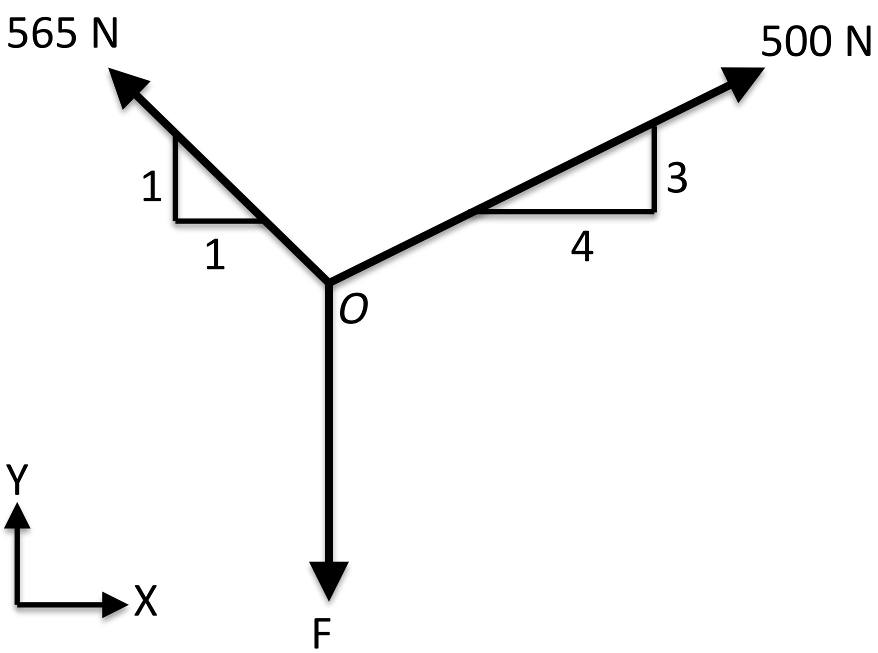
Engineering Mechanics- Morning Session
Approximate 7%-10%
Approximate 15 questions~30 minutes MAX
Review 17 questions (taken from Lindeburg's book, 2009)
March 17th, 2014

Force Equilibrium



QUESTION 1:

What is the most nearly force F when point *O* remains stationary?
(A) 540N (B) 690N (C)860N (D)910N

$$\sum F_X = 0 \Rightarrow F_X = F\cos\theta_X$$

$$\sum F_Y = 0 \Rightarrow F_Y = F\cos\theta_Y$$

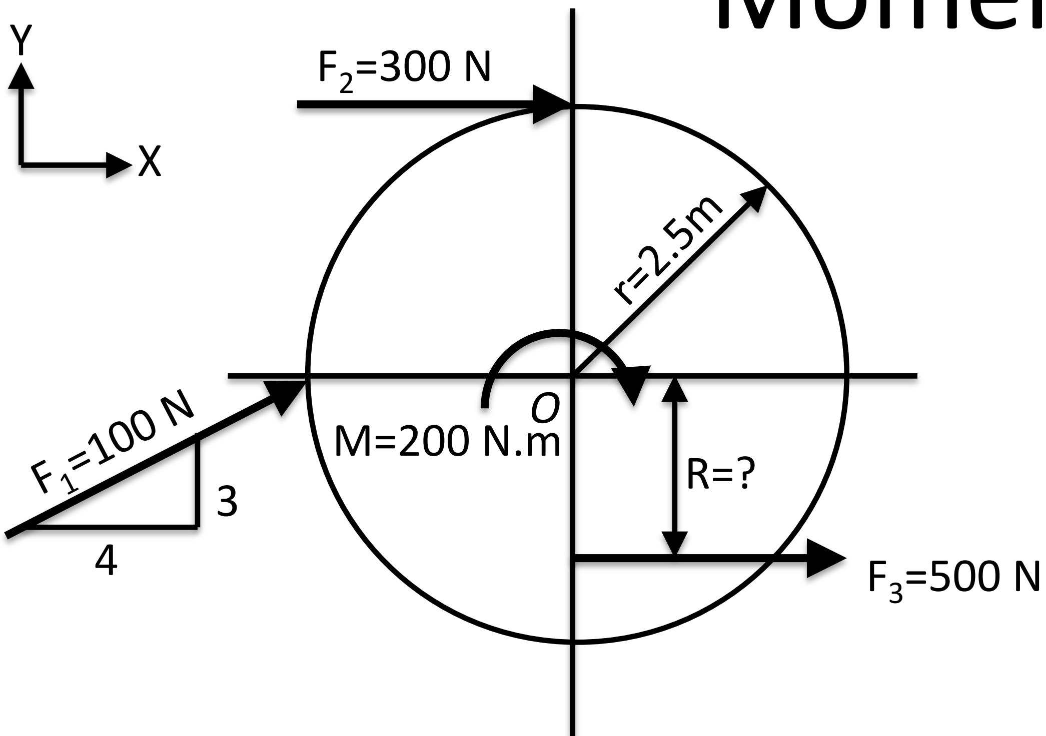
SOLUTION:

$$\sum F_X = (565N)\times(-\frac{1}{\sqrt{2}}) + (500N)(\frac{4}{5}) = 0 \Rightarrow$$ Equilibrium!

$$\sum F_Y = (565N)\times(\frac{1}{\sqrt{2}}) + (500N)(\frac{3}{5}) + F = 0$$

$F = 700N$ in (-y) direction

Moments



QUESTION 2:

A wheel with radius of 2.5 m is pinned at point *O* has two forces applied at its rim and has a third force applied at a distance R from the center. Additionally a moment is applied at the center of wheel.

What is the most nearly distance R for static equilibrium?

(A) 1.4m (B) 1.6m (C) 1.8m (D) 2.2m

SOLUTION:

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

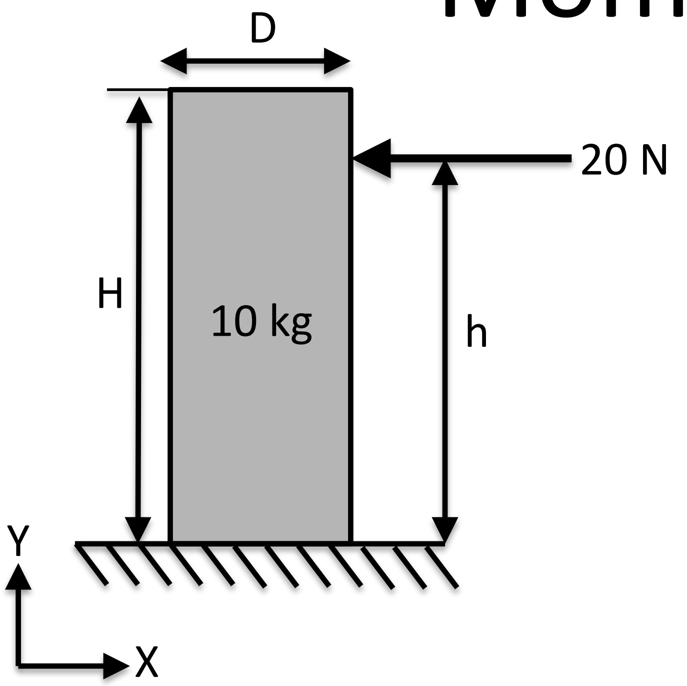
$$M_Z = xF_Y - yF_X$$

$$\sum M_O = 0$$

$$\sum M_O = -(200N \cdot m) - (100N)(\frac{3}{5})(2.5m) - (300N)(2.5m) + R(500N) = 0$$

$$R = 2.2m$$

Moment and Friction



QUESTION 3:

A homogeneous block with a mass of 10 kg with dimensions DxH,H≥4D rests on a level surface with coefficient of friction μ>0.2 What is most nearly the maximum value of h for which no tipping of the block can occur?

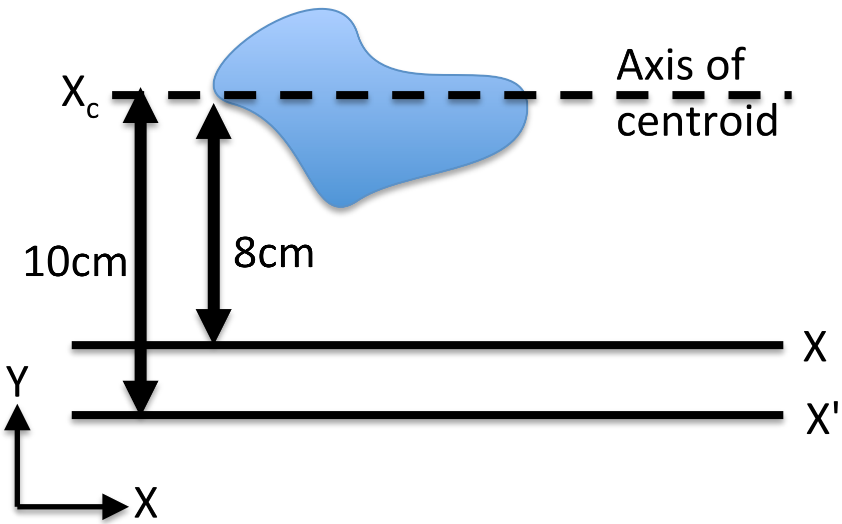
SOLUTION:

$$\sum M_O = 0$$

$$\sum M_O = -(\frac{D}{2})(10kg)(9.81\frac{m}{s^2}) + h(20N) = 0$$

$$h = \frac{\frac{D}{2}(10kg)(9.81\frac{m}{s^2})}{20N} \Rightarrow h = 2.45D \Rightarrow h = \frac{5}{2}D$$

Moment of inertia



QUESTION 4:

The area shown is 60 cm². If the area moment of inertia about x axis is 3870.3 cm4 then what is the area moment inertia about the X'-axis?

$$I_X = \int y^2\, dA \quad I_Y = \int x^2\, dA \quad I_X^{\parallel} = I_X + Ad_X^2$$

SOLUTION:

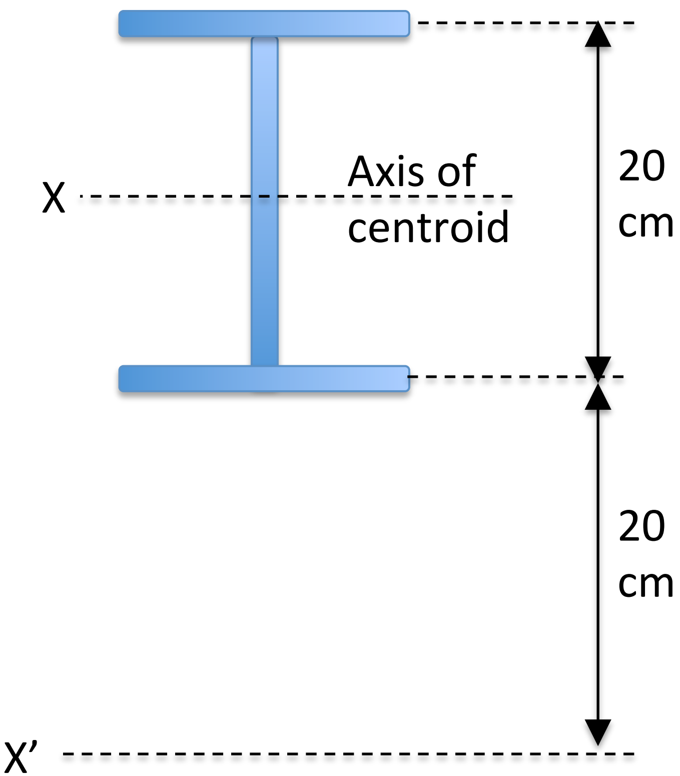
$$I_X = I_C + Ad^2$$

$$I_X = I_C + A(8cm)^2 \Rightarrow I_C = 3870.3cm^4 - (60cm^2)(8cm)^2$$

$$I_{X'} = I_C + A(10cm)^2 = 3870.3cm^4 - (60cm^2)(8cm)^2 + (60cm^2)(10cm)^2$$

$$I_{X'} = 6030.3cm^4$$

Moment of inertia



QUESTION 5:

The moment of inertia about the x' axis of the cross section shown is 245 833 cm^4. If the cross-sectional area is 250 cm^2 and the thickness of the web and the flanges are the same, what is the moment of inertia about the centroidal axis?

(A) $2.1\text{x}10^4$ cm^4
(B) $8.0\text{x}10^4$ cm^4
(C) $1.5\text{x}10^5$ cm^4
(D) $2.5\text{x}10^5$ cm^4

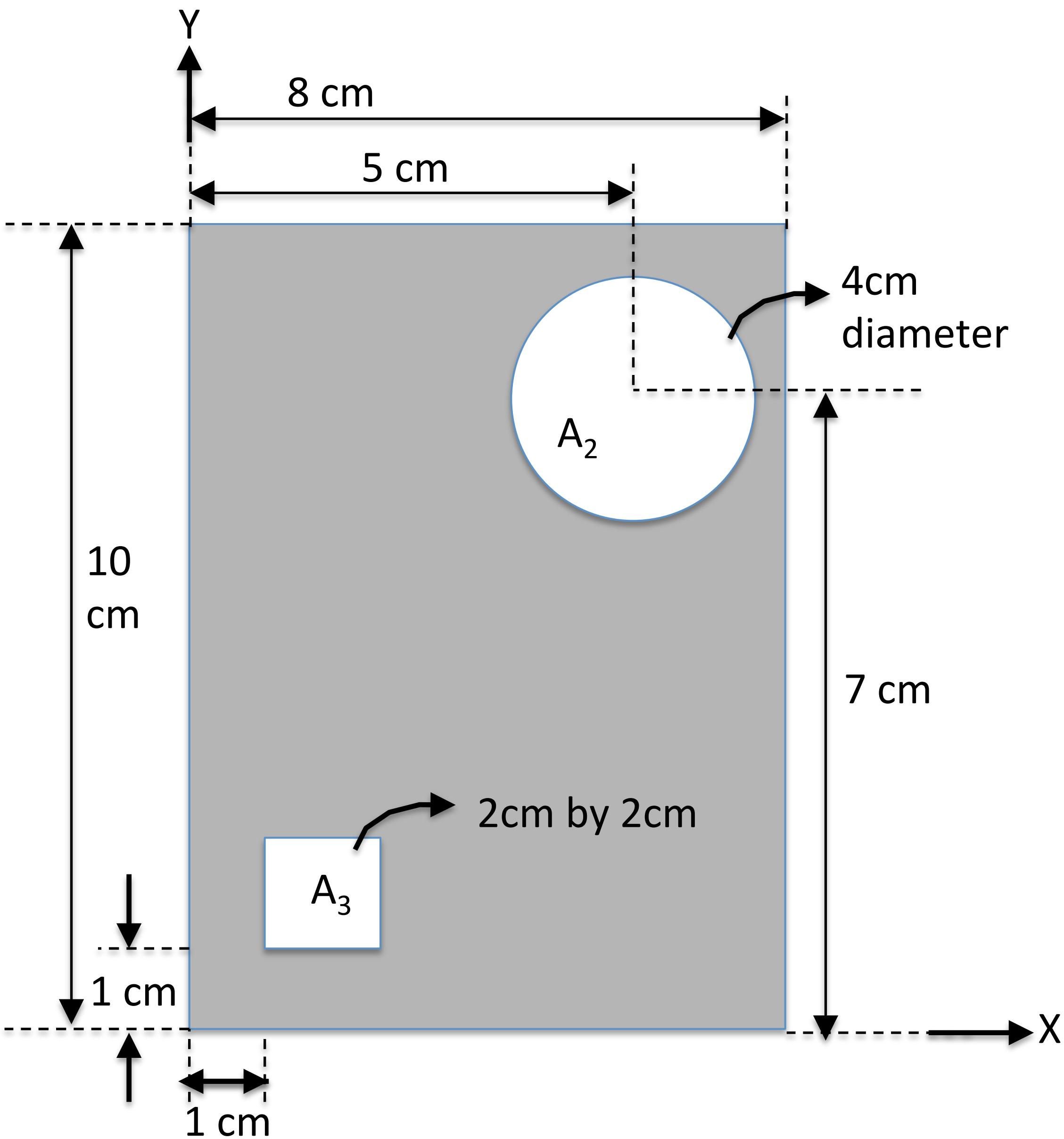
SOLUTION:

$$I'_X = I_{C,X} + Ad^2$$

$$I_{C,X} = I_{X'} - Ad^2 = (245833cm^4) - (250cm^2)(30cm)^2$$

$$= 20833cm^4 \Rightarrow 2.1 \times 10^4 cm^4$$

Centroids



QUESTION 6:

What are the x- and y-coordinates of the centroid of the area?

(A) 3.4 cm; 5.6 cm
(B) 3.5 cm; 5.5 cm
(C) 3.93 cm; 4.79 cm
(D) 4.00 cm; 5.00 cm

Centroids

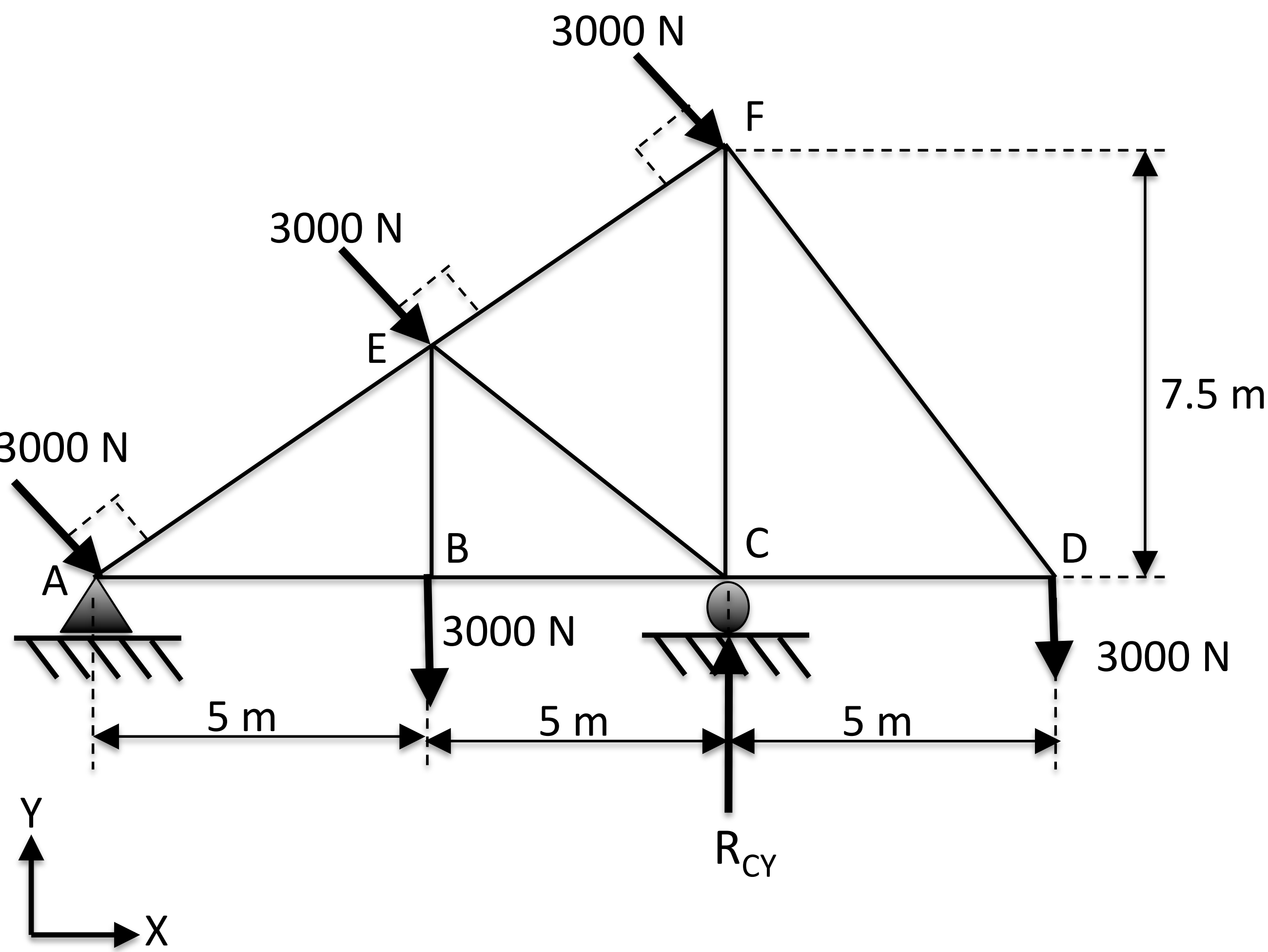
SOLUTION:

$$A = \sum A_n = (8cm)(10cm) - \frac{\pi(4cm)^2}{4} - (2cm)(2cm) = 63.43cm^2$$

$$y_C = \frac{\sum y_{c,n} A_n}{A} = \frac{(5cm)(80cm^2) - (\frac{\pi}{4})(4cm)^2(7cm) - (2cm)(4cm^2)}{63.43cm^2} = 4.79cm$$

$$x_C = \frac{\sum x_{c,n} A_n}{A} = \frac{(4cm)(80cm^2) - (5cm)(\frac{\pi}{4})(4cm)^2 - (2cm)(4cm^2)}{63.43cm^2} = 3.93cm$$

Truss Method of Joints

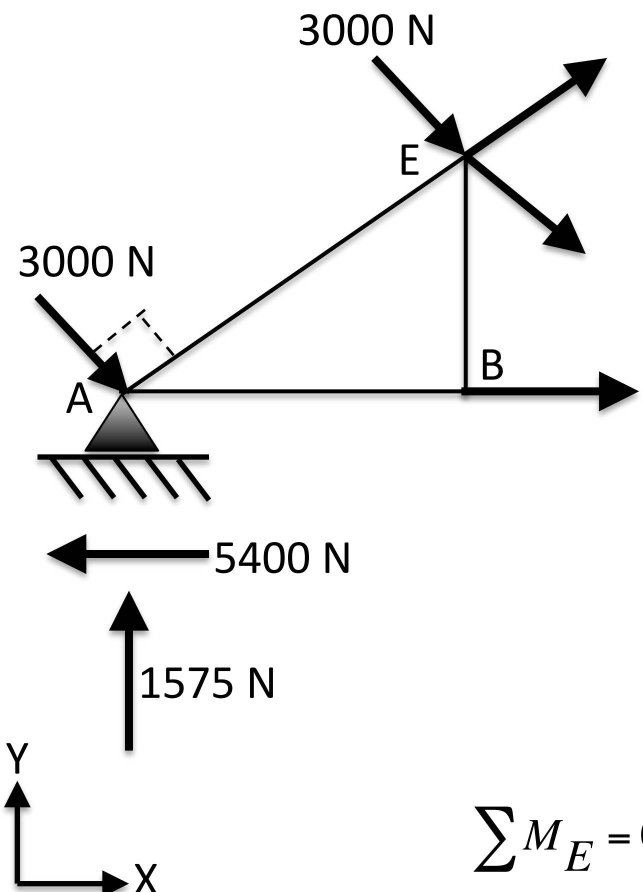


QUESTION 7:

Determine the force in member BC.

(A) 0 N
(B) 1000 N (C)
(C) 1500 N (T)
(D) 2500 N (T)

Truss Method of Joints



SOLUTION:

$$\sum M_A = 0 = (3000N)(6.25) + (3000N)(12.5m) + (3000N)(5m)$$
$$-R_{C_Y}(10m) + (3000N)(15m)$$
$$R_{C_Y} = 11625N(upward)$$
$$\sum F_Y = 0 = R_{A_Y} - (3)(3000N)(\cos 36.87^O) - (2)(3000N) + 11625N$$
$$R_{A_Y} = 1575N(upward)$$
$$\sum F_X = 0 = R_{A_X} - (3)(3000N)(\sin 36.87^O)$$
$$R_{A_X} = -5400N(left)$$

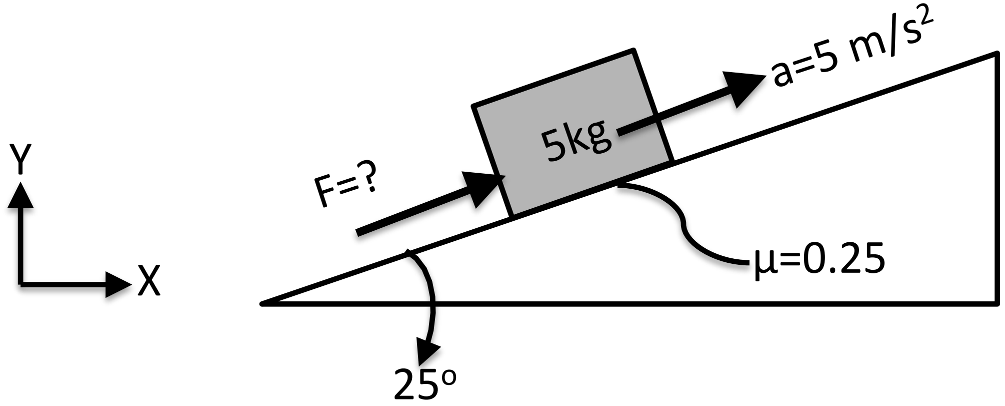
Use method of sections

$$\sum M_E = 0 = (5400N)(3.75m) + (1575N)(5m) - (3000N)(6.25m) - BC(3.75m)$$
$$BC = 2500N$$

Constant Acceleration

QUESTION 8:

A block is pushed up a plane inclined at 25° to the horizontal. If the block has a mass of 5 kg and the coefficient of friction between the plane and the block is 0.25, most nearly how much force is required to accelerate the block up the plane at the rate $5 m/s^2$?
(A) 13 N (B) 57 N (C) 180 N (D) 240 N



SOLUTION:

$$ma = F - mg\sin\theta - \mu N$$

$$F = ma + mg\sin\theta + \mu mg\cos\theta$$

$$F = (5kg)(5\frac{m}{s^2}) + (5kg)(9.81\frac{m}{s^2})\sin 25^o + (0.25)(5kg)(9.81\frac{m}{s^2})\cos 25^o$$

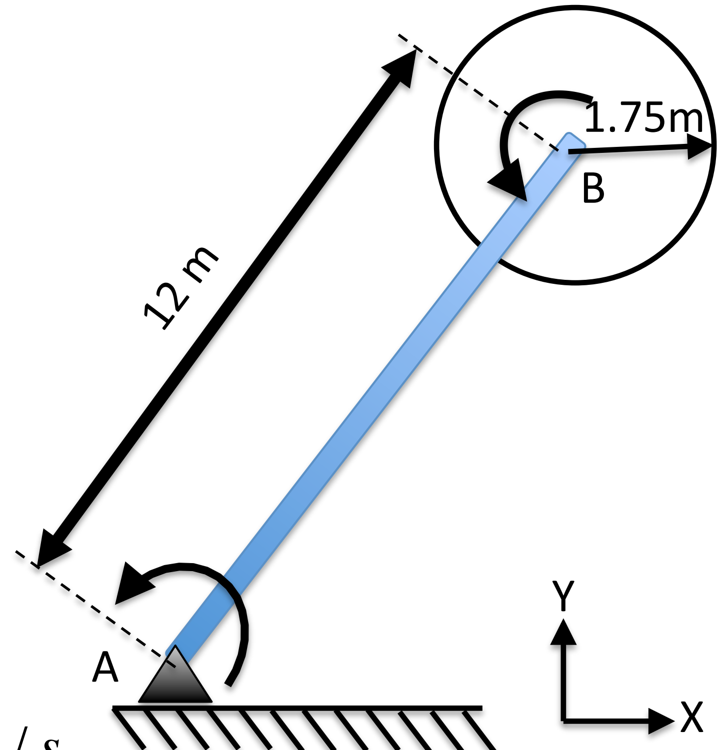
$$F = 56.8N$$

Curvilinear Motion

QUESTION 9:

Rigid link AB is 12 m long. It rotates counterclockwise about point A at 12 rev/min. A thin disk with radius 1.75 m is pinned at its center to the link at point B. The disk rotates counterclockwise at 60 rev/min with respect to point B. What is the maximum tangential velocity seen by any point on the disk?

(A) 6 m/s (B) 26 m/s (C) 28 m/s (D) 45 m/s



SOLUTION:

Maximum tangential velocity of an extension of line AB to an extreme point on the disk due to rotation of the disk only is

$$v_{t,B|A} = r\omega = r(2\pi f)$$

$$v_{t,B|A} = \frac{(12m + 1.75m)(2\pi \frac{rad}{rev})(12 \frac{rev}{\min})}{60 \frac{s}{\min}} = 17.28 m/s$$

Curvilinear Motion

Maximum tangential velocity of the periphery of the disk with respect to point B is

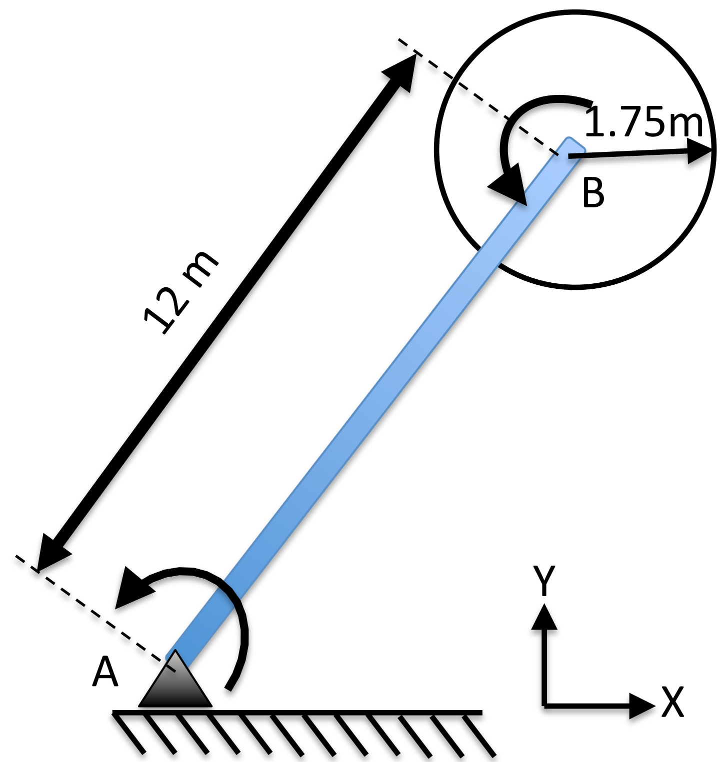
$$v_{t,Disk|B} = r\omega = r(2\pi f)$$

$$v_{t,B|A} = \frac{(1.75m)(2\pi \frac{rad}{rev})(60\frac{rev}{\min})}{60\frac{s}{\min}} = 11.00 m/s$$

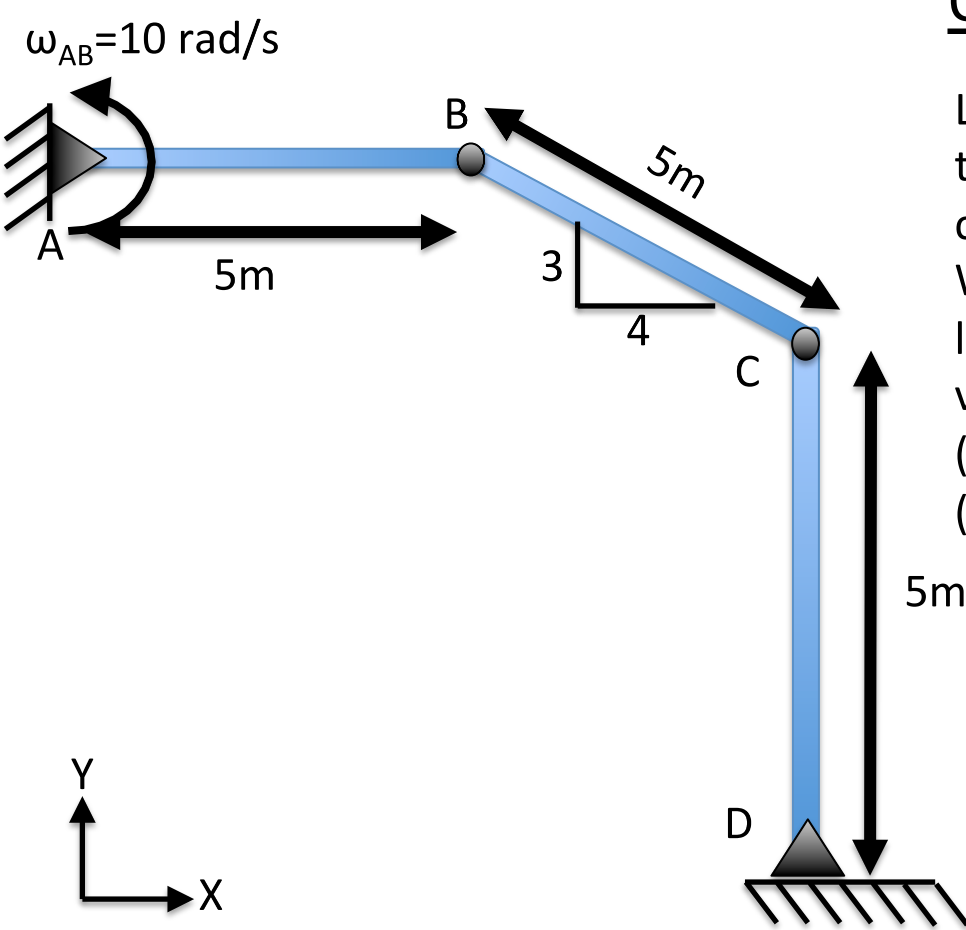
The velocities combine when two velocity vectors coincide in direction. The maximum velocity of the periphery of the disk with respect to point A is the sum of the magnitudes of two velocities

$$v_{t,Disk|A} = v_{t,Disk|B} + v_{t,B|A}$$

$$v_{t,Disk|A} = 17.28\frac{m}{s} + 11.00\frac{m}{s} = 28.28\frac{m}{s}$$



Rotational Motion



QUESTION 10:

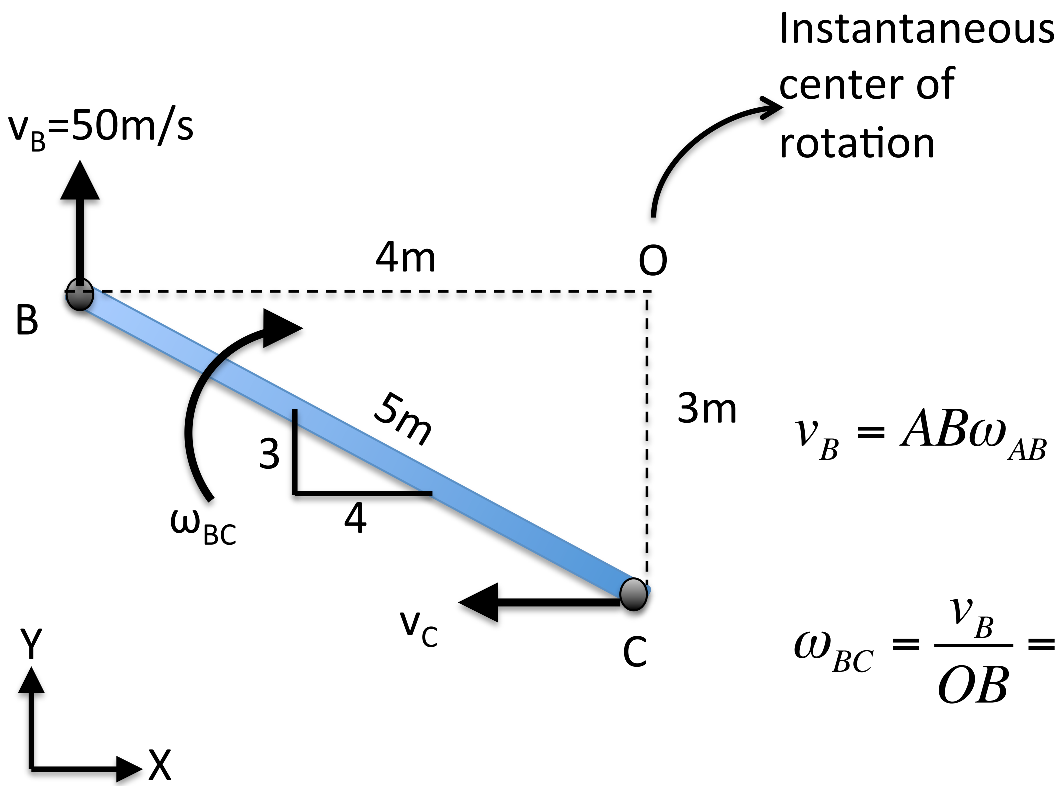
Link AB of the linkage mechanism shown in the illustration rotates with an instantaneous counterclockwise angular velocity of 10 rad/s. What is the instantaneous angular velocity of link BC when link AB is horizontal and CD is vertical?

(A) 2.25 rad/s cw (B) 3.25 ccw rad/s

(C) 5.50 rad/s cw (D) 12.5 rad/s cw

Rotational Motion

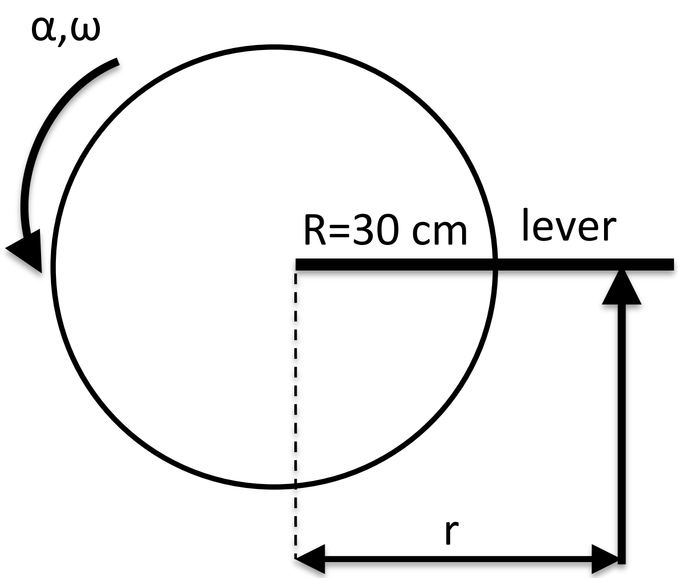
SOLUTION:



$$v_B = AB\omega_{AB} = (5m)(10\frac{rad}{s}) = 50m/s$$

$$\omega_{BC} = \frac{v_B}{OB} = \frac{50\frac{m}{s}}{4m} = 12.5rad/s(clockwise)$$

Rotational Motion



QUESTION 11:

A uniform thin disk has a radius of 30 cm and a mass of 2 kg. A constant force of 10N is applied tangentially at a varying , but unknown, distance from the center of the disk. The disk accelerates about its axis at 3t rad/s^2. What is the distance from the center of the disk at which the force is applied at t=12 s ?

(A) 32.4 cm (B) 36.0 cm (C) 54.0 cm (D) 108 cm

SOLUTION:

$$I = \frac{1}{2} mR^2 = (0.5)(2kg)(0.3m)^2 = 0.09 kg.m^2$$

$$\alpha = 3t = (3\frac{rad}{s})(12s) = 36\frac{rad}{s^2}$$

$$M_0 = Fr = I\alpha \Rightarrow r = \frac{I\alpha}{F}$$

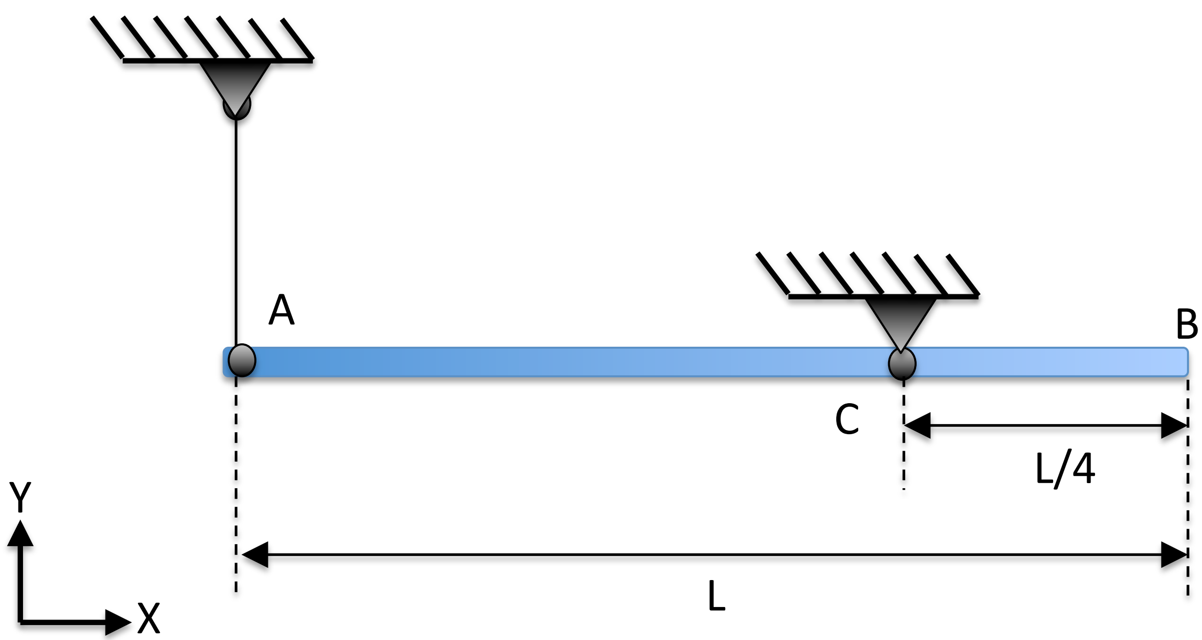
$$r = \frac{(0.09kg.m^2)(36\frac{rad}{s^2})}{10N} = 0.324m = 32.4cm$$

Rotational Motion

QUESTION 12:

A uniform rod (AB) of length L and weight W is pinned at point C and restrained by cable OA. The cable is suddenly cut. The rod starts to rotate about point C, with point A moving down and point B moving up. What is the instantaneous linear acceleration of point B?

(A) 3g/16 (B) g/4 (C) 3g/7 (D) 3g/4



Rotational Motion

SOLUTION:

First calculate mass moment of inertia about point C. Then write the moments on the rod, and from that calculate angular acceleration. From angular acceleration tangential acceleration of point B can be found.

$$I_C = I_{CG} + md^2 = (\frac{1}{12})mL^2 + m(\frac{L}{4})^2 = mL^2(\frac{1}{12} + \frac{1}{16}) = (\frac{7}{48})mL^2$$

$$\sum M_C = \sum Fr = (\frac{3W}{4})(\frac{\frac{3L}{4}}{2}) - (\frac{W}{4})(\frac{\frac{L}{4}}{2}) = \frac{WL}{4} = \frac{mgL}{4}$$

$$\alpha = \frac{\sum M_C}{I_C} = \frac{\frac{mgL}{4}}{(\frac{7}{48})mL^2} = \frac{12g}{7L}$$

$$\alpha_{t,B} = r\alpha = (\frac{L}{4})(\frac{12g}{7L}) = \frac{3g}{7}$$

Linear Momentum

QUESTION 13:

A 2kg clay ball moving at a rate of 40 m/s collides with a 5 kg ball of clay moving in the same direction at a rate of 10 m/s. What is the final velocity of both balls if they stick together after colliding?

(A) 11.8 m/s (B) 12.4 m/s (C) 15.3 m/s (D) 18.6 m/s

SOLUTION:

$$e = 0 = \frac{v'_1 - v'_2}{v_2 - v_1} \Rightarrow v'_1 = v'_2 = v' \Rightarrow \text{Sticking after collision}$$

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v' \Rightarrow \text{Conservation of momentum}$$

$$v' = \frac{(2kg)(40\frac{m}{s}) + (5kg)(10\frac{m}{s})}{(5kg) + (2kg)} = 18.6\frac{m}{s}$$

Centrifugal Force

QUESTION 14:

A 1000 kg car enters an unbanked curve of 1 km radius with a velocity of 80 km/h. What is the minimum coefficient of friction between the road and the car that will allow the car to travel through the turn without breaking?

(A) 0.05 (B) 0.25 (C) 0.35 (A) 0.65

SOLUTION:

$$F_f = F_C \Rightarrow \mu N = ma = m(r\omega^2) = \frac{mv^2}{r} \Rightarrow$$

Frictional force must counteract the centrifugal force

$$\mu mg = \frac{mv^2}{r}$$

$$\mu = \frac{v^2}{gr} = \frac{((80\frac{km}{h})(1000\frac{m}{km})(\frac{1h}{3600s}))^2}{(9.81\frac{m}{s^2})(1000m)} = 0.05$$

Projectile motion

QUESTION 15:

A projectile is launched from a level plane at 30º from horizontal with an initial velocity of 1250 m/s. What is most nearly the maximum height above the plane the projectile will reach?
(A) 20 km (B) 40 km (C) 60 km (A) 80 km

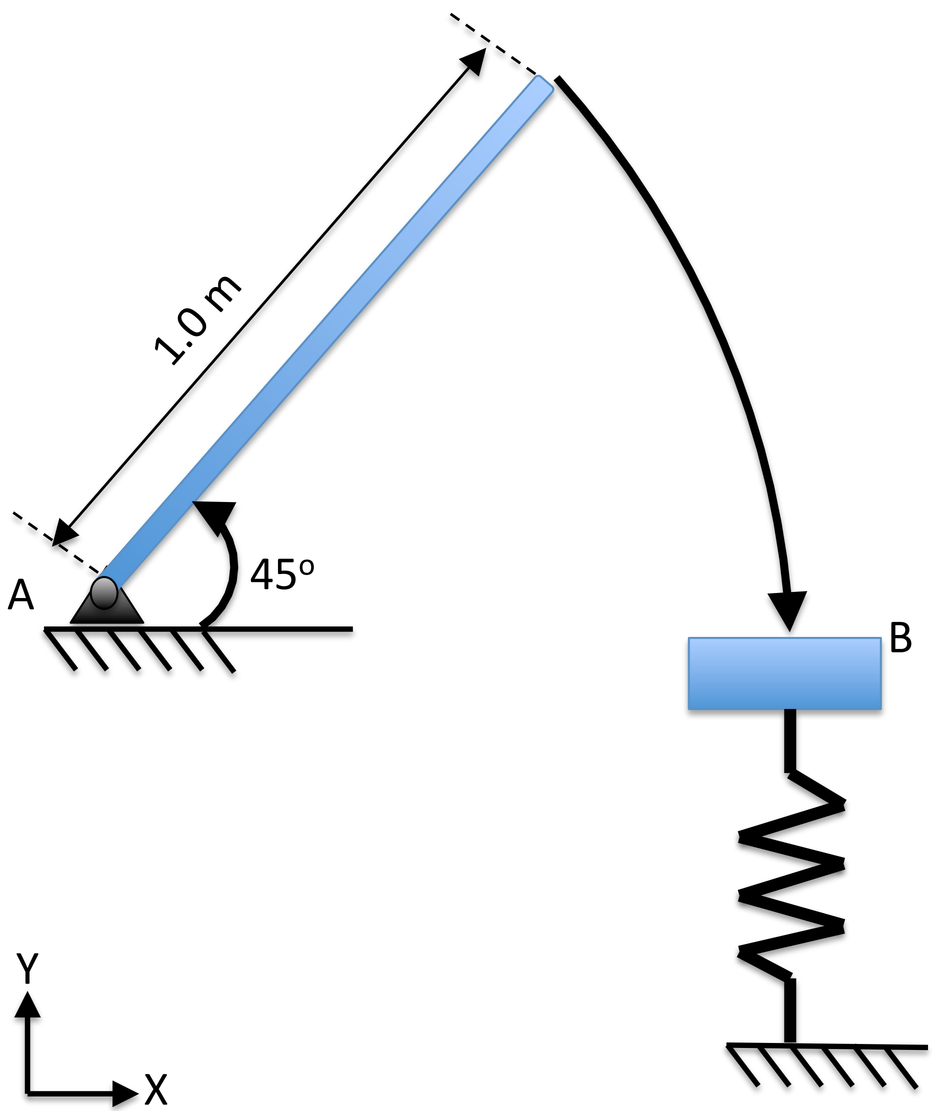
SOLUTION:

$$t = \frac{v_0 \sin\theta}{g}$$

$$y = -\frac{gt^2}{2} + v_0(\sin\theta)t + y_0 = -\frac{v_0^2(\sin\theta)^2}{2g} + \frac{v_0^2(\sin\theta)^2}{g} = \frac{v_0^2(\sin\theta)^2}{2g}$$

$$y = \frac{(1250\frac{m}{s})^2(\sin 30^O)^2}{2(9.81\frac{m}{s^2})(1000\frac{m}{km})} = 19.9km$$

Energy



QUESTION 16:

A uniform rod is 1 m long and has a mass of 10 kg. It is pinned at point A, frictionless pivot. The rod is released from a rest position 45° from the horizontal. The top of an undeflected ideal spring is located at point B to contact the tip of the rod when the rod is horizontal. The spring has a constant of 98 kN/m.

What is the velocity of the rod's tip when the rod just becomes horizontal?

(A) 2.6 m/s (B) 4.6 m/s

(C) 6.9 m/s (D) 8.9 m/s

Energy

SOLUTION:

Consider all of the mass of rod is concentrated at its centroid. After finding the height, it is possible to find the initial potential energy. This energy becomes kinetic, when rod's end reaches to B. After finding the rotational velocity linear velocity can be found.

$$h = \frac{1}{2}L(\sin 45^O)$$

$$I_{rod} = \frac{mL^2}{3}$$

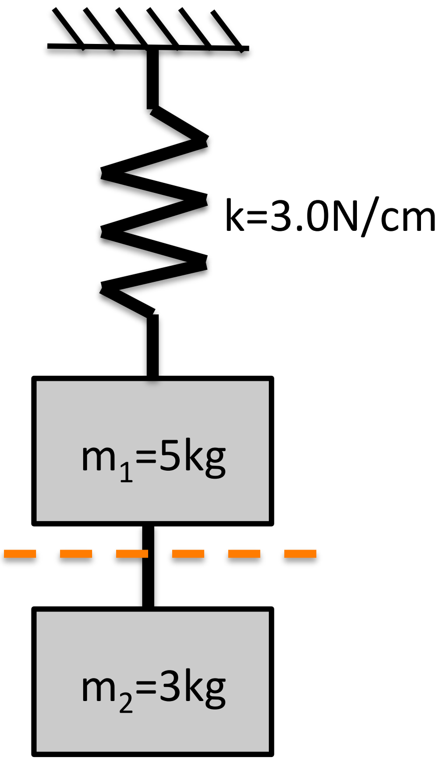
$$mgh = \frac{I\omega^2}{2}$$

$$mg(\frac{1}{2}L\sin 45^O) = (\frac{mL^2}{3})(\frac{\omega^2}{2})$$

$$\omega = \sqrt{\frac{3g\sin 45^O}{L}} = \sqrt{\frac{3(9.81\frac{m}{s^2})(\sin 45^O)}{1m}} = 4.56\frac{rad}{s}$$

$$v = r\omega = (1m)(4.56\frac{rad}{s}) = 4.56\frac{m}{s} \Rightarrow 4.6\frac{m}{s}$$

Vibration

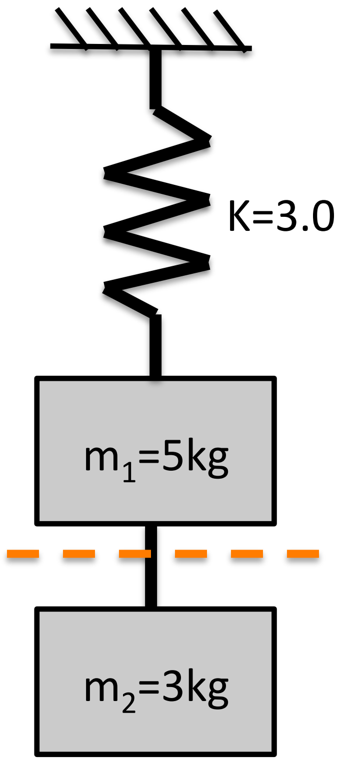


QUESTION 17:

Two masses are connected to a spring as shown. Both masses are motionless. The string connecting the masses is suddenly cut. Assuming that the spring never goes "solid", what is most nearly the amplitude of oscillation for m_1?

(A) 9.81 cm (B) 16.4 cm (C) 19.6 cm (A) 26.2 cm

Vibration



SOLUTION:

Difference between total deflection and deflection of m_1 only will be the amplitude of oscillation.

$$F = k\delta = W = mg \Rightarrow \delta_0 = \frac{(5kg + 3kg)(9.81\frac{m}{s^2})}{300\frac{N}{m}} = 0.2616m = 26.16cm$$

$$\delta_{st,1} = \frac{m_1 g}{k} = \frac{(5kg)(9.81\frac{m}{s^2})}{300\frac{N}{m}} = 0.1635m = 16.35cm$$

$$x_0 = \delta_0 - \delta_{sr,1} = 26.16cm - 16.35cm = 9.81cm$$