Spring 2022

EMISSARY

Mathematical Sciences Research Institute

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Complex Dynamics: From Special Families to Natural Generalizations in One and Several Variables

Núria Fagella, Charles Favre, and Liz Vivas

A transcendental meromorphic function with a superattracting fixed point (basin shown in orange), an attracting 3-cycle (basin in red, homeomorphic to the Douady rabbit), and a Cantor bouquet — a Cantor set of curves of points which escape to infinity under iteration.

More than twenty-five years after the vivid and scientifically exciting program in Complex Dynamics and Hyperbolic Geometry at MSRI (organized by B. Branner, S. Kerckhoff, M. Lyubich, C. McMullen, and J. Smillie in 1995), the complex dynamics community is back at MSRI with new ideas on old problems and new sets of questions. Special families of complex maps (either polynomial, rational, or transcendental) are now better understood, and the field is naturally moving towards generalizations in one and several variables, finding unexpected connections with other areas of mathematics (like group theory, statistical mechanics, and even arithmetic geometry).

Fatou–Julia Theory

Let us start by reviewing the basic steps of the theory of iteration of holomorphic maps as developed by Fatou and Julia in the first part of the twentieth century. Let $f\colon \mathbb{C} \to \mathbb{C}$ be any holomorphic map. We are interested in the iterates of f which are defined inductively

(continued on page 13)

Celebrating David Eisenbud, Outgoing Director

David Eisenbud's term as Director will end on July 31, 2022, and **Tatiana Toro** will become MSRI's new Director on August 1. David explains **what's so great about being at MSRI** in his View column (page 2) and Hélène Barcelo honors David with an **appreciation of his dedication to service** (page 3).

Also in the issue...

MSRI will host a **May 12 Celebration of Women in Mathematics Workshop** (page 20); the **2022 Mathical Book Prize** winners are honored (page 11); the **National Math Festival** hosts spring events online (page 10); the **Puzzles Column** features problems from Ukrainian Math Olympiads (page 17) . . . and much more*!*

The View from MSRI

David Eisenbud, Director

What's so great about being at MSRI?

This is a question that could be for the members, workshop participants, those who come to Summer Research or ADJOINT programs, and the students who come to summer graduate schools or MSRI-UP. I have some access to their points of view through exit surveys, and we generally see great satisfaction. For example, on the survey taken by all members as they left MSRI last fall, responses to the item "Professionally, my overall satisfaction with MSRI was..." averaged 4.82 out of a possible 5.

But I don't have to analyze statistics to see (and hear) that the members who have come to MSRI this year are positively joyful to be chatting math in person once again! Deputy Director Hélène Barcelo and I take our postdocs to lunch in small groups at this time of year to hear how things are going and to see if course-corrections are needed. The responses this year are particularly gratifying.

I will retire as Director of MSRI (for the second time) on August 1, 2022, though I'll remain a Professor at UC Berkeley, so I've been thinking about my own answer to the question. Part of my satisfaction comes simply from meeting so many mathematicians and supporters of mathematics, each with his or her own enthusiasm, like those shown in the photographs on page 3.

Perhaps the greatest source of satisfaction is a consequence of the powerful support of the math community, the NSF, and more recently a community of private foundations and individual donors: because of them, MSRI is a place where good ideas can be realized with a minimum of bureaucracy, and where the results are visible in colleagues' enjoyment of the work and the setting.

Here are some of the projects that the wonderful resources of MSRI made it possible to realize through my first two terms (1997–2007) and after I returned to the Directorship in 2013: the creation of the Banff International Research Station (BIRS); the "Hot Topics," "Critical Issues in Math Education," and "Connections" workshop series; the great increase in the number of summer graduate schools; the founding of the Blackwell–Tapia Prize and workshop series; the MSRI-UP undergraduate research program; the MSRI–CME Group prize for mathematical contributions in finance and economics; the doubling of the AMS's Congressional Briefings in Washington, DC; the increased emphasis on mentoring postdocs and the creation of a number of endowed postdoctoral fellowships; and, for the public, the National Math Festival, the Mathical Book Prize, and the YouTube channel *Numberphile*.

I worked on all these projects, but none could have been realized by me alone. Indeed, part of the fun was in the necessary collaborations! For example, BIRS came about because I was able to jump

in and abet Nassif Ghoussoub's effort; the math education activities were developed with Deborah Ball; and the Mathical Book Prize sprang from an idea of Roger Strauch and has been developed by Kirsten Bohl.

In impact, in scope, in the number of people whose enthusiasm and work was involved, one of my biggest projects was the extension of MSRI's building, now named Chern Hall. (There are four photos that show the process and the result on page 3.)

Among those whose collaboration was essential in this undertaking were architect Bill Glass, Academic Sponsors Chair Doug Lind, Development Director Jim Sotiros, Board Chair Dusa MacDuff, Deputy Directors Hugo Rossi, Michael Singer, and Bob Megginson, and of course the donors — Jim and Marilyn Simons, Will Hearst, Ed Baker and many, many others.

It gives me great pleasure now to look back at the many projects and collaborations that I've formed, and the many mathematicians who have enjoyed the results, over the last 25 years. Currently I'm busy with an exciting MSRI project that looks to the future, almost ready to reveal. Watch your email for an announcement on May 18!

In so many ways, my association with MSRI has been one of the great experiences of my life.

In addition to his service as Director — celebrated by Hélène Barcelo on the next page — David's photography has helped document MSRI's life over the years. You can enjoy the cycle of the seasons in the (physical!) views from MSRI above, and feel the energy and growth of the institute in the set of photos he selected to accompany this issue's View column, also on page 3.

Contents

Mathematicians and their enthusiasm. *Two great number theorists: Peter Scholze explains a new idea to Richard Taylor (top left); Postdocs at play — I mean work! (bottom left).* ***The evolution of Chern Hall.*** *(clockwise from top middle) A classic, foggy view of the original entryway; Celebration outside the old library at the groundbreaking; Construction site for the Simons Auditorium; New entryway at night; Roger Penrose gives the inaugural Lecture in the new Simons Auditorium; The new auditorium is also a concert hall: Davitt Moroney performs Bach.*

Celebrating David Eisenbud's Service

Hélène Barcelo, Deputy Director

David Eisenbud, a distinguished and prolific researcher, first joined MSRI in 1997 to oversee the institute and expand its mission as one of the world's preeminent centers for collaborative research. After decades of service, he will retire from the Directorship of MSRI in August 2022.

David's leadership has had a profound impact on MSRI and the mathematical profession at large. As he wrote in a recent prospectus, "After four decades of innovation and growth, MSRI's potential to lead, shape, and diversify the future of mathematics is greater than ever." His contributions toward these goals have been fundamental and he leaves us well-poised for continued success far into the future.

David has spearheaded a multitude of innovative projects to engage a diverse and energetic next generation of mathematical talent. From workshops on insect Navigation, game theory, and the theory of neural computation, to the National Math Festival and award-winning YouTube channel *Numberphile*, his uncanny flair for improving the public perception of mathematics is indisputable.

Also indisputable is David's phenomenal success with fundraising, both for the expansion of MSRI's building and its endowment. What was once considered a dream is now a reality thanks to David's unparalleled talent for communicating MSRI's value and vision to our magnanimous benefactors.

Of course, beside many a great leader is a great partner. An esteemed psychiatrist, Monika Eisenbud has been a tireless supporter of both David and MSRI, devoting time, energy, and sage counsel toward the institute's success. She has also graciously hosted and supported MSRI's events, many of which have taken place in their beautiful home in the Berkeley Hills.

Serving alongside David during his transformative tenure has been a privilege. Among many other lessons, he has taught me how to intersperse rapid and bold decisions amidst careful long-range planning. David is an exemplar of both leadership and scientific excellence and his legacy will endure. Indeed, the mathematical community owes him a *fière chandelle*!

Analysis and Geometry of Random Spaces

Mario Bonk, Ellen Powell, Steffen Rohde, Eero Saksman, Alan Sola, and Fredrik Viklund

This program is devoted to the investigation of analytic and geometric objects that arise from natural probabilistic constructions, often motivated by models in mathematical physics. An important example of such an object is one-dimensional Brownian motion, which can be used to describe the random motion of small particles in a liquid. One-dimensional Brownian motion can be thought of mathematically as a random real-valued function of one (time) variable. It is "universal" as it appears in many different, and *a priori* unrelated, contexts in probability and analysis.

Random objects depending on *two* variables are related to random surfaces. They appeared in the physics literature already in the early 1980s in connection with conformal field theory and what is known as two-dimensional Liouville quantum gravity (LQG), but a solid mathematical understanding of random surfaces has been established only very recently. Now there are many different ways to define these objects rigorously, for instance, as scaling limits of random surface triangulations.

Random surfaces and other related geometric structures exhibit a rich non-smooth and fractal geometry. While some amount of regularity is still present, they frequently fall (just) outside the scope of standard analytic machinery. One of the main goals of this semester's program is to develop new analytic, probabilistic, and geometric tools necessary for their study.

From Random Functions to Random Surfaces

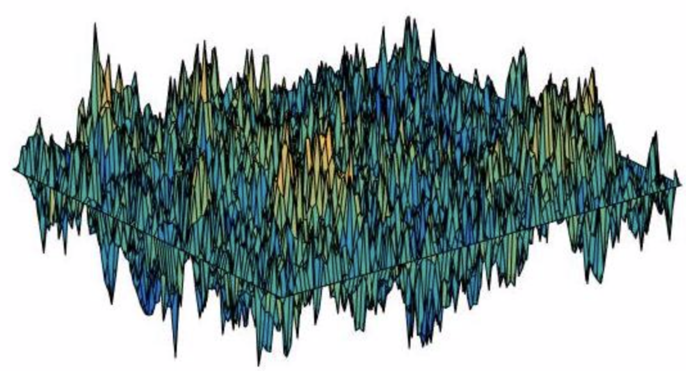
In the late 19th century, the mathematical community was shocked when Weierstrass produced examples of continuous functions that are nowhere differentiable. Now we understand that a "typical" continuous function always shows this seemingly exotic behavior. One can use one-dimensional Brownian motion $t \mapsto B_t$ to illustrate this point and at the same time make the previous statement precise.

For this we recall that one-dimensional Brownian motion is described by Gaussian random variables B_t, $t \in [0, \infty)$, with mean 0. Here $B_0 = 0$ and the random variables B_t are tied together to form a continuous process $t \mapsto B_t$ such that increments $B_{t+h} - B_t$, $h \geqslant 0$, are independent of B_s for $0 \leqslant s \leqslant t$ with variance $\mathbf{E}[(B_{t+h} - B_t)^2] = h$. These requirements uniquely characterize the process; existence can be shown by using random Fourier series or scaling limits of simple random walks on the non-negative integers.

Each sample path $t \mapsto B_t$ of Brownian motion is a continuous function selected, informally speaking, at random among all continuous functions. One can show that with probability 1 that this function is nowhere differentiable; so for a continuous function it is indeed "typical" to show this non-smooth behavior.

By now the mathematical theory of Brownian motion is highly developed and has culminated in theories such as Ito's stochastic calculus. There are many variants of the process, such as the Brownian bridge (a process starting at 0 and conditioned to return to zero at time $t = 1$), and Brownian motion in higher dimensions. In dimension 2 the theory is particularly rich as conformal invariance properties of Brownian motion gives a natural connection to the theory of analytic functions.

Just as the mathematical theory of Brownian motion is inspired by physical processes, similar considerations lead to the study of random surfaces, again having an underlying Gaussian structure. In this context, the role of Brownian motion (or, more precisely, of the Brownian bridge) giving a natural way of sampling one-variable functions is played by the Gaussian free field (GFF).



A discrete approximation of the Gaussian Free Field on a square (image credit: Ellen Powell).

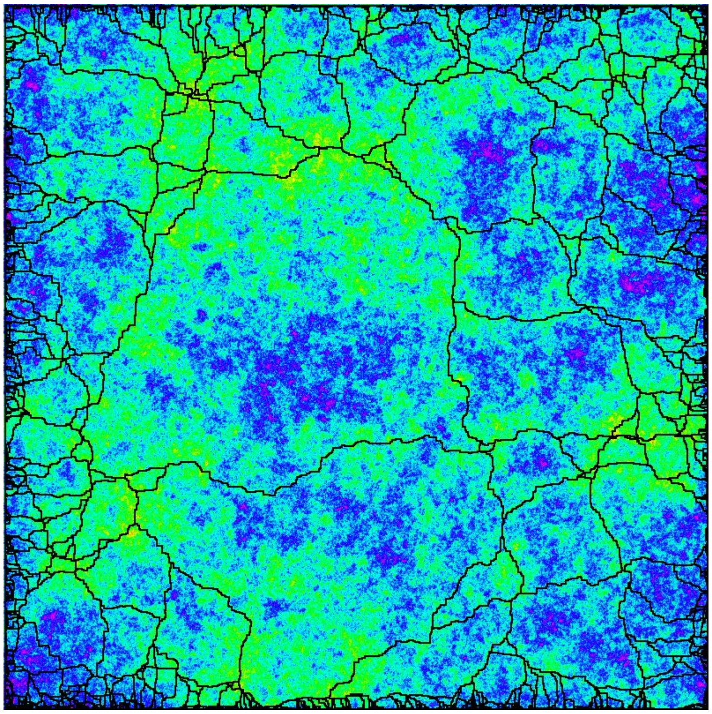
One would like to characterize this random object as a "field" $\{\Gamma(p)\}_{p \in D}$ of Gaussian random variables $\Gamma(p)$ indexed by points $p = (x, y) \in \mathbb{R}^2$ in a two-dimensional (bounded) region $D \subset \mathbb{R}^2$. Here one assumes that the Gaussians $\Gamma(p)$ have expectation 0 and covariance given by

$$\mathbf{E}[\Gamma(p)\Gamma(q)] = G_D(p, q),$$

where G_D denotes the Green function in D (associated with the Dirichlet problem for the Laplacian on D). See the figure above for an illustration. In principle, by the general theory of Gaussian processes, these requirements should be enough to characterize the random function $p \in D \mapsto \Gamma(p)$ uniquely, if it exists. Unfortunately, the Gaussian free field in fact does *not* exist as a random *function*. An intuitive explanation is that the Green function $G_D(p, q)$ blows up as $p \to q$, requiring each $\Gamma(p)$ to have infinite variance, which is impossible.

Fortunately, one *can* make sense of the GFF as a random *generalized function*, that is, a random *distribution*. Based on some probabilistic and functional analytic machinery, existence of the GFF can be shown, as in the one-dimensional case, by using random infinite series or by looking at scaling limits of discrete versions.

With this solid mathematical foundation, the Gaussian free field has been studied intensively in the last two decades. Conformal invariance takes on a central role: if D and D' are conformally equivalent regions in the plane, then the law of the GFF on D' can be obtained from that on D.



An LQG-surface for $\gamma = \sqrt{8/3}$ simulated with a GFF. The black curves are geodesics on the surface. Light colors indicate where the GFF has low (negative) values, and dark colors indicate where the GFF has high values (image credit: Wei Qian).

With the GFF in hand, a definition of "random surfaces" can be established rigorously. This is inspired by the work of the physicist Polyakov and others in the 1980s, and the mathematical treatment of Gaussian multiplicative chaos by Kahane in the same decade. One would like to consider two-dimensional manifolds equipped with a (random) Riemannian metric given by the expression

$$ds^2 = e^{\gamma\Gamma(p)}(dx^2 + dy^2), \tag{†}$$

with $\gamma > 0$ serving as a parameter. Since the Gaussian free field is a distribution and not a function, this expression is ill-defined: there is no known way of exponentiating a general distribution. Nevertheless, by considering regularizations Γ_ϵ of the GFF, it is possible, when $0 < \gamma < 2$, to make sense of a random area measure as

$$\lim_{\epsilon \to 0} \epsilon^{\gamma^2/2} e^{\gamma\Gamma_\epsilon(p)} dx\, dy.$$

With substantial further work, which is based on very recent advances, one can now give a precise meaning to the random metric in (†). The associated geometric structures, known as γ-LQG surfaces (see the figure above for an illustration), are too rough to be Riemannian manifolds in the usual sense; indeed, viewed as metric spaces, they have Hausdorff dimensions strictly greater than 2. Thus the search for "typical" or "randomly selected" surfaces once more leads to exotic objects, and has provided impetus for the development of new probabilistic and geometric tools to define and handle such spaces.

The random spaces one obtains as LQG-surfaces can be identified with scaling limits of certain discrete objects, just as Brownian motion is a limit of random walks. For instance, for certain natural models of triangulations chosen "uniformly at random," rescaling the resulting graph distances in a suitable way as the number of faces goes to infinity yields a scaling limit known as the Brownian map. This limiting random metric space has now been identified, in law, with a certain LQG-surface with parameter $\gamma = \sqrt{8/3}$. Remarkably, it has also been proven that if one conformally embeds in a canonical way the planar uniform triangulations in the limiting sequence, then the rescaled counting measure on the faces converges to a $\sqrt{8/3}$-LQG area measure. Obtaining such results for other values of γ and "non-uniform" triangulations or random planar maps is an important open direction for research.

There are also deep connections of this theory with conformal field theory. Indeed, the correlation functions in this theory can be expressed in terms of negative moments of certain Gaussian multiplicative chaos measures. Recently, what is known as Liouville conformal field theory was given a precise mathematical definition and this led to establishing several long-standing predictions made originally by physicists (such as the celebrated DOZZ formula).

While a theory of random two-dimensional spaces is taking shape, one might also ask about analogous higher-dimensional "random manifolds" with Gaussian features. Much less appears to be known here, illustrating the special nature of two-dimensional analysis and geometry.

From Random Surfaces to Random Curves

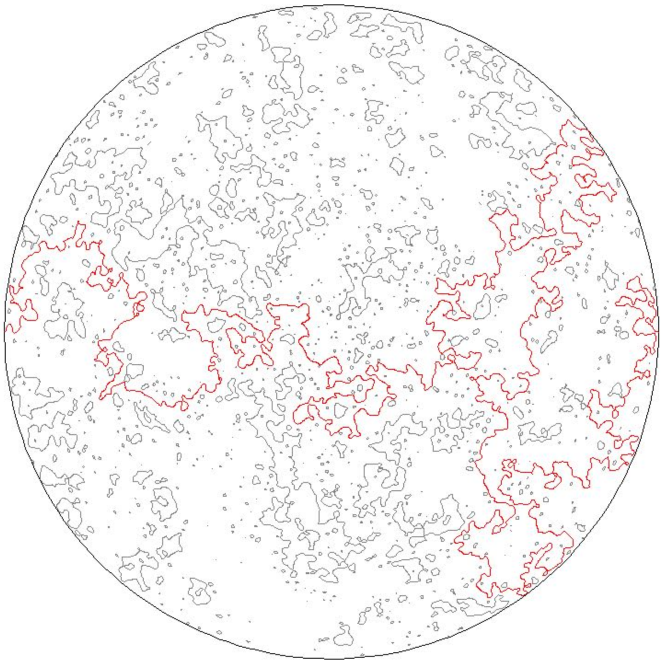
One way of exploring a smooth surface, given locally as a graph of a function, is by following its level lines—the sets of points of given equal height on the surface. If the surface is very irregular, one may expect these curves (perhaps defined in a more abstract way) to be quite rough. Even in the degenerate situation described above for the Gaussian free field, the idea of a level set gives a powerful way of exploring the geometry of the space. It is particularly intriguing that these ideas are related to an important conformally invariant process in the plane, namely the Schramm–Loewner Evolutions SLE_κ.

This process was discovered around 25 years ago by Schramm. It generates random planar curves by feeding a one-dimensional Brownian motion into the Loewner differential equation from classical conformal mapping theory. In the setting of the upper half-plane $\mathbb{H} = \{z \in \mathbb{C} \colon \operatorname{Im}(z) > 0\}$, we consider the ODE

$$\partial_t g(z,t) = \frac{2}{g(z,t) - \lambda(t)} \tag{‡}$$

with the initial condition $g(z,0) = z \in \mathbb{H}$ and a given "driving term" $t \mapsto \lambda(t) \in \mathbb{R}$. If λ is sufficiently regular, one can show that there exists a generated "trace curve" γ in $\mathbb{H}$ so that one can solve the ODE pathwise for $z \in \mathbb{H} \setminus \gamma[0,t]$ until time $t \geqslant 0$. Moreover, $g(\cdot,t)$ is a conformal map from $\mathbb{H} \setminus \gamma[0,t]$ onto $\mathbb{H}$.

If $\lambda(t) = \sqrt{\kappa} B_t$ is Brownian motion with speed $\kappa > 0$, then this ODE defines the SLE_κ-process and produces random curves γ (showing this is by no means easy and requires a delicate combination of complex analysis and stochastic calculus). The SLE_κ-curves are random fractals: with probability 1, each curve γ has Hausdorff dimension given by $\min\{1 + \kappa/8, 2\}$.



Level lines of the Gaussian free field obtained as SLE-curves (image credit: Nam-Gyu Kang).

The SLE_κ-formalism has led to breakthroughs for several important open problems such as identifying scaling limits of discrete random models at criticality or computing the Hausdorff dimension for the boundary of planar Brownian motion. The geometric properties of SLE-curves have been studied in detail as well: now we have precise estimates on harmonic measure, or hitting distribution of planar Brownian motion, and have pathwise Hölder continuity properties of SLE_κ-curves depending on the parameter κ. Moreover, important connections between SLE-curves and random surfaces have been uncovered. For instance, it is possible to view SLE_4-curves as 0-level lines of the Gaussian free field (see the figure above for an illustration). Similarly, there is a way of conformally fusing two LQG-surfaces together in such a way that their interface is given by an SLE-curve.

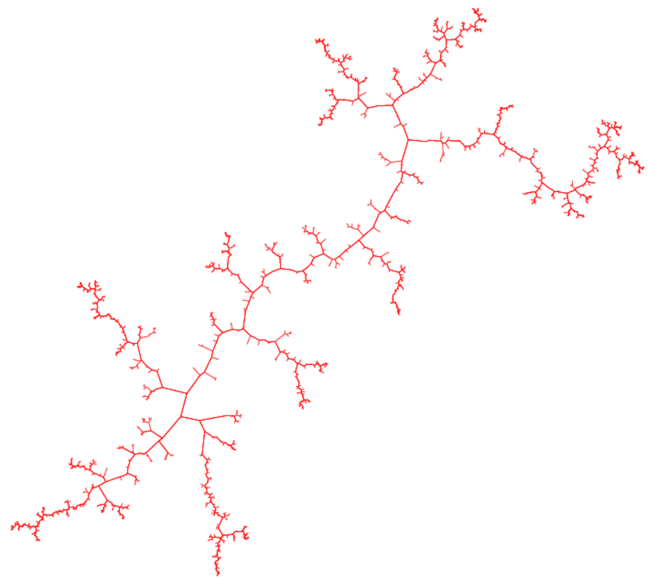
The *Loewner energy* of a curve γ produced by the Loewner equation is the integral of the square of the derivative of the driving term λ. It has recently been identified as an important quantity that, among other things, allows for a new characterization of the so-called Weil–Peterson quasicircles. This connects the Loewner theory to Teichmüller theory and to the theory of quasiconformal maps (homeomorphisms that map infinitesimal circles onto infinitesimal ellipses with uniformly bounded eccentricity).

A generalization of (‡) is the Loewner–Kufarev equation, which is obtained by, roughly speaking, feeding into the equation a general (possibly random) time-dependent measure as driving term instead of a point mass located at $\lambda(t)$. It generates conformal maps onto evolving planar domains $D_t \subset \mathbb{H}$ and allows one to describe processes where growth occurs at multiple points simultaneously.

Manifold Connections: Growth Processes, Dynamics, and Conformality

In *conformal welding*, one seeks to glue Riemann surfaces together along their boundaries. This problem has been studied extensively in complex analysis. For example, suppose that for a given planar Jordan domain Ω, f is the conformal map of the unit disk $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ onto D, and g is the conformal map of the complement of (the closure of) $\mathbb{D}$ to the complement of D (in the Riemann sphere). Both f and g have homeomorphic extensions to the unit circle $\mathbb{S}^1 = \{z \in \mathbb{C} : |z| = 1\}$, and so one can form the welding homeomorphism $\varphi := f^{-1} \circ g$ on $\mathbb{S}^1$. The interesting question is the converse: Given a homeomorphism $\varphi\colon \mathbb{S}^1 \to \mathbb{S}^1$, when does it arise as a welding homeomorphism and when is the solution (essentially) unique?

Conformal welding ties in with the study of the group $\mathrm{Diff}_+(\mathbb{S}^1)$ of orientation-preserving diffeomorphisms on the circle. Conformal field theory motivates the study of the Virasoro algebra, the central extension of the Witt algebra which is closely related to the quotient group $\mathrm{Diff}_+(\mathbb{S}^1)/\mathbb{S}^1$. The infinite-dimensional nature of these objects poses considerable challenges, since one needs to go beyond the usual Lie setting to make sense of basic notions such as Brownian motion on these spaces.



A sample of the continuum random tree embedded in the plane via a welding procedure (image credit: Steffen Rohde).

One can pose welding and mating problems for several random and dynamical objects such as random trees or graphs, or Julia sets of rational functions, but often the existing analytic techniques are not sufficient to obtain the existence results one would like to have. Proofs can become delicate and require a careful probabilistic or dynamical analysis. Often the arguments reveal that while the object in question is too rough for standard arguments to go through, "stochastic self-similarity" becomes key. As an example, one can show that the continuum random tree, a probabilistic object that can be derived from one-dimensional Brownian motion, admits a conformal welding (see the figure above). It can be identified with a scaling limit associated with a certain class of random polynomials, called Shabat polynomials, as their degrees tend to infinity. There are important connections with laminations and other dynamical systems notions, illustrating how random geometry can benefit from developments in areas such as complex dynamics and hyperbolic geometry.

Some problems that involve randomness can be directly motivated by visual observations in nature. Examples include growth of bacterial colonies and dielectric breakdown phenomena; the former growth processes result in circular shapes while the latter give rise to fractal-looking dendritic patterns.

Continuum mathematical models for these processes in the plane can be formulated using a version of the Loewner–Kufarev equation. Unlike the SLE-setting, a driving term for the differential equation is no longer prescribed, but is dynamically updated depending on the evolution of the growing shape via the derivatives of the Loewner maps.

Closely related to these conformal aggregation models is the quantum Loewner evolution (QLE), which combines the SLE-formalism with derivative dependence and the Gaussian free field. The resulting stationarity properties in QLE allow one to get a handle on the process and have been exploited in several settings, but in general a complete understanding of conformal aggregation models remains elusive.

Conclusion

Many key questions of the research program require expertise from several different areas of mathematics. This program brings together analysts, geometers, and probabilists in order to meet, discuss, and learn about each other's techniques. In addition to research seminars and informal exchanges, the program features several learning seminars, for instance on conformal field theory aspects of Liouville Quantum Gravity, and on the geometry of the group of circle diffeomorphisms $\mathrm{Diff}_+(\mathbb{S}^1)$. We expect that new collaborations will be initiated, in particular between mathematicians with different backgrounds, leading to exciting new connections and viewpoints.

Focus on the Scientist: Nikolai Makarov

Nikolai Makarov is a Research Professor and Clay Senior Scholar in this semester's program on Analysis and Geometry of Random Spaces. He received his Ph.D. in 1986 under the direction of Nikolskii at the Steklov Institute of Mathematics in St. Petersburg (at that time Leningrad), joined the faculty at CalTech in 1991 and has been the Richard Merkin Distinguished Professor of Mathematics since 2013.

Nikolai Makarov

Nick's early work was a milestone in the study of harmonic measure in the plane and triggered an avalanche of influential and celebrated work. Harmonic measure is a fundamental object in analysis that arises in different contexts, for instance in probability theory as the hitting distribution of Brownian motion, in potential theory as the Riesz measure associated with the Dirichlet problem, and in conformal mapping as the image of Lebesgue measure under conformal maps of the unit disc.

A recurrent theme of Nick's work is the interplay between complex analysis and probability theory. Already in the late 1980s he promoted and exploited the close connections between martingale theory and conformal mappings, a viewpoint that has become more and more important in probability theory since the discovery of Schramm–Loewner evolution (SLE) around the year 2000. Another manifestation of the interplay between complex analysis and probability theory is the seemingly intractible diffusion limited aggregation (DLA): In many natural processes, as well as in computer simulations such as the Witten–Sander model, particles (for instance charged ions in electrolysis) that perform a random walk (or Brownian motion in the continuous setting) form randomly growing clusters that display intriguing patterns of branching "arms." Despite hundreds of publications in the physics literature, very little is known rigorously about this process. Together with Lennart Carleson, Nick investigated certain non-branching variants of this problem and developed powerful techniques to treat stability questions and the number of surviving arms.

This is a technical tour-de-force and deep work in its own right, but it also highlights another impresseve aspect of Nick's work, namely that of a true visionary: Indeed, Carleson and Makarov introduced a discrete version of DLA that was re-discovered many years later by the physicists Hastings and Levitov, making headlines in the physics community under the name Hastings–Levitov model. Even the celebrated Schramm–Loewner evolution is in some sense (at least philosophically) related to the Carleson–Makarov growth model, and according to oral communication with Oded Schramm, this circle of ideas had been in the back of his mind when he discovered SLE.

In recent years, Nick has made fundamental contributions to random matrix theory, the uncertainty principle in harmonic analysis, anti-holomorphic dynamics and connections to Hele–Shaw flows, quadrature domains and Schwarz reflection, and also to conformal field theory from the perspective of the Gaussian free field and SLE. His extraordinary breadth, depth, and vision has left deep marks on many of the areas represented in the Analysis and Geometry of Random Spaces program.

Nick was an invited speaker at the 1986 ICM and has received several distinguished honors, notably the Salem prize and the Rolf Schock prize. His impact on the field is also clearly visible through the accomplishments of his former students, many of whom have received high honors (notably ICM invitations for Smirnov and Polteratski, Salem prizes for Zhan and Smirnov, and the fields medal for Smirnov). Nick also had, and keeps having, an impact on a large number of junior mathematicians, by consistently hosting and collaborating with a large number of postdoctoral researchers at CalTech.

— Steffen Rohde

Named Postdocs — Spring 2022

Strauch

Yilin Wang is this semester's Strauch Postdoctoral Fellow in the Analysis and Geometry of Random Spaces program. She studied at the École Normale Supérieure and earned her bachelor's degree from the Université Paris Diderot and two master's degrees from the Universités Pierre-et-Marie-Curie and Paris-Sud before pursuing a Ph.D. in Mathematics under the direction of Wendelin Werner at the ETH in Zurich. For her outstanding thesis, Yilin was awarded a prestigious Swiss Innovation Prize and an ETH medal. Subsequently, Yilin was a Moore Instructor at MIT, and after the MSRI random spaces program she will be a Junior Professor at the IHES in Paris. A few months ago, she was awarded a Mirzakhani New Frontiers Prize by the Breakthrough Prize Foundation. Yilin's research interests are in complex analysis and probability theory. She introduced the concept of the Loewner energy and discovered fundamental relations with the theory of stochastic Schramm–Loewner evolutions (SLEs). Her theory of Loewner energy is one of the most important developments in complex analysis in the last decade. *The Strauch fellowship is funded by a generous annual gift from Roger Strauch, Chairman of The Roda Group. He is a member of the Engineering Dean's College Advisory Boards of UC Berkeley and Cornell University, and is also currently the chair of MSRI's Board of Trustees.*

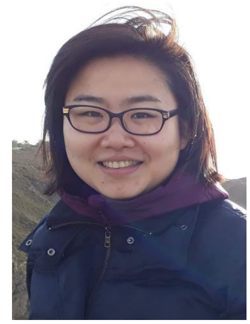
Viterbi

Janne Junnila was born and studied in Helsinki, Finland both for his undergraduate degree and Ph.D., which he completed in 2018 under the direction of Eero Saksman. After this he moved to EPFL in Lausanne, Switzerland, working with Juhan Aru as a member of the Chair of Random Geometry. Much of his work is connected with fundamental questions concerning logarithmically correlated random fields and their associated Gaussian multiplicative chaos. Such fields are ubiquitous in both pure and applied mathematics. One of Janne's remarkable results, dating back to his Ph.D., was to show that Gaussian multiplicative chaos is unique all the way up to and including the critical point where it can be defined. Recently he has obtained beautiful results on the imaginary chaos associated to the Gaussian free field; some of these results hinge on a surprising connection with Malliavin calculus, while others show a deep connection to the critical XOR-Ising model from statistical mechanics. Janne's research thus covers a very wide spectrum of themes at the heart of modern probability theory. *The Viterbi postdoctoral fellowship is funded by a generous endowment from Dr. Andrew Viterbi, well known as the co-inventor of Code Division Multiple Access based digital cellular technology and the Viterbi decoding algorithm, used in many digital communication systems.*

S. Della Pietra

Peter Lin is the Steven Della Pietra Postdoctoral fellow in the Analysis and Geometry of Random Spaces program. He obtained his Ph.D. in 2019 from the University of Washington under the supervision of Steffen Rohde. Before joining MSRI he was a Detlef Gromoll Instructor at Stony Brook University. Peter's research interests span both analysis and probability. Among other things, he has made important contributions to the theory of conformal laminations. For instance, with Rohde he developed a new method for how to construct solutions given rough lamination data and characterized certain particularly natural laminations. Applications include a proof that the continuum random tree admits conformal lamination. *The Stephen Della Pietra fellowship was established in 2017 by the Della Pietra Family Foundation. Stephen received his Ph.D. in mathematical physics from Harvard University. He is a partner at Renaissance Technologies, a board member of the Simons Center for Geometry and Physics, and treasurer of the National Museum of Mathematics in New York. He is also currently a trustee of MSRI.*

Huneke



Yan Mary He is the Huneke Postdoc in the Complex Dynamics program. She received her Ph.D. in 2018 from the University of Chicago under the direction of Danny Calegari and Peter Shalen. She then spent three years as a postdoctoral fellow at the University of Luxembourg (2019) and the University of Toronto (2020-2021) before joining the University of Oklahoma as a tenure-track assistant professor in Fall 2021. Mary studies problems in (higher) Teichmueller theory, complex dynamics, thermodynamic formalism and interactions between these fields. *The Huneke postdoctoral fellowship is funded by a generous endowment from Professor Craig Huneke, who is internationally recognized for his work in commutative algebra and algebraic geometry.*

Berlekamp

Mikhail Hlushchanka is the Berlekamp Postdoctoral Fellow in the program on Complex Dynamics. Misha received his University Diploma in 2013 from Belarusian State University, and his Ph.D. in 2017 from Jacobs University in Germany under the supervision of Dierk Schleicher and Daniel Meyer. From 2017–20 he was a Hedrick Assistant Adjunct Professor at UCLA. Since 2020, he has been on the faculty of Utrecht University as a Junior Assistant Professor. In his dissertation, Misha constructs natural combinatorial models for various classes of rational maps and uses them to study growth properties of the associated iterated monodromy groups and to provide a complete classification of critically fixed rational maps. In general, he seeks to gain further insight into the characterization theorems for rational maps, and establish new relations between dynamical aspects of rational maps and various notions of complexity of the associated Julia sets and iterated monodromy groups. *The Berlekamp fellowship was established in 2014 by a group of Elwyn Berlekamp's friends, colleagues, and former students whose lives he touched in many ways. He was well known for his algorithms in coding theory, important contributions to game theory, and his love of mathematical puzzles.*

V. Della Pietra

Leticia Pardo Simón is the Vincent Della Pietra Fellow in this semester's Complex Dynamics program. After completing her Ph.D. at the University of Liverpool in 2019 under the supervision of Lasse Rempe, she held a postdoctoral position at the Institute of Mathematics of the Polish Academy of Sciences. She is currently a research fellow at the University of Manchester. Leticia's research

mainly focuses on the study of Julia sets of transcendental entire functions. She has shown the existence of Cantor bouquets in the Julia set of a large class of these maps and, by developing novel techniques, she has provided a complete description of the Julia set of certain functions with escaping critical values, as a collection of landing dynamic rays. *The Vincent Della Pietra fellowship was established in 2017 by the Della Pietra Foundation. Vincent received his Ph.D. in mathematical physics from Harvard University. He is a partner at Renaissance Technologies, co-founder of the Della Pietra Lecture Series at Stony Brook University, and a board member of PIVOT. He is also currently a trustee of MSRI.*

Forthcoming Workshops

May 2–6, 2022: *Adventurous Berkeley Complex Dynamics*

Aug 25–26, 2022: *Connections Workshop: Analytic and Geometric Aspects of Gauge Theory*

Aug 29–September 02, 2022: *Introductory Workshop: Analytic and Geometric Aspects of Gauge Theory*

Sep 8–9, 2022: *Connections Workshop: Floer Homotopy Theory*

Sep 12–16, 2022: *Introductory Workshop: Floer Homotopy Theory*

Oct 24–28, 2022: *New four-dimensional gauge theories*

November 14–18, 2022: *Floer homotopical methods in low dimensional and symplectic topology*

Summer Activities

Jun 13–Jul 22, 2022: *MSRI-UP 2022: Algebraic Methods in Mathematical Biology*

June 6–July 15, 2022: *Summer Research in Mathematics*

June 20–July 1, 2022: *African Diaspora Joint Mathematics Workshop (ADJOINT)*

Summer Graduate Schools

Jun 6–17, 2022: *Integral Equations and Applications (MSRI)*

Jun 20–Jul 1, 2022: *New Directions in Representation Theory (AMSI, Brisbane, Australia)*

Jun 20–Jul 1, 2022: *Geometric Flows (Athens, Greece)*

Jul 5–15, 2022: *Random Graphs (MSRI)*

Jul 5–15, 2022: *Algebraic Theory of Differential and Difference Equations, Model Theory, and their Applications (St. Mary's College, Moraga, CA)*

Jul 11–22, 2022: *Metric Geometry and Geometric Analysis (Oxford, United Kingdom)*

Jul 11–22, 2022: *Séminaire de Mathématiques Supérieures 2022: Floer Homotopy Theory (Vancouver, Canada)*

Jul 17–Aug 6, 2022: *2022 Joint PCMI School: Number Theory Informed by Computation (Park City, Utah)*

Jul 18–29, 2022: *Recent Topics in Well Posedness (Taipei, Taiwan)*

Jul 25–Aug 5, 2022: *Mathematics of Machine Learning (INdAM, Italy)*

Jul 25–Aug 5, 2022: *Topological Methods for the Discrete Mathematician (St. Mary's College, Moraga, CA)*

Jul 25–Aug 5, 2022: *Sums of Squares Method in Geometry, Combinatorics, and Optimization (Alberta, Canada)*

Aug 1–12, 2022: *Tropical Geometry (St. Mary's College, Moraga, CA)*

For more information about any of MSRI's scientific activities, please see msri.org/scientific.

Clay Senior Scholars

The Clay Mathematics Institute (www.claymath.org) has announced the 2022–23 recipients of its Senior Scholar awards. The awards provide support for established mathematicians to play a leading role in a topical program at an institute or university away from their home institution. Here are the Clay Senior Scholars who will work at MSRI in 2022–23.

Analytic and Geometric Aspects of Gauge Theory (Fall 2022)
Tomasz Mrowka, MIT

Floer Homotopy Theory (Fall 2022)
Ivan Smith, University of Cambridge

Algebraic Cycles, L-Values, and Euler Systems (Spring 2023)
Henri Darmon, McGill University

Diophantine Geometry (Spring 2023)
Mark Kisin, Harvard University

Public Understanding of Mathematics Updates

2022 National Math Festival — Spring Events at NCSciFest

What do children's books, snowflakes and fractals, and happy numbers have in common? All are on offer this spring as part of the 2022 National Math Festival, with free, live online events for all ages.

This spring's offerings included a series of live online events on Saturday, April 23, as part of the North Carolina Science Festival. The goal was to infuse the festival with expanded offerings for the "M" in STEM. Also partnering with MSRI to present the lineup of free programming for all ages was North Carolina Central University.

⋆ **Pamela E. Harris** of Williams College (and cofounder of lathisms.org) presented a talk on "Happy Numbers," hosted by **Kimberly Weems**, professor of mathematics at NC Central.

⋆ Two hands-on **Julia Robinson Mathematics Festival** activities, River Crossings and Prime Cubes, were held for ages 8–13.

⋆ **Natural Math** led a math playtime for children ages 2–7 so they could explore making snowflakes and fractals.

⋆ **Shelley Pearsall**, author of *ALL OF THE ABOVE*, a middle-grade novel featuring students attempting to break a tetrahedron-building world record, gave a Mathical Book Prize author reading, hosted by **Gail Hollowell**, associate professor of biology at NC Central, and her 13-year-old son, Isaiah.

Gail Hollowell played a key role in planning the full range of the day's events, and she and her son were instrumental in choosing *ALL OF THE ABOVE* for the live author reading. For more information and recordings of the Happy Numbers talk and the Mathical Book event, visit nationalmathfestival.org/join/2022-festival.

AMS/MSRI Congressional Briefing, December 2021

MSRI and the American Mathematical Society hosts Congressional briefings on mathematical topics in Washington, DC, to inform members of Congress and Congressional staff about new developments made possible through federal support of basic science research. On Dec 2, 2021, **Cédric Villani** presented a briefing on "Mitigating Climate Change: Science and Policy." The recipient of numerous awards including the Fields Medal and the AMS Doob Prize, Villani is currently Professor at the University of Lyon, and, since 2017, an elected member of the French Parliament. He is a member of the French Academy of Sciences, as well as the Pontifical Academy of Sciences. Learn more and view upcoming events: msri.org/congress.

Mathical Authors Join Read Across America Events

The Mathical Book Prize partnered with the San Bernardino City Unified School District in southern California to celebrate National Read Across America Day on March 2, 2022, with online author visits enjoyed by students and families. Authors **Jen Arena** (*Marta! Big and Small*, an Honor Book for pre-K), **Ana Crespo** (*Lia & Luís: Who Has More?*, Award Winner for pre-K), and **Traci Sorell** (*Classified: The Secret Career of Mary Golda Ross, Cherokee Aerospace Engineer*, Honor Book for Grades 3–5) joined this year's event, which emphasized multicultural and multilingual titles. Launched in 1998 by the National Education Association (NEA), Read Across America is the nation's largest celebration of reading.

Traci Sorell also shared her book — and some of her research into the life of Mary Golda Ross — with students in DC Public Schools on March 2, and with students in the Bluebird Math Circle of the Alliance for Indigenous Math Circles on April 11.

Inspiring Voices in a New Video Series for Teachers

How are teachers reimagining the field of mathematics education during the pandemic? Tune into **Kristopher J. Childs**' new video series, "Inspiring Voices from the Classroom," this spring to find out. The pilot series was filmed live in Atlanta, GA, and premiered on Facebook, Instagram, and YouTube in April. Childs, who is chief equity and social justice officer at OpenUp.org, a national educational nonprofit, comes to the project with a commitment to "highlight current mathematics classroom teachers and provide a positive platform that allows their authentic voices to be heard."

"The show focuses on three pillars: educating, inspiring, and advocating," says Childs. Early episodes span middle school and high school math teaching, connecting culture and teaching, and teaching math to students with disabilities. Childs is a former public school mathematics teacher in urban school settings at the secondary level as well as a former assistant professor of STEM education at Texas Tech University. He holds a Ph.D. in mathematics education from the University of Central Florida.

The series was supported by MSRI as part of the 2022 National Math Festival. Visit kristopherchilds.com/shows for more information.

Datathon4Justice Brings Data Analysis to High Schools

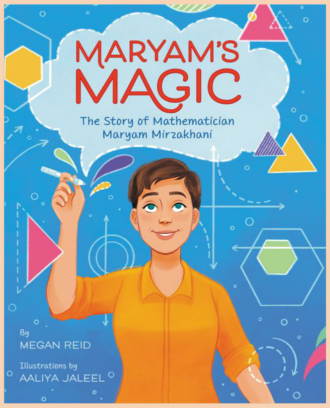
QSIDE Institute

On April 9, 2022, the QSIDE Institute teamed up with the National Math Festival to present the first-ever High School Datathon4Justice. The event was coordinated with high school math and data science programs around the US, and focused on building skills and analyzing real-world

Mathical Continues to Celebrate Math in Youth Literature

MSRI has announced the 2022 winners of the Mathical Book Prize, which recognizes outstanding fiction and literary nonfiction for youth ages 2–18. The Mathical Prize, now in its eighth year, is selected annually by a committee of Pre-K–12 math and language arts teachers, librarians, mathematicians, early childhood experts, and others.

2022 Winners!



Pre-K: *1 Smile, 10 Toes*, by Nelleke Verhoeff (Barefoot Books)

Grades K–2: *Uma Wimple Charts Her House*, by Reif Larsen and Ben Gibson (Random House Children's Books)

Grades 3–5: *Maryam's Magic: The Story Of Mathematician Maryam Mirzakhani*, by Megan Reid, with illustrations by Aaliya Jaleel (HarperCollins Children's Books)

Grades 6–8: *AfterMath*, by Emily Barth Isler (Lerner Publishing Group)

The selection committee was co-chaired this year by **Elizabeth (Betsy) Bird**, collection development manager of the Evanston (IL) Public Library system and reviewer for Kirkus and the New York Times, **Katie Hendrickson**, president of the Code.org advocacy coalition, and a former secondary school mathematics teacher, and **Candice Price**, assistant professor of mathematics at Smith College and co-founder of the website Mathematically Gifted and Black.

2022 Honor Books

Pre-K: *What Will Fit?*, by Grace Lin (Charlesbridge); *Circle Under Berry* by Carter Higgins (Chronicle Books); *Shape Up, Construction Trucks*, by Victoria Allenby (Pajama Press); and *Marta! Big and Small*, by Jen Arena, with illustrations by Angela Dominguez (Roaring Brook Press / Macmillan).

Grades K–2: *We Are One: How the World Adds Up*, by Susan Hood, with illustrations by Linda Yan (Candlewick Press).

Grades 3–5: *Molly and the Mathematical Mysteries: Ten Interactive Adventures in Mathematical Wonderland*, by Eugenia Cheng, with illustrations by Aleksandra Artymowska (Candlewick Press); *Classified: The Secret Career of Mary Golda Ross, Cherokee Aerospace Engineer*, by Traci Sorell, with illustrations by Natasha Donovan (Lerner Publishing Group); *If the World Were 100 People: A Visual Guide to Our Global Village*, by Jackie McCann, with illustrations by Aaron Cushley (Random House Children's Books); and *Hedy Lamarr's Double Life: Hollywood Legend and Brilliant Inventor*, by Laurie Wallmark, with illustrations by Katy Wu (Union Square & Co.).

Grades 6–8: *In the Red*, by Christopher Swiedler (HarperCollins Children's Books); *Much Ado About Baseball*, by Rajani LaRocca (Little Bee Books); and *It's a Numbers Game! Baseball*, by James Buckley, Jr. (National Geographic Kids Books).

Grades 9–12: *The Quantum Weirdness of the Almost-Kiss*, by Amy Noelle Parks (Harry N. Abrams).

In addition, two new Mathical Hall of Fame titles were selected by the prize committee: *Anno's Counting Book*, by Mitsumasa Anno (1977), and *The Cat in Numberland*, by Ivar Ekeland, with illustrations by John O'Brien (2006).

The 2022 Mathical Book Prize is awarded by the Mathematical Sciences Research Institute (MSRI) in cooperation with the Institute for Advanced Study (IAS). The prize is awarded in partnership with the National Council of Teachers of English (NCTE) and the National Council of Teachers of Mathematics (NCTM), and in coordination with the Children's Book Council (CBC). Mathical is made possible through the generous support of the Firedoll Foundation, Joan and Irwin Jacobs, and Sandor and Faye Straus. MSRI thanks the Simons Foundation for founding support of the Mathical Book Prize. MSRI partners with organizations including First Book, School Library Journal, the Books for Kids Foundation, the DREME Network at Stanford University, and many others to distribute Mathical titles and resources nationally to children in need. Additional resources to support educators, librarians, and families including grade-level flyers, printable bookmarks, and more can be found at mathicalbooks.org.

data and aimed "to create a living community of high school students excited about the applications of mathematics in social justice endeavors."

This year's event used a dataset of the diversity of artists in major U.S. museums in regards to race/ethnicity, gender, date of birth, and region of origin, and participating students used the data to ask and answer questions related to diversity, equity, inclusion, and social justice.

QSIDE explains, "the inaugural High School Datathon4Justice will serve as a pilot program that can be replicated and adapted by other schools, organizations, and groups. This visible program of the National Math Festival calls attention to the power, beauty, and importance of math to shape change in our world and reveal and address injustice using quantitative methods."

You can find more information at the QSIDE website, including the curriculum for free download: qsideinstitute.org/high-school-datathon4justice/.

Program Associates — Spring 2022

*Program Associate fellowships support current graduate students to take part in our research programs, thanks to the generosity of MSRI donors **Stephen Della Pietra and Pamela Hurst-Della Pietra**, **Kristin Lauter**, **The Salgo-Noren Foundation**, and **Marie Vitulli**. These new fellowships allow graduate students to receive financial support so that they can remain in residence at MSRI for the entire semester with their advisor, fully integrated into the semester's research program.*

Salgo-Noren

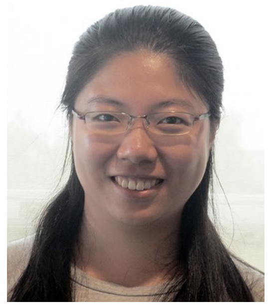
Marco Carfagnini is the Salgo-Noren Program Associate in the Analysis and Geometry of Random Spaces program. In 2017 he received his M.S. from the University of Rome Tor Vergata, working on random fields on the sphere. He is currently a Ph.D. candidate at the University of Connecticut working with Masha Gordina. In Fall 2022, Marco will join the University of California, San Diego, as a Stefan E. Warschawski Visiting Assistant Professor. His research is in stochastic analysis and its interplay with differential geometry and functional analysis. In particular, he studies limit laws for degenerate stochastic processes, especially processes on Lie groups, and analysis and geometry of path spaces.

Vitulli

Caroline Davis is the Marie Vitulli Program Associate in the Complex Dynamics program. She is currently a third-year graduate student at Indiana University, advised by Kevin Pilgrim and Dylan Thurston, and before that received a bachelor's degree in math and philosophy at the University of Michigan. She works on Sierpiński carpet matings of polynomials, as well as liftings of rational maps.

Lauter

Liangbing Luo is the Lauter Program Associate in the Analysis and Geometry of Random Spaces program. She graduated from Wuhan University, China, and is currently a Ph.D. candidate at the University of Connecticut under the supervision of Masha Gordina. Liangbing's research interests lie primarily in analysis, probability, and geometry. She is particularly interested in functional inequalities such as logarithmic Sobolev inequalities and Poincaré inequalities. More specifically, her research investigates logarithmic Sobolev inequalities with respect to a hypoelliptic heat kernel measure on finite-dimensional and infinite-dimensional Heisenberg groups, and how the constants depend on the geometry of their underlying spaces. In Fall 2022, Liangbing will join Lehigh University as a C.C. Hsiung Visiting Assistant Professor.

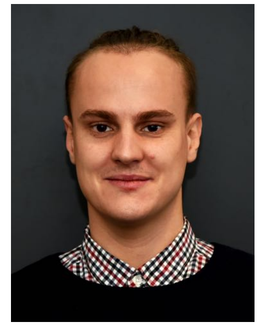


Lauter

Malavika Mukundan is the Lauter Program Associate in the Complex Dynamics program. She is currently a fourth-year graduate student at the University of Michigan, Ann Arbor; her advisor is Sarah Koch. Malavika earned her Bachelor's degree at the Chennai Mathematical Institute in India. For her graduate work, she studies dynamical embeddings of complex polynomials, from combinatorial, algebraic, and geometric perspectives. Malavika also serves as the Vice President of the AWM Michigan Student Chapter.

Salgo-Noren

Nikolai "Kolya" Prochorov is the Salgo-Noren Program Associate in this spring's program on Complex Dynamics. He is currently a Ph.D. student at Aix-Marseille University under the direction of Dierk Schleicher. He grew up in Minsk, Belarus, where he obtained his first university degree in mathematics. Kolya's research focuses on transcendental Thurston theory: in the 1980s, Fields medalist William Thurston (MSRI Director from 1992–97) introduced a panorama of results connecting the geometry of 3-manifolds, surface automorphisms, and rational maps, and Kolya is working to extend this theory from rational to transcendental maps as dynamical systems.



Math Circles Library

Vol. 28 in the AMS/MSRI Mathematical Circles Library will be released in June 2022, inspired by the Julia Robinson Mathematics Festival (jrmf.org). *A Festival of Mathematics: A Sourcebook*, by Alice Peters and Mark Saul, aims to engage students in mathematical discovery through fun and approachable problems that reveal deeper mathematical ideas. Each chapter starts with a gentle on-ramp that requires no more than simple arithmetic or intuitive concepts of symmetry. Follow-up problems and activities require intuitive logic and reveal more sophisticated notions of strategy and algorithms. It's appropriate for students aged 8–18, as well as for teachers wanting to hone their skills. *MSRI and the AMS are publishing books in the Mathematical Circles Library series as a service to young people, their parents and teachers, and the mathematics profession.*

Complex Dynamics: From Special Families to Natural Generalizations in One and Several Variables

(continued from cover)

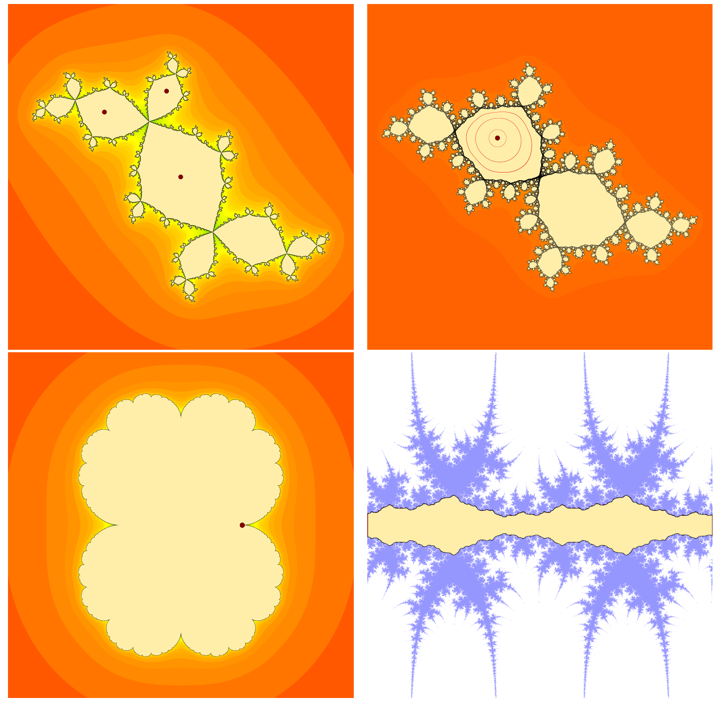
by the requirements $f^{n+1} := f^n \circ f$ and $f^1 = f$. Our (idealistic) hope would be to describe all orbits — that is, the behavior of all sequences of the form $\{f^n(z)\}_{n\in\mathbb{N}}$ for any given initial condition $z \in \mathbb{C}$.

If f is an affine map then the dynamics can be easily described: either f is conjugate to a translation $z \mapsto z+1$ or to a homothety $z \mapsto \lambda z$ for some $\lambda \in \mathbb{C}$, and one can compute all iterates of f by a closed formula and describe explicitly the orbits of all points. From now on, we shall assume that f is either a polynomial of degree at least 2, or an entire transcendental function (that is, with an essential singularity at infinity). It is then arguably impossible to describe all orbits since these maps always exhibit chaotic behaviors in parts of the complex plane.

To analyze the dynamics of f, one introduces its Fatou set $F(f)$ as the open set where the sequence $\{f^n\}$ locally forms an equicontinuous family (in the spherical metric), and call its complement the Julia set $J(f)$. It is a fact that $J(f)$ is always a non-empty closed and (completely) invariant set, which might be the whole complex plane but otherwise has empty interior. When f is a polynomial, points with sufficiently large modulus have orbits which tend to infinity uniformly and hence belong to the Fatou set. In particular, $J(f)$ is compact in that case.

In the Julia set, sensitivity to initial conditions is the rule: periodic orbits and points having dense orbits in $J(f)$ both form dense subsets so that the dynamics is chaotic on that set.

In the Fatou set, on the other hand, points behave asymptotically as their neighbors do, and connected components of the Fatou set (called *Fatou components*) are permuted. If such a component U is fixed by f, then either (a) f is analytically conjugate to an irrational rotation on U and we say that U is a *Siegel disk*, (b) all points tend towards an attracting fixed orbit lying in U, or (c) they converge towards a point z_0 in the boundary of U. The last case breaks up further into two possibilities: (c_1) z_0 is a fixed point at which the multiplier $f'(z_0)$ is a root of unity (in which case z_0 is a *parabolic* fixed point), or (c_2) $z_0 = \infty$ and it is the essential singularity of f (in which case f is transcendental and U is called a *Baker domain*). If U is periodic of period p, an analogous classification can be deduced by considering f^p.



Fatou and Julia sets of three polynomials of degree two and one entire transcendental function. Clockwise from top left, in yellow: the basin of an attracting 3-cycle (known as the Doudady rabbit); a Siegel disk; a Baker domain; the basin of a parabolic fixed point. The Julia sets are shown in black, and the shades of orange show the basin of attraction of infinity.

Wandering Domains in One and Several Dimensions

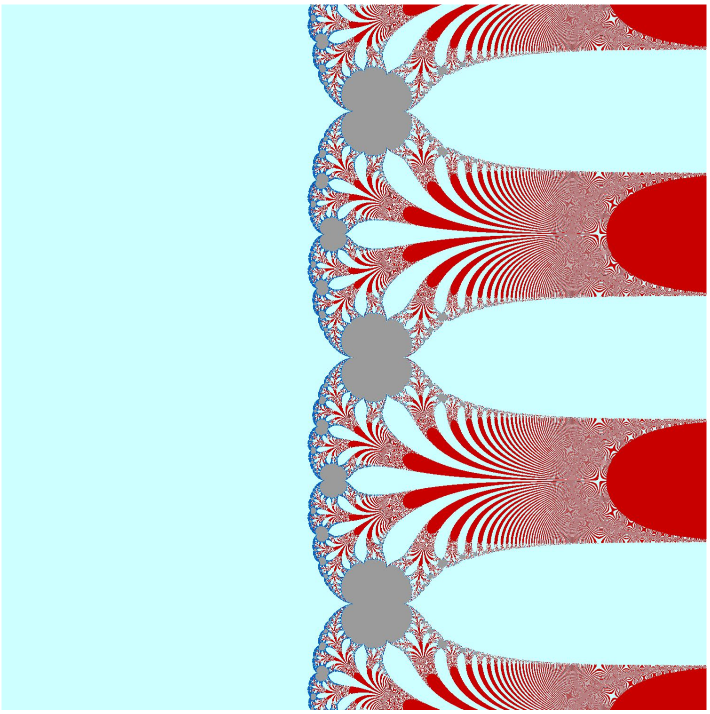
In a landmark paper in 1982, D. Sullivan proved that given a rational map f, all Fatou components are eventually mapped to a periodic component, in surprising parallelism with a theorem of Ahlfors for Kleinian groups. Sullivan's theorem attracted a lot of attention at the time since Sullivan's method was based on quasiconformal surgery, forever linking complex dynamics to hyperbolic geometry. Since then, quasiconformal techniques have lead to many of the most significant results in the field, notably by A. Douady and J.H. Hubbard, M. Lyubich, C. McMullen, M. Shishikura, J.C. Yoccoz, and many others.

When f is transcendental, the story is substantially different. Special classes of entire maps, including for example the exponential family $z \mapsto \lambda e^z$, do not have wandering domains. More generally A. Eremenko and M. Lyubich, and L. Goldberg and L. Keen, proved that the same is true for transcendental maps having a finite number of singularities of their inverse branches. However, general entire maps may exhibit wandering domains, and their internal dynamics was only recently understood by A. Benini, V. Evdoridou, N. Fagella, P. Rippon and G. Stallard. In the meantime an important intriguing question remains unsolved: *Are there any wandering domains for which no subsequence of iterates converges to infinity?*

The geometry of wandering Fatou components and the dynamics on their boundary is also the focus of intense research. Recent work of D. Martí-Pete, L. Rempe, and J. Waterman uses approximation techniques in complex analysis going back to Runge, to build a counterexample to a 40-year-old conjecture of A. Eremenko that asked whether every connected component of the set of escaping points of a transcendental entire function is unbounded.

The Fatou–Julia theory can also be defined for holomorphic maps in higher dimensions, and exciting developments on polynomial mappings on $\mathbb{C}^2$ have taken place recently. Consider any polynomial map $f\colon \mathbb{C}^2 \to \mathbb{C}^2$ which can be extended holomorphically to the complex projective space $\mathbb{P}^2_{\mathbb{C}}$. Then M. Astorg, X. Buff,

R. Dujardin, H. Peters and J. Raissy (following an idea by Lyubich) recently proved that such dynamical systems may also have wandering domains. In fact, they exhibited an explicit polynomial map of $\mathbb{C}^2$ which admits a wandering bounded Fatou component. This result provides a striking difference from the one-dimensional case! Interestingly enough, their analysis was based on the phenomenon of parabolic implosion in *one* dimension, a theory originally developed by A. Douady, aimed at understanding the behaviour of the Julia set near a parabolic fixed point when f is slightly perturbed.



The dynamical plane of $z \mapsto z^2 e^{1-z}$. The large blue component is a Baker domain while orbits of wandering domains are in grey. In red, Cantor sets of curves in the escaping set.

Another special feature in higher dimensions is that invertible maps may show chaotic behaviour. The most prominent example of such maps is given by the polynomial automorphism of $\mathbb{C}^2$ defined by

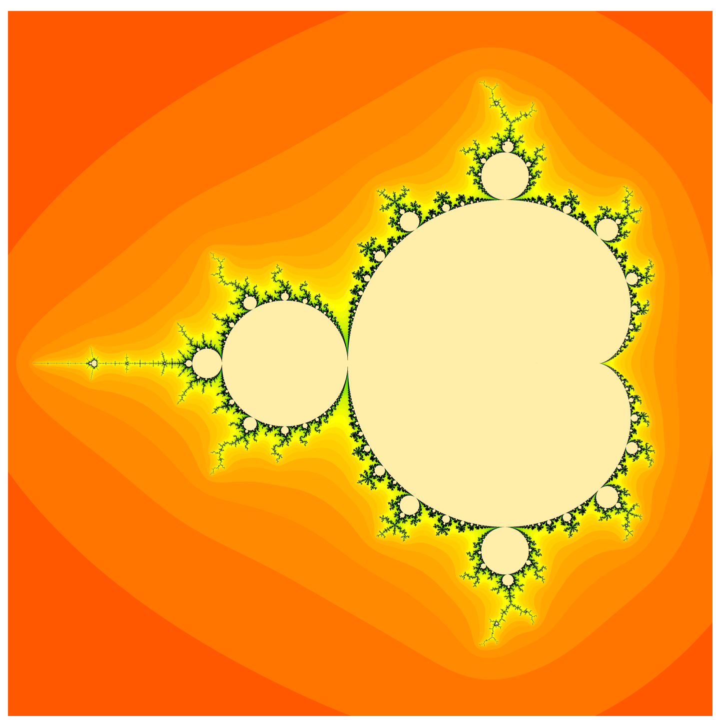
$$f(z,w) = (P(z) - w, bz),$$

where P is a polynomial of degree at least 2 and $b \neq 0$. This family, introduced by Hénon in the 1970s, was thoroughly studied in the complex domain in a series of celebrated papers by E. Bedford and J. Smillie with additional contributions by J. Hubbard, J. Fornaess, M. Lyubich, and N. Sibony. When f has constant Jacobian $|b| = 1$, it preserves the Lebesgue measure, and Fatou components need to be bounded and periodic. But recently, S. Biebler and P. Berger announced the existence of dissipative Hénon maps (with $b \ll 1$) with real coefficients that contain a wandering Fatou component. Their breakthrough paper builds on a geometric model for the dynamics in the real domain, and proceeds with a careful extraction of parameters. The mechanism they use for producing the wandering domain does not rely on parabolic implosion. It is a great challenge to further understand under what conditions a polynomial map may have a wandering domain in higher dimensions.

Post-Critically Finite Dynamics: From Thurston's Theorem to Special Families

Pick any polynomial $f\colon \mathbb{C} \to \mathbb{C}$ of degree d at least 2. Its critical set is defined as the locus where f is not locally injective: this set is defined by the equation $f' = 0$ and has thus $(d-1)$ points counted with multiplicity. It is a fact that the geometry of the Julia set $J(f)$ and the dynamics on the Fatou set $F(f)$ are very closely determined by the orbit of the critical points (for example, for any periodic Fatou component U, there exists at least one critical point whose orbit accumulates on $\overline{U}$). Polynomials for which the orbit of every critical point is finite are called *postcritically finite* polynomials (PCF for short), and form the class of maps for which the dynamics is the simplest.

Undoubtedly, one of the results which has shaped the theory of complex dynamics is a celebrated theorem proved by Thurston in the 1980s, which gives necessary and sufficient conditions under which a topological PCF branched covering of the Riemann sphere is homotopically equivalent to a rational map. This result threw light on the fundamental question of when combinatorics determines dynamics. Several generalizations have subsequently been developed, opening a wide area of research: The case of infinite branched coverings was initiated within the exponential family during the MSRI program 25 years ago by J. Hubbard, M. Shishikura, and D. Schleicher, while G. Zhang, Y. Jiang, G. Cui, Tan Lei, and M. Rees treated geometrically finite covers beyond the PCF case. And quite recently, L. Bartholdi and V. Nekrashevych solved the twisted rabbit problem for degree two polynomials (an intricate combinatorial problem related to Thurston's theorem) and revealed an unexpected connection with the theory of iterated monodromy groups.



The Mandelbrot Set (light yellow and black): the set of parameters c for which the Julia set of $z^2 + c$ is connected. Its boundary, shown in black, is the set parameters for which structural stability fails, and it is known as the Bifurcation Locus.

Focus on the Scientist: Alex Kapiamba

Alex Kapiamba plays ultimate frisbee at the national and semi-professional level and is also a (not so) amateur baker and a magician, but his real passion is for mathematics. He is a program associate in this semester's program on Complex Dynamics.

Alex began his studies at Oberlin College in Ohio; his first foray into mathematical research came during his final year there, with a research project studying how the topology of a space influences the properties of dynamical systems on it.

Alex Kapiamba

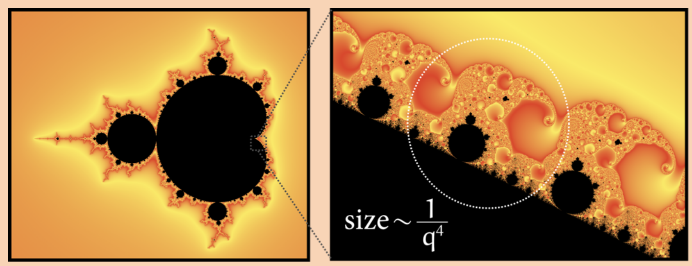
Interested in learning more about dynamical systems, Alex started working with Sarah Koch (lead organizer of this semester's program) and studying the Sullivan dictionary, a collection of analogies and correspondences relating complex dynamics to groups acting on hyperbolic spaces. In 2018 he joined the Ph.D. program in mathematics at the University of Michigan and will be graduating in 2023.

One of Alex's most exciting results is a so-called strong Yoccoz inequality for near-parabolic quadratic polynomials. The Mandelbrot set is a fractal that encodes information about the dynamics of quadratic polynomials, and is a central object of study in complex dynamics. While studying parabolic implosions in a 2018 reading course, Alex realized that there was a discrepancy between what was proved in the literature and what one can observe about the Mandelbrot set. The size of parts of the Mandelbrot set, usually called "elephants" due to their resemblance to the animal (see the figure below), appeared significantly smaller than the size given by the most precise estimates. He set out to understand and resolve this discrepancy. After a delicate analysis of parabolic implosion and careful computation, he produced a sharp new bound for the size of the qth elephant!

While unraveling the secrets of the elephants proved a difficult and delicate task, it is not the only research Alex has already completed. With John Hubbard, one of the pioneers in the modern study of complex dynamics, he is developing a complete model for the topology of Julia sets and their limits for quadratic and cubic polynomials. Besides continuing his research work, Alex has also already started mentoring undergraduate research projects.

We expect only more exciting news from him in the future (about mathematics and more), and we hope for all his math dreams to come true!

— Jasmin Raissy and Dylan Thurston



Thurston's theorem is also familiar to many MSRI members in the companion program on the Analysis and Geometry of Random Spaces. One instance of the connection is the monograph by M. Bonk and D. Meyer on the dynamics of expanding topological PCF branched coverings from the point of view of fractal geometry.

In a different spirit, let us now discuss the distribution of PCF polynomials in the space of polynomial maps. This moduli space Poly_d consists of all monic and centered polynomials of the form $f_a(z) = z^d + a_1 z^{d-2} + \cdots + a_{d-1}$ and can thus be identified (up to a ramified cover) with $\mathbb{C}^{d-1}$. The well known Mandelbrot set, whose boundary is the locus where the Julia set does not vary continuously under perturbation, is contained in Poly_2. Branner and Hubbard observed that degree d PCF polynomials are contained in a bounded set in Poly_d for any $d \geqslant 2$. To impose that a critical point of f_a has a finite orbit of a fixed length is equivalent to the vanishing of a polynomial in the variable $a \in C^{d-1}$ having integral coefficients. This in turn implies that PCF maps must have coefficients in $\mathbb{Q}^{\mathrm{alg}}$.

This observation was the starting point that triggered a series of works applying tools from arithmetic geometry to complex dynamics, in particular in the context of an intriguing conjecture set forth by J. Silverman. Suppose V is an algebraic irreducible subvariety in Poly_d that contains a Zariski dense subset of PCF polynomials. The conjecture states that V should be defined by *critical relations*, like the collision of two critical orbits. The first significant case was proved by M. Baker and L. Demarco in an influential paper where they set the overall strategy for attacking the conjecture. Building on contributions by D. Ghioca and H. Ye, C. Favre and T. Gauthier recently proved the conjecture when V is a (complex) curve. One compelling aspect of their proof is that it mixes classical aspects from complex dynamical systems (rigidity or similarities between Julia sets and the bifurcation locus) with more algebraic and arithmetic techniques (dynamical height or Ritt theory of decomposition of polynomials).

Conclusion

We have not touched upon many other interesting subjects that were discussed during workshops and informal meetings, notably renormalization theory. M. Lyubich gave an overview on the multiple revolutions that have quaked the subject during the past 25 years. The whole community of Douady's rabbit lovers now awaits a solution to the big one — that is, to the famous so-called MLC conjecture stipulating that the Mandelbrot set should be locally connected. A positive answer would have far-reaching consequences, such as the density of hyperbolic maps in the parameter space, a conjecture which dates back to Fatou.

CME Group–MSRI Prize

The 15th annual CME Group–MSRI Prize in Innovative Quantitative Applications will be awarded to Nancy L. Stokey, the Frederick Henry Prince Distinguished Service Professor of Economics at the University of Chicago. A panel discussion on "Macro Risks and Deglobalization" will be held as part of the award event in Chicago, IL, in May 2022.

Stokey is a member of the National Academy of Sciences and of the American Academy of Arts and Sciences, a Fellow of the Econometric Society, a Distinguished Fellow of the American Economic Association, and the 2021 President of the Society for the Advancement of Economic Theory. She is co-author of the influential monograph *Recursive Methods in Economic Dynamics* (1989), which has provided the mathematical basis for much of modern macroeconomics. She is also co-developer of a model of dynamic taxation and debt policy and is author of *The Economics of Inaction* (2009), which treats models that involve fixed costs of adjustment. Stokey has contributed to various areas of microeconomics, with the first rigorous proof of the famous Coase conjecture, and as co-developer of the no-trade theorem. Her recent work has focused on economic growth and development, especially on the role of trade and technology transfers in accelerating growth in middle-income countries.

The annual CME Group–MSRI Prize is awarded to an individual or a group to recognize originality and innovation in the use of mathematical, statistical, or computational methods for the study of the behavior of markets, and more broadly of economics. You can read more about the CME Group–MSRI Prize at tinyurl.com/cme-msri.

Call for Membership

MSRI invites membership applications for the 2023–24 academic year in these positions:

Research Professors	by October 1, 2022
Research Members	by December 1, 2022
Postdoctoral Fellows	by December 1, 2022

In the academic year 2023–24, the research programs are:

Algorithms, Fairness, and Equity
Aug 14–Dec 15, 2023

Organized by Rediet Abebe, Vincent Conitzer, Moon Duchin, Bettina Klaus, Jonathan Mattingly, Wesley Pegden

Mathematics and Computer Science of Market and Mechanism Design
Aug 14–Dec 15, 2023

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Jan 16–May 24, 2024

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MSRI uses **MathJobs** to process applications for its positions. Interested candidates must apply online at mathjobs.org. For more information about any of the programs, please see msri.org/scientific/programs.

Named Positions, Spring 2022

MSRI is grateful for the generous support that comes from endowments and annual gifts that support faculty and postdoc members of its programs each semester.

Chern, Eisenbud, and Simons Professors

The Analysis and Geometry of Random Spaces
Kari Astala, University of Helsinki
Mario Bonk, University of California, Los Angeles
Gregory Lawler, University of Chicago
Steffen Rohde, University of Washington
Eero Saksman, University of Helsinki
Nageswari Shanmugalingam, University of Cincinnati

Complex Dynamics: From Special Families to Natural Generalizations in One and Several Variables
Núria Fagella, Universitat de Barcelona
Sarah Koch, University of Michigan
Curtis McMullen, Harvard University
Dylan Thurston, Indiana University
Sebastian van Strien, Imperial College London

Named Postdoctoral Fellows

The Analysis and Geometry of Random Spaces
S. Della Pietra: Peter Lin, Stony Brook University
Strauch: Yilin Wang, Massachusetts Institute of Technology
Viterbi: Janne Junnila, École polytechnique fédérale de Lausanne

Complex Dynamics: From Special Families to Natural Generalizations in One and Several Variables
Berlekamp: Mikhail Hlushchanka, Utrecht University
Huneke: Mary Yan He, University of Toronto
V. Della Pietra: Leticia Pardo Simon, University of Manchester

Named Program Associates

The Analysis and Geometry of Random Spaces
Salgo Noren: Marco Carfagnini, University of Connecticut
Lauter: Liangbing Luo, University of Connecticut

Complex Dynamics: From Special Families to Natural Generalizations in One and Several Variables
Vitulli: Caroline Davis, Indiana University, Bloomington
Lauter: Malavika Mukundan, University of Michigan
Salgo Noren: Nikolao Prochorov, Aix-Marseille University

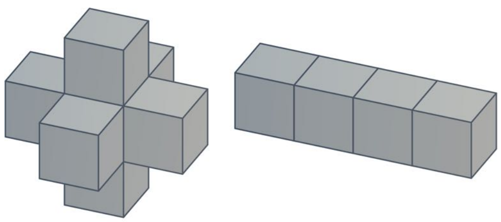
Puzzles Column

Joe P. Buhler and Tanya Khovanova

Ukraine is one of the many Eastern European countries with a rich and vibrant history of Mathematical Olympiads for high school students. All of the problems in this issue of the Emissary are chosen from Ukrainian Olympiads, including some from the Soros Olympiads, which were held in Ukraine for several years as an independently supported event. We thank Pasha Pylyavskyy for helping us with these problems.

1. *56th Ukrainian Olympiad, 2016, 10th grade.* Let $[x]$ denote the greatest integer that is less than or equal to x. Solve the equation $x[x] = 2016$.

2. *40th Ukrainian Olympiad, 2000, 11th grade.* Is it possible to exactly tile an $11 \times 11 \times 11$ cube using copies of the two shapes shown below?



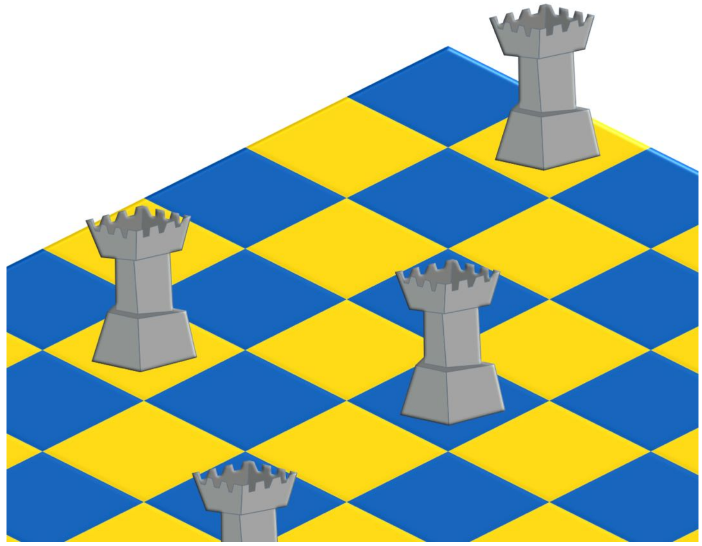
3. *1st Ukrainian Soros Olympiad, 1994, 9th grade.* What is the minimum number of rooks that can be placed on an 8×8 chessboard so that every cell is attacked by at least two rooks? (Assume that a rook does *not* attack the cell on which it sits.)

4. *56th Ukrainian Olympiad, 2016, 10th grade.* Consider a natural number n and its factorization: $n = p_1^{\alpha_1} \dots p_k^{\alpha_k}$, where p_i are prime numbers. We let the degree of n, written $\deg(n)$, be the sum of the exponents $\alpha_1 + \dots + \alpha_k$. Prove that there exist 2016 consecutive natural numbers such that among them there are exactly 1000 with degrees less than 11.

5. *5th Ukrainian Soros Olympiad, 1998, 10th grade.* Find all functions f from $\mathbb{R}$ to $\mathbb{R}$ such that

$$f(x+y) = \max(f(x), y) + \min(x, f(y))$$

for all real numbers x and y.



6. *55th Ukrainian Olympiad, 2015, 11th grade.* A sequence of natural numbers a_n is defined as follows: $a_1 = s$, $a_2 = t$, where s and t are natural numbers, and for $n \geqslant 2$, we have a_{n+1} is the number of indices i, where $1 \leqslant i \leqslant n$, such that $a_i = a_n$. For example, for $s = 2$ and $t = 1$, the sequence begins $2, 1, 1, 2, 2, 3, \dots$ Find all pairs of s and t such that the sequence $a_n + a_{n+1}$ starting from someplace is non-decreasing.

7. *5th Ukrainian Soros Olympiad, 1998, 9th grade.* Show that there are infinitely many positive integers n such that the floor of $2^n/n$ is odd.

8. *2nd Ukranian Soros Olympiad, 1995, 11th grade.* Ten distinct points are placed in the plane so that out of any five, one can find four that lie on one circle. Let k be the maximum number of points that lie on a circle. What is the smallest possible value of k?

9. *5th Ukranian Soros Olympiad, 1998, 9th grade.* Is there a finite set of points S in the plane such that the points do not all lie on a line, and for any three points a, b, c in S that do not lie on a line, the circumcenter of the triangle with vertices a, b, c is also in S?

10. *35th Ukrainian Olympiad, 1995, 10th grade.* Does there exist a finite set of (at least two) points in the plane such that for every pair p, q of distinct points in the set, the perpendicular bisector of the line segment pq contains exactly two other points in the set?

Send your thoughts to the authors at puzzles@msri.org. Solutions will be posted online by August 2022.

Call for Proposals

All proposals can be submitted to the Director or Deputy Director or any member of the Scientific Advisory Committee with a copy to proposals@msri.org. For detailed information, please see the website msri.org.

Thematic Programs

The Scientific Advisory Committee (SAC) of the institute meets in January, May, and November each year to consider letters of intent, pre-proposals, and proposals for programs. The deadlines to submit proposals of any kind for review by the SAC are **March 1**, **October 1**, and **December 1**. Successful proposals are usually developed from the pre-proposal in a collaborative process between the proposers, the Directorate, and the SAC, and may be considered at more than one meeting of the SAC before selection. For complete details, see tinyurl.com/msri-progprop.

Hot Topics Workshops

Each year MSRI runs a week-long workshop on some area of intense mathematical activity chosen the previous fall. Proposals should be received by **March 1**, **October 1**, and **December 1** for review at the upcoming SAC meeting. See tinyurl.com/msri-htw.

Summer Graduate Schools

Every summer MSRI organizes several two-week long summer graduate workshops, most of which are held at MSRI. Proposals must be submitted by **March 1**, **October 1**, and **December 1** for review at the upcoming SAC meeting. See tinyurl.com/msri-sgs.

2021 Annual Report

We gratefully acknowledge the supporters of MSRI whose generosity allows us to fulfill MSRI's mission to advance and communicate the fundamental knowledge in mathematics and the mathematical sciences; to develop human capital for the growth and use of such knowledge; and to cultivate in the larger society awareness and appreciation of the beauty, power, and importance of mathematical ideas and ways of understanding the world.

This report acknowledges grants and gifts received from January 1 – December 31, 2021. In preparation of this report, we have tried to avoid errors and omissions. If any are found, please accept our apologies, and report them to development@msri.org. If your name was not listed as you prefer, let us know so we can correct our records. If your gift was received after December 31, 2021, your name will appear in the 2022 Annual Report. For more information on our giving program, please visit www.msri.org.

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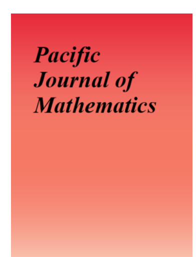
Pacific Journal of Mathematics



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MSRI Hosts Celebration of Women in Mathematics Workshop

The **May 12 Celebration of Women in Mathematics** is part of an international initiative to inspire women everywhere to celebrate their achievements in mathematics, and to encourage an open, welcoming, and inclusive work environment for everybody. May 12 was chosen for this annual event since it is the birthday of mathematician Maryam Mirzakhani.

For 2022, MSRI will host a workshop aimed at graduate students, with a focus on "How to Build a Career in Math." It will be a hybrid workshop, with online and in-person activities at satellite institutions.

The workshop features a panel discussion of women mathematicians from the U.S., Argentina, Ghana, Japan, and the United Kingdom, and special focus on how to become an independent researcher; finding mentors; how to build a network and collaborations; and how to balance teaching, research, administrative duties, and life. Learn more at www.tinyurl.com/msri-may12-2022.

Panelists, from left: ***Elizabeth Donovan****, Murray State University, USA;* ***June Barrow-Green****, The Open University, United Kingdom;* ***Yukari Ito****, University of Tokyo, Japan;* ***Andrea Solotar****, University of Buenos Aires, Argentina;* ***Angela Tabiri****, African Institute for Mathematical Sciences, Ghana*