# Energy loss of light in interstellar and <br> intergalactic space <br> reproduced with kind permission of Società Italiana di Fisica original publication in Italian on Il Giornale di Fisica, Vol. 44, issue 2, 2003 https://www.sif.it/riviste/sif/gdf/econtents/2003/044/02/article/4 <br> Copyright(c)(2003) by the Italian Physical Society 

M. Missana<br>via Libertà 40, Ceriale (SV)

## 1 Introduction

The study of energy loss of electromagnetic waves passing through matter, interstellar or intergalactic, was suggested to me about in 1965 by L. Rosino and G. Righini because Rosino had tried, unsuccessfully, to measure the so-called cosmological redshift by the $\operatorname{Lyman}_{\alpha}$ line of hydrogen atom in galactic spectra. It seems to me that this kind of study can be done by two methods.

The one I've been following, for about 35 years, considers diffusion (and absorbtion) of electromagnetic waves by free or bound electrons in atoms; study suggested in those years also by L. Pasinetti to interpret spectra of stars like 41 Tauri, by A. Masani to study the spectrum of the Orion Nebula and by other colleagues.

I calculated cross-sections of diffusion by quantum mechanics and by quantum theory of electromagnetic field.

Cross-section of diffusion of an electromagnetic wave by an electron or another particle is the ratio between the scattered (diffused) wave intensity and the incident wave intensity, when there is one electron per unit volume, electrons being on a plane surface of thickness one.

Absorption cross-section of an electromagnetic wave by an electron or another particle is the ratio between (incident wave intensity minus transmitted wave intensity) and incident wave intensity, minus the total cross-section of the backward diffusion, when there is one electron per unit volum, electrons being on a planar surface of thickness one.

Absorption cross-sections are tabulated for a lot of elements by Wiese 1].
Dr. Roberto Monti (Tesre, Bologna), about 20 years ago, explained to me a second method, mathematically much more simpler, but it requires some ad hoc hypotheses. It simply consists in introducing in the equations of electromagnetic waves propagating in interstellar or intergalactic matter, an appropriate index of refraction $n$, different from one, an appropriate current $\vec{J}$ and an appropriate charge density Q . The value of these quantities is determined a posteriori by Monti imposing that the solution of electromagnetic wave equation

$$
\begin{equation*}
\left[-(\vec{\nabla})^{2}+\left(\frac{n}{c}\right)^{2} \frac{d^{2}}{d t^{2}}\right] A^{\alpha} \cong J^{\alpha} \tag{1}
\end{equation*}
$$

gives the cosmological red shift and the observed attenuations in intensity; with $J^{\alpha} \equiv(\vec{J}, Q), A^{\alpha}$ 4-potential of electromagnetic wave, $\alpha \equiv(1,2,3,4), c$ constant of the speed of light in vacuum; $(x, y, z)$ Cartesian coordinates, $t$ time and

$$
\vec{\nabla} \equiv\left(\frac{d}{d x}, \frac{d}{d y}, \frac{d}{d z}\right)
$$

Cosmological red shift is the shift toward longer wavelenghts of the wavelenghts of spectral lines observed in the spectra of distant galaxies with respect to the corresponding laboratory wavelenght; shift that on average is as greater as smaller is the luminous intensity of the galaxy.

Attenuation is the decreasing in intensity of the luminosity of galaxies when increases their distance, distance determined by some astronomical method (variable stars, etc.).

## 2 The equation of radiative transfer

Coming back to the method I followed, once obtained cross-sections for diffusion and absorption of electromagnetic radiation by atoms, molecules and corpuscoles of the interstellar medium, they are included in the equation of radiative transfer, equation that should be solved with the help of the computer as nobody has managed to solve it with algebraic or special functions except in simple cases; in fact we find ourselves solving an integral equation to partial derivatives containing finite differences.

Now a polar coordinate system $x^{j} \equiv(r, \theta, \phi), j \equiv(1,2,3)$ is introduced, and a Cartesian coordinate system $x^{j} \equiv(x, y, z)$ superimposed on it; for convenience of writing it is indicated $\mu=\cos \theta$ and $z=r \mu$, where $z$ is the distance of the light source along the polar axis, which is directed towards the terrestrial observer.

In fig. 1 the main plane of these coordinates ( $x$ - $y$ plane) coincides with the surface of the star or galaxy (taken flat for simplicity) in which the spectrum, which is then absorbed and diffused, is generated.

In this study I've used the equation of transfer of Chandrasekhar [2], for plane waves, substituting the cross-section of the isotropic diffusion with the most likely Thomson cross-section and adding only absorption [3]; it is

$$
\begin{align*}
&\left(\frac{\sqrt{3}}{2} \mu \frac{d}{d \tau}+1\right) I(\tau, \mu, \lambda)=-\frac{\sigma_{a} D_{a}}{\sigma_{T} D} \cdot I(\tau, \mu, \lambda)+\frac{3}{16 \pi} \\
& \cdot \int_{0}^{2 \pi} d \phi^{\prime} \int_{-\pi}^{\pi} d \mu^{\prime}\left(1+\cos ^{2} \Theta\right) \\
& \cdot I\left[\tau, \mu^{\prime}, \lambda-\gamma(1-\cos \Theta)\right] \tag{2}
\end{align*}
$$

with

$$
\begin{gather*}
\cos \Theta=\mu \mu^{\prime}+\sqrt{1-\mu^{2}} \sqrt{1-\mu^{\prime 2}} \cdot \cos \left(\phi-\phi^{\prime}\right) \\
\tau=\frac{\sqrt{3}}{2} \sigma_{T} \int_{0}^{z} D d z \tag{2a}
\end{gather*}
$$

(different from that of Chandrasekhar) $D$ number of diffusion centres per unit volume, $D_{a}$ number of absorbing centres per unit volume, $I(\tau, \mu, \lambda)$ electromagnetic wave intensity, $\lambda$ electromagnetic wavelength, $\gamma=2,4 \times 10^{-12} \mathrm{~m}$ is the


Figure 1 Interstellar medium between the planes $(x, y)$ and $\left(x^{\prime}, y^{\prime}\right)$. (Figure of D. Garegnani)

Compton wavelength of the electron, $\sigma_{a}$ is the total absorbtion cross-section, $\sigma_{T}$ the total Thomson cross-section for diffusion $=6,7 \times 10^{-29} \mathrm{~m}^{2}$ in the case of free electron at rest, for the wavelengths of the visible spectrum under conditions of linear diffusion; in the non-linear theory the cross-section is proportional to the wavelenght according to Prof. Mario Verde of the Turin University (1974).

Be

$$
\begin{equation*}
\tau_{S}=\frac{\sqrt{3}}{2} \sigma_{T} \int_{0}^{R} D d z \cong \frac{\sqrt{3}}{2} \sigma_{T} D R \tag{2b}
\end{equation*}
$$

with $Z=R$ the distance crossed by light on the way from the source to the Earth, the boundary conditions are

$$
\left\{\begin{array}{l}
I(0, \mu, \lambda)=\Psi(\lambda) \text { per } \mu>0(z=0)  \tag{3}\\
I\left(\tau_{S}, \mu, \lambda\right)=0 \text { per } \mu<0(z=R)
\end{array}\right.
$$

where $\Psi(\lambda)$ is an arbitrary function imposed by the physical conditions.
A solution of this equation, valid for plane waves that propagate along the direction of $z$ axis, in a Cartesian coordinate system, can be expressed by the integral of algebraic and special functions given in the formula below, in the approximation of pure absorption and Thomson scattering, namely

$$
\sigma=\frac{3}{16 \pi} \sigma_{T}\left(1+\cos ^{2} \Theta\right)
$$

and

$$
d \lambda=\gamma(1-\cos \Theta)
$$

as specified in equation (2), in presence of Compton effect with electrons at rest [4, 5] (errata corrige [6]).

Assuming for simplicity of calculation $\sigma_{a}=0, \sigma_{T}=$ cost and that in the conditions (3) at the source the profile of spectral lines is a Gaussian of amplitude $W_{0}$, i.e. [7]

$$
\begin{align*}
\Psi(\lambda)=I_{0} \exp \left[\frac{-\left(\lambda-\lambda_{0}\right)^{2}}{W_{0}^{2}}\right]= & \\
\frac{1}{2 \pi} \int_{-\infty}^{\infty} I_{0} \sqrt{\pi} W_{0} \exp \left[\frac{-\alpha^{2} W_{0}^{2}}{4}\right] & \\
& \cdot \exp \left[i \alpha\left(\lambda-\lambda_{0}\right)\right] d \alpha, \tag{3a}
\end{align*}
$$

in the approximation of a single intensity that propagates forward and only one that turns back; the solution, for the intensity that propagates forward, is given by

$$
\begin{equation*}
I\left(\tau, \frac{1}{\sqrt{3}}, \lambda\right)=\operatorname{Re}\left\{\frac{1}{\pi} \int_{0}^{\infty} F_{+}(\tau, \alpha) \exp \left[i \alpha\left(\lambda-\lambda_{0}\right)\right] d \alpha\right\} \tag{4}
\end{equation*}
$$

with $i$ imaginary unit and

$$
\begin{gather*}
F_{+}\left(\tau_{s}, \alpha\right)=\frac{-2 \omega \sqrt{\pi} I_{0} W_{0} \exp \left[\frac{-\alpha^{2} W_{0}^{2}}{4}\right]}{\left[P_{1} \exp \left(\omega \tau_{s}^{*}\right)-P_{2} \exp \left(-\omega \tau_{s}^{*}\right)\right]}  \tag{5}\\
\omega=\left(C^{* 2}-C^{2}-2 K C^{*}+K^{2}\right)^{1 / 2}
\end{gather*}
$$

$I_{0}$ spectral line intensity at the source, defined in the equation (3a), $W_{0}$ spectral line width ( $\cong 0,6 \mathrm{FWHM})$, at the source,

$$
\begin{gathered}
K=2 \exp [i \alpha \gamma], \\
\tau_{s}^{*}=\tau_{s} \exp [-i \alpha \gamma]=\tau_{s} 2 / K, \\
P_{1}=C^{*}-K-\omega, \\
P_{2}=C^{*}-K+\omega,
\end{gathered}
$$

$C^{*}$ is the complex conjugate of $C$

$$
C^{*}=\exp [i \alpha \gamma / 3]\left[J_{0}(2 \alpha \gamma / 3)-J_{2}(2 \alpha \gamma / 3) / 6+i J_{1}(2 \alpha \gamma / 3)\right]
$$

where $J_{n}(x)=(-1)^{n} J_{n}(-x)$ is the Bessel function of the first kind, of order $n$, in the variable $x$ [8]. Remember that to get the formulas (4) and (5) the Gaussian quadrature formulas in the case $n=2$ were used [2]

$$
\begin{equation*}
\int_{-1}^{1} f(\mu) d \mu \sim \sum_{i=1}^{n} f\left(\mu_{i}\right) a_{i} \sim f\left(\mu_{-1}\right) a_{-1}+f\left(\mu_{1}\right) a_{1} \tag{5a}
\end{equation*}
$$

with $\mu_{ \pm 1}= \pm 1 / \sqrt{3}, a_{ \pm 1}=1$ and of the remarkable integral 9 ]

$$
\int_{0}^{2 \pi} \exp [i t \cos x] \cos (n x) d x=2 \pi \exp \left[\frac{i n \pi}{2}\right] J_{n}(t), \quad n=0,1,2 \ldots
$$

| $W_{o}$ |  |  |  |  |  |  |  |  |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\tau$ | 500 | 1000 | 2000 | 6000 | 10000 | 20000 | 50000 | 100000 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 37,6 | 38,4 | 38,6 | 38,7 | 38,7 | 38,7 | 38,7 | 38,7 |
| 2 | 108 | 113,3 | 115,1 | 115,6 | 115,6 | 115,6 | 115,6 | 115,6 |
| 3 | 200 | 216,6 | 223,5 | 225,8 | 225,8 | 225,8 | 225,8 | 225,8 |
| 6 | 553,5 | 642 | 707 | 745 | 750 | 751,2 | 752,5 | 752,5 |
| 9 | 1008 | 1172,5 | 1349 | 1520 | 1549 | 1562 | 1567 | 1567 |
| 12 | 1590 | 1795 | 2089 | 2495 | 2591 | 2649 | 2668 | 2670 |
| 15 | 2327 | 2531 | 2916 | 3610 | 3834 | 3991 | 4049 | 4058 |
| 18 | 3230 | 3408 | 3835 | 4830 | 5228 | 5559 | 5707 | 5732 |

Table I

The expression (4) of the diffuse intensity can be studied by the program INOXC.f in Fortran77, obtainable from the author upon request.

From these formulas it can be deduced by that program above, that the central intensity of the broad lines and hence the intensity of the continuous spectrum of value $I\left(0,1 / \sqrt{3}, \lambda_{0}\right)=I_{c}$ at the origin, varies in good approximation by the following formula [3, 7, 10]:

$$
\begin{gather*}
I\left(\tau_{s}, 1 / \sqrt{3}, \lambda_{0}^{\prime}\right) \sim \frac{I\left(0, \frac{1}{\sqrt{3}}, \lambda_{0}\right)}{1+\tau_{s}}=\frac{I\left(0, \frac{1}{\sqrt{3}}, \lambda_{0}\right)}{1+\frac{R}{R_{0}}},  \tag{6}\\
\lambda_{0}^{\prime}>\lambda_{0}+\gamma \tau_{s} 2 / \sqrt{3} \tag{6a}
\end{gather*}
$$

with $\tau_{s}$ optical thickness of the diffusing medium defined in the formula (2b), $R_{0}=\frac{2}{\sqrt{3} \sigma_{T} D}, D$ number of diffusion centers per unit volume, as defined above, with $\lambda_{0}^{\prime}$ wavelenght of a line observed in a distant galaxy (after diffusion) and $\lambda_{0}$ is the corresponding wavelength of a line observed in a laboratory spectrum.

This formula (6), can be obtained for the intensity of the continuous spectrum, without numerical calculations, using distributions [8, putting the Fourier transform of $\Psi(\lambda)$, defined in eq.(3), equal to $I_{c} \delta(\alpha), \delta(\alpha)$ is the Dirac delta function; this is also reported in article [7], where however in eq. (12) there is erroneously $\overrightarrow{\Phi_{0}}(\vec{x}, t)$ instead of $\overrightarrow{\Phi_{0}}(\vec{x}, \omega)$ and then $\overrightarrow{\Phi_{0}}(\vec{x}, \omega) \sim$ cost. instead of $\overrightarrow{\Phi_{0}}(\vec{x}, \omega) \sim \delta(\omega)$, as has been pointed out by P. Mantegazza of the Observatory Brera in Merate.

For thinner lines, on the other hand, numerical results indicate a decrease in intensity much more accentuated with increasing optical thickness $\tau_{s}$, as follows from the numerical results reported in table II, taken from [7]; these were obtained with the program INOXC.f, which uses double precision.

It is worth noting that the verification of the numerical stability of results was done partially, only for those optical thickness $\tau_{s}$ lower than unity that can interpret the redshift of the lines observed in the sun [4, 11.

In Table I there are the wavelength shifts of spectral lines $\lambda_{0}^{\prime}-\lambda_{0}$ in m $\AA=$ $10^{-13} \mathrm{~m}$ as a function of the amplitudes $W_{0}$ given in the first raw, always in $\mathrm{m} \AA$ and of the optical thickness $\tau$ (dimensionless) in the first column.

| $W_{o}$ |  |  |  |  |  |  |  |  |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\tau$ | 500 | 1000 | 2000 | 6000 | 10000 | 20000 | 50000 | 100000 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0,495 | 0,499 | 0,4995 | 0,4995 | 0,5 | 0,5 | 0,5 | 0,5 |
| 2 | 0,319 | 0,329 | 0,333 | 0,333 | 0,333 | 0,333 | 0,333 | 0,333 |
| 3 | 0,2245 | 0,242 | 0,248 | 0,25 | 0,25 | 0,25 | 0,25 | 0,25 |
| 6 | 0,0926 | 0,1195 | 0,134 | 0,142 | 0,1425 | 0,1425 | 0,1428 | 0,1428 |
| 9 | 0,043 | 0,0655 | 0,0841 | 0,0972 | 0,099 | 0,0995 | 0,1 | 0,1 |
| 12 | 0,0221 | 0,0378 | 0,0548 | 0,0718 | 0,0748 | 0,0763 | 0,0768 | 0,0769 |
| 15 | 0,0125 | 0,0229 | 0,0366 | 0,0547 | 0,0589 | 0,0615 | 0,0623 | 0,0624 |
| 18 | 0,0076 | 0,0145 | 0,0251 | 0,0424 | 0,0475 | 0,051 | 0,0523 | 0,0525 |

Table II

In Table II there are the central intensities of spectral lines, assuming $1=I_{0}$ the intensity of the lines before diffusion, as a function of the amplitudes $W_{0}$ given in the first row, in units of $\mathrm{m} \AA=10^{-13} \mathrm{~m}$ and in function of the optical thickness $\tau$ (dimensionless) given in the first column; the fourth decimal place is not significant.

They have been obtained only for not very large values of optical thickness; they also are not satisfactory because, having assumed a constant $\sigma_{T}$ to integrate the transfer equations, give a cosmological redshift constant, independent of the wavelength of the lines observed, in contrast to the measures that give a redshift proportional to the wavelength.

A discussion of the dependence of the cross section of diffusion $\sigma_{T}$ on the wavelength will be subject to further communication.

Including now the effect of absorption, the intensity of the lines and of the continuous spectrum, in the case of plane waves, can be obtained with the formulas of the publication [3; the calculations are quite complex, however here it is assumed as a first approximation $\sigma_{a}=$ cost., independent of $\lambda$ and the following formula is obtained, always in the two-flux approximation:

$$
\begin{align*}
& I\left(\tau_{s}, \frac{1}{\sqrt{3}}, \lambda^{\prime}\right)=I\left(0, \frac{1}{\sqrt{3}}, \lambda_{0}\right) \\
& \cdot \exp \left[-\sigma_{a} D_{a} R\right] \\
& \cdot \operatorname{Re}\left\{\frac{1}{\pi} \int_{0}^{\infty} F_{+}(\tau, \alpha) \exp \left[i \alpha\left(\lambda-\lambda_{0}\right)\right] d \alpha\right\}  \tag{7}\\
& I\left(\tau_{s}, \frac{1}{\sqrt{3}}, \lambda_{0}^{\prime}\right)=\frac{I\left(0, \frac{1}{\sqrt{3}}, \lambda_{0}\right) \exp \left[-\sigma_{a} D_{a} R\right]}{1+\tau_{s}} \\
& \lambda_{0}^{\prime}>\lambda_{0}+\gamma \tau_{s} 2 / \sqrt{3} \tag{6b}
\end{align*}
$$

$D_{a}$ is the number of absorption centers per unit volume and $\lambda^{\prime}{ }_{0}$ is the value of $\lambda^{\prime}$ for which the maximum intensity of the spectral line is obtained after being diffused for the interaction with the interstellar matter.

Actually the results of Table $I$ show that $\lambda_{0}^{\prime}-\lambda_{0}$ increases with progressive non-linear law with the increase of $\tau_{s}$.

## 3 Olbers' paradox and the background radiation

In order to solve the Olbers' paradox and study the background radiation, the transfer equation in polar coordinates must be used, in a medium with spherical symmetry. I have studied this problem in the article [12] with a program of which perhaps there is a copy at the Department of Astronomy, University of Oxford while the copy of Brera was unfortunately destroyed. This program is much more complex than the program INOXC.f but doesn't differ significantly from it in the results, apart from factor $1 / R^{2}$, in the case of only two fluxes of radiation, for large values of $R$. In fact, the transfer equation in polar coordinates, in the above cases, is [2]:

$$
\begin{align*}
& \left(\frac{\sqrt{3}}{2} \mu \frac{\delta}{\delta \tau^{\prime \prime}}+\frac{1-\mu^{2}}{D \sigma_{T} r} \frac{\delta}{\delta \mu}+1\right) \\
& \quad \cdot I^{\prime \prime}\left(\tau^{\prime \prime}, \mu, \lambda\right)=-\frac{\sigma_{a} D_{a}}{\sigma_{T} D} I^{\prime \prime}\left(\tau^{\prime \prime}, \mu, \lambda\right)+ \\
& \quad+\frac{3}{16 \pi} \int_{0}^{2 \pi} d \phi^{\prime} \int_{-\pi}^{\pi} d \mu^{\prime}\left(1+\cos ^{2} \Theta\right) \cdot \\
& \quad \cdot I^{\prime \prime}\left[\tau^{\prime \prime}, \mu^{\prime}, \lambda-\gamma(1-\cos \Theta)\right] \tag{8}
\end{align*}
$$

with

$$
\tau^{\prime \prime}=\frac{\sqrt{3}}{2} \sigma_{T} \int_{0}^{r} D d r
$$

From eq.(1) for spherically symmetric waves in absence of diffusion and absorption there are solutions of the type: $A(r, \theta, \phi, t)=f(r, \theta, \phi, t) / r$, where $f(r, \theta, \phi, t)$ describes a spherical wave, etc..; therefore, since the intensity I is obtained from A with Poynting's formula, it was put tentatively in eq. (8)

$$
\begin{equation*}
I^{\prime \prime}\left(\tau^{\prime \prime}, \mu, \lambda\right)=I^{\prime}\left(\tau^{\prime \prime}, \mu, \lambda\right) / r^{2} \tag{9}
\end{equation*}
$$

with $I^{\prime}\left(\tau^{\prime \prime}, \mu, \lambda\right)$ unknown function.
Remembering also that for the formula (7) of article 12, (in the same approximation of the Gauss quadrature formulas) indicating $I\left(\mu_{i}\right)=I_{i}$, introducing the discrete values of $\mu$ defined in eq.(5a), it follows

$$
\begin{equation*}
\frac{\delta}{\delta \mu} I\left(\mu_{J}\right) \sim \sum_{i=1}^{n} I_{i} d_{i, j} \tag{6}
\end{equation*}
$$

with $d_{i, j}$ constants in this case, therefore from eq.(8) written above, multiplied
by $r^{2}$ one gets

$$
\begin{align*}
& \left(\frac{\sqrt{3}}{2} \mu_{j} \frac{\delta}{\delta \tau^{\prime \prime}}+1\right) I^{\prime}\left(\tau^{\prime \prime}, \mu_{j}, \lambda\right)+ \\
& \quad \frac{1}{D \sigma_{T}}\left[\left(1-\mu_{j}^{2}\right) \sum_{i=1}^{n} I^{\prime}\left(\tau^{\prime \prime}, \mu_{i}, \lambda\right)\right. \\
& \left.\cdot d_{i, j}-2 \mu_{j} I^{\prime}\left(\tau^{\prime \prime}, \mu_{j}, \lambda\right)\right] / r= \\
& \quad-\frac{\sigma_{a} D_{a}}{\sigma_{T} D} I^{\prime}\left(\tau^{\prime \prime}, \mu_{j}, \lambda\right)+\frac{3}{(16 \pi)} \\
& \quad \cdot \int_{0}^{2 \pi} d \phi^{\prime} \int_{-\pi}^{\pi} d \mu^{\prime}\left(1+\cos ^{2} \Theta\right) \\
& \quad \cdot I^{\prime}\left[\tau^{\prime \prime}, \mu^{\prime}, \lambda-\gamma(1-\cos \Theta)\right] \tag{8a}
\end{align*}
$$

It is evident that for large $r$ eq.(8a) asymptotically coincides with eq.(2) in $\tau^{\prime \prime}$ and therefore $I^{\prime}=I$; it follows that the central intensity of the wide lines and hence the intensity of continuous spectrum, due to a flux of radiation in spherical symmetry, for the formulas (9) and (7), is given in good approximation by the following formula:

$$
\begin{align*}
& I^{\prime \prime}\left(\tau_{s}, \frac{1}{\sqrt{3}}, \lambda_{0}^{\prime}\right) \sim \\
& \sim \frac{I\left(0, \frac{1}{\sqrt{3}}, \lambda_{0}\right) \exp \left[-\sigma_{a} D_{a} R\right]}{R^{2}\left(1+\tau_{s}\right)}= \\
& =\frac{I\left(0, \frac{1}{\sqrt{3}}, \lambda_{0}\right) \exp \left[-\sigma_{a} D_{a} R\right]}{R^{2}\left(1+\frac{R}{R_{0}}\right)}  \tag{9a}\\
& \lambda_{0}{ }^{\prime}>\lambda_{0}+\gamma \tau_{s} 2 / \sqrt{3} \tag{6b}
\end{align*}
$$

with $\tau_{s}$ optical thickness of the diffusing medium defined in formula (2b),

$$
R_{0}=\frac{2}{\sqrt{3} \sigma_{T} D}
$$

as previously written, with $\lambda_{0}^{\prime}$ wavelength of a line observed in a distant galaxy and with $\lambda_{0}$ wavelength of a line observed in a laboratory spectrum; in absence of absorption and diffusion $\left(D_{a}=0, R_{0}=\infty\right)$ formula (9a) is that of a spherical wave and justifies a posteriori the equation (9).

Although the result of this formula is approximated, it allows to deduce that the visible spectrum, besides decreasing with the increase in distance, also tends to disappear completely because it moves to another region of the spectrum. Therefore with these formulas it is possible to solve the so-called Olbers' paradox and demonstrate that even if the sky is endless, the brightness of the night sky is finite, as long as there is intergalactic and interstellar matter.

This paradox says that the average brightness of the night sky, of course far away from stars and galaxies, that is indicated $I_{K}$, should be infinite if the sky
is infinite being given by

$$
\begin{equation*}
I_{K}(\lambda)=\lim _{R \rightarrow \infty}\left(4 \pi R I_{A m} N\right)=\infty \tag{10}
\end{equation*}
$$

where N is the average number of stars per unit volume, $I_{A m}$ is the average intensity of one of them, which can be obtained from the absolute magnitude and $R$ is the distance from Earth.

Formula (10) can be deduced with the following simple considerations: - the average intensity that comes per unit area from a source of avarage intensity $I_{A m}$ at a distance $R$, in absence of intergalactic medium, is $I_{A m} / R^{2}$; - the average brightness that comes from all the stars at the distance between $R$ and $R+d R$ is

$$
\begin{equation*}
d I_{K}(\lambda)=N 4 \pi R^{2} d R I_{A m} / R^{2} \tag{11}
\end{equation*}
$$

it follows that integrating between zero and infinity we have the Olbers'formula (10).

However, if absorption and diffusion are introduced by the formula (9a), (where $I\left(0,1 / \sqrt{3}, \lambda_{0}\right)$ is replaced with $I_{A m}$ ) thus putting in eq. (11)

$$
I_{A m} \exp \left[-\sigma_{a} D_{a} R\right] /\left[R^{2}\left(1+R / R_{0}\right)\right]
$$

instead of $I_{A m} / R^{2}$, the following equation is obtained:

$$
\begin{equation*}
d I_{K}(\lambda)=\frac{N 4 \pi d R I_{A m} \exp \left[-\sigma_{a} D_{a} R\right]}{1+R / R_{0}} \tag{11a}
\end{equation*}
$$

From this equation, in the case of pure absorption, neglecting the effects of diffusion and integrating, the intensity is always finite and is

$$
\begin{equation*}
I_{K}(\lambda) \sim N I_{A m} 4 \pi /\left(\sigma_{a} D_{a}\right) \tag{12}
\end{equation*}
$$

If there is no absorption, but only diffusion, always for the formula (11a), neglecting the effects of absorption and integrating, one has a logarithmic infinity, very weak and that is

$$
\begin{align*}
& I_{K}\left(\lambda^{\prime}\right) \\
& \quad \sim  \tag{13}\\
& \quad \sim \frac{I_{A m} N}{\sigma_{T} D} \lim _{R \rightarrow \infty} \log \left[1+\frac{\sqrt{3}}{2} \sigma_{T} D R\right]
\end{align*}
$$

and from eq.(6b) it follows

$$
\begin{equation*}
\lambda_{m}^{\prime}>\lambda_{m}+\gamma \lim _{R \rightarrow \infty} \sigma_{T} D R \tag{13a}
\end{equation*}
$$

where $\lambda_{m}$ is an average value of the wavelength of a certain spectral region, before diffusion in the interstellar matter, and $\lambda_{m}^{\prime}$ indicates an average value of the wavelength in the corresponding spectral range observed after the diffusion of light in intergalactic space. Concluding, there is also in this case a total extinction of visible light because, for the formula (13a), the spectrum moves in the extreme infrared, from a given distance forward. With this formula (11a)
we demonstrate now that according to Eddington's theory 13 the background radiation (also known as 3 K ) originates within 700 pc from Earth, $1 \mathrm{pc}=$ $3,08567810^{16} \mathrm{~m}$.

Remember that in Eddington's theory it is assumed that the interstellar matter is in thermodynamical equilibrium with the radiation coming from the stars and radiates like a blackbody. With this simple hypothesis, from the density of the observed light, the temperature of the interstellar medium and the brightness of the background radiation are obtained; with the data of that time Eddington had found for the interstellar matter a temperature of $3,18 \mathrm{~K}$, close to the temperature of $2,96 \mathrm{~K}$ recently measured by Woody et al. [14].

Since from Allen [15] $\sigma_{a} \sim 10^{-13} \mathrm{~m}^{2}, D_{a} \sim 0,5 \times 10^{-6}$ granules $\mathrm{m}^{-3}$, placing these numerical values of the formula (11a), neglecting the effect of diffusion which is not known and is not given by Allen, it follows that the interstellar medium, which radiates the 3 K , and is at a distance equal to or greater than 1 kpc from us, gives a contribution to the illumination of our night sky less than $0,21 N 4 \pi d R I_{A m}$, i.e. is marginal and most of the 3 K radiation that is observed is produced within 700 pc from the galactic plane, on which the Earth is roughly located.

Remembering that the thickness of the galactic disk around the Earth is about $1 \mathrm{kpc}(2 \mathrm{kpc}$ total thickness of the disc) it follows that the 3 K is of local galactic origin. However, it would be advisable to obtain again the values of $\sigma_{a}$ and $D_{a}$ with a theory that distinguishes between absorption and diffusion.

## 4 Discussion

Mario Carpino:
Where the infinite background energy ends up at long wavelengths?
Answer:
In part, it goes to the stars that emitted it at short wavelengths, in part it goes to the intergalactic matter that reemits it as the background radiation (with the same formulas of Eddington [13] and the observative data of Allen [15], the temperature of the intergalactic matter can be calculated assuming that it is in a state of thermodynamic equilibrium with the optical radiation and in a first approximation I remember to had found, several years ago, that it is less than $2,7 \mathrm{~K}$ ) and then maybe it is absorbed for other effects.

It should not be forgotten also that the cross section of Thomson used has an approximate value and therefore the results are a first approximation.

Luigi Guzzo:
Does the cross section of the diffusion $\sigma_{T}$ vary with the wavelength?
Answer:
Data on cross sections of diffusion of light are scarce, in the case of the free electron $\sigma$, according to the current literature, it is given by Klein-Nishima formula; for the bound electron additional terms must be added due to interaction with other electrons, with the atomic nucleus, with the other atoms in addition to non-linear terms which, according to Mario Verde, are proportional to the wavelenght as in the Breemstrahlung effect.

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