Electromagnetic Ring Launcher

A Major Qualifying Project

Submitted to the Faculty of the

WORCESTER POLYTECHNIC INSTITUTE

In partial fulfillment of the requirements for the degree of

Bachelor of Science

by

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Submitted: April 30, 2014

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Abstract

Electromagnetics have a wide variety of applications in today's world. Developments such as the railgun and the Hyperloop demonstrate the use of electromagnetism in projectile launchers. The objective of this MQP is to understand the variables and possible issues associated with an electromagnetic launcher through the construction of a ring launcher similar to a coilgun. This project also seeks to measure the optimum parameters to achieve the highest efficiency for a launch and design a self-triggering circuit to allow the launcher to perform without human interaction.

Acknowledgements

We would like to extend our sincerest gratitude and thanks to Professor Alexander Emanuel whose vision and support guided us throughout this project. His experience and intellect were a major part of the driving force that led to this project's success.

We would also like to thank Robert Boisse for his support in the acquisition of parts for the project and his willingness to help through the Electrical and Computer Engineering shop in Atwater Kent.

Authorship

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Introduction:

Electromagnetics have a wide variety of uses today. Our inspiration for this MQP stemmed from our research into the possible real world applications of electromagnetic fields. We initially researched the military weaponized structure known as the Railgun depicted in figure I; we looked at the parameters and methods used to build it to get a better understanding of the approach that was used to complete this projectile launcher, using the electromagnetic effects of current passing through parallel conducting rails.

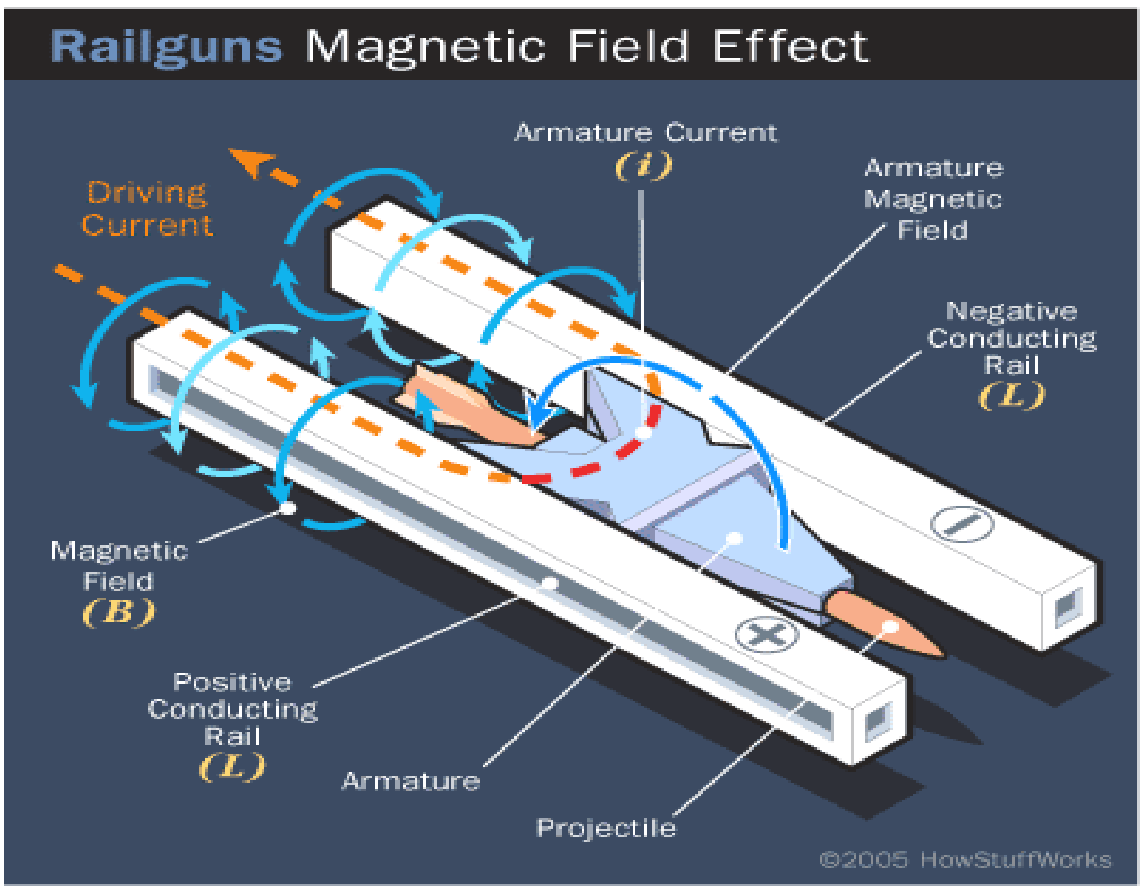

Figure I: Railgun Structure

We then expanded our research into transportation and delivery systems and found the conceptually theorized Hyperloop. This train system, shown in figure II below, is envisioned for a travel time of 35 minutes between the cities of Los Angeles and San Francisco and consists of pressurized capsules riding on cushions of air, driven by a combination of linear induction motors. We also briefly researched the much simpler coilgun, that consists of one or more coils being used as magnets to accelerate a conducting projectile.

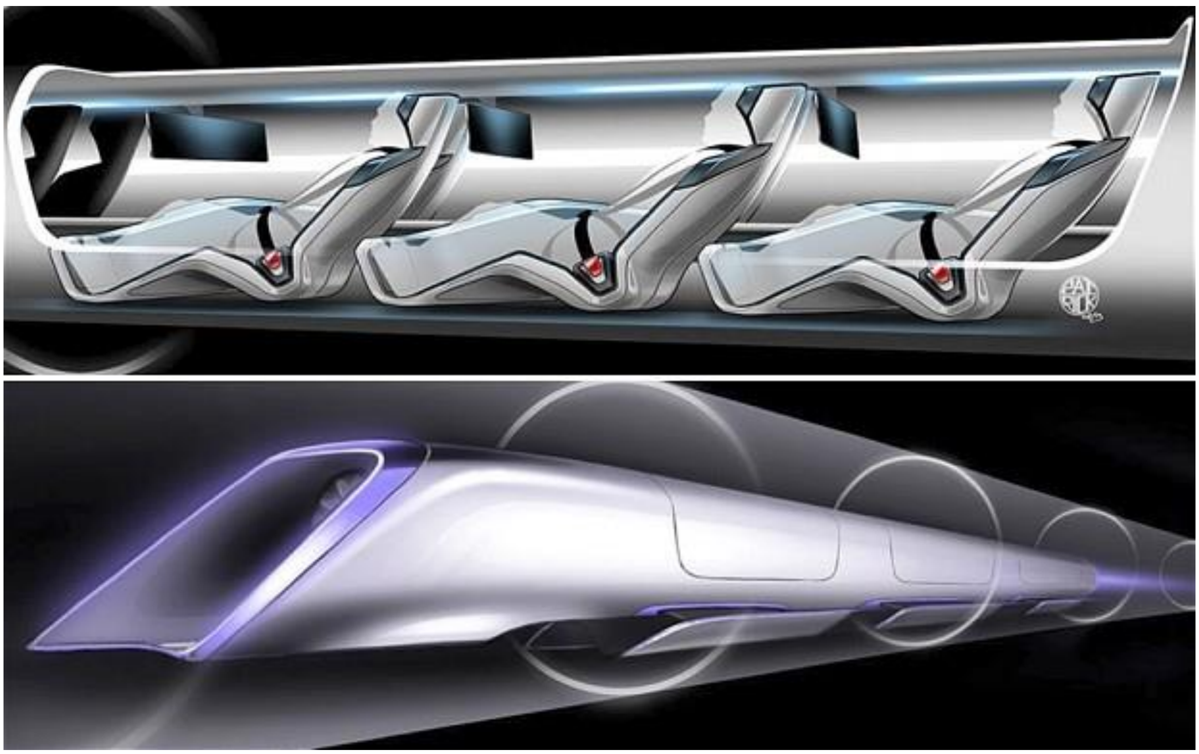

Figure II: Hyperloop System

The research into these various applications of electromagnetics inspired our group to make this topic the focus of our MQP, specifically the use of electromagnetic projectile launchers. Once we decided our focus, we brainstormed all possible aspects of the launcher we could investigate as part of our project and what improvements we could make on the existing structure that could have significant uses in the real world.

Overview:

Once the focus of our MQP was decided, the major tasks and accomplishments to be achieved within the one year span had to be outlined. Since our goal was to construct a simpler version of a projectile launcher, it was decided to use small metallic rings as the projectile. This would allow safe conduction of experiments and easier manipulation of the projectile to understand the significance of its dimensions to the success of the overall launch.

When constructing any major appliance without extensive prior knowledge, it is important to understand its mechanisms thoroughly. This became the first objective of this MQP; to understand the various parameters involved with the electromagnetic ring launcher and how each of these variables contributes to a launch. These include each of the components of the actual launcher, their interactions with each other and the electromagnetic characteristics of the apparatus.

Once the parameters and the launcher's structure were understood, the objective became to expand on the existing model with additions that can prove beneficial in existing applications. Initially the idea was to expand the launcher by adding a secondary launch mechanism; however, due to budget restrictions and the lack of availability of certain parts, this became a very difficult task to achieve. Another useful addition envisioned for the electromagnetic ring launcher was the removal of any human interaction with the launcher, in regards to the trigger and having the device self-trigger. In larger applications, this can significantly remove any safety hazards associated with the launcher and in the case of weaponized structures, ensure minimum risk. This became one of the major objectives for the project.

Once the components of the launcher, their interactions with each other and the numerous physical characteristics involved in this project were understood, it was important to learn the effects of changes in physical structures and placements on a specific launch. This was partly achieved through the use of a speed testing circuit, which was used in conjunction with the self-triggering mechanism to observe the change in speed of a launch under different conditions. Studying these effects involved using theoretical knowledge and calculations made of the system and testing them practically. This not only validated the theory behind the launcher but also helped us acquire the optimum parameters for a launch. This ensures maximum efficiency for the device and is essential knowledge when applying larger projectile structures.

The final objective of this project was to use the theory and knowledge at hand and simulate the launcher using PSpice software. This not only helped better understand the interactions involved in the launcher but also allowed for testing of conditions that would be difficult to achieve experimentally. Although a secondary launch system was not constructed physically, simulations in PSpice also allowed us to see the effects such an addition would have on the existing structure and the optimum parameters for such an installation to occur.

Chapter 1: System Design and Component Specifications

This section of the report lists the components used to construct the electromagnetic ring launcher and the specifications used for each of the parts. This chapter also introduces the overall system of the project and how it works, as well as some of the thought and ideas behind the design process. The ring launcher consists of a power supply, resistor, capacitor, switch, magnetic coil and acrylic tube as well as the ring used as a projectile.

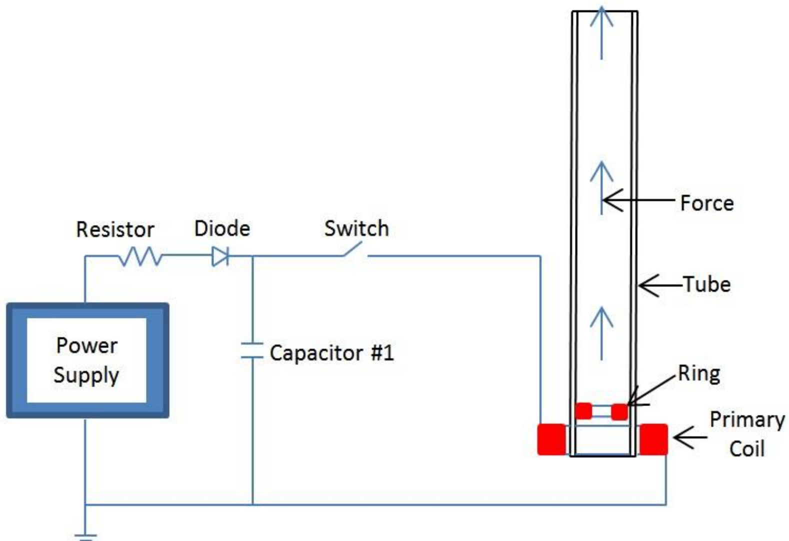

Figure 1.1: System Design

Figure 1.1 above depicts the overall structure of the launcher and how the circuit components attached to the magnetic coil are set up. The electromagnetic launcher functions based upon the principles of Faraday's law of induction and Lenz's law. The power supply in figure 1.1 charges the capacitor when turned on. The diode and resistor are used to limit the current flowing through and to control its direction. Once the capacitor is charged, the switch is closed, allowing the capacitor to discharge into the magnetic coil. The current flowing into the coil causes a change in the magnetic flux through the coil and this, according to Faraday's law of induction, induces an electromotive force (emf). According to Lenz's law, the induced emf gives rise to a current whose magnetic field opposes the original change in magnetic flux. As such, the current through the coil will induce a current in the ring in the opposite direction. These two currents will repel each other and cause the ring to launch upwards through the tube.

Power Supply

The power supply used for this launcher had a range of 350 – 500V. This variation in voltage was used in various tests explained later in the report. For the standard launch used to demonstrate the functionality of the launcher, an output of 400V from the power supply was used.

Diode and Resistor

The diode in figure 1.1 is used to control the direction of the current flow from the power supply and ensure its moves flows to the capacitor. A resistor of 4.7 kΩ is used to limit the current going into the capacitors.

Capacitor

A combination of capacitors was used in this circuit to achieve the desired launch. The configuration of the capacitors produced 300 μF and these capacitors were specifically chosen to have low internal resistances and be capable of charging and discharging in relatively short periods of time, so as to increase the efficiency of the launcher.

Switch

The purpose of the switch in figure 1.1 is to trigger the launch of the ring through the tube. The switch is left open while the capacitor is charging and when required, it is closed, allowing the current to pass through to the coil and launch the ring. The switch's implementation in this circuit had to be modified as part of the project to have the launcher self-trigger and this is discussed in chapter 2 of the report.

Acrylic Tube

The acrylic tube used was an essential part of the launcher. The clear and sturdy structure of the tube allows for careful monitoring of the ring's launch as well as a stable foundation for the overall device. The tube was available in the following heights: 1 ft., 3 ft. and 5 ft. For this project a height of 3 feet was sufficient. Figure 1.2 below shows the dimensions of the tube and figure 1.3 shows an actual horizontal image of the tube used:

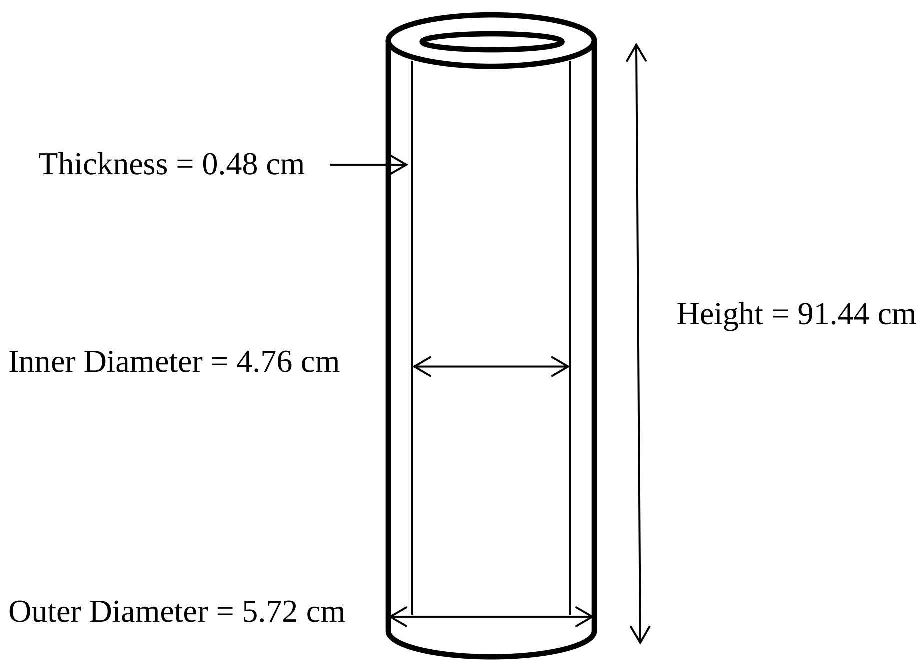

Figure 1.2: Dimensions of Tube

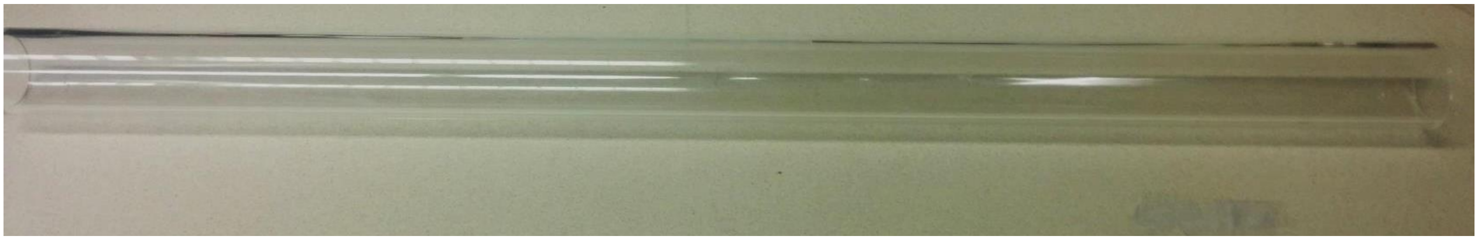

Figure 1.3: Acrylic Tube

Magnetic Coil

The magnetic coil used in the launcher was made using 12 AWG copper magnet wire. The coil was formed by winding the wire 50 turns around a wooden cylinder of the desired circumference in a mechanical vice. Layers of the coil were separated using thin paper as an insulator. Figures 1.4 and 1.5 below show the dimensions of the coil and the coil itself:

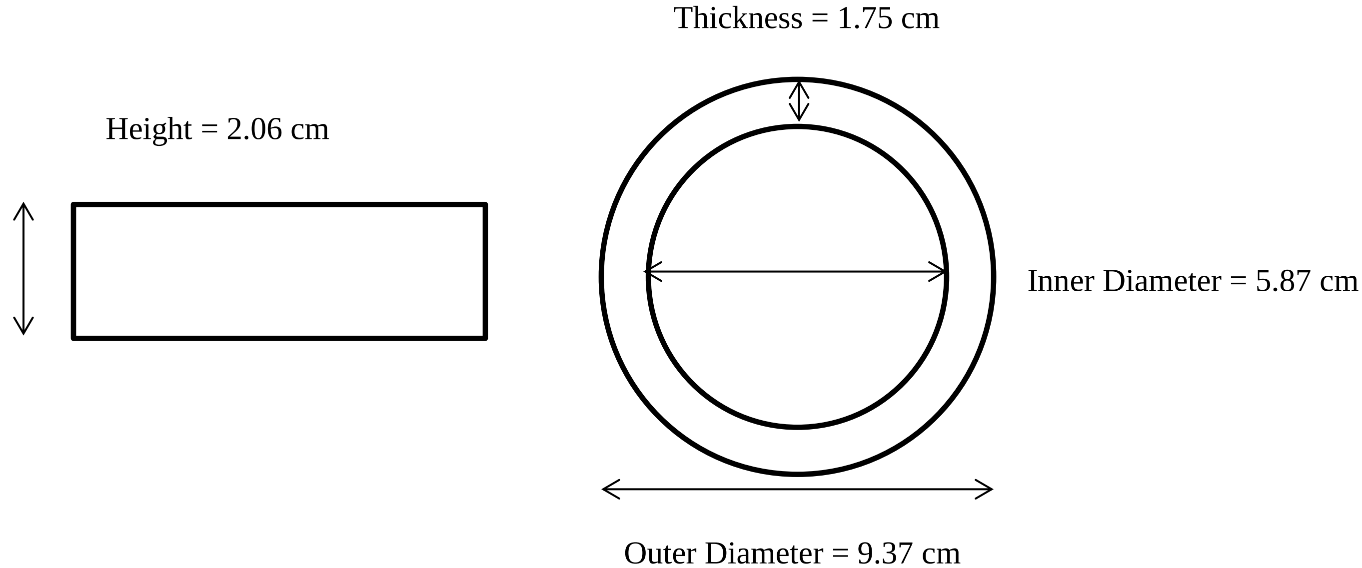

Figure 1.4: Side view and Top view of coil showing dimensions

Figure 1.5: Magnetic Coil

The dimensions of the coil can be used to calculate its resistance. This is done using the equations below:

$$R_c = \rho_{copper} \frac{l_{coil}}{A_{wire}} \qquad \text{(Equation 1.1)}$$

$$l_{coil} = N_{coil} * 2\pi r_{coil} \qquad \text{(Equation 1.2)}$$

$$A_{wire} = \pi r_{wire} \qquad \text{(Equation 1.3)}$$

As is shown in Equation 1.1, the resistance of the coil was calculated by multiplying the resistivity of copper (ρ_{copper}) by the length of the coil (l_{coil}) divided by the area of the wire itself (A_{wire}). The resistivity of copper is $1.68*10^{-8}$ Ω.m. The length of the coil is calculated using the average circumference of the coil and multiplying it by the number of turns in the coil, 50. Using Equation 1.2, the length of the coil is:

$$l_{coil} = N_{coil} * 2\pi r_{coil} = 50 * 2\pi * 0.038\,m = 11.94\,m$$

Using Equation 1.3, the area of the wire itself is calculated as:

$$A_{wire} = \pi r^2{}_{wire} = \pi * 0.1cm^2 = 3.14 * 10^{-6}\,m^2$$

Substituting these values into Equation 1.1, the resistance of the coil is:

$$R_c = \rho_{copper}\frac{l_{coil}}{A_{wire}} = (1.68 * 10^{-8}) * \frac{11.94m}{3.14 * 10^{-6}m^2} = 63.87m\Omega$$

Therefore, the resistance of the coil is approximately 64 mΩ. Since the ring launcher involves electromagnetic reactions, it is important to calculate the inductance of the coil as well. This can be done using the circuit configuration in figure 1.6 below:

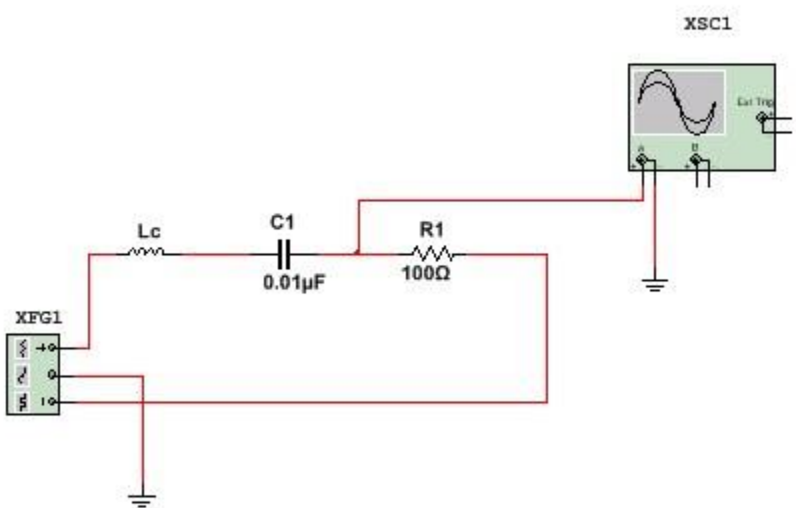

Figure 1.6: Coil Inductance Circuit

The circuit in figure 1.6 consists of a function generator, the copper coil (L_C), a 0.01µF capacitor (C1), a 100Ω resistor (R1) and an oscilloscope. The function generator was set to produce a sine wave signal and the oscilloscope was used to measure the output current. The point of this method was to find the frequency at which the current resonates. Resonance refers to when the system oscillates at greater amplitudes. In simpler terms, the frequency at which the current has the highest peak is what was needed to be found. After that frequency was found, it was to be entered in to equation 1.4 & 1.5 below:

$$L_{coil} = \frac{1}{\omega^2 C} \qquad \text{Equation 1.4}$$

$$\omega = 2\pi f \qquad \text{Equation 1.5}$$

The frequency at which the current had the greatest resonance was 105 kHz. This value can be seen in Appendix A where AC analysis was performed on the circuit in Figure 1.6. Entering this value into the equations above produces the following results:

$$\omega = 2\pi f = 2\pi * 105kHz = 659735.457\, rad/s$$

$$L_{coil} = \frac{1}{\omega^2 C} = \frac{1}{659735.457^2 * 0.01\mu F} = 230\mu H$$

Thus the inductance for the coil used in the launcher is 230μH. The significance of this value is explained in more detail in chapter 4 of this report.

Aluminum Rings

Perhaps the most important component during the manufacturing process was the projectile to be used. First we looked at the resistance of different materials that we might use to construct our projectile from, and then we weighed them to see the difference between them. While testing and researching it came down to copper and aluminum as possible candidates for our rings, but the lightness of the aluminum and its low resistance made it the ideal choice when it came down to the final decision. As part of the objective to find the optimum parameters, not one but five different aluminum rings were manufactured using a machine lathe. Starting with a 'standard' ring, two of the rings had a progressively larger thickness while the other two has a progressively larger height. The testing of these rings is covered in chapter 3. The rings were numbered 1 through 5, with the ‘standard’ ring being 1, the thicker rings being 2 and 3 and the taller rings being 4 and 5. The only dimension kept the same between the rings was the outer diameter of the ring. The schematic in figure 1.7 outlines the defining features of each of the rings and table 1.1 shows how these values differ for each ring:

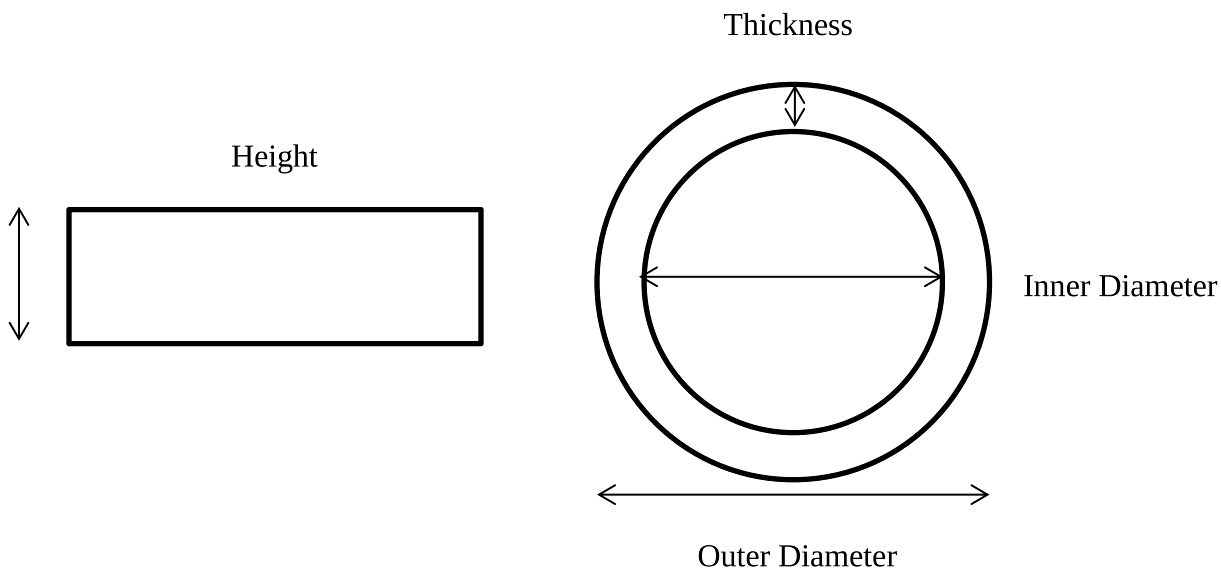

Figure 1.7: Side view and Top view of ring showing dimensions

Ring Number	Height (cm)	Outer Diameter (cm)	Inner Diameter (cm)	Thickness (cm)
1	0.635	4.763	4.445	0.159
2	0.635	4.763	3.970	0.397
3	0.635	4.763	3.335	0.714
4	0.794	4.763	4.445	0.159
5	1.111	4.763	4.445	0.159

Table 1.1: Dimensions of Aluminum Rings

As can be seen from table 1.1 above, the difference in heights has a ratio of:

1: 1.25: 1.75

And the thicknesses have a ratio:

1: 2.5: 4.5

Figures 1.8 and 1.9 show more clearly how these ratios affected the size of the rings:

Figure 1.8: Ring 3, Ring 2 and Ring 1 comparison

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Figure 1.9: Ring 4, Ring 1 and Ring 5 comparison

The aluminum rings are used as conductive projectiles in the launcher and so it is important to calculate their resistances and inductances. The resistances of the ring can be calculated using equation 1.6 below:

$$R_R = \rho_{aluminum} \frac{l_{ring}}{A_{ring}} \quad \text{(Equation 1.6)}$$

Where $\rho_{aluminum}$ is the resistivity of aluminum; $2.82 * 10^{-8}$ Ω.m, *l* is the length of the ring, which in this case is the circumference and the cross-sectional area A can be calculated as the product of the ring's height and thickness. The resistance of the ring can then be used to calculate its inductance. According to equation 1.7, the inductance of the ring simply requires knowledge of the radius of the ring and the coil's inductance, radius and number of turns;

$$L_{ring} = \frac{{r_{ring}}^2}{{N_{coil}}^2 * {r_{coil}}^2} * L_{coil} \quad \text{(Equation 1.7)}$$

Table 1.2 below shows the rings and their calculated resistances and inductances:

Ring	Length (m)	Cross – Sectional Area (m^2)	Resistance (mΩ)	Inductance (nH)
1	0.1446	1.01 E-5	0.404	33.76
2	0.1372	2.52 E-5	0.154	30.38
3	0.1272	4.53 E-5	0.079	26.11
4	0.1446	1.26 E-5	0.324	33.76
5	0.1446	1.77 E-5	0.230	33.76

Table 1.2: Resistances and Inductances of the Rings

The significance of these values is discussed in chapter 3 of the report.

The next set of calculations involved determining the mutual inductance of the coil and ring (standard ring). These calculations were performed using the formulas in the "Inductance Calculations" book by Frederick W. Grover. The initial method used to calculate this inductance is shown in equation 1.8 below. The values for *a* and *A*, the radii of the ring and coil respectively, were measured. Then, for increasing values of d, values of the parameter $(k')^2$ were calculated. Using these values, the corresponding values of *f* were found through the tables in "Inductance Calculations".

$$(k')^2 = \frac{\left(1-\frac{a}{A}\right)^2+\left(\frac{d}{A}\right)^2}{\left(1+\frac{a}{A}\right)^2+\left(\frac{d}{A}\right)^2} \qquad \text{(Equation 1.8)}$$

The calculation of the mutual inductance was then performed using equation 1.9 and the mutual inductance was then graphed against distance, as shown in figure 1.10. These calculations and the corresponding graph were all done using Microsoft Excel

$$M = f \times A\sqrt{\frac{a}{A}} \qquad \text{(Equation 1.9)}$$

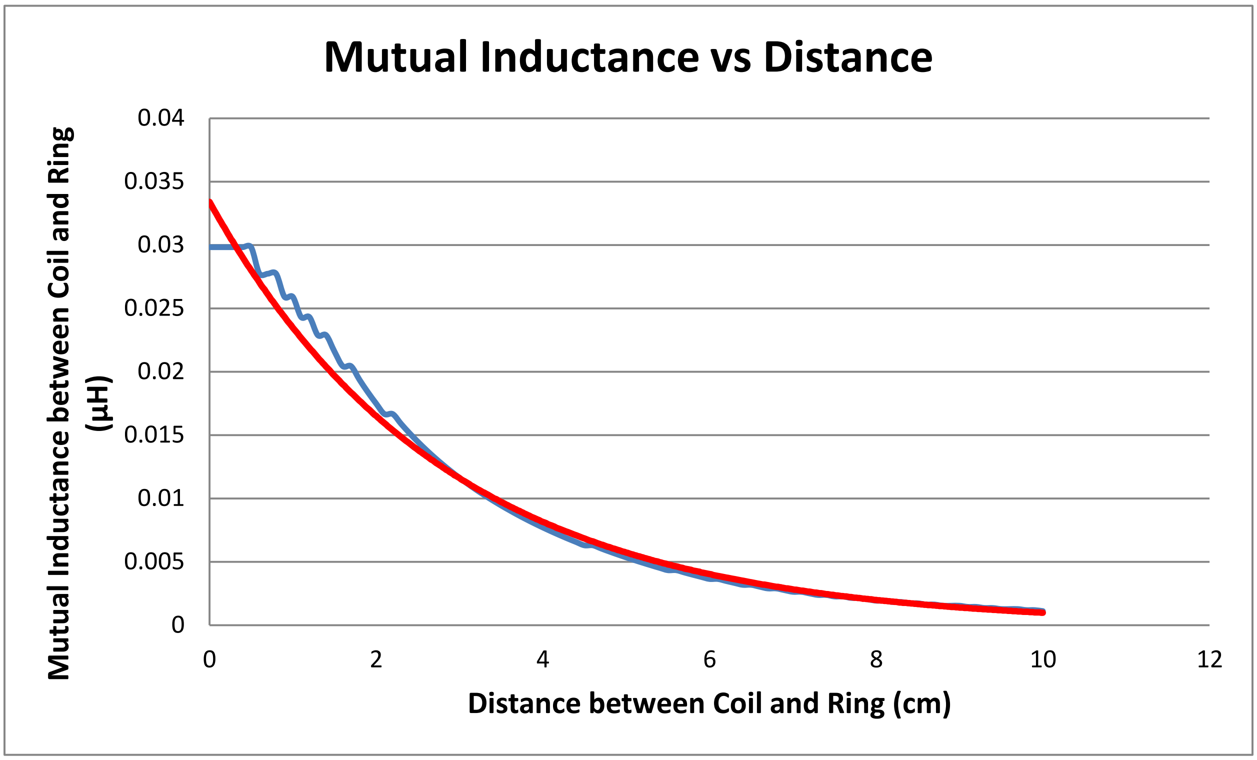


Figure 1.10: Mutual Inductance vs Distance graph

While the mutual inductance calculated using the equations above is correct, a more accurate method of calculating mutual inductance is through the Lyle Method. The Lyle Method involves replacing the coil or ring with two filaments, 1 and 2, and the distance apart of the filaments being the equivalent breadth, 2β, of the coil.

The value of β was calculated using the following equation:

$$\beta^2 = \frac{b^2 - c^2}{12} \qquad \text{(Equation 1.10)}$$

The equivalent radius of A was calculated using equation 1.11:

$$r = a\left(1 + \frac{c^2}{24a^2}\right) \qquad \text{(Equation 1.11)}$$

Using the formulae above, the coil and ring were broken down into two filaments each, with four different radii. The mutual inductance, using the Lyle Method, is the sum of the mutual inductances between the four filaments, multiplied by the number of turns in the coil (50) and the number of turns in the ring (1). The equation is shown below:

$$M = N_1 N_2 M_0 \qquad \text{(Equation 1.12)}$$

In equation 1.12, Mo is the sum of the mutual inductances of M_{13}, M_{14}, M_{23} and M_{24}. The calculations using the Lyle Method formed a much more accurate graph on excel, shown in figure 1.11. The full set of values calculated for this graph can be found in Appendix B. According to the graph, the maximum value of the mutual inductance was found to be 1.52μH for a distance of 0 to 0.4cm. Theoretically the maximum value for mutual inductance should be at 0 cm only. This slight error is due to the Lyle Method being based on numerous approximations.

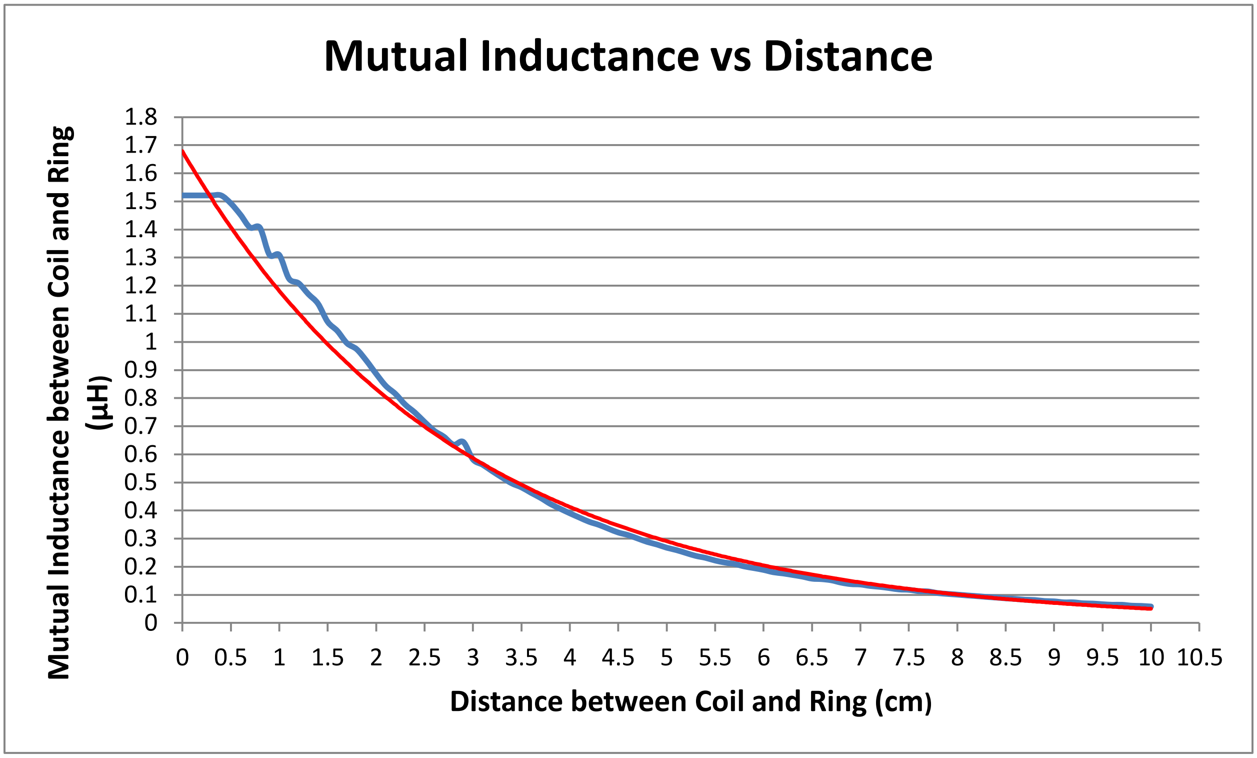


Figure 1.11: Mutual Inductance vs Distance using Lyle's Method

Once the launcher once constructed and the various characteristics were calculated, the focus became extending on the existing structure and optimizing the launches. Figure 1.10 shows the circuit configuration of the launcher in multisim

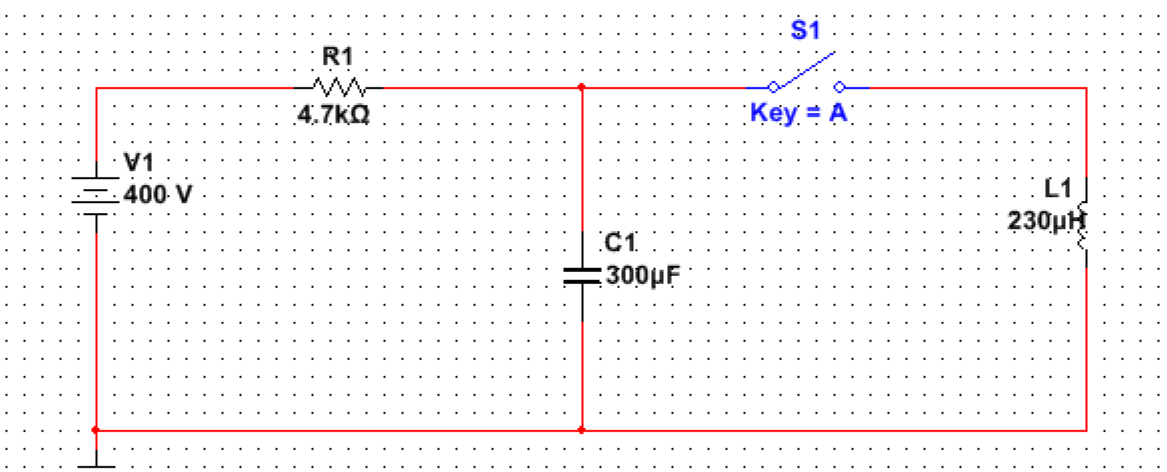


Figure 1.10: Launcher Circuit Configuration

Chapter 2: The Self – Triggering Circuit

One of our major objectives for this MQP was to have the circuit self-trigger without the intervention of any human or outside sources. With the use of electromagnetic technology in military applications like in the case of the Railgun, it became apparent to us that such a mechanism will have a big impact on the safety aspect of the launcher and will insure that in the case of an error in the system humans don't have to pay the price.

For the launcher to be self-triggering, it would have to work off the movement of the ring. This mechanism was achieved through the use of transmitter and receiver circuits. In the case of the transmitter, an input DC voltage signal is used to turn on an infrared LED that will send a signal to the receiver. The receiver contains a phototransistor that will be receiving the signal continuously causing a high output of the circuit. When the traveling ring passes through the tube it will cause a break in the line of sight between the transmitter and the receiver. This in contrast will cause the output to have a low pulse. Since the launcher must be triggered when the ring passes through the tube, the output of the phototransistor must be inverted. This is achieved through the use of an inverting op-amp which will cause the signal to invert to a positive pulse with a voltage gain dependent on the supplied voltage to the op-amp. The inverted signal will then be used to trigger the gate of an SCR acting as a switch and cause it to turn on, allowing it to send the current from the capacitors to the coil, launching the ring further into the tube.

SCR

A silicon controlled rectifier or SCR is a four layer current controlling device. Unlike regular transistors or BJTs, SCRs have the advantage of being able to conduct large amounts of current while only requiring a minimal amount to trigger it on. This made the SCR the ideal candidate for our ring launcher, replacing the original switch. Figure 2.1 shows the SCR used for this project.

Figure 2.1: Westinghouse 2N3890 SCR

The indented round end of the SCR is the anode and is connected to the capacitor in the original circuit, while the thick red lead is the cathode and is connected to the coil. The thin white lead is the gate of the SCR and is connected to the output of the triggering circuit. Once triggered the SCR remains on until a significant drop in the current passing through. Figure 2.2 shows the implementation of the SCR into the original circuit. The open lead in the middle would be connected to the output from the op-amp in the receiver circuit.

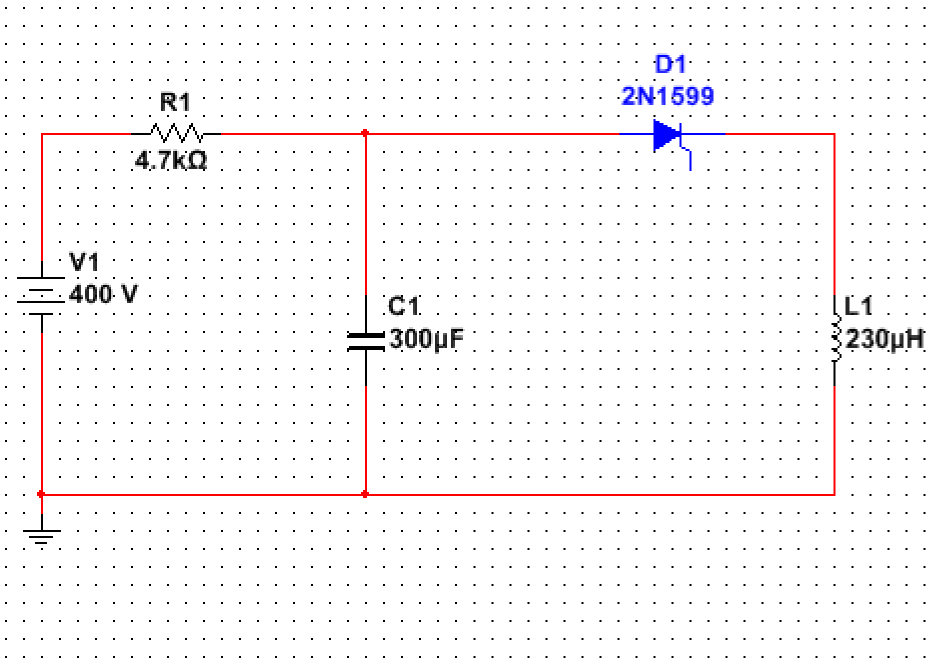


Figure 2.2: SCR Circuit Diagram

Transmitter and Receiver Circuit

The transmitter and receiver circuits are the essential parts of the self-triggering mechanism. Figures 2.3 and 2.4 depict the basic idea behind these circuits and how they are attached to the launcher.

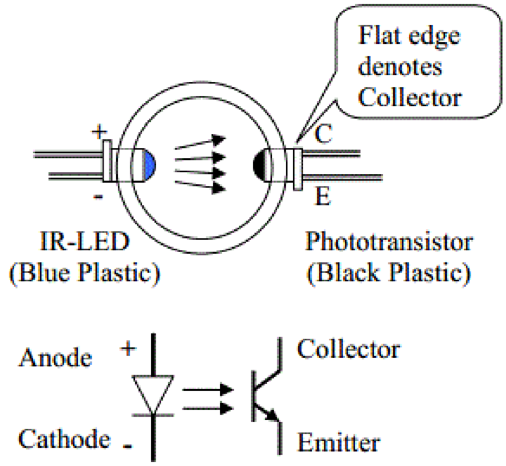


Figure 2.3: Transmitter and Receiver placement on the tube

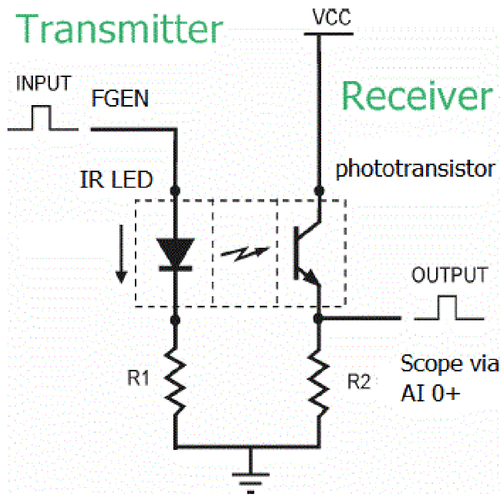


Figure 2.4: Transmitter and Receiver Circuit Configurations

As shown in the diagrams above the circuit's functionality is based upon the interaction between the infrared LED and infrared phototransistor. This works perfectly for the requirement of a self-triggering mechanism since it can be based around the movement of a projectile, such as a ring. The LED transmitter circuit can be attached on one end of the tube as shown in figure 2.3 and the receiver on the opposite end with the phototransistor facing the LED. Normally, the LED transmits photons to the phototransistor which converts this to a current. If an oscilloscope is attached to the output like in figure 2.4, it would display a small positive reading. However, for the transmitter and receiver to be constantly configured and to the have the mechanism only be triggered by the movement of the ring, the output has to be attached to the gate of the SCR somehow. Since this is a very small output value it will not trigger the gate of the SCR.

As the ring passes through the tube, it will momentarily block the line of sight between the LED and phototransistor. To have the trigger be based on the ring's motion, this is the point in time when the SCR should be switched on and by having the coil attached to the tube just under the triggering circuit the ring will launch successfully. The issue is that when the ring blocks the line of sight, it diminishes the already small output of the phototransistor even further. We need it to do the opposite so that the ring blocking the line of sight causes a sudden rise in voltage and this pulse can be applied to the gate of the SCR to trigger it. This is why an inverting op-amp is used in the receiver circuit. The circuit diagram of the op amp attached to the receiver circuit is shown in figure 2.5. The lead coming from the output of the op amp would be attached to the gate of the SCR.

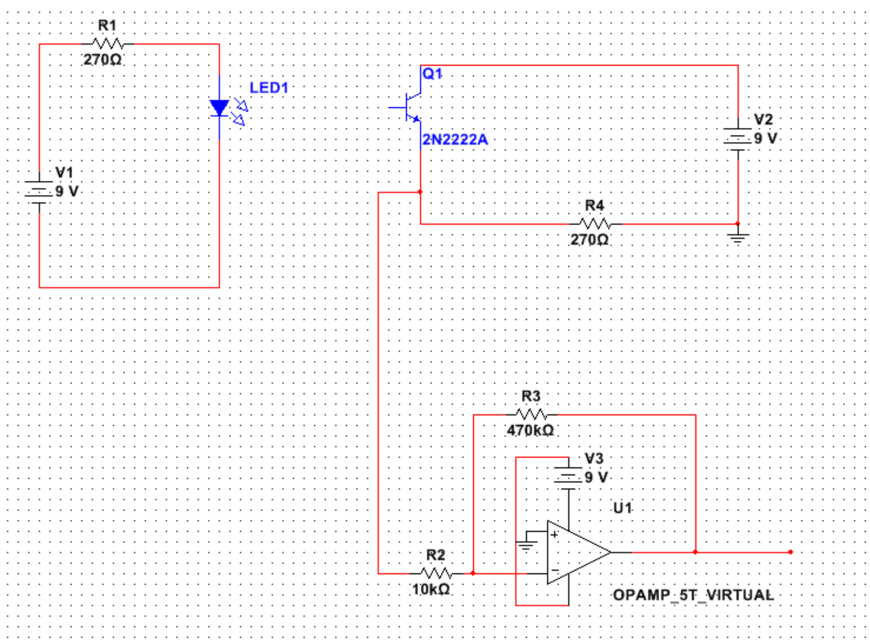


Figure 2.5: Op-amp connected receiver circuit

Since the transmitter and receiver circuits have to be attached directly to the sides of the tubes, portable 9V batteries were used as the voltage source, with the receiver circuit containing an extra one to power the op-amp. The actual soldered circuits are shown in figures 2.6 and 2.7. The wires coming off the receiver circuit are attached to the gate of the SCR. The transmitter and receiver circuits also helped us construct the speed testing circuit outline in chapter 3.

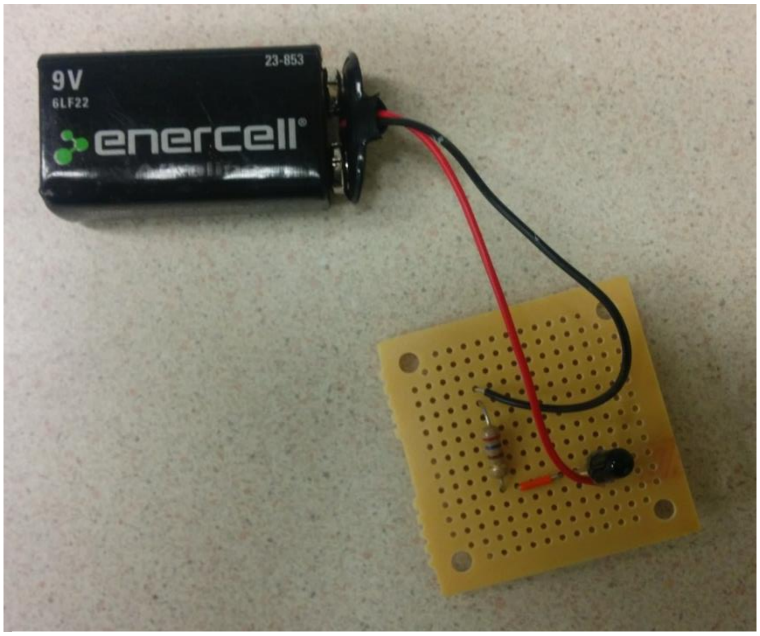


Figure 2.6: Transmitter Soldered Circuit

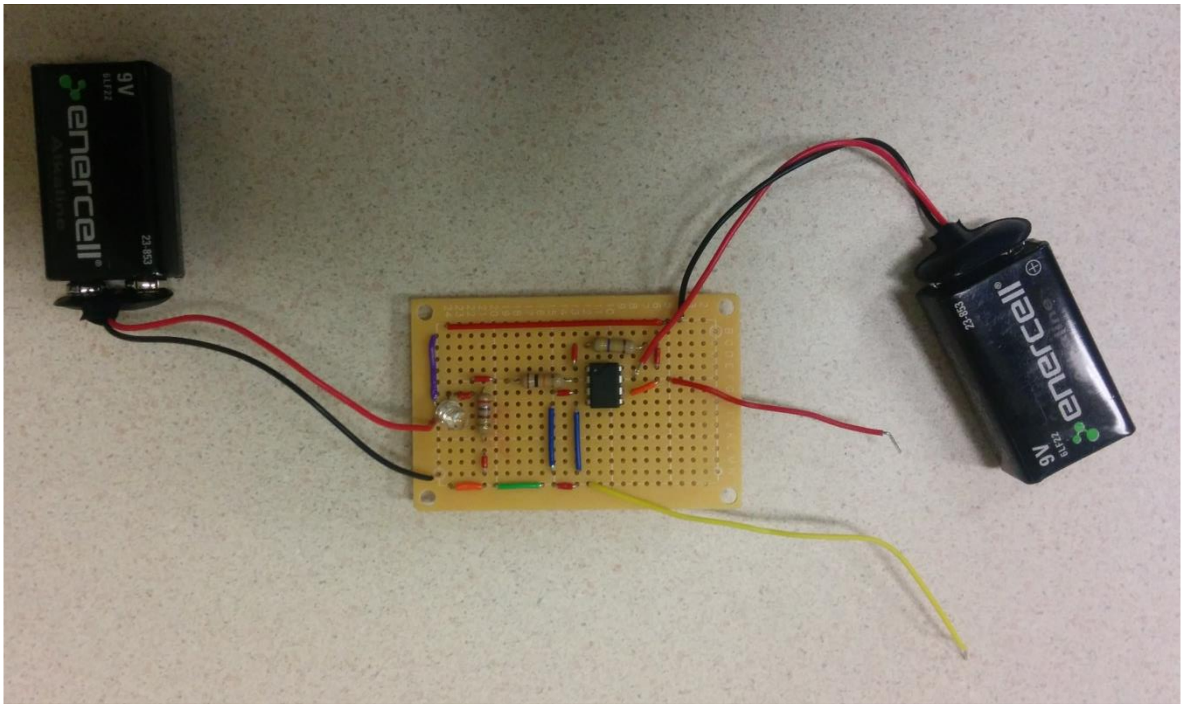


Figure 2.7: Receiver Soldered Circuit

Chapter 3: Optimum Parameters

To accurately construct this project, we spent most of our time understanding how different components react to each other and what is the best set up or positioning to get the best possible results. In this chapter we will take our theories on electromagnetic fields and the force generated by flux to the test. We began by manufacturing the rings that will be put to the test, then testing the optimum position between the triggering mechanism and the rings to reach the heights distance. We also built the velocity measuring circuit to see the fastest speed that can be achieved by our rings relative to our second coil position. This chapter highlights our trial and error period in which most of our results were experimentally obtained.

Finding the Ideal Height for Launch:

Figuring out the best parameters for our conducting projectile took a lot of calculating and testing. We initially began using theoretical and inductance calculations. These calculations were obtained by Grover's book on induction which helped us estimate what might be the optimum cross section for the ring. After the theoretical calculations we started with the trial and error process. This included the careful carving and manufacturing of aluminum rings with different thickness and heights ranging from our standard ring. After successfully manufacturing those rings we went through a triggering trial and error period that included using the different rings at different heights in the tube and seeing the longest travel distance by the ring under its ideal position relative to the coil wrapped around the tube. As shown in table 3.1 and the figures 3.1-6. Distance x here refers to the distance between the bottom of the ring and the bottom of the coil. A supply voltage of 400V was used for all launches

Distance (x)	Height of Ring Launched (in)				
	Ring 1	Ring 2	Ring 3	Ring 4	Ring 5
0	0	0	0	0	0
0.125	0	0	0	0	0
0.250	0	0	4.188	0	1.250
0.375	0.875	20.375	8.500	4.250	1.750
0.500	9.125	33.750	37.875	11.125	12.750
0.625	10.750	37.500	44.500	12.500	11
0.750	9	21.500	32.375	15.375	9.375
0.875	8.125	15.875	25.375	9.875	8
1	7.625	14.813	16.625	4.625	3
1.125	4	9	9	3.750	1.375
1.250	2	3.625	4.375	2.125	0.938
1.375	1.125	1.125	1.375	1.750	0
1.500	0	0	0	0	0

Table 3.1: Heights Reached from Different Ring Launches

The graphs below show the measurements of from table 3.1 to give a clearer view on when the max height was achieved.

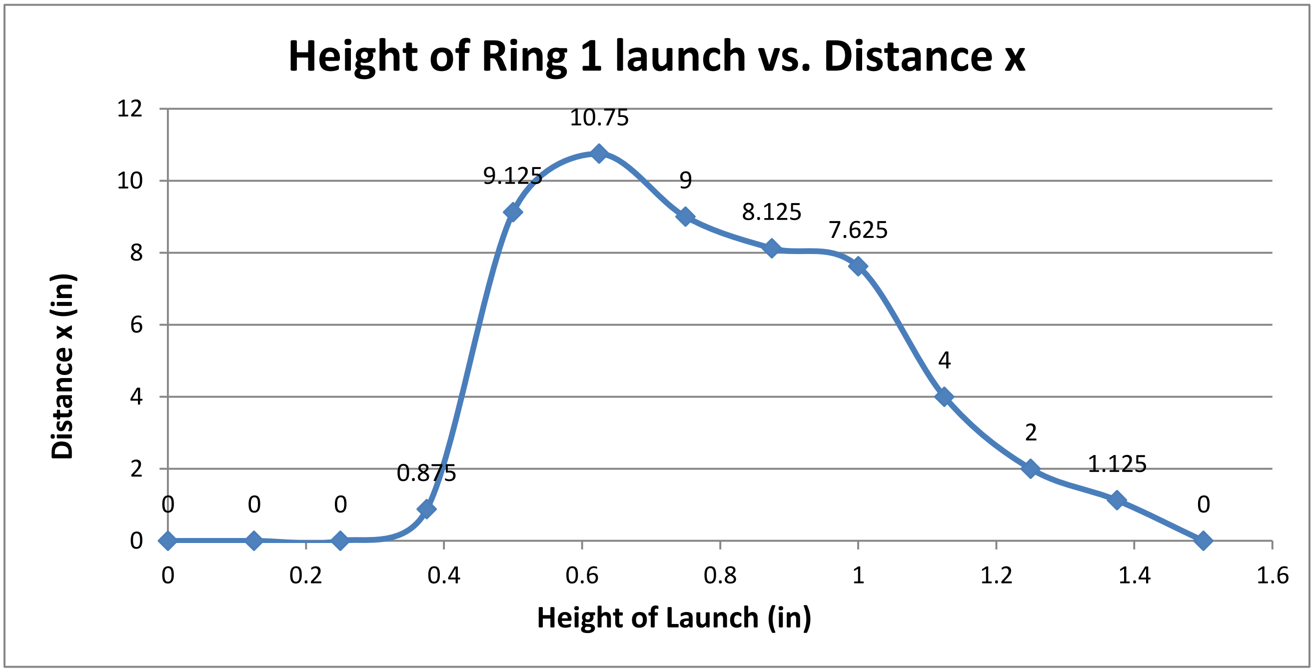


Figure 3.1: Height of Ring 1 launch vs Distance x

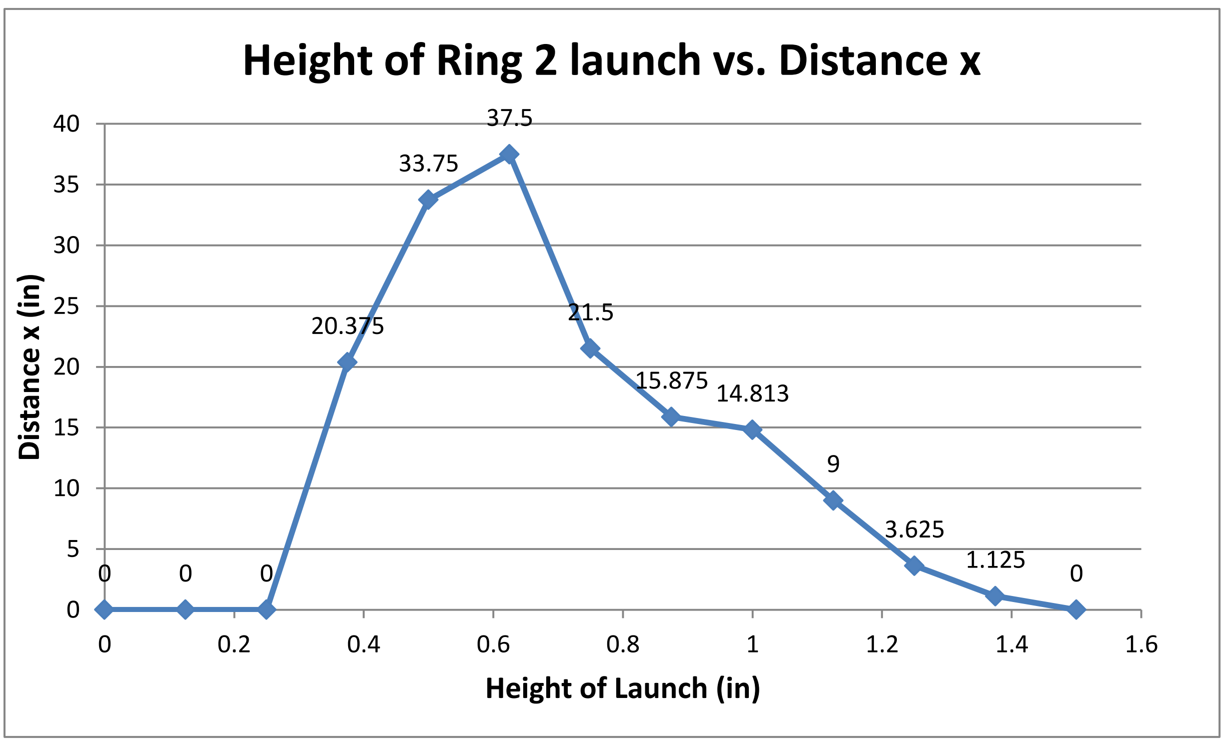


Figure 3.2: Height of Ring 2 launch vs. Distance x

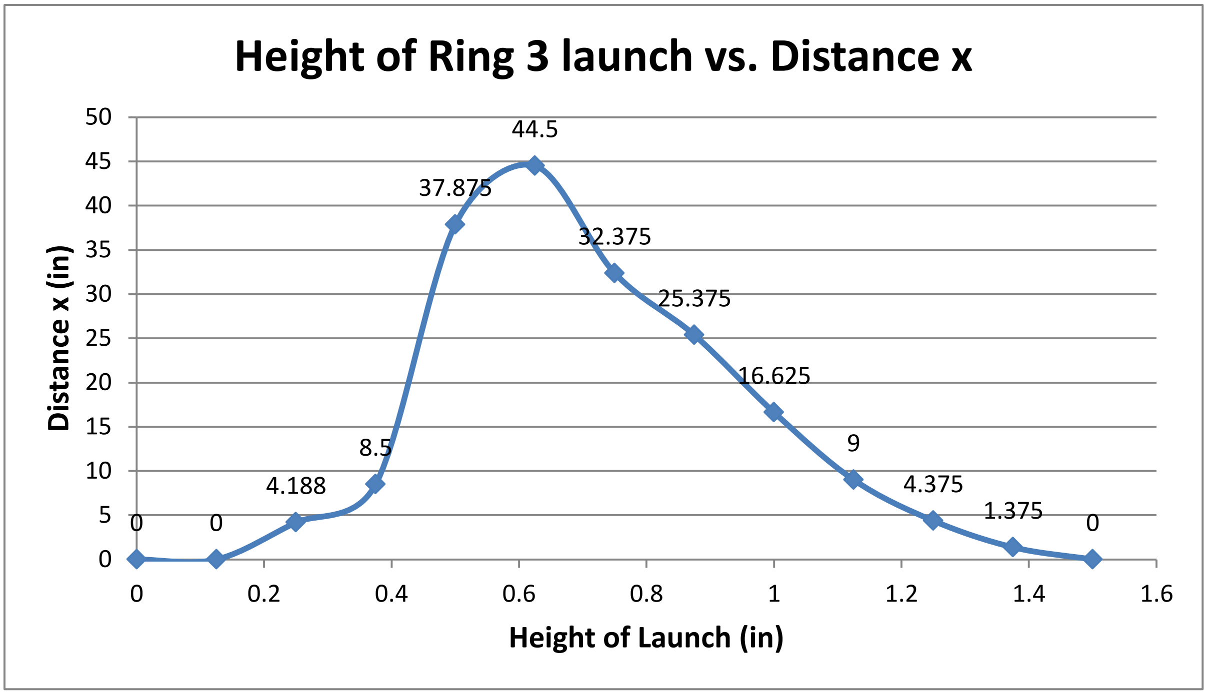


Figure 3.3: Height of Ring 3 launch vs Distance x

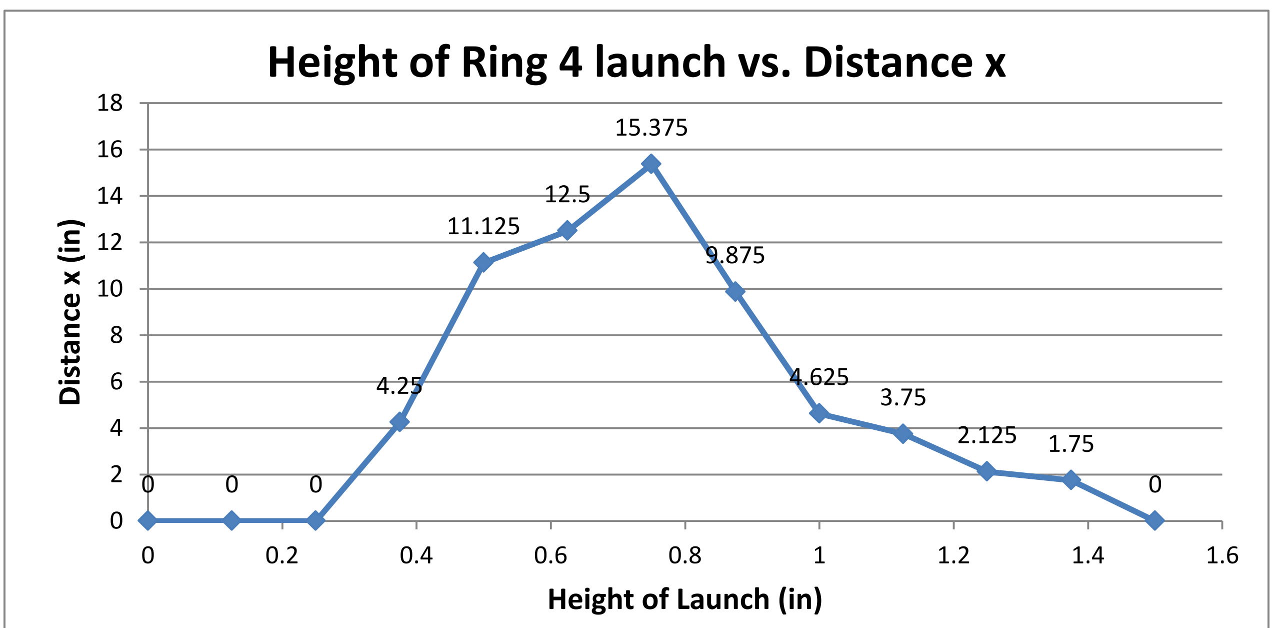


Figure 3.4: Height of Ring 4 launch vs Distance x

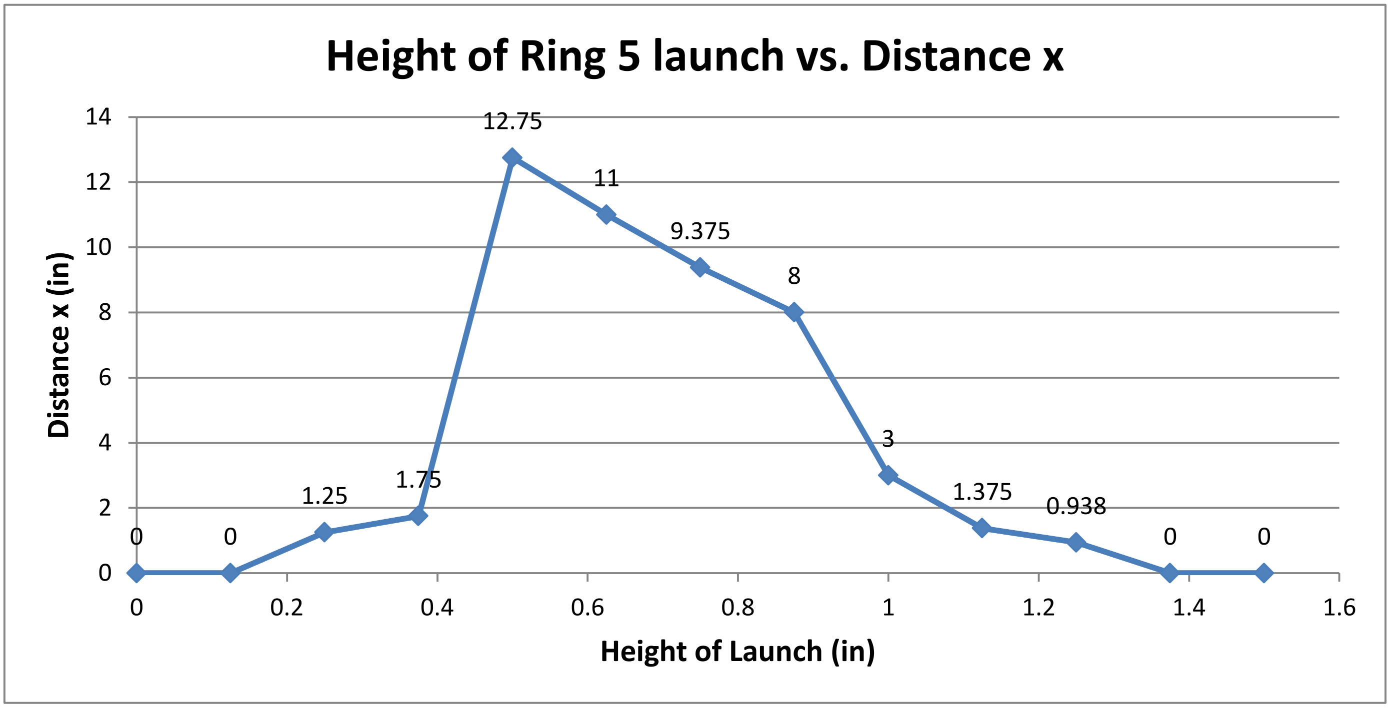


Figure 3.5: Height of Ring 5 launch vs Distance x

The results from the graphs above were used to find the ideal distance x between the bottom of the coil and the bottom of the ring. This distance was found to be 0.625 in. The different rings were then tested at this height to compare the launch heights achieved by each ring. The results are shown in figure 3.6 below

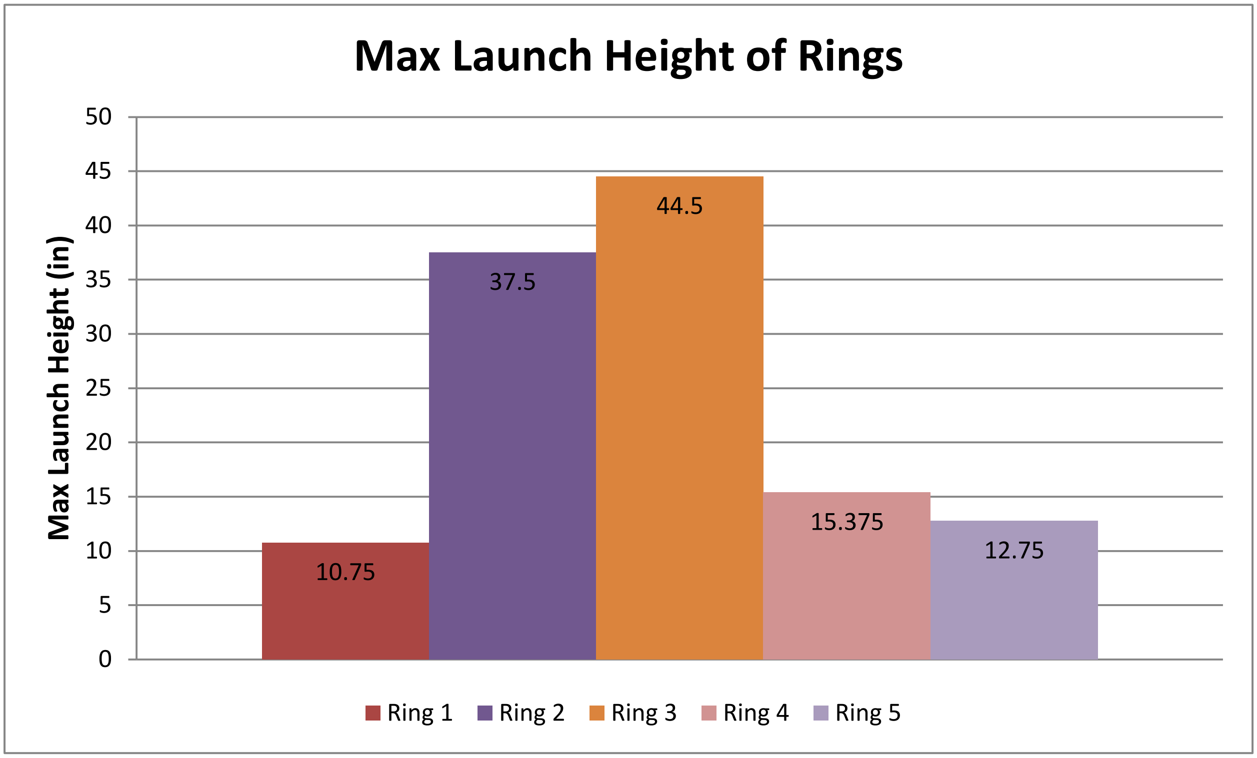


Figure 3.6: Comparison of Max Heights Reached by Rings

The graph above shows that the ideal ring from testing that achieved the highest launch was ring 3. This validates the resistance calculations from chapter 1 that showed ring 3 to have the lowest resistance. A lower resistance would mean a higher current induced in the ring, allowing it to travel further up the tube.

After finding idea ring shown in the graphs above, we began experimenting with different voltages like in our simulation in PSpice. The power supply used ranged from 350-500 volts and we tested these voltages at increments of 25 volts. Using 500 volts gave us the highest launch of our ideal ring as predicted using Faraday's law. Figure 3.7 and Table 3.2 below shows the test results.

Voltages (V)	Height of Ring Launch (in)
350	22.250
375	30.625
400	36.875
425	43.500
450	54.250
475	71.625
500	81.125

Table 3.2: Heights reached at different voltages

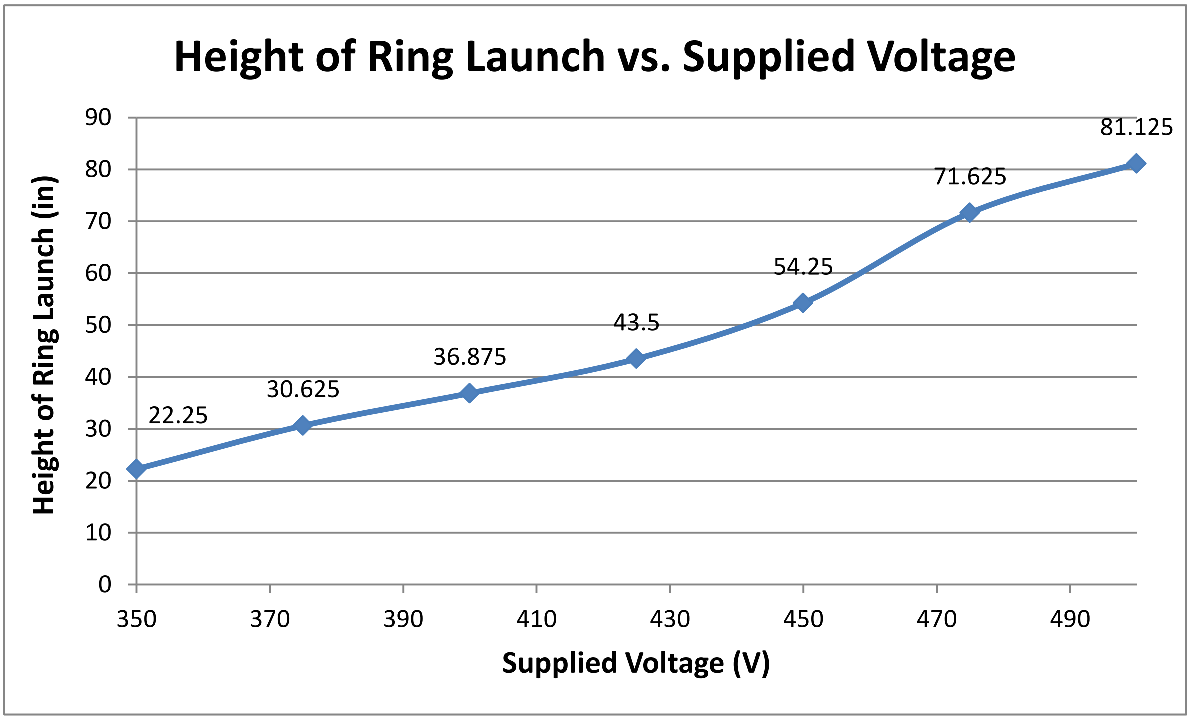


Figure 3.7: Heights at Different Voltages

After finding the ideal ring to use as our projectile, we also conducted a test using velocity measuring circuits that were constructed to ensure that the ideal ring that was picked was the fastest. While testing, ring three had the highest velocity and that was theoretically proven before by the inductance and resistance tables in chapter 1. In the previous table it showed that the resistance of ring 3 was the lowest. Table 3.3 below shows the measured velocity of our five rings.

Ring	Distance between circuits (m)	Time elapsed (ms)	Measured Velocity (m/s)
1	0.61	340	1.794
2	0.61	300	2.033
3	0.61	280	2.179
4	0.61	360	1.694
5	0.61	380	1.605

Table 3.3: Measured Velocity of Rings

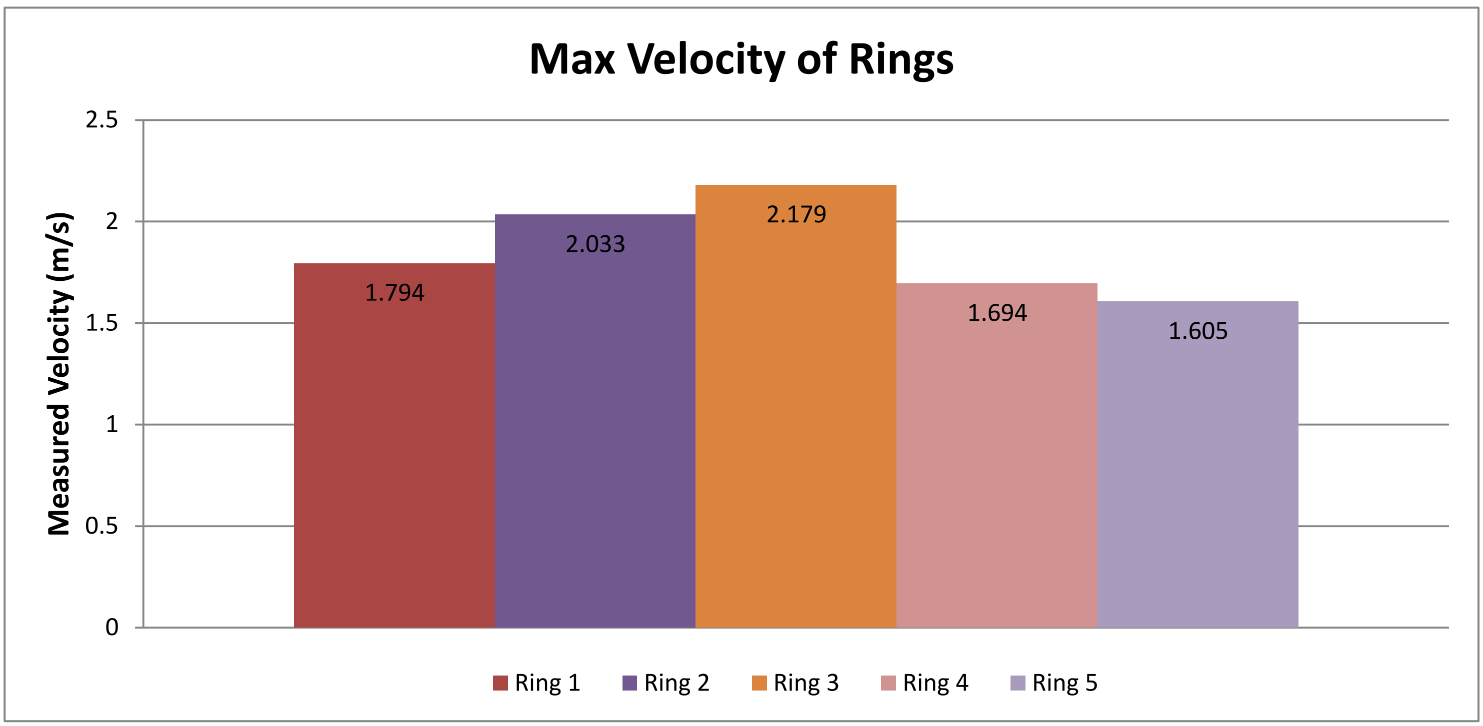


Figure 3.8: Max. velocity of Rings

Our final testing for the optimum parameters for this project included finding the ideal positioning of the coil in contrast with the triggering mechanism mentioned earlier in the report. This testing process was mainly a trial and error process where the distance between the coil and the trigger circuit was changed multiple times until the position that produced the highest possible force and made the ring travel at its highest speed was discovered. This can be seen in Table 3.4 and Figure 3.9.

Distance between coil and circuit 1 (in)	Distance between circuit 1 and 2 (in)	Time elapsed (ms)	Measured Velocity (m/s)
5	0.61	320	1.906
6	0.61	320	1.906
7	0.61	300	2.033
8	0.61	280	2.179
9	0.61	300	2.033
10	0.61	320	1.906
11	0.61	340	1.794

Table 3.4: Measured Velocity at Different Coil Positions

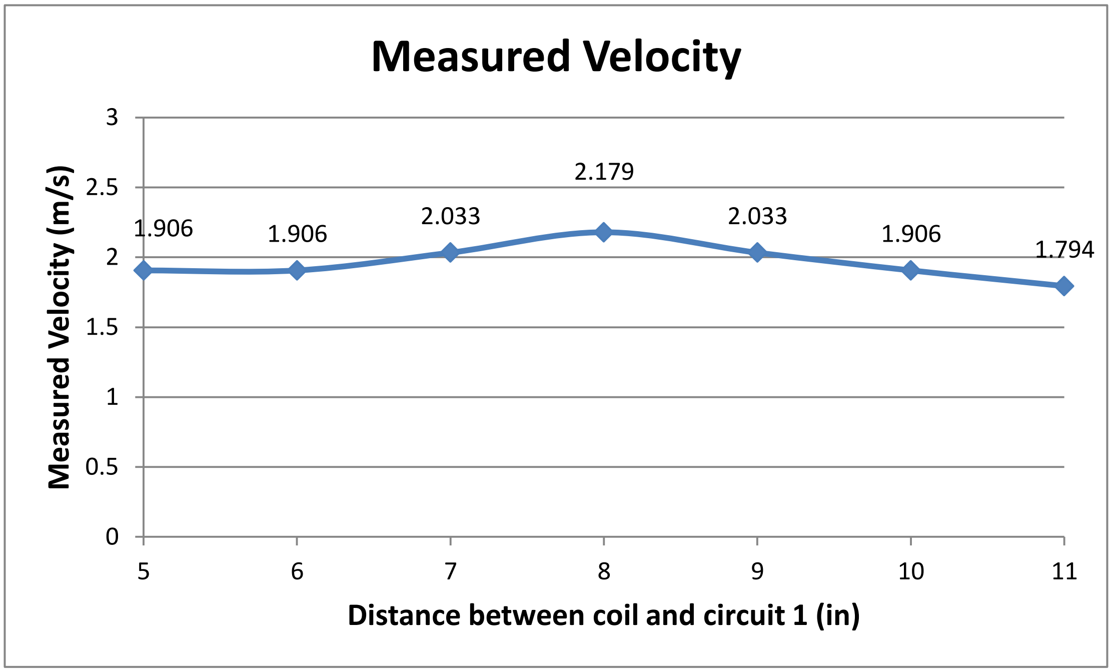


Figure 3.9: Measured Velocity at Different Coil Positions

Speed testing circuit:

The velocity tests conducted in this chapter were done through the use of an extended transmitter and receiver circuit first introduced in chapter 2. The speed circuit was used to measure the ideal position for the coil relative to the triggering circuit and was used to reconfirm the ideal ring found from the height experiments did in fact travel the fastest through the tube. The set up for this circuit was parallel to the triggering circuit further above the tube. It also uses an LED transmitter and a receiver. But unlike the triggering circuit it does not contain an inverting op-amp as the output from the phototransistor simply needs to be monitored for a change when the ring passes and is not used as a voltage source for a separate circuit. When the ring passes through the tube it creates two consecutive pulses; one on the output of the triggering circuit's phototransistor and another on the output of the speed circuit's phototransistor. Knowing the distance between these two circuits and the time elapsed between the pulses on an oscilloscope, the velocity can be measured. The setup for this velocity testing and the modified receiver circuit can be seen in figures 3.10 and 3.11 below

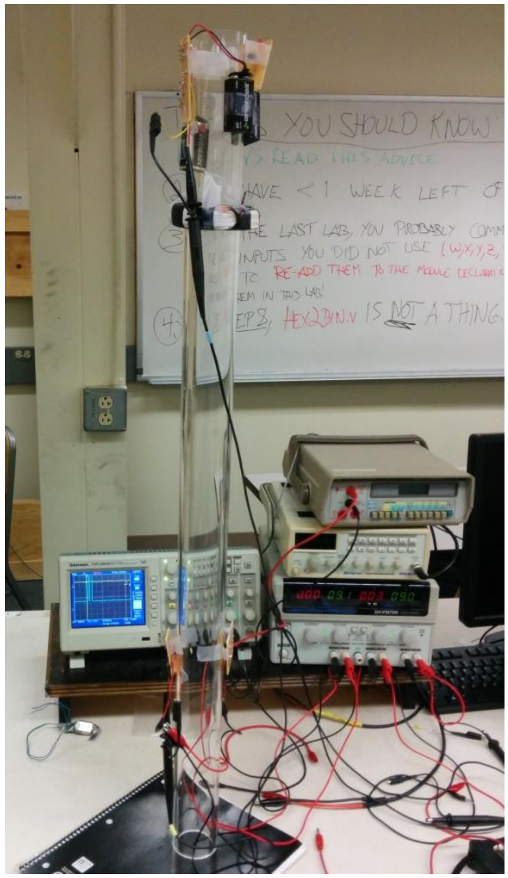


Figure 3.10: Setup for Velocity testing

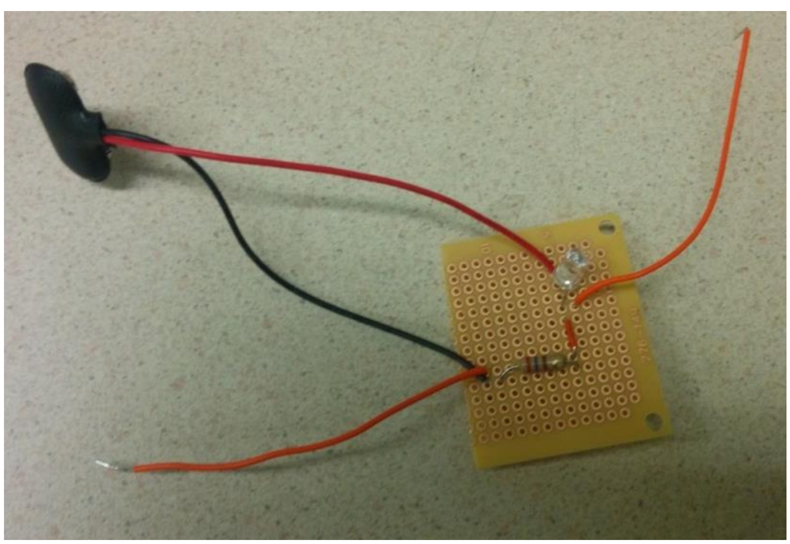


Figure 3.11: Modified receiver circuit

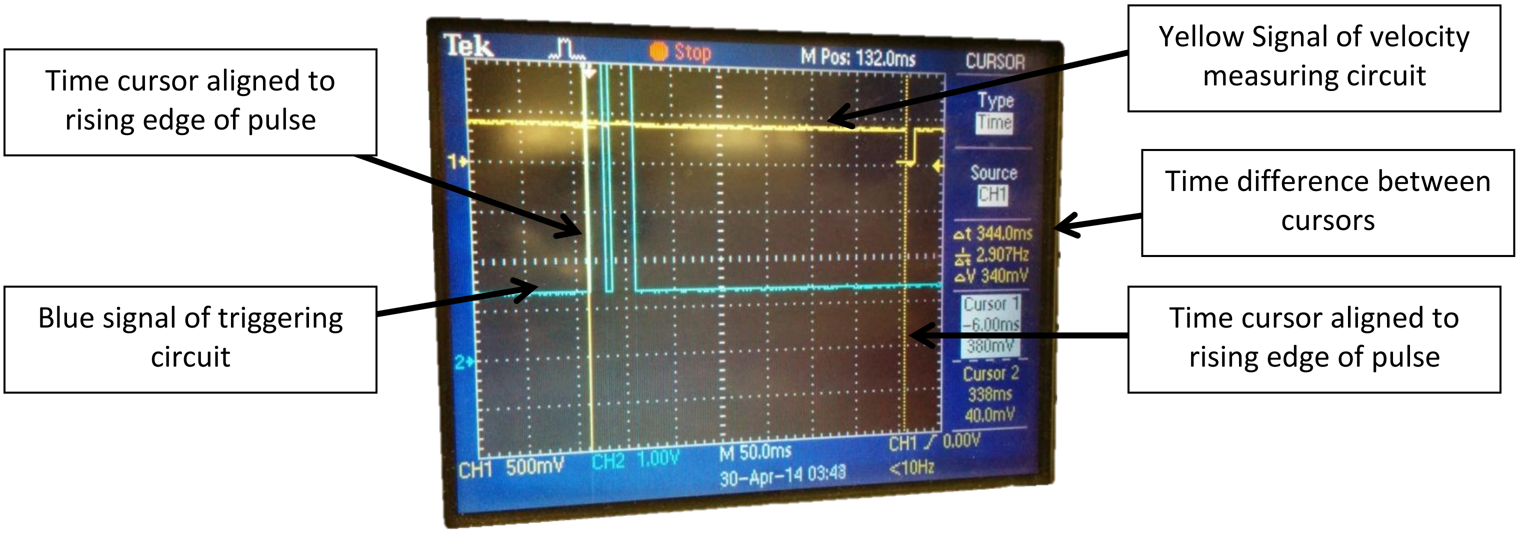


Figure 3.12: Measurement of Velocities Using Oscilloscope

Figure 3.12 above shows the method used to measure the velocities on the oscilloscope. The output from the two receiver circuits are shown as the blue and yellow signals above. As the ring passes through the tube it produces the two pulses shown and the time elapsed between the rise of each pulse, measured using the oscilloscope's cursors, in conjunction with the distance between the receiver circuits is used to measure the velocity

Chapter 4 – Project Simulation

PSpice Simulation

Figure 4.1 below is an overview of the system for this Major Qualifying Project (MQP):

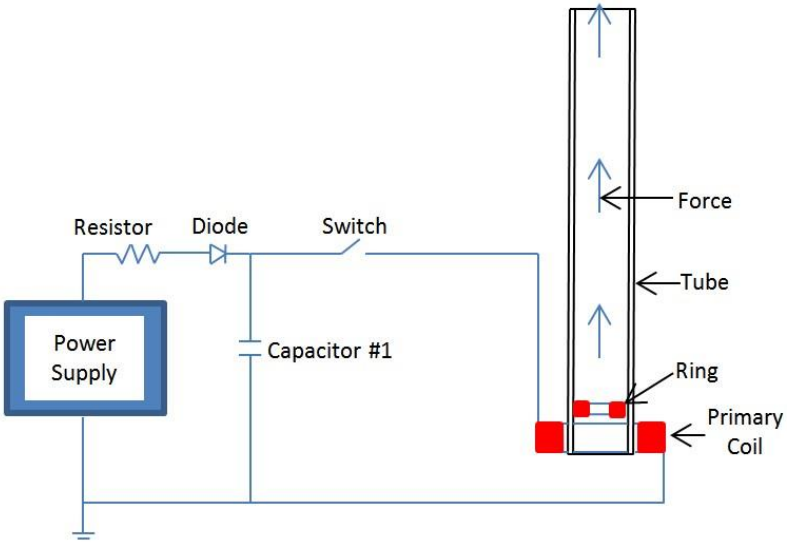


Figure 4.1-MQP System Overview

The DC power supply is regulated through a diode along with a resistor to limit the current. The power supply then charges the capacitor. When the switch is closed, the capacitor charge is released and the coil gets energized. The voltage in the coil induces a voltage in the ring, thus producing a current flow within the ring itself. The current flowing through the ring and coil oppose each other, creating a similar effect of how two identical poles of a magnet oppose each other. This repulsion between the coil and ring causes the ring to launch out of the tube.

A computer program called PSpice was used to simulate this system. This program creates electrical circuits by mapping out the nodes between electrical components. Figure 4.2 below is the electrical circuit for this MQP that was created in PSpice.

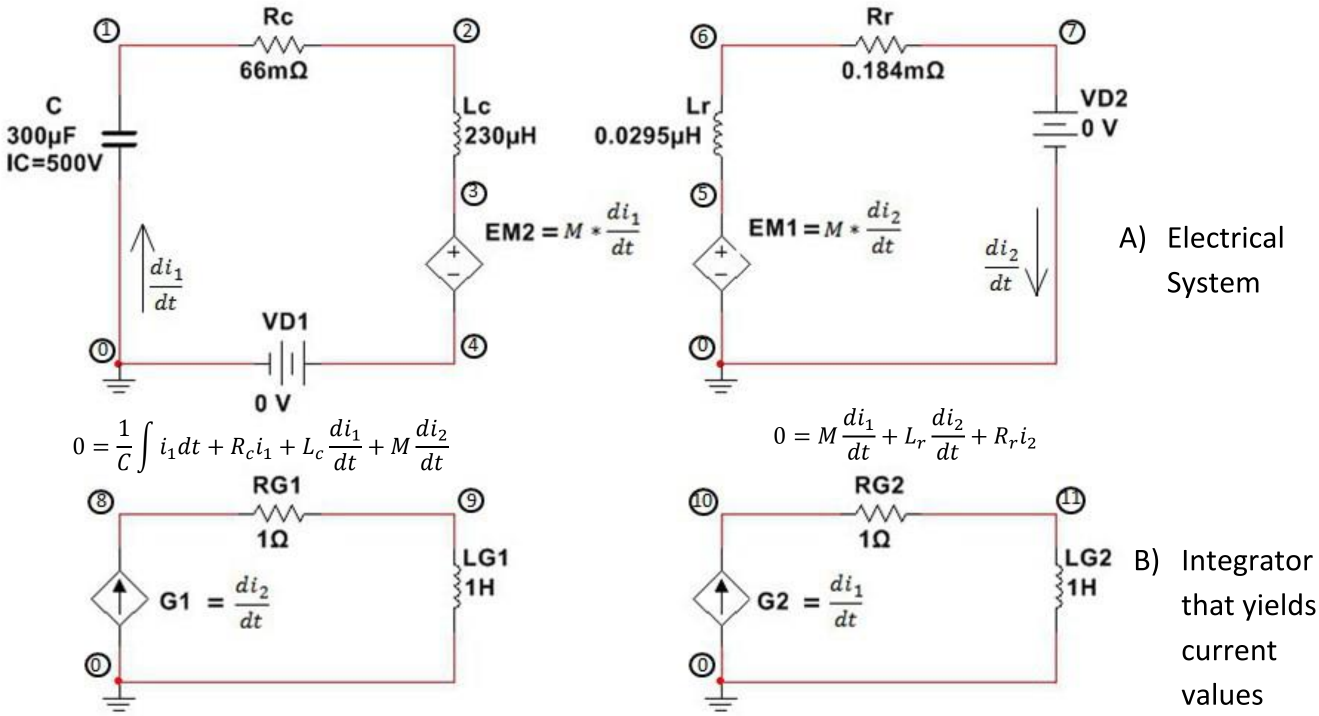


Figure 4.2-PSpice Electrical Circuit

Figure 4.2a above consists of two circuits. The left circuit represents the coil and has the following voltage loop equation:

$$0 = \frac{1}{C}\int i_1 dt + R_c i_1 + L_c \frac{di_1}{dt} + M\frac{di_2}{dt}$$

The circuit on the right presents the ring as has the following voltage loop equation:

$$0 = M\frac{di_1}{dt} + L_r\frac{di_2}{dt} + R_r i_2$$

The simulation took in effect that the capacitor C was fully charged to 500V so the diode and resistor connected to the source were not needed in the simulation. The capacitor has a capacitance of 300µF.

Figure 4.3 is an overview of the coil:

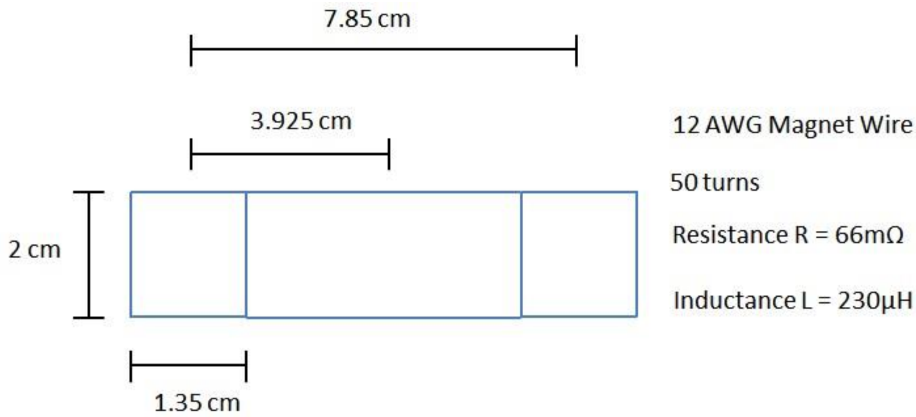


Figure 4.3-Coil Overview

The resistor R_c in Figure 4.2a represents the resistance of the coil. The value of the coil resistance was calculated using the equations below:

$$R_c = \rho_{copper} \frac{l_{coil}}{A_{wire}} \qquad \text{(Eq. 4.1)}$$

$$l_{coil} = N_{coil} * 2\pi r_{coil} \qquad \text{(Eq. 4.2)}$$

$$A_{wire} = \pi r_{wire} \qquad \text{(Eq. 4.3)}$$

As is shown in Equation 4.1, the resistance of the coil was calculated by multiplying the resistivity of copper (ρ_{copper}) by the length of the coil (l_{coil}) divided by the area of the wire itself (A_{wire}). The resistivity of copper is $1.68*10^{-8}$ g/cm^3. The length of the coil requires you to take the average circumference of the coil and multiply it by the number of turns in the coil. Using Equation 4.2, the length came out to be:

$$l_{coil} = N_{coil} * 2\pi r_{coil} = 50 * 2\pi * 3.925cm = 12.3308\ m$$

Using Equation 4.3, the area of the wire itself came out to be:

$$A_{wire} = \pi {r^2}_{wire} = \pi * 0.1cm^2 = 3.14 * 10^{-6} m^2$$

Putting all of these values back into Equation 4.1, the resistance came out to be:

$$R_c = \rho_{copper}\frac{l_{coil}}{A_{wire}} = (1.68 * 10^{-8}) * \frac{12.3308m}{3.14 * 10^{-6}m^2} = 66m\Omega$$

Below in Figure 4.4 is an overview of the ring that was used for this project:

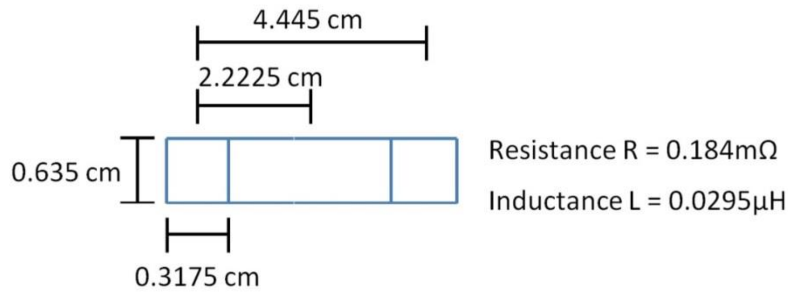


Figure 4.4-Ring Overview

The resistance of the ring (R_r) was calculated using Equations 4.1, just using different values relating to the ring. Using Equation 4.2, the length of the ring was calculated below:

$$l_{ring} = N_{ring} * 2\pi r_{ring} = 1 * 2\pi * 2.2225cm = 0.1398\ m$$

Since the ring was just one turn of a rectangular shape, the area of the ring would be calculated using Equation 4.4, which is the product of the rings' height (h_{ring}) and thickness (t_{ring}):

$$A_{ring} = h_{ring} * t_{ring} \qquad \text{(Eq. 4.4)}$$

$$A_{ring} = h_{ring} * t_{ring} = 0.00635m * 0.003175m = 0.00002016125\ m^2$$

Putting these two values above into Equation 4.1 gives the following results:

$$R_r = \rho_{aluminum}\frac{l_{ring}}{A_{ring}} = 2.65 * 10^{-8} * \frac{0.1398}{0.00002016125} = 0.184m\Omega$$

The inductor labeled L_c in Figure 4.2a is the inductance of the coil. The inductance of the coil required the use of an oscilloscope and a separate circuit as shown in Figure 4.5:

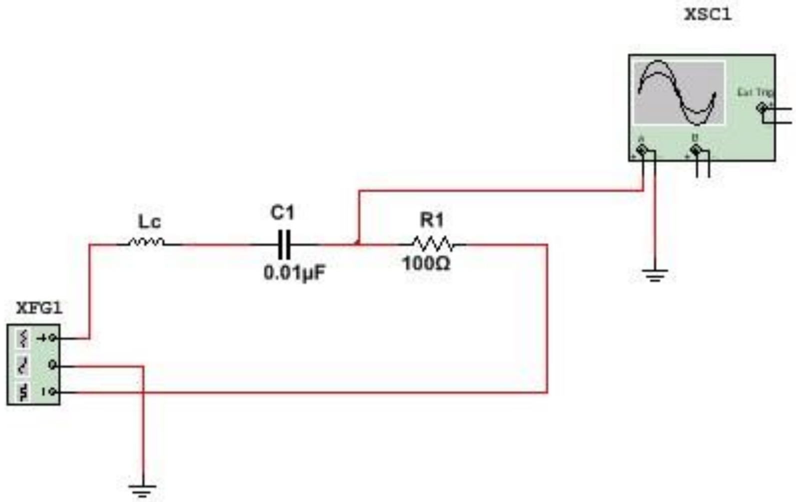


Figure 4.5-Measuring Coil Inductance

The circuit consists of a function generator, the copper coil (L_C), a 0.01µF capacitor (C1), a 100Ω resistor (R1) and an oscilloscope. The function generator was set to produce a sine wave signal and the oscilloscope was used to measure the output current. The point of this method was to find the frequency at which the current resonates. Resonance refers to when the system oscillates at greater amplitudes. In simpler terms, the frequency at which the current has the highest peak is what was needed to be found. After that frequency was found, it was to be entered in to Equation 4.5 & 4.6 below:

$$L_{coil} = \frac{1}{\omega^2 C} \qquad \text{(Eq. 4.5)}$$

$$\omega = 2\pi f \qquad \text{(Eq. 4.6)}$$

The frequency at which the current had the greatest resonance was 105 kHz. This value can be seen in Appendix A where AC analysis was performed on the circuit in Figure 4.5. Entering this value into the equations above produces the following results:

$$\omega = 2\pi f = 2\pi * 105kHz = 659735.457\, rad/s$$

$$L_{coil} = \frac{1}{\omega^2 C} = \frac{1}{659735.457^2 * 0.01\mu F} = 230\mu H$$

The inductor labeled L_r in Figure 4.2a represents the inductance of the ring. The inductance of the ring was found using Equation 4.7 below:

$$L_{ring} = \frac{{r_{ring}}^2}{{N_{coil}}^2 * {r_{coil}}^2} * L_{coil} \qquad \text{(Eq. 4.7)}$$

$$L_{ring} = \frac{{r_{ring}}^2}{{N_{coil}}^2 * {r_{coil}}^2} * L_{coil} = \frac{2.2225cm^2}{50^2 * 3.925cm^2} * 230\mu H = 0.0295\mu H$$

VD1 and VD2 are dummy voltages used to graph the currents through the coil and the ring.

Since all of the values of the components are known, the simulation in PSpice can be made. Figure X.6 below is the PSpice code for the electrical system of the MQP:

```
MQP Circuit Diagram 2 (Mutual Induction Values)
*
*Coil and Ring Simulation
C      1  0   300u      IC=500
Rc     1  2   66m
Lc     2  3   230u      IC=0
EM2    3  4   VALUE = {(1.52e-6)*V(9)}
EM1    5  0   VALUE = {(1.52e-6)*V(11)}
Lr     5  6   0.0295u   IC=0
Rr     6  7   0.184m
VD1    4  0   0
VD2    7  0   0
*
*Applying Mutual Induction and di/dt
G1     0  8   VALUE = {V(6)/0.000184}
G2     0 10   VALUE = {V(1,2)/0.066}
RG1    8  9   1
RG2   10 11   1
LG1    9  0   1         IC=0
LG2   11  0   1         IC=0
```

Figure 4.6-PSpice Electrical Code

Figure 4.6 contains all of the information for the components that were explained above along with a few separate other components which include EM1, EM2, G1 and G2.

EM1 and EM2 are voltage controlled voltage supplies. They are used to implement the mutual induction within the system and have the follow formulas:

$$EM1 = M * \frac{di_2}{dt}$$

$$EM2 = M * \frac{di_1}{dt}$$

The symbol M stands for Mutual Inductance which is the inductance created between the coil and ring. As seen in Figure 4.6 and previously shown in Chapter 1, the mutual induction is 1.52μH.

The values for $\frac{di_1}{dt}$ and $\frac{di_2}{dt}$ are found using the voltage controlled current sources G1 and G2 that are seen in Figure X.2b. As seen in Figure 4.6, G1 has the following value:

$$G1 = \frac{V(6)}{0.000184} = G1 = \frac{di_2}{dt}$$

The value V(6) stands for the voltage at Node 6 of the circuit shown in Figure X.2. The value 0.000184 is the resistance of the ring. Together these two values are used to find the current flowing through the ring, which is the value of G1.

Also from Figure X.6, G2 has the following value:

$$G2 = \frac{V(1,2)}{0.066} = G2 = \frac{di_1}{dt}$$

The value V(1,2) is the voltage between nodes 1 and 2 in the circuit shown in Figure X.2. The resistance of the coil is represented by the value 0.066. Together these values are used to find the current in the coil, which is the value of G2.

Running the simulation, the following graphs were produced:

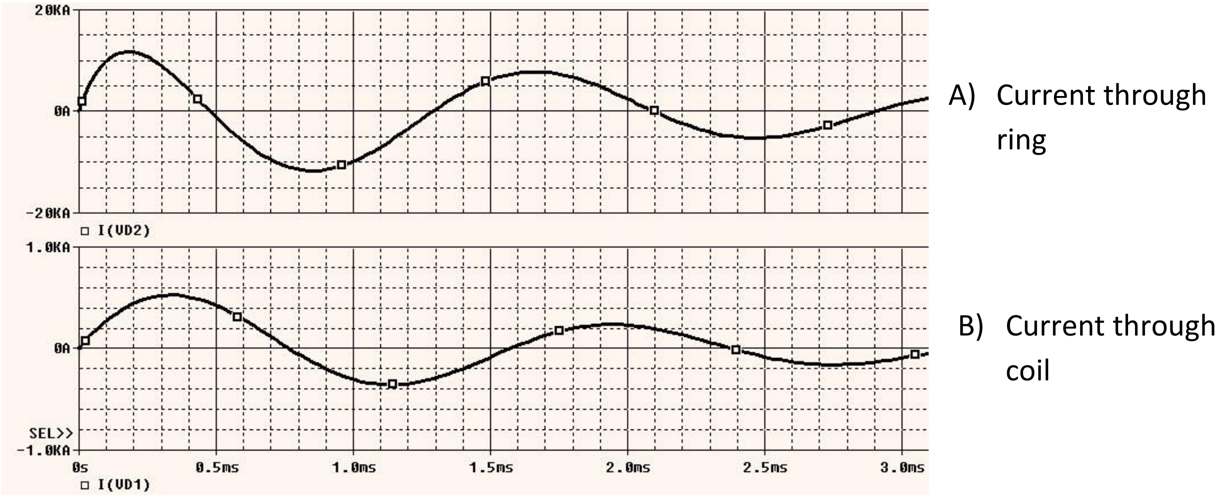


Figure 4.7-Ring and Coil Currents vs. Time

Figure 4.7A represents the current going through the ring which is around 12k amps. Figure 4.7B represents the current in the coil which is about 500 amps. This is exactly what is needed for this system. The current through the ring is a lot higher than the current in the coil because the ring coil is going through only one turn while the coil have 50 turns that the current has to go through. The greater the current through the ring, the greater the opposing forces between the ring and coil will be.

Optimizing the System – Force, Velocity, and Height

In order to optimize the system to produce the greatest force, velocity and height, the best position for the ring to be at was to be found. This process required the use of the following diagram:

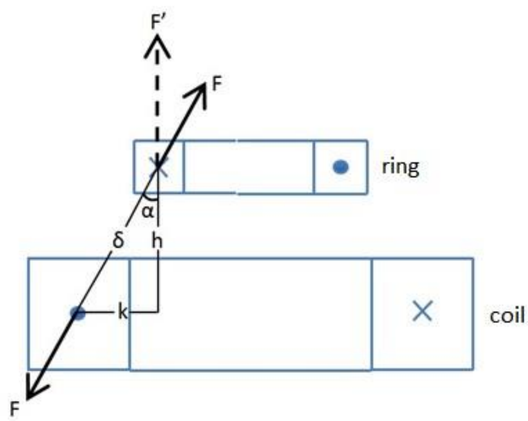


Figure 4.8-Optimizing the System (Resultant Force Calculations)

In Figure 4.8, the bottom block represents the coil and the top block represents the ring. The distance between the coil and ring centers is h, k is the difference between the radius of the coil and the radius of the ring, δ is the diagonal distance between the coil and ring and α is the angle between the ends of the coil and ring. Since the radii of the coil and ring are always constant, k also has a constant value as shown below:

$$k = r_{coil} - r_{ring} = 3.925 - 2.2225 = 1.7cm$$

The diagonal arrows labeled F represent the direct force between the coil and the ring and F' represents the vertical force, which is the force value that needs to be optimized. The greater the vertical force, the higher and faster the ring will launch. Equations 4.8 & 4.9 below are needed to calculate the direct force F:

$$F = 2 * 10^7 * \frac{N_{coil} * i_{coil} * i_{ring}}{\delta} * l_{ring} \qquad \text{(Eq. 4.8)}$$

$$\delta = \sqrt{h^2 + k^2} \qquad \text{(Eq. 4.9)}$$

However, the force that was needed was the vertical force (F') which is similar to Equation 4.8 but includes the multiplication by cos(α):

$$F' = 2 * 10^7 * \frac{N_{coil} * i_{coil} * i_{ring}}{\delta} * l_{ring} * \cos(\alpha) \qquad \text{(Eq. 4.10)}$$

$$\cos(\alpha) = \frac{h}{\delta} \qquad \text{(Eq. 4.11)}$$

The values of h ranged from 0cm to 10cm in increments of 0.25cm. Using Excel, the following graph was produced, mapping out the vertical force in relation to the value of h by using Equations 4.9-4.11:

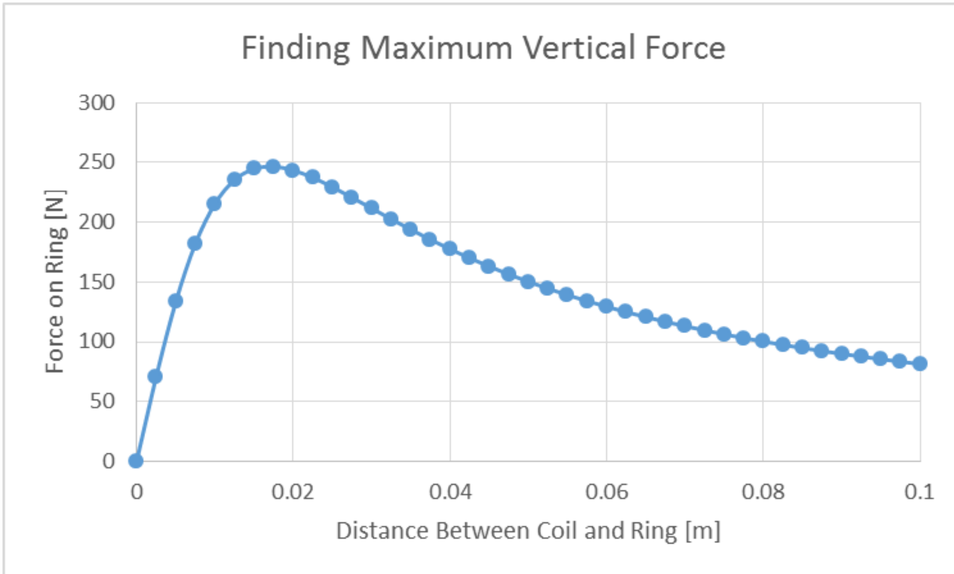


Figure 4.9-Finding Maximum Force

According to Figure 4.9, the graph peaks around 1.75cm which would be the best h value to achieve maximum vertical force. To test out this force is PSpice, the following circuit was created:

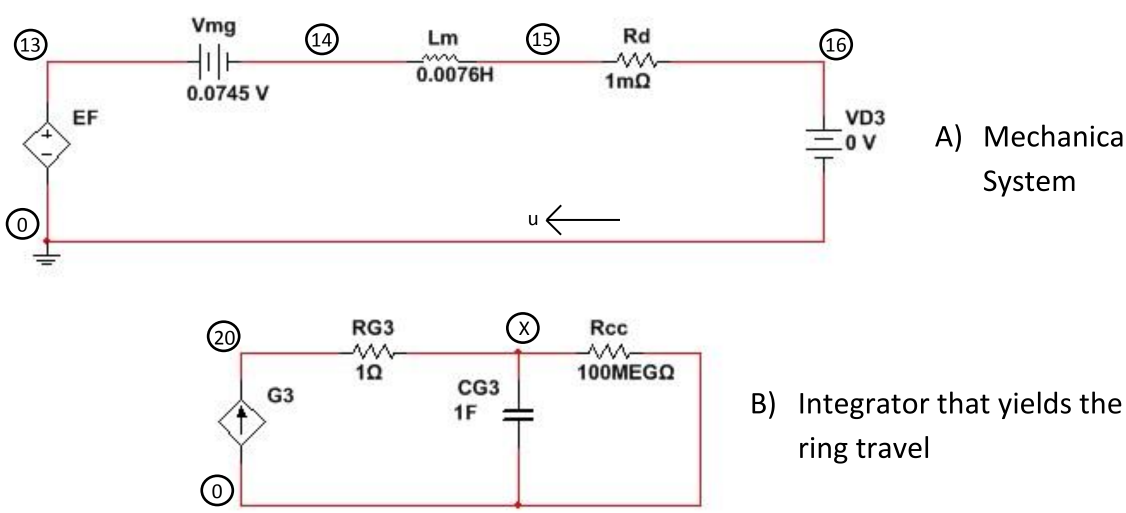


Figure 4.10-Electrical Equivalent to Mechanical System

Figure 4.10 above is the electrical equivalent of the mechanical system to this project. It has the equation of:

$$F' = mg + m\frac{dx^2}{dt^2} + D\frac{dx}{dt} \qquad \text{(Eq. 4.12)}$$

Equation 4.12 is another equation for the total vertical force. The value mg stands for downward force caused by gravity, $m\frac{dx^2}{dt^2}$ accounts for the acceleration and $D\frac{dx}{dt}$ factors in the friction as the ring launches through the tube.

In Figure 4.10a, the voltage controlled voltage source (EF) represents the total vertical force between the coil and ring. The inductor (L_m) represents the mass of the ring. It is found using Equation 4.13 below:

$$m = \left(\pi * D_{rin\ g} * h_{ring} * t_{ring} * \sigma_{aluminum}\right) \qquad \text{(Eq. 4.13)}$$

Using Equation 4.13, the mass of the ring is calculated below:

$$m = \left(\pi * D_{rin\ g} * h_{ring} * t_{ring} * \sigma_{aluminum}\right)$$

$$= \left(\pi * 4.445cm * 0.635cm * 0.3175cm * 2.7\ \frac{g}{cm^3}\right)$$

$$= 0.0076kg = 7.6g$$

The voltage source V_{mg} represents the downward force of the weight of the ring. Equation 4.14 below shows how that value is calculated:

$$m * g = \left(\pi * D_{rin\ g} * h_{ring} * t_{ring} * \sigma_{aluminum}\right) * g \qquad \text{(Eq. 4.14)}$$

$$= \left(\pi * 4.445cm * 0.635cm * 0.3175cm * 2.7\ \frac{g}{cm^3}\right) * 9.8$$

$$= 0.0076kg * 9.8$$

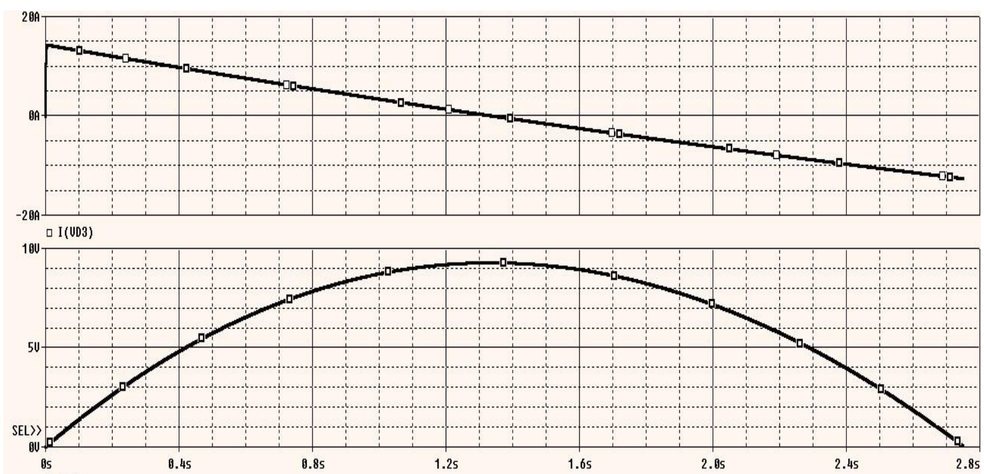
$$= \ 0.0745kg = 74.5g$$

` The resistor R_d is a dampening resistor. The voltage labeled VD3 is a dummy voltage used to get the current within this circuit. The current flowing through this circuit represents the velocity of the ring (u) as it is launched. Figure 4.10b is used to get the maximum height that the ring reaches. The following is the PSpice code for the mechanical circuit:

```
*Finding Force
EF   13   0   VALUE = {(2*10^-7)*(50)*(I(VD1))*(I(VD2))*(0.1398/0.0247)*(0.729)}
Vmg  13  14   0.0745
Lm   14  15   0.0076    IC=0
Rd   15  16   1m
VD3  16   0   0
*
*Max height
G3    0  20   VALUE = {I(VD3)}
RG3  20   X   1
CG3   X   0   1         IC=0.0175
Rcc   X   0   100Meg
```

Figure 4.11-Mechanical System Code

As seen in Figure 4.11, the component EF takes the value of Equation 10. The values for δ and cos(α) were calculated using the optimum value of h which came out to be 1.75cm. The rest of the code consists of the components in Figure 4.10. From the PSpice code above, the following graphs for the velocity and height were obtained:



Key
1.0V=1.0 m

A) Velocity of the ring

B) Height traveled by the ring

Figure 4.12-Velocity and Height of Ring vs Time

Figure 4.12A represents the velocity of the ring and Figure 4.12B represents the height traveled by the ring. Once the ring is launched, it reaches a top speed of about 14 m/s. The velocity then decreases linearly and eventually reaches 0 m/s. While the velocity is positive, that signifies when the ring is traveling upward, which can be seen in the height graph. When the velocity falls to 0 m/s, that is when the ring is at its' peak height of about 9.25 meters. When the velocity goes negative, that signifies when the ring is traveling downward. This can also be seen on the height graph.

All of these graphs represent the greatest force, velocity, and height that the ring can achieve with a 500V source. Experimentations can also be taken with different source voltages.

Experimenting With the Source Voltage

As what was stated earlier, different source voltages would result in different values of force, velocity and height. Using the same optimal position of the ring that was found earlier, the same methods were performed to discover the currents and height for source voltages of 100V, 300V and 1000V.

Figure 4.13 below displays the current values for the varying voltage sources. Figure 4.13A represents the current through the ring and Figure 4.13B represents the current through the coil.

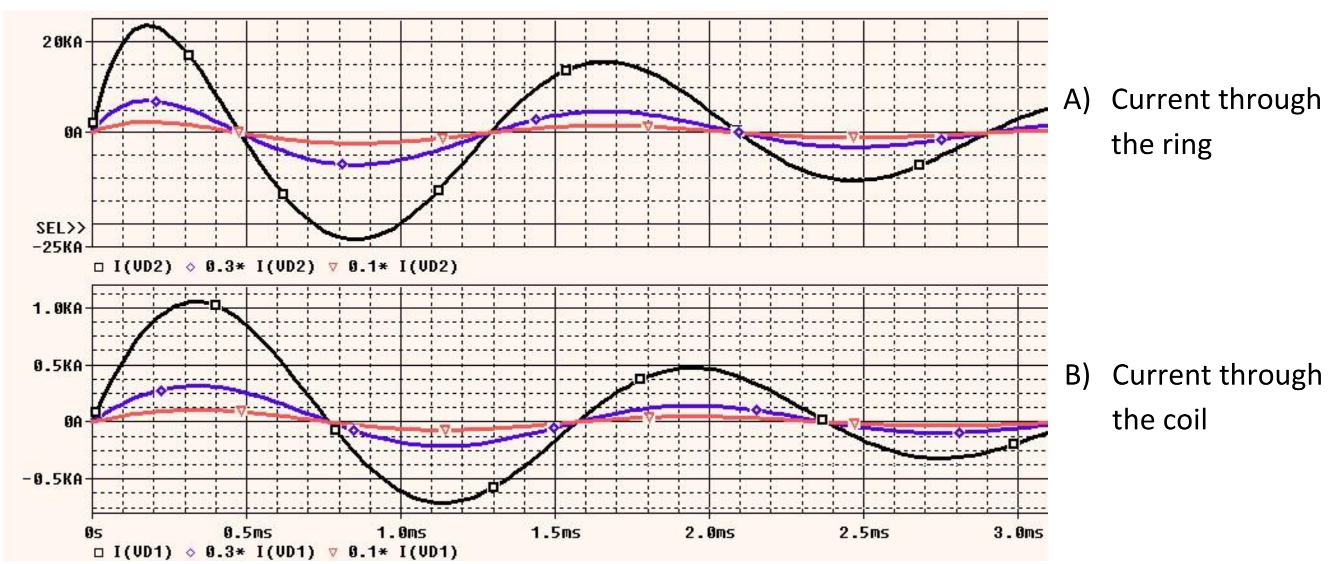


Figure 4.13-Currents for Varying Voltage Sources

The black line represents the current with the 1000V source, the blue line represents the current with the 300V source and the red line represents the current with the 100V source. As shown, the 100V source has the highest current rating. Since the currents are proportional with the voltage, the current with the 100V source would be $\frac{1}{10}$ of the current with the 1000V source. The same goes with the 300V source except that the current will be $\frac{3}{10}$ the value of the current with the 1000V source.

Next the height of the ring was observed for the different source voltages. Figure 4.14 shows the height traveled by the ring with a 100V source

Key
10.0 mV=1.0 cm

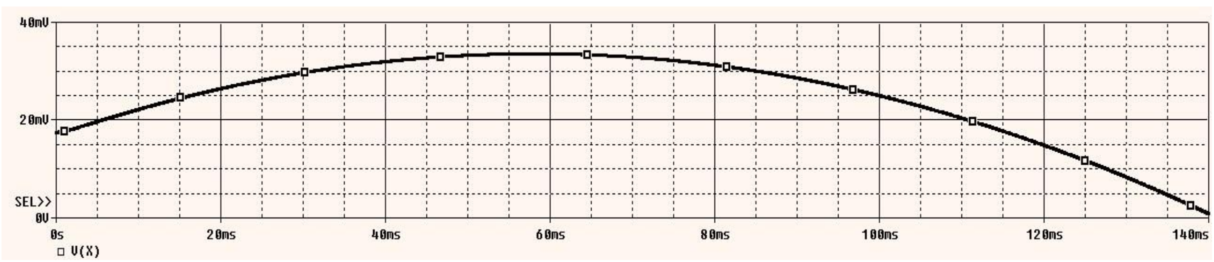


Figure 4.14-Height Traveled With 100V Source

Figure 4.14 starts at 1.75cm because that is the initial height that the ring has. The ring reaches its maximum height of 3.4cm around 60ms and falls back to the ground around 1.275 seconds. This height however is not enough to launch out the tube.

Figure 4.15 below is the height graph for the 300V source:

Key
1.0V=1.0 m

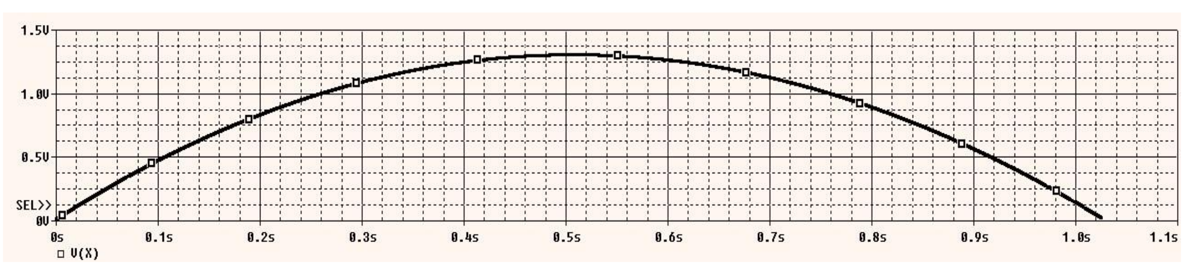


Figure 4.15-Height Traveled With 300V Source

Figure 4.15 show that the ring reaches a peak of about 1.3 meter. It reaches this peak height at about 0.5 seconds and lands on the ground after about 1 second. This height is enough to launch the ring out of the tube but not by much.

Figure 4.16 is the height traveled by the ring with the 1000V source:

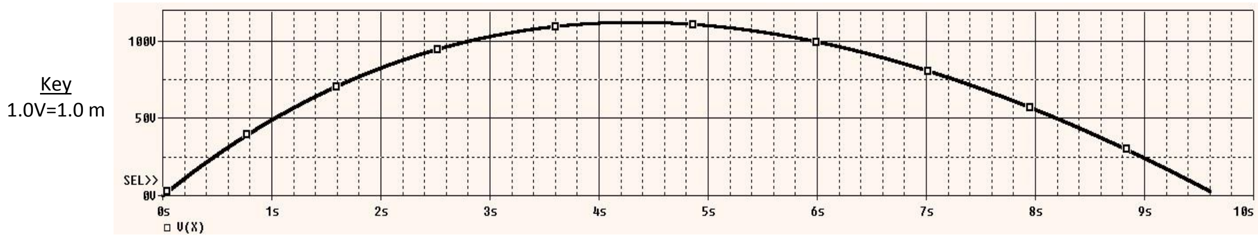


Figure 4.16-Height Traveled With 1000V Source

As shown in Figure 4.16, at around 4.5 seconds, the ring reaches a maximum height of about 110 meters and falls back to the ground after about 9.5 seconds.

Below is a clearer representation of the relation between source voltage and height traveled:

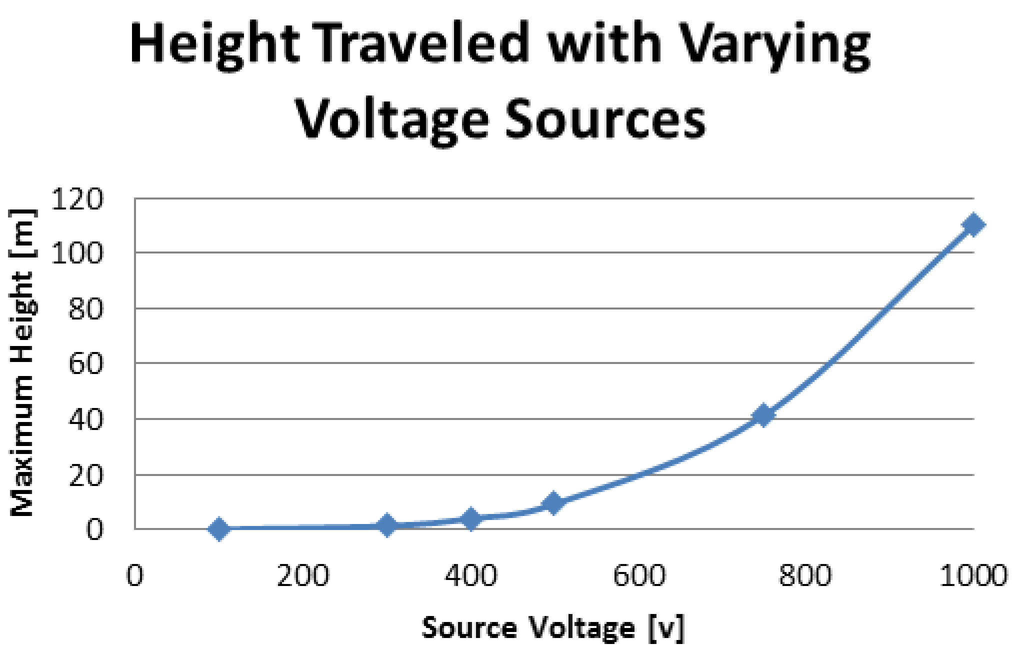


Figure 4.17-Height Traveled with Varying Voltage Sources

Figure 4.17 above illustrates that the height traveled depends exponentially by the source voltage.

Implementing a Second Station

To optimize the system even more a second identical coil was added to the simulation of the system. The addition of this second station would limit the interaction from humans, increasing the safety of the system. The new system diagram is as follows:

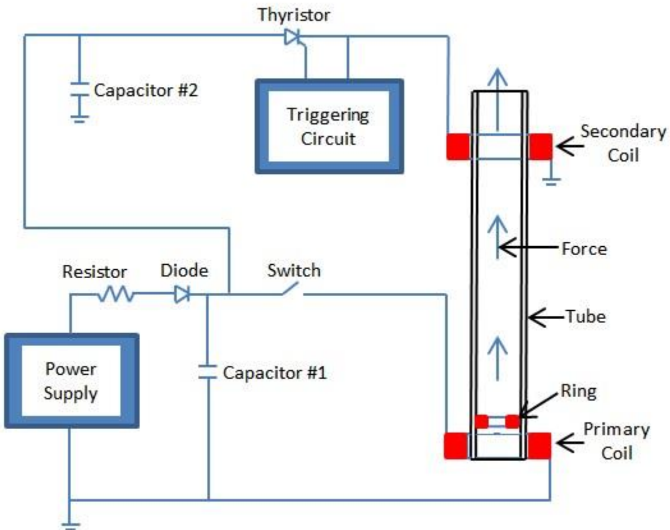


Figure 4.18-Second Station Overview

Figure 4.18 is the same as Figure 4.1 but with the second station added. The second station is identical to the first except that instead of using a mechanical switch to energize a coil, the second station uses of a thyristor and a triggering circuit. A thyristor is a type of diode that only lets current flow through its anode and cathode if it receives an input on its gate pin as seen in Figure 4.19:

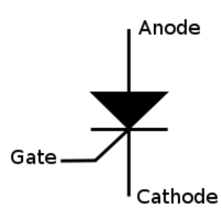


Figure 4.19-Thyristor symbol

The triggering circuit is explained below in Figure 4.20:

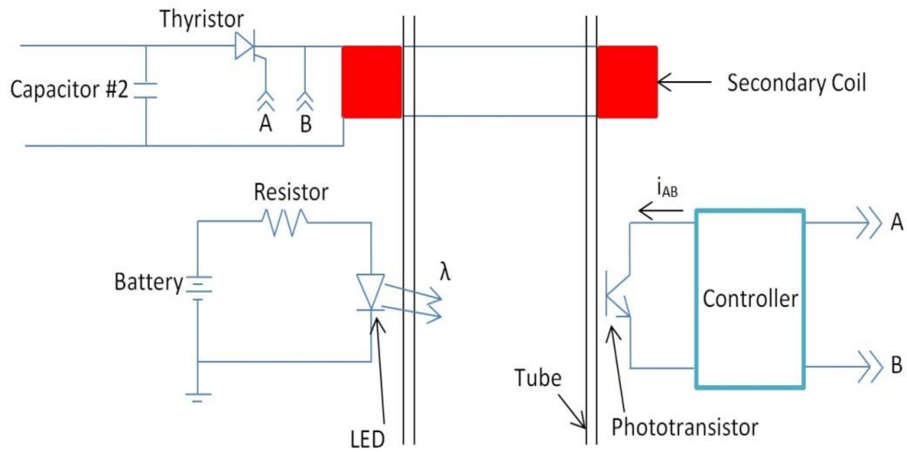


Figure 4.20-Triggering Circuit

The triggering mechanism in Figure 4.20 consists of a light emitting diode (LED), a phototransistor, and a controller. When the ring travels up the ring, the light path between the LED and phototransistor is broken. When that path is broken, the controller will create a pulse. That pulse is then sent to the thyristor, thus activating the thyristor and the secondary coil. Figure 4.21 is a graph of this concept:

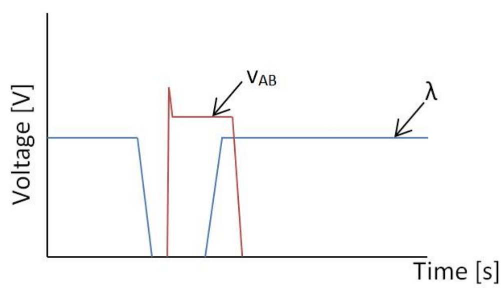


Figure 4.21-Triggering Graph

Along with adding the second station, the best position at which the ring should be when it receives the charge from the second coil to achieve maximum force was also to be found. This is similar to the diagram in Figure 4.8 but this time the value h is referred to as h'. The same methods were performed to find the maximum force output from the second coil. The following is the graph produced when the second coil was simulated to be 50 cm above the first and the ring has an initial velocity of 13.895 m/s. This initial velocity was put as the initial condition for the inductor L_m. As seen in Figure 4.11, the initial condition for L_m was originally 0 because the ring starts off at 0 m/s. However, this is not the case for this next simulation.

The following graph is the force from the second coil in relation to the value of h':

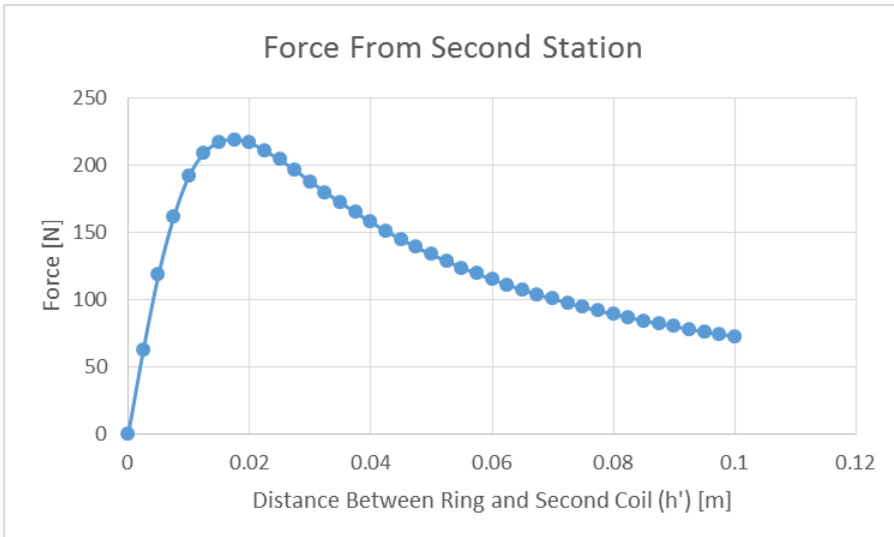


Figure 4.22-Maximum Force from Second Station

Figure 4.22 is very similar to Figure 4.9 and they also yield the same results. The ring has to be a 1.75 cm above either coil to achieve maximum force. The graph in Figure 4.22 however does not reach the same peak value of about 250 Newtons as it did in Figure 4.9. This is because since the ring has an initial velocity of 13.895 m/s, the ring is still flying upwards, the mutual induction between the coil and the ring is getting lower and the ring still has an induced current. It is unsure whether the induced current in the ring from the first coil would increase or decrease the current applied to the ring from the second coil, thus creating loses in the second station. To limit any loses, it would be best to have the second station kick in when the ring is at its peak height and has a velocity of 0 m/s. The following graphs explain the travel of the ring:

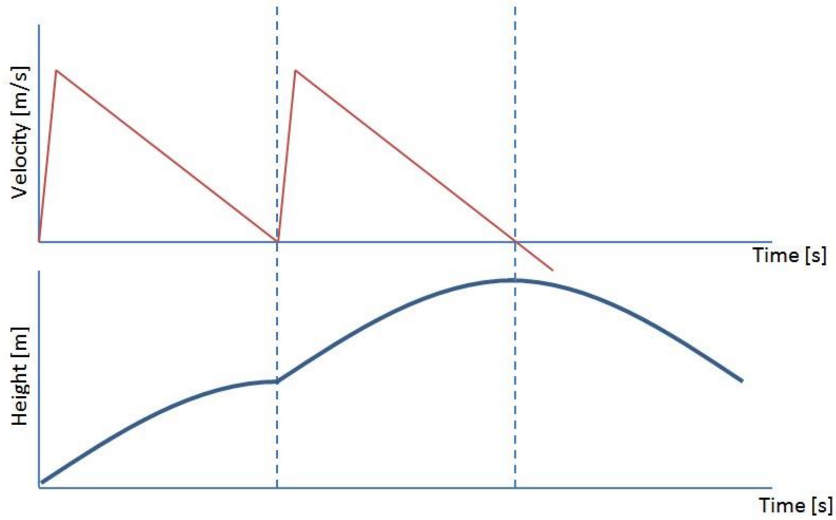


Figure 4.23-Second Station results

As seen above in Figure 4.23, the first coil is activated and the velocity jumps up and then linearly decreases. When the ring is at its peak and has a velocity of 0 m/s, this is when the second coil is activated, essentially doubling the height traveled by the ring with little to no loses in the system.

Conclusion:

In our modern world we are seeing a lot of changes from the conventional ways to using creative new ways to help better improve our lives with higher efficiency. Our major qualifying project represents the possibilities of this technology to be applied in our daily lives, as in the case of faster and more efficient transportation system through the Hyper-loop. All though we are not condoners of wars or weaponized applications we were still able to recognize the potential of using electricity, magnetic flex and electromagnetic fields to project conducting projectiles at speeds never thought possible without the use of conventional chemical reactions. We also managed to successfully remove the need for any human interaction in the launcher that may otherwise cause significant safety risks in a variety of applications. We also managed to understand some of the parameters that affect a launch and its efficiency, such as the size and shape of the projectile, the applied voltage and subsequent current and electromagnetic force. In the end we believe that our understanding of electromagnetics and applied forces has improved extensively. We are able to identify real world problems concerning the subject and use the right skills and tools to find the desired results.

Appendix A

The induction of the coil was found using the circuit in Figure X.5. Through this circuit, the current resonates at 105 kHz and that corresponds to an induction value of 230μH. This was proved in simulation by running AC analysis on the above circuit. Below in Figure X.A2 is the simulation code:

```
Frequency and Inductatance
**************************
V   1  0  ac  1
L   1  2  230u    IC=0
R   2  3  100
C   3  0  0.01u   IC=0
.PROBE
.AC DEC 2000 50k 200k
.END
```

Figure A1-AC Analysis code

The code in Figure A1 has an AC source with amplitude of 1V. The coil has the inductance value of 230μH, the resistance of 100 Ω and a capacitor of 0.01μF. Running this simulation yields the following graph:

Figure A3-Current vs Frequency from AC analysis

Figure A3 shows the current through the circuit at the corresponding frequency values. The current peaks at 10 mA and the corresponding frequency is 105 kHz. Figure X.A4 below is a table that shows these values clearer:

Trace Name	V1
X Values	105.076K
I(R)	10.000m

Figure A4-Table from AC analysis

Appendix B - Calculations of Lyle's Method

a	A	a/A	k'^2	x		d/A	$(d/A)^2$	$(1-a/A)^2$	$(1+a/A)^2$		f		M
2.23	3.85	0.579221	0.070994	0		0	0	0.177055	2.493938		0.010179		0.029826
2.23	3.85	0.579221	0.071245	0.1		0.025974	0.000675	0.177055	2.493938		0.010179		0.029826
2.23	3.85	0.579221	0.071998	0.2		0.051948	0.002699	0.177055	2.493938		0.010179		0.029826
2.23	3.85	0.579221	0.073251	0.3		0.077922	0.006072	0.177055	2.493938		0.010179		0.029826
2.23	3.85	0.579221	0.074998	0.4		0.103896	0.010794	0.177055	2.493938		0.010179		0.029826
2.23	3.85	0.579221	0.077235	0.5		0.12987	0.016866	0.177055	2.493938		0.010179		0.029826
2.23	3.85	0.579221	0.079954	0.6		0.155844	0.024287	0.177055	2.493938		0.009464		0.02773
2.23	3.85	0.579221	0.083147	0.7		0.181818	0.033058	0.177055	2.493938		0.009464		0.02773
2.23	3.85	0.579221	0.086804	0.8		0.207792	0.043178	0.177055	2.493938		0.009464		0.02773
2.23	3.85	0.579221	0.090914	0.9		0.233766	0.054647	0.177055	2.493938		0.008843		0.025911
2.23	3.85	0.579221	0.095463	1		0.25974	0.067465	0.177055	2.493938		0.008843		0.025911
2.23	3.85	0.579221	0.100439	1.1		0.285714	0.081633	0.177055	2.493938		0.008297		0.024311
2.23	3.85	0.579221	0.105826	1.2		0.311688	0.09715	0.177055	2.493938		0.008297		0.024311
2.23	3.85	0.579221	0.111609	1.3		0.337662	0.114016	0.177055	2.493938		0.00781		0.022884
2.23	3.85	0.579221	0.117771	1.4		0.363636	0.132231	0.177055	2.493938		0.00781		0.022884
2.23	3.85	0.579221	0.124295	1.5		0.38961	0.151796	0.177055	2.493938		0.007371		0.021598
2.23	3.85	0.579221	0.131163	1.6		0.415584	0.17271	0.177055	2.493938		0.006974		0.020435
2.23	3.85	0.579221	0.138357	1.7		0.441558	0.194974	0.177055	2.493938		0.006974		0.020435
2.23	3.85	0.579221	0.145857	1.8		0.467532	0.218587	0.177055	2.493938		0.006611		0.019371
2.23	3.85	0.579221	0.153646	1.9		0.493506	0.243549	0.177055	2.493938		0.006278		0.018395
2.23	3.85	0.579221	0.161703	2		0.519481	0.26986	0.177055	2.493938		0.00597		0.017493
2.23	3.85	0.579221	0.17001	2.1		0.545455	0.297521	0.177055	2.493938		0.005685		0.016658
2.23	3.85	0.579221	0.178547	2.2		0.571429	0.326531	0.177055	2.493938		0.005685		0.016658
2.23	3.85	0.579221	0.187295	2.3		0.597403	0.35689	0.177055	2.493938		0.00542		0.015881
2.23	3.85	0.579221	0.196235	2.4		0.623377	0.388598	0.177055	2.493938		0.005173		0.015157
2.23	3.85	0.579221	0.205348	2.5		0.649351	0.421656	0.177055	2.493938		0.004941		0.014478
2.23	3.85	0.579221	0.214616	2.6		0.675325	0.456063	0.177055	2.493938		0.004723		0.013839
2.23	3.85	0.579221	0.224022	2.7		0.701299	0.49182	0.177055	2.493938		0.004518		0.013238
2.23	3.85	0.579221	0.233547	2.8		0.727273	0.528926	0.177055	2.493938		0.004325		0.012673
2.23	3.85	0.579221	0.243175	2.9		0.753247	0.567381	0.177055	2.493938		0.004142		0.012136
2.23	3.85	0.579221	0.252889	3		0.779221	0.607185	0.177055	2.493938		0.003969		0.01163
2.23	3.85	0.579221	0.262674	3.1		0.805195	0.648339	0.177055	2.493938		0.003805		0.011149
2.23	3.85	0.579221	0.272514	3.2		0.831169	0.690842	0.177055	2.493938		0.003649		0.010692
2.23	3.85	0.579221	0.282395	3.3		0.857143	0.734694	0.177055	2.493938		0.0035		0.010255
2.23	3.85	0.579221	0.292303	3.4		0.883117	0.779895	0.177055	2.493938		0.003359		0.009842
2.23	3.85	0.579221	0.302224	3.5		0.909091	0.826446	0.177055	2.493938		0.003224		0.009447
2.23	3.85	0.579221	0.312147	3.6		0.935065	0.874346	0.177055	2.493938		0.003095		0.009069

References

1. Grover, Frederick W. (1946). *Inductance Calculations: Working Formulas and Tables*. New York: Van Nostrand-Reinhold