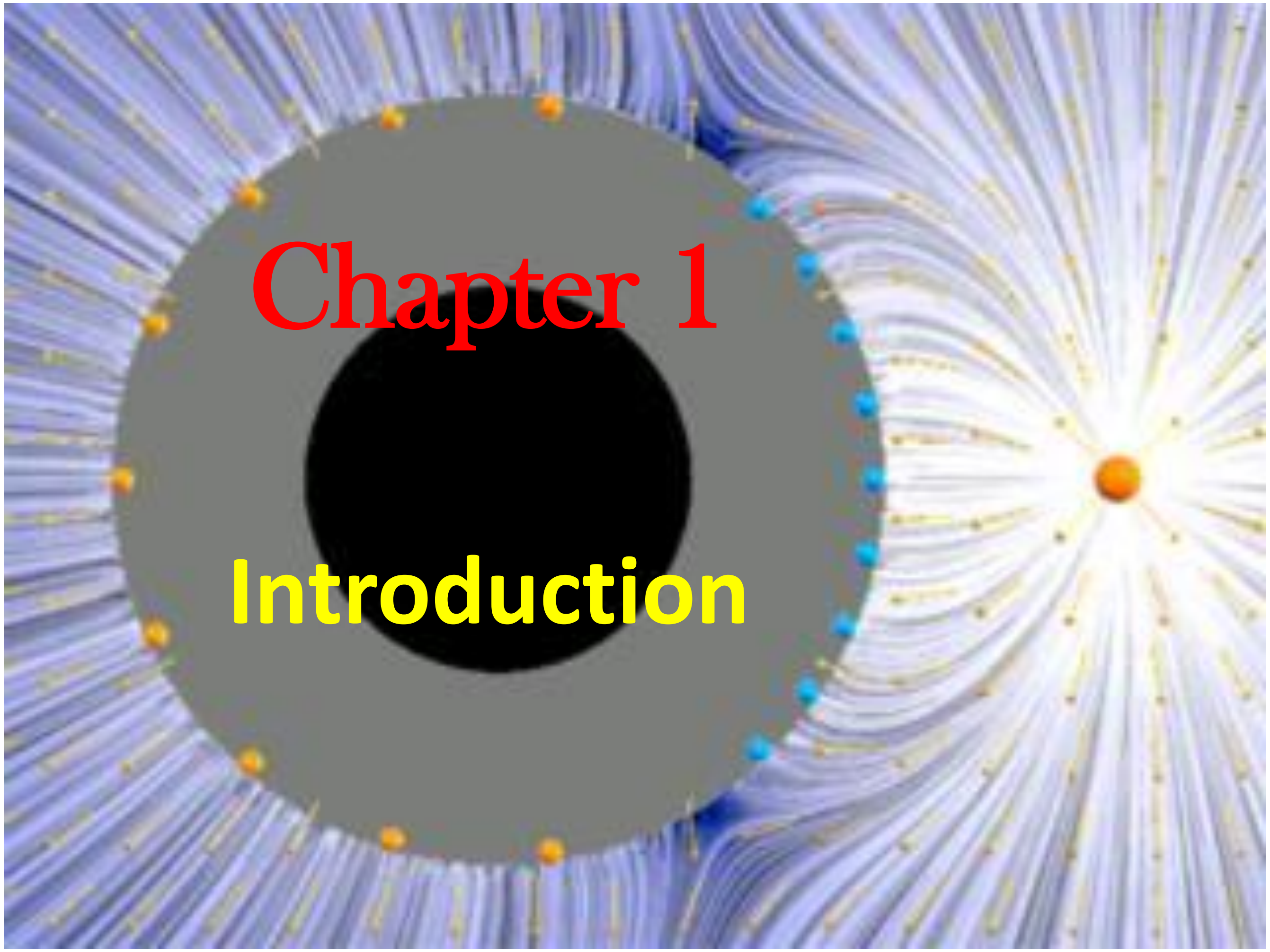
Electromagnetics



Chapter 1

Introduction

1.3 The Nature of Electromagnetism (EM)

There are four fundamental forces in nature :

1- The ***nuclear force*** : is the strongest of the four and limited to ***submicroscopic*** systems(nuclei).

2- The ***electromagnetic force :*** its strength is 10^{-2} of the nuclear force and is found in ***microscopic*** systems, such as atoms and molecules .

3- The ***weak-interaction force*** : its strength is 10^{-14} of the nuclear force and plays a role in interactions involving radioactive particles.

4- The ***gravitational force*** : is the weakest of the four forces (about 10^{-41} of the nuclear force), but it is the dominant force in ***macroscopic*** system ,such as the solar system.

*** our interest in this course is with electromagnetic force which consists of electrical force F_e and magnetic force F_m.

,but

We first look at the gravitational force properties because they provide a useful analogue to those of the electromagnetic force .

The Gravitational Force: A Useful Analogue

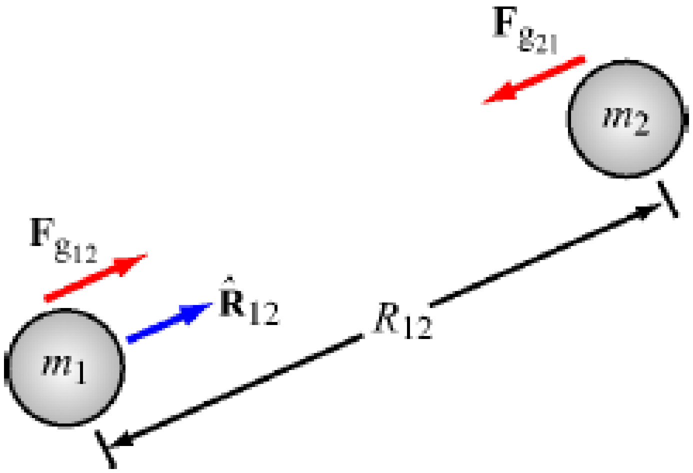


Figure 1-2 Gravitational forces between two masses.

According to Newton's law of gravity, the gravitational force $F_{g_{21}}$ acting on mass m_2 due to mass m_1 at distance R_{12} from m_2 is given by :

$$\vec{F}_{g_{21}} = -\hat{R}_{12}\frac{Gm_1m_2}{(R_{12})^2} \qquad \text{(N)}$$

,where

G : is the universal gravitational constant.

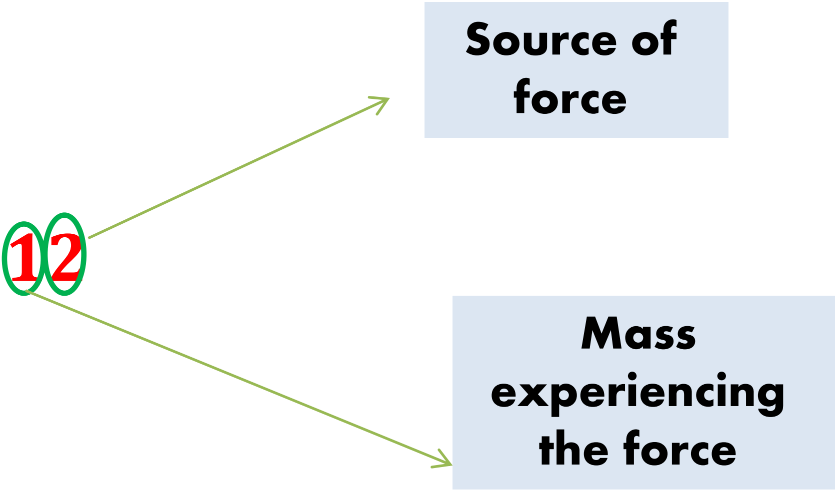
G= $6.67 \times 10^{-11} \quad N.m^2/kg^2$

$\hat{R}_{12}$ **:** is a unit vector that points from m_1 to m_2 .

*** The minus sign indicates that the gravitational force is attractive.

,so

$$\vec{F}_{g_{21}} = - \vec{F}_{g_{12}}$$



Source of force

Mass experiencing the force

,where $\vec{F}_{g_{12}}$ is the force acting on mass m_1 due to gravitational pull of mass m_2 .

- The gravitational force does not require any contact between the interacting particles. This is called ***action-at-a-distance.***

- This phenomenon of direct action at a distance led to a concept called ***fields.***

Figure 1-3 shows a gravitational field ψ_1 induced by a mass m_1 that does not physically emanate(release) from the object, but its influence exists at every point in space.

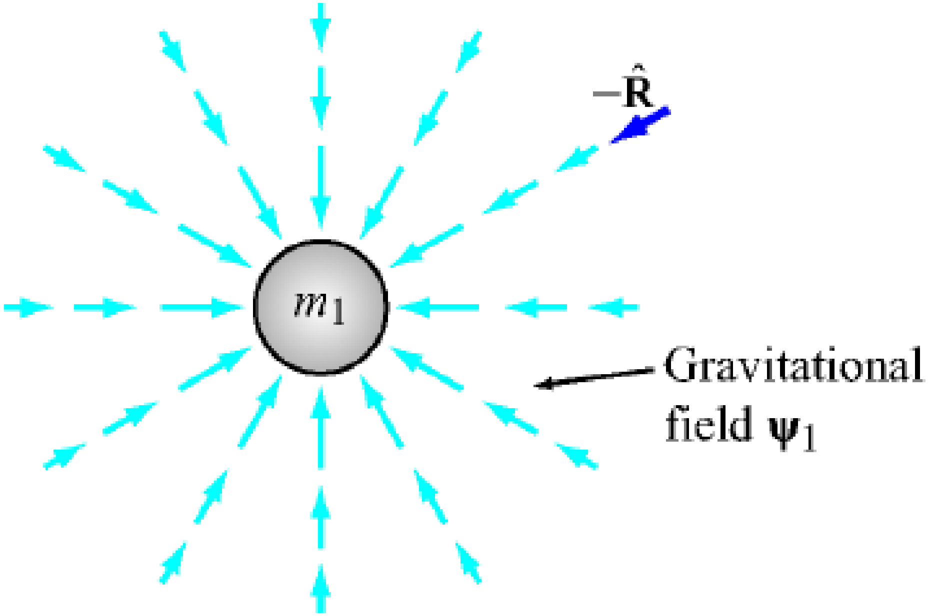


Figure 1-3: Gravitational field $\boldsymbol{\psi}_1$ induced by a mass m_1.

- At a distance R from m_1, the field ψ_1 is a vector defined as :

$$\boldsymbol{\psi_1 = -\hat{R}\frac{Gm_1}{R^2}} \qquad (\boldsymbol{N/kg})$$

,where $\boldsymbol{\hat{R}}$ is a unit vector which points in the radial direction from m_1 and therefore -$\boldsymbol{\hat{R}}$ points toward m_1.

- The force due to $\boldsymbol{\psi_1}$ acting on a mass m_2 at a distance $R = R_{12}$ along the direction $\hat{R} = \hat{R}_{12}$ is:

$$\boldsymbol{\overrightarrow{F}_{g_{21}} = \psi_1 m_2 = -\widehat{R}_{12}\,\frac{Gm_1m_2}{(R_{12})^2}} \quad \textbf{(N)}$$

- At any point in space, the force $\boldsymbol{F_g}$ acting on a mass m placed at that point <u>is related to $\boldsymbol{\psi}$ </u>by:

$$\boldsymbol{\psi = \frac{F_g}{m}}$$

*** The force $\boldsymbol{F_g}$ is due to a single mass or a distribution of many masses.

Electric Fields

Until now, we have learned that mass is the source of the gravitational field ,but what is about the electric field?! ,what is its source?!what is the difference between its source and the gravitational field source?! Is it a big difference?!

- The source of electrical field is electric charge.

- It is as gravitational field varies inversely with the square of the distance from its respective source.
- Electric charge may have positive or negative polarity whereas mass does not exhibit such a property.

- According to atomic physics, all matter contains of **neutrons**, **positively charged protons** and **negatively charged electrons.**

- Fundamental quantity of charge is that of a single electron denoted by the letter $\boldsymbol{e}$.

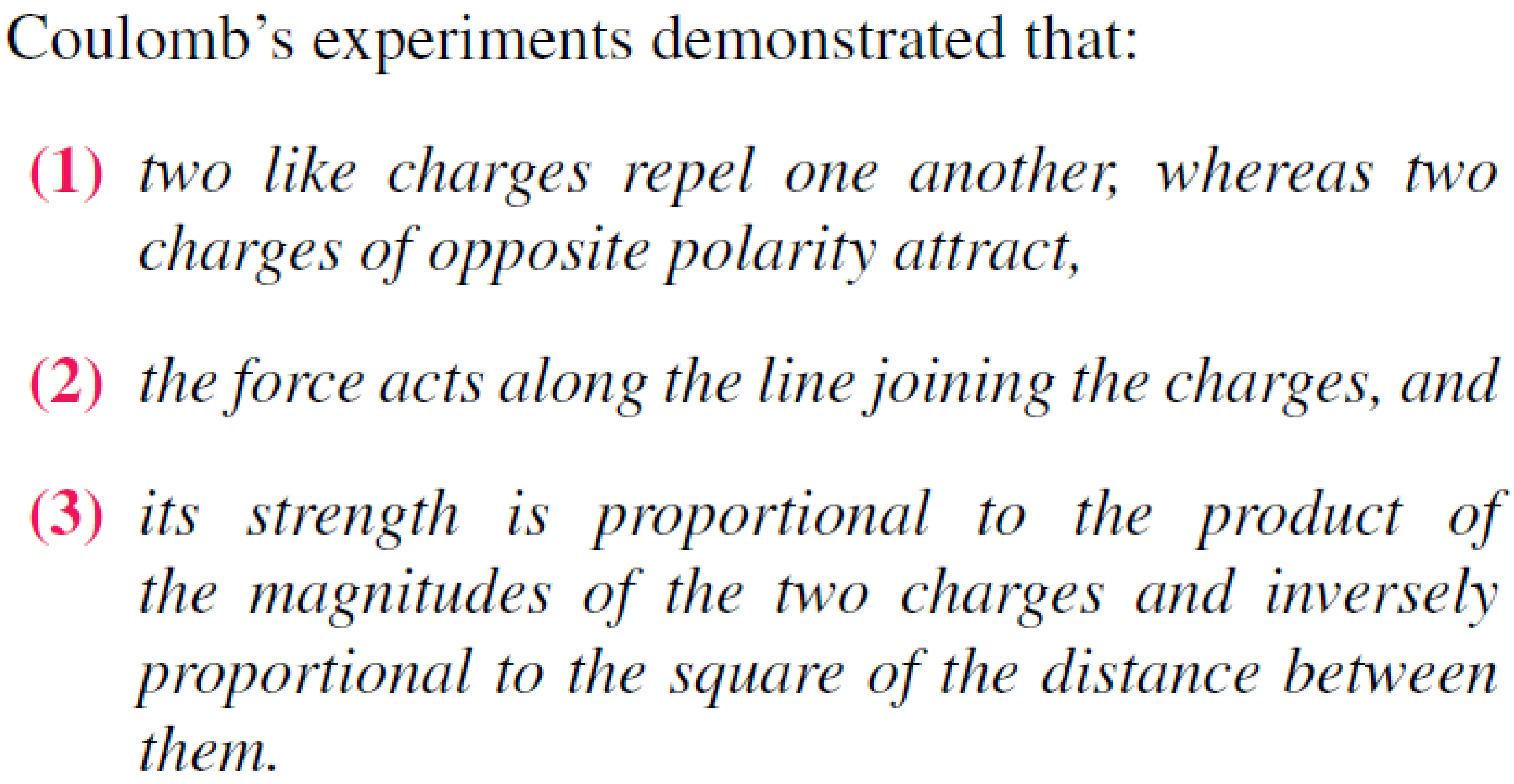
- The unit of electric charge is coulomb (C),named in the honor of 18th century French scientist Charles Augustin de Coulomb.

- The magnitude of $\boldsymbol{e}$ is :

$$\boldsymbol{e = 1.6 \times 10^{-19} \quad (C)}.$$

- Electron charge: $q_e = -e$
- Proton charge: $q_\mathrm{p} = e$

Coulomb’s Law

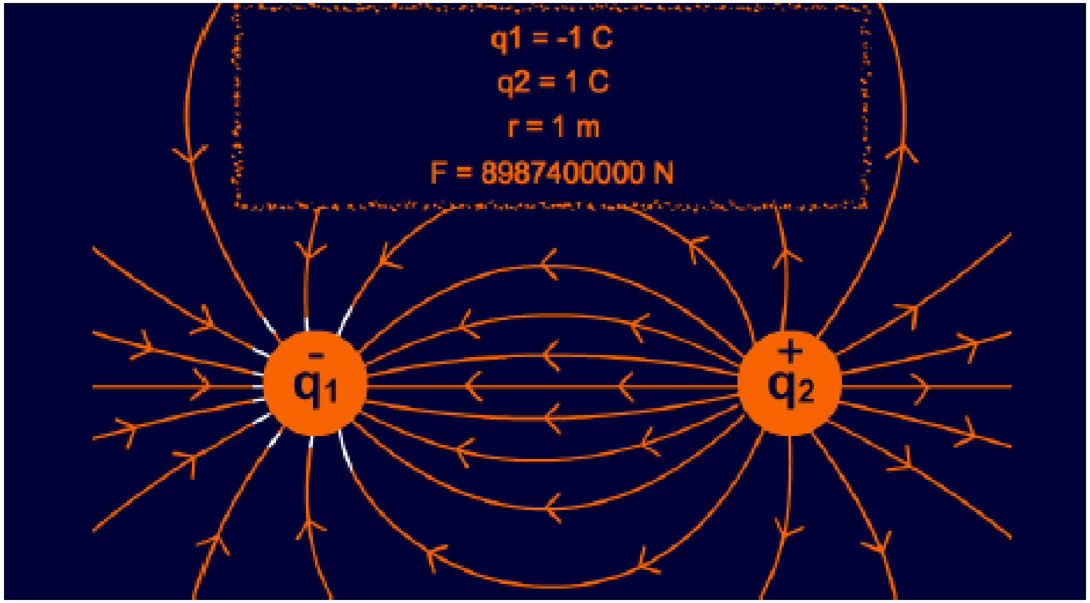


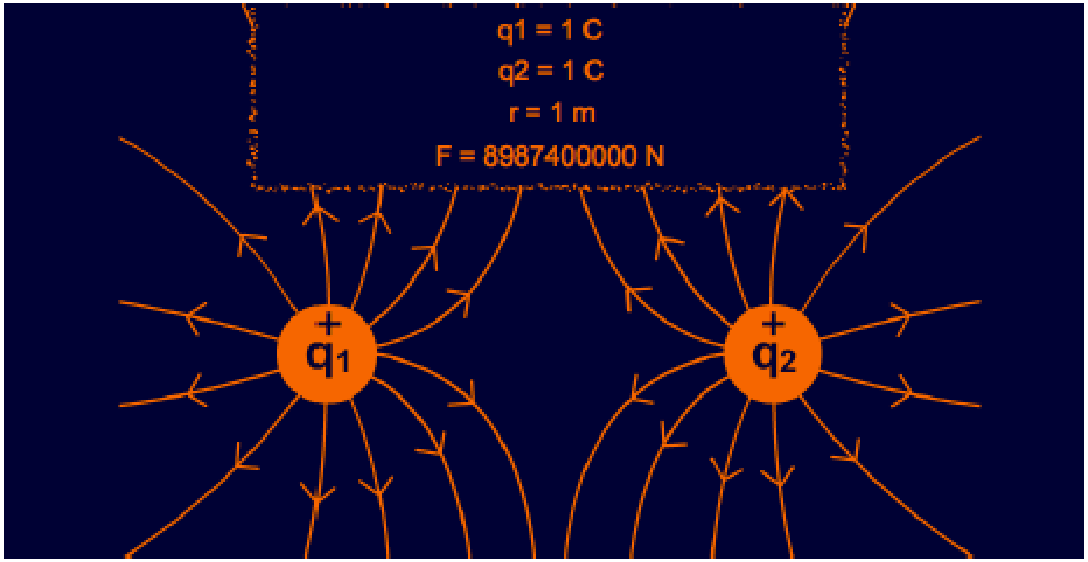
Coulomb’s experiments demonstrated that:

(1) *two like charges repel one another, whereas two charges of opposite polarity attract,*

(2) *the force acts along the line joining the charges, and*

(3) *its strength is proportional to the product of the magnitudes of the two charges and inversely proportional to the square of the distance between them.*





two like charges repel one another, whereas two charges of opposite polarity attract,

- These properties constitute Coulomb’s law which can be expressed mathematically by the following equation:

$$\vec{\mathbf{F}}_{e_{21}} = \hat{\mathbf{R}}_{12} \frac{q_1 q_2}{4\pi \varepsilon_0 R_{12}^2} \quad \text{(N)} \quad \text{(in free space)},$$

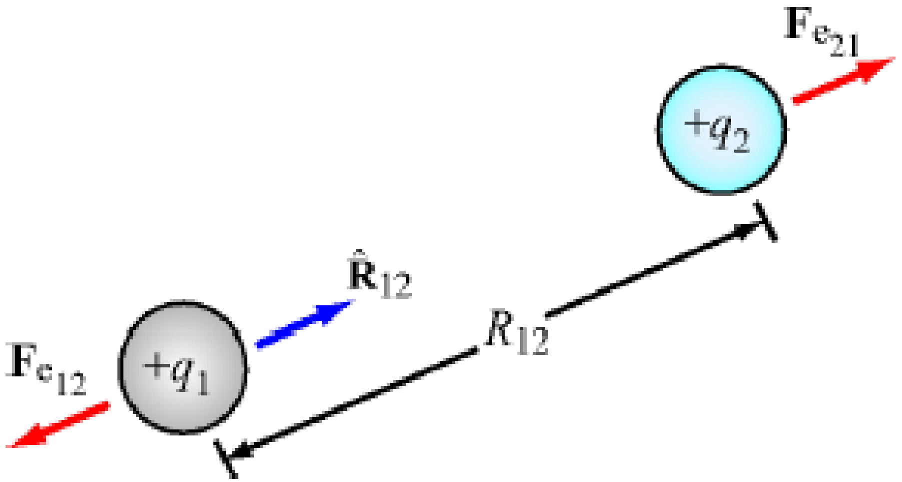


Figure 1-4: Electric forces on two positive point charges in free space.

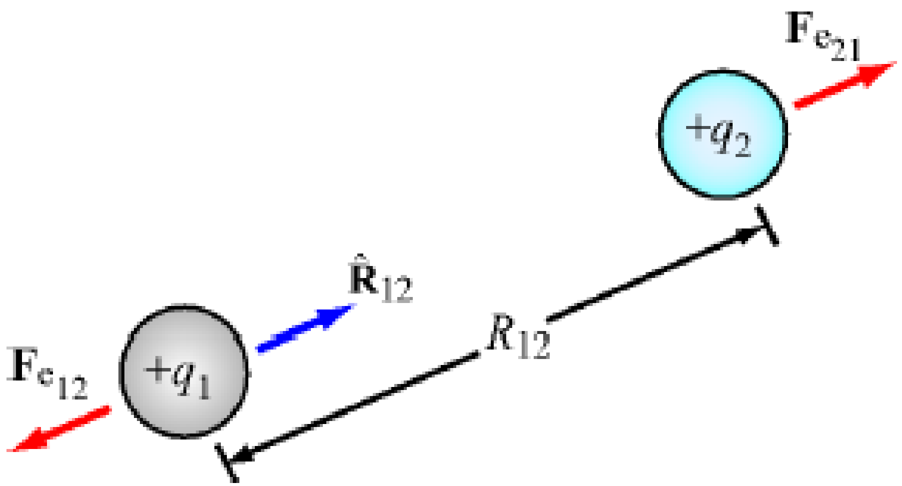


Figure 1-4: Electric forces on two positive point charges in free space.

$$\boxed{\vec{F}_{e_{12}} = - \vec{F}_{e_{21}}}$$

,where

$\vec{F}_{e_{21}}$: is the electrical force acting on charge q_2 due to charge q_1 .

R_{12} : is the distance between the two charges .

$\hat{R}_{12}$: is a unit vector pointing from charge q_1 to charge q_2 .

ε_0 : is the electrical permittivity of free space.

($\varepsilon_0 = 8.845 \times 10^{-12}$ farad per meter (F/m))

- In analogy to gravitational field ψ, the electric field intensity $\vec{\boldsymbol{E}}$ due to any charge q can be expressed as:

$$\vec{E} = \hat{R}\frac{q}{4\pi\varepsilon_o R^2} \qquad (V/m) \qquad \text{(in free space)}$$

Where,

R : is the distance the charge and observation point.

$\hat{R}$: is the radial unit vector pointing away from the charge.

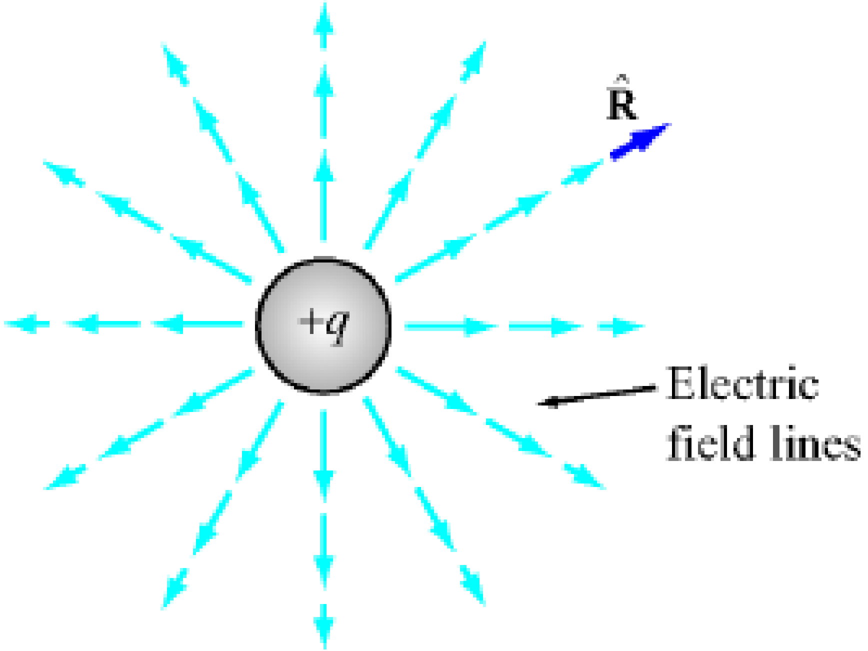


Figure 1-5: Electric field E due to charge q.

- Electric charge exhibits two important properties:

1- the low of conservation of electric charge which states that *the net electric charge can neither be created nor destroyed.*

if a volume contains n_p protons and n_p electrons, then the total charge is :

$$\boldsymbol{q = n_p e - n_e e = \left(n_p - n_e\right)e \qquad (C).}$$

Note that .....

Even if some of the protons were to combine with an equal number of electrons to produce neutrons or other particles, the net charge $\boldsymbol{q}$ remains unchanged.

2- The principle of linear superposition, which states that the total vector electric field at a point in space due to a system of point charges is equal to the vector sum of the electric fields at that point due to the individual charges.

We will use this property in future chapters to compute the electric field due to complex distributions of charge.

- Until now, we have described the electric field induced by an electric charge when in free space.

- What happens if a positive point charge is placed in a material composed of atoms?!

- Before adding a test charge (positive charge) inside the material, the material is electrically neutral.

- When a test charge is placed, the atoms are distorted and polarized as shown in figure(1-6).

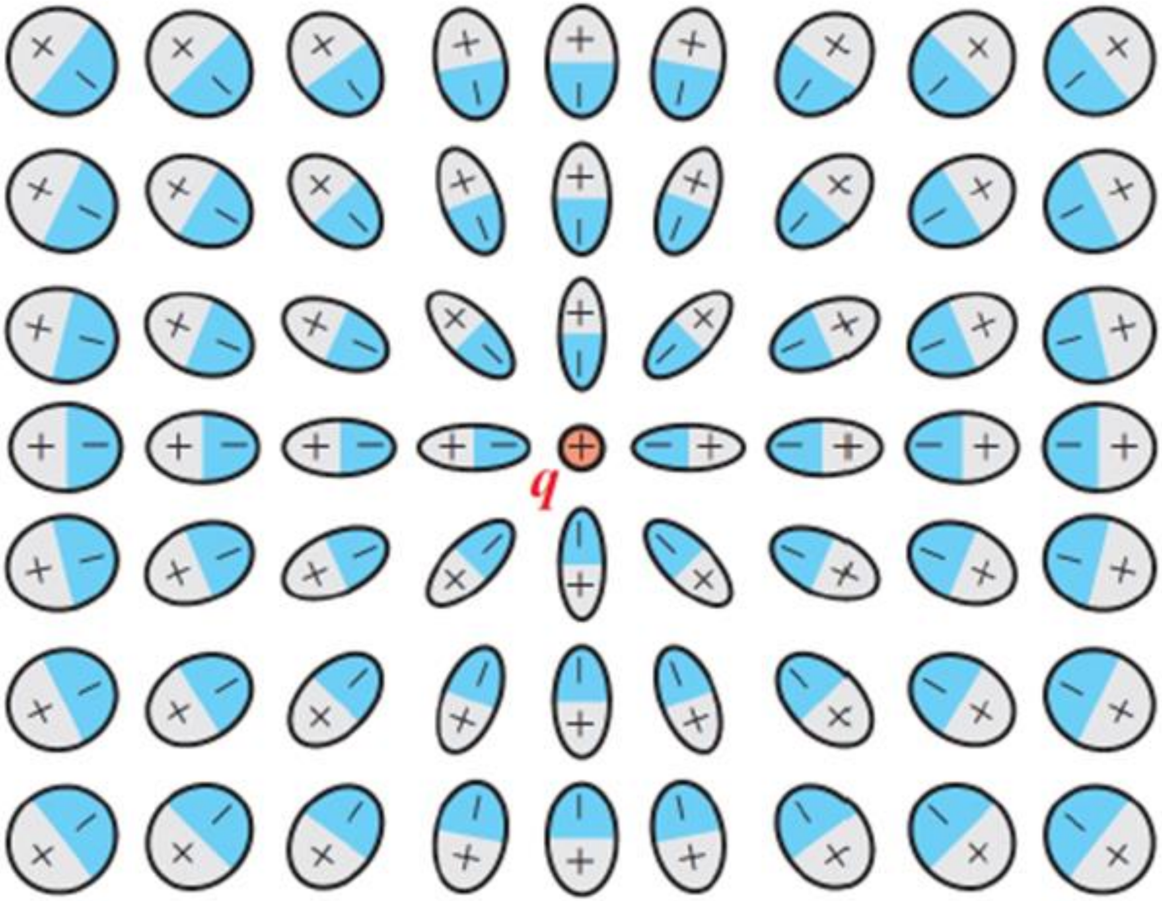


Figure 1-6: Polarization of the atoms of a dielectric material by a positive charge q.

- The degree of polarization depends on the distance between the atom and the isolated point charge.

- Polarization is the process of electric dipole formation.

- Dipoles of the atoms or (molecules) counteract the field due to the point charge.

- Therefore, the electric field inside the material is different from that of free space.

- To extend our equation for electric field from free space to inside the material, we replace ε_0 *by* ε :

$$\boxed{\vec{\mathbf{E}} = \hat{\mathbf{R}} \frac{q}{4\pi\varepsilon R^2} \qquad \text{(V/m)}}$$

- ε is defined as :

$$\boldsymbol{\varepsilon = \varepsilon_r \varepsilon_0 \quad (F/m)}$$

,where $\boldsymbol{\varepsilon_r}$ is a dimensionless quantity called the *"relative permittivity"* or *"dielectric constant"* of the material.

*** For vacuum, $\boldsymbol{\varepsilon_r}$=1

*** For air near earth's surface, $\boldsymbol{\varepsilon_r}$=1.0006

Electric Flux Density:

- In addition to the electric field intensity **E**, we will often find it convenient to also use a related quantity called the electric flux density **D**, given by :

$$\vec{\boldsymbol{D}} = \boldsymbol{\varepsilon}\vec{\boldsymbol{E}} \qquad (\boldsymbol{C}/\boldsymbol{m}^2)$$

Note that ….

E and D constitute one of two fundamental pairs of electromagnetic field quantities.

Magnetic field

- As early as 800 B.C.,first magnetic stones were discovered by Greeks. We call these stones now “***magnetite***” $(F_{e_3}O_4)$.

- Each magnet has two poles:

  north and south poles.

- It is impossible to separate N and S. If the magnet is broken into pieces, each piece has its own north and south poles.

- The magnetic lines encircling a magnet are called *magnetic field lines*.

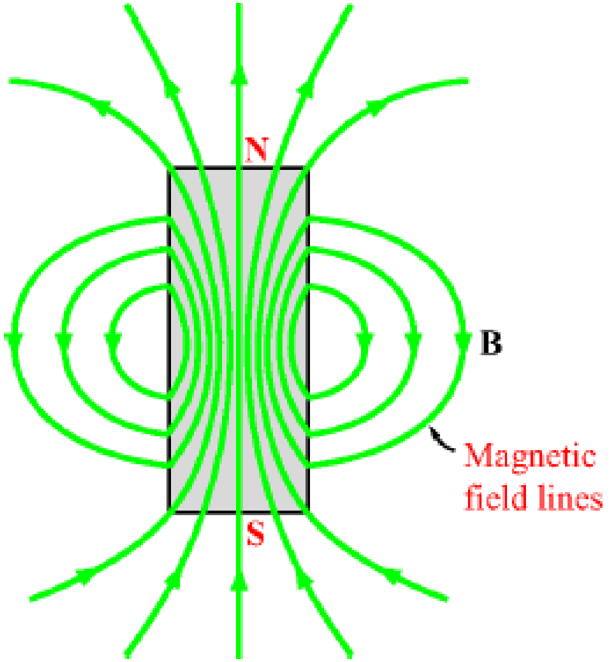


Figure 1-7: Pattern of magnetic field lines around a bar magnet.

- These lines represent the existence of a magnetic field called the ***magnetic flux density B.***

- A magnetic field not only exists around permanent magnets but can also be created by electric current.

- This connection between electricity and magnetism was first discovered by Danish scientist Hans Oersted in 1819.

- Shortly after oersted,French scientists Jean Baptiste Biot and Felix Savart developed an expression relating I and B.

- This expression is now called “Biot-Savart law”:

$$\boxed{\vec{\mathbf{B}} = \hat{\boldsymbol{\phi}}\,\frac{\mu_0 I}{2\pi r} \qquad (\mathrm{T})}$$

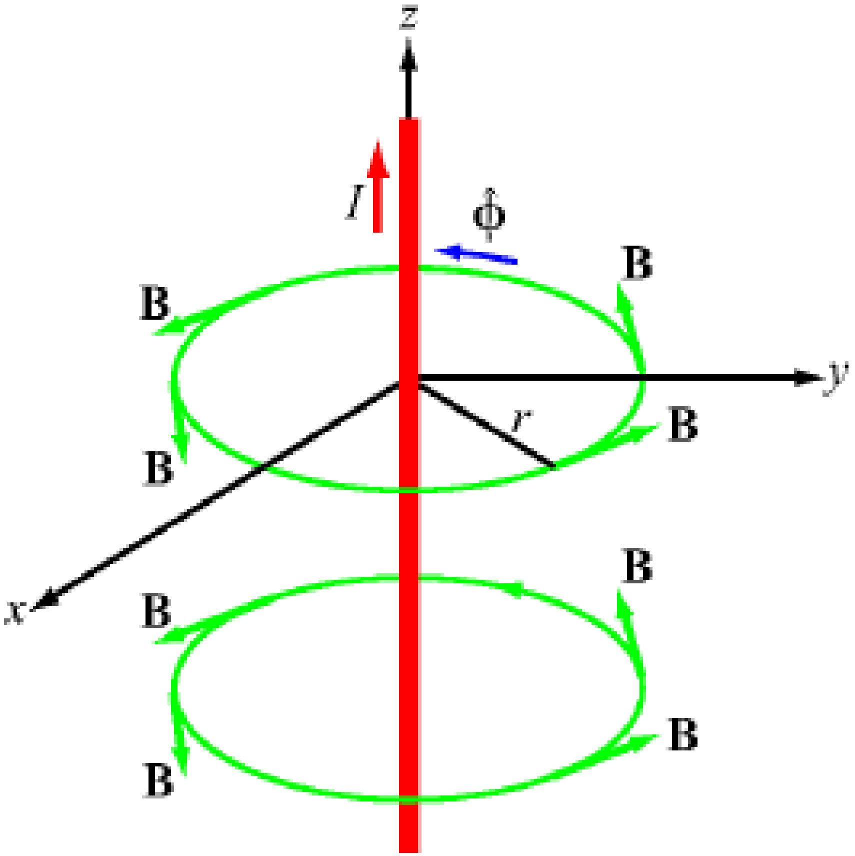


Figure 1-8: The magnetic field induced by a steady current flowing in the z-direction.

$$\boxed{\vec{\mathbf{B}} = \hat{\boldsymbol{\phi}}\, \frac{\mu_0 I}{2\pi r} \qquad (\mathrm{T})}$$

- The unit of B is tesla (T).

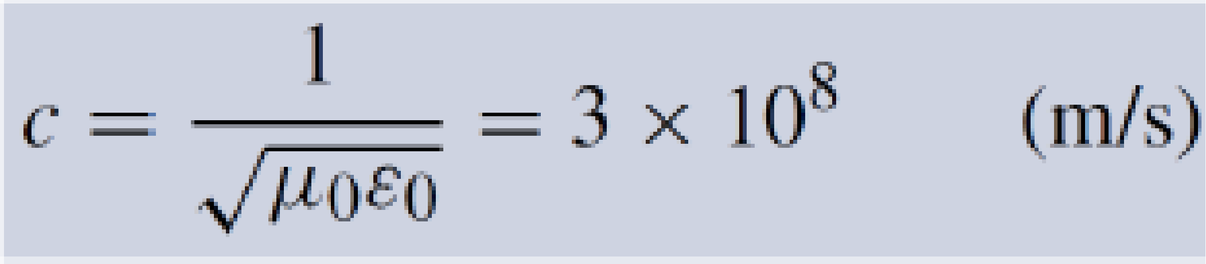
- μ_0 : is the magnetic permeability of free space.

$$\boldsymbol{\mu_0 = 4\pi \times 10^{-7}}$$

- r: is the radial distance from the current.

- $\hat{\phi}$ is an azimuthal unit vector denoting the fact that the magnetic field direction is tangential to the circle surrounding the current.

- Electric and magnetic fields are connected through the speed of light:



$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \qquad \text{(m/s)}$$

- The magnetic permeability μ accounts for magnetization properties of a material.

*** *A Nonmagnetic material* has $\mu = \mu_0$.

*** *A Ferromagnetic material* like iron has $\mu \gg \mu_0$.

- In analogy to permittivity of material $\boldsymbol{\varepsilon}$, μ can be defined as :

$$\boldsymbol{\mu = \mu_r \mu_0} \qquad \boldsymbol{(H/m)}$$

Where, μ_r is dimensionless quantity called ***relative magnetic permeability*** of the material.

- We have learned before that **E** and **D** constitute one of two pairs of electromagnetic field quantities.

- The second pair is **B** and the magnetic field intensity **H**,which are related to each other through μ:

$$\boldsymbol{B} = \boldsymbol{\mu H} \qquad (\boldsymbol{T})$$

Static and Dynamic Fields

- There are three branches of electromagnetics:

1- Electrostatics

2- Magnetostatics

3- Dynamics (Time-varying fields).

- The electric field **E** is governed by charge q. So if the charge doesn’t change with time, **E** remains constant.

- This corresponds to **“Electrostatics”**:

$$\frac{\partial q}{\partial t} = 0 \;\Rightarrow\; E \text{ is constant}$$

- Also,we have learned that the magnetic field ***H*** is governed by the current I. So if I is constant **B** is constant.

- This corresponds to **“Magnetostatics”:**

$$\frac{\partial I}{\partial t} = 0 \Rightarrow \ B \ is \ constant$$

- Now we consider the case when I changes with time.

Since $\boldsymbol{I = \frac{dq}{dt}}$, it means that amount of charge present in a given section of a wire ,varies also with time. So **E** varies with time .

☺

In general, *A time-varying electric field will generate a time-varying magnetic field and vice versa.*

Summary of the three branches of electromagnetics

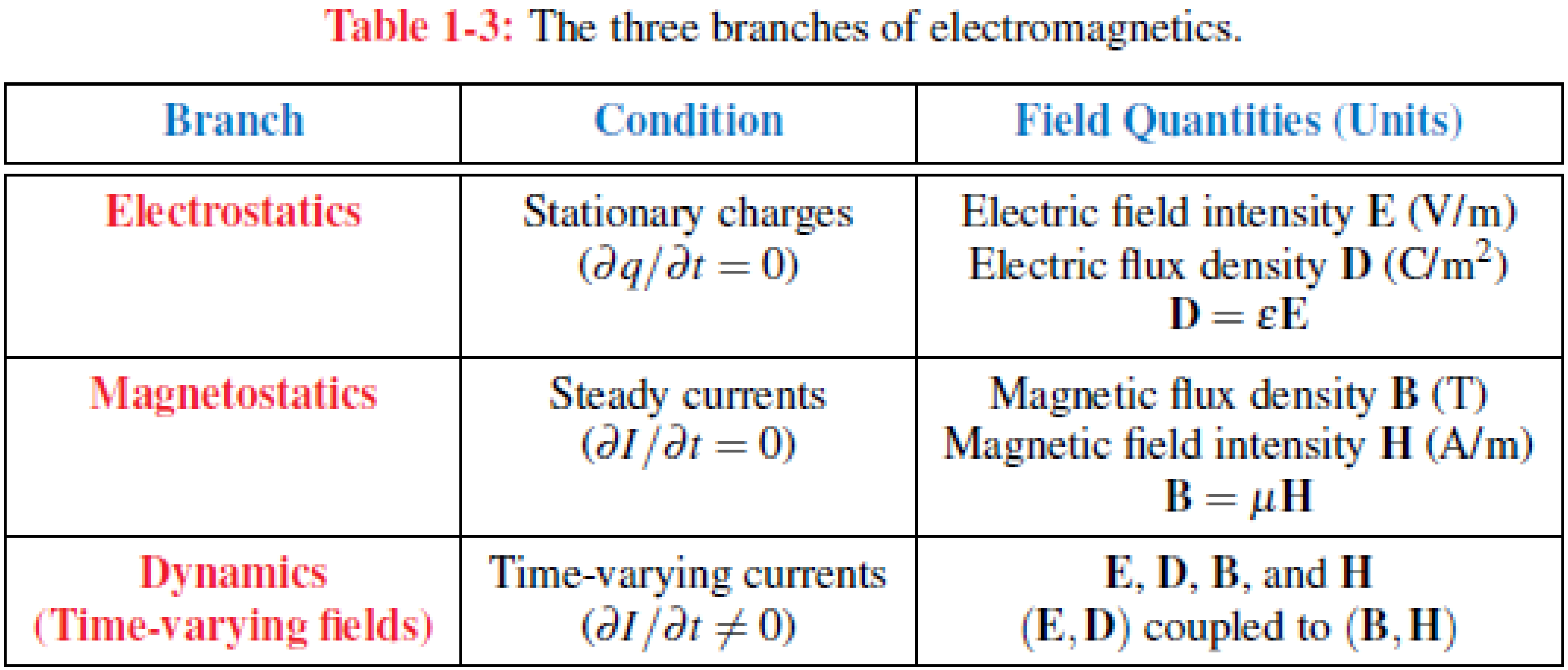


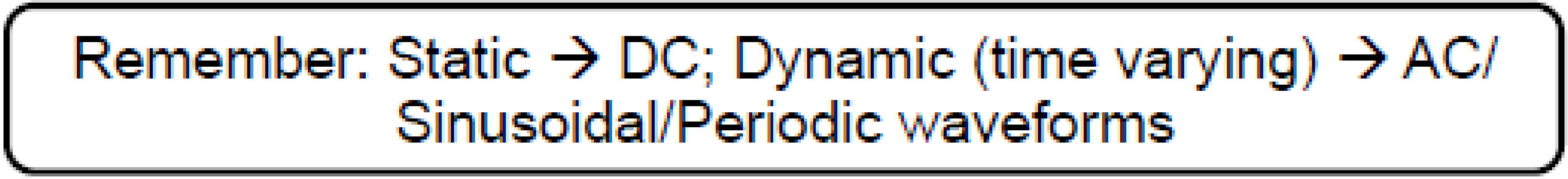
Table 1-3: The three branches of electromagnetics.

Branch	Condition	Field Quantities (Units)
Electrostatics	Stationary charges $(\partial q/\partial t = 0)$	Electric field intensity **E** (V/m) Electric flux density **D** (C/m^2) $\mathbf{D} = \varepsilon \mathbf{E}$
Magnetostatics	Steady currents $(\partial I/\partial t = 0)$	Magnetic flux density **B** (T) Magnetic field intensity **H** (A/m) $\mathbf{B} = \mu \mathbf{H}$
Dynamics **(Time-varying fields)**	Time-varying currents $(\partial I/\partial t \neq 0)$	**E**, **D**, **B**, and **H** $(\mathbf{E}, \mathbf{D})$ coupled to $(\mathbf{B}, \mathbf{H})$

Note that….

Under static conditions, induced electric and magnetic fields are independent;
under Dynamic conditions, fields become coupled.

Also,



Remember: Static → DC; Dynamic (time varying) → AC/
Sinusoidal/Periodic waveforms

Conductivity σ:

The conductivity indicates how easy charges move in material and it is measured in **Siemens per meter(S/m).**

** If $\boldsymbol{\sigma = 0}$, charges do not move more than atomic distances and the material is called ***"perfect dielectric”.***

** If $\boldsymbol{\sigma} = \infty$,the charges can move freely inside the material which is then called a ***perfect conductor.***

ε, μ and σ are the **“constitutive parameters”** of a material.

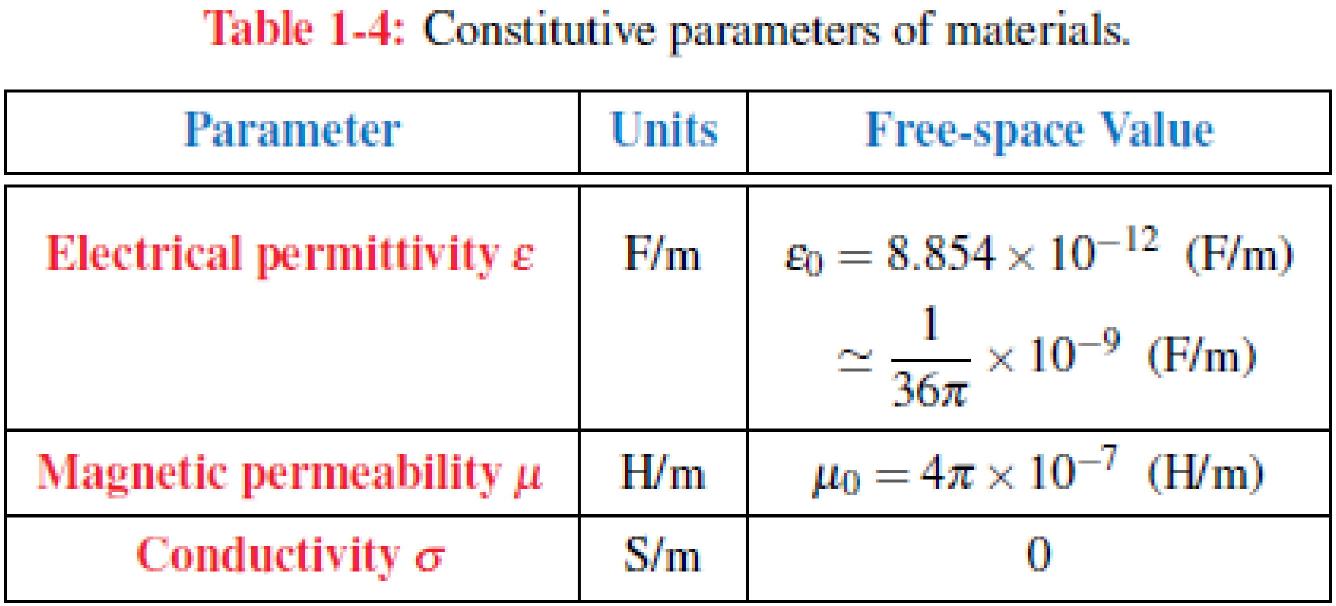


Table 1-4: Constitutive parameters of materials.

Parameter	Units	Free-space Value
Electrical permittivity ε	F/m	$\varepsilon_0 = 8.854 \times 10^{-12}$ (F/m) $\simeq \dfrac{1}{36\pi} \times 10^{-9}$ (F/m)
Magnetic permeability μ	H/m	$\mu_0 = 4\pi \times 10^{-7}$ (H/m)
Conductivity σ	S/m	0

A medium is said to be ***homogeneous*** if its constitutive parameters are constant throughout the medium.