Electrohydrodynamic (EHD) Instabilities

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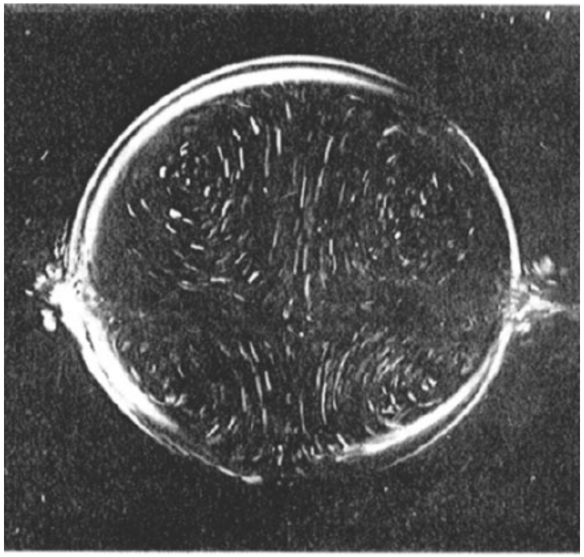
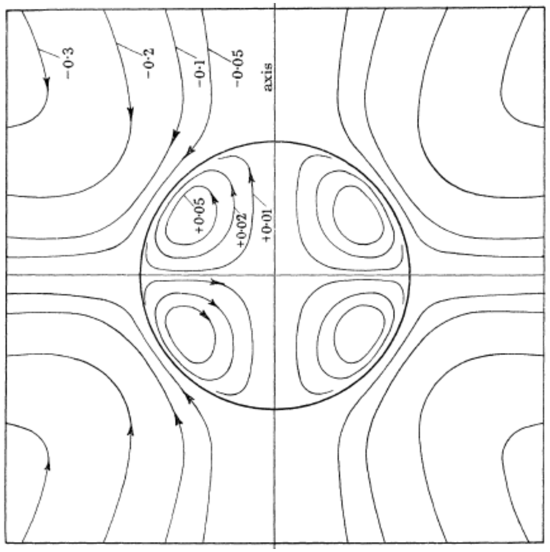
EHD History: G. I. Taylor (1966)

- “The elongation of a drop of one dielectric fluid in another owing to the imposition of an electric field has previously been studied assuming that the interface is uncharged and the fluids at rest.”
- “For a steady field this is unrealistic, because however small the conductivity of either fluid the charge associated with steady currents must accumulate at the interface till the steady state is established.”
- “It is shown that equilibrium can only be established in a drop when circulations are set up both in the drop and its surroundings.”

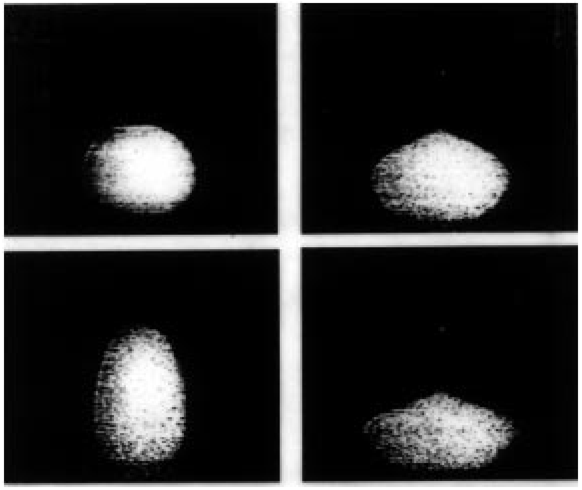


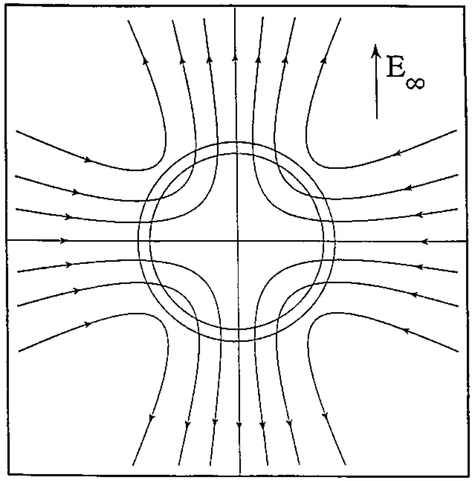


G. I. Taylor 1966, Proc. Roy. Soc. Lon. A, *291*, 159.

Developments in Electrohydrodynamics

- Melcher and Taylor 1969, "Electrohydrodynamics: a review of the role of interfacial shear stresses," Annu. Rev. Fluid Mech., *1*, 111.
- Melcher 1981, *Continuum Electromechanics*, MIT Press.
- Saville 1997, "Electrohydrodynamics: the Taylor-Melcher leaky dielectric model," Annu. Rev. Fluid Mech. *29*, 27.
  - » Recent experimental studies are in better agreement with the theory.
  - » Leaky-dielectric model applies to sharp as well as diffuse interfaces.
  - » Electrohydrodynamics and electrokinetics began to merge since Taylor's 1966 paper.

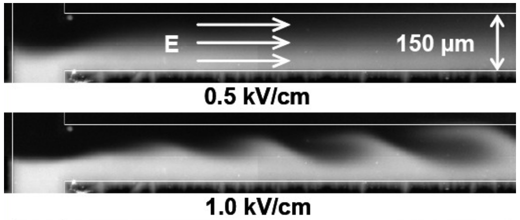




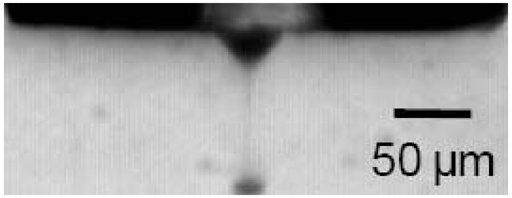
Saville 1997, Annu. Rev. Fluid Mech. *29*, 27.

Outline

- **Leaky-dielectric model**
  - » **Ohmic model derivations**
  - » **Maxwell stresses**
  - » **Jump conditions**
  - » **Applications in microsystems: the high-conductivity, small-scale limit**
- **Electrokinetic flow instabilities**
  - » **Bulk-coupled model**
  - » **Temporal, convective and absolute instabilities**
  - » **EHD instabilities with electroosmotic convection**
  - » **Applications in electrokinetic assays and micromixing**



- **Electrohydrodynamic cone-jets**
  - » **Surface-coupled model**
  - » **Choking: supercritical flow and pulsating jet**
  - » **Varicose and whipping instabilities**
  - » **Applications in droplet microfluidics and electrospinning**



Primary References

- **Electrodynamics**

  Leaky-Dielectric Model

  - » Melcher and Taylor 1969, Annu. Rev. Fluid Mech. *1*, 111.
  - » Melcher 1974, IEEE T. Educ. *E-17*, 100 (and the corresponding film).
  - » Melcher 1981, Continuum Electromechanics, MIT Press.
  - » Saville 1997, Annu. Rev. Fluid. Mech, *29*, 27.
  - » Castellanos 1998, Electrohydrodynamics, Springer.

  Flow Instabilities

  - » Saad 1993, Compressible Fluid Flow, 2nd Ed. Prentice Hall.
  - » Huerre and Rossi 1998, Ch.2 in Hydrodynamics and Nonlinear Instabilities (ed. Goreche and Manneville), Cambridge
  - » Eggers and Villermaux 2008, Rep. Prog. Phys. *71*, 036601.

- **Electrokinetic Flow Instabilities**

  - » Melcher and Schwarz 1968, Phys. Fluids, *11*, 2604.
  - » Hoburg and Melcher 1976, J. Fluid Mech. *73*, 333.
  - » Baygents and Baldessari 1998, Phys. Fluids. *10*, 301.
  - » Lin, Storey, Oddy, Chen and Santiago 2004, Phys. Fluids, *16*, 1922.
  - » Chen, Lin, Lele and Santiago 2005, J. Fluid Mech. *524*, 263.
  - » Posner and Santiago 2006, J. Fluid Mech. *555*, 1.

- **Electrohydrodynamic Cone-Jets**

  - » Melcher and Warren 1971, J. Fluid Mech. *47*, 127.
  - » Cloupeau and Prunet-Foch 1994, J. Aerosol Sci. *25*, 1021.
  - » Fernandez de la Mora 1996, J. Colloid Interf. Sci. *178*, 209.
  - » Ganan-Calvo 1997, J. Fluid Mech. *97*, 165.
  - » Hohman, Shin, Rutledge and Brenner 2001, Phys. Fluids. *13*, 2201; 2221.
  - » Chen, Saville and Aksay 2006, Appl. Phys. Lett. *89*, 124103.

Electrohydrodynamic (EHD) Instabilities

Lectures 1-2: Leaky Dielectric Model

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Lectures 1-2: Leaky Dielectric Model

- Ohmic model derivation
  - » Electro-diffusion of individual species
  - » Conservation laws of bulk properties
- Maxwell stresses
  - » Kelvin and Helmholtz force densities
  - » Maxwell stress for incompressible flow
- Jump conditions
  - » Surface-coupled model
  - » Bulk-coupled model
- Ohmic model for microsystems
  - » Instantaneous charge relaxation (high conductivity limit)
  - » Diffusive processes (small length scales)

Electro-Diffusion of Ions*

Electro-diffusion equations assuming

- Binary, monovalent electrolyte
- Dilute solution
- No reactions (see Saville 1997 for derivation involving reactions)

$$\frac{\partial c_+}{\partial t} + \mathbf{v} \cdot \nabla c_+ = D_+ \nabla^2 c_+ - m_+ F \nabla \cdot (c_+ \mathbf{E}) \qquad \textbf{(1.1)}$$

$$\frac{\partial c_-}{\partial t} + \mathbf{v} \cdot \nabla c_- = D_- \nabla^2 c_- + m_- F \nabla \cdot (c_- \mathbf{E}) \qquad \textbf{(1.2)}$$

Advection **Diffusion** **Electro-migration**

*Levich VG, 1962, *Physicochemical Hydrodynamics*, Prentice-Hall.

Bulk Properties

Charge density:

$$\rho_f = F(c_+ - c_-) \qquad \textbf{(1.3)}$$

Conductivity:

$$\sigma = F^2(c_+ m_+ + c_- m_-) \qquad \textbf{(1.4)}$$

Einstein's Relation:

$$D_\pm = RTm_\pm \qquad \textbf{(1.5)}$$

Electro-neutrality <u>if</u>

$$\Theta = Fm_+ \frac{\rho_f}{\sigma} = \frac{c_+ - c_-}{c_+ + \frac{m_-}{m_+} c_-} \ll 1 \qquad \textbf{(1.6)}$$

$$\boxed{c_+ \approx c_- = c}$$

Not exactly equal, so there will be a charge density!

Algebraic Manipulation

Eq. (1.1) – Eq. (1.2); applying Einstein's Relation

$$(D_+ + D_-)F\nabla\cdot(c\mathbf{E}) = RT(D_+ - D_-)\nabla^2 c \qquad (1.7)$$

Substituting (1.7) into (1-1):

$$\frac{\partial c}{\partial t} + (\mathbf{v}\cdot\nabla)c = D_{eff}\nabla^2 c \qquad (1.8)$$

Effective diffusivity

$$D_{eff} = \frac{2D_+D_-}{D_+ + D_-} \qquad (1.9)$$

For binary, asymmetric electrolyte assuming electro-neutrality

$$c = \frac{c_+}{z_-} = \frac{c_-}{z_+} \qquad (1.10)$$

“Conservation” of Conductivity

With electro-neutrality:

$$\sigma = F^2(m_+ + m_-)c \qquad \textbf{(1.11)}$$

Eq. (1.8) becomes

$$\boxed{\frac{\partial\sigma}{\partial t} + (\mathbf{v}\cdot\nabla)\sigma = D_{eff}\nabla^2\sigma} \qquad \textbf{(1.12)}$$

<u>Interpretation of Eq. (1.12):</u> under the electro-neutrality assumption,

- Each cation is <u>almost always</u> paired with an anion;
- Conductivity (an weighted sum of cationic and anionic concentrations) can be treated as a material property that is “conserved” in the same manner as uncharged species.

Conservation of Charge

Eq. (1.1) – Eq. (1.2); In an exact manner

On RHS, $c_+ \approx c_- = c$ applied for simplicity; See Saville 97 for exact derivation.

$$\frac{\partial \rho_f}{\partial t} + \mathbf{v} \cdot \nabla \rho_f = (D_+ - D_-) F \nabla^2 c - (m_+ + m_-) F^2 \nabla \cdot (c\mathbf{E}) \qquad \textbf{(1.13)}$$

Rearranging (1.13); Applying incompressibility condition

$$\frac{\partial \rho_f}{\partial t} + \nabla \cdot \left\{ \rho_f \mathbf{v} - \underbrace{(D_+ - D_-) F \nabla c} + \sigma \mathbf{E} \right\} = 0 \qquad \textbf{(1.14)}$$

Charge Storage | **Charge Convection** | **Diffusive Current** | **Ohmic Current**

Charge Conservation in the Ohmic Limit

If diffusive current is negligible compared to conduction current

$$\frac{\mathbf{i}_D}{\mathbf{i}_O} \sim O\left(\frac{D_+ - D_-}{D_+ + D_-}\frac{RT/F}{E_a\delta}\frac{\Delta c}{\bar{c}}\right) = \frac{E_d}{E_a} \ll 1 \qquad \textbf{(1.15)}$$

δ: characteristic interfacial (diffusion) length; E_a: applied E-field

Eq. (1.14) reduces to

$$\boxed{\frac{D\rho_f}{Dt} + \nabla\cdot(\sigma\mathbf{E}) = 0} \qquad \textbf{(1.16)}$$

<u>Interpretation of Eq. (1.16):</u>

- Charge storage in a volume of fixed identity is brought by Ohmic current.
- Negligible diffusive current is a valid assumption when the thermal voltage is small compared to a characteristic external voltage.

Electro-Diffusion Eqns Recasted as the Ohmic Model

$$\frac{\partial c_+}{\partial t} + \mathbf{v}\cdot\nabla c_+ = D_+\nabla^2 c_+ - m_+ F\nabla\cdot(c_+\mathbf{E}) \qquad \textbf{(1.1)}$$

$$\frac{\partial c_-}{\partial t} + \mathbf{v}\cdot\nabla c_- = D_-\nabla^2 c_- + m_- F\nabla\cdot(c_-\mathbf{E}) \qquad \textbf{(1.2)}$$

If electro-neutral, i.e. $\Theta = Fm_+\dfrac{\rho_f}{\sigma} \ll 1$ **(<u>only</u> needed for bulk-coupled model)**

$$\frac{\partial \sigma}{\partial t} + (\mathbf{v}\cdot\nabla)\sigma = D_{eff}\nabla^2\sigma \qquad \textbf{(1.12)}$$

If negligible diffusive current, i.e. $\dfrac{\mathbf{i}_D}{\mathbf{i}_O} \sim \dfrac{E_d}{E_a} \ll 1$ **(for surface- <u>&</u> bulk- coupled models)**

$$\frac{D\rho_f}{Dt} + \nabla\cdot(\sigma\mathbf{E}) = 0 \qquad \textbf{(1.16)}$$

Chen, Lin, Lele and Santiago 2005, J. Fluid Mech. *524*, 263.

Alternative Derivation of the Ohmic Model

Conduction current in the laboratory frame,

$$\mathbf{i} = \mathbf{i}^*(\rho_f, \mathbf{E}) + \rho_f \mathbf{v} = \sigma \mathbf{E} + \rho_f \mathbf{v}$$

where $\mathbf{i}^* = \sigma \mathbf{E}$ is <u>postulated</u> as the constitutive law for current

> "It is evident from recently reported research that this simplest of all conduction laws can be used to understand a surprisingly wide range of electrohydrodynamic phenomena." (Melcher & Taylor 1969)

Conservation of charge:

$$\frac{\partial \rho_f}{\partial t} + \nabla \cdot \mathbf{i} = 0 \quad \Longrightarrow \quad \frac{D\rho_f}{Dt} + \nabla \cdot (\sigma \mathbf{E}) = 0$$

Melcher and Taylor 1969, Annu. Rev. Fluid Mech. *1*, 111.

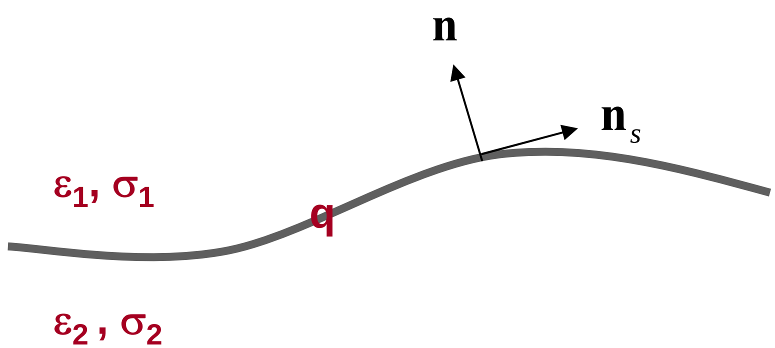
Electric Jump Conditions

Differential Laws:

$$\nabla \times \mathbf{E} = 0$$

$$\nabla \cdot (\varepsilon \mathbf{E}) = \rho_f$$

$$\frac{D\rho_f}{Dt} + \nabla \cdot (\sigma \mathbf{E}) = 0$$



- **Surface charge density: q**
- **Jump of A across the interface:** $[\![\mathrm{A}]\!] = \mathrm{A}_1 - \mathrm{A}_2$
- **Surface gradient:** $\nabla_s = \nabla - \mathbf{n}\mathbf{n} \cdot \nabla$

Jump Conditions:

$$\mathbf{n} \times [\![\mathbf{E}]\!] = 0$$

$$\mathbf{n} \cdot [\![\varepsilon \mathbf{E}]\!] = q$$

$$\frac{\partial q}{\partial t} + (\mathbf{n} \cdot \mathbf{v}) \nabla \cdot (q\mathbf{n}) + \nabla_s \cdot (\mathbf{K}_s) + \mathbf{n} \cdot [\![\sigma \mathbf{E}]\!] = 0$$

Melcher and Taylor 1969, Annu. Rev. Fluid Mech. *1*, 111.
Castellanos & Gonzalez 1998, IEEE T. Dielectrics Elec. Insul., *5*, 334.

Charge Conservation on the Interface

$$\underbrace{\frac{\partial q}{\partial t}}_{\text{Storage of Surface Charge}} + \underbrace{(\mathbf{n}\cdot\mathbf{v})\nabla\cdot(q\mathbf{n})}_{\text{Deformation of Interface}} + \underbrace{\nabla_s\cdot(\mathbf{K}_s)}_{\text{Surface Current}} + \underbrace{\mathbf{n}\cdot[\![\sigma\mathbf{E}]\!]}_{\text{Bulk Conduction}} = 0$$

- Interfacial deformation and its consequence on surface charge conservation is discussed in details by Castellanos 1998; Castellanos & Gonzalez 1998.

$$(\mathbf{n}\cdot\mathbf{v})\nabla\cdot(q\mathbf{n}) = (\mathbf{n}\cdot\mathbf{v})(\mathbf{n}\cdot\nabla q) + q(\mathbf{n}\cdot\mathbf{v})(\nabla\cdot\mathbf{n})$$

- The simplest form of surface current is pure charge advection: $\mathbf{K}_s = q\mathbf{v}_s$

  Other effects such as surface conduction are typically neglected.
- Bulk conduction is the free current density in the frame of reference moving with the boundary. Bulk charge advection never reaches the boundary (Melcher 1981).

Castellanos 1998, Electrohydrodynamics, Springer, pp. 13-15;
Castellanos & Gonzalez 1998, IEEE T. Dielectrics Elec. Insul., *5*, 334.
Melcher 1981, Continuum Electromechanics, MIT Press, p 2.18.

Consequence of the Ohmic Law

$$\boxed{\frac{D\rho_f}{Dt} = \frac{\rho_f}{\tau_e} - \sigma \mathbf{E} \cdot \left(\frac{\nabla \sigma}{\sigma} - \frac{\nabla \varepsilon}{\varepsilon} \right)}$$

- Charge relaxation time $\tau_e = \varepsilon / \sigma$
- In regions of uniform conductivity and permittivity, for an observer following a particle of fixed identity, the net free charge decays with the relaxation time:

$$\rho_f = \rho_{f,0} \exp(-t / \tau_e)$$

- "Unless an element of material having uniform properties can be traced along a particle line to a source of net charge, it supports no net charge."

Melcher 1974, IEEE T. Educ. *E-17*, 100.

Charge Density Scale from Ohmic Law

$$\boxed{\frac{D\rho_f}{Dt} = \frac{\rho_f}{\tau_e} - \sigma \mathbf{E} \cdot \left(\frac{\nabla \sigma}{\sigma} - \frac{\nabla \varepsilon}{\varepsilon} \right)}$$

- If ρ_f results from conductivity gradient

$$\rho_f \sim \varepsilon \mathbf{E} \cdot \frac{\nabla \sigma}{\sigma} \left(\sim \frac{\varepsilon E_a}{\delta} \right)$$

- If ρ_f results from permittivity gradient

$$\rho_f \sim \mathbf{E} \cdot \nabla \varepsilon$$

Maxwell Stress in Vacuum

Electric force density (due to free charge) in terms of Maxwell stress:

$$\mathbf{f} = \rho_f \mathbf{E} = \nabla \cdot (\varepsilon_0 \mathbf{E}) \mathbf{E}$$

$$= \nabla \cdot (\varepsilon_0 \mathbf{E}\mathbf{E}) - \varepsilon_0 \mathbf{E} \cdot \nabla \mathbf{E} \overset{(\nabla \times \mathbf{E} = 0)}{=} \nabla \cdot (\varepsilon_0 \mathbf{E}\mathbf{E}) - \frac{1}{2} \nabla \varepsilon_0 \mathbf{E} \cdot \mathbf{E}$$

$$= \nabla \cdot \left(\varepsilon_0 \mathbf{E}\mathbf{E} - \frac{1}{2} \varepsilon_0 E^2 \boldsymbol{\delta} \right) = \nabla \cdot \mathbf{T}^e$$

$$\mathbf{T}^e = \varepsilon_0 \begin{bmatrix} \frac{1}{2}\left(E_x^2 - E_y^2 - E_z^2\right) & E_x E_y & E_x E_z \\ E_x E_y & \frac{1}{2}\left(E_y^2 - E_z^2 - E_x^2\right) & E_y E_z \\ E_x E_z & E_y E_z & \frac{1}{2}\left(E_z^2 - E_x^2 - E_y^2\right) \end{bmatrix}$$

Panofsky and Phillips 1962, Classical Electricity and Magnetism, Addison-Wesley, Ch. 6.5.

Maxwell Stress in Dielectrics

- **Kelvin force density (assuming the force on each dipole is transmitted to macroscopic medium)**

$$\mathbf{f}_K = \rho_f \mathbf{E} + \mathbf{P} \cdot \nabla \mathbf{E} = \nabla \cdot \left(\varepsilon \mathbf{E} \right) \mathbf{E} + \left(\varepsilon - \varepsilon_0 \right) \mathbf{E} \cdot \nabla \mathbf{E}$$

$$= \nabla \cdot \left(\varepsilon \mathbf{E}\mathbf{E} - \frac{1}{2} \varepsilon_0 E^2 \boldsymbol{\delta} \right)$$

- **Helmholtz force density (accounting for dipole interactions)**

$$\mathbf{f}_H = \mathbf{f}_K - \nabla \left(\frac{1}{2} (\varepsilon - \varepsilon_0) E^2 - \frac{1}{2} \rho \left(\frac{\partial \varepsilon}{\partial \rho} \right)_T E^2 \right)$$

$$= \nabla \cdot \left(\varepsilon \mathbf{E}\mathbf{E} - \frac{1}{2} \varepsilon E^2 \boldsymbol{\delta} + \underbrace{\frac{1}{2} \rho \left(\frac{\partial \varepsilon}{\partial \rho} \right)_T E^2 \boldsymbol{\delta}}_{} \right)$$

Electrostriction
(typically absorbed in pressure)

Equivalence of Kelvin and Helmholtz Concepts

- In incompressible flow, any two force densities differing by the gradient of a scalar pressure will give rise to the same incompressible deformation.
  - » In incompressible flow, pressure becomes a "left-over" variable. It is whatever it must be to satisfy the incompressibility condition: $\nabla \cdot \mathbf{v} = 0$
- Both force densities, if used consistently, will yield the same answer as far as incompressible mechanical deformation is concerned;
  - » Note the electric force density distributions are very different. See a classic example in Melcher 1981, Section 8.3.
  - » Kelvin force density is useful for appreciating the underlying microscopic electromechanics;
  - » Helmholtz force density is useful for predicting the consequences of electromechanical coupling.

Melcher 1981, Continuum Electromechanics, MIT Press, Sec. 8.3.
Melcher 1974, IEEE T. Educ. *E-17*, 100.

Maxwell Stress in Incompressible Dielectrics

Helmholtz force density:

$$\mathbf{f} = \rho_f \mathbf{E} - \frac{1}{2} E^2 \nabla \varepsilon = \nabla \cdot \left(\varepsilon \mathbf{E}\mathbf{E} - \frac{1}{2} \varepsilon E^2 \boldsymbol{\delta} \right) = \nabla \cdot \mathbf{T}^e$$

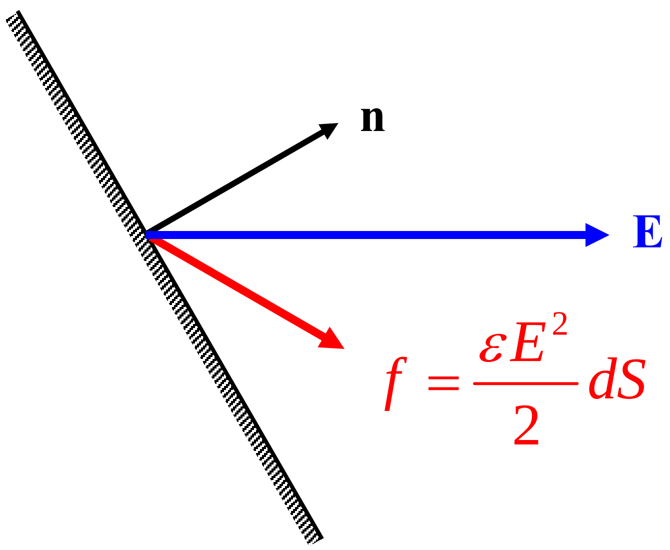
$$\mathbf{T}^e = \varepsilon \begin{bmatrix} \frac{1}{2}\left(E_x^2 - E_y^2 - E_z^2\right) & E_x E_y & E_x E_z \\ E_x E_y & \frac{1}{2}\left(E_y^2 - E_z^2 - E_x^2\right) & E_y E_z \\ E_x E_z & E_y E_z & \frac{1}{2}\left(E_z^2 - E_x^2 - E_y^2\right) \end{bmatrix}$$

The force density reduces to that in vacuum if $\varepsilon = \varepsilon_0$.

Direction of Maxwell Stress

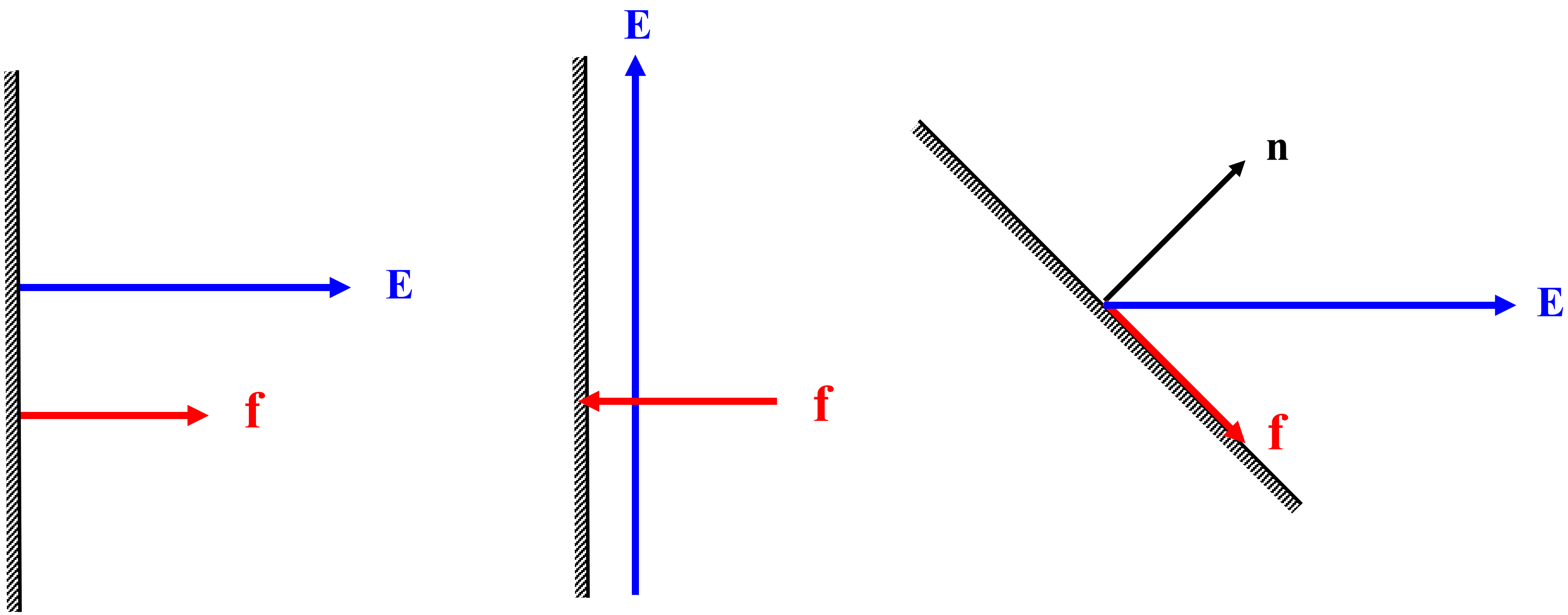
Choose a coordinate system in which the x-axis is parallel to the direction of the field (E_y=E_z=0).

$$\mathbf{T}^e = \varepsilon \begin{bmatrix} \frac{1}{2}E^2 & 0 & 0 \\ 0 & -\frac{1}{2}E^2 & 0 \\ 0 & 0 & -\frac{1}{2}E^2 \end{bmatrix}$$



The electric field bisects the angel between the normal to the surface and the direction of the resultant force acting on the surface.

Examples of Maxwell Stresses



Panofsky and Phillips 1962, Classical Electricity and Magnetism, Addison-Wesley, Ch. 6.5.

Hydrodynamic Equations

Conservation of mass (incompressible):

$$\boxed{\nabla \cdot \mathbf{v} = 0}$$

Conservation of momentum:

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p^* + \mu \nabla^2 \mathbf{v} + \nabla \cdot \mathbf{T}^e$$

$$\boxed{\rho \frac{D\mathbf{v}}{Dt} = -\nabla p^* + \mu \nabla^2 \mathbf{v} + \rho_f \mathbf{E} - \frac{1}{2} E^2 \nabla \varepsilon}$$

- **p* may have absorbed forces of electric origin;**
- *** dropped in future eqns**

Electric force due to <u>free</u> charge

Polarization force due to <u>pairs</u> of charges

Melcher and Taylor 1969, Annu. Rev. Fluid Mech. *1*, 111.
Saville 1997, Annu. Rev. Fluid Mech. *29*, 27.

Hydrodynamic Jump Conditions

$$\mathbf{n} \times [\![\mathbf{v}]\!] = 0$$

$$\mathbf{n} \cdot [\![\mathbf{v}]\!] = 0$$

$$\mathbf{n} [\![p]\!] = \mathbf{n} \cdot \left[\!\!\left[\mathbf{T}^m + \mathbf{T}^e \right]\!\!\right]$$ **where,**

$$\mathbf{T}^m = \mu(\nabla \mathbf{v} + \nabla \mathbf{v}^T)$$

$$\mathbf{T}^e = \varepsilon \mathbf{E}\mathbf{E} - \frac{1}{2} \varepsilon E^2 \boldsymbol{\delta}$$

$$[\![\mathbf{T}^e \cdot \mathbf{n}]\!] \cdot \mathbf{n} = \frac{1}{2} [\![\varepsilon (\mathbf{E} \cdot \mathbf{n})^2 - \varepsilon (\mathbf{E} \cdot \mathbf{t}_1)^2 - \varepsilon (\mathbf{E} \cdot \mathbf{t}_2)^2]\!]$$

$$[\![\mathbf{T}^e \cdot \mathbf{n}]\!] \cdot \mathbf{t}_i = q \mathbf{E} \cdot \mathbf{t}_i \quad \left(\text{using } \mathbf{n} \times [\![\mathbf{E}]\!] = 0, \ \mathbf{n} \cdot [\![\varepsilon \mathbf{E}]\!] = q \right)$$

Melcher and Taylor 1969, Annu. Rev. Fluid Mech. *1*, 111.
Saville 1997, Annu. Rev. Fluid Mech. *29*, 27.

Consequence of Helmholtz Force Density

$$\mathbf{f} = \rho_f \mathbf{E} - \frac{1}{2} E^2 \nabla \varepsilon = \nabla \cdot \left(\varepsilon \mathbf{E}\mathbf{E} - \frac{1}{2} \varepsilon E^2 \boldsymbol{\delta} \right) = \nabla \cdot \mathbf{T}^e$$

- The electric field bisects the angel between the normal to the surface and the direction of the resultant force acting on the surface.
- Leaky dielectric model is required to generate <u>tangential shear stress</u> at interfaces
  - » Interface between perfect conductors ($\sigma \to \infty$): supports no tangential electric stress (not even free charge)
    - ➢ **As far as electrostatics is concerned, the two regions can be joined together as a single "perfect" conductor with $\sigma \to \infty$.**
  - » Interface between perfect dielectrics ($\sigma = 0$): force density is perpendicular to the surface (in the direction of $-\nabla\varepsilon$)

Melcher and Taylor 1969, Annu. Rev. Fluid Mech. *1*, 111.

Electromechanical Coupling

- Surface-coupled model
  - » Electromechanical coupling is through interfacial electric stresses (i.e. jump conditions)
  - » Volumetric force density is zero for piecewise homogeneous media

$$\mathbf{n}[\![p]\!] = \mathbf{n} \cdot [\![\mathbf{T}^m + \mathbf{T}^e]\!]$$

- Bulk-coupled model
  - » Electromechanical coupling is through volumetric electric forces
  - » Property variations are continuous (i.e. no jump conditions)

$$\mathbf{f} = \rho_f \mathbf{E} - \frac{1}{2} E^2 \nabla \varepsilon$$

Electrohydrodynamics in Microsystems

- Instantaneous charge relaxation
  - Typically aqueous electrolyte (high-conductivity limit)

$$\cancel{\frac{D\rho_f}{Dt}} + \nabla \cdot (\sigma \mathbf{E}) = 0 \qquad \rightarrow \qquad \nabla \cdot (\sigma \mathbf{E}) = 0$$

- Diffusive processes become important
  - Small length scales
  - Bulk-coupled model for <u>miscible</u> interfaces

$$\frac{\partial \sigma}{\partial t} + (\mathbf{v} \cdot \nabla)\sigma = 0 \qquad \rightarrow \qquad \frac{\partial \sigma}{\partial t} + (\mathbf{v} \cdot \nabla)\sigma = D_{eff} \nabla^2 \sigma$$