End-of-Year AP Calculus Projects

Congratulations! You've conquered the AP Calculus BC Exam! For the next three weeks, you will be responsible for completing three projects. The first two projects will count as test grades for fourth quarter. Your final project will count as your exam grade. Please note that each project has an expected due date as well as a rubric. Please pay special attention to the rubric as it will explain how each project will be graded.

Project 1 – Can Can!: An Investigation* – Due Friday, May 19th at the beginning of class

You will probably never walk into a grocery store and say, "oh I need to use calculus to get through this shopping trip!" That's still going to be true. But that isn't to say there *isn't* calculus hidden behind everyday objects. For this group problem set, you may work in groups of up to three to analyze cans.

Guiding Question: How "volume optimized" are the cans in the store?

Each can in the store is made of a certain amount of metal.[1] Could you melt that metal down, reforge it into a different sized cylinder which holds even *more volume*? In other words, does that metal enclose the *most* volume it could?

Your process:

You're going to find 3 different sized cans (e.g. soda can, soup cans, red bull cans, Arizona iced tea cans, tuna fish cans, etc.) and calculate the amount of material used to make these cans (the surface area will suffice – we aren't going to take into account the thickness of the cans).

For each can, you need to calculate what the maximum amount of volume the metal could hold *in theory*.

Then you're going to calculate how "volume optimized" each can is, by calculating

$$\frac{\text{volume of can}}{\text{best possible volume of can}}$$

This decimal can be converted to a percent, which represents how close the actual can is to being the "highest volume" can.

Your product:

You will need to make a visually arresting, colorful poster (a) with photographs of each of your cans, (b) with the height, radius, surface area, and volume of each can labeled, (c) with a clear explanation of how you algebraically calculated the *maximum* possible volume for your cans, and (d) with a calculation of how "volume optimized" each can is. You will want to (for each can) produce a graph of the *can's volume* versus the *can's radius*, and mark the point on the graph with the maximum possible volume, and mark the point on the graph which represents your actual can.

Advice

1. Please do all your work in centimeters, round to the nearest *tenth*, and keep π in all calculations until the end. (Use units!)

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[1] We'll use *surface area* to talk about how much metal is used to build the can. The assumption is that all cans are made of metal with the same thickness, which I know is not true. It's a simplifying assumption.

2. When measuring the radius of each can, you need to be as accurate as possible. You can get the most accurate radius if you measure the circumference of the can (wrap a piece of paper around the can and mark the circumference, and then measure the amount you've marked off) and then calculate the radius using $C = 2\pi r$.
3. When graphing, you should use www.desmos.com (an online graphing calculator). Type the equation in on the left hand window, and click on *settings* to set the x-min/x-max, y-min/y-max. To save the graph, click on *save/share* (you may need to create an account to do this).
4. You may not want to write out the calculation for *each* can on your poster. Instead, after working out the problem for a couple cans, can you come up with a way to show how you'd get the answer for a can with *any surface area*? Putting that on the poster, with some explanation of what you're doing in each stop, will be useful.

Checklist/Rubric: This project will be graded as a percentage of 83 pts as distributed below.

1. A clever title (more clever than the title of this worksheet) _____/2
2. A photograph of each can _____/6
3. Can 1: ____/20
   a. Height, radius, surface area, and volume of each can (in cm, to the nearest tenth) _____/4
   b. Derivation for how to calculate the maximum possible volume for each can
      i. Derivation must be neatly written; each step must be clearly explained. ______/10
   c. A Calculation of how "volume optimized" each can is (the percent) _____/2
   d. A graph with the Volume vs. Radius graphed _____/4
      i. Graph must have a proper Xmin, Xmax, Ymin, Ymax and axes must be labeled.
      ii. Graph has a clearly marked point which represents the most "volume optimized"
      iii. the graph has a clearly marked point which represents the actual can
4. Can 2: ____/20
   a. Height, radius, surface area, and volume of each can (in cm, to the nearest tenth) _____/4
   b. Derivation for how to calculate the maximum possible volume for each can
      i. Derivation must be neatly written; each step must be clearly explained. ______/10
   c. A Calculation of how "volume optimized" each can is (the percent) _____/2
   d. A graph with the Volume vs. Radius graphed _____/4
      i. Graph must have a proper Xmin, Xmax, Ymin, Ymax and axes must be labeled.
      ii. Graph has a clearly marked point which represents the most "volume optimized"
      iii. the graph has a clearly marked point which represents the actual can
5. Can 3: ____/20
   a. Height, radius, surface area, and volume of each can (in cm, to the nearest tenth) _____/4
   b. Derivation for how to calculate the maximum possible volume for each can
      i. Derivation must be neatly written; each step must be clearly explained. ______/10
   c. A Calculation of how "volume optimized" each can is (the percent) _____/2
   d. A graph with the Volume vs. Radius graphed _____/4
      i. Graph must have a proper Xmin, Xmax, Ymin, Ymax and axes must be labeled.
      ii. Graph has a clearly marked point which represents the most "volume optimized"
      iii. the graph has a clearly marked point which represents the actual can
6. The poster is neatly put together, organized in a logical, coherent way _____/5
7. The poster is free from calculation/algebraic/calculus errors. _____/10

- Edited from https://samjshah.com/2012/05/31/a-calculus-optimization-poster-project/

<u>Project 2 – Calculus Recipe Project – Due Friday, May 26th</u>

Each student or pair of students will find a recipe with at least 10 numerical values in the recipe (ingredients amount, cooking temperature, baking time, serving size, etc.). You must re-write *all* numerical values in the recipe using calculus problems that represent the correct amounts in the recipe.

For example:

Old Recipe	$\Rightarrow$	**New Recipe**
2 eggs	$\Rightarrow$	$\lim_{x\to\infty}\frac{16x^2+3x}{8x^2-5}$ **eggs**
yields 8 dozen	$\Rightarrow$	**yields** $\int_0^2 (3x^2)dx$ **dozen**

<u>All students</u> must include each of the following types of problems once:

1. Limit evaluation
2. Definite Integral
3. Average Value
4. Slope of a Tangent Line
5. Volume of a Solid (by cross-sections OR revolution)

Additionally, you must choose three from the list below:

6. Speed of a parametric particle
7. Arc Length (in any coordinate system)
8. Converging Improper Integral Value
9. Radius of Convergence for an Infinite Power Series
10. Converging Infinite Series Value (geometric series, power series, etc.)

Other options inlcude:

- Relative Minimum/Maximum Value
- Absolute Minimum/Maximum Value
- Inflection Point
- Area bounded by curves
- Related Rates
- Polar area
- Approximation using Euler's Method
- Values corresponding to Logistic Models (carrying capacity, population at greatest growth rate, etc.)
- Error Bound

ADDITIONAL CRITERIA:

- You must include a copy of the original recipe, separate from your typed calculus version of the recipe.
- Your group must also hand in an answer sheet that shows **FULL SOLUTIONS** for every value. Your work does not need to be typed, but it must be neat and organized.
- You are not required to actually make the recipe…but it definitely sweetens the deal by 5 points! ☺

This project is worth 50 points. The points will be divided among the individual problems you create and graded for accuracy. If you have a recipe that has the minimum 10 problems, each will be worth 5 points.

<u>Project 3 – Mathematics Interest Project</u>

- All papers due Tuesday, May 30th
- Presentations for exam exempt seniors will begin on Tuesday, May 30th
- Presentations for non-exempt students will be during the teacher-made exam period.

Final Project – Mathematics Interest Project

"Research" a topic that interests you that pertains to any mathematical concept. (It does not necessarily have to be a high-level math concept…sometimes the simplest connections are the most interesting!).

- Write a synopsis of your interest topic to be submitted. (31 points—see attached rubric)
- Present your interest project to the class, including audio/visual aids. (19 points—see attached rubric)

Possible Interest Project Topics

- Fractals
- Tessellations (and MC Escher)
- Mobius strip
- The geometry of Tetris
- Rubik's Cube
- Logic Problems (Lewis Carroll)
- Sudoku
- Card counting
- Casino games
- Origami
- Optical illusions
- Card tricks
- Fibonacci Sequence/The Golden Ratio (math in nature)
- Cryptography and codes
- Board games
- Voting methods
- Graph theory
- Alternative numbering systems/the history of numbers and numbering systems
- Abacus/slide rule (calculating before the calculator)
- The history of Calculus (Newton and Liebniz)
- Any other topic approved by Ms. Anthony

Interest Topic Rubric Name ______________________________________

Synopsis (33 points):	Formatting				2	1	0

- Typed
- 12 point Times New Roman font
- Double-spaced
- Standard 1" margins
- 2–3 pages

Sources		4	3	2	1	0

- must have 4 credible sources listed on a *Works Cited* page, and cite each source at least once in your paper

Citations				2	1	0

- properly included in MLA format in body of paper

Mathematical Relevancy	5	4	3	2	1	0
Knowledge of Topic	5	4	3	2	1	0
Ability to Explain Topic	5	4	3	2	1	0
Ability to Explain Math	5	4	3	2	1	0
Mechanics/Grammar	5	4	3	2	1	0

- no personal pronouns (I, me, you, us, we, etc.)
- must include an introduction and conclusion
- standard requirements for en essay
- 1 point deduction per spelling error
- 1 point deduction per punctuation/capitalization error

Subtotal: __________/ 33

Presentation (19 points):	Eye Contact			3	2	1	0
	Speaking Clearly			3	2	1	0
	Enthusiasm			3	2	1	0
	Comprehensibility	5	4	3	2	1	0
	Audio/Visual Aid	5	4	3	2	1	0

Subtotal: __________/ 19

TOTAL _____________/ 52