Engineering Mathematics-III

(For DCRUST, Murthal, Sonipat, GJU & ST, Hisar, MDU, Rohtak, KUK, Kurukshetra and other Indian Universities)

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To
My wife Manju
&
Daughters
Bhumika & Vanshika
for their
love & support

B.S. Vashisth

Preface

This book has written as per the new syllabus of M.D.U., G.J.U. and K.U.K., D.C.R.U.S.T. (Murthal) Haryana. For the third semester students of B.E. / B.Tech. An attempt has been made to make the subject matter easily intelligible in all respect. The questions have been carefully selected and properly graded. The language used in solving the questions is made as simple as simple as possible. I do feel that the students will find the book very useful and rewarding.

Although every care has been taken to keep the book free from errors and misprints but still I look forward to receive constructive suggestions and corrections which might have escaped my scrutiny.

—Authors
B.S. Vashisth
Dr. Sanjay Kumar

SYLLABUS

D.C.R.U.S.T.

L	T	P	Credits	MATH-201 B.Tech. Semester-III)	MATHEMATICS-III (Common for all Branches)	
3	2	–	5		Class work	: 50 Marks
					Examination	: 100 Marks
					Total	: 150 Marks
					Duration of Examination	: 3 Marks

Part-A

Fourier Series and Fourier Trasnsform: Euler's formulae, conditions for a Fourier expansion, change of interval, Fourier expansion of odd and even functions, Fourier expansion of square wave, rectangular wave, saw-toothed wave, half and full rectified wave, half range sine and cosine series.

Fourier integrals, Fourier transforms, Shifting theorem (both on time and frequency axes), Fourier transform of derivatives, Fourier transforms of integrals, Convolution theorem, Fourier transform of Dirac-delta function.

Part-B

Functions of Complex Variable: Definition, Exponential function, Trognometric and Hyperbolic functions, Logrithmic functions, Limit and Continuity of a function, Differentiability and Analyticity.

Cauchy-Riemann equations, necessary and sufficient conditions for a function to be analytic, polar form of the Cauchy-Riemann equations. Harmonic functions, application to flow problems. Integration of complex functions. Cauchy-Integral theorem and formula.

Power series, radius and circle of convergence, Taylor's Maclaurin's and Laurent's series. Zeroes and singularities of complex functions, residues. Evaluation of real integrals using residues (arround unit and semi circle only).

Part-C

Probability Distributions and Hypothesis Testing: Conditional probability, Bayes theorem and its applications, expected value of a random variable. Properties and application of Bonimial, Poisson and Normal distributions.

Testing of a hypothesis, tests of significance for large samples, student's t-distribution (applications only), Chi-square test of goodness of fit.

Linear Programming: Linear programming problems formulation, Solving linear programming problems using (*i*) Graphical method (*ii*) Simplex method (*iii*) Dual simplex method.

SYLLABUS

(K.U., Kurukshetra)

Class work : 100 Marks
Sessional : 50 Marks
Total : 150 Marks
Duration of Exam. : 3 Hours

Unit-I

Fourier Series: Euler's formulae. Conditions for Fourier expansions, Fourier expansion of functions having points of discontinuity, change of interval. Odd and even functions, Half-range series.

Fourier Transforms: Fourier integrals, Fourier transforms, Fourier cosine and sine transforms. Properties of Fourier transforms, Convolution theorem, Perseval's identity, Relation between Fourier and Laplace transforms, Fourier transforms of the derivatives of a function, Application to boundary value problems.

Unit-II

Functions of a Complex Variable: Functions of a complex variable, Exponential function, Trigonometric, Hyperbolic and Logarithmic functions, limit and continuity of a function, Differentiability and analyticity.

Cauchy-Riemann Equations: Necessary and sufficient conditions for a function to be analytic, Polar Form of the Cauchy-Riemann equations, Harmonic functions, Application to flow problems, conformal transfor-mation, Standard transformations (Translation. Magnification and rotation, inversion and reflection Bilinnear)

Unit-III

Probability Distributing: Probability, Baye's theorem, Discrete and Continuous probability distributions, Moment generating function. Probability generating function, Properties and applications of Binomial, Poisson and normal distributions.

Unit-IV

Linear-Programming: Linear programming problems formulation, Solution of Linear Programming Problem using Graphical method. Simplex Method, Dual-Simplex Method.

SYLLABUS

(M.D.U., Rohtak)

MAT-201-F Mathematics-III

L	T	P		
3	1	–	Class work	: 50 Marks
			Sessional	: 100 Marks
			Total	: 150 Marks
			Duration of Exam.	: 3 Hours

Note: Examiner will set 9 questions in total, with two questions from each section and one questions covering all sections which will be Q.1. This Q.1 is compulsory and of short answers type. Each question carries equal mark (20 marks). Students have to attempt 5 questions in total at least one question from each section.

Section-A

Fourier Series and Fourier Transforms: Euler's formulae, conditions for a Fourier expansion, change of interval, Fourier expansion of odd and even functions, Fourier expansion of square wave, rectangular wave, saw-toothed wave, half and full rectified wave, half range sine and cosine series.

Fourier integrals, Fourier transforms, Shifting theorem (both on time and frequency axes), Fourier transforms of derivatives, Fourier transforms of integrals, Convolution theorem, Fourier transform of Dirac-delta function.

Section-B

Functions of a Complex Variable: Definition, Exponential function, Trigonometric and Hyperbolic functions, Logarithmic functions. Limit and Continuityof a function, Differentiability and Analyticity.

Cauchy-Riemann Equations, necessary and sufficient conditions for a function to be analytic, Polar Form of the Cauchy-Riemann equations. Harmonic functions, application to flow problems. Integration of complex functions. Cauchy-integral theorem and formula.

Section-C

Power series, radius and circle of convergence, Taylor's, Maclaurin's and Laurent's series. Zeroes and singularities of complex functions, Residues. Evaluation of real integrals using residues (around unit and semi circle only).

Probability Distributions and Hypothesis Testing: Conditional probability, Bayes theorem and its applications, expected value of a random variable. Properties and application of Binomial, Poisson and Normal distributions.

Section-D

Testing of a hypothesis, tests of significance for large samples, Student's t-distribution (applications only), Chi-square test of goodness of fit.

Linear-Programming: Linear programming problems formulation, Solution of Linear Programming Problem using Graphical method. Simplex Method, Dual-Simplex Method.

LIST OF IMPORTANT FORMULAE

1. Trigonometry

(*i*) $\sin 0^\circ = 0 \qquad \cos 0^\circ = 1 \qquad \tan 0^\circ = 0$

$\sin 30^\circ = \frac{1}{2} \qquad \cos 30^\circ = \frac{\sqrt{3}}{2} \qquad \tan 30^\circ = \frac{1}{\sqrt{3}}$

$\sin 45^\circ = \frac{1}{\sqrt{2}} \qquad \cos 45^\circ = \frac{1}{\sqrt{2}} \qquad \tan 45^\circ = 1$

$\sin 60^\circ = \frac{\sqrt{3}}{2} \qquad \cos 60^\circ = \frac{1}{2} \qquad \tan 60^\circ = \sqrt{3}$

$\sin 90^\circ = 1 \qquad \cos 90^\circ = 0 \qquad \tan 90^\circ \to \infty$

(*ii*) $\sin(n\pi + \theta) = (-1)^n \sin\theta, \ n \in Z$

$\cos(n\pi + \theta) = (-1)^n \cos\theta, \ n \in Z$

$\cos(n\pi + 0) = \cos n\pi = (-1)^n, \ n \in Z$

$\sin n\pi = 0, \ n \in Z$

(*iii*) If $\sin\theta = 0$, then $\theta = n\pi, \ n \in Z$

If $\cos\theta = 0$, then $\theta = (2n+1)\frac{\pi}{2}, \ n \in Z$

If $\tan\theta = 0$, then $\theta = n\pi, \ n \in Z.$

(*iv*) If $\sin\theta = \sin\alpha$, then $\theta = n\pi + (-1)^n \alpha, n \in Z$

If $\cos\theta = \cos\alpha$, then $\theta = 2n\pi \pm \alpha, \ n \in Z$

If $\tan\theta = \tan\alpha$, then $\theta = n\pi + \alpha, \ n \in Z.$

(*v*) $\left.\begin{aligned} \sin^2\theta = \sin^2\alpha \\ \cos^2\theta = \cos^2\alpha \\ \tan^2\theta = \tan^2\alpha \end{aligned}\right] \Rightarrow \theta = n\pi \pm \alpha, n \in Z$

(*vi*) $\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

(*vii*) $\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

$\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B$

$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

$\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$$

$$\cos(A+B) = \frac{\cot A \cdot \cot B - 1}{\cot A + \cot B}, \cot(A-B) = \frac{\cot A \cdot \cot B + 1}{\cot B - \cot A}$$

$$\tan(45^\circ + A) = \frac{1 + \tan A}{1 - \tan A}, \tan(45^\circ - A) = \frac{1 - \tan A}{1 + \tan A}$$

$$\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$$

$$\cos(A+B) \cdot \cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A.$$

$$\tan(A+B+C) = \frac{\tan A + \tan B + \tan C - \tan A \cdot \tan B \cdot \tan C}{1 - \tan A \cdot \tan B - \tan B \cdot \tan C - \tan C \cdot \tan A}$$

(*viii*) ($A - B$) Formulae

$$2\sin A \cdot \cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A \cdot \sin B = \sin(A+B) - \sin(A-B)$$

$$2\cos A \cdot \cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A \cdot \sin B = \cos(A-B) - \cos(A+B)$$

$(C - D)$ Formulae

$$\sin C + \sin D = 2\sin\frac{C+D}{2} \cdot \cos\frac{C-D}{2}$$

$$\sin C - \sin D = 2\cos\frac{C+D}{2} \cdot \sin\frac{C-D}{2}$$

$$\cos C + \cos D = 2\cos\frac{C+D}{2} \cdot \cos\frac{C-D}{2}$$

$$\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{D-C}{2}\right)$$

(*ix*) $\sin^2\theta + \cos^2\theta = 1$

$\sec^2\theta - \tan^2\theta = 1$

$\text{cosec}^2 - \cot^2\theta = 1$

$$\sin 2\theta = 2\sin\theta\cdot\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1 = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$1 - \cos 2\theta = 2\sin^2\theta, \left(i.e., \sin^2\theta = \frac{1-\cos 2\theta}{2}\right)$$

$$1 + \cos 2\theta = 2\cos^2\theta, \left(i.e., \cos^2\theta = \frac{1+\cos 2\theta}{2}\right)$$

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$$

2. Hyperbolic Functions

(*i*) $\sinh x = \dfrac{e^x - e^{-x}}{2}$ (*ii*) $\cosh x = \dfrac{e^x + e^{-x}}{2}$

(*iii*) $\cosh^2 x - \sinh^2 x = 1$ (*iv*) $\sinh^{-1} z = \log\left(z + \sqrt{1+z^2}\right)$

(*v*) $\cosh^{-1} z = \log\left(z + \sqrt{z^2 - 1}\right)$ (*vi*) $\tanh^{-1} z = \dfrac{1}{2}\log\left(\dfrac{1+z}{1-z}\right)$

(*vii*) $e^x = \cosh x + \sinh x$ (*viii*) $e^{-x} = \cosh x - \sinh x$

3. Limits

(*i*) $\underset{x\to 0}{Lim}\dfrac{\sin x}{x} = 1$ (*ii*) $\underset{x\to 0}{Lim}\dfrac{\tan x}{x} = 1$

(*iii*) $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$ (*iv*) $\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^x = e^a$

4. Differential Calculus

(*i*) $\frac{d}{dx}(x^n) = n\cdot x^{n-1}$ (*ii*) $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$

(*iii*) $\frac{d}{dx}(e^x) = e^x \log_e e = e^x$ (*iv*) $\frac{d}{dx}(a^x) = a^x \log_e a$

(*v*) $\frac{d}{dx}(\sin x) = \cos x$ (*vi*) $\frac{d}{dx}(\cos x) = -\sin x$

(*vii*) $\frac{d}{dx}(\tan x) = \sec^2 x$ (*viii*) $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

(*ix*) $\frac{d}{dx}(\sec x) = \sec x\cdot\tan x$ (*x*) $\frac{d}{dx}(\text{cosec } x) = -\text{cosec } x\cdot\cot x$

(*xi*) $\frac{d}{dx}\left(\sin^{-1}x\right) = \frac{1}{\sqrt{1-x^2}}$ (*xii*) $\frac{d}{dx}\left(\cos^{-1}x\right) = \frac{-1}{\sqrt{1-x^2}}$

(*xiii*) $\frac{d}{dx}\left(\tan^{-1}x\right) = \frac{1}{1+x^2}$ (*xiv*) $\frac{d}{dx}\left(\cot^{-1}x\right) = \frac{-1}{1+x^2}$

(*xv*) $\frac{d}{dx}\left(\sec^{-1}x\right) = \frac{1}{x\cdot\sqrt{x^2-1}}$ (*xvi*) $\frac{d}{dx}\left(\text{cosec}^{-1}x\right) = \frac{-1}{x\cdot\sqrt{x^2-1}}$

(*xvii*) $\frac{d}{dx}(\log_a x) = \frac{1}{x}\log_a e$ (*xviii*) $\frac{d}{dx}(\log x) = \frac{1}{x}$

(*a*) Product Rule and Quotient Rule

(*i*) $\frac{d}{dx}(u\cdot v) = u\cdot\frac{dv}{dx} + v\cdot\frac{du}{dx}$

(*ii*) $\frac{d}{dx}(u\cdot v\cdot w) = vw\cdot\frac{du}{dx} + wu\cdot\frac{dv}{dx} + uv\frac{dw}{dx}$

(*iii*) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\cdot\frac{dv}{dx}}{v^2}$

(*b*) Chain Rule

If $y = f(u)$ is a function of u and $u = \phi(x)$ is a function of x, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

(*c*) Differentiation of Implicit Functions

A equation of the form $f(x, y) = 0$ in which y cannot be expressed directly in terms of x is known as implicit equation. The function determined by an implicit equation is called implicit function of x and y.

Here $f(x, y) = 0$ where, 'y' is also a function of x.

e.g. $x^4 + y^4 + 4xy + 100 = 0$

Differention w.r. to x.

$$4x^3 + 4y^3 \cdot \frac{dy}{dx} + 4y \cdot 1 + 4x + \frac{dy}{dx} = 0$$

$$(y^3 + x)\frac{dy}{dx} = -\left(x^3 + y\right)$$

$$\Rightarrow \qquad \frac{dy}{dx} = -\frac{\left(x^3 + y\right)}{y^3 + x}$$

(*d*) Differentiation in Case of Parametric Functions

If $x = f(t)$, $y = \phi(t)$ be any two functions of t

Then, $$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

(*e*) Derivative of a Function with Respect to Another Function

If $f(x)$ and $\phi(x)$ are two functions of x, then to find the derivative of $f(x)$ with respect to $\phi(x)$,

We put, $y = f(x)$ and $z = \phi(x)$

Then, $$\frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}}$$

5. Integral Calculus

(*a*) Standard Elementary Integrals

We now give some standard integrals which the students must learn by heart.

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$
2. $\int \frac{1}{x} dx = \log|x| + c$
3. $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c, \quad (n \neq -1)$
4. $\int \frac{dx}{ax+b} = \frac{1}{a} \log|ax+b| + c$
5. $\int e^x \cdot dx = e^x + c$
6. $\int a^x \, dx = \frac{a^x}{\log a} + c$
7. $\int \sin x \, dx = -\cos x + c$
8. $\int \cos x \, dx = \sin x + c$
9. $\int \sec^2 x \, dx = \tan x + c$
10. $\int \text{cosec}^2 x \, dx = -\cot x + c$
11. $\int \sec x \cdot \tan x \, dx = \sec x + c$
12. $\int \text{cosec}\, x \cdot \cot x = -\text{cosec}\, x + c$
13. $\int \sin(ax+b) dx = -\frac{1}{a} \cos(ax+b) + c$
14. $\int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b) + c$

15. $\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b)+c$

16. $\int e^{mx}\cdot dx = \frac{e^{mx}}{a}+c$

17. $\int a^{mx}\cdot dx = \frac{a^{mx}}{m\log a}+c$

18. $\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}x+c = -\cos^{-1}x+c$

19. $\int \frac{1}{1+x^2}dx = \tan^{-1}x+c = -\cot^{-1}x+c$

20. $\int \frac{1}{x\cdot\sqrt{x^2-1}}dx = \sec^{-1}x+c$

21. $\int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1}+c,\ n\neq -1$

22. $\int \frac{f'(x)}{f(x)}dx = \log|f(x)|+c$

23. $\int \tan x\,dx = -\log|\cos x|+c = \log|\sec x|+c$

24. $\int \cot x\,dx = \log|\sec x|+c = -\log|\operatorname{cosec} x|+c$

25. $\int \sec x\,dx = \log|\sec x+\tan x|+c = \log\left|\log\left(\frac{x}{4}+\frac{x}{2}\right)\right|+c$

26. $\int \operatorname{cosec} x\,dx = \log|\operatorname{cosec} x-\cot x|+c = \log\left|\log\frac{x}{2}\right|+c$

27. $\int \sin h\ x\,dx = \cos h\,x+c$

28. $\int \cos h\,x = \sin h\,x+c$

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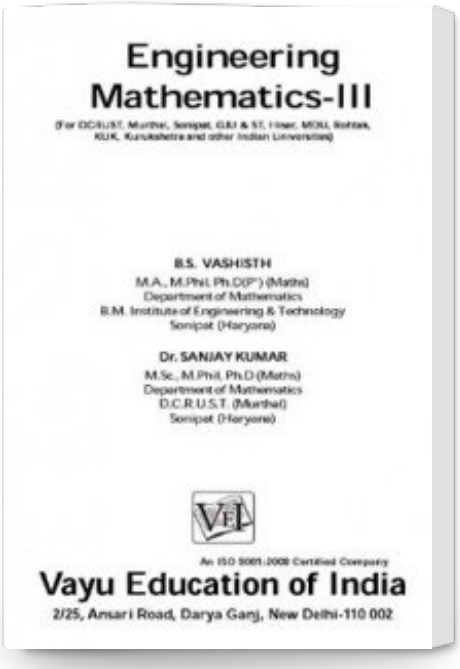

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