



# ENGINEERING MATHEMATICS

## ESE | GATE | PSUs

### 2018



- ▶ COMPLETE THEORY (ESE & GATE PATTERN)
- ▶ 1386 QUESTIONS (LAST 25 YEARS OF ESE & GATE EXAMS) WITH DETAILED SOLUTION
- ▶ 672 EXAMPLES FOR CONCEPTUAL CLARITY
- ▶ 289 TOPICS COVERED

# **ENGINEERING MATHEMATICS**

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- Comprising conceptual questions marked with '\*' to save the time while revising**



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**First Edition : 2017**

# PREFACE

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We, the IES MASTER, have immense pleasure in placing the first edition of “**Engineering Mathematics**” before the aspirants of GATE & ESE exams.

Dear Students, as we all know that in 2016 UPSC included Engineering Mathematics as a part of syllabus of common paper for ESE exam as well as of a technical paper for EC/EE branch, while Engineering Mathematics already has 15% weightage in GATE exam. We have observed that currently available books cover neither all the topics nor all previously asked questions in GATE & ESE exams. Since most of the books focus on only some selected main topics, students have not been able to answer more than 60-65% of 1386 questions that have been asked in GATE & ESE exams so far. Hence to overcome this problem, we have tried our best by covering more than 289 topics under 31 chapters in 8 units. (One should not be in dilemma that 289 topics are more than sufficient. These are the minimum topics from where GATE & ESE have already asked questions). Since we have covered every previous year questions from last 25 years of each topic, students can easily decide, how much time to allocate on each chapter based on the number of questions asked in that particular exam. Again, we have included only those proofs that are necessary for concept building of topics and we have stressed on providing elaborate solution to all the questions.

It is the only book in the market which has complete theory exactly on ESE & GATE Pattern. After each topic there are sufficient number of solved examples for concept building & easy learning. The book includes such types of 672 examples. It also covers all the previously asked questions in which conceptual questions are marked with ‘\*’ sign so that students can save their time, while revising.

Having incorporated my teaching experience of more than 13 years, I believe this book will enable the students to excel in Engineering Mathematics.

My source of inspiration is Mr. Kanchan Thakur Sir (Ex-IES). He has continuously motivated me while writing this book.

My special thanks to the entire IES MASTER Team for their continuous support in bringing out the book. I strongly believe that this book will help students in their journey of success. I invite suggestions from students, teachers & educators for further improvement in the book.

**Dr. Puneet Sharma**  
(M.Sc., Ph.D.)  
IES Master Publications  
New Delhi

# CONTENTS

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## UNIT 1 : CALCULUS

<b>1.1</b>	<b>Limits, Continuity and Differentiability .....</b>	<b>01 – 44</b>
(i)	Function .....	01
(ii)	Limit of a Function .....	01
(iii)	Theorem on Limits .....	02
(iv)	Indeterminate Forms .....	04
(v)	L-Hospital Rule .....	05
(vi)	Fundamentals of Continuity .....	08
(vii)	Kinds of Discontinuities .....	10
(viii)	Properties of Continuous Functions .....	15
(ix)	Saltus of a Function .....	16
(x)	Function of Two Variables .....	17
(xi)	Limit of a Function of Two Variables .....	17
(xii)	Continuity of Function of Two Variables .....	22
(xiii)	Differentiability .....	23
	<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>29</b>
<b>1.2</b>	<b>Partial Differentiation .....</b>	<b>45 – 65</b>
(i)	Partial Derivatives .....	45
(ii)	Homogeneous Functions .....	49
(iii)	Euler's Theorem on Homogeneous Function .....	50
(iv)	Total Differential Coefficients .....	52
(v)	Change of Variables .....	52
(vi)	Jacobian .....	56
(vii)	Chain Rule of Jacobian .....	59

# GATE 2018

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(viii) Functional Dependence .....	60
<b>Previous Years GATE &amp; ESE Questions .....</b> <span style="float: right;">63</span>	
<b>1.3 Infinite Series .....</b>	<b>66 – 78</b>
(i) Expansion of Functions .....	66
(ii) Maclaurin's theorem .....	66
(iii) Taylor's Theorem .....	66
(iv) Convergence and Divergence of Infinite Series .....	69
(v) Methods to Find Convergence of Infinite Series .....	70
(vi) Power Series .....	71
<b>Previous Years GATE &amp; ESE Questions .....</b> <span style="float: right;">72</span>	
<b>1.4 Mean Value Theorems .....</b>	<b>79 – 86</b>
(i) Rolle's Theorem .....	79
(ii) Lagrange's First Mean Value Theorem .....	80
(iii) Cauchy's Mean Value Theorem .....	80
(iv) Bolzanos Theorem .....	81
(v) Intermediate Value Theorem .....	81
(vi) Darboux Theorem .....	81
<b>Previous Years GATE &amp; ESE Questions .....</b> <span style="float: right;">84</span>	
<b>1.5 Maxima and Minima .....</b>	<b>87 – 114</b>
(i) Increasing and Decreasing Functions .....	87
(ii) Maxima and Minima of Function of Single Variable .....	88
(iii) Sufficient Condition (Second Derivate Test) .....	89
(iv) Maxima-Minima of Functions of Two Variables .....	89
(v) Sufficient Condition (Lagrange's Conditions) .....	91
(vi) Lagrange's Method of Undetermined Multipliers .....	94
<b>Previous Years GATE &amp; ESE Questions .....</b> <span style="float: right;">98</span>	
<b>1.6 Multiple Integral .....</b>	<b>115 – 176</b>
(i) Concept of Integration .....	115
(ii) Some Standard Formulae .....	115

(iii) Some Important Integration and their Hints .....	116
(iv) Definite integrals .....	117
(v) Fundamental Properties of Definite Integrals .....	117
(vi) Fundamental Theorem of Integral Calculus .....	118
(vii) Definite integral as the Limit of a Sum .....	118
(viii) Double Integrals .....	121
(ix) Double Integrals in Polar Coordinates .....	122
(x) Triple Integrals .....	127
(xi) Change of Order of Integration .....	130
(xii) Use of Jacobian in Multiple Integrals .....	133
(xiii) Multiple Integral using Change of Variables Concept .....	134
(xiv) Area and Volume in Different Coordinates Systems .....	136
(xv) Arc Length of Curves (Rectification) .....	139
(xvi) Intrinsic Equation of a Curve .....	139
(xvii) Volumes of Solids of Revolution .....	141
(xviii) Surfaces of Solids of Revolution .....	143
(xix) Beta and Gamma Functions .....	144
(xx) Properties of Beta & Gamma Functions .....	145
(xxi) Leibnitz Rule of Differentiation under the sign of Integration.....	152
(xxii) Improper Integral .....	153
<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>154</b>
<b>1.7 Functions and their Graphs .....</b>	<b>177 – 191</b>
(i) Cartesian Product .....	177
(ii) Relations .....	177
(iii) Types of Relations .....	177
(iv) Functions .....	178
(v) Types of Functions .....	178
(vi) Some Basic Graphs .....	179
(vii) Graph Transformation .....	185
<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>186</b>

## UNIT 2 : VECTOR CALCULUS

<b>2.1 Vector &amp; their Basic Properties .....</b>	<b>192 – 202</b>
(i) Vector .....	192
(ii) Addition or Subtraction of Two Vectors .....	192
(iii) Dot Product.....	193
(iv) Angle between Two Vectros .....	193
(v) Triangle Inequality.....	194
(vi) Cross Product.....	194
(vii) Area of a Triangle.....	194
(viii) Area of a Paraellelogram .....	194
(ix) Direction Cosines of a Vector.....	195
(x) Direction Ratio of a Line joining Two Points .....	196
<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>199</b>
<b>2.2 Gradient, Divergence and Curl.....</b>	<b>203 – 229</b>
(i) Partial Derivatives of Vectors .....	203
(ii) Point Functions .....	203
(iii) Del Operator .....	204
(iv) The Laplacian Operator .....	204
(v) Gradient.....	205
(vi) Directional Derivative .....	205
(vii) Level Surface .....	206
(viii) Divergence .....	211
(ix) Curl (Rotation) .....	211
(x) Vector Identities .....	213
<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>217</b>
<b>2.3 Vector Integration .....</b>	<b>230 – 248</b>
(i) Line Integral .....	230
(ii) Surface Integral .....	232
(iii) Volume Integral .....	233

(iv) Gauss' Divergence Theorem .....	234
(v) Stoke's Theorem .....	236
(vi) Green's Theorem .....	237
<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>239</b>

### **UNIT 3 : COMPLEX ANALYSIS**

<b>3.1 Complex Number .....</b>	<b>249 – 264</b>
(i) Complex Numbers as Ordered Pairs .....	249
(ii) Euler Notation .....	249
(iii) Algebraic Properties .....	249
(iv) Geometrical Representation .....	250
(v) Modulus and Argument .....	250
(vi) Complex Conjugate Numbers .....	252
(vii) Cube Roots of Unity .....	252
<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>255</b>
<b>3.2 Analytic Functions .....</b>	<b>265 – 280</b>
(i) Neighbourhood of Complex Number $z_0$ .....	265
(ii) Function of a Complex Variable .....	265
(iii) Types of Complex Function .....	266
(iv) Continuity of Complex Function .....	266
(v) Differentiability of Complex Function .....	266
(vi) Analytic Functions .....	267
(vii) Cauchy-Riemann Equations .....	268
(viii) Polar Form of Cauchy-Riemann Equations .....	269
(ix) Harmonic Functions .....	271
<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>276</b>
<b>3.3 Complex Integration .....</b>	<b>281 – 310</b>
(i) Contour .....	281
(ii) Elementary Properties of Complex Integrals .....	282
(iii) Cauchy Integral Theorem .....	285

(iv) Cauchy's Integral Formula .....	286
(v) Taylor Series of Complex Function .....	291
(vi) Laurent's Series .....	291
(vii) Zeros of an Analytic Function .....	292
(viii) Singularities of an Analytic Function .....	293
(ix) Types of Isolated Singular Points .....	293
(x) Residue .....	294
(xi) Cauchy-Residue Theorem .....	295
(xii) Methods of Evaluating Residues .....	295
<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>298</b>

#### **UNIT 4 : DIFFERENTIAL EQUATIONS**

<b>4.1 Ordinary Differential Equations .....</b>	<b>311 – 341</b>
(i) Definition of Differential Equation .....	311
(ii) Order and Degree of Ordinary Differential Equation .....	311
(iii) Non-Linear Differential Equation .....	312
(iv) Solution of Differential Equation .....	312
(v) Formation of Differential Equation .....	313
(vi) Wronksian .....	313
(vii) Linearly dependent & Linearly independent solutions .....	314
(viii) Methods of Solving Differential Equations .....	314
(ix) Differential Equations of First Order and First Degree .....	315
(x) Exact Differential Equation .....	316
(xi) Equations Reducible to Exact Form .....	318
<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>322</b>
<b>4.2 Linear Differential Equations of Higher Order with Constant Coefficients .....</b>	<b>342 – 384</b>
(i) Complementary Function .....	342
(ii) Rules for Finding the Complementary Function .....	342
(iii) Particular Integral .....	345

(iv) Methods of Evaluating Particular Integral .....	345
(v) Cauchy's Homogeneous Linear Differential Equation .....	353
(vi) Legendre's Homogeneous Linear Differential Equation .....	353
(vii) Simultaneous Linear Differential Equation .....	357
(viii) Variation of Parameters .....	357
(ix) Differential Equation of First Order and Higher Degree.....	358
(x) Clairaut's Equation .....	361
<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>363</b>
<b>4.3 Partial Differential Equations .....</b>	<b>385 – 395</b>
(i) Order and Degree of Partial Differential Equation.....	385
(ii) Linear Partial Differential Equation .....	385
(iii) Classification of 2nd Order Linear P.D.E. in Two Variables .....	386
(iv) One Dimensional Wave Equations .....	387
(v) One Dimensional Heat Equation .....	388
(vi) Two Dimensional Heat Equation .....	389
(vii) Laplace Equation .....	390
<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>392</b>

## UNIT 5 : NUMERICAL METHODS

<b>5.1 Numerical Solutions of Linear Equations .....</b>	<b>396 – 404</b>
(i) Gauss-Jacobi Method .....	396
(ii) Gauss-Seidel Method .....	399
<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>404</b>
<b>5.2 Numerical Solution of Algebraic and Transcendental Equations .....</b>	<b>405 – 431</b>
(i) Graphical Method .....	405
(ii) Bisection Method .....	406
(iii) Regula-Falsi Method .....	410
(iv) The Secant Method .....	412
(v) Newton-Raphson Method .....	413
<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>417</b>

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<b>5.3</b>	<b>Interpolation .....</b>	<b>432 – 440</b>
(i)	Interpolation and Extrapolation .....	432
(ii)	Methods of Interpolation .....	432
(iii)	Notation of Finite Difference Calculus .....	432
(iv)	Newton-Gregory Forward Interpolation Formula (for equal intervals) .....	433
(v)	Newton-Gregory Backward Interpolation Formula (for equal intervals) .....	434
(vi)	Newton's Divided Difference Interpolation Formula (for Unequal Intervals) .....	436
(vii)	Lagrange's Interpolation Formula (for Unequal Intervals) .....	437
	<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>439</b>
<b>5.4</b>	<b>Numerical Integration (Quadrature) .....</b>	<b>441 – 456</b>
(i)	General Quadrature Formula .....	441
(ii)	The Trapezoidal Rule .....	441
(iii)	Simpson's One-Third Rule .....	442
(iv)	Simpsons 3/8 <sup>th</sup> Rule .....	443
	<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>449</b>
<b>5.5</b>	<b>Numerical Solution of a Differential Equation .....</b>	<b>457 – 475</b>
(i)	Picard's Method .....	457
(ii)	Euler's Method .....	460
(iii)	Improved Euler's Method .....	460
(iv)	Modified Euler's Method .....	461
(v)	Runge-Kutta Methods .....	466
	<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>471</b>

## **UNIT 6 : TRANSFORM THEORY**

<b>6.1</b>	<b>Laplace Transform .....</b>	<b>476 – 501</b>
(i)	Integral Transform .....	476
(ii)	Definition of Laplace Transform .....	476
(iii)	Properties of Laplace Transform .....	476
(iv)	Laplace Transform of Periodic Function .....	477
(v)	Types of Laplace Transform .....	477

(vi) Evaluation of Integrals with the help of Laplace Transform .....	478
(vii) Inverse Laplace Transform .....	478
(viii) Properties of Inverse Laplace Transform .....	479
(ix) Types of Inverse Laplace Transform.....	479
(x) Convolution of Two Functions .....	480
(xi) Unit Step Function .....	480
(xii) Unit Impulse Function (Dirac Delta Function) .....	480
(xiii) Existence Theorem for Laplace Transform.....	482
(xiv) Initial Value Theorem .....	482
(xv) Final Value Theorem .....	482
(xvi) Application of Laplace Transform in Differential Equation .....	483
<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>484</b>

<b>6.2 Fourier Series .....</b>	<b>502 – 513</b>
(i) Periodic Function .....	502
(ii) Fourier Series (Main Definition) .....	503
(iii) Euler's Formula of Fourier Series .....	503
(iv) Dirichlet's Conditions .....	505
(v) Fourier Expansion of Even and Odd Function .....	506
(vi) Half Range Fourier Series .....	508
<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>510</b>

## **UNIT 7 : LINEAR ALGEBRA**

<b>7.1 Algebra of Matrices .....</b>	<b>514 – 548</b>
(i) Definition of Matrix .....	514
(ii) Types of Matrices .....	514
(iii) Product of Matrix by a Scalar .....	516
(iv) Addition and Subtraction of Matrices .....	516
(v) Multiplication of Matrices .....	516
(vi) Minors of Matrix .....	518
(vii) Cofactors of a Matrix .....	518

(viii) Properties of Determinants .....	518
(ix) Adjoint of Matrix .....	519
(x) Inverse of a Matrix .....	519
(xi) Conjugate of a Matrix .....	520
(xii) Conjugate Transpose of a Matrix .....	520
(xiii) Special Types of Matrices .....	520
<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>527</b>
<b>7.2 Rank of Matrices .....</b>	<b>549 – 567</b>
(i) Definition of Rank.....	549
(ii) Elementary Transformations .....	550
(iii) Equivalent Matrices .....	550
(iv) Properties of Rank .....	550
(v) Echelon Form .....	551
(vi) Normal Form .....	553
(vii) Elementary Matrices .....	556
(viii) Computation of the Inverse of Matrix by Elementary Transformation .....	557
(ix) Row Rank and Column Rank of a Matrix .....	560
<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>562</b>
<b>7.3 System of Equations .....</b>	<b>568 – 613</b>
(i) Linear Dependence and Independence of Vectors .....	568
(ii) System of Linear Equations .....	570
(iii) Methods of Solving Non-Homogenous System .....	572
(iv) Methods of Solving Homogenous System .....	579
(v) Diagonally Dominant System .....	583
(vi) Gauss Elimination Method with Pivoting .....	583
(vii) Gauss-Jordan Method .....	585
(viii) LU Decomposition (Factorization) Method .....	587
(ix) Dolittle Method .....	587
(x) Crout's Method .....	590
<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>593</b>

<b>7.4 Eigen Values and Eigen Vectors .....</b>	<b>614 – 659</b>
(i) Definition .....	614
(ii) Properties of Eigen Values & Eigen Vectors .....	615
(iii) Similar Matrices .....	622
(iv) Diagonalisation .....	622
(v) Cayley-Hamilton Theorem .....	624
<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>628</b>

## **UNIT 8 : PROBABILITY & STATISTICS**

<b>8.1 Permutation and Combination .....</b>	<b>660 – 697</b>
(i) Set .....	660
(ii) Types of Set .....	660
(iii) Venn Diagrams of Different Sets .....	661
(iv) Fundamental Principle of Counting .....	663
(v) Permutation .....	663
(vi) Combination .....	664
(vii) Difference between Permutation and Combination .....	666
(viii) Total Number of Combinations .....	666
(ix) Permutation of Alike Items .....	666
(x) Division of Different Items into Groups of Equal Size .....	666
(xi) Circular Permutation .....	667
(xii) Sum of All Numbers formed from Given Digits .....	668
(xiii) Division of Identical Items into Groups .....	668
(xiv) Rank of a Word .....	669
(xv) Number of Dearrangements .....	669
(xvi) Number of Factors .....	670
<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>695</b>

<b>8.2 Probability .....</b>	<b>698 – 742</b>
(i) Some Basic Concepts .....	687
(ii) Definition of Probability .....	700

(iii) Addition Theorem of Probability .....	703
(iv) Odds in Favour and Odds Against .....	704
(v) Compound Events .....	705
(vi) Conditional Probability .....	705
(vii) Important Results on Independence of Events .....	706
(viii) Binomial Theorem on Probability .....	709
(ix) Total Probability Theorem .....	710
(x) Baye's Theorem .....	711
<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>713</b>
<b>8.3 Statistics - I (Probability Distribution) .....</b>	<b>743 – 790</b>
(i) Random Experiment .....	743
(ii) Discrete Random Variable .....	743
(iii) Cumulative Distribution Function for Discrete Random Variable .....	743
(iv) Bernoulli Random Variable .....	746
(v) Binomial Random Variable .....	746
(vi) Geometric Random Variable .....	748
(vii) Poisson Random Variable .....	748
(viii) Continuous Random Variable .....	749
(ix) Cumulative Distribution Function for Continuous Random Variable .....	750
(x) Uniform Random Variable .....	751
(xi) Exponential Random Variable .....	752
(xii) Gamma Random Variable .....	752
(xiii) Normal Random Variable .....	752
(xiv) Standard Normal Distribution .....	753
(xv) Skewness in Normal Curve .....	754
(xvi) Expectation of Random Variable .....	755
(xvii) Variance .....	756
(xviii) Standard Deviation .....	756
(xix) Covariance .....	757
(xx) Mean .....	757

(xxi) Median .....	758
(xxii) Mode .....	758
<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>759</b>
<b>8.4 Statistics - II (Correlation &amp; Regression) .....</b>	<b>791 – 802</b>
(i) Curve Fitting .....	791
(ii) Methods of Least Squares .....	791
(iii) Correlation .....	795
(iv) Methods of Estimating Correlation .....	796
(v) Regression Analysis .....	797
(vi) Line of Regression of y on x .....	798
(vii) Line of Regression of x on y .....	798
(viii) Properties of Regression Coefficients .....	799
(ix) Angle between Two Lines of Regression .....	800
<b>Previous Years GATE &amp; ESE Questions .....</b>	<b>801</b>

## PREVIOUS YEARS GATE & ESE QUESTIONS

**Questions marked with asterisk (\*) are Conceptual Questions**

1.\*  $\lim_{x \rightarrow 0} \frac{x(e^x - 1) + 2(\cos x - 1)}{x(1 - \cos x)} = \underline{\hspace{2cm}}$

[GATE-1993 (ME)]

2.  $\lim_{x \rightarrow 0} \frac{1}{10} \frac{1 - e^{-j5x}}{1 - e^{-ix}} = \underline{\hspace{2cm}}$

- (a) 0 (b) 1.1  
(c) 0.5 (d) 1

[GATE-1993]

3. The value of  $\left[ \lim_{x \rightarrow \infty} \frac{\sin x}{x} \right]$  is:

- (a)  $\infty$  (b) 2  
(c) 1 (d) 0

[GATE-1994; 1 Mark]

4. The value of  $\left[ \lim_{x \rightarrow \infty} \left( \frac{1}{\sin x} - \frac{1}{\tan x} \right) \right]$  is:

- (a) 0 (b) 1  
(c) 2 (d)  $\infty$

[GATE-1994; 1 Mark]

5. The limit of  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta}$  is

- (a) 1  
(b) 0  
(c)  $\infty$   
(d) None of the above

[GATE-1994; 1 Mark]

6.\*  $\lim_{x \rightarrow \infty} \frac{x^3 - \cos x}{x^2 + (\sin x)^2}$  equal

- (a)  $\infty$  (b) 0  
(c) 2 (d) does not exist

[GATE-1995-CS; 2 Marks]

7.\* The function  $f(x) = |x + 1|$  on the interval  $[-2, 0]$  is

- (a) continuous and differentiable  
(b) continuous on the integers but not differentiable at all points  
(c) neither continuous nor differentiable  
(d) differentiable but not continuous

[GATE-1995; 1 Mark]

8.  $\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right)$  is

- (a)  $\infty$  (b) 0  
(c) 1 (d) Non-existent

[GATE-1995; 1 Mark]

9. If a function is continuous at a point its first derivative

- (a) may or may not exist  
(b) exists always  
(c) will not exist  
(d) has unique value

[GATE-1996; 1 Mark]

10. If  $y = |x|$  for  $x < 0$  and  $y = x$  for  $x \geq 0$ , then

- (a)  $\frac{dy}{dx}$  is discontinuous at  $x = 0$   
(b)  $y$  is discontinuous at  $x = 0$   
(c)  $y$  is not defined at  $x = 0$   
(d) None of these

[GATE-1997; 1 Mark]

11.  $\lim_{\theta \rightarrow 0} \frac{\sin m\theta}{\theta}$ , where  $m$  is an integer, is one of the

following:

- (a)  $m$  (b)  $m\pi$   
(c)  $m\theta$  (d) 1

[GATE-1997; 1 Mark]

12.\* Find the limiting value of the ratio of the square of the sum of a natural numbers to  $n$  times the sum of squares of the  $n$  natural number as,  $n$  approaches infinity.

[GATE-1997]

13.\* The profile of a cam in a particular zone is given by  $x = \sqrt{3} \cos \theta$  and  $y = \sin \theta$ . The normal to the cam profile at  $\theta = \frac{\pi}{4}$  is at an angle (with respect to  $x$  axis):

- (a)  $\frac{\pi}{4}$  (b)  $\frac{\pi}{2}$   
(c)  $\frac{\pi}{3}$  (d) 0

[GATE-1998]

14. Limit of the function  $\lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^2 + n}}$  is

- (a)  $\frac{1}{2}$
- (b) 0
- (c)  $\infty$
- (d) 1

[GATE-1999; 1 Mark]

15. Value of the function  $\lim_{x \rightarrow a} (x - a)^{(x-a)}$  is

- (a) 1
- (b) 0
- (c)  $\infty$
- (d) a

[GATE-1999; 1 Mark]

16.  $\lim_{x \rightarrow 1} \frac{(x^2 - 1)}{(x - 1)}$  is

- (a)  $\infty$
- (b) 0
- (c) 2
- (d) 1

[GATE-2000; 1 Mark]

17. The limit of the function  $f(x) = \left[ 1 - \frac{a^4}{x^4} \right]$  as  $x \rightarrow \infty$  is

- given by
- (a) 1
  - (b)  $\exp[-a^4]$
  - (c)  $\infty$
  - (d) Zero

[GATE-2000; 1 Mark]

18. What is the derivative of  $f(x) = |x|$  at  $x = 0$ ?

- (a) 1
- (b) -1
- (c) 0
- (d) Does not exist

[GATE-2001; 1 Mark]

19.  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin 2\left(x - \frac{\pi}{4}\right)}{x - \frac{\pi}{4}} = \underline{\hspace{2cm}}$

- (a) 0
- (b)  $\frac{1}{2}$
- (c) 1
- (d) 2

[GATE-2001 (IN)]

20.\* Which of the following functions is not differentiable in the domain  $[-1, 1]$ ?

- (a)  $f(x) = x^2$
- (b)  $f(x) = x - 1$
- (c)  $f(x) = 2$
- (d)  $f(x) = \max(x, -x)$

[GATE-2002; 1 Mark]

21.\* The limit of the following sequence as  $n \rightarrow \infty$  is:

$$x_n = n^{1/n}$$

- (a) 0
- (b) 1
- (c)  $\infty$
- (d)  $-\infty$

[GATE-2002-CE; 1 Mark]

22.  $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x}$  is equal to

- (a) 0
- (b)  $\infty$
- (c) 1
- (d) -1

[GATE-2003; 1 Mark]

23. The value of the function  $f(x) = \lim_{x \rightarrow 0} \frac{x^3 + x^2}{2x^3 - 7x^2}$  is

- (a) zero
- (b)  $-\frac{1}{7}$
- (c)  $\frac{1}{7}$
- (d) infinite

[GATE-2004; 1 Mark]

24. If  $x = a(\theta + \sin \theta)$  and  $y = a(1 - \cos \theta)$ , then  $dy/dx$  will be equal to

- (a)  $\sin\left(\frac{\theta}{2}\right)$
- (b)  $\cos\left(\frac{\theta}{2}\right)$
- (c)  $\tan\left(\frac{\theta}{2}\right)$
- (d)  $\cot\left(\frac{\theta}{2}\right)$

[GATE-2004]

25.\* With a 1 unit change in b, what is the change in x in the solution of the system of equations  $x + y = 2$ ,  $1.01x + 0.99y = b$ ?

- (a) zero
- (b) 2 units
- (c) 50 units
- (d) 100 units

[GATE-2005]

26.\* Equation of the line normal to function  $f(x) = (x - 8)^{2/3} + 1$  at  $P(0, 5)$  is

- (a)  $y = 3x - 5$
- (b)  $y = 3x + 5$
- (c)  $3y = x + 15$
- (d)  $3y = x - 5$

[GATE-2006; 2 Marks]

27. If  $f(x) = \frac{2x^2 - 7x + 3}{5x^2 - 12x - 9}$ , then  $\lim_{x \rightarrow 3} f(x)$  is

- (a)  $-1/3$
- (b)  $5/18$
- (c) 0
- (d)  $2/5$

[GATE-2006; 2 Marks]

28.\* If,  $y = x + \sqrt{x + \sqrt{x + \sqrt{x + \dots \infty}}}$ , then  $y(2) =$

## ANSWER KEY

1. (See Sol.)	21. (b)	41. (b)	61. (c)
2. (c)	22. (a)	42. (d)	62. (c)
3. (d)	23. (b)	43. (c)	63. (c)
4. (d)	24. (c)	44. (c)	64. (c)
5. (a)	25. (c)	45. (b)	65. (c)
6. (a)	26. (b)	46. (b)	66. (0.5)
7. (b)	27. (b)	47. (a)	67. (1)
8. (b)	28. (b)	48. (b)	68. (d)
9. (a)	29. (b)	49. (d)	69. (25)
10. (a)	30. (a)	50. (a)	70. (b)
11. (a)	31. (c)	51. (c)	71. (d)
12. (0.75)	32. (c)	52. (c)	72. (b)
13. (c)	33. (b)	53. (a)	73. (a)
14. (d)	34. (a)	54. (c)	74. (d)
15. (a)	35. (b)	55. (-2)	75. (c)
16. (c)	36. (c)	56. (c)	76. (1)
17. (a)	37. (c)	57. (-0.33)	
18. (d)	38. (c)	58. (d)	
19. (d)	39. (a)	59. (a)	
20. (d)	40. (c)	60. (c)	

## SOLUTIONS

**Sol-1:**

Using L' Hospital Rule

$$\text{Req. Limit} = \lim_{x \rightarrow 0} \frac{(e^x - 1) + xe^2 - 2\sin x}{(1 - \cos x) + x(\sin x)} \left( \frac{0}{0} \right)$$

Again using L' Hospital rule

$$= \lim_{x \rightarrow 0} \frac{(e^x + e^x + xe^2 - 2\cos x)}{\sin x + \sin x + x\cos x} \left( \frac{0}{0} \right)$$

Again using L' Hospital Rule

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{e^x + e^x + e^x + xe^x + 2\sin x}{\cos x + \cos x + \cos x - x\sin x} \\ &= \frac{1+1+1+0+0}{1+1+1-0} = 1 \end{aligned}$$

**Sol-2: (c)**

Using L' Hospital rule

$$\lim_{x \rightarrow 0} \frac{1 - e^{-j5x}}{10(1 - e^{-jx})} = \lim_{x \rightarrow 0} \frac{1}{10} \times \frac{5je^{-j5x}}{je^{-jx}} = \frac{5}{10} = 0.5$$

**Sol-3: (d)**

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{\sin x}{x} &= \lim_{y \rightarrow 0} \frac{\sin 1/y}{1/y} \\ &= \lim_{y \rightarrow 0} y \sin \left( \frac{1}{y} \right) = 0 \end{aligned}$$

$$\boxed{\lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0}$$

( $\because |\sin x| \leq 1$ )

**Sol-4: (d)**

# ADMISSION OPEN

FOR CE, ME, EE, ECE

 SESSION 2017-2018  
**IITES/GATE/PSUs**



Batches start on

**6<sup>th</sup> August**  
8:30AM - 2:30PM

GS &  
Engineering Aptitude  
Regular Batch

**11<sup>th</sup> August**  
8:30AM - 2:30PM

GS &  
Engineering Aptitude  
Regular Batch

**3<sup>rd</sup> July**  
3:30PM - 9:30PM

**Regular Batch**  
(ME)

**24<sup>th</sup> July**  
1:30PM - 8:00PM

**Regular Batch**  
(EE)

**27<sup>th</sup> July**  
3:30PM - 9:30PM

**Regular Batch**  
(CE)

**1<sup>st</sup> August**  
1:30PM - 8:00PM

**Regular Batch**  
(EE)

**1<sup>st</sup> August**  
1:30PM - 8:00PM

**Regular Batch**  
(ECE)

$$\begin{aligned}\lim_{x \rightarrow \infty} \left( \frac{1}{\sin x} - \frac{1}{\tan x} \right) &= \lim_{x \rightarrow \infty} \left( \frac{1 - \cos x}{\sin x} \right) \\&= \lim_{x \rightarrow \infty} \frac{2 \sin^2 \frac{x}{2}}{2 \sin \frac{x}{2} \cos \frac{x}{2}} \\&= \lim_{x \rightarrow \infty} \tan \frac{x}{2} = \infty\end{aligned}$$

**Sol-5: (a)**

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$$

**Sol-6: (a)**

$$\begin{aligned}\lim_{x \rightarrow \infty} \frac{x^3 - \cos x}{x^2 + (\sin x)^2} &= \lim_{x \rightarrow \infty} \frac{1 - \frac{\cos x}{x^3}}{\frac{x^2}{x^3} + \frac{(\sin x)^2}{x^3}} \\&= \lim_{x \rightarrow \infty} \frac{1 - \frac{\cos x}{x^3}}{\frac{1}{x} + \frac{(\sin x)^2}{x^3}} \\&= \frac{1 - 0}{\frac{1}{\infty} + 0} = \frac{1}{0}\end{aligned}$$

$$\boxed{\lim_{x \rightarrow \infty} \frac{x^3 - \cos x}{x^2 + (\sin x)^2} = \infty}$$

**Sol-7: (b)**

$$\begin{aligned}f(x) &= |x + 1| \\&= -(x + 1) \quad \text{for } x < -1 \\&= (x + 1) \quad \text{for } x \geq -1\end{aligned}$$

 Only concern is  $x = -1$ 

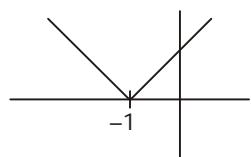
$$\text{Left limit} = \lim_{x \rightarrow -1^-} -(x + 1) = 0$$

$$\text{Right limit} = \lim_{x \rightarrow -1^+} (x + 1) = 0$$

$$\boxed{f(x) \text{ is continuous in the interval } [-2, 0]}$$

$$LD = -1$$

$$\text{Right Derivative} = +1$$

$$\boxed{\text{Hence } f(x) \text{ is not differentiable at } x = -1}$$
**Alternative Solution :**


It is clear from graph  $f(x)$  is continuous every where but not differentiate at  $x = -1$  because there is sharp change in slope at  $x = -1$ .

**Sol-8: (b)**

$$\begin{aligned}\lim_{x \rightarrow 0} x \sin \frac{1}{x} &= 0 \cdot \sin \infty \\&= 0 \times [-1, 1] \\&= 0\end{aligned}$$

**Sol-9: (a)**

If  $f(x)$  is continuous at a point  $x = a$  then it may or may not be differentiable at  $x = a$ .

**Sol-10: (a)**

$$y = |x|$$

$$= \begin{cases} -x, & \text{for } x < 0 \\ x, & \text{for } x > 0 \end{cases} \quad \text{and} \quad \frac{dy}{dx} = \begin{cases} -1, & x < 0 \\ 1, & x > 0 \end{cases}$$

$$\text{left limit} = \lim_{x \rightarrow 0} (-x) = 0$$

$$\text{Right limit} = \lim_{x \rightarrow 0} (x) = 0$$

$$\therefore \text{Left limit} = \text{Right limit}$$

$$\boxed{y \text{ is continuous at } x = 0}$$

$$\text{Left derivative} = -1$$

$$\text{Right derivative} = +1$$

$$\therefore \text{Left derivative} \neq \text{Right derivative}$$

$$\boxed{\frac{dy}{dx} \text{ is discontinuous at } x = 0}$$
**Sol-11: (a)**

$$\lim_{\theta \rightarrow 0} \frac{\sin m\theta}{\theta} \left( \frac{0}{0} \right)$$

Apply LH Rule

$$\lim_{\theta \rightarrow 0} \frac{m \cos m\theta}{1} = m$$

**Alternative Solution:**

$$\lim_{\theta \rightarrow 0} \frac{\sin m\theta}{\theta} = \lim_{\theta \rightarrow 0} m \frac{\sin m\theta}{m\theta} = m(1) = m$$

**Sol-12: (0.75)**

 Sum of the squares of natural numbers =  $S_2$ 

$$S_2 = 1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2$$

$$S_2 = \frac{n(n+1)(2n+1)}{6}$$

 Sum of natural numbers =  $S_1$ 

$$S_1 = 1 + 2 + 3 + 4 + \dots + n$$

$$S_1 = \frac{n(n+1)}{2}$$

Hence,

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{\left[ \frac{n(n+1)}{2} \right]^2}{n \times \left[ \frac{n(n+1)(2n+1)}{6} \right]}$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{\frac{n^2(n+1)^2}{4}}{n \times n(n+1)(2n+1)} = \frac{1}{6}$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{3(n+1)}{2(2n+1)}$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{3}{2} \left( \frac{1 + \frac{1}{n}}{2 + \frac{1}{n}} \right)$$

$$\Rightarrow \frac{3}{4} = 0.75$$

**Sol-13: (c)**

$$x = \sqrt{3} \cos \theta$$

$$y = \sin \theta$$

Slope of cam profile at an angle  $\theta$  is

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{d\theta} \times \frac{1}{\left(\frac{dx}{d\theta}\right)} \\ &= \frac{\cos \theta}{-\sqrt{3} \sin \theta} = \frac{-\cot \theta}{\sqrt{3}} \end{aligned}$$

Slope of Normal to cam profile at angle  $\theta$  is

$$m = \frac{-1}{\left(\frac{dy}{dx}\right)} = \sqrt{3} \tan \theta$$

$$m = \sqrt{3} \tan\left(\frac{\pi}{4}\right)$$

$$m = \sqrt{3}$$

Angle of Normal with x-axis is

$$\alpha = \tan^{-1}(m) = \tan^{-1}(\sqrt{3})$$

$$\boxed{\alpha = 60^\circ = \frac{\pi}{3}}$$

**Sol-14: (d)**

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^2+n}} &= \lim_{n \rightarrow \infty} \frac{n}{n\sqrt{1+\frac{1}{n}}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1+\frac{1}{n}}} = 1 \end{aligned}$$

**Sol-15: (a)**

$$\text{Let } y = \lim_{x \rightarrow a} (x-a)^{(x-a)}$$

Taking logarithm on both sides.

$$\begin{aligned} \log y &= \lim_{x \rightarrow a} \log(x-a)^{(x-a)} \\ &= \lim_{x \rightarrow a} (x-a) \log(x-a) \end{aligned}$$

$$= \lim_{x \rightarrow a} \frac{\log(x-a)}{\left(\frac{1}{x-a}\right)}$$

$$\text{Apply L'H Rule} = \lim_{x \rightarrow a} \frac{\frac{1}{(x-a)}}{-\frac{1}{(x-a)^2}}$$

$$\log y = \lim_{x \rightarrow a} -(x-a)$$

$$\log y = 0$$

$$\Rightarrow y = e^0 = 1$$

**Sol-16: (c)**

$$\begin{aligned} \lim_{x \rightarrow 1} \left( \frac{x^2-1}{x-1} \right) &= \lim_{x \rightarrow 1} \frac{(x-1)(x+1)}{(x-1)} \\ &= \lim_{x \rightarrow 1} (x+1) = 1+1 \end{aligned}$$

$$\boxed{\lim_{x \rightarrow 1} \left( \frac{x^2-1}{x-1} \right) = 2}$$

**Sol-17: (a)**

$$\begin{aligned} \lim_{x \rightarrow \infty} f(x) &= \lim_{x \rightarrow \infty} \left[ 1 - \frac{a^4}{x^4} \right] \\ &= 1 - \frac{a^4}{(\infty)} = 1 - 0 = 1 \end{aligned}$$

**Sol-18: (d)**

$$f(x) = |x| = \begin{cases} x, & x > 0 \\ -x, & x < 0 \end{cases}$$

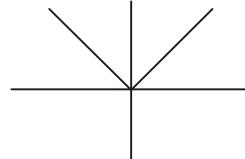
Left derivative = -1

Right derivative = +1

$\therefore LD \neq RD$

The derivative does not exist at  $x = 0$ .

**Alternative Solution :**



It is clear from graph  $f(x)$  is not differentiable at  $x = 0$ .

**Sol-19: (d)**

$$\lim_{x \rightarrow \frac{\pi}{4}} \cdot \frac{\sin\left[2\left(x - \frac{\pi}{4}\right)\right]}{\left(x - \frac{\pi}{4}\right)} \left( \frac{0}{0} \text{ form} \right)$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \cdot \frac{2\cos\left[2\left(x - \frac{\pi}{4}\right)\right]}{1} = 2$$



# ENGINEERING MATHEMATICS

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### 2018



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