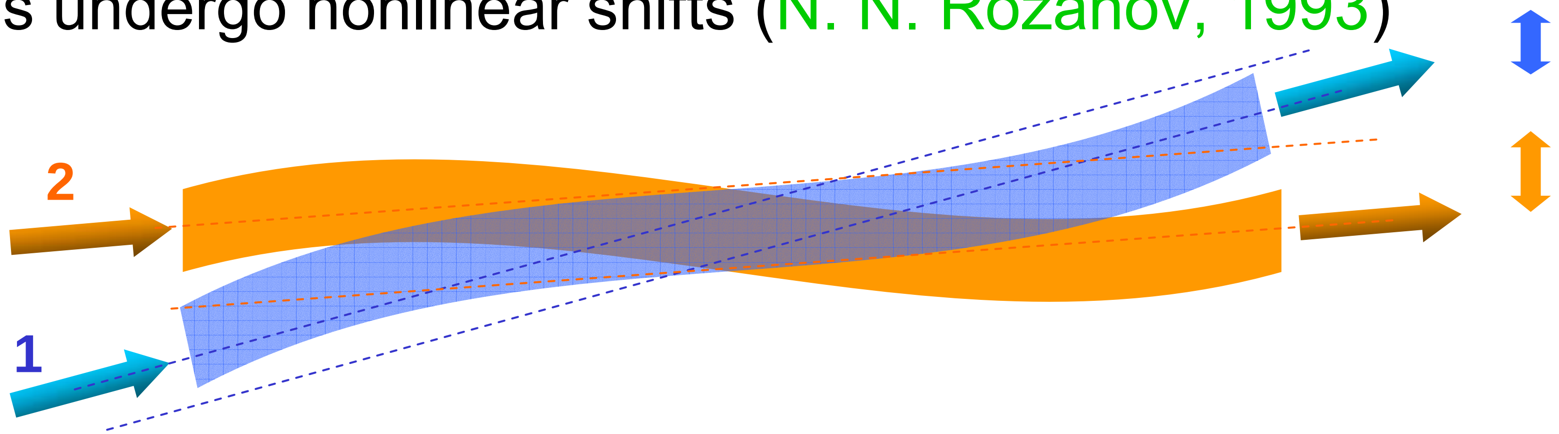
with $n_2 = 7(e^4 \hbar / 45 \pi m_e^4 c^7)$

These equations yield for $E_j = \sqrt{I_j} \exp(i\Phi_j) \quad j = 1,2$

$$I_1 = I_1(u), \quad I_2 = I_2(v), \quad u = z - t, \quad v = z + t$$

$$\Phi_1(u,v) = \Phi_1(u,v_0) + (n_2\omega_1/2)\int_{v_0}^{v} I_2(s)ds; \; \Phi_2(u,v) = \Phi_2(u_0,v) - (n_2\omega_2/2)\int_{u_0}^{u} I_1(s)ds$$

The pulse profiles transport without any distortion and the phases undergo nonlinear shifts (N. N. Rozanov, 1993)



There is no self-focusing of the plane EM wave because both the invariants, $F = (B^2 - E^2)/2$ and $G = (E \cdot B)$, vanish

Two counter-propagating EM waves undergo induced focusing because in this case $F \neq 0$

The critical power for transverse nonlinear effects is

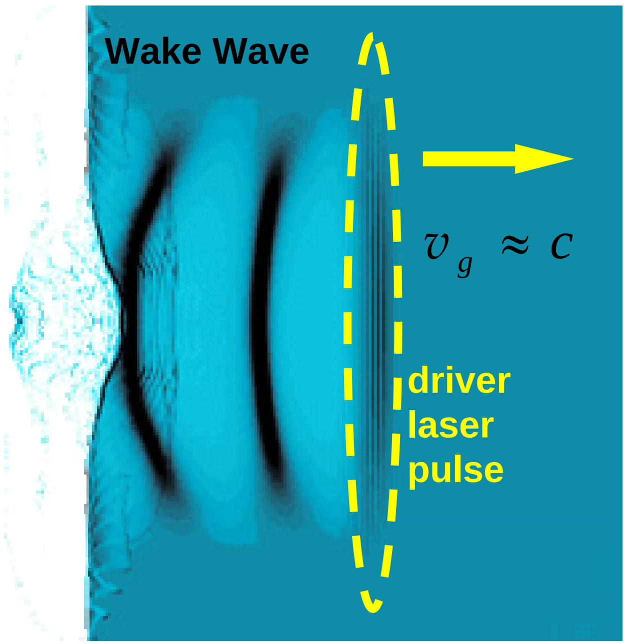
$$\mathcal{P}_{cr} = \frac{45\pi^2}{\alpha} \frac{cE_{QED}^2 \lambda^2}{4\pi}$$

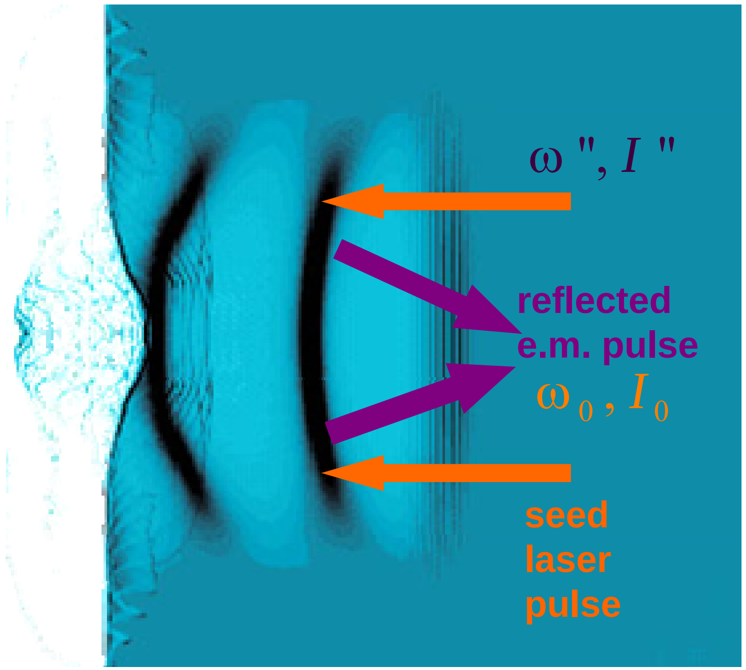
for $\lambda = 1\mu m$ it is equal to $\mathcal{P}_{cr} \approx 2.5 \times 10^{24} W$

In order to approach the "nonlinear vacuum frontier" we must increase either the EM wave power or decrease its wavelength.

EM Pulse Intensification and Shortening in Flying Mirror Light Intensification (FMLI) process

(RELATIVISTIC ENGINEERING)





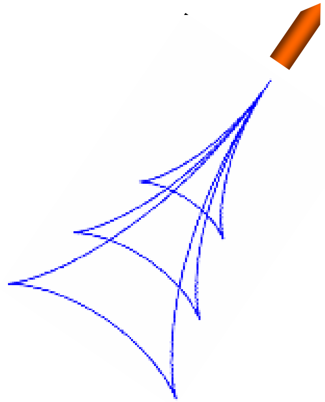
$$\omega'' = \frac{1 + v_{ph}/c}{1 - v_{ph}/c}\omega \approx 4\gamma_{ph}^2 \omega_0$$

$$I''_{max} \approx R(\gamma_{ph})\, \gamma_{ph}^6\, I_0$$

S. V. Bulanov, T. Esirkepov, T. Tajima, Phys. Rev. Lett. 91, 085001 (2003)

$$R(\gamma_{ph}) \sim \gamma_{ph}^{-3}$$

Wake Waves



Kelvin Ship Waves

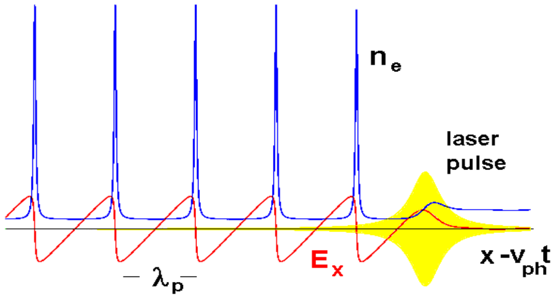
$$\omega = \sqrt{kg}$$

$$x = X_1 \cos\theta \left(1 - \frac{1}{2}\cos^2\theta\right)$$

$$y = X_1 \cos^2\theta \sin\theta$$

$$-\pi/2 < \theta < \pi/2$$

Wake Plasma Waves



$$\lambda_p = 2\pi / k_p \qquad k_p v_{ph} = \omega_{pe}$$

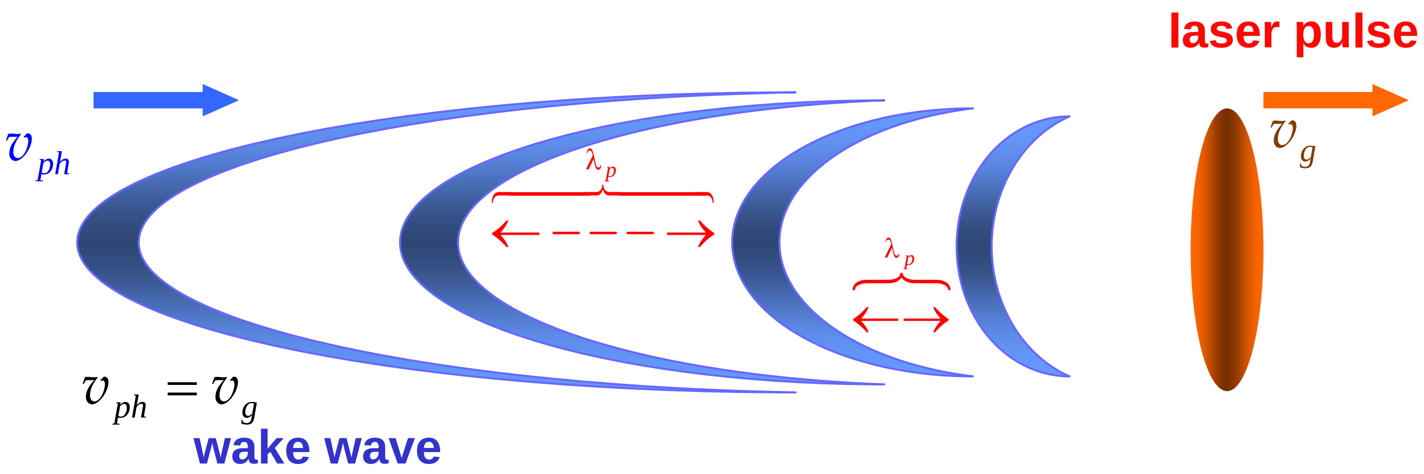
$$\omega_{pe} = \left(4\pi n e^2 / m_e\right)^{1/2}$$

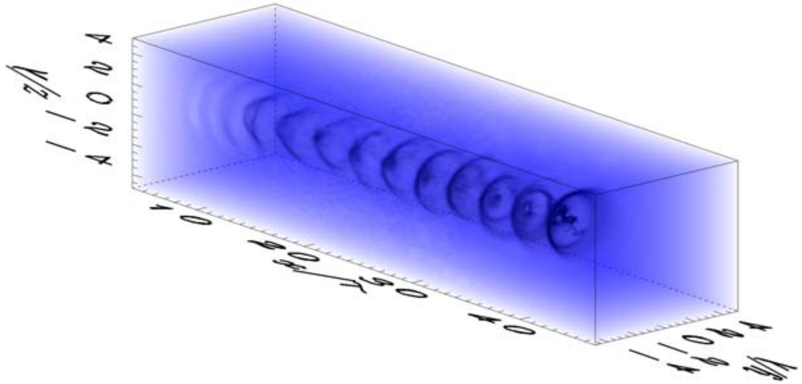
Wave Break

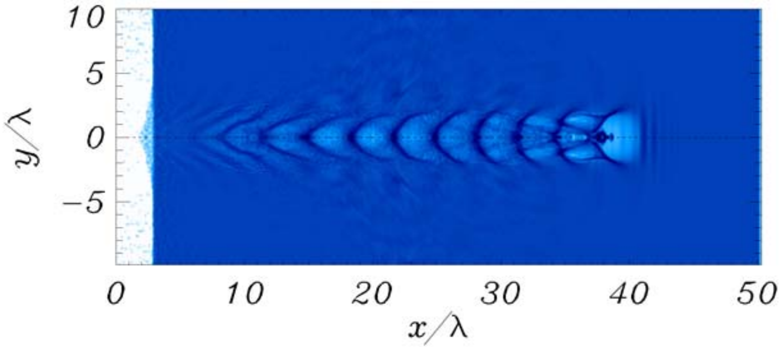
Paraboloidal Form of the Wake Wave

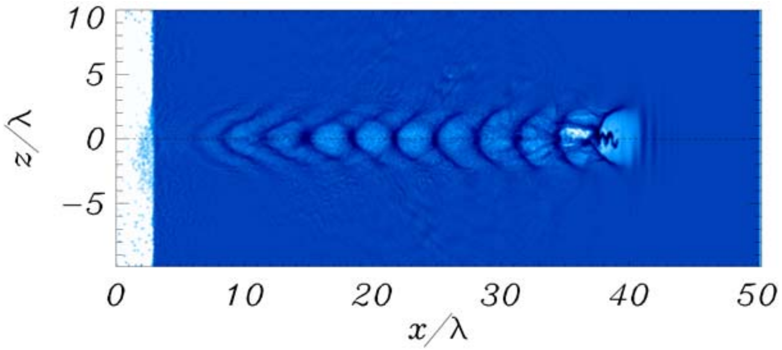
Wake-Wave-Breaking can be destructive or it can develop in a gradual (gentle), i.e. in a controllable way, which, in the case of the wake wave, provides a mechanism for the electron injection into the acceleration phase.

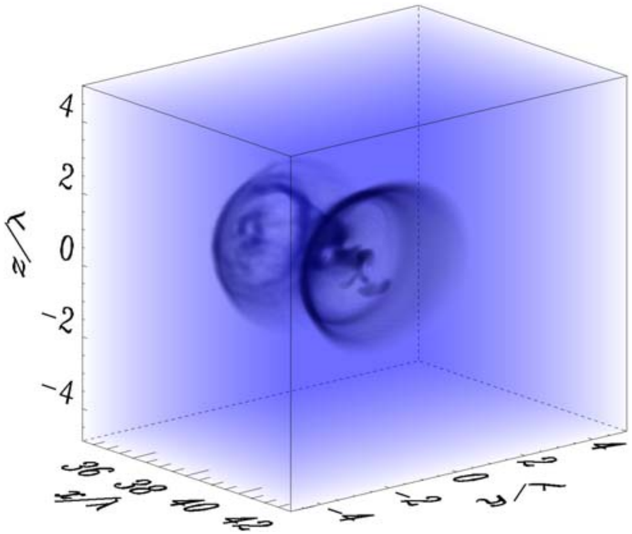
3D relativistically strong wake wave has a paraboloidal form







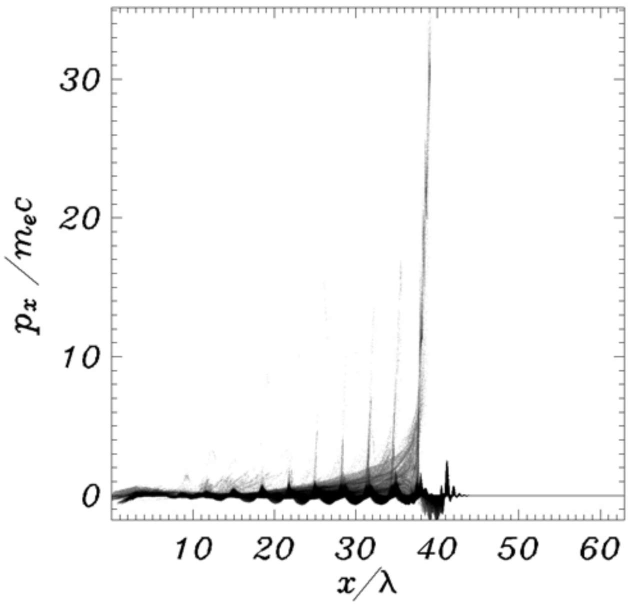




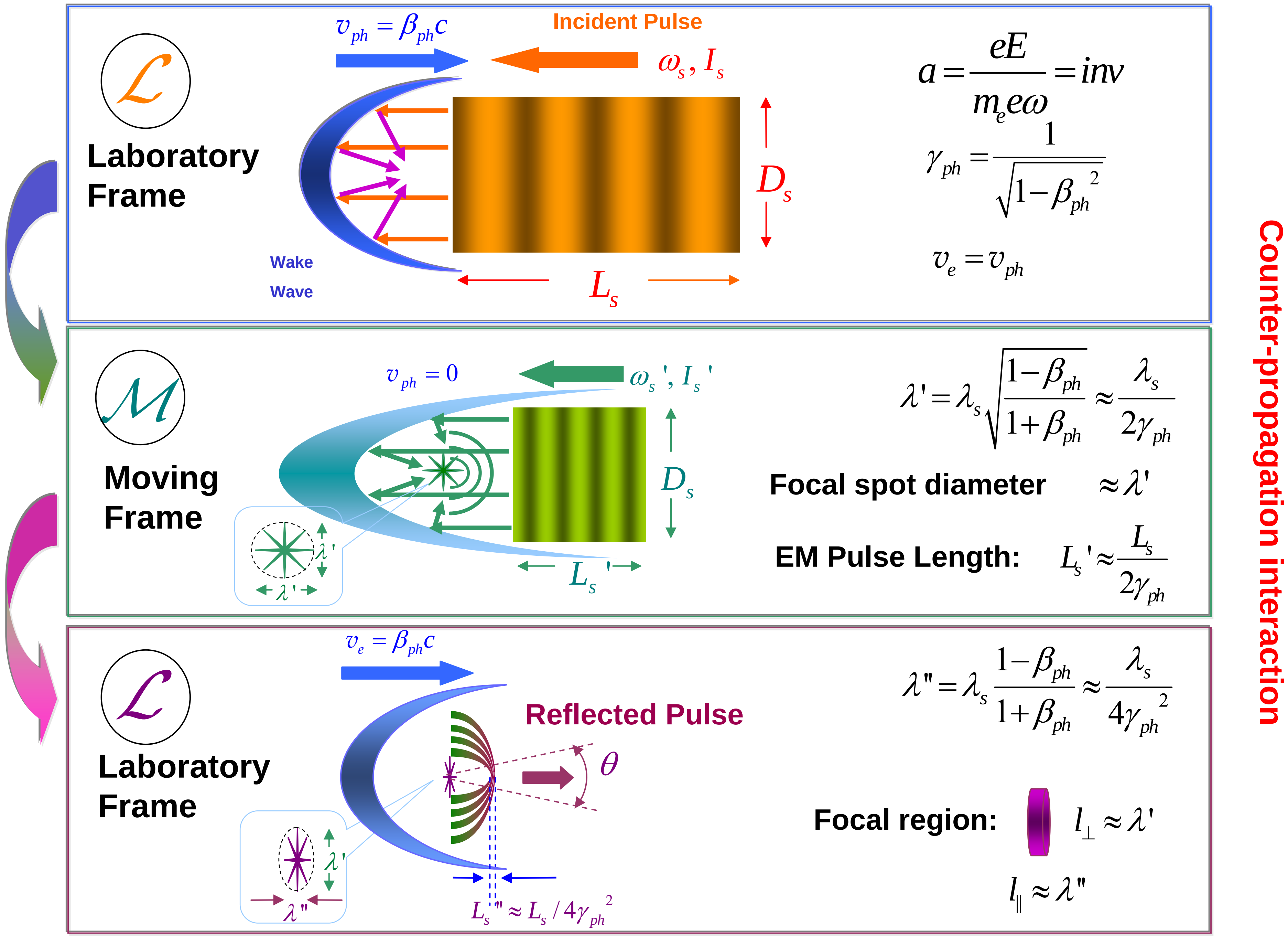
$$\omega / \omega_{pe} = 5$$

$$a = 2$$

$$l = 12\lambda$$

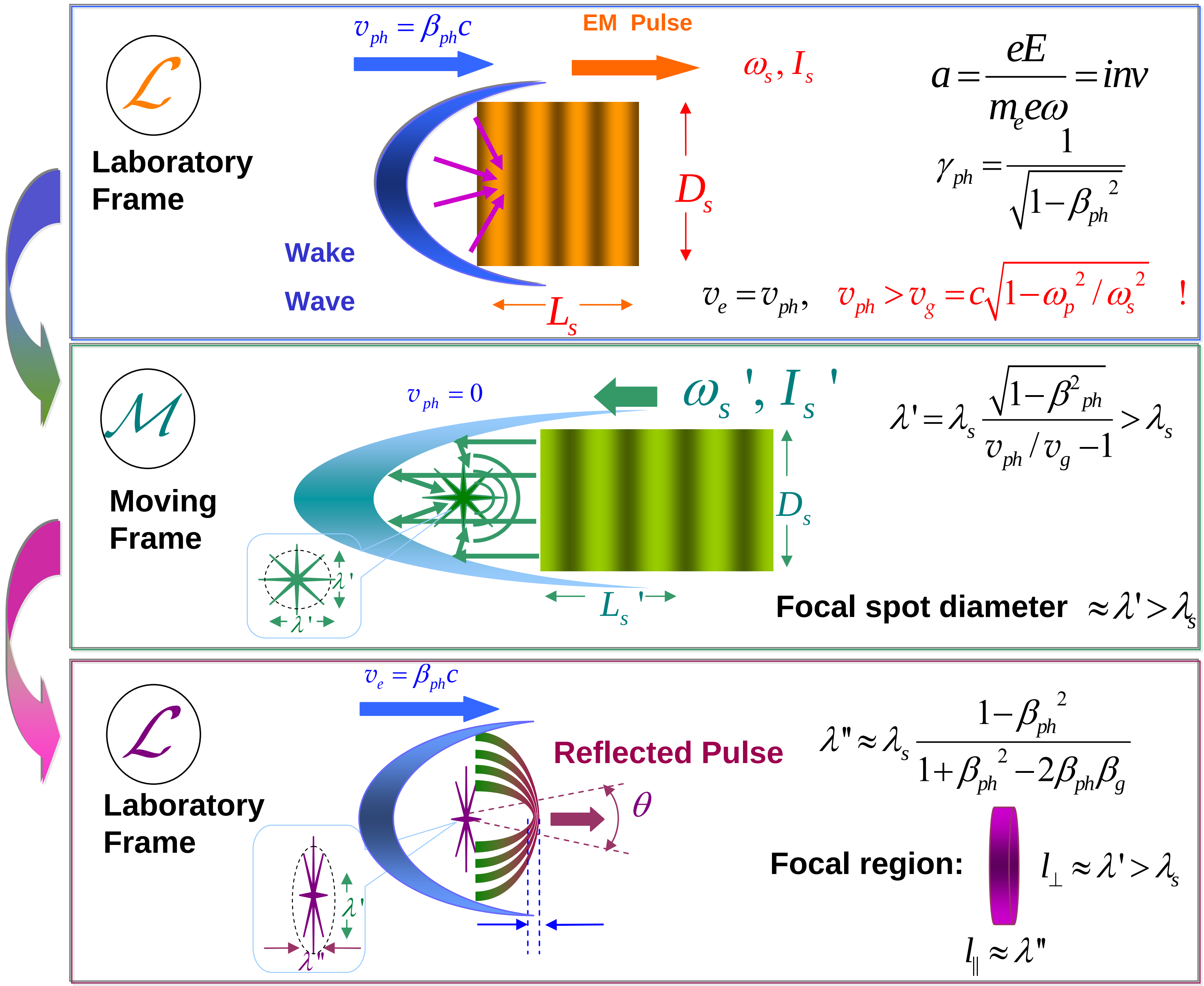


Transverse Wake-wave Breaking

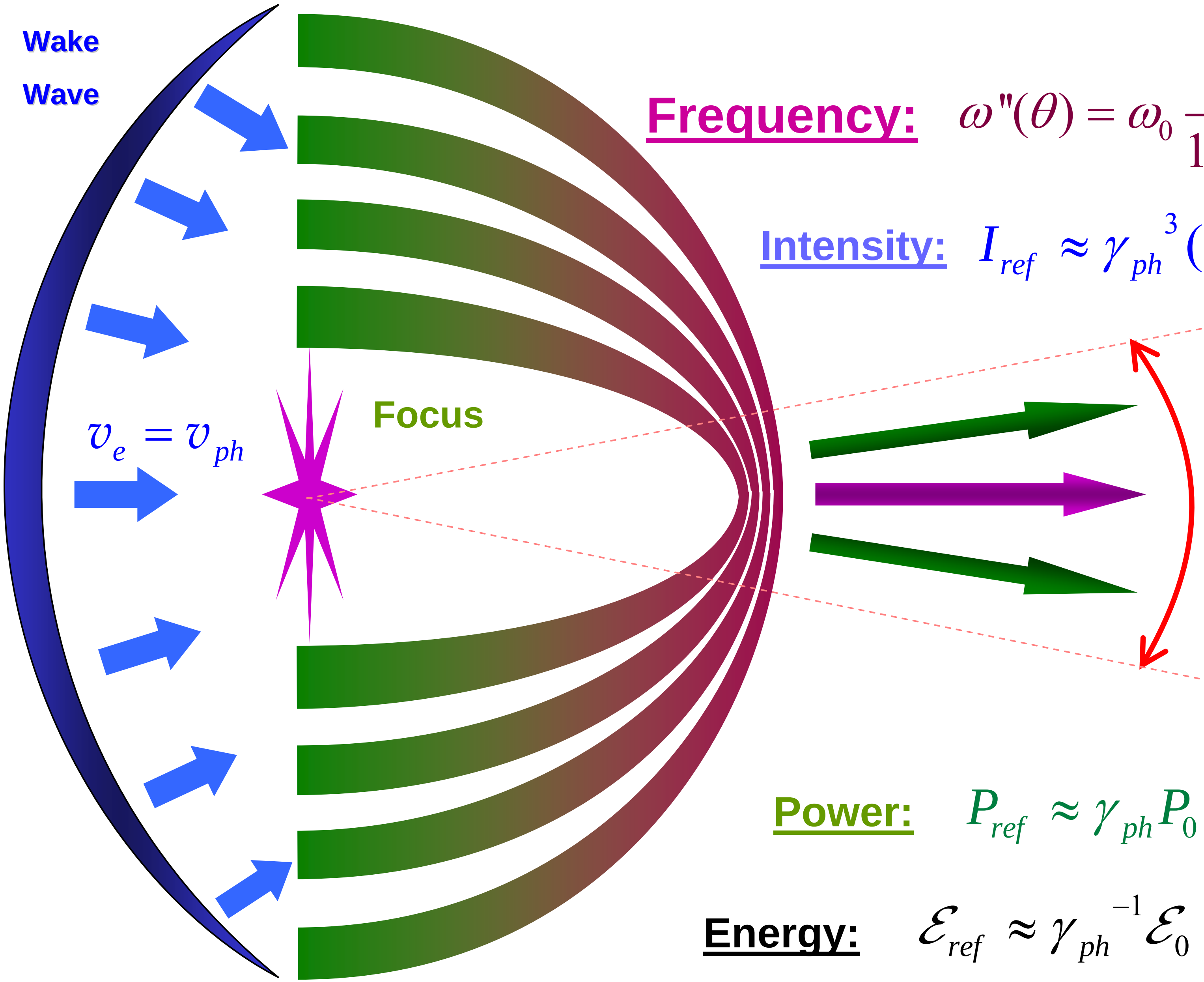

Laboratory Frame
$v_{ph} = \beta_{ph} c$
Incident Pulse
ω_s, I_s
D_s
L_s
Wake Wave
$a = \frac{eE}{m_e e \omega} = inv$
$\gamma_{ph} = \frac{1}{\sqrt{1-\beta_{ph}^2}}$
$v_e = v_{ph}$
Moving Frame
$v_{ph} = 0$
ω_s', I_s'
D_s
L_s'
λ'
$\lambda' = \lambda_s \sqrt{\frac{1-\beta_{ph}}{1+\beta_{ph}}} \approx \frac{\lambda_s}{2\gamma_{ph}}$
Focal spot diameter $\approx \lambda'$
EM Pulse Length: $L_s' \approx \frac{L_s}{2\gamma_{ph}}$
Laboratory Frame
$v_e = \beta_{ph} c$
Reflected Pulse
θ
λ'
λ''
$L_s'' \approx L_s / 4\gamma_{ph}^2$
$\lambda'' = \lambda_s \frac{1-\beta_{ph}}{1+\beta_{ph}} \approx \frac{\lambda_s}{4\gamma_{ph}^2}$
Focal region: $l_\perp \approx \lambda'$
$l_\parallel \approx \lambda''$
Counter-propagation interaction

Co-propagation interaction: photon accelerator

S.V. Bulanov and A. S. Sakharov, JETP Lett. 54, 203 (1991)
S. V. Bulanov, F. Pegoraro, A. M. Pukhov, Phys. Rev. Lett. 74, 710 (1995)
Z-M. Sheng, Y. Sentoku, K. Mima, K. Nishihara, Phys. Rev. E 62, 7258 (2000)



Reflected E.M. Beam



Frequency: $\omega''(\theta) = \omega_0 \dfrac{1+\beta_{ph}}{1-\beta_{ph}\cos\theta}$

Intensity: $I_{ref} \approx \gamma_{ph}{}^3 (D_s/\lambda_s)^2 I_0$

Collimation:

$\theta : 1/\gamma_{ph}$

Almost all the energy is emitted into narrow angle

Power: $P_{ref} \approx \gamma_{ph} P_0$

Energy: $\mathcal{E}_{ref} \approx \gamma_{ph}{}^{-1} \mathcal{E}_0$

Critical QED Electric Field

Intensity: $$I_{ref} \approx \gamma_{ph}^3 (D_s / \lambda_s)^2 I_0$$

$$E_{ref} = \sqrt{\frac{4\pi I_{ref}}{c}} \rightarrow E_{QED}$$

The electric field can not be larger than the Schwinger field in the co-moving with the mirror reference frame, M.

In the laboratory reference frame, L, we have for the EM field invariant

$$F = \frac{B^2 - E^2}{2} \approx -\frac{E_{QED}^2}{2\gamma_{ph}^2}$$

i.e. the upper limit for the electric field is

$$E_{ref} \leq \gamma_{ph} E_{QED}$$

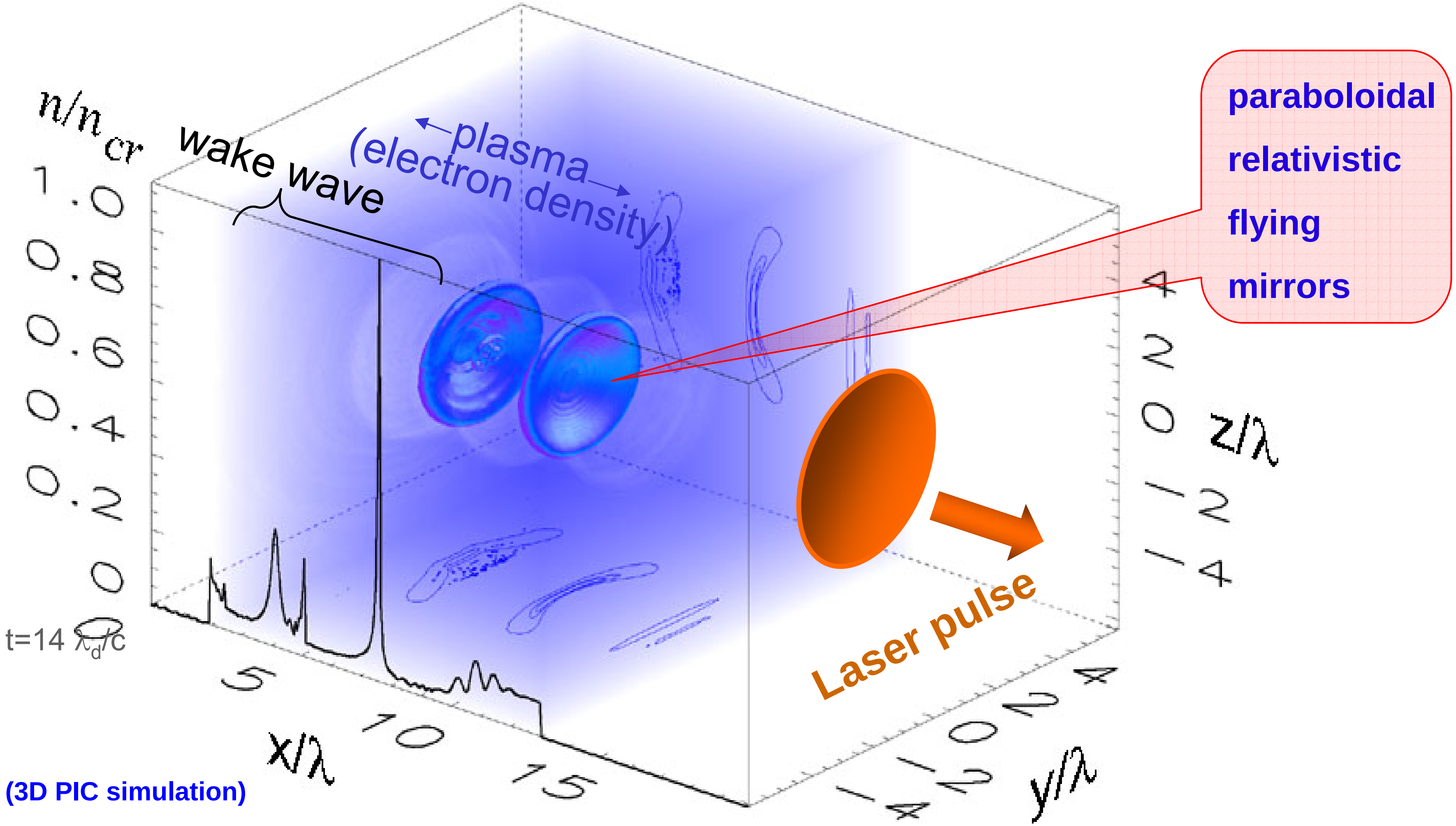
Needed Laser Pulse Energies

Take an example of the wakefield excitation in $10^{18} cm^{-3}$ plasma by the EM wave with $a_0 = 15$. This means the Lorentz factor associated with the phase velocity of the wakefield is related to ω_{pe} / ω, as $a_0^{1/2}(\omega_{pe}/\omega)$, ≈ 125. Thus the laser pulse intensification of the order of 465 may be realized. The counter-propagating $1 \mu m$, 2×10^{19} W/cm^2 laser pulse is partially reflected and focused by the wakefield cusp. If the reflected beam diameter is $40 \mu m$, the final intensity is 5×10^{28} W/cm^2.

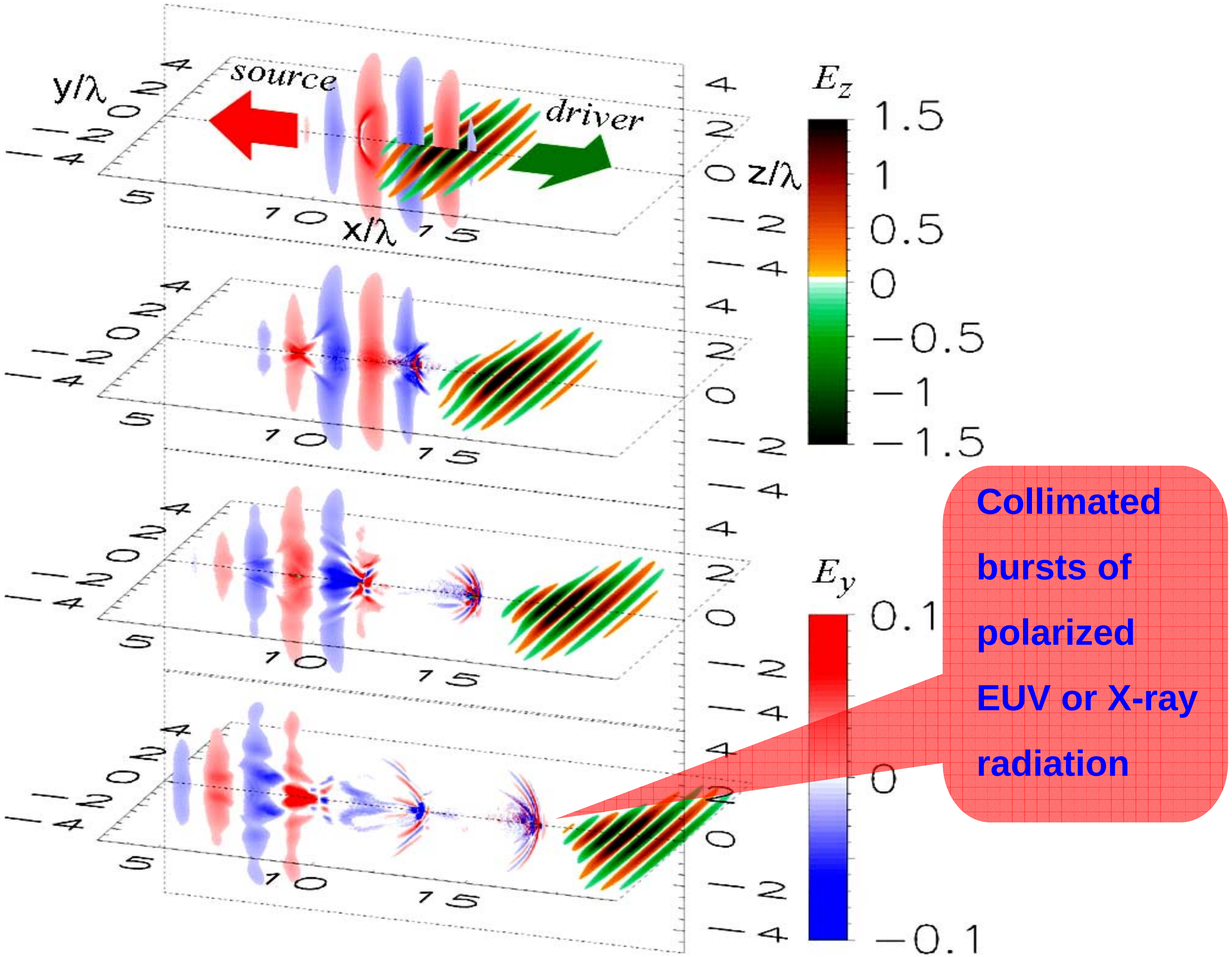
We used the wavebreaking condition: $\gamma_e \approx a_0^2$. The driver pulse intensity should be sufficiently high and its beam diameter should be enough to give such a wide mirror, i.e. to be 4×10^{20} W/cm^2 with the diameter $40 \mu m$. Thus, the driver and source must carry 6 kJ and 30 J, respectively.

Reflected intensity can approach the Schwinger limit. In this range of the electromagnetic field intensity it becomes possible to investigate such the fundamental problems of nowadays physics using already available laser, as e.g. the electron-positron pair creation in vacuum and the photon-photon scattering WITH the ELI PARAMETERS.

3D Particle-In-Cell simulation



3D Particle-In-Cell simulation



3D Particle-In-Cell simulation gives:

parabolidal mirror velocity	$\beta_{ph} \approx 0.87$
γ_{ph}-factor	$\gamma_{ph} \approx 2$
frequency upshift	$\dfrac{\omega_f}{\omega_i} \approx 14 \quad \boxed{= \dfrac{1+\beta_{ph}}{1-\beta_{ph}}}$
reflected EM wave amplitude	$\dfrac{E_{f\,\max}}{E_0} \approx 16$
reflected EM wave intensity	$\dfrac{I_{f\,\max}}{I_0} \approx 256$

Laser-Plasma Interaction
in the Radiation Dominated Regime
(a_0>316)

Laboratory Astrophysics → Laboratory Astrophysics with the High Power Lasers

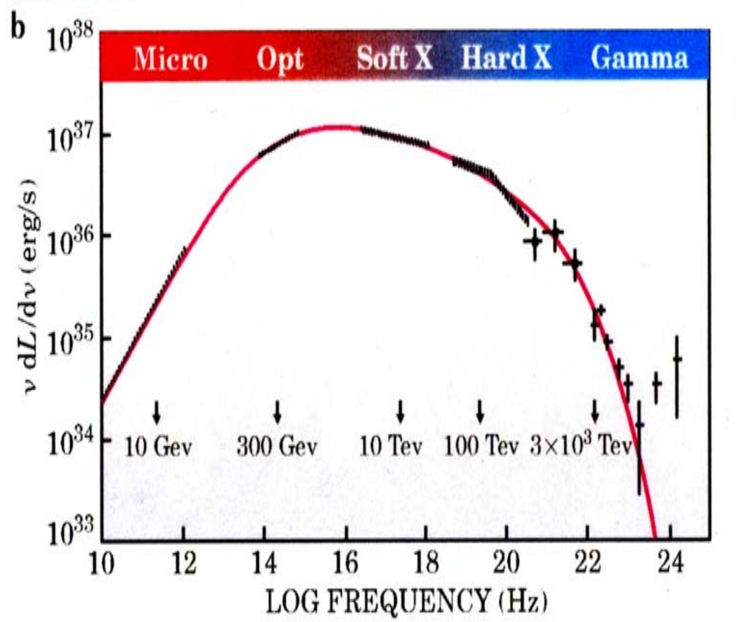
Paticle Physics → Astro-Particle Physics

Astrophysics → Laboratory Laser Physics

What can we learn from the Astrophysics of Cosmic Rays?

V. S. Berezinskii, S. V. Bulanov, V. L. Ginzburg, V. A. Dogiel, V. S. Ptuskin,
Astrophysics of cosmic rays.
(North Holland Publ.Co. Elsevier Sci. Publ. Amsterdam, 1990)

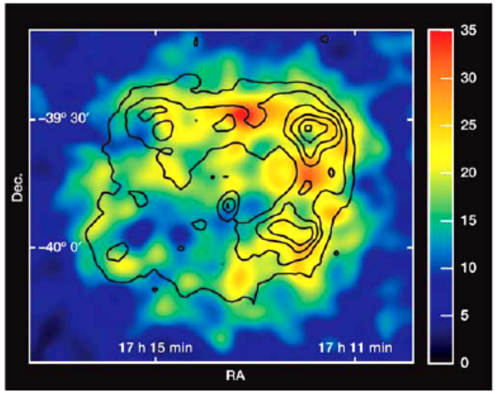
PeV γ from Crab Nebula

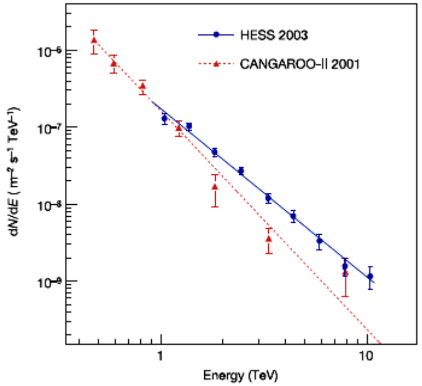


The Crab Pulsar, lies at the center of the Crab Nebula. The picture combines optical data (red) from the Hubble Space Telescope and x-ray images (blue) from the Chandra Observatory. The pulsar powers the x-ray and optical emission, accelerating charged particles and producing the x-rays.

However these high energy electrons can not reach the Earth!

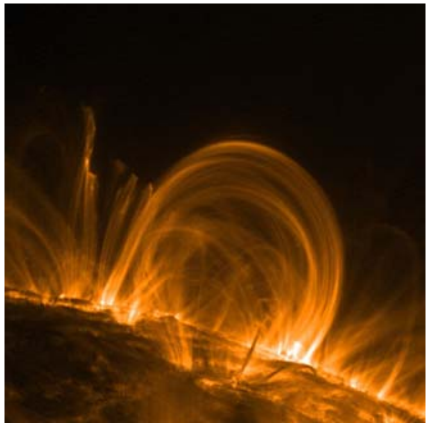
Supernova Shock Wave Acceleration of Charged Particles





Gamma-ray image, HESS telescopes, of the SNR RX J1713.7 2 3946, and the gamma-ray spectrum (F. A. Aharonian, et al., Nature, 432, 75 (2004)).

Solar Cosmic Ray Acceleration during Magnetic Field Line Reconnection



In solar flares the synchrotron losses become dominant for relativistic electrons with the energy above 1 GeV

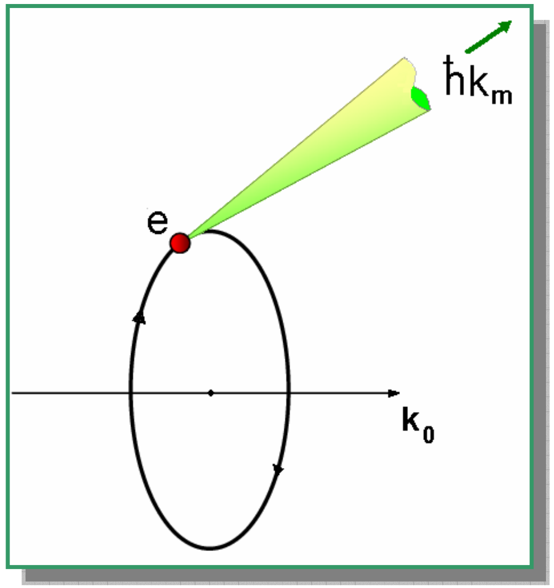
Radiation Losses of Ultrarelativistic Electrons in the EM Wave – Plasma Interaction

In the circularly polarized EM the charged particle moves along a circle trajectory. We may borrow the expressions for the properties of the radiation emitted by the particle from the theory of synchrotron radiation. Equations of the electron motion are:

$$m_e c \frac{du^i}{ds} = \frac{e}{c} F^{ik} u_k + g^i$$

Where the radiation force is given by

$$g^i = \frac{2e^2}{3c}\left(\frac{d^2 u^i}{ds^2} - u^i u^k \frac{d^2 u_k}{ds^2} \right)$$



In the relativistic limit the radiation losses are described by the formula,

$$W = \frac{2e^2}{3m_e^2 c^3}\left(\frac{dp_\mu}{d\tau}\frac{dp_\mu}{d\tau}\right)$$

which gives

$$W = \frac{2e^2 c}{3R^2}\beta^4\gamma^4$$

For synchrotron losses we have $R = pc/eB$ and

$$W = \frac{16\pi e^4 c}{3m_e^2 c^3}\frac{B^2}{8\pi}\gamma^2$$

In the case of the charged particle interaction with the electromagnetic wave (circular polarization) the orbit radius and the particle momentum are

$$R = \frac{c}{\omega_0} = \frac{\lambda}{2\pi} \qquad \text{and} \qquad p = m_e c a_0$$

It yields the emitted intensity

$$W = \frac{4\pi r_e}{3\lambda} \omega_0 \, m_e c^2 a_0{}^4$$

The energy gain rate $\approx \omega_0 \, m_e c^2 a_0$

is larger than the energy losses if

$$a_0 << \left(\frac{4\pi r_e}{3\lambda} \right)^{-1/3} \approx 316$$

Zel'dovich, Ya. B., 1975, Sov. Phys. Usp. 18, 79

Zhidkov, A., Koga, J., Sasaki, A., and Uesaka, M., 2002, Phys, Rev. Lett. 88, 185002

S. V. Bulanov, T. Esirkepov, J. Koga, T. Tajima, Plasma Phys. Rep. 30, 221 (2004)

For the transverse components of the momentum we have the equation which describes a balance between the particle acceleration and its slowing down due to the radiation damping force

$$a_0^2 = \left(\frac{p}{m_e c}\right)^2 + \varepsilon_{rad}^2 \frac{p^2}{(m_e^2 c^2 + p^2)} \left[a_0^2 + \left(\frac{p}{m_e c}\right)^4\right]^2$$

Here $\varepsilon_{rad} = \dfrac{4\pi r_e}{3\lambda}$ **with** $r_e = \dfrac{e^2}{m_e c^2}$ **the classical electron radius**

In the limit of relatively low amplitude of the laser pulse, when

$1 \ll a_0 \ll a_{rad} = \varepsilon_{rad}^{-1/3}$ **or** $10^{18} < I < 10^{23} W/cm^2$

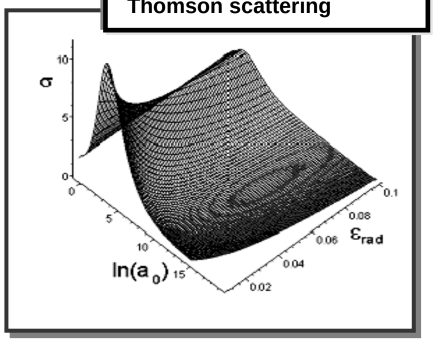
$$p = m_e c a_0$$

In the limit $a_{rad} \ll a_0$ **or** $10^{23} < I < 10^{25} W/cm^2$

the momentum dependence is

$$p = m_e c\left(a_0/\varepsilon_{rad}\right)^{1/4}$$

Cross section of nonlinear Thomson scattering



PIC Simulation Method

Vlasov equation for collisionless plasma

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}\cdot\frac{\partial f_\alpha}{\partial \mathbf{x}} + q_\alpha\left(\mathbf{E}(\mathbf{x},t) + \frac{\mathbf{v}}{c}\times\mathbf{B}(\mathbf{x},t)\right)\cdot\frac{\partial f_\alpha}{\partial \mathbf{p}} = 0$$

one-particle distribution function

Maxwell equations

$$\partial_t \mathbf{E} = c \quad \nabla\times\mathbf{B} - 4\pi \quad \mathbf{J}$$

$$\partial_t \mathbf{B} = -c \quad \nabla\times\mathbf{E}$$

$$\operatorname{div}\mathbf{E} = 4\pi \quad \rho$$

$$\operatorname{div}\mathbf{B} = 0$$

$$\rho = \sum_\alpha q_\alpha \int f_\alpha(\mathbf{x},\mathbf{p},t)\,d\mathbf{p}$$

$$\mathbf{J} = \sum_\alpha q_\alpha \int \mathbf{v} f_\alpha(\mathbf{x},\mathbf{p},t)\,d\mathbf{p}$$

$$\frac{\partial f_\alpha}{\partial t} + \dot{\mathbf{x}}\cdot\frac{\partial f_\alpha}{\partial \mathbf{x}} + \dot{\mathbf{p}}\cdot\frac{\partial f_\alpha}{\partial \mathbf{p}} = \frac{\mathrm{d}}{\mathrm{d}t} f_\alpha\left(\mathbf{x}(t), \mathbf{p}(t), t\right) = 0$$

where $\mathbf{x}(t)$, $\mathbf{p}(t)$ meet the characteristic equations

$$\begin{cases} \dot{\mathbf{x}} = \dfrac{\mathbf{p}}{\gamma m_\alpha}, \quad \gamma = \sqrt{1 + |\mathbf{p}/m_\alpha|^2} \\ \dot{\mathbf{p}} = q_\alpha\left(\mathbf{E}(\mathbf{x},t) + \dfrac{\mathbf{v}}{c}\times\mathbf{B}(\mathbf{x},t)\right) \end{cases}$$

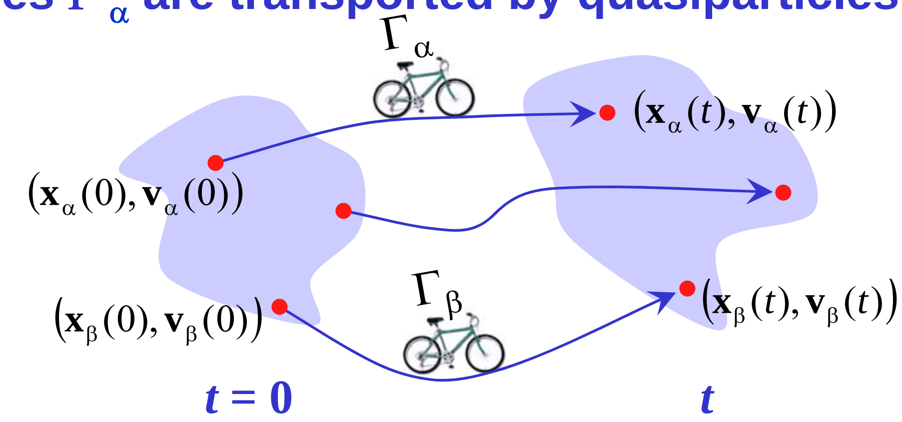
$$f_\alpha(\mathbf{x},\mathbf{p},t) = \sum_\alpha \Gamma_\alpha\cdot\delta(\mathbf{x}-\mathbf{x}_\alpha(t))\cdot\delta(\mathbf{p}-\mathbf{p}_\alpha(t))$$

$$\Gamma_\alpha = f_\alpha(\mathbf{x}_\alpha(0), \mathbf{p}_\alpha(0), 0)$$

α counts a point in the phase space (x,v)

$\Gamma_\alpha\cdot\delta(\mathbf{x}-\mathbf{x}_\alpha(t))\cdot\delta(\mathbf{p}-\mathbf{p}_\alpha(t))$ – quasiparticle

Values Γ_α are transported by quasiparticles



PIC Simulation Method

Vlasov equation for collisionless plasma

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}\cdot\frac{\partial [illegible]}{\partial \mathbf{x}} [illegible] q_\alpha\left(\mathbf{E}(\mathbf{x},t)+\frac{\mathbf{v}}{c}\times\mathbf{B}(\mathbf{x},t)\right)\cdot\frac{\partial f_\alpha}{\partial \mathbf{p}} = 0$$

one-particle distribution function

Maxwell equati[illegible]

$$\partial_t \mathbf{E} = c \quad \nabla\times\mathbf{B} - 4\pi \quad \mathbf{J}$$

$$\partial_t \mathbf{B} = -c \quad \nabla\times\mathbf{E}$$

$$\operatorname{div}\mathbf{E} = 4\pi \quad \rho$$

$$\operatorname{div}\mathbf{B} = 0$$

$$\rho = \sum_\alpha q_\alpha [illegible] f_\alpha(\mathbf{x},\mathbf{p},t)\,d\mathbf{p}$$

$$\mathbf{J} = \sum_\alpha q_\alpha \int \mathbf{v} f_\alpha [illegible] t)\,d\mathbf{p}$$

$$\frac{\partial f_\alpha}{\partial t} + \dot{\mathbf{x}}\cdot\frac{\partial f_\alpha}{\partial \mathbf{x}} + \dot{\mathbf{p}}\cdot\frac{\partial f_\alpha}{\partial \mathbf{p}} = \frac{\mathrm{d}}{[illegible]} [illegible] \mathbf{x}(t)\ ,\ \mathbf{p}(t)\ ,\ t\ ) = 0$$

where [illegible], $\mathbf{p}(t)$ meet the characteristic equations

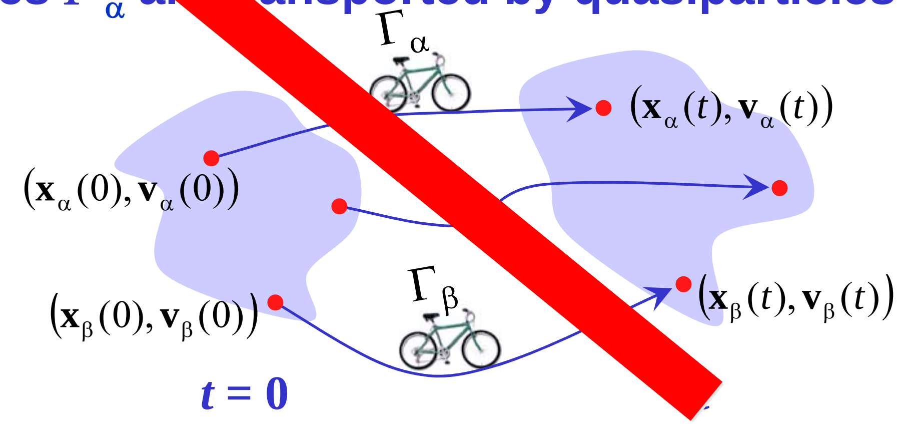
$$\dot{\mathbf{x}} = \frac{\mathbf{p}}{\gamma m_\alpha}\ ,\quad \gamma = \sqrt{1+|\mathbf{p}/m_\alpha|^2}$$

$$\dot{\mathbf{p}} = q_\alpha\left(\mathbf{E}(\mathbf{x},t)+\frac{\mathbf{v}}{c}\times\mathbf{B}(\mathbf{x},t)\right)$$

$$f_\alpha(\mathbf{x},\mathbf{p},t) = \sum_\alpha \Gamma_\alpha\cdot\delta(\mathbf{x}-\mathbf{x}_\alpha(t))\cdot\delta(\mathbf{p}-[illegible])$$

$$\Gamma_\alpha = f_\alpha(\mathbf{x}_\alpha(0),\ \mathbf{p}_\alpha(0),\ 0)$$

Values Γ_α a[illegible] transported by quasiparticles



α counts a point in [illegible] phase space (x,v)

$$\Gamma_\alpha\cdot\delta(\mathbf{x}-\mathbf{x}_\alpha([illegible])[illegible](\mathbf{p}-\mathbf{p}_\alpha(t))$$ – quasiparticle

Quantum Effects

become important at the electron energy, when the recoil due to the photon emission becomes of the order of the electron momentum, i.e. at

$$\gamma \geq \gamma_q = \left(m_e c^2 / \hbar\omega\right)^{1/2}$$

The electron gamma factor $\gamma_e = \left(a_0 / \varepsilon_{rad}\right)^{1/4}$

That is why the quantum limit is $a_0 > a_q = 2e^2 m_e c / 3\hbar^2 \omega$

For the equivalent electric field of EM wave it yields

$$E_q = \frac{2e m_e^2 c^2}{3\hbar^2} = \frac{2\alpha E_{QED}}{3}, \quad \alpha \approx \frac{1}{137}$$

Laser intensity:

$$10^{25} < I < 10^{29} W / cm^2$$

In the quantum limit, when $\Upsilon = \left(\frac{\hbar\omega}{m_e c^2}\right)\left(\frac{p}{m_e c}\right)^2 \gg 1$

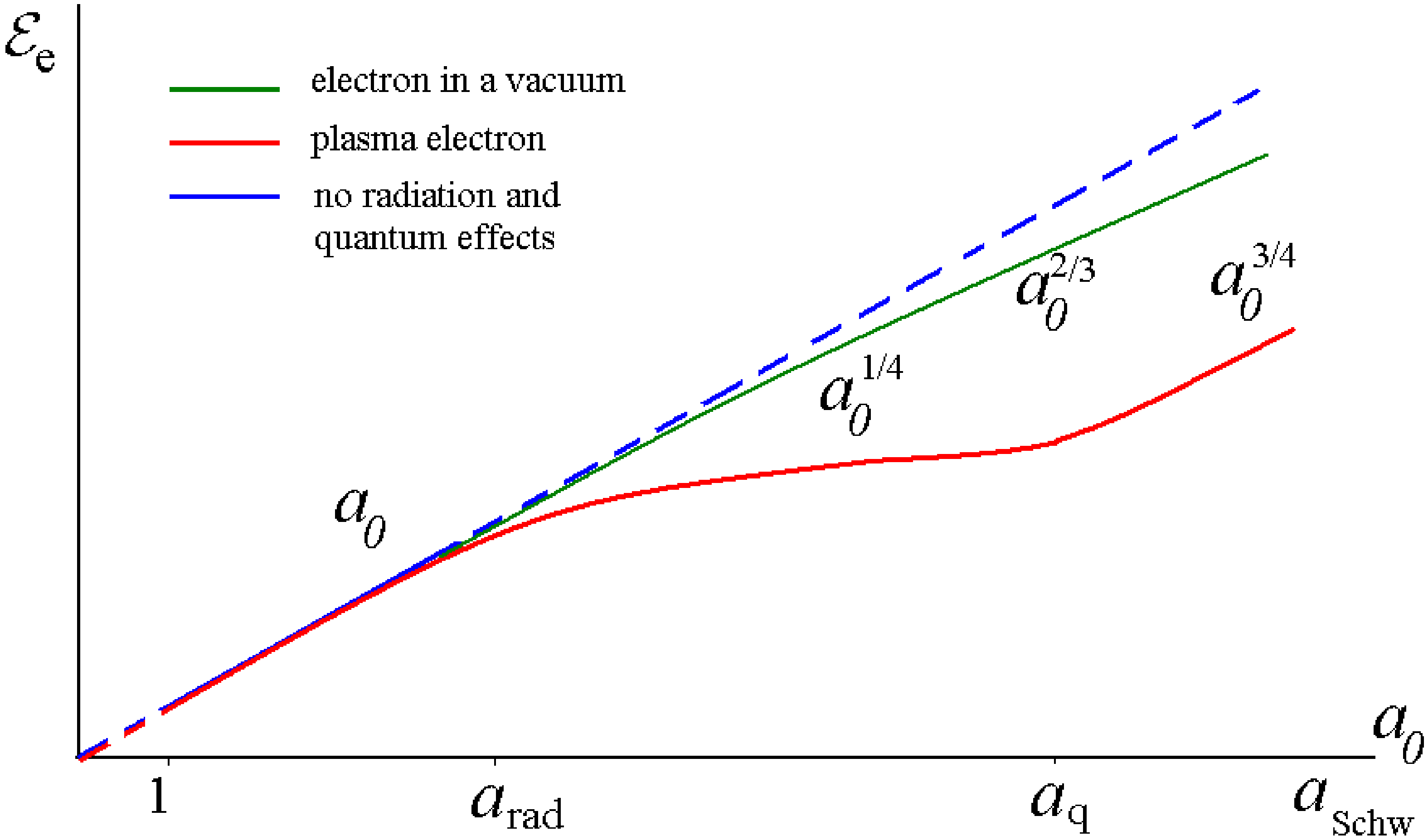
Equation

$$a_0^2 - \left(\frac{p}{m_e c}\right)^2 = \varepsilon_{rad}^2 \left(\frac{p}{m_e c}\right)^8 \mathrm{I}^2(\Upsilon)$$

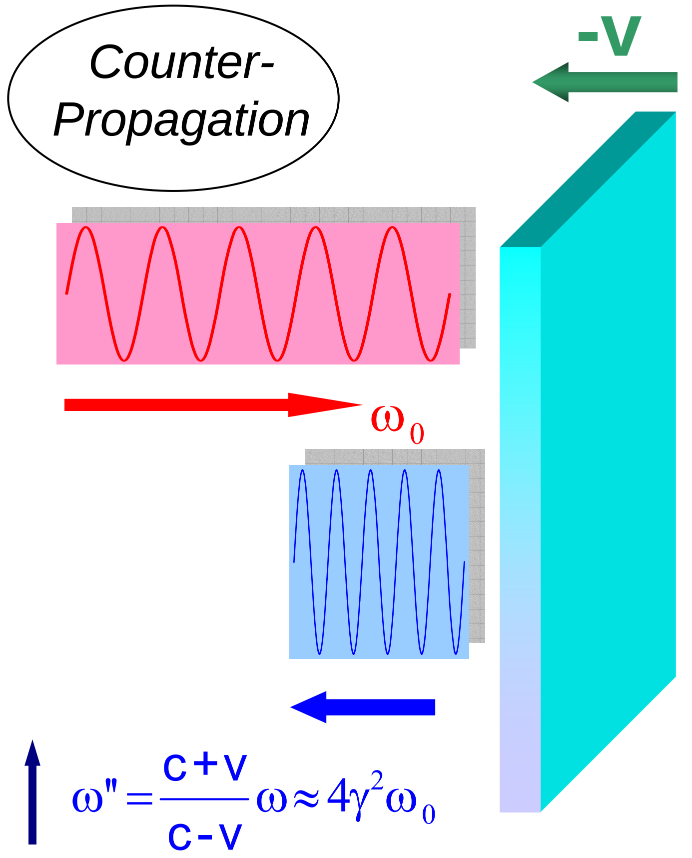
gives

$$p \approx m_e c \left(\frac{\hbar\omega}{m_e c^2}\right)^{1/2} \left(\frac{0.34\, a_0}{\varepsilon_{rad}}\right)^{3/8}$$

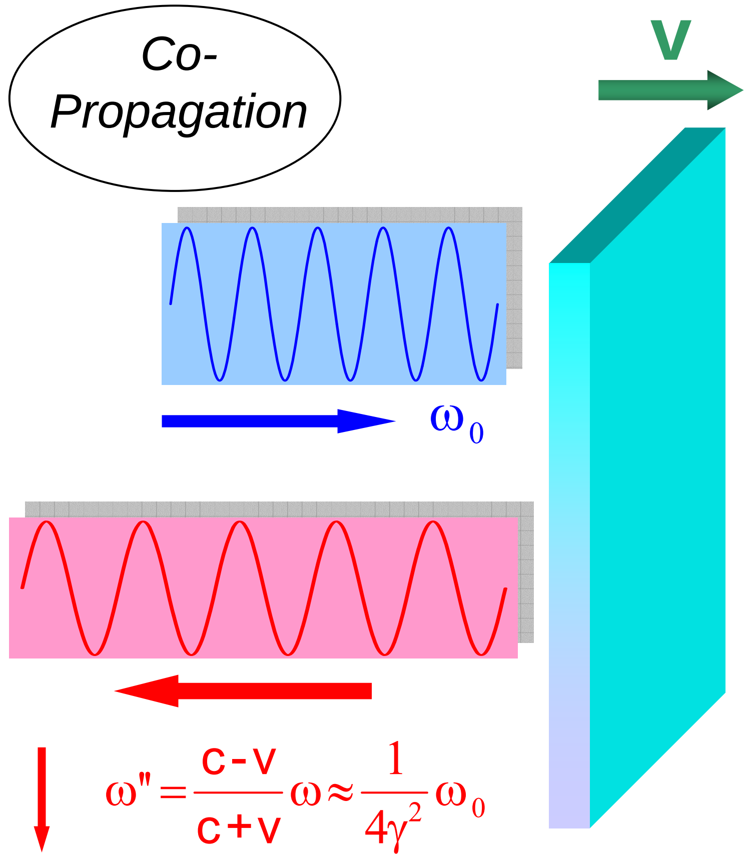
Scaling of electron energy



Reflection of EM Wave at the Relativistic Mirror

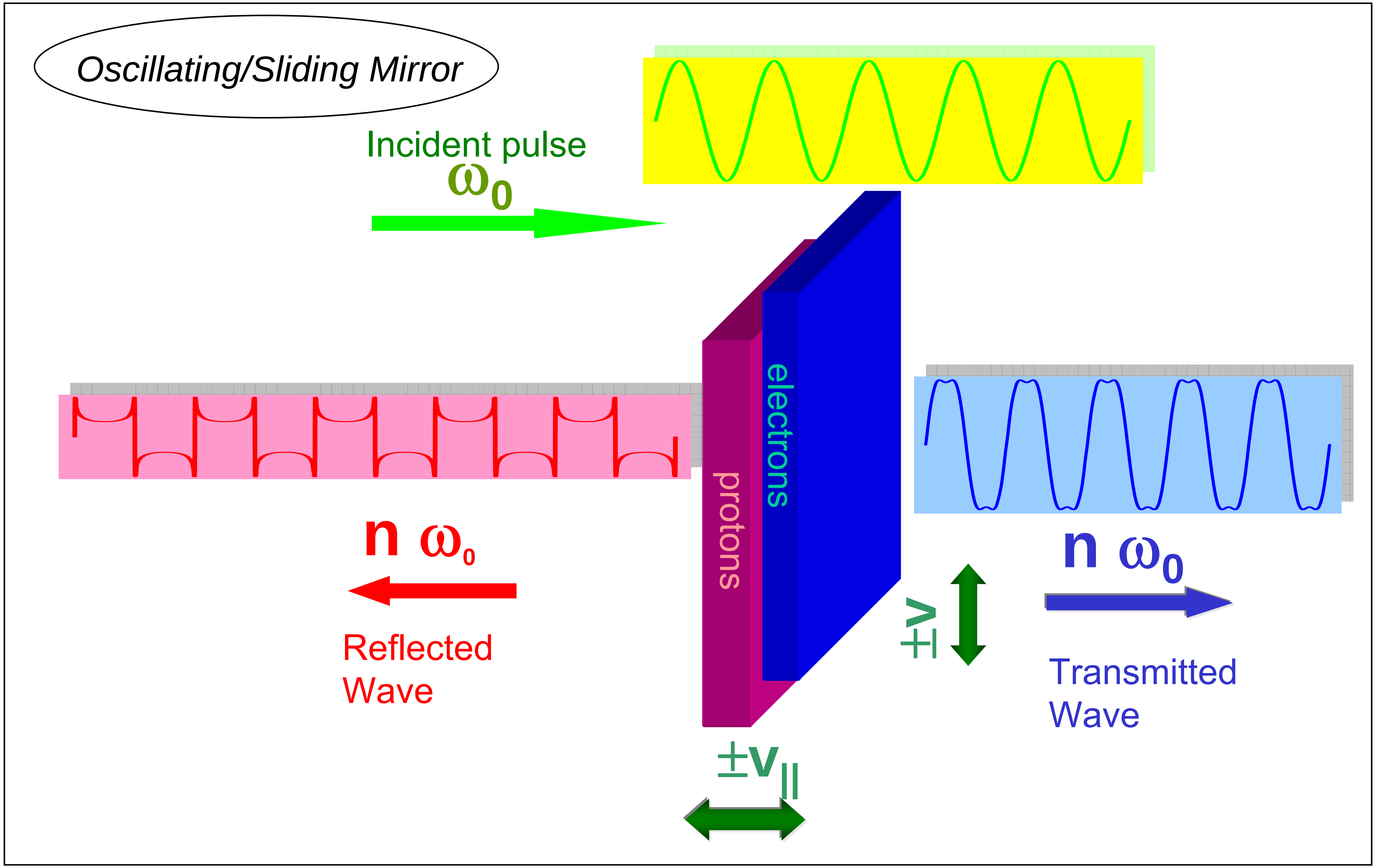


The mirror “compresses” the EM wave



The EM wave pushes the mirror

High Order Harmonics from the Oscillating Relativistic Mirror

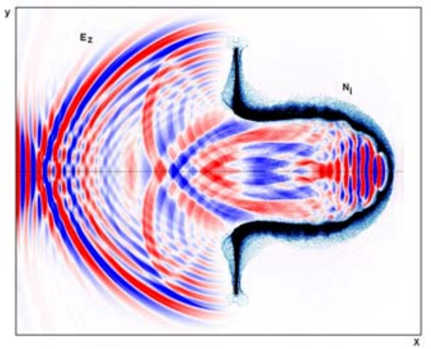


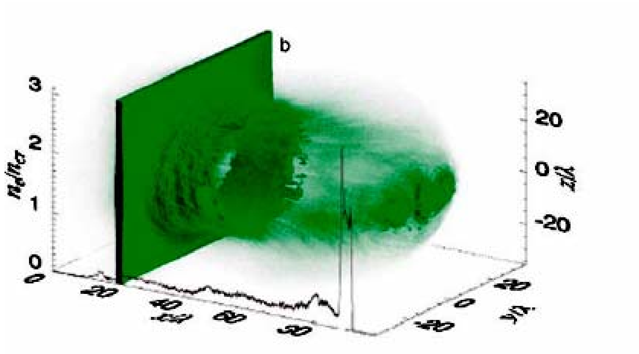
Muti-GeV Ion Generation via Radiation Pressure Dominated Acceleration (RPDA) Mechanism

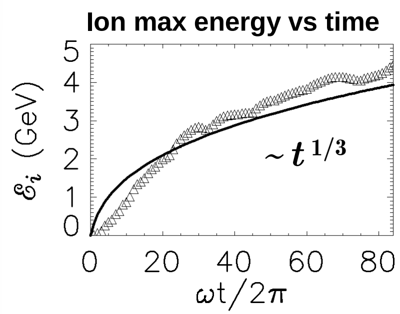
In RPDA the Laser pulse is confined inside the relativistic cocoon. Long laser pulse almost completely transforms its energy into the fast ion energy.

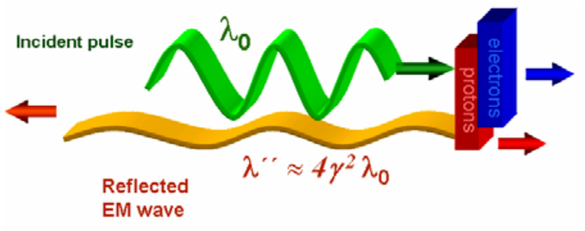
T. Esirkepov, M. Borghesi, S.V. Bulanov, G. Mourou, and T. Tajima, Phys. Rev. Lett. 92, 175003 (2004).

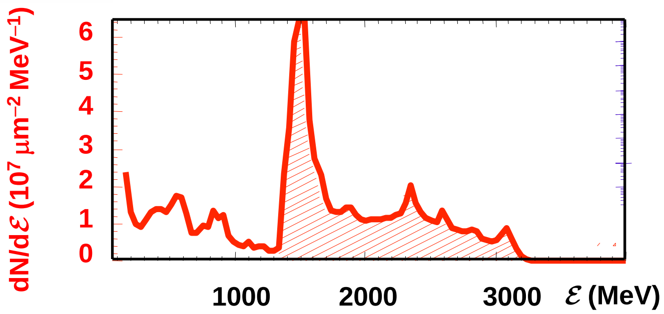
Ion momentum depends on time as $$p \approx m_p c \left(\frac{3}{2} a_0^{\,2} \frac{m_e}{m_p} \left(\frac{\omega_0}{\omega_{pe}} \right)^2 \frac{ct}{l_0} \right)^{1/3}$$











Final ion energy

Radiation pressure on the front part of the "cocoon" is equal to

$$P = \frac{E_0^{\,2}}{2\pi}\left(\frac{\omega''}{\omega_0}\right)^2 = \frac{E_0^{\,2}}{2\pi}\frac{1-\beta_M}{1+\beta_M}$$

It yields

$$\frac{dp}{dt} = \frac{E_0^{\,2}}{2\pi n_0 l_0}\left(\frac{\left(m_p^{\,2}c^2 + p^2\right)^{1/2} - p}{\left(m_p^{\,2}c^2 + p^2\right)^{1/2} + p}\right)$$

Solution to this equation gives for energy balance

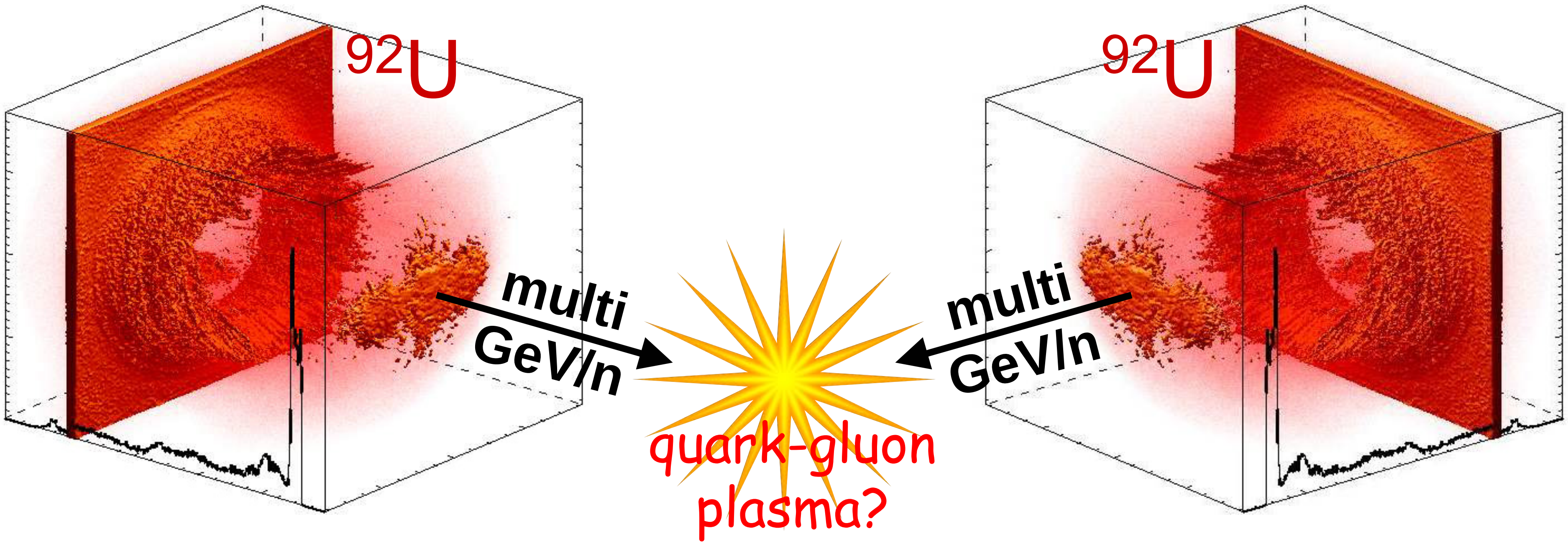
$$\frac{p}{m_p c} = \frac{2W(W+1)}{2W+1}$$

FLUENCE

$$W\left(t-\frac{x}{c}\right) = \int_{-\infty}^{t-x/c} \frac{E_0^{\,2}(\xi)d\xi}{2\pi n_0 l_0 m_i c}$$

Final ion energy depends on the laser pulse energy as $\mathcal{E}_i = \mathcal{E}_{las} / N_{tot}$
Efficiency of the laser energy conversion into the fast ion energy can be formally up to 100%: 30 KJ laser pulse can accelerate 10^{12} protons up to the energy equal to 200 GeV

Laser heavy-ion collider



$$\mathcal{N}_{events} = \frac{N_p^{\,2}\sigma}{S} = \frac{(10^{12})^2 10^{-23}}{\pi(10\mu m)^2} \approx 3\times 10^6$$

Compare with RHIC/BNL collider (Au+Au,100 GeV/nucleon) per day

THE END

Thanks

for

your attention

THE END

Thanks

for

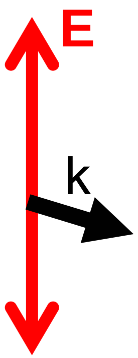
your attention

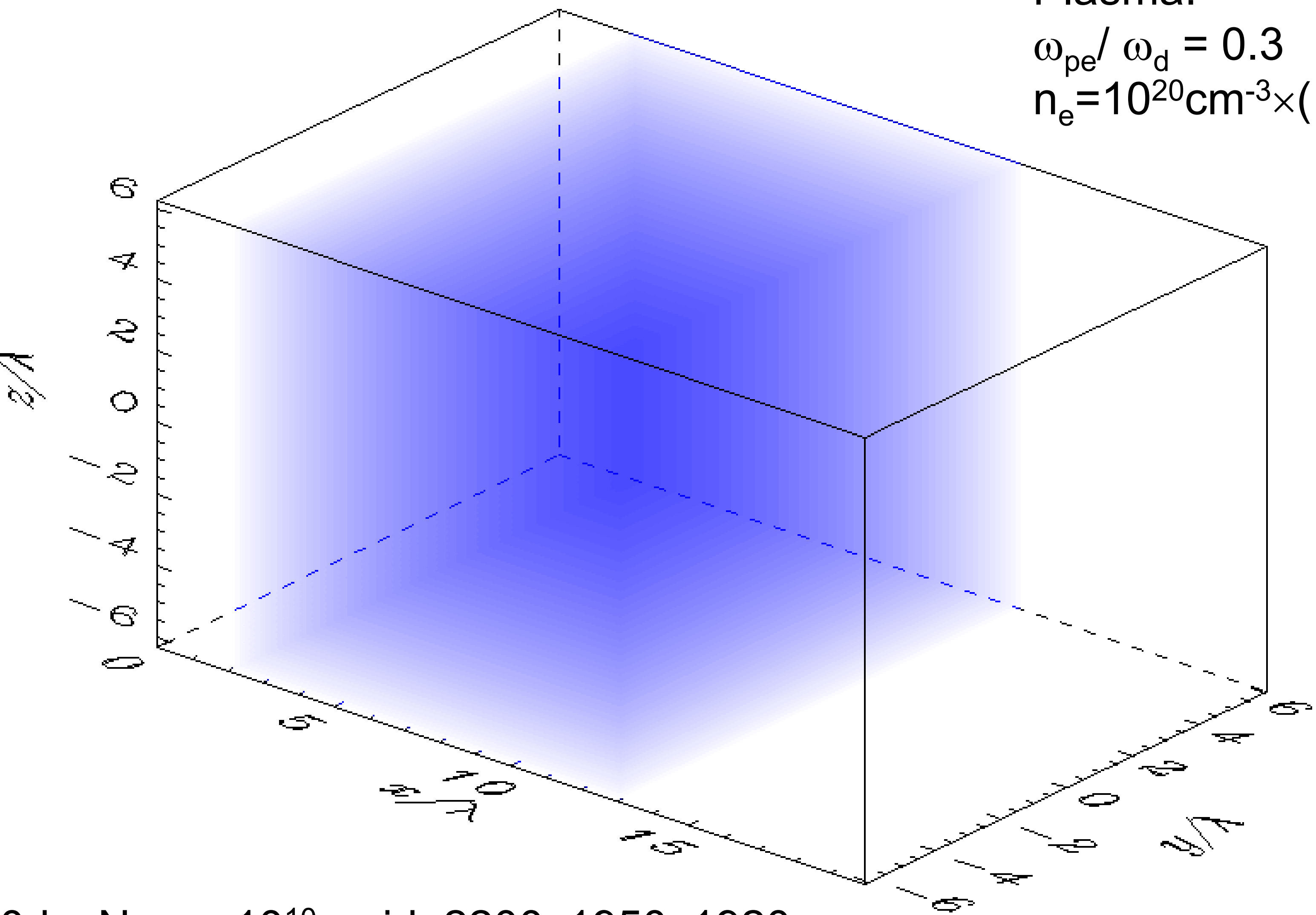
Driver pulse: a=1.7
size=3λx6λx6λ, Gaussian
$I_{peak}=4\cdot10^{18}$ W/cm$^2\times(1\mu m/\lambda)^2$

3D Particle-In-Cell simulation (I)

t= 0.00

Plasma:
$\omega_{pe}/\ \omega_d = 0.3$
$n_e=10^{20}$cm$^{-3}\times(1\mu m/\lambda)^2$





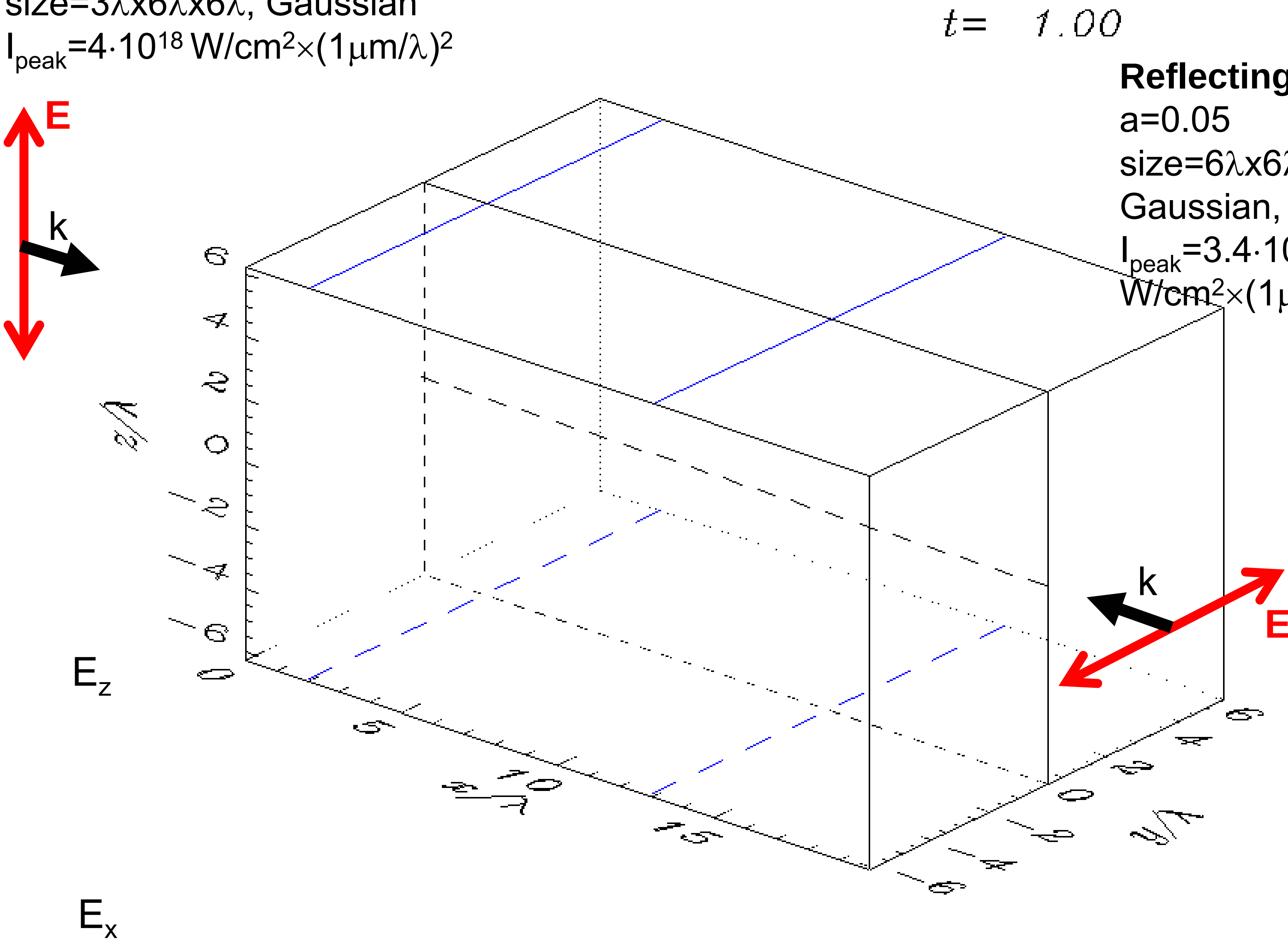
λ= 100dx, N_{part} = 10^{10}, grid: 2200x1950x1920

HP Alpha Server SC ES40/227 (720 CPU)

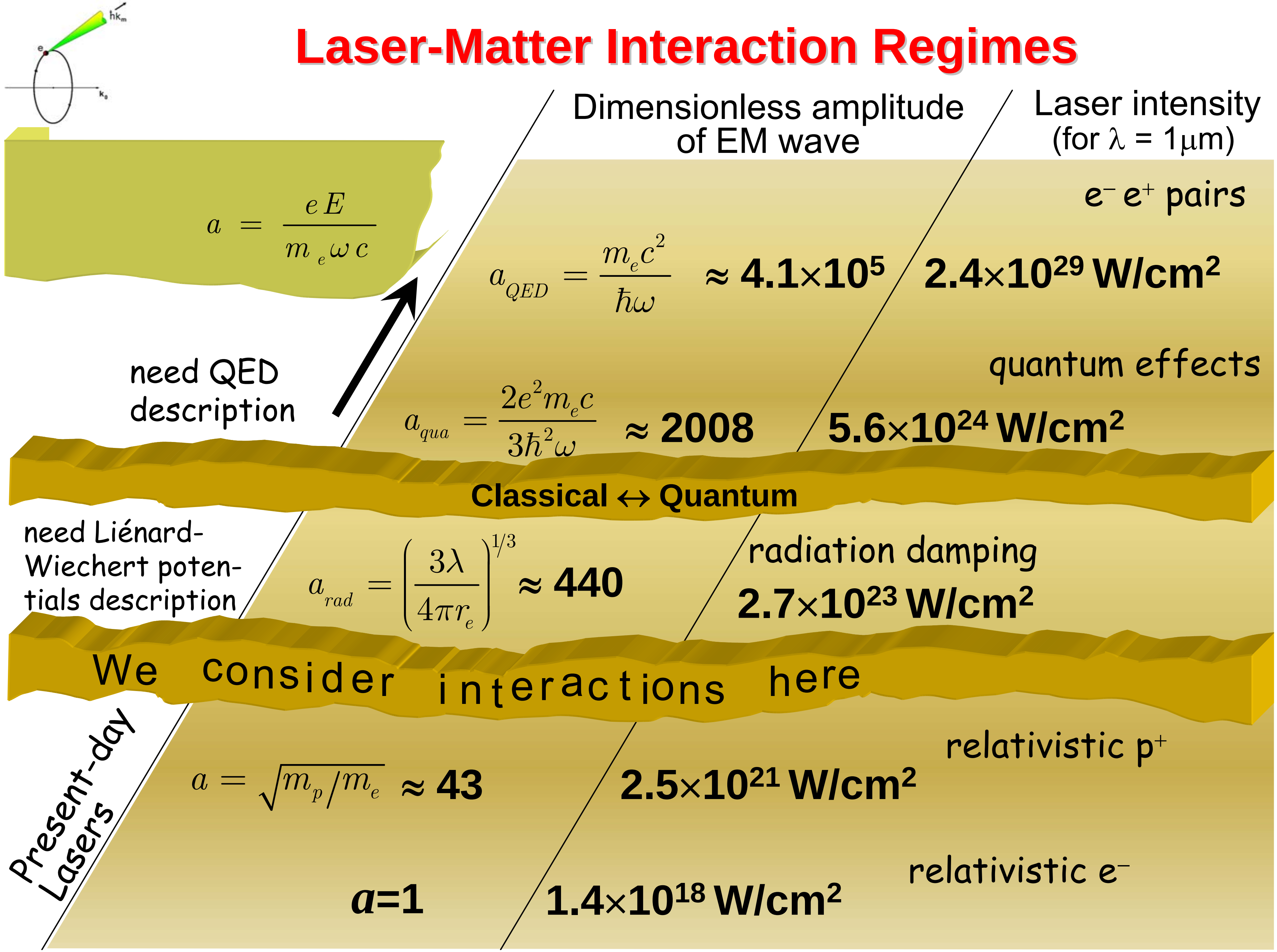
3D Particle-In-Cell simulation (II)

Driver pulse: ***a=1.7***
size=3λx6λx6λ, Gaussian
I_{peak}=4·10^{18} W/cm^2×(1μm/λ)^2

Reflecting pulse:
a=0.05
size=6λx6λx6λ,
Gaussian, λ_s = 2λ
I_{peak}=3.4·10^{15}
W/cm^2×(1μm/λ)^2



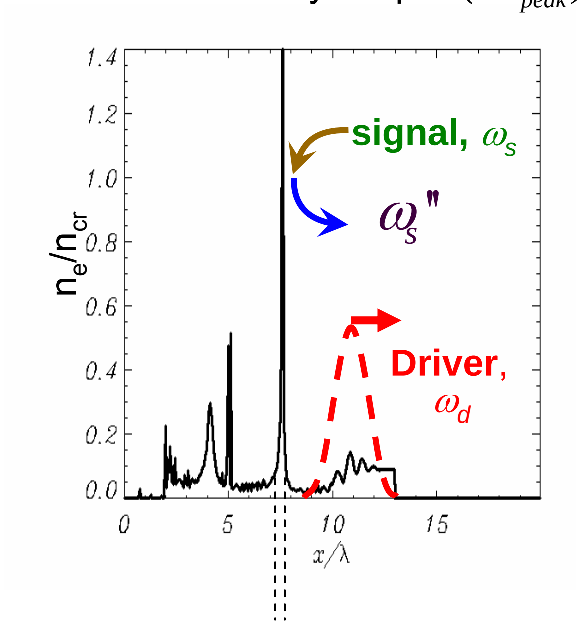
XZ,color: E_y XY,contour: E_z XY,color: E_x at z=0



Laser-Matter Interaction Regimes
ħk_m
e
k_0
$a = \frac{eE}{m_e \omega c}$
Dimensionless amplitude of EM wave
Laser intensity (for λ = 1μm)
e⁻ e⁺ pairs
$a_{QED} = \frac{m_e c^2}{\hbar\omega}$ ≈ 4.1×10^5
2.4×10^29 W/cm^2
need QED description
quantum effects
$a_{qua} = \frac{2e^2 m_e c}{3\hbar^2\omega}$ ≈ 2008
5.6×10^24 W/cm^2
Classical ↔ Quantum
need Liénard-Wiechert potentials description
radiation damping
$a_{rad} = \left(\frac{3\lambda}{4\pi r_e}\right)^{1/3}$ ≈ 440
2.7×10^23 W/cm^2
We consider interactions here
Present-day Lasers
relativistic p⁺
$a = \sqrt{m_p/m_e}$ ≈ 43
2.5×10^21 W/cm^2
relativistic e⁻
a=1
1.4×10^18 W/cm^2

Reflection at the "Flying Mirror"

Electron density cusp $\propto (x-x_{peak})^{-2/3}$



Here are ½ of all electrons
(in the wake wave period)

Wave equation for the vector-potential A_z of EM pulse

$$\partial_{tt}A_z - c^2\Delta A_z + \frac{4\pi e^2 n_e\left(x - v_{ph}t\right)}{m_e\gamma_e}A_z = 0, \quad n_e \approx \frac{1}{2}n_0\left(1 + \lambda_p\delta\left(x - v_{ph}t\right)\right)$$

In the moving frame we seek solution of this Eq.

$$\frac{d^2 A_z}{dx'^2} + q^2 A_z = \chi\delta(x')A_z \quad \textbf{with} \quad q^2 = \left(\frac{\omega_s'}{c}\right)^2 - k_\perp'^2 - \frac{\omega_{pe}^2}{2c^2\gamma_{ph}} > 0$$

where $A_z = \exp(iqx') + \rho(q)\exp(-iqx')$

It yields $\rho(q) = -\dfrac{\chi}{\chi + 2iq}$

In the strongly nonlinear wake:

$$\lambda_p \approx 4\frac{c}{\omega_{pe}}\left(2\gamma_{ph}\right)^{1/2} \Rightarrow \chi \approx 4\frac{\omega_{pe}}{c}\left(2\gamma_{ph}\right)^{1/2}$$

We find the reflection coefficient $R(q) = |\rho(q)|^2 \approx \left(\dfrac{\omega_d}{\omega_s}\right)^2 \dfrac{1}{2\gamma_{ph}^3}$