UNIT – IV

ELECTROMAGNETIC THEORY

Electromagnetic theory is a discipline concerned with the study of charges at rest and in motion. Electromagnetic principles are fundamental to the study of electrical engineering and physics. Electromagnetic theory is also indispensable to the understanding, analysis and design of various electrical, electromechanical and electronic systems. Some of the branches of study where electromagnetic principles find application are: RF communication, Microwave Engineering, Antennas, Electrical Machines, Satellite Communication. Atomic and nuclear research, Radar Technology, Remote sensing, EMI EMC, Quantum Electronics, VLSI, Electromagnetic theory is a prerequisite for a wide spectrum of studies in the field of Electrical Sciences and Physics.

4.1. TERMS AND DEFINITIONS:

Electric Field:

The space around an electric charge in which the effect of the charge is felt is termed as electric field or electrostatic field. A static electric field may be due to a positive charge or a negative

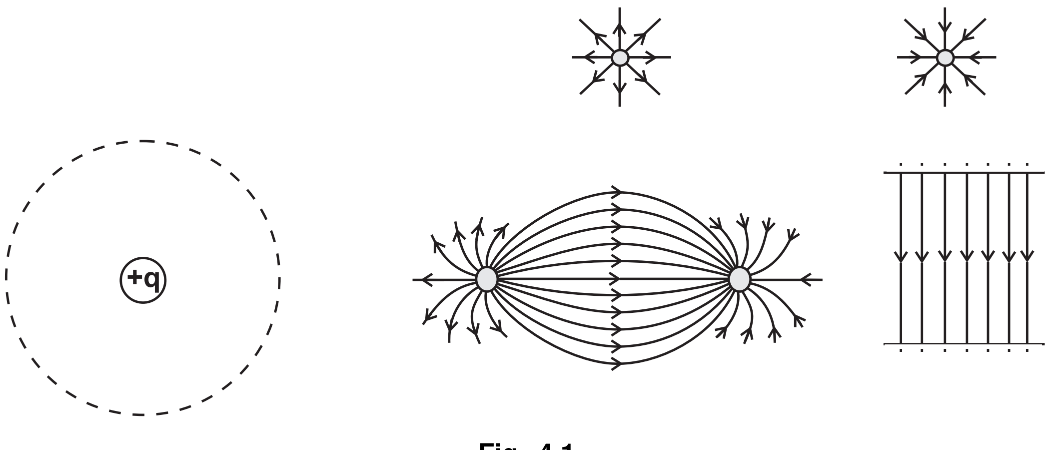


Fig. 4.1

Point Charge:

A point charge means that electric charge which is separated on a surface or space whose geometrical dimensions are very very small compared to other dimensions, in which the effect of electric field to be studied.

One Coulomb:

One coulomb of charge is defined as the charge possessed by $(1/1.602 \times 10^{-9})$ i.e. 6×1018 number of electrons.

4.2. COULOMB'S LAW:

Coulomb's law: The force between two electrostatic charges q_1 and q_2 is proportional to the product $q_1 q_2$ and is inversely proportional to the square of the distance 'r' separating the charges. The force acts along the line joining the two charges. Mathematically we can write it in different forms:

(i) $\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_1^2} \hat{r}_1$

Here $\hat{r}_1$ is the unit vector pointing in the direction from q_1 toward q_2

(ii) $\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^3} \vec{r}_{12}$

Here $\vec{r}_{12}$ is the radius vector connecting charges q_1 and q_2:

$\left| \vec{r}_{12} \right| = r$

(iii) Normally we are following the scalar form.

i.e. $F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$ newton

Remarks: (1) This equation applies to a pair of point charges situated in a vacuum; This equation is true when the dimensions of the charges are negligible compared to 'r'.

(2) The quantity '4π' is introduced to simplify the equations or problems in electrostatics. The constant 'ε_0' is called the permittivity of free space.

$$\varepsilon_0 = 8.85 \times 10^{-12} \text{ farad/metre}$$

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{\text{newton metre}^2}{\text{coulomb}^2} = \text{metre/farad.}$$

It is also connected with velocity of light in free space i.e. velocity of light in free space ‘C’ $= \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8$ m/s

where μ_0 = permeability of free space $= 4\pi \times 10^{-7}$ henry/m

(3) In a dielectric medium, $F = \frac{q_1 q_2}{4\pi \varepsilon_0 \varepsilon_r r^2}$

where ε_r is the relative permittivity of the medium (or) dielectric constant of the medium.

(4) The force is repulsive if q_1 and q_2 are of the same sign and is attractive if they are of different sign.

Coulomb is the unit of charge and is also defined as the amount of charge that flows through a conductor in one second when a current of one ampere flows through it.

4.3. ALTERNATING SYSTEMS:

Voltage & current generators find wide application in instrumentation and communication systems.

Alternating current and voltage generators are called alternating systems.

Generation of alternating emf:

When a closed coil rotates with a constant angular velocity in a magnetic field, the induced emf in the coil is sinusoidal. When such an emf is fed to a circuit, it can be shown that the resulting current is also sinusoidal.

A current which flows first in one direction in a circuit, called the positive direction, then in the reverse or negative direction as shown in **Fig. 4.2 (a, b)** is called alternating current. This cycle is repeated continuously and has an average value of zero over a period. Since the variation of the current strengths is sine wave. The value of current at any instant is given by

$$I = I_{max} \sin wt.$$

Where I is the value of current at time t and I_{max} is the maximum value of the current and w is the angular velocity ($= 2\pi f$, f being frequency)

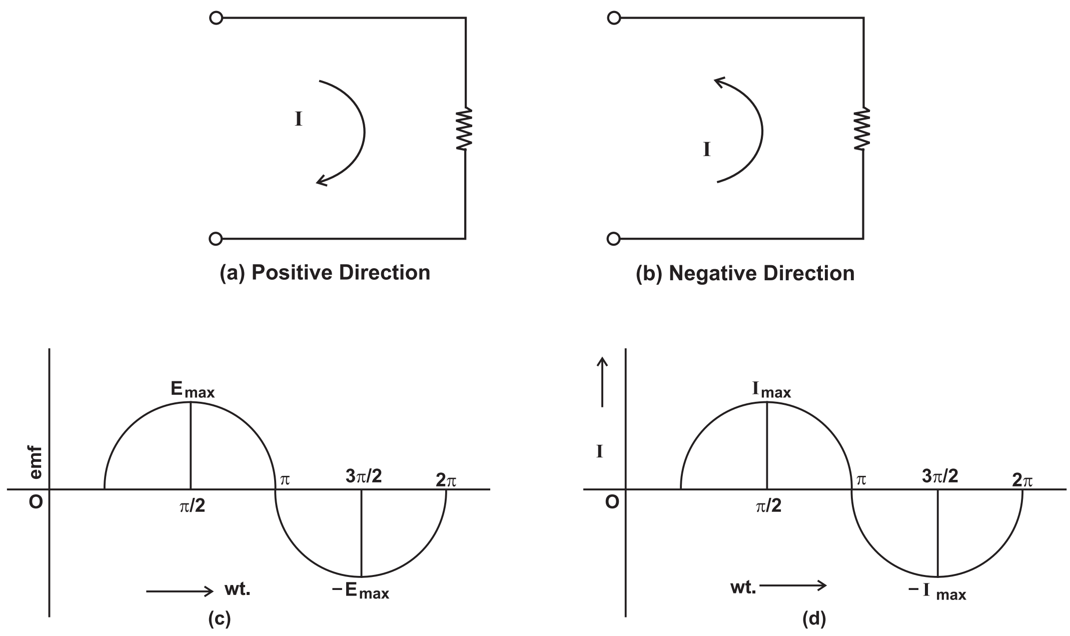


Fig. 4.2

Both the emf and current undergo a complete cycle of changes, having positive and negative values, every time wt. changes by 360° or 2 π radians (**Fig. 4.2(c), (d)**).

Wave forms: The shape of the curve obtained by plotting the instantaneous values of voltage or currents ordinate against time as x-axis gives the wave form of an alternating quantity and is called wave form or wave shapes.

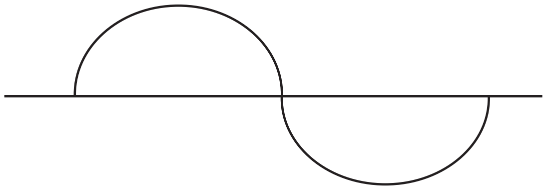


Fig. 4.3: (a) Sine wave

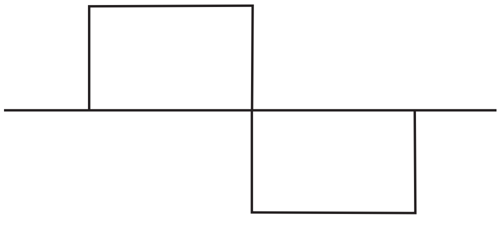


Fig. 4.3: (b) Square wave

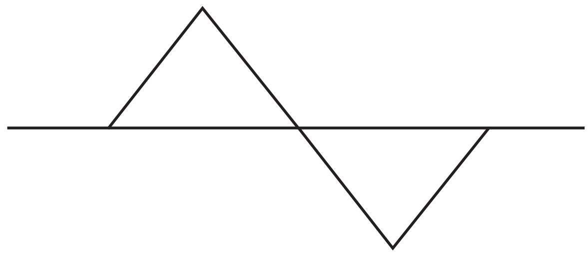


Fig. 4.3: (c) Triangular wave

Terms and Definitions:

Amplitude: The maximum value of the current or voltage, whether positive or negative is called is amplitude.

Cycle: One complete set of positive and negative values of an alternating quantity is known as a cycle.

Time period: The time period of an alternating quantity is the time required to complete one cycle.

Frequency: The number of cycles per second of an alternating quantity is called frequency. Hence the frequency of an alternating quantity is the reciprocal of the time period and is expressed in Hertz (Hz) or cycles / second.

Phase: Phase of an alternating quantity at any instant represents the fraction of the time period, of that alternating quantity. In other words phase is a direction of any alternating quantity.

Phase difference: It refers lag or lead of an alternating quantity with respect to other.

4.4. RMS VALUE OF AN ALTERNATING QUANTITY:

The r.m.s. value of an alternating current is the value of the direct current which produces same amount of heat in the same time in the same conductor. An alternating current is not steady but varies from instant to instant.

If I_1, I_2, I_3, ... In are instantaneous currents r.m.s value will be

$$I_{rms} = \frac{I_1^2 + I_2^2 + \ldots I_n^2}{n}$$

We can also find expression for I_{rms} mathematically as follows

$$\text{Average value of } I^2 = \frac{1}{\pi} \int_o^{\pi} I^2 \, d\theta$$

Substituting the value of $I = I_{max} \sin wt$.

$$\left(I_{rms}\right)^2 = \frac{1}{\pi} \int_o^{\pi} \left(I_{max} \sin wt\right)^2 d\theta$$

$$I_{rms}^2 = \frac{I_{max}^2}{\pi} \int_0^{\pi} \sin^2 wt \, d\theta \quad \text{Since } wt = \theta$$

$$= \frac{I_{max}^2}{\pi} \int_o^{\pi} \sin^2 \theta \, d\theta$$

$$= \frac{I_{max}^2}{2} \int_o^{\pi} \left(\frac{1 - \cos 2\theta}{2}\right) d\theta$$

(as $\cos 2\theta = 1 - 2\sin^2\theta$)

$$= \frac{I_{max}^2}{2\pi} \int_o^{\pi} d\theta + \cos 2\theta \, d\theta$$

$$= \frac{I_{max}^2}{2\pi} [(\pi - 0) - (0 - 0)]$$

$$= \frac{I_{max}^2}{2\pi} \times \pi$$

$$= \frac{I_{max}^2}{2}$$

$$I_{rms} = \sqrt{\frac{I_{max}^2}{2}} = \frac{I_{max}}{\sqrt{2}} = 0.1707 \, I_{max}$$

Similarly for voltage, we have $V_{rms} = 0.707 \, V_{max}$.

Mean or Average Value of an alternating quantity:

The mean value or average value (I_{av}) of an alternating current is equal to its steady current which transfers across any circuit the same charge as it transferred by the alternating current during the same time.

In case of sinusoidal or alternating current, the average value over complete cycle is zero. Hence in these cases the average value is obtained by integrating the instantaneous values of alternating quantity over half cycle only.

However in case of unsymmetrical alternating current i.e., whose two half cycles are not exactly similar, the average value is taken over the whole cycle. The alternating current is given by the following relation.

$$I = I_{max} \sin wt$$

$$\therefore \; I_{av} = \frac{1}{\pi} \int_{o}^{\pi} I \, d\,\theta$$

$$I_{av} = \frac{1}{\pi} \int_{o}^{\pi} I_{max} \sin d\,\theta$$

$$= \frac{I_{max}}{\pi} \left[-\cos\theta \right]_{o}^{\pi}$$

$$= \frac{I_{max}}{\pi} [+1+1]$$

$$= \frac{I_{max}}{\pi} 2$$

$$= \frac{2\,I_{max}}{\pi} = \frac{\text{Twice the maximum circuit}}{\pi}$$

$$I_{av} = 0.637\, I_{max}$$

Average value of current $= 0.637 \times$ maximum value.

4.5. RESISTORS:

The resistor is a passive electrical component to create resistance in the flow of electric current. In almost all electrical networks and electronic circuits they can be found. The resistance is measured in ohms.

Fig. 4.4

In the preparation of any resistor to be used in an electrical circuit, the following major considerations are kept in view:

1. The resistance should not change with time and should have the same value for a wide range of applied voltages.

2. The material of the resistor should have a low temperature coefficient of resistance so that its variation in resistance.

4.6. PRINCIPLE OF CONDENSER (OR) CAPACITOR:

A condenser consists of two conductors, one charged and the other usually earth connected. The principle of a condenser is to increase the capacity of a conductor.

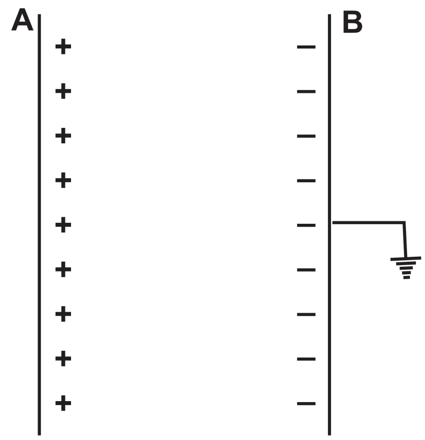


Fig. 4.5

Let A be the charged conductor and B the earth connected conductor. In the absence of B, let the charge on A be +Q coulomb and the potential be 'V' volt. So the capacity of the conductor 'A' = Q/V. If B is kept near A, electrostatic induction takes place. So B will have – Q coulomb on the inner side and +Q coulomb on the outer side. But due to earthing, all the outer side positive charges will go to earth. Consequently the potential of A decreases and its capacity increases. This is due to the presence of B, the amount of work done in bringing an unit positive charge from infinity to A decreases as there will be a force of repulsion due to A and force of attraction due to B. So the resultant force of repulsion on an unit positive charge is reduced, the amount of work done is less and the potential of A decreases. Therefore the capacity of A is increased.

A capacitor (or) condenser has a capacity of one farad if one coulomb of charge is transferred from one conductor to the other when the difference of potential between the two conductors is one volt.

4.7. INDUCTORS:

For a circuit carrying current I produces a magnetic field 'B' which causes a flux,

$$\psi \int \overline{B}\, dS$$

If the circuit has 'N' identical turns, we define the flux linkage λ as

$$\lambda = N^{\psi} .$$

If the medium surrounding the circuit is linear $\lambda \, \alpha \, I$

$$\text{i.e., } \lambda = LI$$

L = Inductance of the circuit.

L is the physical property of the circuit. **A circuit or part of a circuit that has inductance is called an inductor. Thus inductance 'L' of an inductor is defined as the ratio of the magnetic flux linkage λ to the current 'I' through the inductor.**

i.e., $$L = \frac{\lambda}{I} = \frac{N\psi}{I}.$$

Unit of inductance is Henry (H) which is same as Wb/A. Since the linkages are produced by the inductor itself, it is knows as self inductance.

Inductance is a measure of how much magnetic energy is stored in an inductor. The magnetic energy (in J) stored in an inductor is expressed in circuit theory as,

$$\omega_m = \frac{1}{2} LI^2$$

$$\text{or} \quad L = \frac{2\,\omega_m}{I^2}.$$

4.8. EXPRESSION FOR THE ENERGY REQUIRED TO CHARGE A CONDUCTOR WITH ELECTRIC CHARGE Q AT A POTENTIAL 'V' AND ENERGY STORED IN IT:

During the charging process, let the charge on the conductor at any instant be q and its potential 'v'.

As the potential is v, the workdone in bringing an unit positive charge from infinity to the conductor against the electric force is v joule.

Therefore the workdone in bringing a charge $dq = v\,dq = \frac{q}{C}\,dq$ joule

where C is the capacity of the conductor.

Hence the total workdone in bringing a charge 'Q' to the conductor

$$= \int_0^Q \frac{q}{C}\,dq = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

This work represents the energy required to charge the conductor and is stored in the conductor as its potential energy of charge. We may, however, also say that this energy resides in the electric field created by the assembled charges.

For our present calculation, let us take a spherical conductor whose radius is 'R'.

The potential on the surface of the conductor $= \frac{Q}{4 \pi \varepsilon_0 R}$

where Q is the charge given to the conductor.

But the wordone in charging the conductor $= W = \frac{1}{2} QV = \frac{Q^2}{8 \pi \varepsilon_0 R}$

This can be written as

$$W = \frac{\varepsilon_0}{2} \int_R^\infty \left(\frac{Q}{4 \pi \varepsilon_0 r^2} \right)^2 4 \pi r^2 \, dr$$

Here $\frac{Q}{4 \pi \varepsilon_0 r^2} =$ Electric field intensity due to this conductor at any point 'r'.

Therefore $W = \frac{\varepsilon_0}{2} \int E^2 \, d\tau$

where $d\tau$ is the volume element, equal to $4 \pi r^2 \, dr$.

Energy density or energy stored in the electric field

$$= \frac{dW}{d\tau} = \frac{1}{2} \varepsilon_0 E^2 \text{ joule/m}^3$$

4.9. RESISTANCE, CAPACITANCE AND INDUCTANCE IN SERIES A.C. CIRCUIT.

Consider a circuit containing inductance L, capacitance C and resistance R in series as shown in **fig. 4.5.**

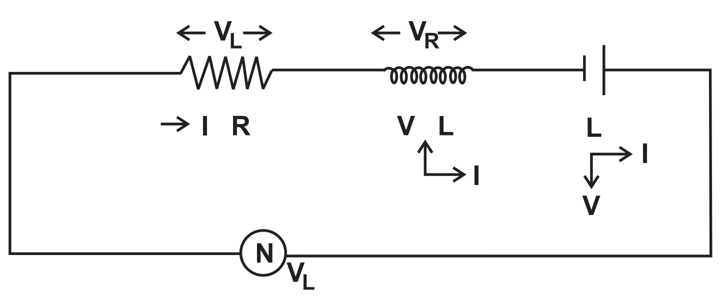


Fig. 4.5(a)

When an alternate emf $V_i = V_{max} \sin wt$ is applied to the circuit an alternate current flows in the circuit. Let I be the current at any instant.

Let voltage drop across resistance R be V_R in phase with current, voltage drop across inductance L be V_L leading the current by 90° and voltage drop across capacitance C be V_C lagging the current by 90°. The resultant of V_L and V_C, $(V_L - V_C)$ leads the current by 90° provided $(V_L > V_C)$ and will lag current by 90° if $r_C > r_L$.

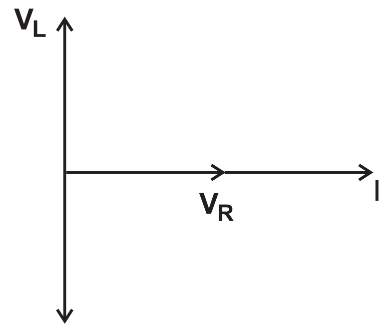


Fig. 4.5(b)

The applied potential difference will be given by

$$V^2 = V_R^2 + \left(V_L + V_C \right)^2$$

$$(IZ)^2 = (IR)^2 + I^2 (L\,\omega - I/c\,\omega)^2$$

$$Z = \sqrt{R^2 + (wL - I/c\,\omega)^2}$$

Where Z is the impedance of the circuit

$$\text{Now } I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (\omega L - I/c\,\omega)^2}}$$

the current I lags behind V by an angle ϕ such that

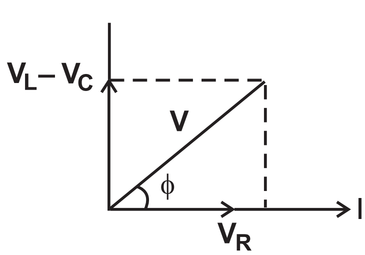


Fig. 4.5(c)

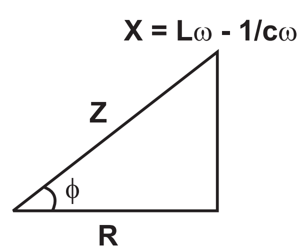


Fig. 4.5(d)

Three cases are possible

(i) When ωL is greater than $I/c\,\omega$. In this case the net reactance $X = L\,\omega - I/c\,\omega$ is positive quantity hence will also be positive and as a result current will lag behind voltage or voltage will lead current.

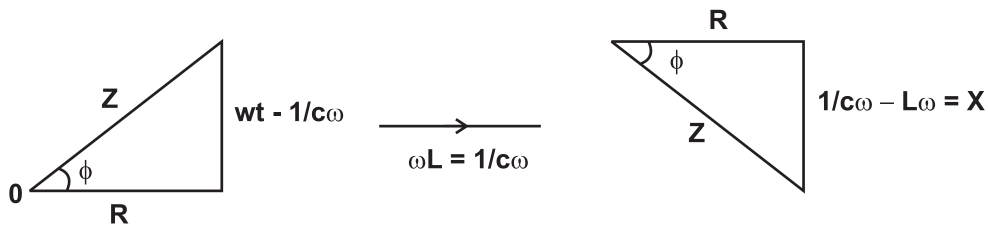


Fig. 4.5(e) Fig. 4.5(f)

(ii) When $L\,\omega = I/c\,\omega$, net reactance $x = 1/c\,\omega - L\,\omega = 0$

i.e., $Z = R$, thus $Z/R = 1$, $\cos\phi = 1, \phi = 0°$

Hence in this case the current and voltage are in phase.

(iii) When $L\,\omega$ is less than $1/c\,\omega$. In this case the net reactance will be a negative quantity and is also negative. The current leads the applied emf as shown in figure.

Series Resonance Circuit:

In a circuit containing R, L and C connected in series the impedance is given by

$$Z = \sqrt{R^2 + (L\,\omega - 1/c\,\omega)^2}$$

The effective reactance in inductive or capacitive depending upon $X_L > X_C$ of $X_L < X_C$. The inductive reactance X_L is directly proportional to the frequency and increases as the frequency increases from zero onward. The capacitive reactance X_C is inversely proportional to the frequency, decrease from an infinite value downwards. At certain frequency both reactants becomes equal and this frequency is called resonant frequency.

therefore $L\,\omega = 1/c\,\omega$

$$\omega^2 = 1/L_C$$

$$2\,\pi\,f = \sqrt{1/L_C} \quad \text{as} \ \ \omega = 2\,\pi\,f$$

$$f = \frac{1}{2\,\pi}\sqrt{\frac{1}{LC}}$$

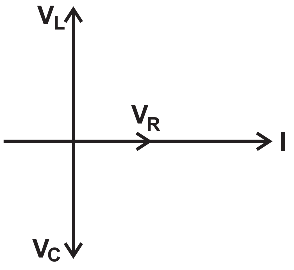


Fig. 4.5(g)

when $X_C = X_L$, $R = Z$, $I = \frac{V}{R}$

since $\cos\phi = \frac{R}{Z} = 1$

$$\phi = 0°$$

$\phi = 0°$ shows that current and voltage are in phase, such a circuit is also called acceptor circuit.

In an a.c. circuit, the ratio of V_L or V_C with applied voltage at resonant frequency is called voltage magnification and denoted by Q factor,

i.e., $Q = \frac{V_L}{V} = \frac{IWL}{IR} = \frac{LW}{R}$

$$Q = 2\,\pi\,f\,\frac{L}{R}$$

as $f = \frac{1}{2\,\pi}\sqrt{\frac{1}{LC}}$

$$Q = 2\,\pi\,\frac{1}{2\,\pi}\sqrt{\frac{1}{LC}}\;\frac{L}{R}$$

$$Q = \frac{1}{R}\sqrt{\frac{1}{C}}$$

Parallel Resonance Circuit:

Here resistance of the inductance coil is negligible.

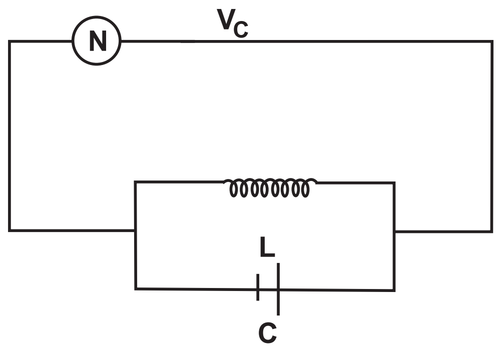


Fig. 4.6

If the supply voltage V and current is I

then $I = I_C + I_L$

$$= \frac{E_{max}}{L\omega} \sin(\omega t - 90) + E_{max}\, c\, \omega \sin(\omega t + 90°)$$

Since, the current in the inductance L will lag in phase by 90° to the applied voltage. The current in the capacitor C will lead in phase by 90° the applied voltage.

$$I = \frac{E_{max}(c\,\omega - 1)\cos\omega t}{L\,\omega}$$

where $I = c\,\omega$, the current $I = 0$

i.e., when $X_C = X_L$ Hence

$$C\,\omega = \frac{1}{L\,\omega}$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \sqrt{\frac{1}{LC}} = 2\,\pi\,f$$

$$f = \frac{1}{2\,\pi}\sqrt{\frac{1}{LC}} \text{ Hertz}$$

For this frequency i.e., at resonant frequency this circuit does not allow the current to flow and works as a perfect choke for a.c. Such circuits are called rejector.

Resonance:

Resonance in AC circuits implies a special frequency determined by the values of the resistance, capacitance, and inductance.

The resonance of a series RLC circuit occurs when the inductive and capacitive reactances are equal in magnitude but cancel each other because they are 180 degrees apart in phase.

4.10. MAGNETISM DEFINITION:

Introduction

Magnetism arises from the magnetic moment or magnetic dipole of the magnetic materials. When the electron revolves around the positive nucleus, orbital magnetic moment arises. Similarly when the electron spins, spin magnetic moment arises. Magnetic materials are all media capable of being magnetized in a magnetic field. i.e. they are capable of creating self magnetic field in the presence of external magnetic field. There are nearly eleven types of magnetic materials. Some of them are diamagnetic, paramagnetic, ferromagnetic, antiferromagnetic and ferromagnetic etc. First let us see the important terms involved in the magnetism.

(i) **Magnetic induction (B)** in any material is the number of lines of magnetic force passing perpendicularly through unit area. Unit : weber / m^2 (or) tesla. It is also equal to the magnetic force experienced by an unit north pole placed in that magnetic field.

(ii) **Magnetic dipoles** are substances in which due to internal atomic currents the substance as a whole possesses a magnetic dipole moment. When an electric current 'i' ampere flows round a circular wire of one turn and area 'a' square metre, it is said to have a magnetic dipole moment 'M' = ia ampere m^2. This magnetic dipole moment or simply magnetic moment is a vector quantity. Its direction is normal to the plane of the loop to the right if the current is clockwise **(figure 4.7)**. The magnetic dipole moment of a current is responsible for magnetic field around the wire. In the case of a magnetic dipole or a bar magnet, the magnetic moment associated with that is the product of its pole strength and magnetic length.

$$\text{i.e., } M = 2l\, m \text{ ampere } m^2$$

where 'm' is the poles strength and '2l' is the distance between north pole and south pole of the magnet.

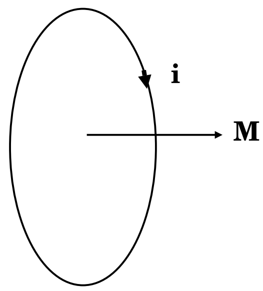


Fig. 4.7: Magnetic moment due to current

(iii) **Intensity of magnetization (I)** of a sample of a material is the magnetic moment per unit volume. Unit : ampere m^{-1}.

(iv) **Magnetic field intensity (H)** is the ratio between the magnetic induction and the permeability of the medium in which the magnetic field exists.

$$\text{i.e., } H = \frac{B}{\mu} \text{ ampere } m^{-1}$$

(v) **Magnetic permeability** (μ) of any material is the ratio of the magnetic induction in the sample to the applied magnetic field intensity.

$$\text{i.e., } \mu = \frac{B}{H} \text{ henry } m^{-1}$$

Thus it measures the amount of produced magnetic induction in the sample per unit magnetic field intensity.

(vi) **Magnetic susceptibility** (ξ) of a material is the ratio between the intensity of magnetisation produced in the sample and the intensity of the applied magnetic field.

$$\text{i.e., } \xi = \frac{1}{H}.$$

It has no unit. Thus the measures the amount of magnetisation produced in the sample during the application of magnetic field.

(vii) **Relation between** μ and ξ

Now $B = \mu H$.

This equation can be written in other way as

$B = \mu_0 (1 + H)$

where μ_o is the permeability of free space.

The relative permeability $\mu_r = \frac{\mu}{\mu_0} = \frac{B/H}{B/(H+I)} = 1 + \frac{I}{H} = 1 + \xi$

where ξ is the susceptibility of the medium. The relative permeability has no unit. The magnetic materials must have high permeability so that large fluxes may be produced.

(viii) Origin of magnetic moment in magnetic materials

The magnetic moment in a material originates from the orbital motion and spinning motion of electrons in an atom.

When a magnetic moment is obtained through the motion of electrons in various orbits of an atom, then it is called **orbital magnetic moment** whose magnitude is always small.

In an atom, generally every two electrons will form a pair such that they have opposite spins. Thus the resultant spin magnetic moment is zero. But in magnetic materials like iron, cobolt, nickel, etc. there are **unpaired electrons** in the 3d orbital. This unpaired electron's spin magnetic moment interacts the adjacent atom's unpaired electron spin magnetic moment to align in a parallel manner resulting enormous spin magnetic moment. Thus these unpaired electron spins are responsible for ferro and paramagnetic behaviour of materials. The value of spin magnetic moment is very large when we compare it with orbital magnetic moment.

4.11. DIFFERENT TYPES OF MAGNETIC MATERIALS:

Diamagnetic materials:

Definition: The number of orientations of electronic orbits in an atom be such that the vector sum of magnetic moments is zero. The external field will cause a rotation action on the individual electronic orbits. This produces an induced magnetic moment which is in the direction opposite to the field and hence tends to decrease the magnetic induction present in the specimen **(figure 4.8(b)).** Thus the diamagnetism is the phenomenon by which the induced magnetic moment is always in the opposite direction of the applied field. The magnetic material having negative susceptibility is called a diamagnetic material. Further for the diamagnetic materials, each atom has no permanent magnetic moment.

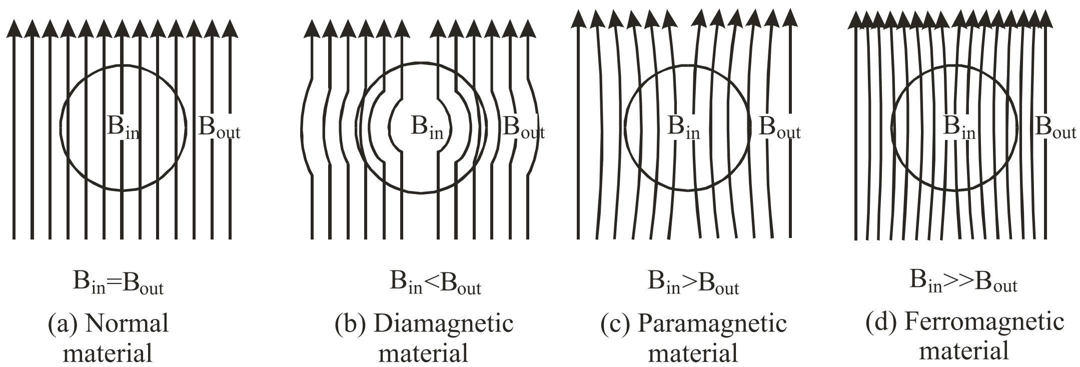


Fig. 4.8: Behaviour of magnetic materials in the presence of magnetic field

Figure 4.8(a) shows the ordinary material which has no repulsion or attraction for the magnetic flux lines when it is placed in an uniform magnetic field. But in **figure 4.8(b)**, there is repulsion of magnetic flux from the centre of the material indicating the diamagnetic behaviour of the magnetic material.

Magnitude of susceptibility	Temperature dependence of susceptibility	Examples
Small, Negative	Independent	Organic materials light elements.
Intermediate, Negative	When the temperature is below 20 K susceptibility varies with external magnetic field and with temperature also.	Alkali earths, Bismuth.
Large, Negative	Diamagnetism exists only below critical temperature 'T_c' and susceptibility varies at this temperature ($T_c < 30$ K).	Superconducting materials like Niobiumand its compounds, copper oxide super-conducting materials, etc.

Special remarks:

1. Diamagnetic materials repel magnetic lines of force.
2. There are no permanent dipoles; consequently magnetic effects are very small.
3. Generally the value of diamagnetic susceptibility is independent of temperature and applied magnetic field strength.

Paramagnetic material

Definition

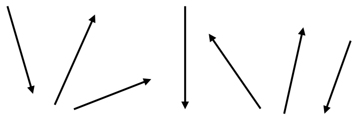
The number of orientations of orbital and spin magnetic moments be such that the vector sum of magnetic moments is not zero and there is a resultant magnetic moment in each atom even in the absence of applied field. If we apply the external magnetic field, there is an enormous magnetic moment along the field direction and the magnetic induction will increase. Thus paramagnetism is the phenomenon by which the orientations of magnetic moments are largely dependent on temperature and applied field. If the applied magnetic energy is greater than the thermal energy, the magnetic moment of the material is finite and large. These materials are used in Lasers and Masers where one can create the required energy levels for transition. Paramagnetic property of oxygen is used in the nuclear magnetic resonance imaging instrument which is used to diagnose the brain tumor or blood clot in the brain.

Magnitude of susceptibility	Temperature dependence of susceptibility	Examples
Small, Positive	Independent	Alkali metals and Transition metals.
Large, Positive	$\chi = \frac{C}{T - \theta}$ where C is equal to curie constant and θ is curie temperature. When $T < \theta$ paramagnetic substance converts into diamagnetic substance	Rare earths like chromium, yttrium, etc.,

Special remarks:

1. Paramagnetic materials attract magnetic lines of force.
2. They possess permanent dipoles.
3. The value of the paramagnetic susceptibility is independent of the applied magnetic field and depends greatly on temperature.

Spin alignment:



Ferromagnetic material

Definition : If a material acquires a relatively high magnetization in a weak field, then it is ferromagnetic. Further even in the absence of applied field, the magnetic moments are enormous. This is due to spontaneous magnetization. Ferromagnetism arises when the exchange energy is favourable for spin alignment.

Magnitude of Susceptibility: very large, positive.

Temperature dependence of susceptibility:

When temperature is greater than curie temperature æ?Æ then

$\xi = \frac{C}{T - \theta}$ (paramagnetic region)

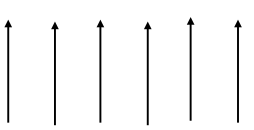
When temperature is less than 'θ', the material is in ferromagnetic state and ξ is very large due to spontaneous magnetization.

Examples: Fe, Ni, Co.

Special remarks:

1. Due to the large internal field, the permanent dipoles are strongly aligned in the same direction and consequently a large spontaneous magnetization results even in the absence of an applied field.
2. They attract the lines of force very strongly.
3. They exhibit magnetization even when the magnetizing field is removed. i.e. they exhibit magnetic hysteresis.
4. During heating they lose their magnetization slowly.

Spin alignment:



Antiferromagnetic materials

Definition: This refers to spin alignment in an antiparallel manner in neighbouring magnetic ions resulting in zero net magnetization.

Magnitude of susceptibility: Small, Positive.

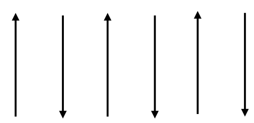
Temperature dependence of susceptibility: When $T > T_N$ (Neel temperature),

$$\chi = \frac{C}{T + \theta}$$

Examples:

FeO (ferrous oxide), MnO (manganese oxide), Cr_2O_3 (chromium oxide) and salts of transition elements.

Spin alignment:



Special remarks:

1. The opposite alignment of adjacent magnetic moments in a solid is produced by an exchange interaction.
2. Initially susceptibility increases slightly as the temperature increases and beyond Neel temperature the susceptibility decreases with the temperature.

Ferrimagnetic materials

Definition: It is a special case of antiferromagnetic in which antiparallel moments are of different magnitudes and a large magnetization arises.

Magnitude of susceptibility: Very large, positive.

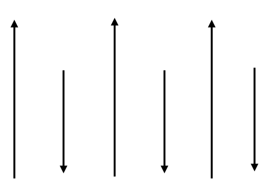
Temperature dependence of susceptibility: At $T > T_N$, $\chi = \frac{C}{T \pm \theta}$

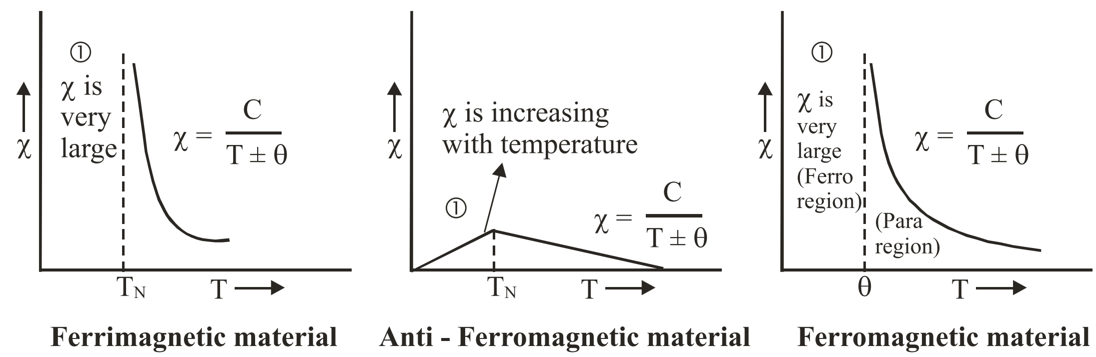
Examples: Ferrous Ferrite and Nickel ferrite

Special remarks:

1. These are composed of two or more sets of different transition metal ions. There are different number of ions in each set. Due to that, unlike antiferromagnetics, there is a net large magnetization.

Spin alignment:





Fig, 4.9: Variation of the susceptibility with temperature in different magnetic materials

4.12. BIOT-SAVART LAW:

In electrostatics we have used the Gauss's law and Coulomb's law to calculate the electric field intensity. If there is high symmetry in the system, using Gauss's law we can easily find $\vec{E}$. But for a dipole we can't apply it where we must use Coulomb's law to calculate $\vec{E}$. So using Coulomb's law, we can compute $\vec{E}$ for any arbitrary charge distribution dividing the distribution into charge elements 'dq' and finding $d\vec{E}$ due to each charge element and finally $\vec{E}$ can be calculated by adding $d\vec{E}$s' or by integration. Similarly here also there are two laws to calculate the magnetic induction '$\vec{B}$' of the system carrying current:

Bio-Savart law is also called Biot-Savart Laplace law). Basically both the laws give the same thing but in different forms. Ampere's law is similar to Gauss's law. That is, it can be used only when there is symmetry in the system. But Biot-Savart law can be used for any arbitrary current element to calculate $\vec{B}$. In that respect it is similar to Coulomb's law in electrostatics.

The magnitude of $d\vec{B}$ produced by a current carrying element of length dl.

(i) directly proportional to the strength of the current 'i' in the element dl and to the length of the element dl.

(ii) inversely proportional to the square of the distance of element from point of observation, and

(iii) directly proportional to the $\sin \theta$ which is the angle between the direction of the current and the line joining the mid point of the element to the point of observation.

Therefore $dB \propto \frac{i\,dl \sin\theta}{r^2}$

$$dB = \frac{\mu_0}{4\pi}\frac{i\,dl \sin\theta}{r^2}$$

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{l} \times \vec{r}}{r^3}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{i\,d\vec{l} \times \vec{r}}{r^3}$$

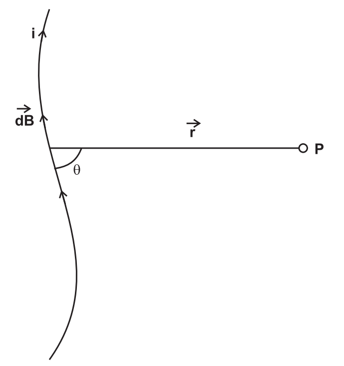


Fig. 4.10: Biot-Savart Law

4.13. ENERGY STORED IN THE MAGNETIC FIELD OF THE COIL:

Suppose that the current through the coil, is rising at a rate di/dt ampere per second, L is its self inductance in henry, the back e.m.f. across it is given by $e = L\frac{di}{dt}$

$$\text{Power} = e\,i = L\,i\frac{di}{dt}$$

The total workdone in bringing the current from zero to a steady value i_0 against this back e.m.f. is given by

$$W = \int p\,dt = \int_0^{i_0} L\,i\frac{di}{dt}\,dt = \frac{1}{2}Li_0^2$$

This is the energy stored in the magnetic field of the coil.

4.14. DOMAIN THEORY OF FERROMAGNETISM:

Weiss proposed this concept of Domains in 1907 to explain the hysteresis effects observed in Ferromagnetic materials as well as to explain the properties of Ferromagnetic materials. A region in a ferro or ferri-magnetic material where all the magnetic moments are aligned in the same direction is called a **domain**. So a ferromagnetic material is divided up into these small regions, called domains, each of which is at all times completely magnetized. The direction of magnetization, however, varies from domain to domain and thus the net macroscopic magnetization is zero in a virgin specimen in the absence of external magnetic field. But when the ferromagnetic material is in the magnetic field, in the initial stages of magnetization in the material, the domains having moments parallel to the magnetic field increases in area; in the final saturation stage the other domains are rotated parallel to the field. Similarly if we demagnetize the material the regular domain arrangement is changed and it is different from the original state. This creates the hysteresis in the ferromagnetic substances.

Further this theory can explain the following

(i) If a magnet is broken into pieces, each piece will be a magnet with a north and a south pole. This is because the domains continue to remain in broken pieces.

(ii) A magnet heated or roughly handled tends to lose its magnetism. This is because the alignment of the domains in the magnet is likely to be distributed during heating and rough handling. Hence magnetism is reduced or lost.

(iii) Domains of soft iron are easily rotated with a comparatively small magnetizing force and hence they are very easily magnetized or demagnetized. In the case of steel a large force is required for rotating the domains which explains the high retentivity of that material.

(iv) A specimen when magnetized suddenly experiences a slight change in its length which is due to rearrangement of domains inside. This is called magnetostriction.

There are two possible ways to align a random domain structure by applying an external magnetic field.

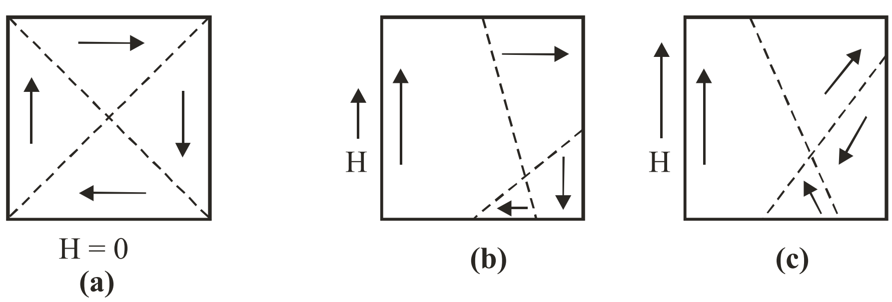


Fig. 4.11: Processes of domain magnetization

(i) **By the motion of domain walls:** i.e., by an increase in the volume of domains that are favourably oriented with respect to the magnetizing field at the cost of those that are unfavourably oriented as shown in **figure 4.11(b)**; **figure 4.11(a)** shows that arrangements of domains for zero resultant magnetic moment in a single crystal or virgin specimen when there is no applied magnetic field.

(ii) **By rotation of domains:** i.e., by the rotation of the direction of magnetization of domain along the direction of field as shown in **figure 4.11(c).** In weak magnetizing fields, the magnetization of the specimen is due to the motion of domain walls and in stronger fields that is due to the rotation of domains.

To study the domain structure clearly, we must know the four types of energy involved in the process of domain growth.

1. Exchange energy

It is the energy associated with the quantum mechanical coupling that aligns the individual atomic dipoles with in a single domain. It arises from interaction of electron spins. It depends upon the interatomic distance.

2. Crystal anisotropy energy

We know that the crystals are anisotropic. Crystal anisotropic energy arises from the difference of energy required for magnetisation along any two different directions in a single crystal. Thus the energy required for magnetisation is a function of crystal orientation. The difference in energy between the hard (111) direction and the easy (100) in BCC iron is about 1.4×10^4 joule/m^3 i.e., the excess work done in magnetizing a specimen to saturation in (111) direction compared with (100) direction is about 1.4×10^4 joule/m^3 and this is the anisotropic energy of iron. In FCC Nickel the reverse is true. For Nickel, (111) direction is the easy direction. i.e., the easy direction of Nickel are body diagonals. This energy is very important in determining the character of domain boundary.

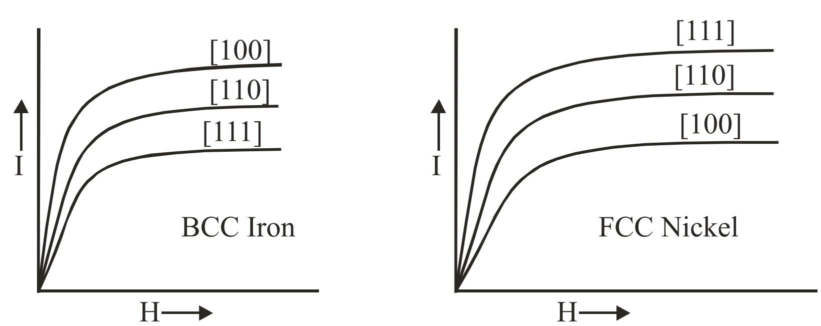


Fig. 4.12: Hard and easy directions for magnetization

Domain wall energy (Bloch wall energy)

It is the sum of contributions from the exchange and crystalline anisotropy energies in the domain wall region. Thickness of the wall is approximately 1000 Å. The boundary between two domains is known as 'Bloch wall'.

3. Magnetostatic energy

When a ferromagnetic substance produces an external field, magnetic potential energy or magnetostatic energy is present in that material. This is due to the presence of resultant dipole moment in that material even in the absence of external magnetic field. This energy is favourable or unfavourable for domain growth if the magnetic moment present in the material is along the direction of magnetisation or not.

4. Magnetostrictive energy

The change in length along the direction of magnetization of a domain solid refers to magnetostriction. Notably we can see this in nickel and ferrites. So if a rod or tube of nickel is brought into a magnetic field parallel to its length, its length changes slightly. The change in length is independent of the sign of field and may be either decrease or an increase depending upon the nature of the material. The change in length of the order of one part in a million. If a nickel rod is brought into an alternating field it is shortened periodically by magnetization. But for permalloy, the length of the rod increases in the presence of magnetic field. The rod thus vibrates with a frequency which is double that of the alternating field. On the other hand if the rod is suitably magnetized before being inserted in the alternating field, then mechanical change in length will be in step with the alternating frequency. In such a case, if the frequency of the alternating field coincides with the natural frequency of the vibration of rod, resonance will occur and the rod will oscillate vigorously. The resonance vibration frequency of the rod depends on its length, the shorter the rod, the higher its resonance frequency. In actual practice it is difficult to make a magnetostrictive vibrator with a very short rod. So in the production of ultrasonic waves, this magnetostriction principle is adopted and using long rods, one can produce ultrasonic waves with low frequency. Further this principle is also followed in magnetostrictive transducers which are used in mechanical filters are used in the single side band transmission to get a very narrow band of frequencies.

Thus the magnetostrictive energy is the energy due to the mechanical stresses generated by magnetostriction in the domains. This energy will affect the growth of domains.

4.15. ELECTROMAGNETIC INDUCTION:

After Ampere and others had investigated the magnetic effects of a current, Faraday attempted to find its converse: he tried to produce a current by means of a magnetic field. His experiments indicate that whenever there is a change in the magnetic induction around a closed coil there is always an induced e.m.f. and hence an induced current in it. This induced current lasts only as long as the magnetic induction changes. Electromagnetic induction is the name given to this process of inducing e.m.f. by changing magnetic flux. If a total number of x lines of induction thread through a coil of y turns-magnetic flux linked with the coil will be xy weber. If the coil has N turns, the total flux linkage ϕ is given by

$$\phi = NAB \cos \theta$$

where B is the magnetic induction or flux density; A is the area of the coil and θ is the angle between the flux density and the normal to the plane of the coil **(figure 4.13).**

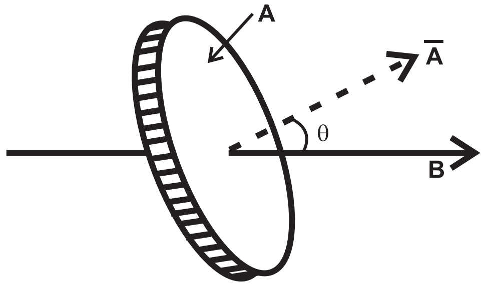


Fig. 4.13: Electromagnetic induction

Faraday's experiments on electromagnetic induction indicate that the induced e.m.f. increases with:

(i) the speed with which we turn the coil keeping the magnet in fixed condition inside the coil.

(ii) the area of the coil.

(iii) the strength of the magnetic field.

iv) the number of turns in the coil.

Results (i) to (iv) above therefore show that the e.m.f. induced in a coil increases with the rate of change of the magnetic flux through it. These results are called Faraday's laws of electromagnetic induction.

4.16. SELF INDUCTION AND MUTUAL INDUCTION:

The laws of electromagnetic induction and the expression for the induced e.m.f. in any closed circuit indicate clearly that whatever be the mechanism by which the number of magnetic lines of force linked with a coil is made to change, the induced e.m.f. or current is always opposing the process of induction.

Self induction:

Let us imagine a solenoid S in which a current is established by a battery circuit as in **figure 4.14**. As current flows it builds up a magnetic field equal to μ_0 ni along the axis where n is the number of turns per metre and i is the current. The magnetic lines of induction begin to thread through the coil and therefore there is an induced e.m.f. in the coil. By Lenz's law this induced e.m.f. must oppose it. The result is that it takes sometime to establish maximum current in the circuit. The growth of current is slow as represented in the graph.

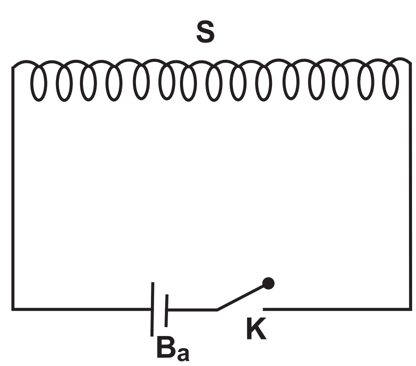


Fig. 4.14: Growth of current

Such a coil S is called an inductor or a self induction coil.

Induced e.m.f. $e = \frac{-d\phi}{dt}$

where ϕ is the flux linked with the circuit at time t. ϕ is proportional to the magnetic induction which is proportional to the current i.

or $\phi = Li$ where L is a constant called the coefficient of self induction of the coil.

$$e = \frac{-d\phi}{dt} = \frac{-d}{dt}(Li) = -L\frac{di}{dt}$$

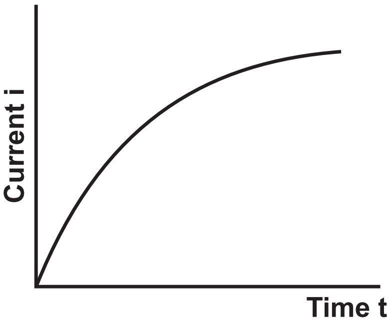


Fig. 4.15: Growth of current

Coefficient of self induction is defined as the flux linked with the coil when unit current flows through or as the e.m.f. induced in the coil when rate of change of current is unity. By the property of inductance of the coil, the change of flux is opposed.

The unit of coefficient of self induction is "henry".

The coefficient of self induction of a long narrow air core solenoid $= \mu_0 An^2 l$ henry where A is its area of section, n is the number of turns per metre and l its length.

A choke is a self inductance coil used to limit alternating currents and to start discharge in the fluorescent tubes lamps.

If the current flowing through the inductor is cut off the magnetic field collapses. It is equivalent to decreasing the number of lines of force. Therefore once again a current is induced, but this time in the opposite direction. The e.m.f. developed will be very high. This is because the rate of change of flux is quicker at break.

Mutual induction:

Let two coils P and S be placed side by side without touching so as to be parallel to each other. Let the first coil P be connected to a battery circuit with a key K and the second coil S be connected to a galvanometer.

When the key is put on, a current (say i) is established in the first or primary coil which builds up a magnetic field. The lines of force begin to thread through the second or secondary coil. Due to the change of flux linked an e.m.f. is induced in the secondary due to which the galvanometer deflects. This e.m.f. is said to be a mutually induced e.m.f. If the key is opered, the galvanometer deflects in the opposite direction. By alternatively closing and opening the key in the primary coil, one can maintain an a.c. voltage or current in the second coil.

If the flux linked with the secondary coil is ϕ due to the current i in the primary coil,

$\phi \propto i$

or $\phi = Mi$ where M is a constant known as the coefficient of mutual induction of the pair of coils P and S.

$$e = \frac{-d\phi}{dt} = -M\left(\frac{di}{dt}\right)$$

Coefficient of mutual induction of a pair of coils is defined as the flux linked with the secondary coil when unit current is established in the primary coil or as the e.m.f. induced in the secondary coil when rate of change of current in the primary coil is unity.

The unit of coefficient of mutual induction is ‘henry’.

The coefficient of mutual induction of a pair of coils depends upon the number of turns in each, the area of the coils, the distance between the coils and the orientation of two coils.

If a self inductance or a mutual inductance coil is wound over an iron core, then the coefficients of induction get increased by very great amount due to the increase in the lines of induction.

[Refer A.C. bridges, for the measurements on self inductance and mutual inductance which are given in the chapter on current electricity.]

Self inductance of a circuit is the property of the circuit by which changing current induces emf in the circuit to oppose the changing current. It is also defined as the ratio of total magnetic flux linkage to the current through the coil.

$$L = \frac{N_1 \phi_1}{I_1} \text{ or } \frac{N_1 \psi_1}{I_1}$$

$$L = \frac{N_2 \phi_2}{I_2} \text{ or } \frac{N_2 \psi_2}{I_2}$$

The mutual inductance between two coils is defined as the ratio of induced, magnetic flux linkage in one coil to the current through in other coil.

Mutual Inductance:

$$M_{12} = \frac{\text{Flux linkage } \lambda_{12} \text{ on circuit L}}{\text{Current } i_2}$$

$$M_{12} = \frac{\lambda_{12}}{I_2} = \frac{N_1 \psi_{12}}{I_2}$$

$$M_{21} = \frac{\lambda_{21}}{I_1} = \frac{N_2 \psi_{21}}{I_1}.$$

For the linear medium surrounding the circuit,

$$M_{12} = M_{21}$$

Examples of Inductors:

Toroids, Solenoids,

Co-axial Transmission Lines,

Parallel Wire Transmission Lines.

Coefficient of coupling is defined as the fraction of the total flux produced by one coil linking a second coil. It is denoted by 'k'

$$K = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2} = \frac{M}{\sqrt{L_1 L_2}}$$

$$K < 1.$$

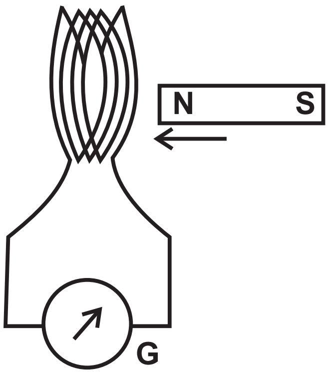


Fig. 4.16: Lenz's Law

4.17. FARADAY'S LAWS:

1. Whenever the number of magnetic lines of force (magnetic flux) threading through a closed circuit changes there is an induced e.m.f. in the circuit.

2. The induced e.m.f. is directly proportional to the time rate of change of flux linked with the circuit: the increase in the flux produces an inverse current

while a decrease in the flux produces a direct current. If e is the induced e.m.f. in a closed circuit in which the flux linked is ϕ.

$$e \propto \frac{-d\phi}{dt} \text{ since } \frac{d\phi}{dt} \text{ is the rate of change of flux.}$$

The actual direction of flow of the induced current is given by Lenz's law.

$$\therefore e = -k\frac{d\phi}{dt}$$

where k is a positive constant and it is shown that $e = -k\frac{d\phi}{dt}$ is consistent with the expression for the force on a conductor $F = Bli$ only if $k = 1$.

$$\therefore e = \frac{-d\phi}{dt}$$

The minus sign expresses Lenz's law.

4.18. LENZ'S LAW:

The direction of the induced e.m.f. is such as to oppose the very process of inducing the current. It means that the induced e.m.f. is in such a direction that if the circuit is closed, the induced current opposes the change of flux. Lenz's law is a natural corollary of the law of conservation of energy. For example, if the north pole of a magnet is introduced in a coil then the direction of the induced current will be anticlockwise as looked from the north pole of the magnet. This current flow, as we know, is equivalent to a north pole being presented to the insertion of the magnet. The resultant repulsion opposes the insertion of the magnet and the mechanical work done against this repulsion in inserting the magnet is converted into electrical energy.

If the magnet is withdrawn the direction of current will be opposite that is clockwise so that a south polarity is presented by the coil attracting the north pole of the magnet. Evidently this opposes the motion of the magnet.

$$\therefore \text{The induced e.m.f. } e = -\frac{d\phi}{dt}$$

If follows that one weber is the flux linking in a circuit, if the induced e.m.f. is one volt when the flux is reduced uniformly to zero in one second.

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