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Emergent critical phase in the frustrated 2D Windmill Antiferromagnet

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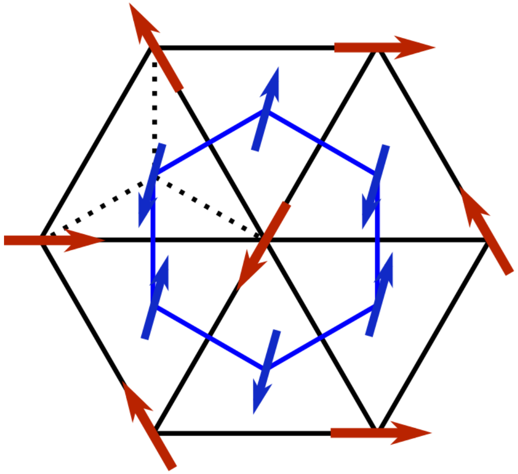
University of Minnesota

Colloquium, Iowa State University, 25 January 2016



References:

- B. Jeevanesan, P. Chandra, P. Coleman, PPO
  Phys. Rev. Lett. **115**, 177201 (2015).
- B. Jeevanesan, PPO
  Phys. Rev. B **90**, 144435 (2014).
- PPO, P. Chandra, P. Coleman, J. Schmalian
  Phys. Rev. B **89**, 0994417 (2014).
  Phys. Rev. Lett. **109**, 237205 (2012).

Magnets: a tale of order and disorder

Order ⟷ Fluctuations ⟷ Disorder

- Ferromagnetic Ising model [1]

$$H = -J \sum_{\langle i,j \rangle} S_i^z S_j^z \qquad J > 0, S_i^z = \pm 1$$

- Mean-field solution $\sum_{\langle i,j \rangle} S_i^z S_j^z \to Nzm^2 + zm \sum_i S_i^z + \ldots$

$$\Rightarrow \quad m = \frac{1}{Z} \sum_{S^z = \pm 1} S^z e^{-\beta H[S^z]} = \frac{e^{\beta J z m} - e^{-\beta J z m}}{e^{\beta J z m} + e^{-\beta J z m}} = \tanh \beta J z m$$

⇒ Phase transition from paramagnetic to ferromagnetic state at finite temperature $T_c = zJ$

[1] E. Ising, Z. Phys. 31, 253 (1925).

Exact solution of 1D Ising model

- Ising model in 1D can be solved exactly [1]

Partition function $Z = \mathrm{Tr}(T^N)$ with transfer matrix $T = \begin{pmatrix} e^{\beta(J+B)} & e^{-\beta J} \\ e^{-\beta J} & e^{\beta(J-B)} \end{pmatrix}$

→ Magnetization $m = -\frac{1}{N}\frac{\partial F}{\partial H} = \frac{\sinh \beta B}{\sqrt{\sinh^2(\beta B) + e^{-4\beta J}}}$

→

$$m(T > 0, B = 0) = 0$$

Paramagnetic at finite temperatures.

$$m(T = 0, B = 0^{\pm}) = \pm 1$$

Ferromagnetic at T=0.

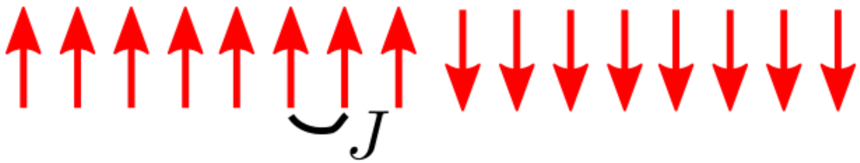
Transition temperature Tc = 0.

[1] E. Ising, Z. Phys. 31, 253 (1925).

Energy-entropy competition

- Peierls' energy-entropy argument

$$H = -J \sum_{\langle i,j \rangle} S_i S_j \quad , S_i = \pm 1$$



Calculate free energy F = E - TS cost of a defect (= domain wall)

$$E = 2J$$
$$S \simeq k_B \log N$$

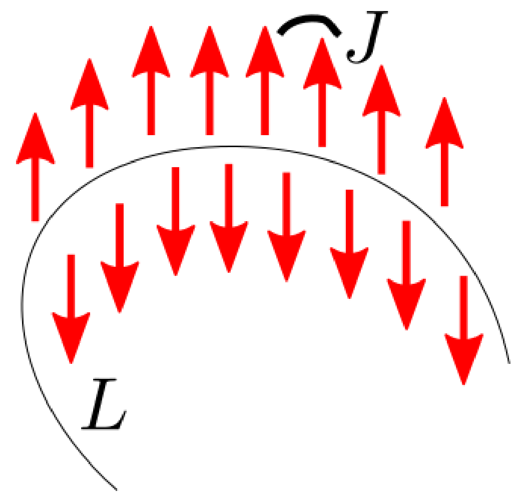
$$F \simeq 2J - k_B T \log N \xrightarrow[N \to \infty]{} -\infty$$

Free energy reduced by generating defects

No long-range discrete order in d=1 at finite T.

Discrete order in two dimensions

- Peierls' argument in two dimensions



$$H = -J \sum_{\langle i,j \rangle} S_i S_j \quad , S_i = \pm 1$$

$$E \simeq JL$$
$$S \simeq k_B \log(z-1)^L$$

$$F \simeq L\left[J - k_B T \log(z-1)\right]$$

Paramagnetic $T > \dfrac{J}{k_B \log(z-1)}$ Ferromagnetic $T < \dfrac{J}{k_B \log(z-1)}$

Phase transition to long-range ordered state at finite Tc.
Lower critical dimension for discrete degrees of freedom $d_{lc} = 1$.

Exact solution of 2D Ising model on square lattice [1] gives Tc = 2.27 J.

[1] L. Onsager Phys. Rev. II **85**, 808 (1944).

Order of continuous spins in two dimensions

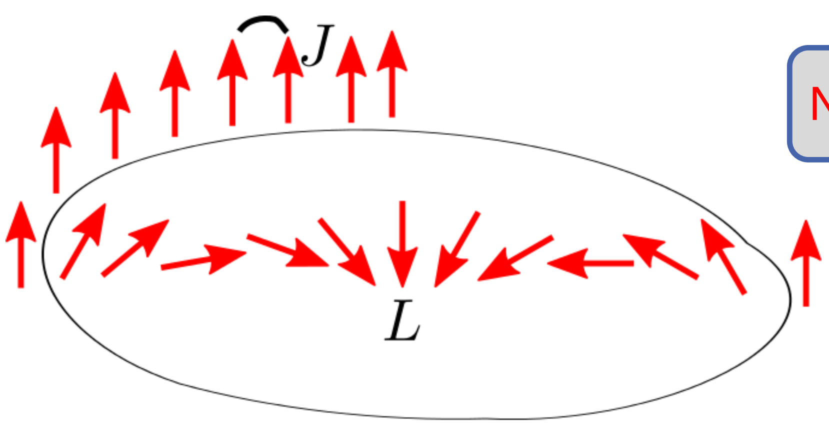
- Continuous spins with two or three components (XY or Heisenberg)

Peierls' argument: no domain wall, instead continuous gradient energy $\nabla\phi \simeq 2\pi/L$

$$E \simeq \int_{L^2} d^2x (\nabla\phi)^2 \simeq \mathcal{O}(1)$$
$$S \simeq k_B \log(z-1)^L$$

$$F \simeq \mathcal{O}(1) - k_B T L \log(z-1)$$

No continuous long-range order in d=2 at T>0.



Thermal fluctuations destroy order. Formal proof provided by **Hohenberg-Mermin-Wagner theorem**.

Lower critical dimension for continuous degrees of freedom is $d_{lc} = 2$.

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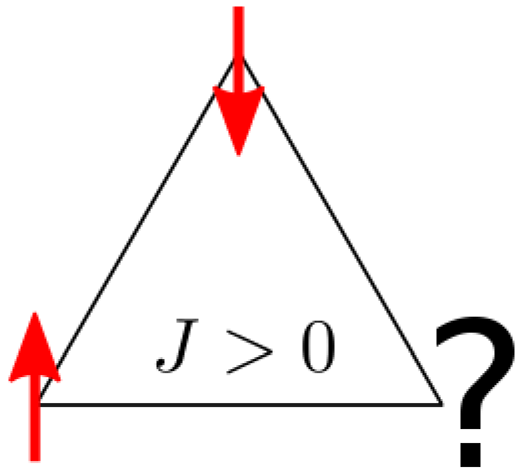
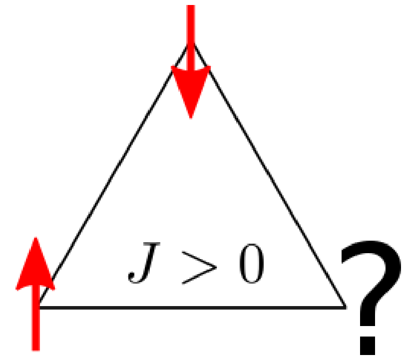
Geometric frustration in antiferromagnets

- Element of frustration: triangle

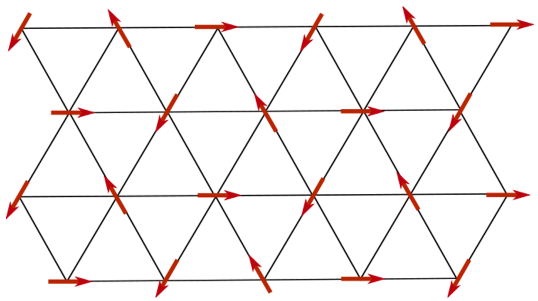


Geometric frustration

- Element of frustration: triangle
  - Edge sharing: Triangular lattice (2D)
  - Corner sharing: Kagome lattice (2D)

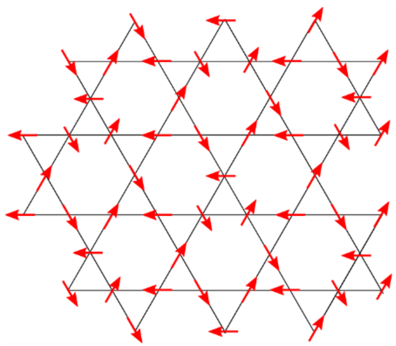


Triangular lattice



- Ising model [1]
  - Ground state degeneracy with extensive entropy $S = 0.323\ N\ k_B$
  - Disordered at T>0
  - Algebraic order at T=0

Kagome lattice

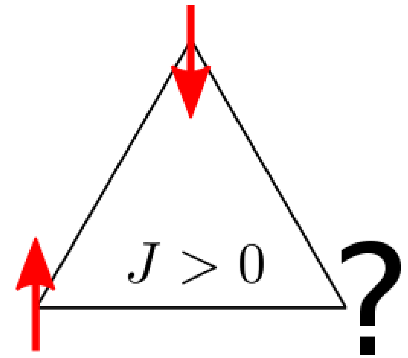


- Ising model [2, 3]:
  - Even larger ground state degeneracy $S = 0.502\ N\ k_B$
  - Disordered at T>0 and T=0

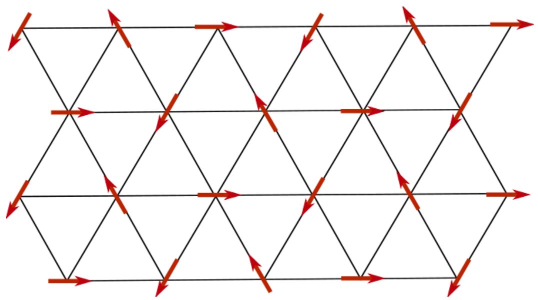
[1] G. H. Wannier, Phys. Rev. **79**, 357 (1950); [2] K. Kano, S. Naya, Prog. Theor. Phys. **10**, 158 (1953); R. Moessner, S. Sondhi, PRB **63**, 224401 (2001).

Geometric frustration

- Element of frustration: triangle
  - Edge sharing: Triangular lattice (2D)
  - Corner sharing: Kagome lattice (2D)

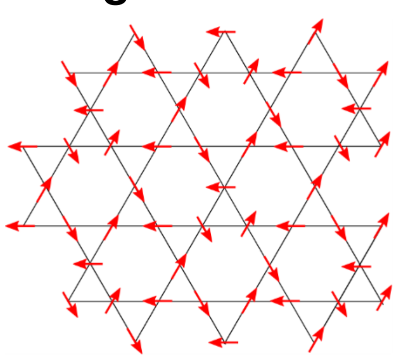


Triangular lattice



- Heisenberg model:
  - 120° order at T=0 (both classical and quantum spin model [1])
  - Disordered at T>0 (Mermin-Wagner theorem)

Kagome lattice



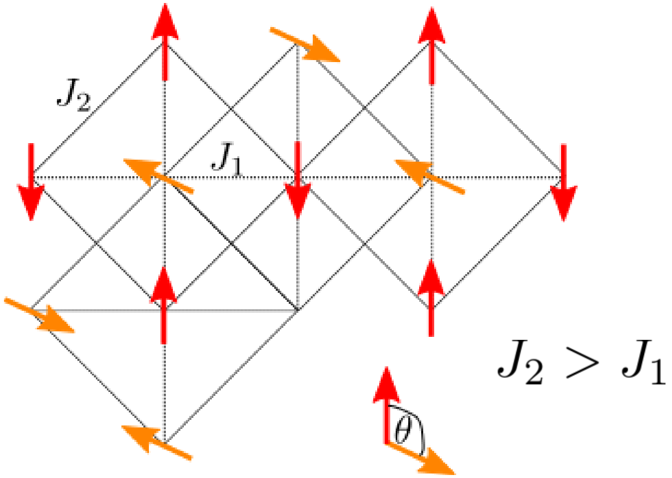
- Heisenberg model:
  - Classical model is disordered at T=0
  - Quantum model seems to be a Quantum Spin Liquid for Spin-1/2: Herbertsmithite [2]

[1] S. R. White, A. L. Chernyshev, PRL **99**, 127004 (2007;
[2] T.-H. Han *et al*., Nature **492**, 406 (2012).

J_1-J_2-model on square lattice

- Fluctuations also induce order: order from disorder [1, 2]
- J_1-J_2-Heisenberg model on square lattice [3, 4]

$$H = J_1 \sum_{\langle i,j \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



- Finite spin correlation length (Mermin-Wagner)

$$\xi(T) \sim a_0 e^{2\pi J S^2 / T}$$

- Neel orders on both sublattices for $J_2 > J_1$
  - Coupled only by fluctuations

[1] J. Villain, J. Phys. Fr **38**, 385 (1977); [2] C. L. Henley, PRL **62**, 2056 (1989); [3] P. Chandra, P. Coleman, A. I. Larkin, PRL **64**, 88 (1990); [4] C. Weber *et al.*, PRL **91**, 177202 (2003);

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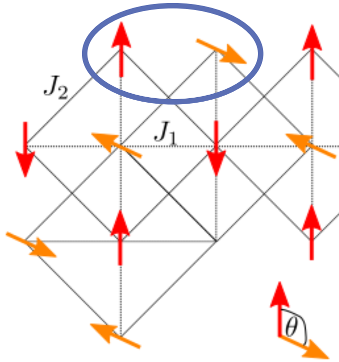
Order from disorder

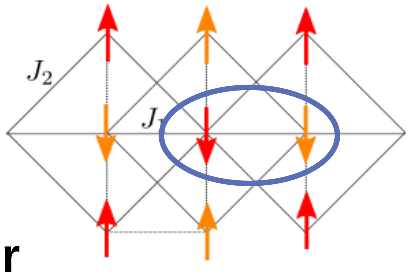
- Fluctuation free energy [1] due to "order from disorder"



$$\delta F = -E(T)[1+\cos^2\theta] \quad \text{minimized for } \theta = 0, \pi$$

$$E(T) = \frac{J_1 S^2}{2J_2}\left(\gamma_Q \frac{1}{S} + \gamma_T \frac{T}{J_2 S^2}\right)$$

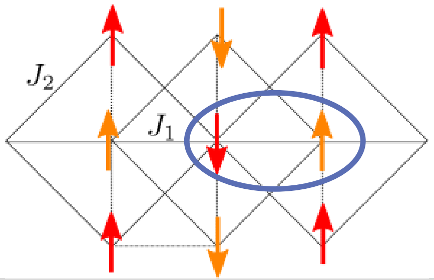
Spins tend to align the fluctuating Weiss' field of the neighbors to their easy plane [3].



$$m_\alpha = +1$$

Emergent discrete Ising $\mathbb{Z}_2$ order parameter

$$m_\alpha \sim S_1 \cdot S_2 = \pm 1$$

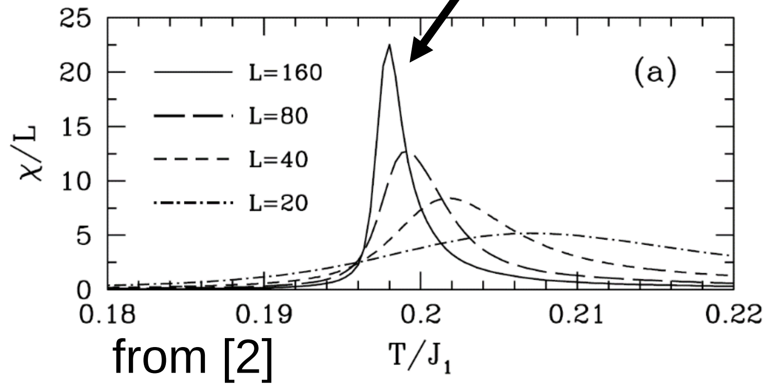


$$m_\alpha = -1$$

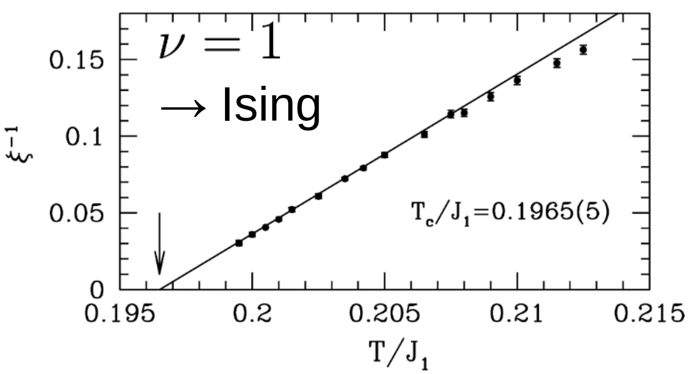
[1] P. Chandra et al., PRL **64**, 88 (1990); [2] J. Villain, J. Phys. Fr **38**, 385 (1977);
[3] C. L. Henley, PRL **62**, 2056 (1989)

Ising phase transition in J_1-J_2-model

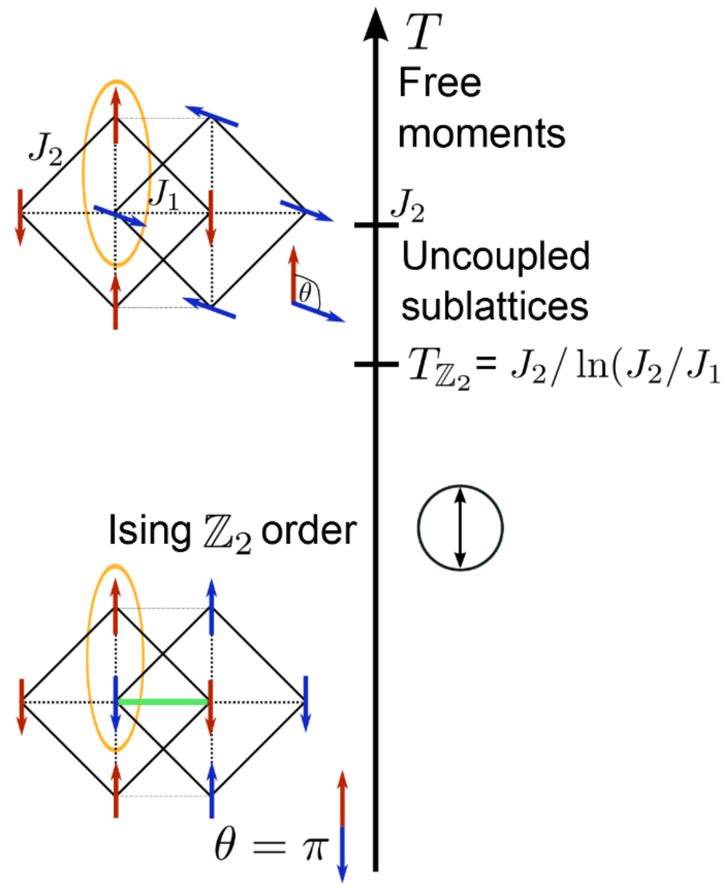
Finite temperature Ising phase transition



from [2]



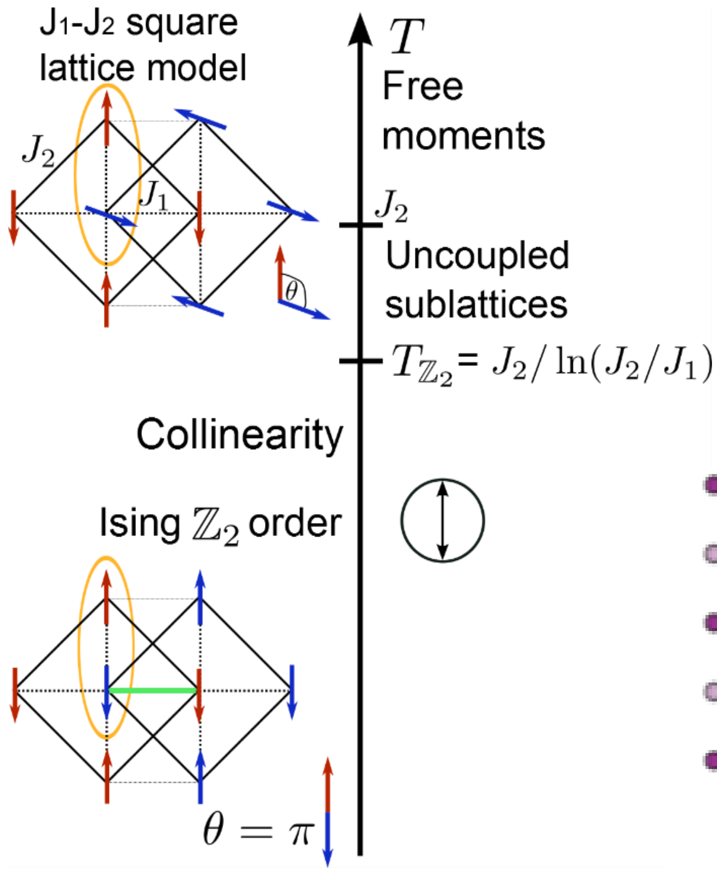
Phase diagram:



[1] P. Chandra, P. Coleman, A. I. Larkin, PRL **64**, 88 (1990); [2] C. Weber *et al.*, PRL **91**, 177202 (2003); [3] R. M. Fernandes *et al.*, PRL **105**, 157003 (2010);

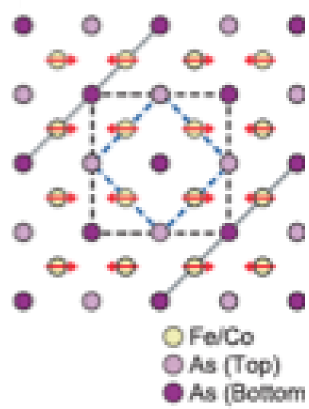
Z_2 order drives structural transition

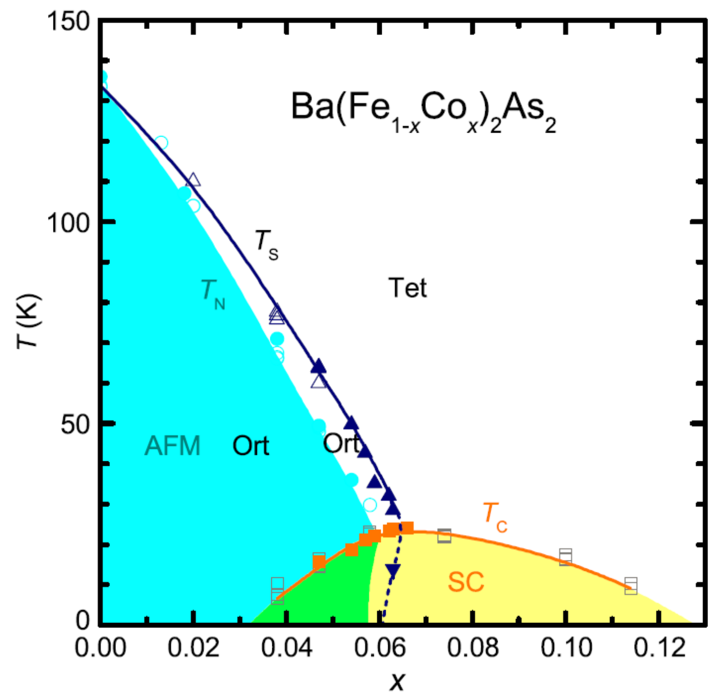
Phase diagram:



Applications in iron-based superconductors [1, 2, 3]

Discrete spin ordering induces structural transition.





From [3]

[1] R. M. Fernandes *et al.*, PRL **105**, 157003 (2010); [2] H. Luetkens, *et al.*, Nat. Mat. **8**, 305 (2009); [3] S. Nandi *et al.*, PRL **104**, 057006 (2010)

Critical phase in 2D Heisenberg model

Can we find
- a **critical phase** with algebraic order
- Berezinskii-Konsterlitz-Thouless (BKT) phase transitions

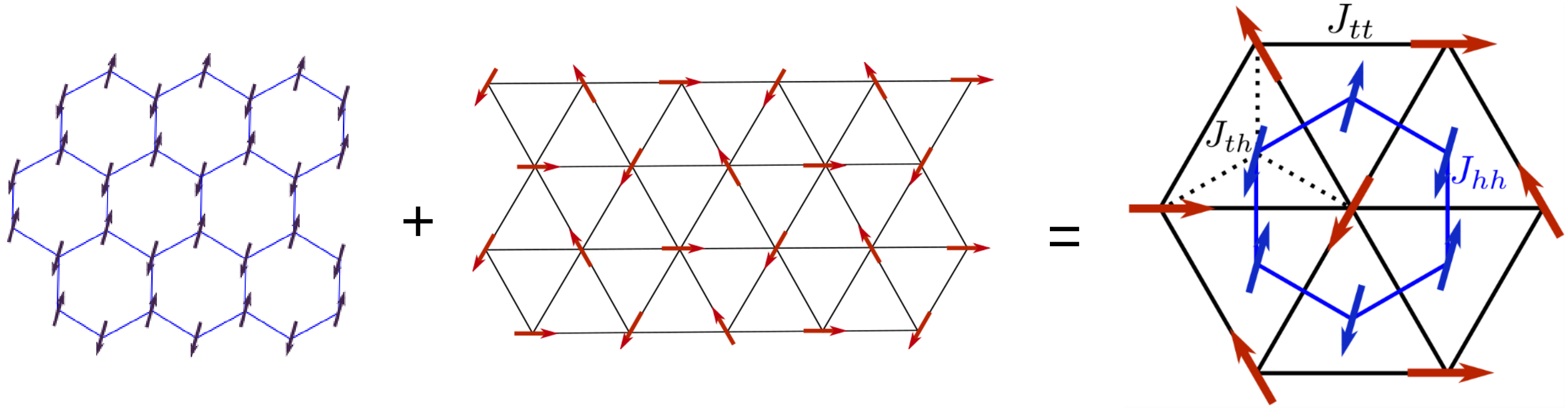
in an **isotropic 2D Heisenberg model** ?

Such physics usually occurs for planar XY spins only.

Strategy:
- Generalize Z_2 to Z_p with $p > 4$
- p-state clock model exhibits critical phase [1,2,3]

[1] J. V. Jose *et al.*, PRB **16**, 1217 (1977); [2] M. S. S. Challa and D. P. Landau, PRB **33**, 437 (1986);
[3] G. Ortiz *et al*., Nucl. Phys. B **854**, 780 (2012).

2D Heisenberg windmill antiferromagnet



- Honeycomb + triangular lattice sites
- Heisenberg spins $\boldsymbol{S}_t(r_j), \boldsymbol{S}_A(r_j), \boldsymbol{S}_B(r_j)$
- Antiferromagnetic nearest-neighbor coupling

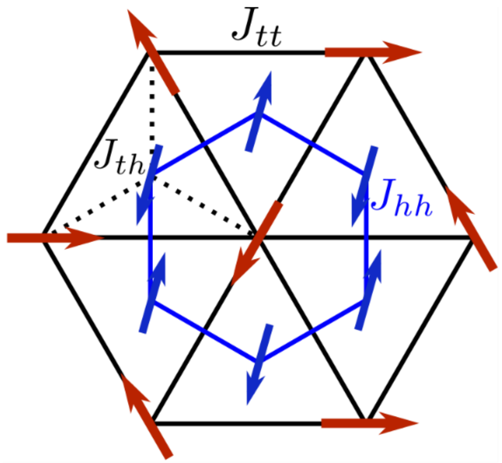
$$H = H_{tt} + H_{AB} + H_{tA} + H_{tB}$$

$$H_{ab} = J_{ab} \sum_{j=1}^{N_L} \sum_{\delta_{ab}} \boldsymbol{S}_a(r_j) \cdot \boldsymbol{S}_b(r_j + \delta_{ab})$$

Windmill in Strangnaes (Sweden)

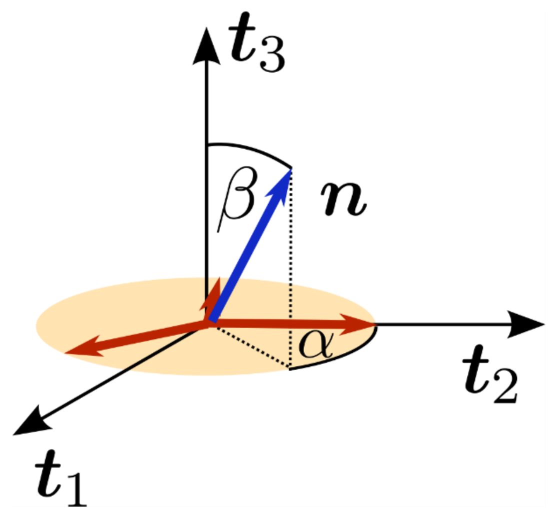
$$a, b \in \{t, A, B\}$$

Ground state of classical spins at small J_{th}



Weak inter-sublattice coupling

$$J_{th} \ll J_{tt}, J_{hh}$$



Neel order on **honeycomb lattice**

→ O(3)/O(2) order parameter $\boldsymbol{n}(\boldsymbol{x})$

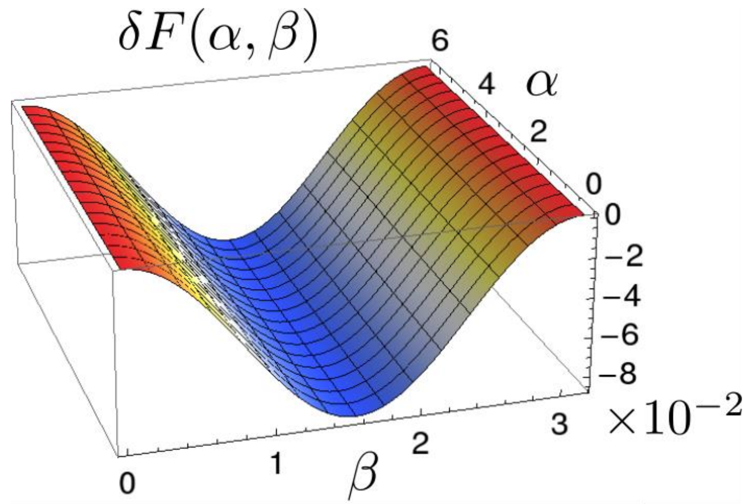
120 degree state on **triangular lattice**

→ SO(3) order parameter $t(\boldsymbol{x}) = (\boldsymbol{t}_1, \quad \boldsymbol{t}_2, \quad \boldsymbol{t}_3)$

Classically at T=0 decoupled even for $J_{th} > 0$

[1] B. Jeevanesan, PPO, PRB **90**, 144435 (2014).

Fluctuation coupling “order from disorder”

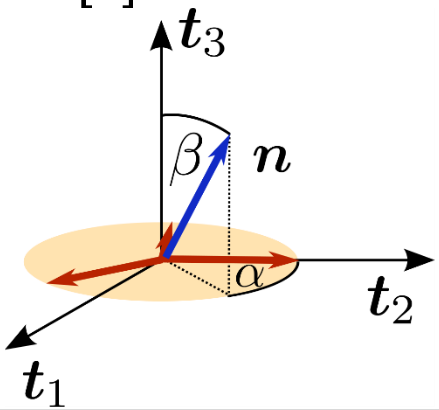


$$J_{th} = 0.4\bar{J}$$
$$\bar{J} = \sqrt{J_{tt} J_{hh}}$$
$$T = 1, S = 1$$

- Fluctuations (quantum and thermal) couple spins on different sublattices
- Spins tend to align perpendicular to fluctuation Weiss field [1]

$$S_c = \frac{1}{2} \int d^2x \left(\gamma \cos^2 \beta \right)$$

Coplanar: $\gamma = (J_{th}/\bar{J})^2 A_\gamma \left(J_{tt}/J_{hh}, \bar{J}/T \right)$



[1] C. L. Henley, PRL **62**, 2056 (1989)

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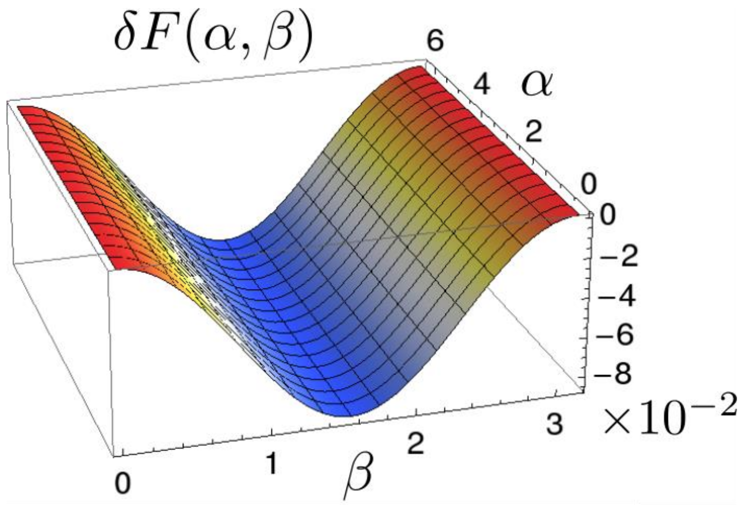

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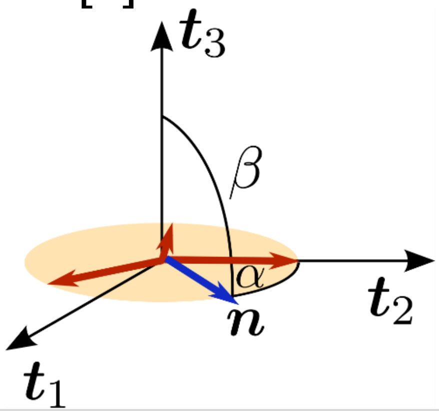
Coplanar: $\gamma \propto (J_{th}/\bar{J})^2$



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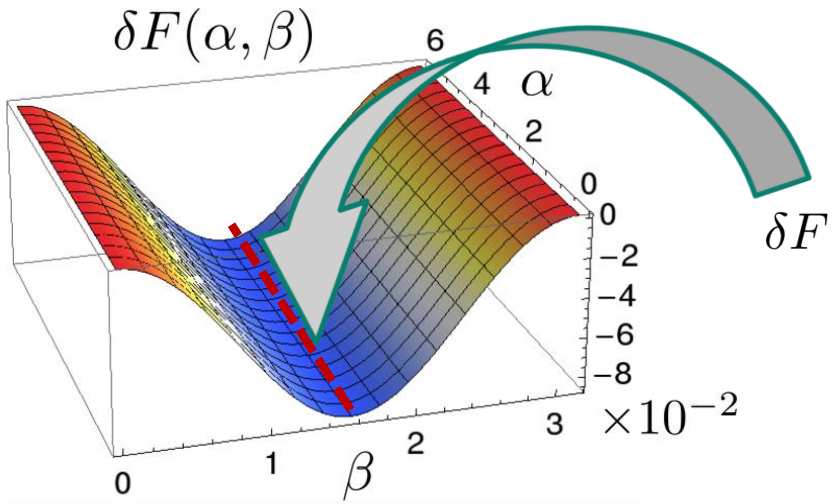

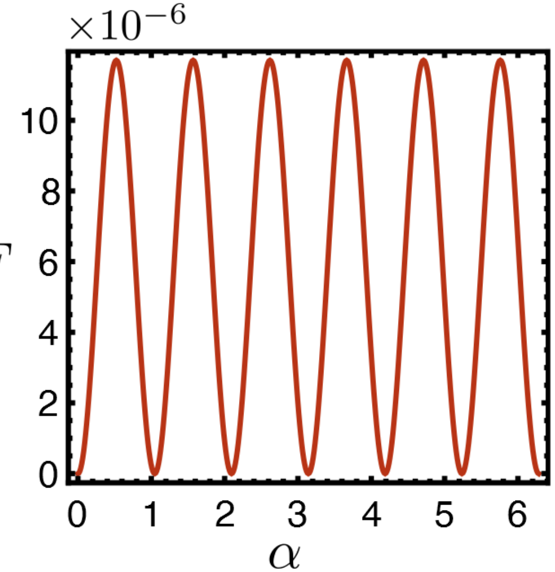

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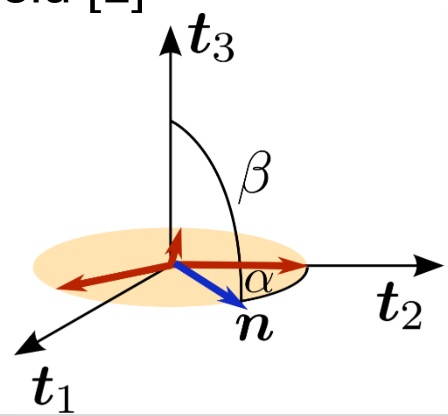
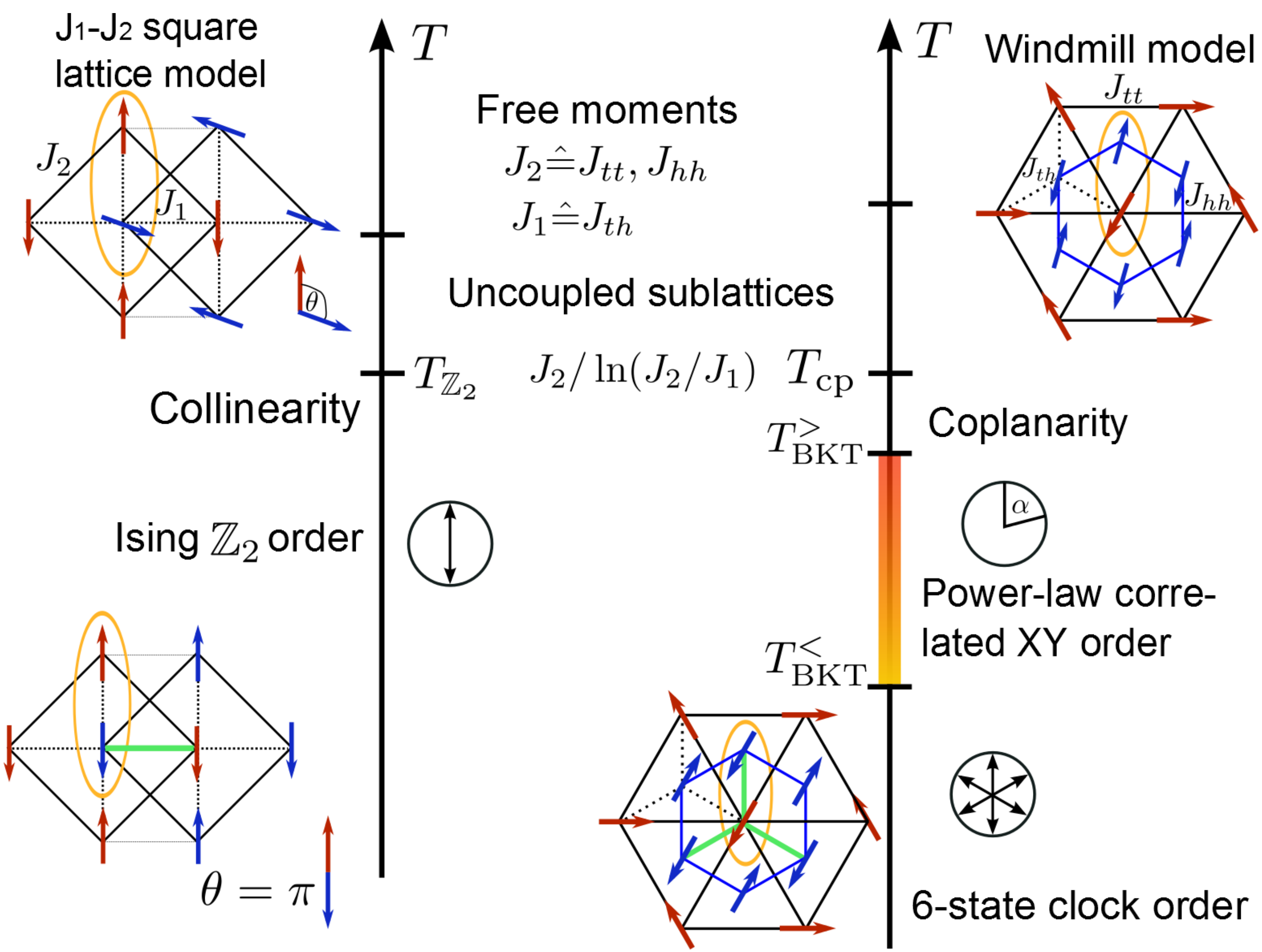


$$S_c = \frac{1}{2}\int d^2x\,\left(\gamma\cos^2\beta + \lambda\sin^6\beta\sin^2(3\alpha)\right)$$

Coplanar: $\gamma \propto (J_{th}/\bar{J})^2$ Z$_6$: $\lambda \propto (J_{th}/\bar{J})^6$

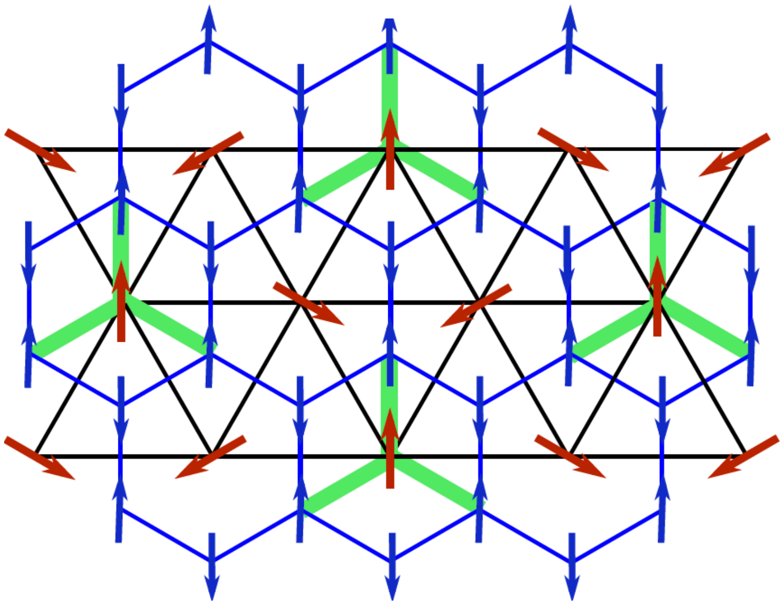
[1] C. L. Henley, PRL **62**, 2056 (1989)

Phase diagram of J1-J2 and Windmill antiferromagnets



Detection of emergent order via coupling to lattice

- Long-range relative spin order induces lattice distortion
- Algebraic relative spin correlations induce lattice softening: elastic modulus vanishes in critical phase



Windmills in Iowa.

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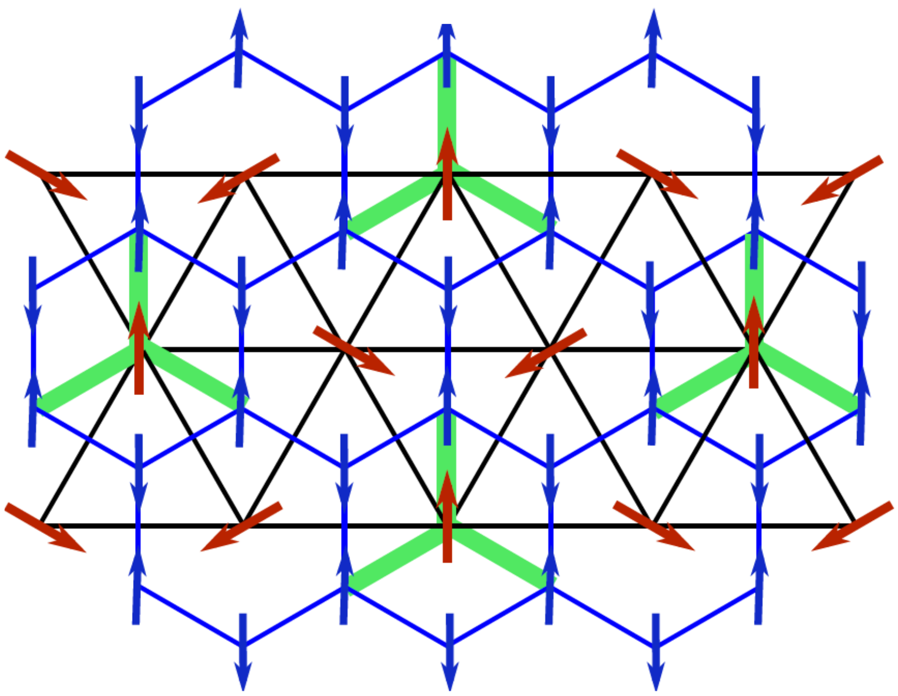
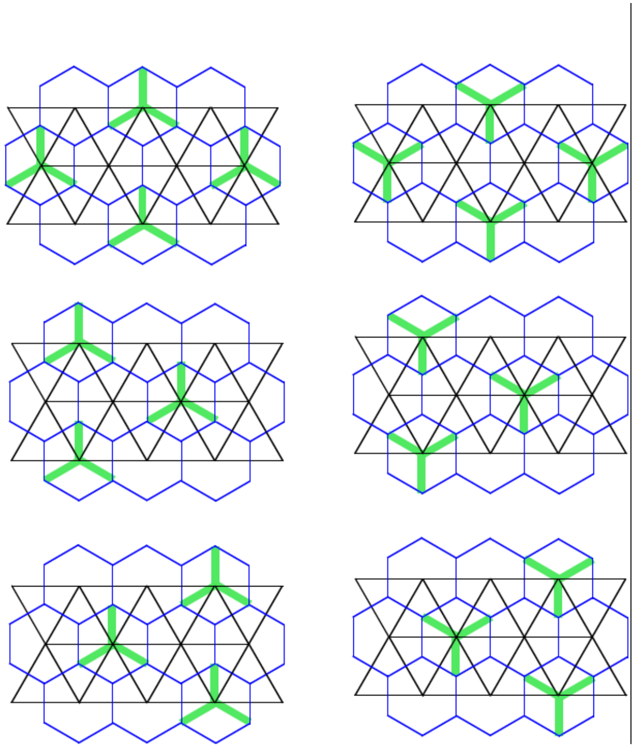
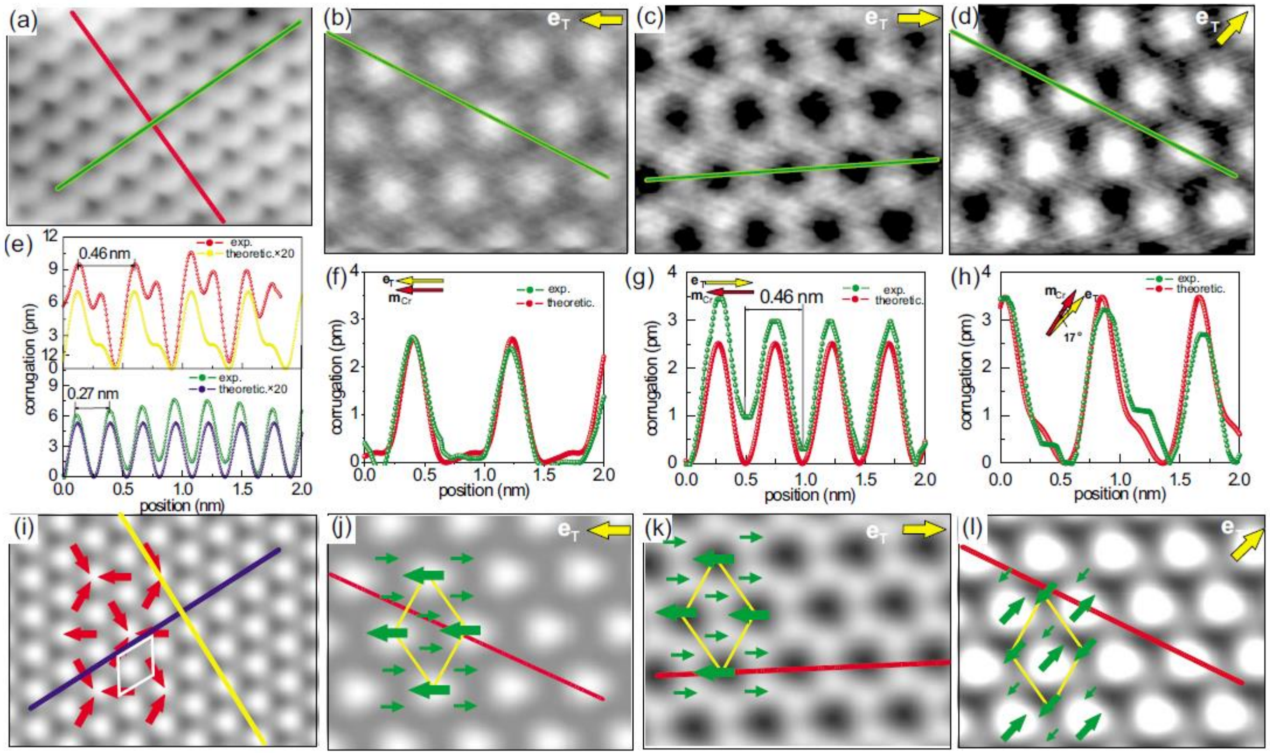




Materials and experimental realization

- Honeycomb antiferromagnet $Na_{1-x}NiSbO_6$ when replacing Sb by magnetic atom [1, 2] is a candidate
- Single layer Cr on Pd 111 surface. Spin polarized STM [3]



Bilayer possible candidate for windmill model realization

[1] J. H Roudebush, R. J. Cava, J. Solid State Chem. **204**, 178 (2013);
[2] K. Ross (private communication); [3] M. Waisniowska *et al.*, PRB **82**, 012402 (2010).

Long-wavelength renormalization group approach

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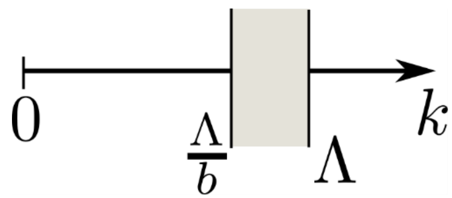
RG of long-wavelength action

Action of O(3)/O(2) x SO(3) Non-Linear Sigma Model (NLSM) plus potential

$$S = \int d^2x \left(\frac{K}{2} (\partial_\mu \boldsymbol{n})^2 + \sum_{j=1}^{3} \frac{K_j}{2} (\partial_\mu \boldsymbol{t}_j)^2 \right) + S_c$$

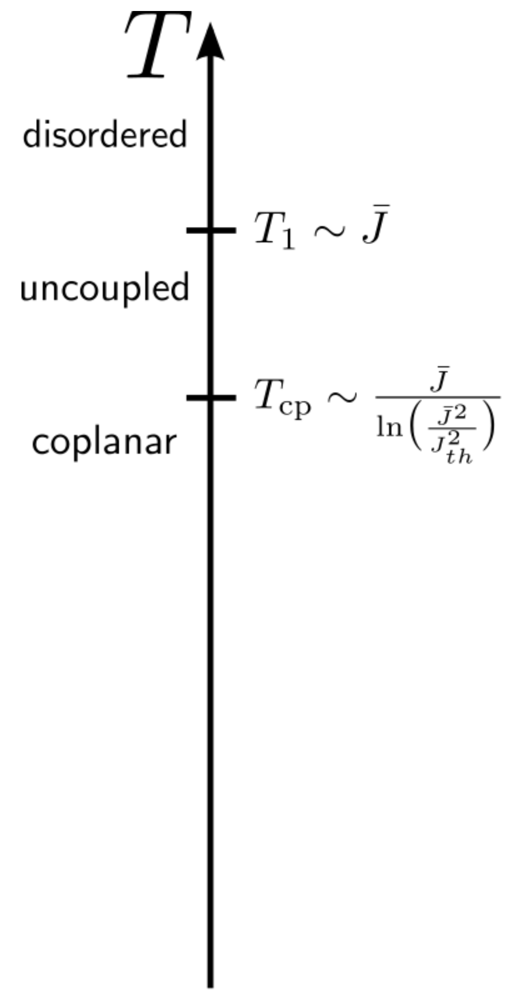
$K = \frac{J_{hh}}{T}$, $K_{1,2} = \frac{J_{tt}}{T}$, $K_3 = 0$

Renormalization group: integration over short-wavelength fluctuations



Spin stiffnesses are reduced at longer lengthscales

$$\frac{dK}{dl} = -\frac{1}{2\pi}; \quad \frac{dK_1}{dl} = -\frac{(1+\eta)^2}{8\pi}; \quad a(l) = a_0 e^l$$



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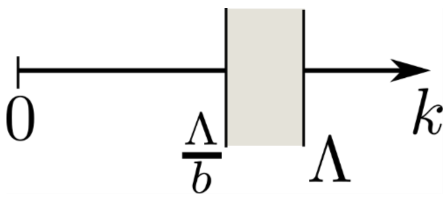
$K = \frac{J_{hh}}{T}$, $K_{1,2} = \frac{J_{tt}}{T}$, $K_3 = 0$

Renormalization group: integration over short-wavelength fluctuations



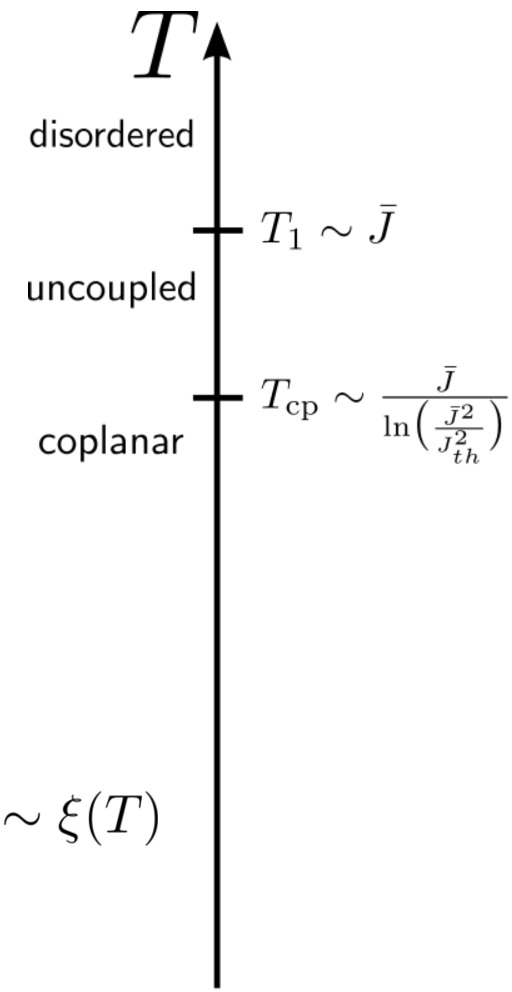
Spin stiffnesses are reduced at longer lengthscales

$$\frac{dK}{dl} = -\frac{1}{2\pi}; \quad \frac{dK_1}{dl} = -\frac{(1+\eta)^2}{8\pi}; \quad a(l) = a_0 e^l$$

Potential terms grow: $\frac{d\gamma}{dl} = 2\gamma; \frac{d\lambda}{dl} = 2\lambda$

Coplanar crossover at temperature: $\gamma(l_\gamma) = 1,\ a_0 e^{l_\gamma} \sim \xi(T)$

$$T_{\rm cp} \sim \frac{\bar{J}}{\ln(\bar{J}^2/J_{th}^2)}$$



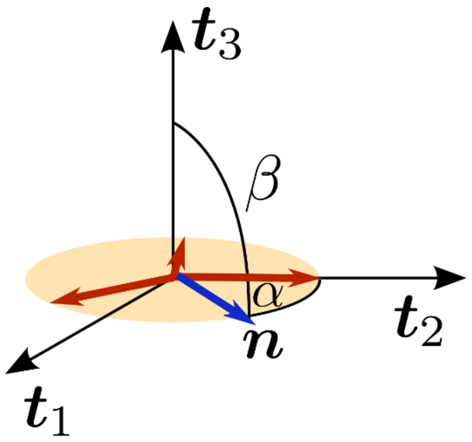
Coplanar regime

- Dynamics is intimately connected $\boldsymbol{n} \perp \boldsymbol{t}_3$

$$h = (\boldsymbol{n}, \quad \boldsymbol{h}_2, \quad \boldsymbol{h}_3) \in SO(3)$$
$$t = (\boldsymbol{t}_1, \quad \boldsymbol{t}_2, \quad \boldsymbol{t}_3) \in SO(3)$$

$$\Longrightarrow \quad t = hU = h\exp(i\alpha\tau_3)$$

- Order parameter symmetry SO(3) x Z_6
  - 3 Euler angles and relative phase $X = (\phi, \theta, \psi, \alpha)$



$$S = \frac{1}{2}\int_x \Big(I_1(\Omega_\mu^1)^2 + I_2(\Omega_\mu^2)^2 + I_3(\Omega_\mu^3)^2 + I_\alpha(\partial_\mu \alpha)^2 + \kappa(\partial_\mu \alpha)\Omega_\mu^3 \Big)$$

Angular velocity: $\Omega_\mu = h^{-1}(\partial_\mu h)$

SO(3) stiffnesses (moments of inertia)

$$I_1 = K_2 + K_3, \; I_2 = K + K_1 + K_3$$
$$I_3 = K + K_1 + K_2$$

U(1) stiffness and SO(3)-U(1) coupling

$$I_\alpha = K_1 + K_2$$
$$\kappa = 2(K_1 + K_2)$$

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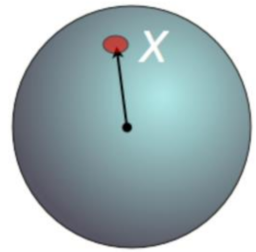
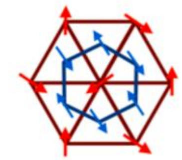
Covariant action

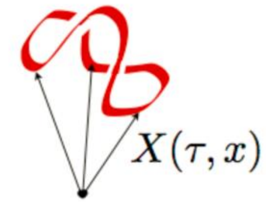
- **Action** of 2D spin system takes form of (Euclidean) **string theory** [1]

3 Euler angles and relative phase

$$X(\tau, x) = (\phi, \theta, \psi, \alpha)$$







Magnetization X = displacement of string in D dimensions

$S_3 \times S_1$

- **Spin stiffnesses define metric tensor**

[1] D. Friedan, PRL **45**, 1057 (1980)

$$S = \frac{1}{2}\int d^2x \, g_{ij}[X(x)]\partial_\mu X^i(x)\partial_\mu X^j(x) + \frac{\lambda}{2}\int d^2x \sin^2(3\alpha)$$

$$g = \begin{pmatrix} g^{SO(3)} & \mathcal{K}^T \\ \mathcal{K} & I_\alpha \end{pmatrix}$$

SO(3) stiffnesses I_1, I_2, I_3

U(1) stiffness

U(1) phase is coupled to non-Abelian sector

Does U(1) sector decouple from SO(3) sector with finite XY stiffness $I_\alpha > 0$?

RG flow = Ricci flow

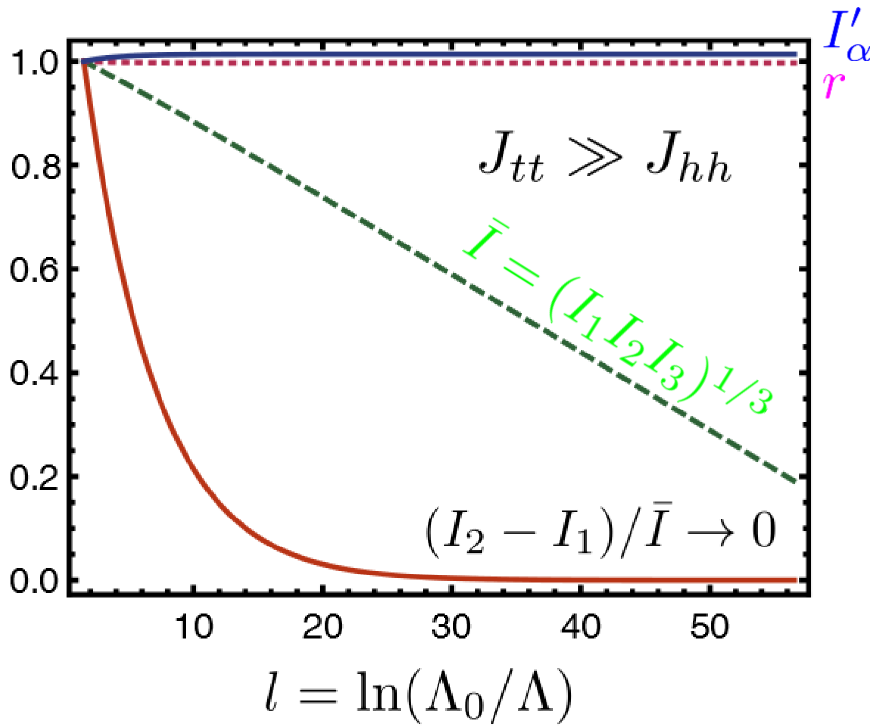
- Action is covariant with stiffness metric tensor
- Covariance is preserved during RG scaling [1]
- RG flow of the metric is given by the Ricci flow [1,2] (two loops)

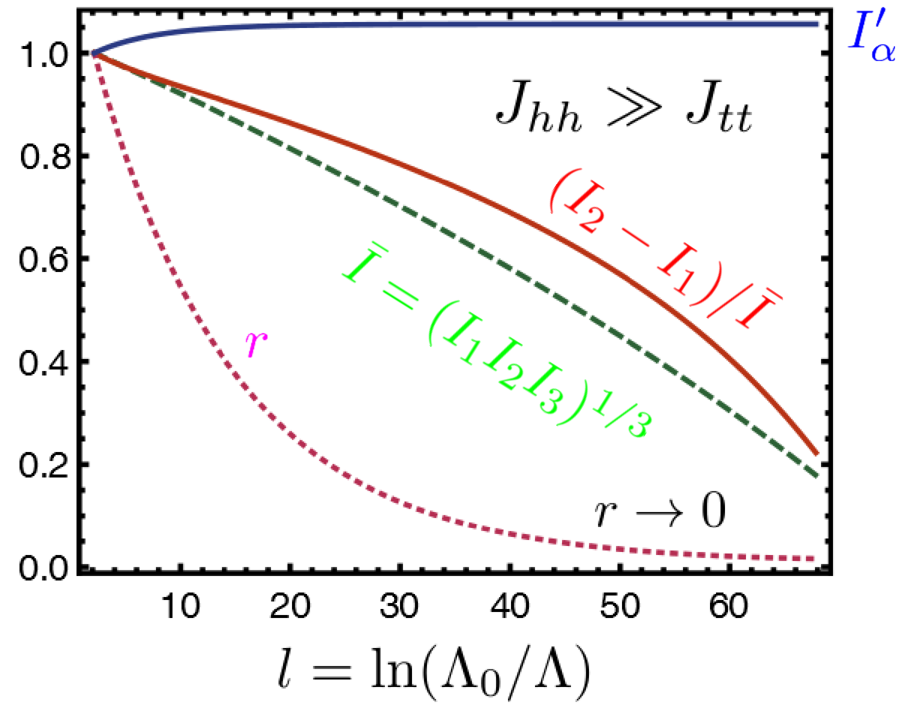
$$\frac{dg_{ij}}{dl} = -\frac{1}{2\pi}R_{ij} - \frac{1}{8\pi^2}R_i{}^{klm}R_{jklm}$$

Ricci and Riemann tensor determined by g_{ij}

[1] D. Friedan, PRL **45**, 1057 (1980); [2] R. S. Hamilton, J. Differential Geom. **17**, 255 (1982)

Phase decoupling RG flow





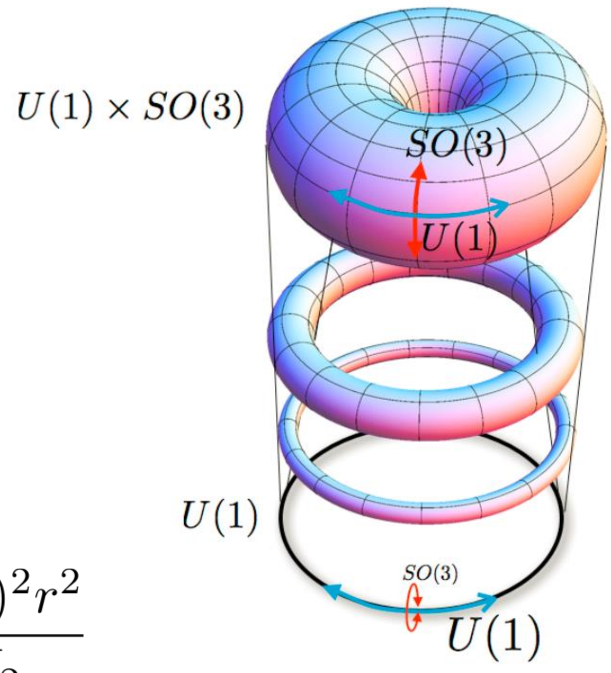
Decoupling of U(1) phase α emerges rapidly

Renormalization of XY stiffness then stops at finite value

$$\frac{d}{dl} I'_\alpha = \beta_\alpha \equiv \frac{(I_1 - I_2)^2 r^2}{4\pi I_1 I_2}$$

Decoupling as toy model for compactification

- One-dimensionsal U(1) part of manifold decouples from 3D non-Abelian SO(3) part



- Ricci scalar grows like

$$R = R^{SO(3)} - \frac{1}{2\pi I'_\alpha}\beta_\alpha$$

$$R^{SO(3)} \sim 1/\bar{I}$$

$$\beta_\alpha = \frac{(I_1 - I_2)^2 r^2}{4\pi I_1 I_2}$$

SO(3) part curles up:

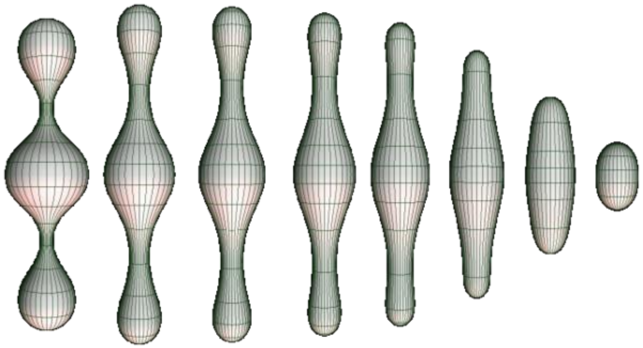
U(1) becomes flat

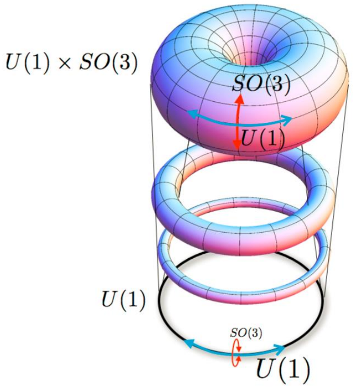
Toy model for compactification [1,2]

[1] M. Gell-Mann and B. Zwiebach, Phys. Lett. B **141**, 333 (1984);
[2] L. Randall and R. Sundrum, PRL **83**, 4690 (1999)

Simulation of exact Ricci flow

- Singularities in two-loop Ricci flow are false Landau poles
- **Use classical magnet to simulate exact Ricci flow**
- Protocol:
  - Suitable magnet realizes given metric
  - Cool system
  - Measure spin correlation functions at various temperatures
  - Extract metric tensor
  - Obtain “surgery-free” generalized Ricci flow of manifold
- Connection to proof of the Poincare conjecture by Perelman (2006): “Every simply connected, closed 3-manifold is homeomorphic to a 3-sphere”





[1] P. Coleman, A. Tsvelik (private communication)

Emergent six-state clock model

- Action of the decoupled U(1) degree of freedom

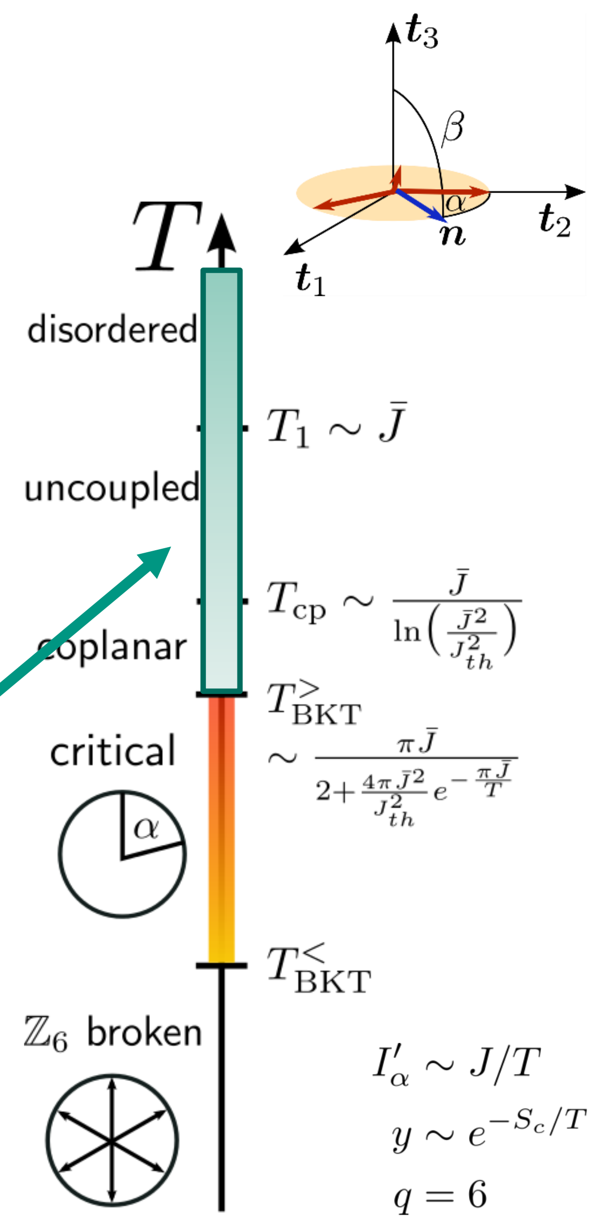
$$S_{\mathbb{Z}_6} = \frac{1}{2}\int d^2x \left[(I'_\alpha (\partial_\mu \alpha)^2 + \lambda \cos(6\alpha) \right].$$

- I'_α renormalizes only due to vortices (BKT-RG)

$$\frac{d(I'_\alpha)^{-1}}{dl} = 4\pi^3 y^2, \quad \frac{dy}{dl} = (2 - \pi I'_\alpha) y$$

- Relevant for $I'_\alpha < 2\pi \Leftrightarrow T > \pi J/2$

→ proliferation of free vortices



[1] J.V. Jose et al., PRB **16**, 1217 (1977)

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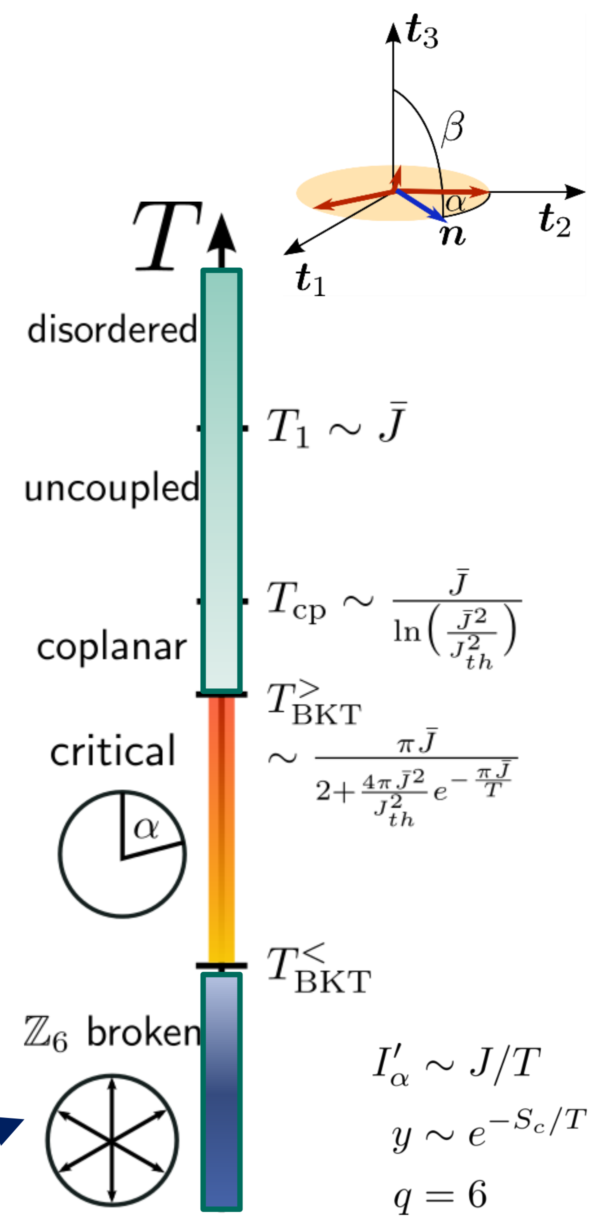
$$\frac{d\lambda}{dl} = (2 - \frac{q^2}{4\pi I'_\alpha})\lambda$$

- Relevant for $I'_\alpha < 2\pi \Leftrightarrow T > \pi J/2$
  → proliferation of free vortices
- Relevant for $I'_\alpha > q^2/8\pi \Leftrightarrow T < 8\pi J/q^2$
  → pinning into one of six minima



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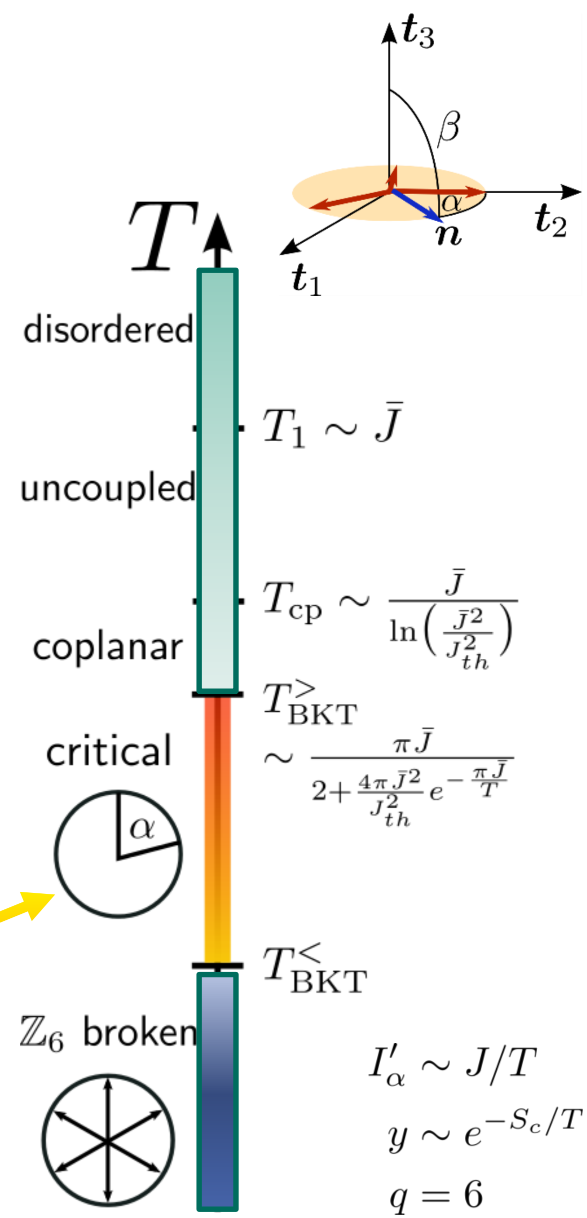
❑ Relevant for $I'_\alpha < 2\pi \Leftrightarrow T > \pi J/2$

➡ proliferation of free vortices

Power-law correlations $\exp[i(\alpha(x) - \alpha(x'))]$

❑ Relevant for $I'_\alpha > q^2/8\pi \Leftrightarrow T < 8\pi J/q^2$

➡ pinning into one of six minima



[1] J.V. Jose et al., PRB **16**, 1217 (1977)

Monte-Carlo simulations

Monte-Carlo simulation of classical model

- Simulate microscopic model of Heisenberg spins on windmill lattice [1]
- Use combination of Monte-Carlo moves
  - parallel-tempering
  - heat-bath step
  - global rotation of honeycomb spins

Define emergent XY spins on each plaquette

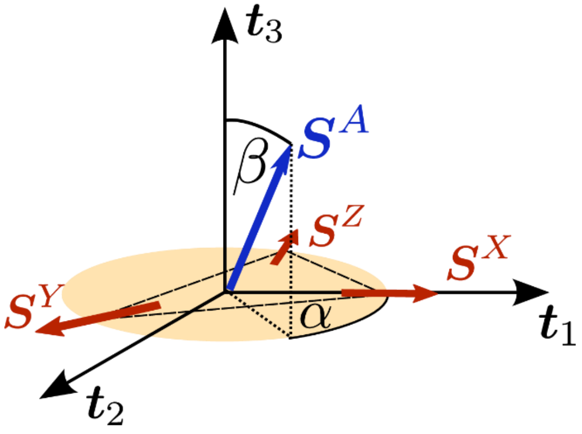
$$\boldsymbol{m}_j = \frac{\left(\boldsymbol{S}_j^A \cdot \boldsymbol{t}_{1,j}, \boldsymbol{S}_j^A \cdot \boldsymbol{t}_{2,j}\right)}{\left\|\left(\boldsymbol{S}_j^A \cdot \boldsymbol{t}_{1,j}, \boldsymbol{S}_j^A \cdot \boldsymbol{t}_{2,j}\right)\right\|} = \left(\cos\alpha_j, \sin\alpha_j\right).$$

Measure magnetization

$$\boldsymbol{m} = \frac{1}{N}\sum_{j=1}^{N}\boldsymbol{m}_j = |\boldsymbol{m}|(\cos\alpha, \sin\alpha).$$

And susceptibility

$$\chi(T,L) = \frac{N}{T}\langle|\boldsymbol{m}|^2\rangle = \frac{1}{NT}\left\langle\left|\sum_j \boldsymbol{m}_j\right|^2\right\rangle$$

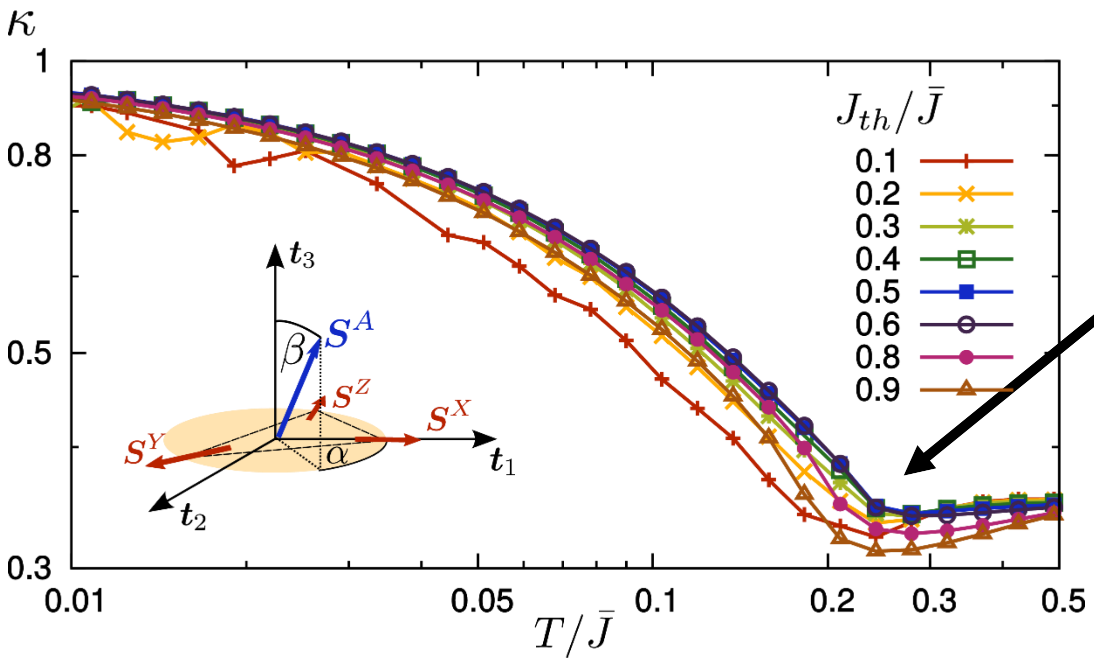


• [1] B. Jeevanesan, P. Chandra, P. Coleman, PPO, PRL **115**, 177201 (2015).

Coplanar crossover

- Estimator for coplanarity

$$\kappa = 1 - \frac{3}{N}\sum_{j=1}^{N}\langle\cos^2\beta_j\rangle = \begin{cases} 1/3\,, & \text{for random relative configuration} \\ 0 & \text{uncorrelated local } 120^\circ \text{ and Néel order} \\ 1\,, & \text{for coplanar configuration}\,. \end{cases}$$



Coplanar crossover at

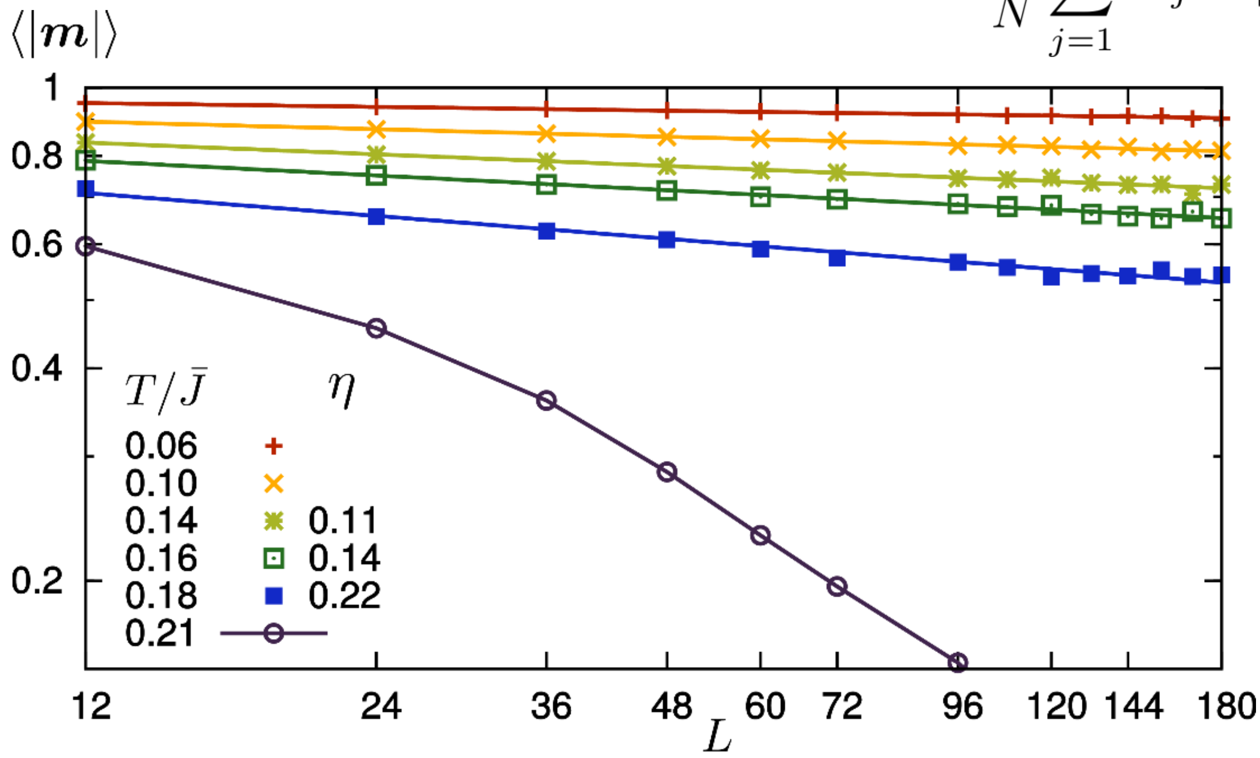
$$T_{\mathrm{cp}} \sim \frac{\bar{J}}{1+\ln(\bar{J}^2/J_{th}^2)}$$

Relatively independent of $J_{th}/\bar{J}$

System size $L = 60 \times 60$

Magnetization versus system size

$$\boldsymbol{m} = \frac{1}{N}\sum_{j=1}^{N}\boldsymbol{m}_j = |\boldsymbol{m}|(\cos\alpha, \sin\alpha)\,.$$



- Vanishes faster than algebraic for large temperatures
- Vanishes algebraically for lower temperatures $\langle|\boldsymbol{m}|\rangle \propto L^{-\eta(T)/2}$
- Temperature dependent exponent $0 < \eta(T) < 0.3$

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Scaling of susceptibility of emergent order

- Finite size scaling of susceptibility at upper transition temperature [1]

Magnetization

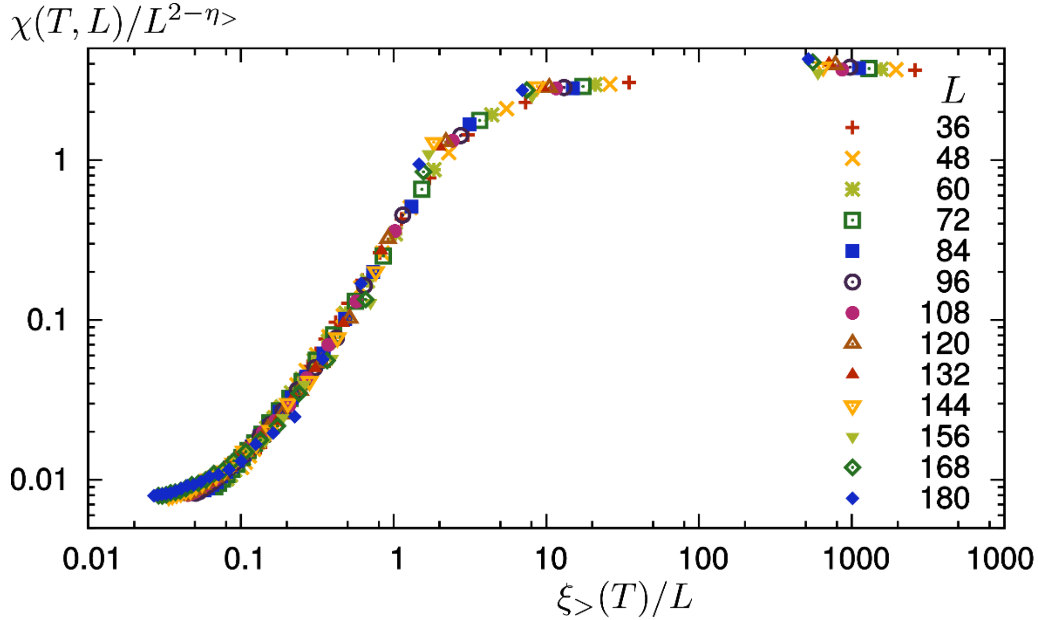
$$m_L = \frac{1}{L^2} \sum_i (\cos\alpha_i, \sin\alpha_i)$$

Susceptibility

$$\chi_L = \langle m_L^2 \rangle$$

Scaling for $T > T_{BKT}^{>}$

$$\xi \sim \exp(a/\sqrt{\Delta T}),\ \chi \sim \xi^{2-\eta}$$



$$\chi_L \sim L^c Y_2(\exp(a/\sqrt{\Delta T}),\ c = 2 - \eta$$

- Confirms BKT nature of upper transition
- Transition temperature $T_{BKT}^{>} = 0.200(4)\bar{J}$
- Exponent $\eta_> = 0.25(1), a_> = 1.9(3)$

Scaling of magnetization of emergent order

- Finite size scaling of magnetization at lower transition temperature

Magnetization

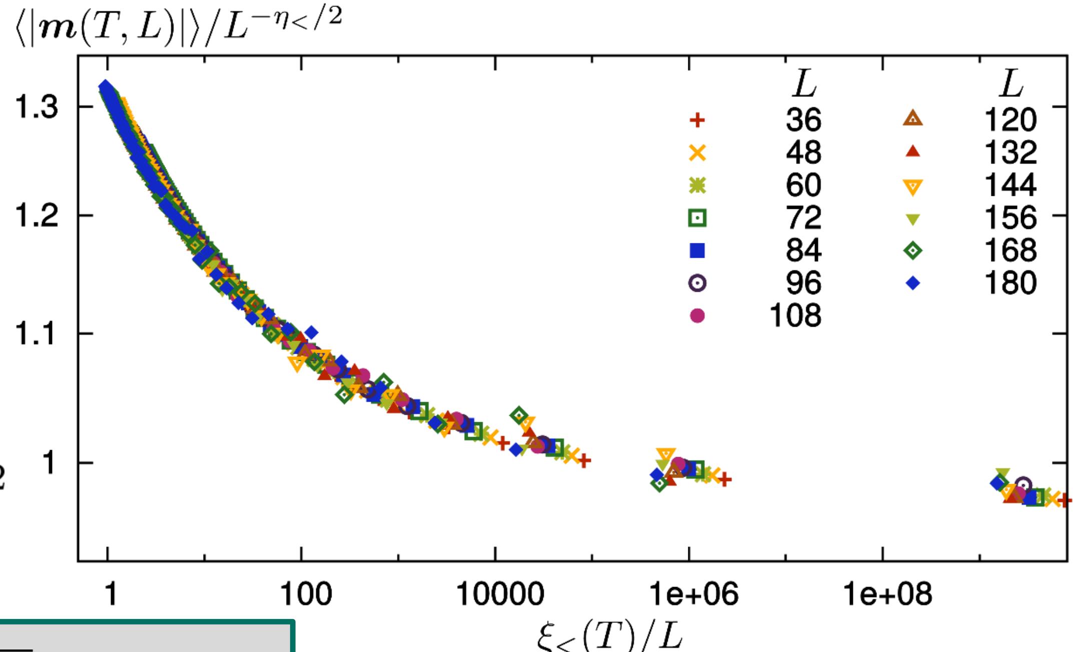
$$m_L = \frac{1}{L^2} \sum_i (\cos\alpha_i, \sin\alpha_i)$$

Susceptibility

$$\chi_L = \langle m_L^2 \rangle$$

Scaling for $T_{BKT}^{<} \leq T \leq T_{BKT}^{>}$

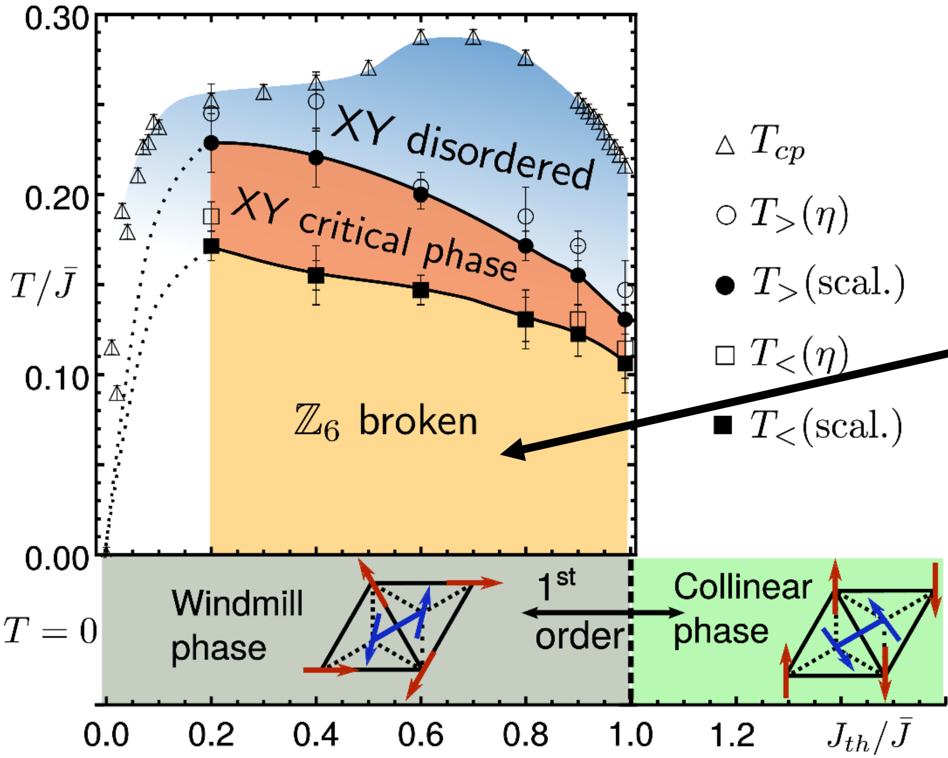
$$\xi \sim \exp(a/\sqrt{\Delta T}),\ m_L \sim \xi^{-\eta/2}$$

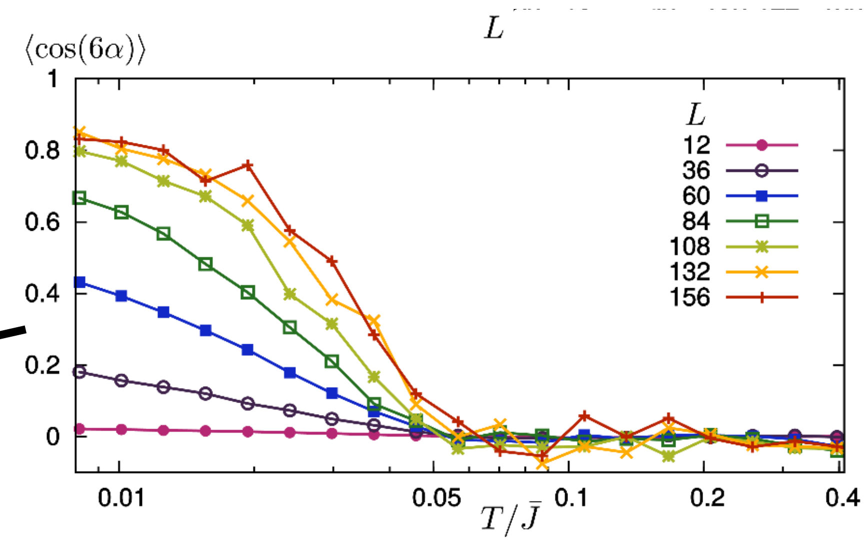


$$m_L \sim L^{-b} Y_1(L^{-1}\exp(a/\sqrt{\Delta T}),\ b = \eta/2$$

- Confirms BKT nature of upper transition
- Transition temperature $T_{BKT}^{<} = 0.18(1)\bar{J}$
- Exponent $\eta_< = 0.11(1), a_< = 5.0(5)$

Phase diagram



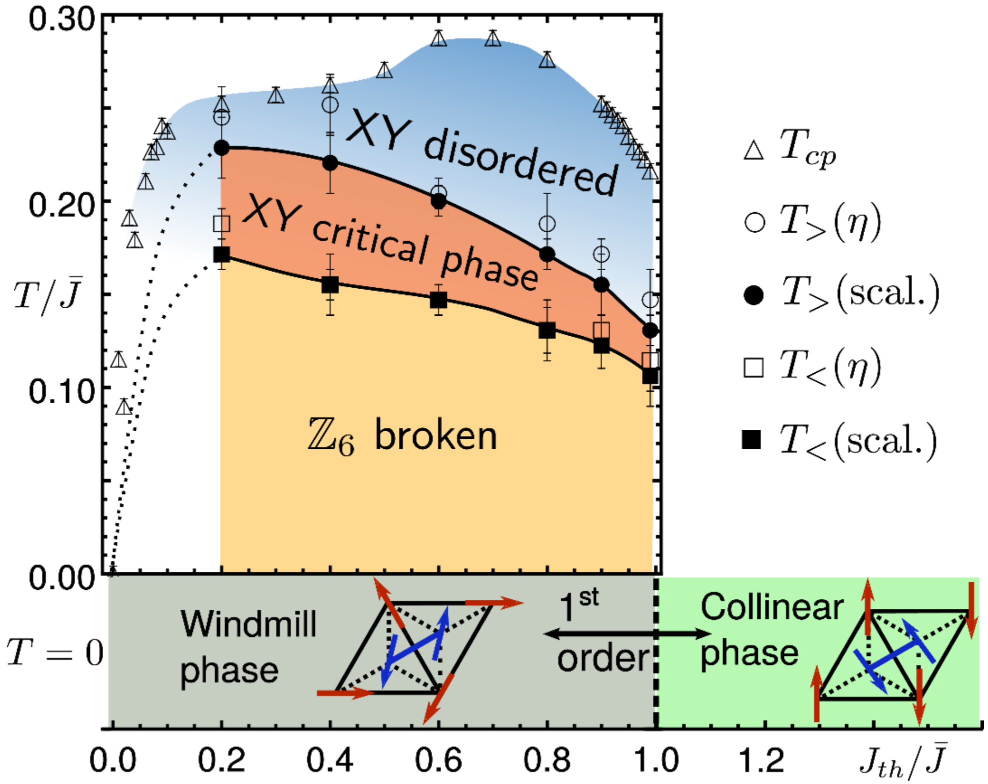


System falls into one of the six minima
Long-range discrete order occurs.

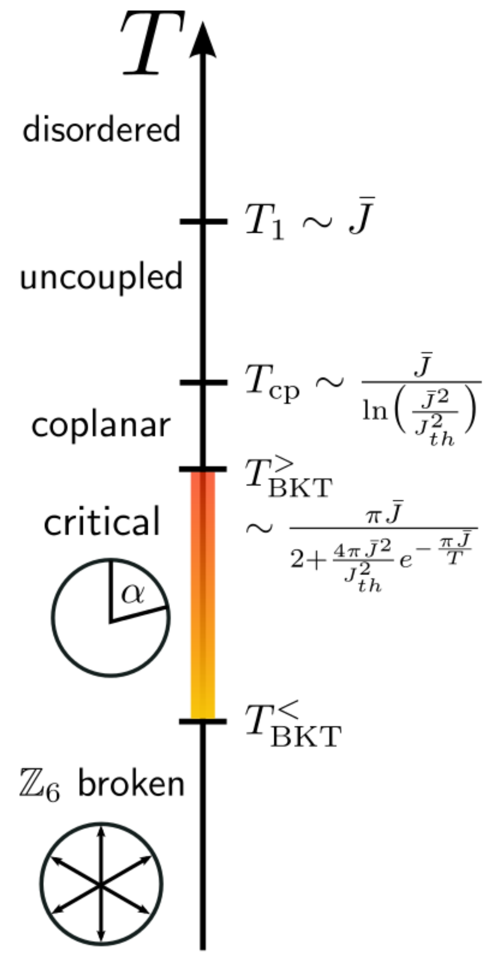
- Direct detection of algebraic and long-range order of emergent XY spins in Heisenberg model.

Phase diagram

Monte-Carlo phase diagram



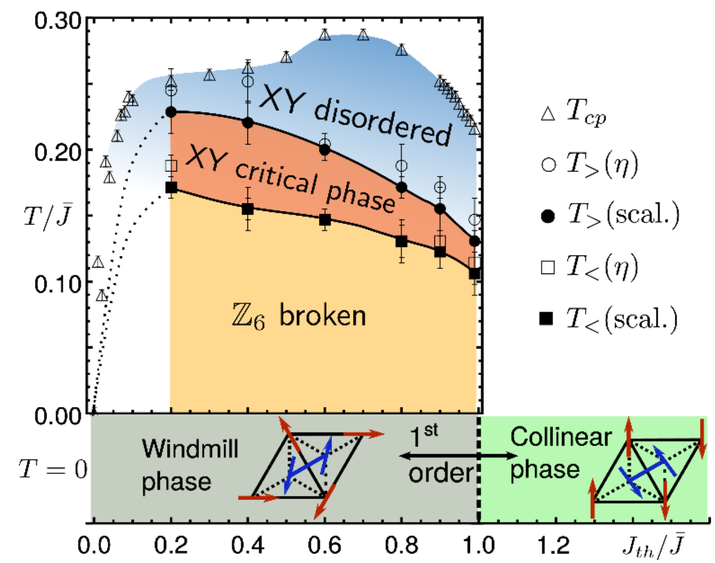
RG phase diagram



- Monte-Carlo simulations provide unbiased verification of long-wavelength picture.

Summary and outlook

- Critical phase in frustrated 2D Heisenberg antiferromagnet
- Algebraic correlations in relative orientation of spins
- Design windmill material
- Implement numerical program to measure spin stiffnesses and Ricci flow
- Apply analysis to other (layered) materials
  - Triangular bilayer, Kagome-Honeycomb, etc…
- Planar XY version



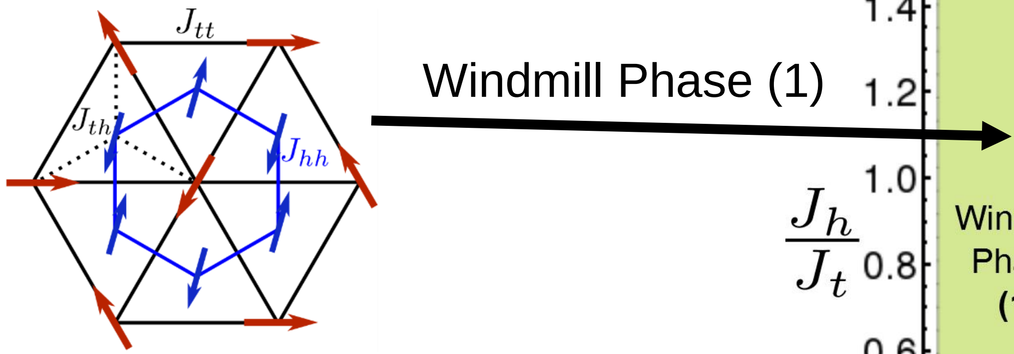
References:

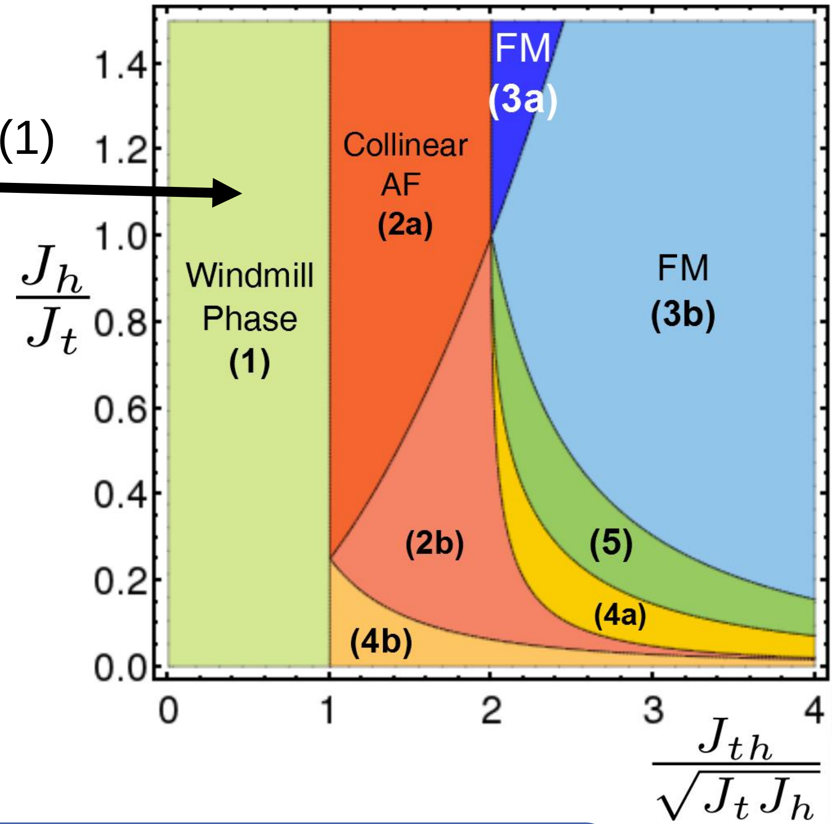
- B. Jeevanesan, P. Chandra, P. Coleman, PPO
  Phys. Rev. Lett. **115**, 177201 (2015).
- B. Jeevanesan, PPO
  Phys. Rev. B **90**, 144435 (2014).
- PPO, P. Chandra, P. Coleman, J. Schmalian
  Phys. Rev. B **89**, 0994417 (2014).
  Phys. Rev. Lett. **109**, 237205 (2012).

Thank you for your attention!

Classical ground state phase diagram

- Complete phase diagram hosts 8 different phases [1]





- Windmill phase stable for non-zero J_{th}
- First order phase transition between (1) and (2a) and (4b) is analogous to J_1-J_2-model

Interested in finite temperature phase diagram above phase (1)

[1] B. Jeevanesan, PPO, PRB **90**, 144435 (2014).