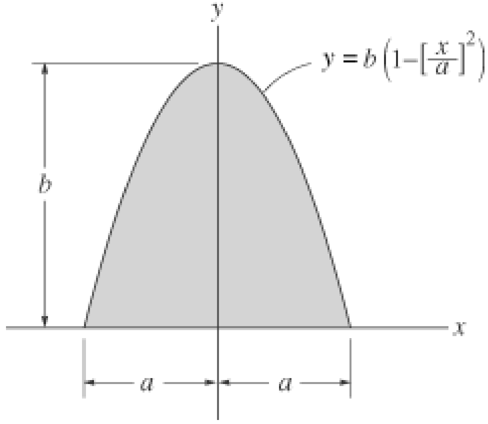
**Problem 10-1**

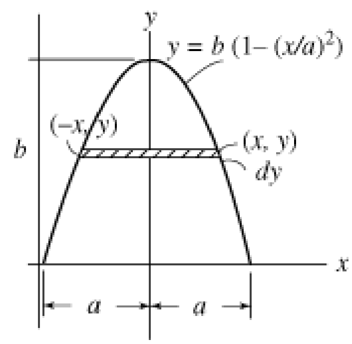
Determine the moment of inertia for the shaded area about the $x$ axis.

Given:

$a = 2\ \text{m}$

$b = 4\ \text{m}$




Solution: $$I_x = 2\int_0^b y^2 a\sqrt{1-\frac{y}{b}}\,dy$$ $$I_x = 39.0\,\text{m}^4$$

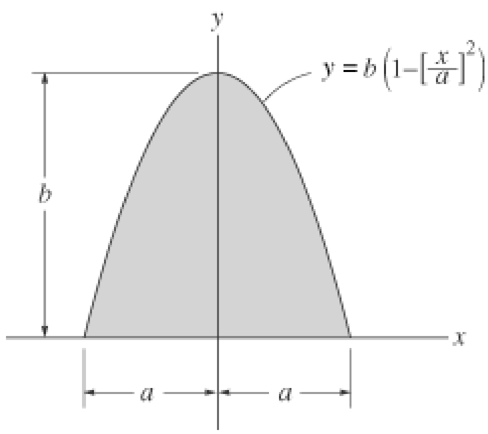
**Problem 10-2**

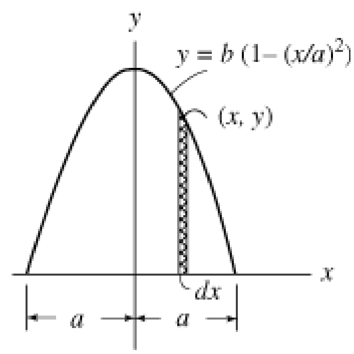
Determine the moment of inertia for the shaded area about the $y$ axis.

Given:

$a = 2\ \text{m}$

$b = 4\ \text{m}$

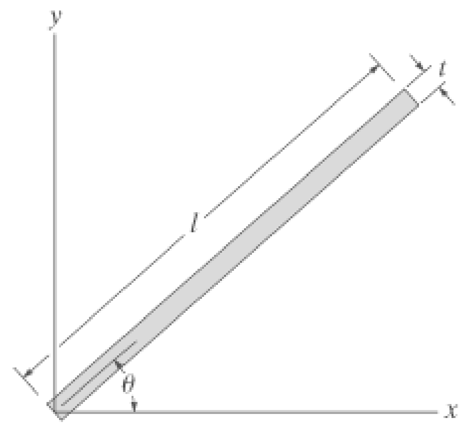



Solution: $$I_y = 2\int_0^a x^2 b\left[1-\left(\frac{x}{a}\right)^2\right]dx$$ $$I_y = 8.53\ \text{m}^4$$

**Problem 10-3**

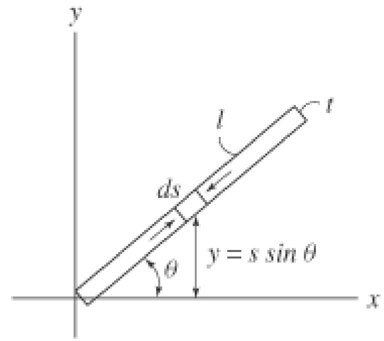
Determine the moment of inertia for the thin strip of area about the $x$ axis. The strip is oriented at an angle $\theta$ from the $x$ axis. Assume that $t << l$.




Solution:

$$I_x = \int_A y^2\, dA = \int_0^l s^2 \sin^2(\theta) t\, ds$$

$$I_x = \frac{1}{3} t l^3 \sin^2(\theta)$$



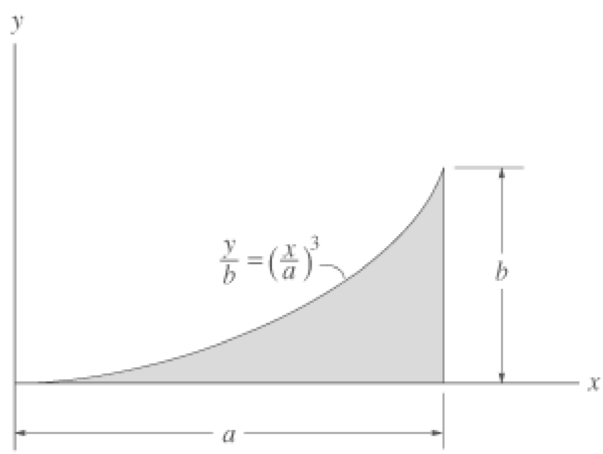

**Problem 10-4**

Determine the moment for inertia of the shaded area about the $x$ axis.

Given:

$a = 4$ in

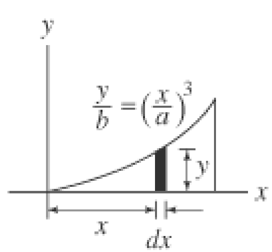
$b = 2$ in




Solution:

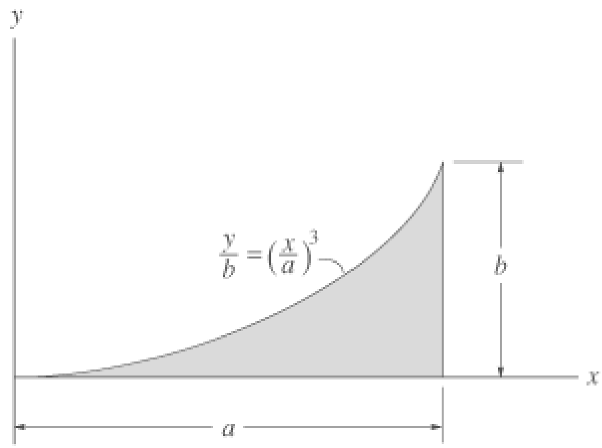
$$I_x = \int_0^a \frac{1}{3}\left[b\left(\frac{x}{a}\right)^3\right]^3 dx$$

$$I_x = 1.07\ \text{in}^4$$



**Problem 10-5**

Determine the moment for inertia of the shaded area about the $y$ axis.




Given:

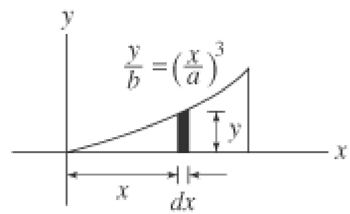
$a = 4\ \text{in}$

$b = 2\ \text{in}$

Solution:

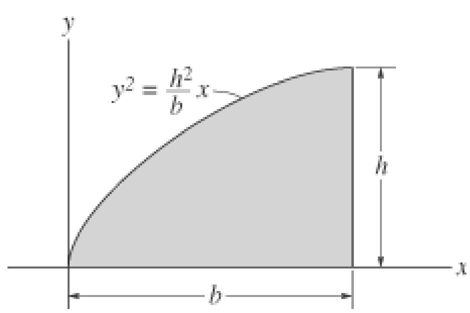
$$I_y = \int_0^a x^2 b\left(\frac{x}{a}\right)^3 \mathrm{d}x$$

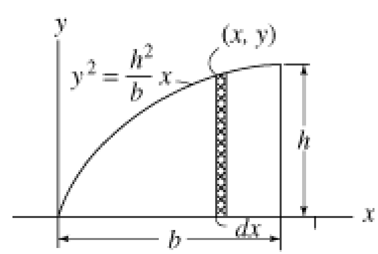
$$I_y = 21.33\ \text{in}^4$$

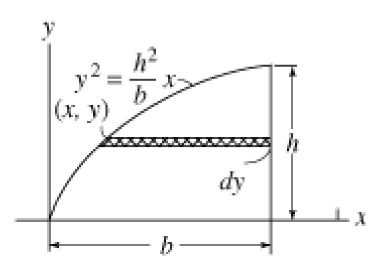



**Problem 10-6**

Determine the moment of inertia for the shaded area about the $x$ axis.




Solution:

$$I_x = \int_0^b \frac{\left(h\sqrt{\frac{x}{b}}\right)^3}{3}\,\mathrm{d}x = \frac{2}{15}\,b\,h^3$$

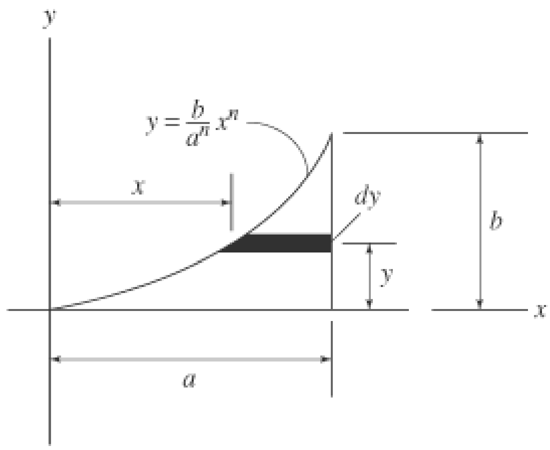
$$I_x = \frac{2}{15}\,b\,h^3$$

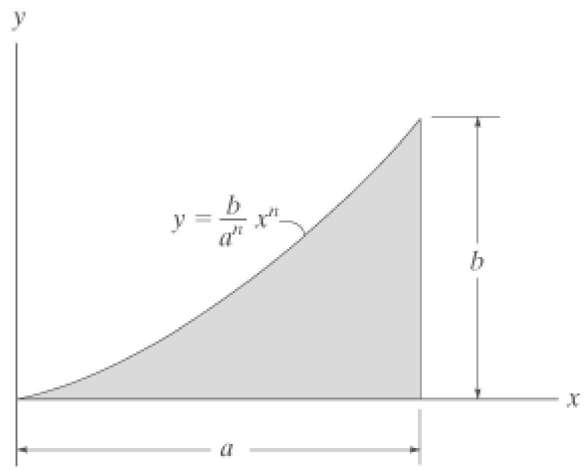
Alternatively

$$I_x = \int_0^h y^2\left(b - b\frac{y^2}{h^2}\right)\,dy = \frac{2}{15}\,b\,h^3 \qquad I_x = \frac{2}{15}\,b\,h^3$$

**Problem 10-7**

Determine the moment of inertia for the shaded area about the *x* axis.



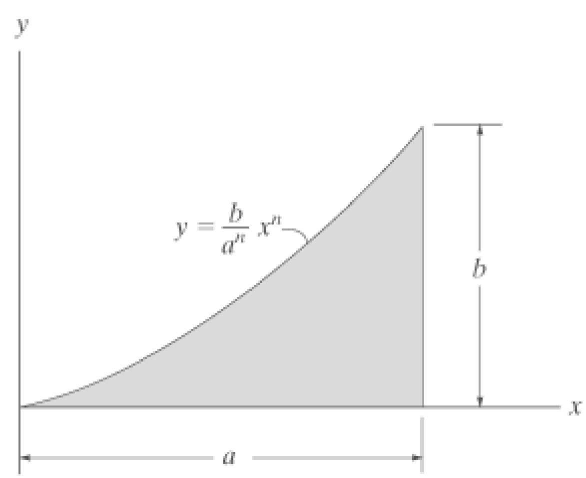




Solution:

$$I_x = \int_0^b A\,y^2\left[a - a\left(\frac{y}{b}\right)^{\frac{1}{n}}\right]dy \qquad I_x = \frac{a\,b^3}{3(1+3n)}$$

**Problem 10-8**

Determine the moment of inertia for the shaded area about the *y* axis.

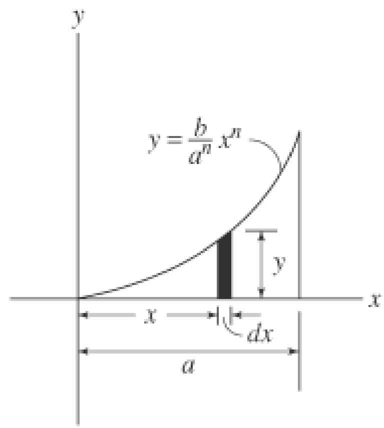


Solution:

$$I_y = \int x^2\, dA = \int_0^a x^2 y\, dx$$

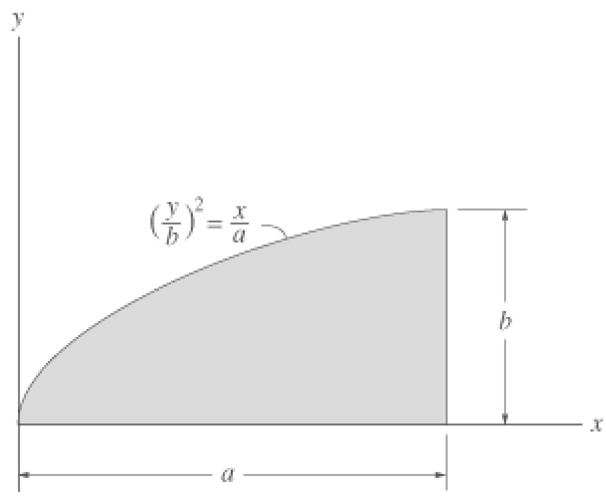
$$I_y = \frac{b}{a^n}\int_0^a x^{n+2}\, dx = \left[\left(\frac{b}{a^n}\right)\frac{x^{n+3}}{n+3}\right]_0^a$$

$$I_y = \frac{b a^3}{n+3}$$




**Problem 10-9**

Determine the moment of inertia for the shaded area about the $x$ axis.




Given:

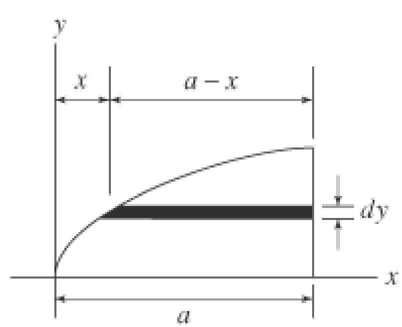
$a = 4$ in

$b = 2$ in

Solution:

$$I_x = \int_0^b y^2\left[a - a\left(\frac{y}{b}\right)^2\right] dy$$

$$I_x = 4.27\ \text{in}^4$$



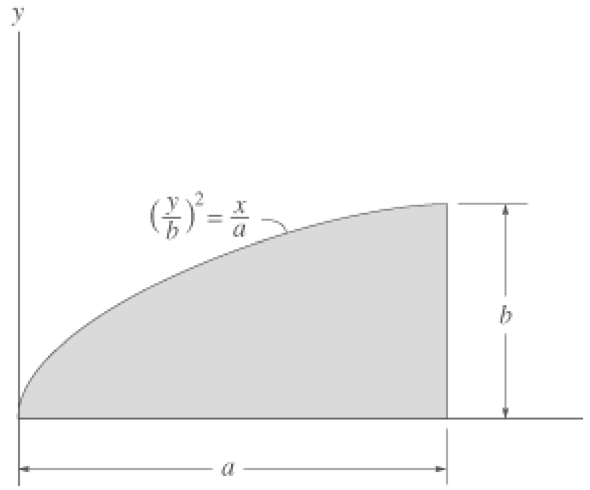

**Problem 10-10**

Determine the moment of inertia for the shaded area about the $y$ axis.

Given:

$a = 4$ in

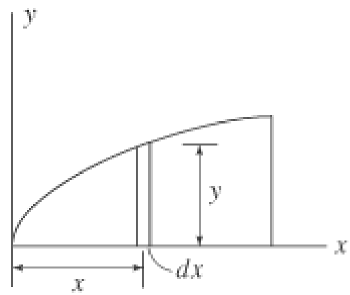
$b = 2$ in




Solution:

$$I_y = \int_0^a x^2 b\sqrt{\frac{x}{a}}\,dx$$

$I_y = 36.6\text{ in}^4$



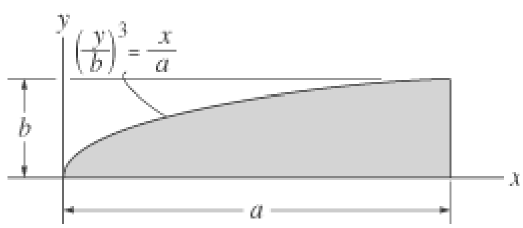

## Problem 10-11

Determine the moment of inertia for the shaded area about the $x$ axis

Given:

$a = 8$ in

$b = 2$ in




Solution:

$$I_x = \int_0^b y^2\left(a - a\frac{y^3}{b^3}\right)dy \qquad I_x = 10.67\text{ in}^4$$

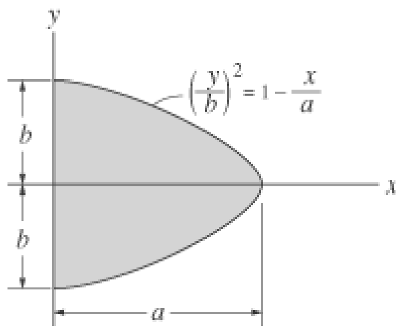
## Problem 10-12

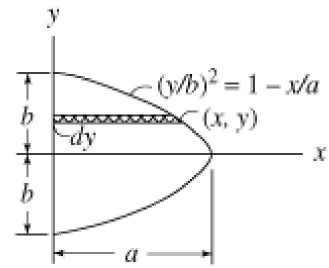
Determine the moment of inertia for the shaded area about the $x$ axis

Given:

$a = 2\ \text{m}$

$b = 1\ \text{m}$




Solution:

$$I_x = \int_{-b}^{b} y^2 a\left(1 - \frac{y^2}{b^2}\right) dy \qquad I_x = 0.53\ \text{m}^4$$

---

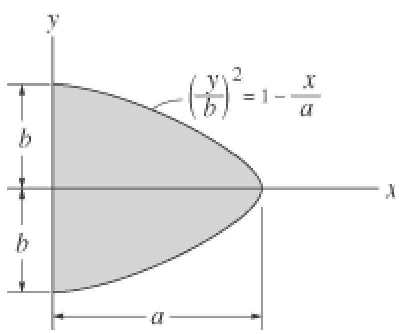
**Problem 10-13**

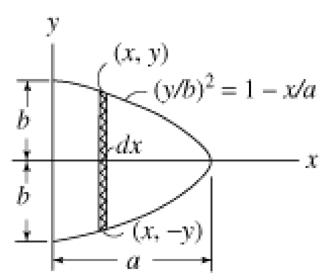
Determine the moment of inertia for the shaded area about the $y$ axis

Given:

$a = 2\ \text{m}$

$b = 1\ \text{m}$




Solution:

$$I_y = \int_0^a x^2 2b\sqrt{1 - \frac{x}{a}}\, dx \qquad I_y = 2.44\ \text{m}^4$$

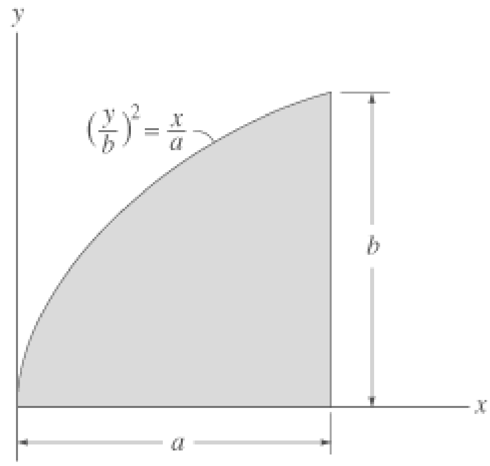
---

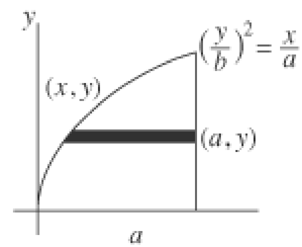
**Problem 10-14**

Determine the moment of inertia for the shaded area about the $x$ axis.

Given:

$a = 4\ \text{in}$ $\quad b = 4\ \text{in}$




Solution:

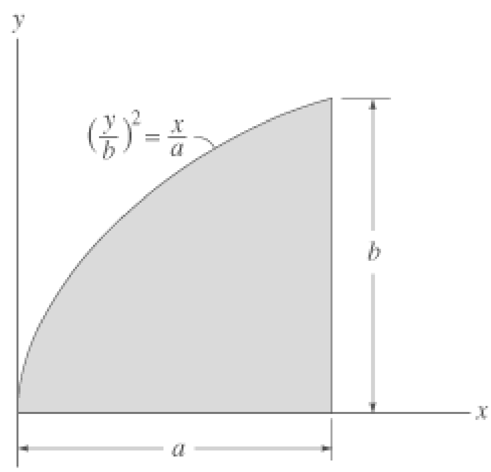
$$I_x = \int_0^b y^2\left[a - a\left(\frac{y}{b}\right)^2\right] dy$$

$$I_x = 34.1 \text{ in}^4$$

---

**Problem 10-15**

Determine the moment of inertia for the shaded area about the $y$ axis.




Given:

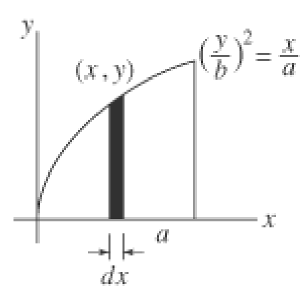
$a = 4 \text{ in}$

$b = 4 \text{ in}$

Solution:

$$I_y = \int_0^a x^2 b\sqrt{\frac{x}{a}}\, dx$$

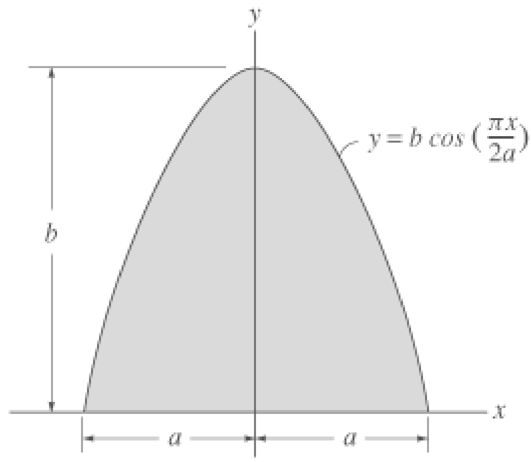
$$I_y = 73.1 \text{ in}^4$$




---

**Problem 10-16**

Determine the moment of inertia of the shaded area about the $x$ axis.



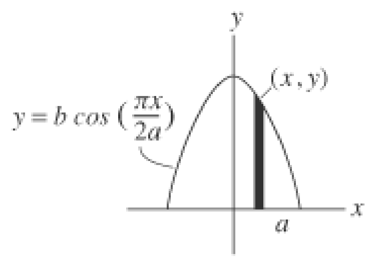

Given:

$a = 2 \text{ in}$

$b = 4 \text{ in}$

Solution:

$$I_x = \int_{-a}^{a} \frac{1}{3}\left(b\cos\left(\frac{\pi x}{2a}\right)\right)^3 dx$$




$$I_x = 36.2 \text{ in}^4$$

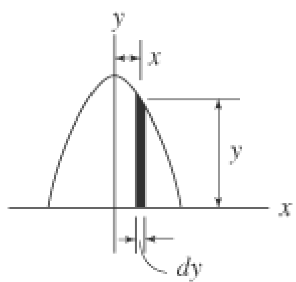
**Problem 10-17**

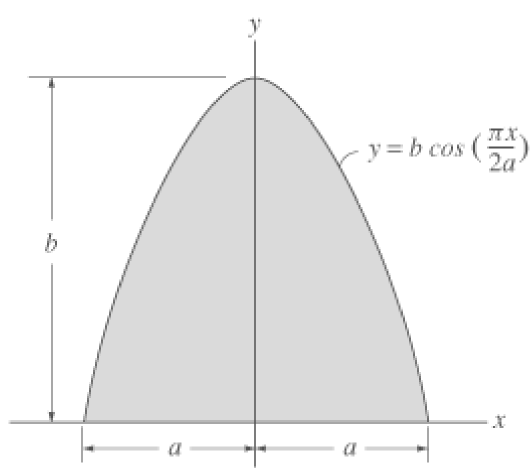
Determine the moment of inertia for the shaded area about the $y$ axis.

Given:

$a = 2 \text{ in}$

$b = 4 \text{ in}$




Solution:

$$I_y = \int_{-a}^{a} x^2 b\cos\left(\frac{\pi x}{2a}\right) dx$$

$$I_y = 7.72 \text{ in}^4$$

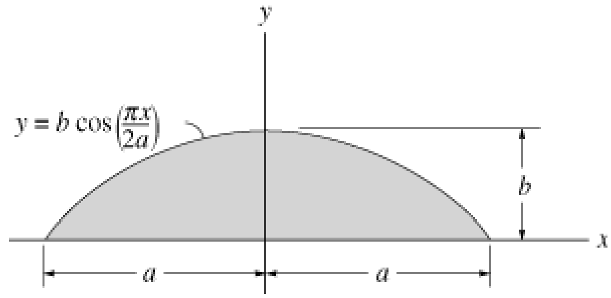
**Problem 10-18**

Determine the moment of inertia for the shaded area about the $x$ axis.

Given:

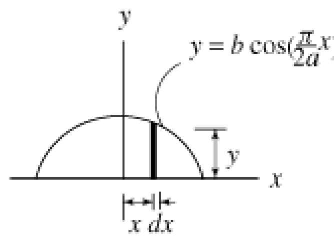
$a = 4\ \text{in}$

$b = 2\ \text{in}$




Solution:

$$I_x = \int_{-a}^{a} \frac{\left(b\cos\left(\frac{\pi x}{2a}\right)\right)^3}{3}\,dx \qquad I_x = 9.05\ \text{in}^4$$




---

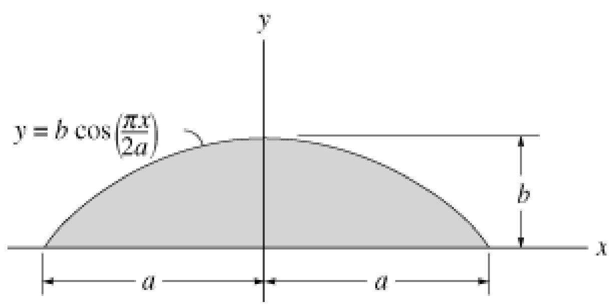
**Problem 10-19**

Determine the moment of inertia for the shaded area about the $y$ axis.

Given:

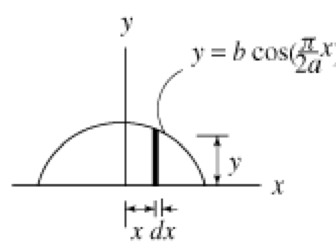
$a = 4\ \text{in}$

$b = 2\ \text{in}$




Solution:

$$I_y = \int_{-a}^{a} x^2 b\cos\left(\frac{\pi x}{2a}\right)\,dx \qquad I_y = 30.9\ \text{in}^4$$




---

**Problem 10-20**

Determine the moment for inertia of the shaded area about the $x$ axis.

Given:

$a = 2\ \text{in}$

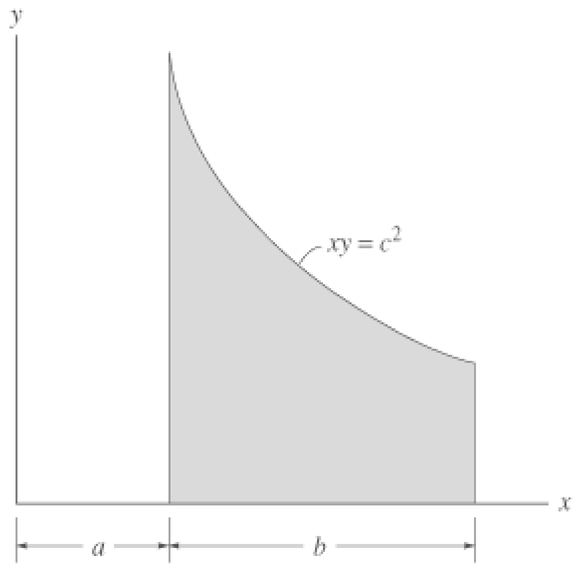
$b = 4\ \text{in}$

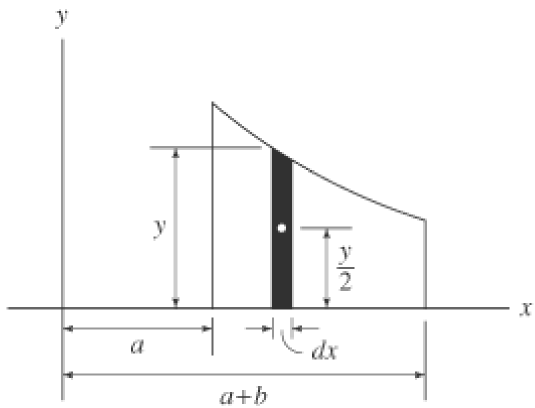
$c = \sqrt{12}\ \text{in}$

Solution:

$$I_x = \int_a^{a+b} \frac{1}{3}\left(\frac{c^2}{x}\right)^3 dx$$

$I_x = 64.0\ \text{in}^4$




**Problem 10-21**

Determine the moment of inertia of the shaded area about the $y$ axis.

Given:

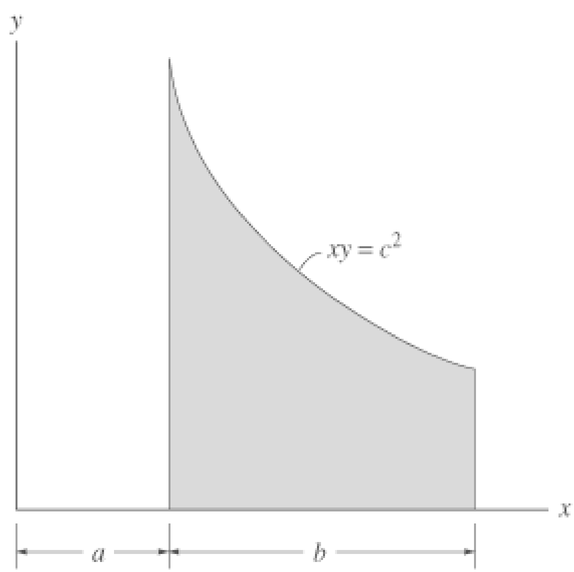
$a = 2\ \text{in}$

$b = 4\ \text{in}$

$c = \sqrt{12}\ \text{in}$

Solution:

$$I_y = \int_a^{a+b} x^2\left(\frac{c^2}{x}\right) dx$$



$I_y = 192.00\,\text{in}^4$

---

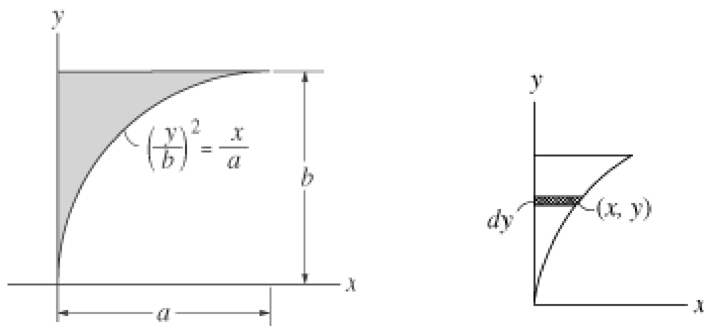
**Problem 10-22**

Determine the moment of inertia for the shaded area about the $x$ axis.

Given:

$a = 2\ \text{m}$

$b = 2\ \text{m}$




Solution:

$$I_x = \int_0^b y^2 a\left(\frac{y^2}{b^2}\right) dy \qquad I_x = 3.20\ \text{m}^4$$

---

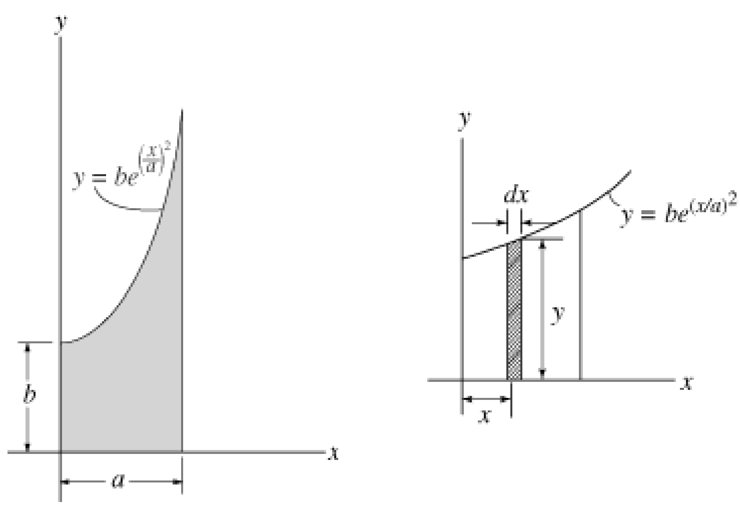
**Problem 10-23**

Determine the moment of inertia for the shaded area about the $y$ axis. Use Simpson's rule to evaluate the integral.

Given:

$a = 1\ \text{m}$

$b = 1\ \text{m}$



Solution:

$$I_y = \int_0^a x^2 b e^{\left(\frac{x}{a}\right)^2} dx \qquad I_y = 0.628 \text{ m}^4$$

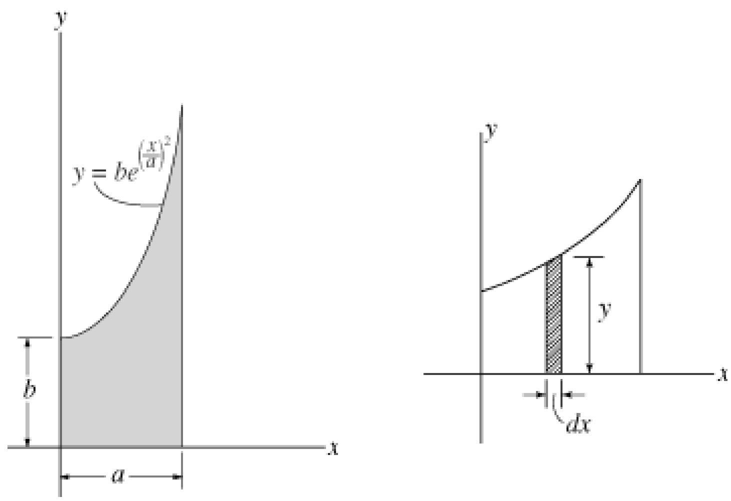
**Problem 10-24**

Determine the moment of inertia for the shaded area about the $x$ axis. Use Simpson's rule to evaluate the integral.

Given:

$a = 1$ m

$b = 1$ m




Solution:

$$I_y = \int_0^a \frac{\left[b e^{\left(\frac{x}{a}\right)^2}\right]^3}{3} dx \qquad I_y = 1.41 \text{ m}^4$$

**Problem 10-25**

The polar moment of inertia for the area is $I_C$ about the $z$ axis passing through the centroid $C$. The moment of inertia about the $x$ axis is $I_x$ and the moment of inertia about the $y'$ axis is $I_{y'}$. Determine the area $A$.

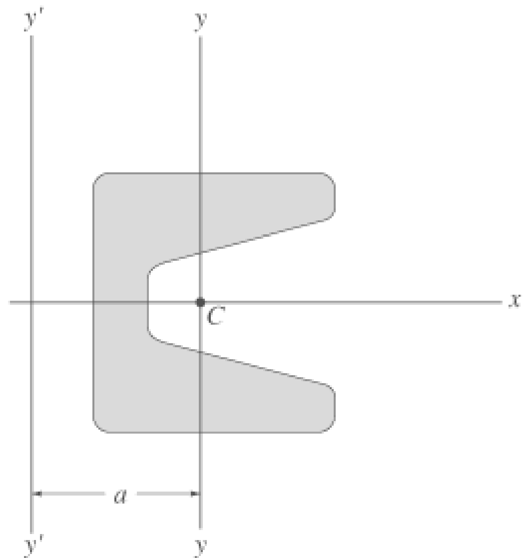
Given:

$I_C = 28 \text{ in}^4$

$I_x = 17\ \text{in}^4$

$I_{y'} = 56\ \text{in}^4$

$a = 3\ \text{in}$




Solution:

$I_C = I_x + I_y$

$I_y = I_C - I_x$

$I_{y'} = I_y + A\,a^2$

$A = \dfrac{I_{y'} - I_y}{a^2}$ $\quad A = 5.00\,\text{in}^2$

---

**Problem 10-26**

The polar moment of inertia for the area is $J_{cc}$ about the $z'$ axis passing through the centroid $C$. If the moment of inertia about the $y'$ axis is $I_{y'}$ and the moment of inertia about the $x$ axis is $I_x$. Determine the area $A$.

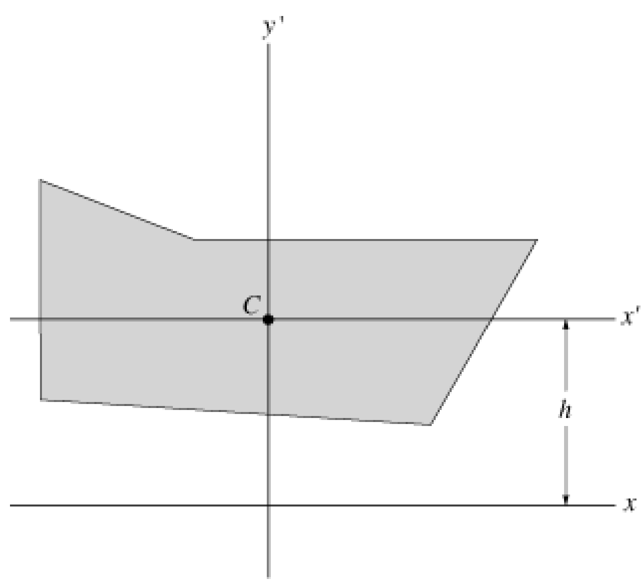
Given:

$J_{cc} = 548 \times 10^6\ \text{mm}^4$

$I_{y'} = 383 \times 10^6\ \text{mm}^4$

$I_x = 856 \times 10^6\ \text{mm}^4$

$h = 250\ \text{mm}$




Solution:

$I_{x'} = I_x - A\,h^2$

$J_{cc} = I_{x'} + I_{y'}$

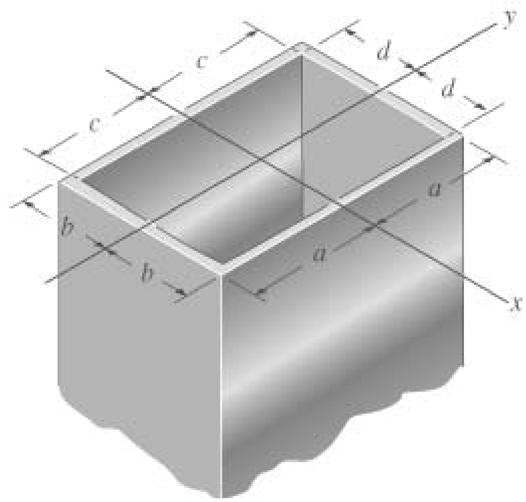
$J_{cc} = I_x - A\,h^2 + I_{y'}$

$$A = \frac{I_x + I_{y'} - J_{cc}}{h^2}$$

$$A = 11.1 \times 10^3 \text{ mm}^2$$

**Problem 10-27**

Determine the radius of gyration $k_x$ of the column's cross-sectional area.




Given:

$a = 100$ mm

$b = 75$ mm

$c = 90$ mm

$d = 65$ mm

Solution:

Cross-sectional area:

$$A = (2b)(2a) - (2d)(2c)$$

Moment of inertia about the $x$ axis:

$$I_x = \frac{1}{12}(2b)(2a)^3 - \frac{1}{12}(2d)(2c)^3$$

Radius of gyration about the $x$ axis:

$$k_x = \sqrt{\frac{I_x}{A}} \qquad k_x = 74.7 \text{ mm}$$

**Problem 10-28**

Determine the radius of gyration $k_y$ of the column's cross-sectional area.

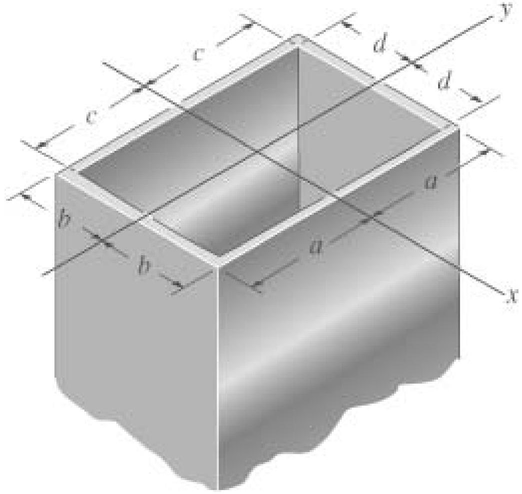
Given:

$a = 100$ mm

$b = 75$ mm

$c = 90 \text{ mm}$

$d = 65 \text{ mm}$




Solution:

Cross-sectional area:

$$A = (2b)(2a) - (2d)(2c)$$

Moment of inertia about the $y$ axis:

$$I_y = \frac{1}{12}(2a)(2b)^3 - \frac{1}{12}(2c)(2d)^3$$

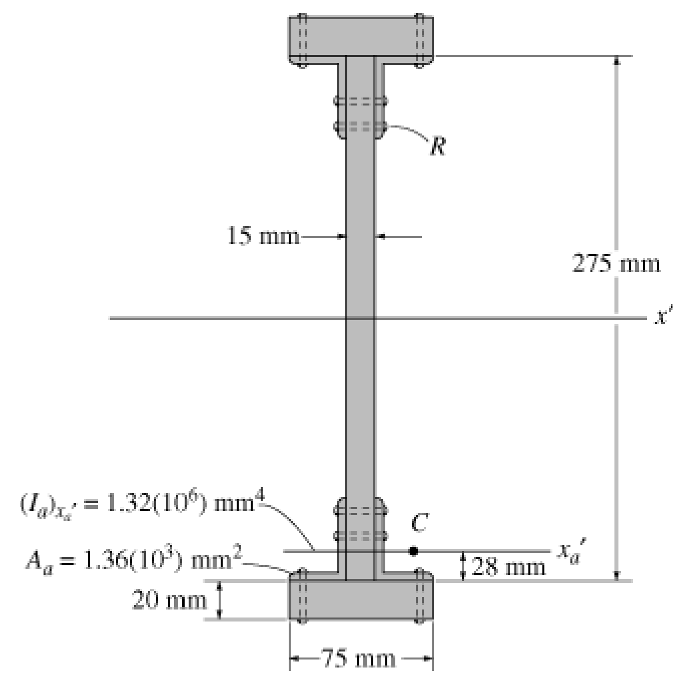
Radius of gyration about the $y$ axis:

$$k_y = \sqrt{\frac{I_y}{A}} \qquad k_y = 59.4 \text{ mm}$$

---

**Problem 10-29**

Determine the moment of inertia for the beam's cross-sectional area with respect to the $x'$ centroidal axis. Neglect the size of all the rivet heads, $R$, for the calculation. Handbook values for the area, moment of inertia, and location of the centroid $C$ of one of the angles are listed in the figure.




Solution:

$$I_E = \frac{1}{12}(15 \text{ mm})(275 \text{ mm})^3 + 4\left[1.32\left(10^6\right)\text{mm}^4 + 1.36\left(10^3\right)\text{mm}^2\left(\frac{275 \text{ mm}}{2} - 28 \text{ mm}\right)^2\right] \ldots$$
$$+ 2\left[\frac{1}{12}(75 \text{ mm})(20 \text{ mm})^3 + (75 \text{ mm})(20 \text{ mm})\left(\frac{275 \text{ mm}}{2} + 10\text{mm}\right)^2\right]$$

$I_E = 162 \times 10^6\ \text{mm}^4$

---

**Problem 10-30**

Locate the centroid $y_c$ of the cross-sectional area for the angle. Then find the moment of inertia $I_{x'}$ about the $x'$ centroidal axis.

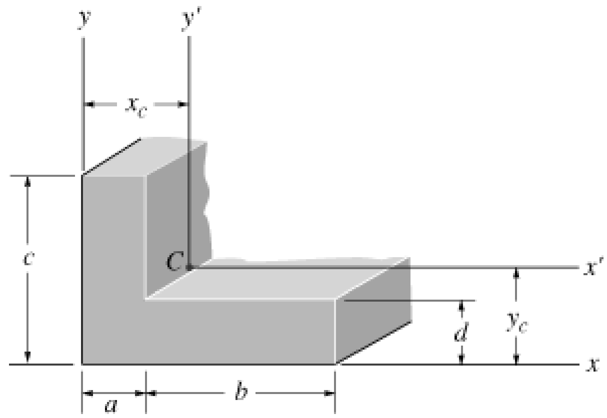
Given:

$a = 2\ \text{in}$

$b = 6\ \text{in}$

$c = 6\ \text{in}$

$d = 2\ \text{in}$




Solution:

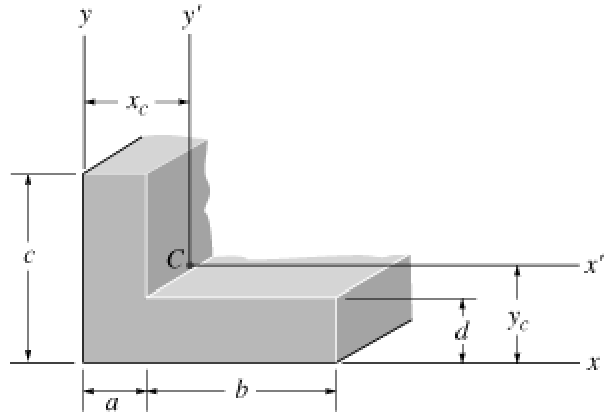
$$y_c = \frac{a c\left(\frac{c}{2}\right) + b d\left(\frac{d}{2}\right)}{a c + b d} \qquad y_c = 2.00\ \text{in}$$

$$I_{x'} = \frac{1}{12} a c^3 + a c\left(\frac{c}{2} - y_c\right)^2 + \frac{1}{12} b d^3 + b d\left(y_c - \frac{d}{2}\right)^2 \qquad I_{x'} = 64.00\ \text{in}^4$$

---

**Problem 10-31**

Locate the centroid $x_c$ of the cross-sectional area for the angle. Then find the moment of inertia $I_{y'}$ about the centroidal $y'$ axis.




Given:

$a = 2$ in

$b = 6$ in

$c = 6$ in

$d = 2$ in

Solution:

$$x_c = \frac{a c\left(\frac{a}{2}\right) + b d\left(a + \frac{b}{2}\right)}{a c + b d}$$

$x_c = 3.00$ in

$$I_{y'} = \frac{1}{12} c a^3 + c a\left(x_c - \frac{a}{2}\right)^2 + \frac{1}{12} d b^3 + d b\left(a + \frac{b}{2} - x_c\right)^2$$

$I_{y'} = 136.00 \text{ in}^4$

---

**Problem 10-32**

Determine the distance $x_c$ to the centroid of the beam's cross-sectional area: then find the moment of inertia about the $y'$ axis.

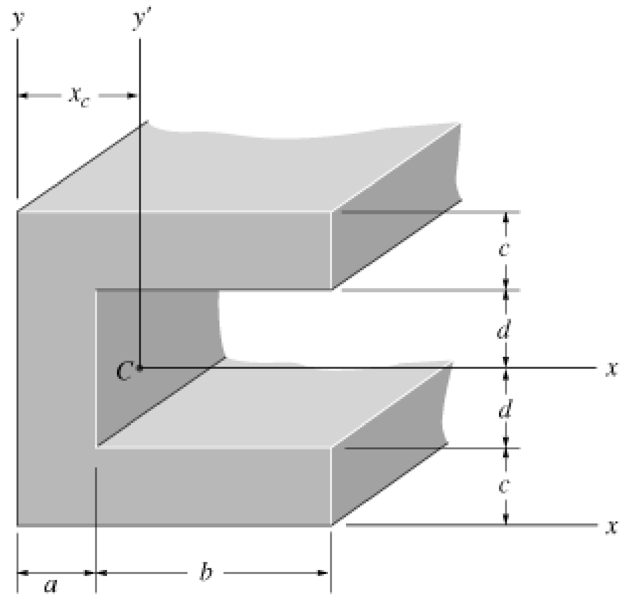
Given:

$a = 40 \text{ mm}$

$b = 120 \text{ mm}$

$c = 40 \text{ mm}$

$d = 40 \text{ mm}$




Solution:

$$x_c = \frac{2(a+b)c\left(\frac{a+b}{2}\right) + 2ad\frac{a}{2}}{2(a+b)c + 2da} \qquad x_c = 68.00 \text{ mm}$$

$$I_{y'} = 2\left[\frac{1}{12}c(a+b)^3 + c(a+b)\left(\frac{a+b}{2} - x_c\right)^2\right] + \frac{1}{12}2da^3 + 2da\left(x_c - \frac{a}{2}\right)^2$$

$$I_{y'} = 36.9 \times 10^6 \text{ mm}^4$$

---

**Problem 10-33**

Determine the moment of inertia of the beam's cross-sectional area about the $x'$ axis.

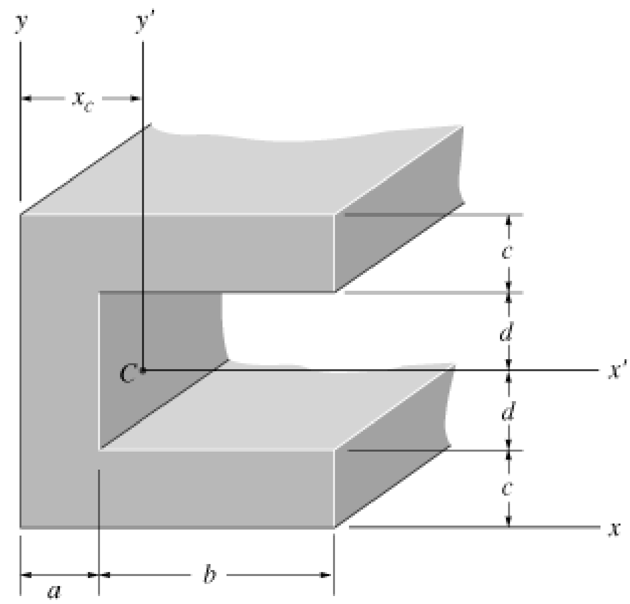
Given:

$a = 40\ \text{mm}$

$b = 120\ \text{mm}$

$c = 40\ \text{mm}$

$d = 40\ \text{mm}$




Solution:

$$I_{x'} = \frac{1}{12}(a+b)(2c+2d)^3 - \frac{1}{12}b(2d)^3$$

$$I_{x'} = 49.5 \times 10^6\ \text{mm}^4$$

---

**Problem 10-34**

Determine the moments of inertia for the shaded area about the *x* and *y* axes.

Given:

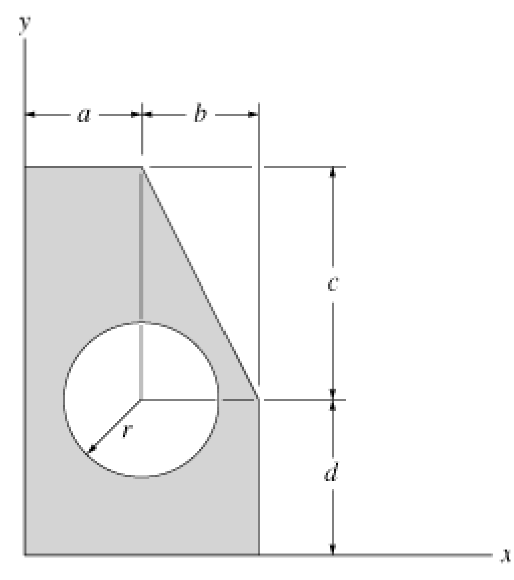
$a = 3\ \text{in}$

$b = 3\ \text{in}$

$c = 6\ \text{in}$

$d = 4\ \text{in}$

$r = 2\ \text{in}$




Solution:

$$I_x = \frac{1}{3}(a+b)(c+d)^3 - \left[\frac{1}{36}bc^3 + \frac{1}{2}bc\left(d+\frac{2c}{3}\right)^2\right] - \left(\frac{\pi r^4}{4} + \pi r^2 d^2\right)$$

$I_x = 1192\,\text{in}^4$

$$I_y = \frac{1}{3}(c+d)(a+b)^3 - \left[\frac{1}{36}cb^3 + \frac{1}{2}bc\left(a + \frac{2b}{3}\right)^2\right] - \left(\frac{\pi r^4}{4} + \pi r^2 a^2\right)$$

$I_y = 364.84\,\text{in}^4$

---

**Problem 10-35**

Determine the location of the centroid $y'$ of the beam constructed from the two channels and the cover plate. If each channel has a cross-sectional area $A_c$ and a moment of inertia about a horizontal axis passing through its own centroid $C_c$, of $I_{x'c}$, determine the moment of inertia of the beam's cross-sectional area about the $x'$ axis.

Given:

$a = 18$ in

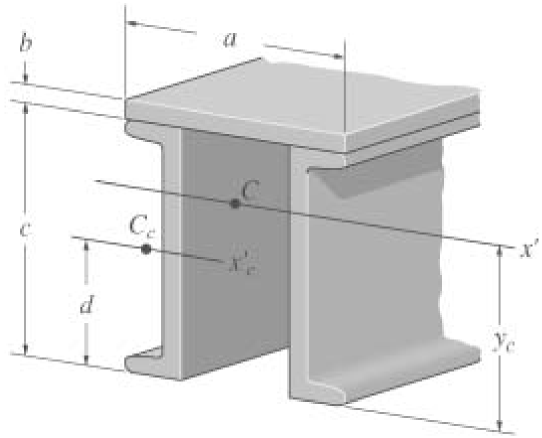
$b = 1.5$ in

$c = 20$ in

$d = 10$ in

$A_c = 11.8\ \text{in}^2$

$I_{x'c} = 349\ \text{in}^4$




Solution:

$$y_c = \frac{2A_c d + ab\left(c + \frac{b}{2}\right)}{2A_c + ab}$$

$y_c = 15.74\,\text{in}$

$$I_{x'} = \left[I_{x'c} + A_c\left(y_c - d\right)^2\right]2 + \frac{1}{12}ab^3 + ab\left(c + \frac{b}{2} - y_c\right)^2$$

$I_{x'} = 2158\,\text{in}^4$

---

**Problem 10-36**

Compute the moments of inertia $I_x$ and $I_y$ for the beam's cross-sectional area about the *x* and *y* axes.

Given:

$a = 30\ \text{mm}$

$b = 170\ \text{mm}$

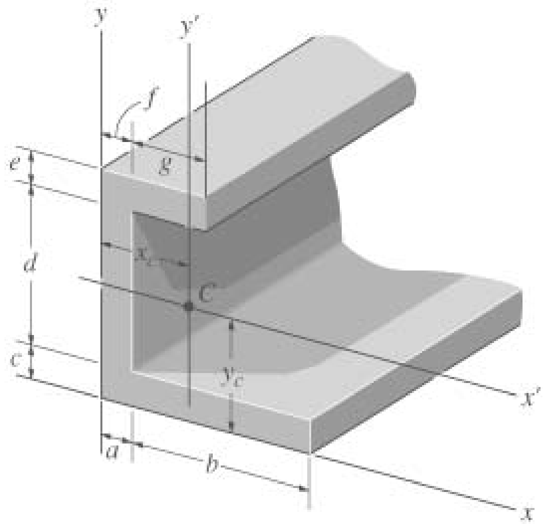
$c = 30\ \text{mm}$

$d = 140\ \text{mm}$

$e = 30\ \text{mm}$

$f = 30\ \text{mm}$

$g = 70\ \text{mm}$




Solution:

$$I_x = \frac{1}{3}a(c+d+e)^3 + \frac{1}{3}bc^3 + \frac{1}{12}ge^3 + ge\left(c+d+\frac{e}{2}\right)^2 \qquad I_x = 154 \times 10^6\ \text{mm}^4$$

$$I_y = \frac{1}{3}c(a+b)^3 + \frac{1}{3}df^3 + \frac{1}{3}c(f+g)^3 \qquad I_y = 91.3 \times 10^6\ \text{mm}^4$$

---

**Problem 10-37**

Determine the distance $y_c$ to the centroid $C$ of the beam's cross-sectional area and then compute the moment of inertia $I_{cx'}$ about the $x'$ axis.

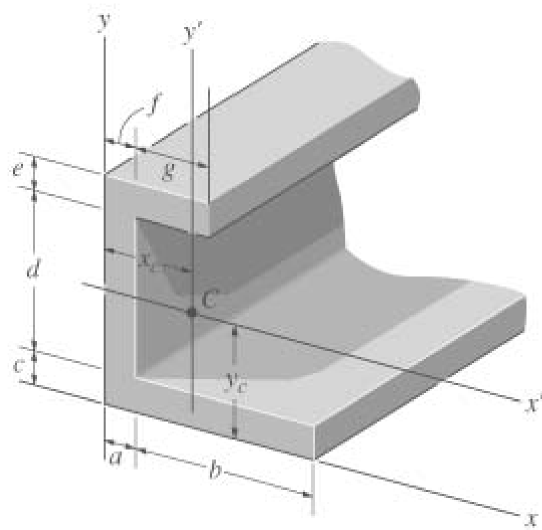
Given:

$a = 30\ \text{mm}$ $\quad e = 30\ \text{mm}$

$b = 170\ \text{mm}$ $\quad f = 30\ \text{mm}$

$c = 30\ \text{mm}$ $\quad g = 70\ \text{mm}$

$d = 140\ \text{mm}$



Solution:

$$y_C = \frac{(a+b)c\left(\frac{c}{2}\right) + d\,f\left(c + \frac{d}{2}\right) + (f+g)e\left(c + d + \frac{e}{2}\right)}{(a+b)c + d\,f + (f+g)e}$$

$$y_C = 80.7\,\text{mm}$$

$$I_{x'} = \frac{1}{12}(a+b)\,c^3 + (a+b)c\left(y_C - \frac{c}{2}\right)^2 + \frac{1}{12}f d^3 + f d\left(c + \frac{d}{2} - y_C\right)^2 \ ...$$
$$+ \frac{1}{12}(f+g)\,e^3 + (f+g)e\left(c + d + \frac{e}{2} - y_C\right)^2$$

$$I_{x'} = 67.6 \times 10^6\,\text{mm}^4$$

---

**Problem 10-38**

Determine the distance $x_c$ to the centroid $C$ of the beam's cross-sectional area and then compute the moment of inertia $I_{y'}$ about the $y'$ axis.

Given:

$a = 30$ mm

$b = 170$ mm

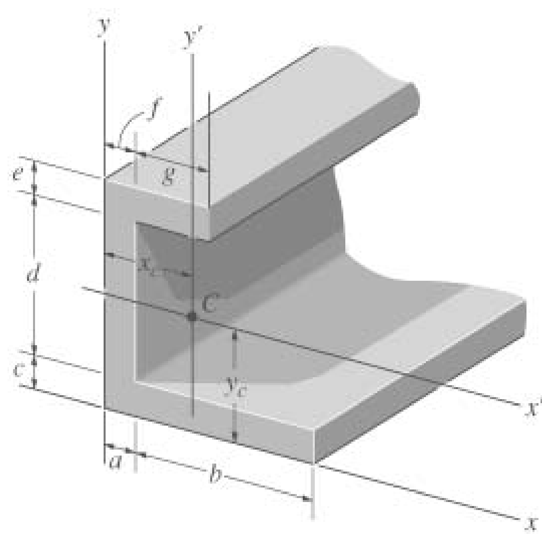
$c = 30$ mm

$d = 140$ mm

$e = 30$ mm

$f = 30$ mm

$g = 70$ mm




Solution:

$$x_C = \frac{b\,c\left(\frac{b}{2} + a\right) + (c+d)\,f\left(\frac{f}{2}\right) + (f+g)e\,\frac{f+g}{2}}{b\,c + b\,c + (f+g)e}$$

$$x_C = 61.6\,\text{mm}$$

$$I_{y'} = \frac{1}{12}c(a+b)^3 + c(a+b)\left(\frac{a+b}{2} - x_c\right)^2 + \frac{1}{12}d f^3 + d f\left(x_c - \frac{f}{2}\right)^2 \ ...$$

$$+ \frac{1}{12}e(f+g)^3 + e(f+g)\left(x_c - \frac{f+g}{2}\right)^2$$

$$I_{y'} = 41.2 \times 10^6 \, \text{mm}^4$$

---

**Problem 10-39**

Determine the location $y_c$ of the centroid $C$ of the beam's cross-sectional area. Then compute the moment of inertia of the area about the $x'$ axis

Given:

$a = 20$ mm

$b = 125$ mm

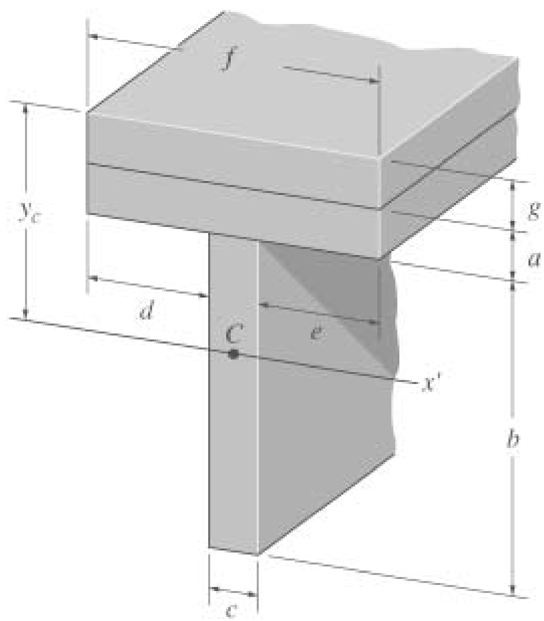
$c = 20$ mm

$f = 120$ mm

$g = 20$ mm

$d = \frac{f-c}{2}$

$e = \frac{f-c}{2}$




Solution:

$$y_c = \frac{(a+g) f\left(\frac{a+g}{2}\right) + c b\left(a+g+\frac{b}{2}\right)}{(a+g) f + c b}$$

$$y_c = 48.25 \, \text{mm}$$

$$I_{x'} = \frac{1}{12} f(a+g)^3 + (f)(a+g)\left(y_c - \frac{a+g}{2}\right)^2 + \frac{1}{12}c b^3 + c b\left(\frac{b}{2} + a + g - y_c\right)^2$$

$$I_{x'} = 15.1 \times 10^6 \, \text{mm}^4$$

---

**Problem 10-40**

Determine $y_c$, which locates the centroidal axis $x'$ for the cross-sectional area of the T-beam, and then find the moments of inertia $I_{x'}$ and $I_{y'}$.

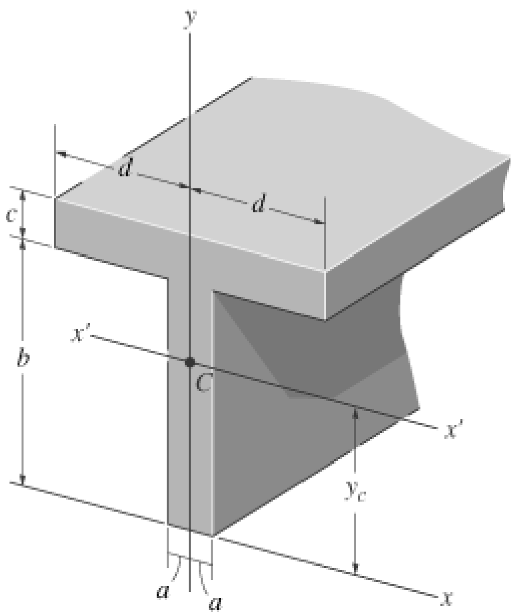
Given:

$a = 25\ \text{mm}$

$b = 250\ \text{mm}$

$c = 50\ \text{mm}$

$d = 150\ \text{mm}$




Solutuion:

$$y_C = \frac{\left(\frac{b}{2}\right)b2a + \left(b + \frac{c}{2}\right)2dc}{b2a + c2d}$$

$$y_C = 207\,\text{mm}$$

$$I_{x'} = \frac{1}{12}2ab^3 + 2ab\left(y_C - \frac{b}{2}\right)^2 + \frac{1}{12}2dc^3 + c2d\left(b + \frac{c}{2} - y_C\right)^2$$

$$I_{x'} = 222 \times 10^6\,\text{mm}^4$$

$$I_{y'} = \frac{1}{12}b(2a)^3 + \frac{1}{12}c(2d)^3$$

$$I_{y'} = 115 \times 10^6\,\text{mm}^4$$

---

**Problem 10-41**

Determine the centroid $y'$ for the beam's cross-sectional area; then find $I_{x'}$.

Given:

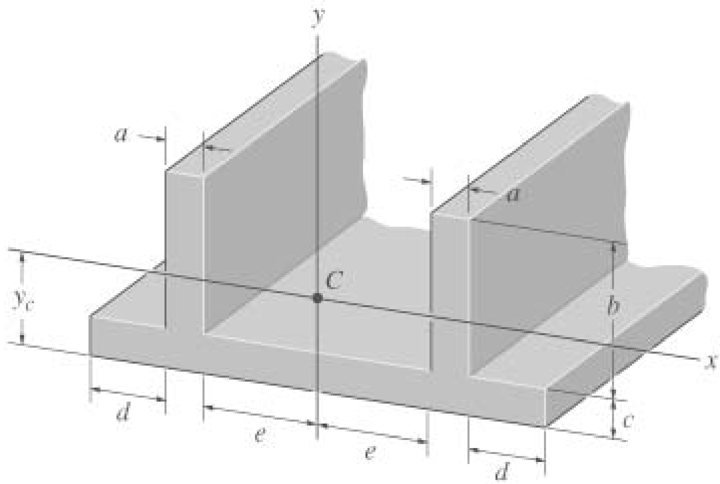
$a = 25\ \text{mm}$

$b = 100\ \text{mm}$

$c = 25\ \text{mm}$

$d = 50\ \text{mm}$

$e = 75\ \text{mm}$




Solution:

$$y_c = \frac{2(a+e+d)c\left(\frac{c}{2}\right) + 2ab\left(c+\frac{b}{2}\right)}{2(a+e+d)c + 2ab}$$

$y_c = 37.50\ \text{mm}$

$$I_{x'} = \frac{2}{12}(a+e+d)\,c^3 + 2(a+e+d)c\left(y_c - \frac{c}{2}\right)^2 \ ...$$
$$+ 2\left[\frac{1}{12}ab^3 + ab\left(c+\frac{b}{2}-y_c\right)^2\right]$$

$I_{x'} = 16.3 \times 10^6\ \text{mm}^4$

---

**Problem 10-42**

Determine the moment of inertia for the beam's cross-sectional area about the *y* axis.

Given:

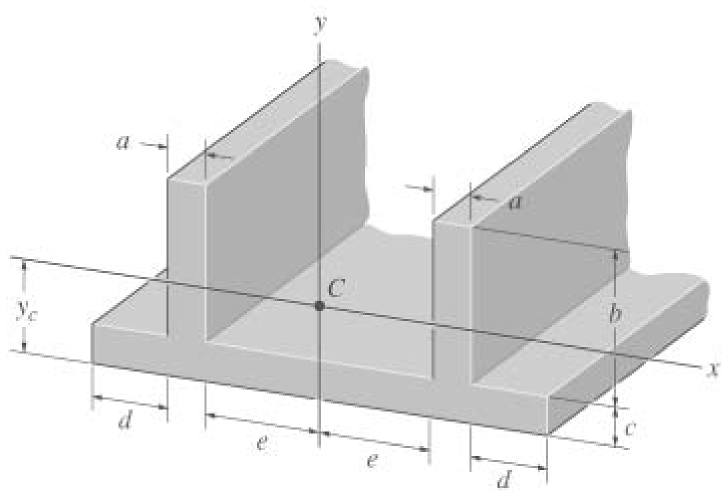
$a = 25\ \text{mm}$

$b = 100\ \text{mm}$

$c = 25\ \text{mm}$

$d = 50\ \text{mm}$

$e = 75\ \text{mm}$



Solution:

$$I_y = \frac{1}{12}2^3(a+d+e)^3c + 2\left[\frac{1}{12}ba^3 + ab\left(e+\frac{a}{2}\right)^2\right]$$

$$I_y = 94.8 \times 10^6\ \text{mm}^4$$

---

**Problem 10-43**

Determine the moment for inertia $I_x$ of the shaded area about the $x$ axis.

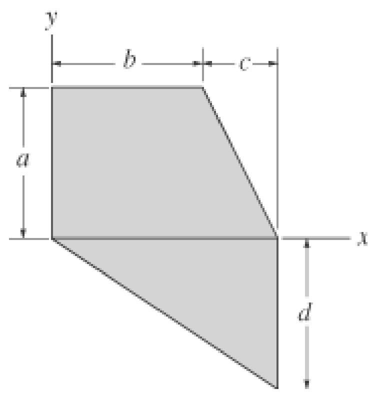
Given:

$a = 6$ in

$b = 6$ in

$c = 3$ in

$d = 6$ in




Solution:

$$I_x = \frac{ba^3}{3} + \frac{1}{12}ca^3 + \frac{1}{12}(b+c)d^3 \qquad I_x = 648\ \text{in}^4$$

---

**Problem 10-44**

Determine the moment for inertia $I_y$ of the shaded area about the $y$ axis.

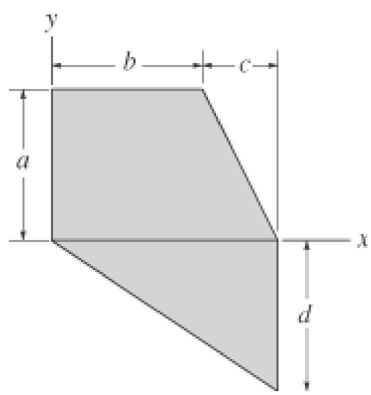
Given:

$a = 6$ in

$b = 6$ in

$c = 3$ in

$d = 6$ in



Solution:

$$I_y = \frac{a\,b^3}{3} + \frac{1}{36}a\,c^3 + \frac{1}{2}a\,c\left(b + \frac{c}{3}\right)^2 + \frac{1}{36}d\,(b + c)^3 + \frac{1}{2}d(b + c)\left[\frac{2(b + c)}{3}\right]^2$$

$$I_y = 1971\,\text{in}^4$$

**Problem 10-45**

Locate the centroid $y_c$ of the channel's cross-sectional area, and then determine the moment of inertia with respect to the $x'$ axis passing through the centroid.

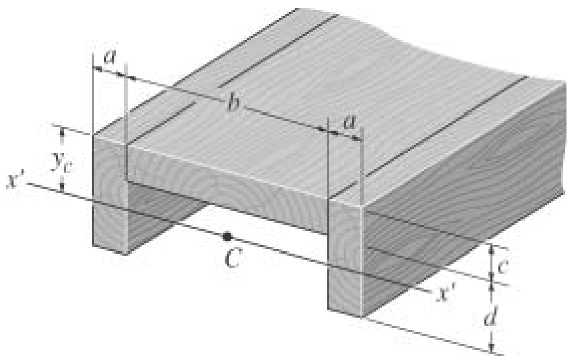
Given:

$a = 2$ in

$b = 12$ in

$c = 2$ in

$d = 4$ in




Solution:

$$y_c = \frac{\frac{c}{2}b\,c + 2\left(\frac{c + d}{2}\right)(c + d)\,a}{b\,c + 2(c + d)\,a}$$

$$y_c = 2\,\text{in}$$

$$I_x = \frac{1}{12}b\,c^3 + b\,c\left(y_c - \frac{c}{2}\right)^2 + \frac{2}{12}a\,(c + d)^3 + 2a(c + d)\left(\frac{c + d}{2} - y_c\right)^2$$

$$I_x = 128\,\text{in}^4$$

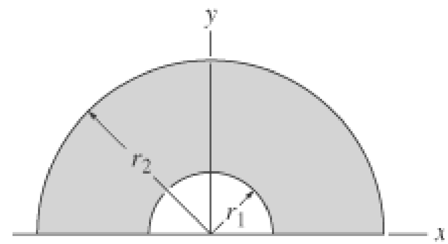
**Problem 10-46**

Determine the moments for inertia $I_x$ and $I_y$ of the shaded area.

Given:

$r_1 = 2$ in

$r_2 = 6\ \text{in}$




Solution:

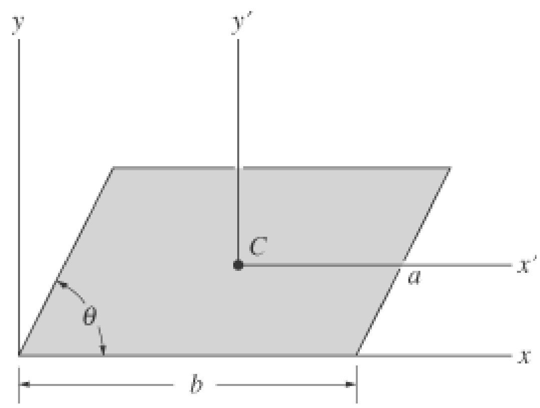
$$I_x = \left(\frac{\pi r_2^{\ 4}}{8} - \frac{\pi r_1^{\ 4}}{8}\right) \qquad I_x = 503\,\text{in}^4$$

$$I_y = \left(\frac{\pi r_2^{\ 4}}{8} - \frac{\pi r_1^{\ 4}}{8}\right) \qquad I_y = 503\,\text{in}^4$$

---

**Problem 10-47**

Determine the moment of inertia for the parallelogram about the $x'$ axis, which passes through the centroid $C$ of the area.




Solution:

$$h = (a)\sin(\theta)$$

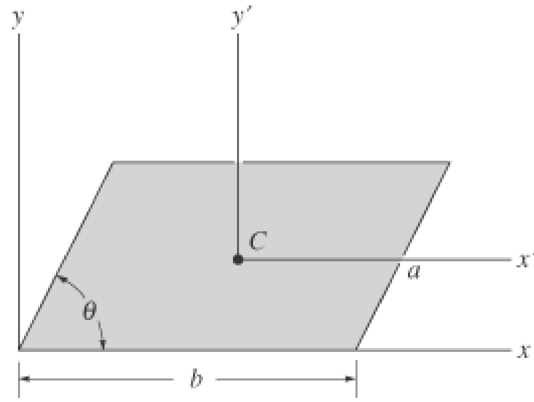
$$I_{xc} = \frac{1}{12}bh^3 = \frac{1}{12}b[(a)\sin(\theta)]^3 = \frac{1}{12}a^3b\sin(\theta)^3$$

$$I_{xc} = \frac{1}{12}a^3b\sin(\theta)^3$$

---

**Problem 10-48**

Determine the moment of inertia for the parallelogram about the $y'$ axis, which passes through the centroid $C$ of the area.




Solution:

$$A = b(a)\sin(\theta)$$

$$x_c = \frac{1}{b(a)\sin(\theta)}\left[\left[b(a)\sin(\theta)\frac{b}{2} - \frac{1}{2}(a)\cos(\theta)(a)\sin(\theta)\frac{(a)\cos(\theta)}{3}\right] \dots \atop + \frac{1}{2}(a)\cos(\theta)(a)\sin(\theta)\left[b + \frac{(a)\cos(\theta)}{3}\right]\right] = \frac{b + (a)\cos(\theta)}{2}$$

$$I_{y'} = \frac{1}{12}(a)\sin(\theta)b^3 + (a)\sin(\theta)b\left(\frac{b}{2} - x_c\right)^2 \dots$$
$$+ -\left[\frac{1}{36}(a)\sin(\theta)\left[(a)\cos(\theta)\right]^3 + \frac{1}{2}(a)\sin(\theta)(a)\cos(\theta)\left[x_c - \frac{(a)\cos(\theta)}{3}\right]^2\right] \dots$$
$$+ \frac{1}{36}(a)\sin(\theta)\left[(a)\cos(\theta)\right]^3 + \frac{1}{2}(a)\sin(\theta)(a)\cos(\theta)\left[b + \frac{(a)\cos(\theta)}{3} - x_c\right]^2$$

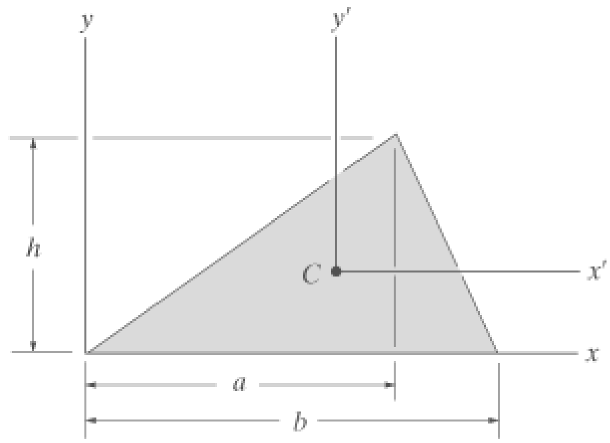
Simplifying we find.

$$I_{y'} = \frac{a\,b}{12}\left(b^2 + a^2\cos(\theta)^2\right)\sin(\theta)$$

---

**Problem 10-49**

Determine the moments of inertia for the triangular area about the $x'$ and $y'$ axes, which pass through the centroid $C$ of the area.




Solution:

$$I_{x'} = \frac{1}{36} b h^3$$

$$x_c = \frac{\frac{2}{3}a\frac{1}{2}ha + \left(a + \frac{b-a}{3}\right)\frac{1}{2}h(b-a)}{\frac{1}{2}ha + \frac{1}{2}h(b-a)} = \frac{b+a}{3}$$

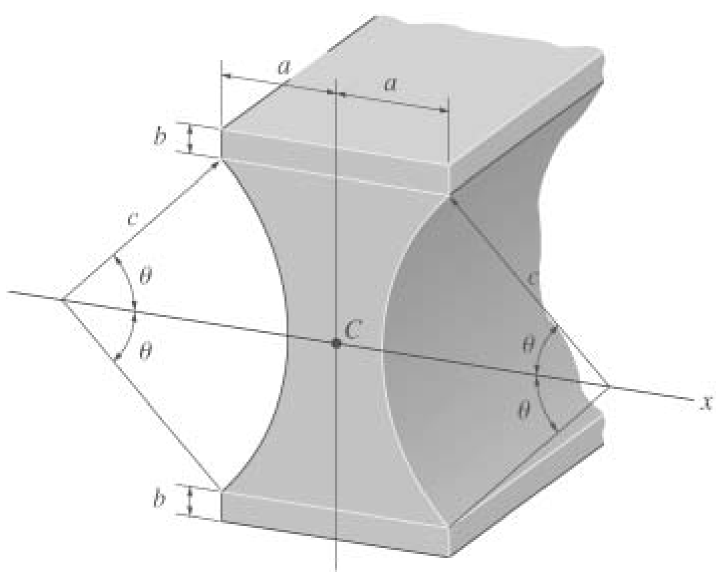
$$I_{y'} = \frac{1}{36}ha^3 + \frac{1}{2}ha\left(\frac{b+a}{3} - \frac{2}{3}a\right)^2 + \frac{1}{36}h(b-a)^3 + \frac{1}{2}h(b-a)\left(a + \frac{b-a}{3} - \frac{b+a}{3}\right)^2$$

$$I_{y'} = \frac{1}{36}hb\left(b^2 - ab + a^2\right)$$

---

**Problem 10-50**

Determine the moment of inertia for the beam's cross-sectional area about the $x'$ axis passing through the centroid $C$ of the cross section.




Given:

$a = 100$ mm

$b = 25$ mm

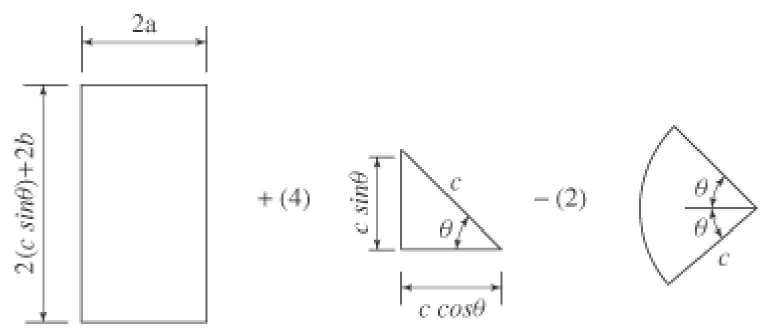
$c = 200$ mm

$\theta = 45$ deg

Solution:

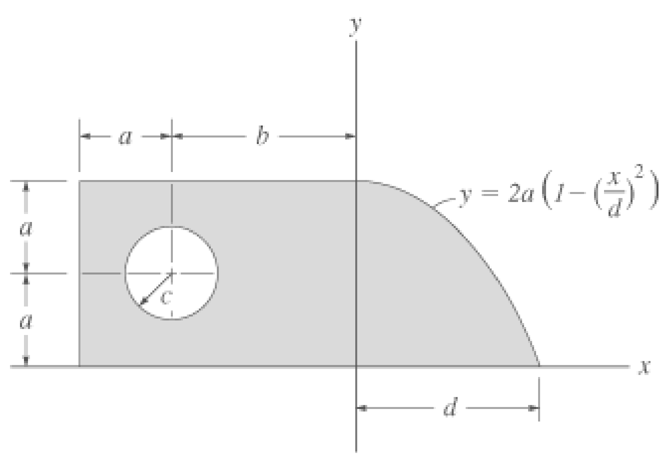
$$I_{x'} = \frac{1}{12}\left[2a\left[2\left(c\sin(\theta) + b\right)\right]^3\right] \dots$$
$$+ 4\left[\frac{1}{12}\left(c\cos(\theta)\right)\left(c\sin(\theta)\right)^3\right] - 2\left[\frac{1}{4}c^4\left(\theta - \frac{1}{2}\sin(2\theta)\right)\right]$$

$I_{x'} = 520 \times 10^6\ \text{mm}^4$




**Problem 10-51**

Determine the moment of inertia of the composite area about the $x$ axis.




Given:

$a = 2\ \text{in}$

$b = 4\ \text{in}$

$c = 1\ \text{in}$

$d = 4\ \text{in}$

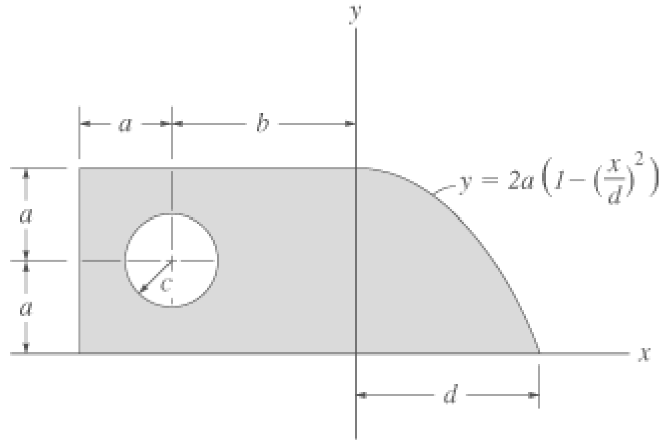
Solution:

$$I_x = \frac{1}{3}(a+b)\,(2a)^3 - \left(\frac{\pi c^4}{4} + \pi c^2\,a^2\right) + \int_0^d \frac{1}{3}\left[2a\left[1 - \left(\frac{x}{d}\right)^2\right]\right]^3 \mathrm{d}x$$

$I_x = 153.7\ \text{in}^4$

**Problem 10-52**

Determine the moment of inertia of the composite area about the $y$ axis.




Given:

$a = 2\ \text{in}$

$b = 4\ \text{in}$

$c = 1\ \text{in}$

$d = 4\ \text{in}$

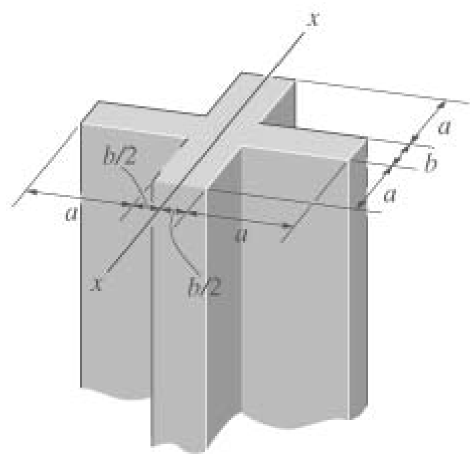
Solution:

$$I_y = \frac{1}{3}(2a)(a+b)^3 - \left(\frac{\pi c^4}{4} + \pi c^2 b^2\right) + \int_0^d x^2 2a\left[1 - \left(\frac{x}{d}\right)^2\right] dx$$

$$I_y = 271.1\ \text{in}^4$$

**Problem 10-53**

Determine the radius of gyration $k_x$ for the column's cross-sectional area.




Given:

$a = 200\ \text{mm}$

$b = 100\ \text{mm}$

Solution:

$$I_x = \frac{1}{12}(2a+b)b^3 + 2\left[\frac{1}{12}b a^3 + b a\left(\frac{a}{2} + \frac{b}{2}\right)^2\right]$$

$$k_x = \sqrt{\frac{I_x}{b(2a+b) + 2ab}} \qquad k_x = 109\,\text{mm}$$

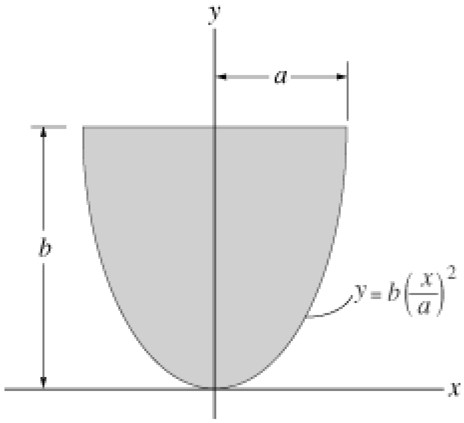
**Problem 10-54**

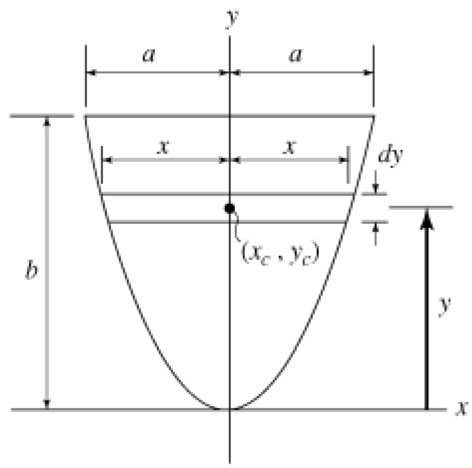
Determine the product of inertia for the shaded portion of the parabola with respect to the *x* and *y* axes.

Given:

$a = 2$ in

$b = 1$ in



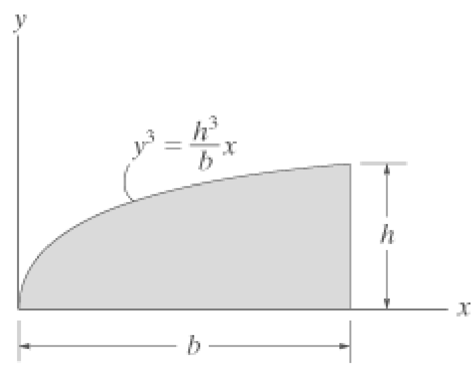




$$I_{xy} = \int_{-a}^{a}\int_{b\left(\frac{x}{a}\right)^2}^{b} x\,y\,dy\,dx \qquad I_{xy} = 0.00\ \text{m}^4$$

Also because the area is symmetric about the *y* axis, the product of inertia must be zero.

**Problem 10-55**

Determine the product of inertia for the shaded area with respect to the *x* and *y* axes.




Solution:

$$I_{xy} = \int_{0}^{b}\int_{0}^{h\left(\frac{x}{b}\right)^{\frac{1}{3}}} x\,y\,dy\,dx = \frac{3}{16}\ b^2\ h^2 \qquad I_{xy} = \frac{3}{16}b^2\,h^2$$

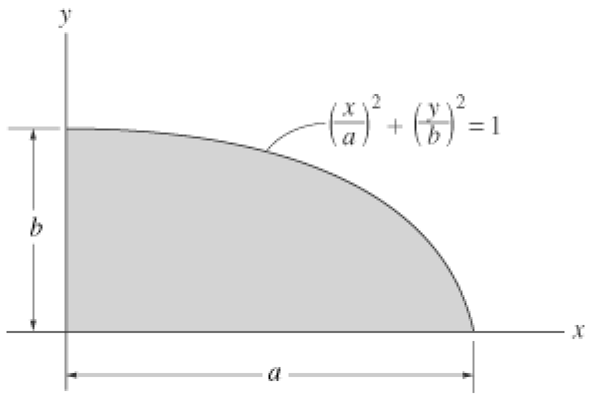
**Problem 10-56**

Determine the product of inertia of the shaded area of the ellipse with respect to the $x$ and $y$ axes.

Given:

$a = 4\text{ in}$

$b = 2\text{ in}$



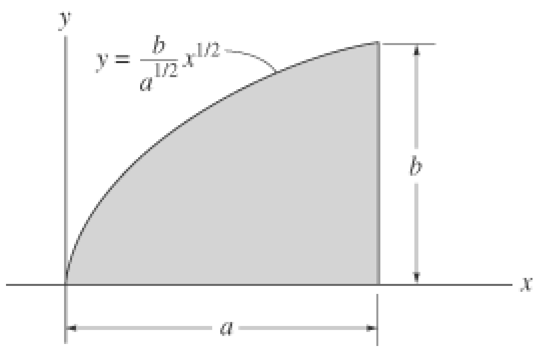

Solution:

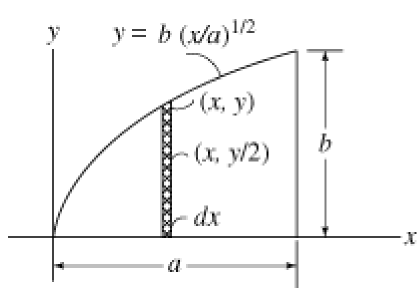
$$I_{xy} = \int_0^a x\left[\frac{b\sqrt{1-\left(\frac{x}{a}\right)^2}}{2}\right] b\sqrt{1-\left(\frac{x}{a}\right)^2}\, dx \qquad I_{xy} = 8.00\text{ in}^4$$

---

**Problem 10-57**

Determine the product of inertia of the parabolic area with respect to the $x$ and $y$ axes.



Solution:

$$I_{xy} = \int_0^a x\left(\frac{b\sqrt{\frac{x}{a}}}{2}\right)b\sqrt{\frac{x}{a}}\,dx = \frac{1}{6}a^3\frac{b^2}{a} \qquad I_{xy} = \frac{1}{6}a^2b^2$$

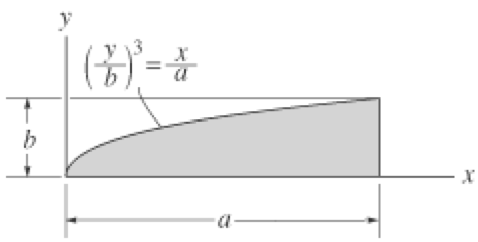
**Problem 10-58**

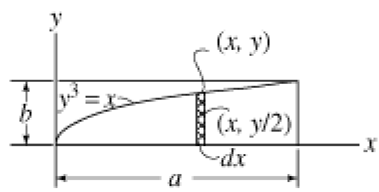
Determine the product of inertia for the shaded area with respect to the $x$ and $y$ axes.

Given:

$a = 8$ in

$b = 2$ in




Solution:

$$I_{xy} = \int_0^a x\frac{b\left(\frac{x}{a}\right)^{\frac{1}{3}}}{2}b\left(\frac{x}{a}\right)^{\frac{1}{3}}\,dx \qquad I_{xy} = 48.00\,\text{in}^4$$

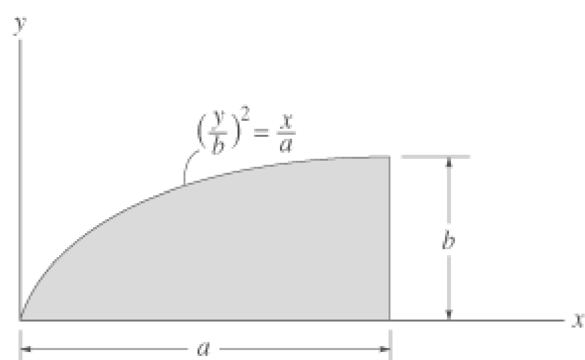
**Problem 10-59**

Determine the product of inertia for the shaded parabolic area with respect to the $x$ and $y$ axes.

Given:

$a = 4$ in

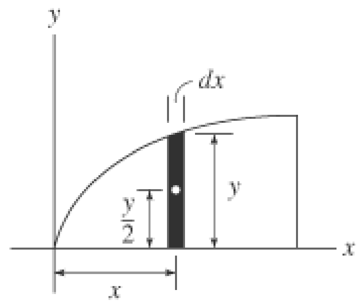
$b = 2$ in




Solution:

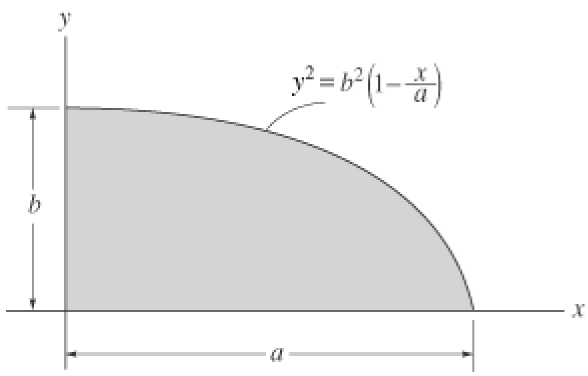
$$I_{xy} = \int_0^a x\frac{b}{2}\sqrt{\frac{x}{a}}\,b\sqrt{\frac{x}{a}}\,dx$$

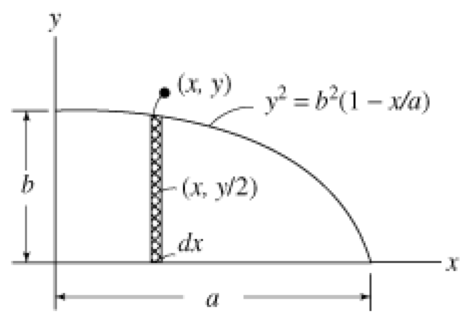
$I_{xy} = 10.67\ \text{in}^4$




**Problem 10-60**

Determine the product of inertia for the shaded area with respect to the $x$ and $y$ axes.




Given:

$a = 2\ \text{m}$

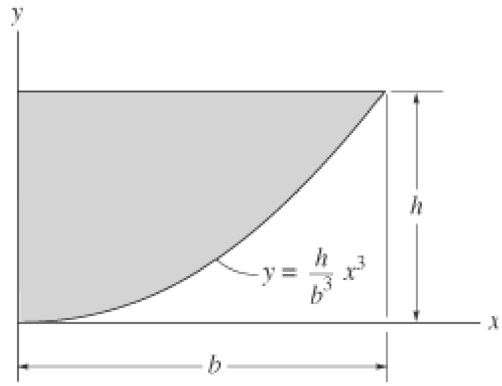
$b = 1\ \text{m}$

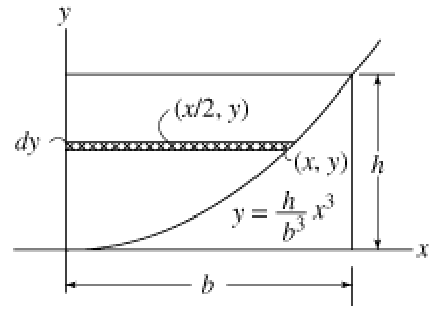
Solution:

$$I_{xy} = \int_0^a x\left(\frac{b}{2}\sqrt{1 - \frac{x}{a}}\right) b\sqrt{1 - \frac{x}{a}}\ dx \qquad I_{xy} = 0.333\ \text{m}^4$$

**Problem 10-61**

Determine the product of inertia for the shaded area with respect to the $x$ and $y$ axes.




Solution:

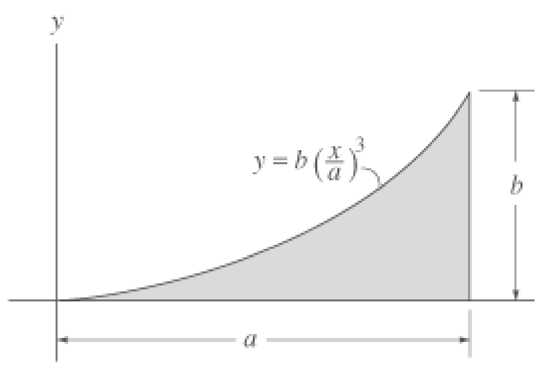
$$I_{xy} = \int_0^h y\frac{1}{2}\left[b\left(\frac{y}{h}\right)^{\frac{1}{3}}\right]^2 \mathrm{d}y = \frac{3}{16}\ b^2\ h^2$$

$$I_{xy} = \frac{3}{16}h^2 b^2$$

---

**Problem 10-62**

Determine the product of inertia of the shaded area with respect to the *x* and *y* axes.




Given:

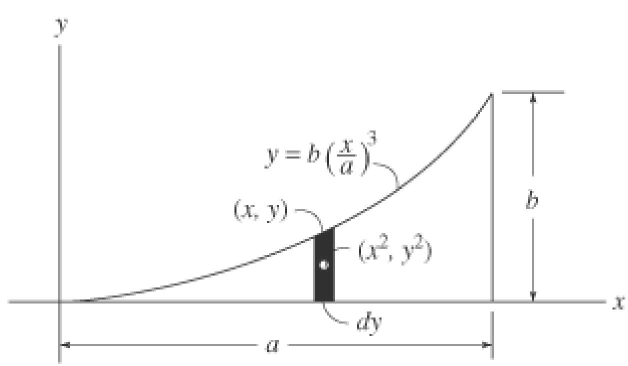
$a = 4\ \text{in}$

$b = 2\ \text{in}$

Solution:

$$I_{xy} = \int_0^a x\left(\frac{b}{2}\right)\left(\frac{x}{a}\right)^3 b\left(\frac{x}{a}\right)^3 \mathrm{d}x$$

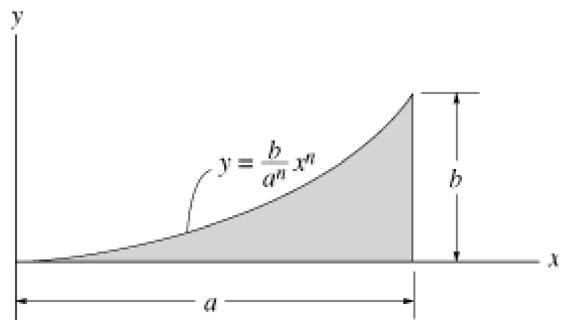
$$I_{xy} = 4.00\ \text{in}^4$$




---

**Problem 10-63**

Determine the product of inertia for the shaded area with respect to the $x$ and $y$ axes.

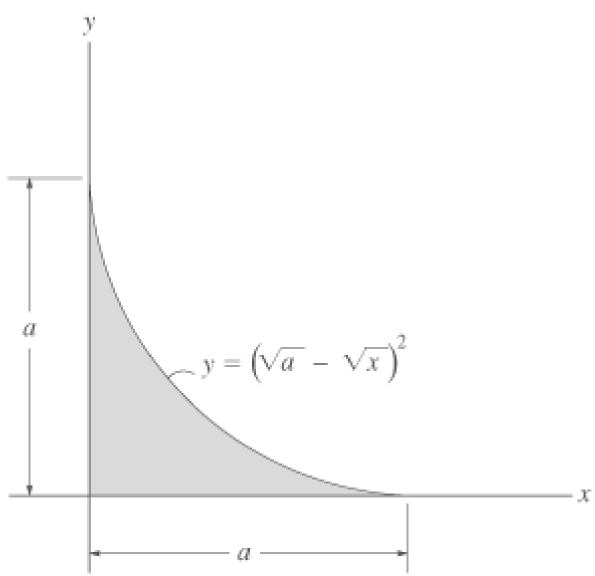



Solution:

$$I_{xy} = \int_0^a x\left(\frac{b}{2}\frac{x^n}{a^n}\right)b\frac{x^n}{a^n}\,dx \qquad I_x = \frac{a^2 b^2}{4(n+1)} \qquad \text{provided } n \neq -1$$

**Problem 10-64**

Determine the product of inertia for the shaded area with respect to the $x$ and $y$ axes.




Given:

$a = 4$ ft

Solution:

$$I_{xy} = \int_0^a x\frac{\left(\sqrt{a}-\sqrt{x}\right)^2}{2}\left(\sqrt{a}-\sqrt{x}\right)^2\,dx$$

$I_{xy} = 0.91 \text{ ft}^4$

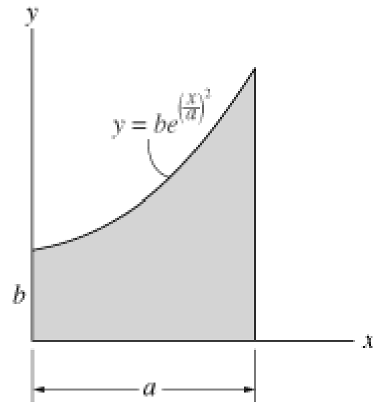
**Problem 10-65**

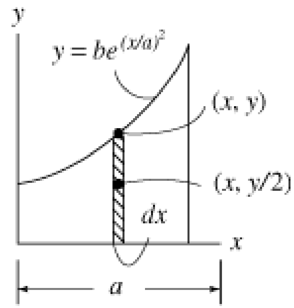
Determine the product of inertia for the shaded area with respect to the *x* and *y* axes. Use Simpson's rule to evaluate the integral.

Given:

$a = 1\ \text{m}$

$b = 0.8\ \text{m}$



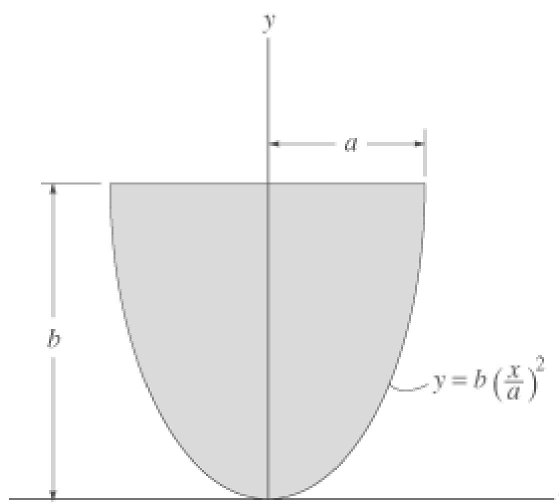

Solution:

$$I_{xy} = \int_0^a x\left(\frac{b}{2}\right)e^{\left(\frac{x}{a}\right)^2} b\,e^{\left(\frac{x}{a}\right)^2}\,dx \qquad I_{xy} =$$

---

**Problem 10-66**

Determine the product of inertia for the parabolic area with respect to the *x* and *y* axes.




Given:

$a = 1\ \text{in}$

$b = 2\ \text{in}$

Solution:

Due to symmetry about *y* axis

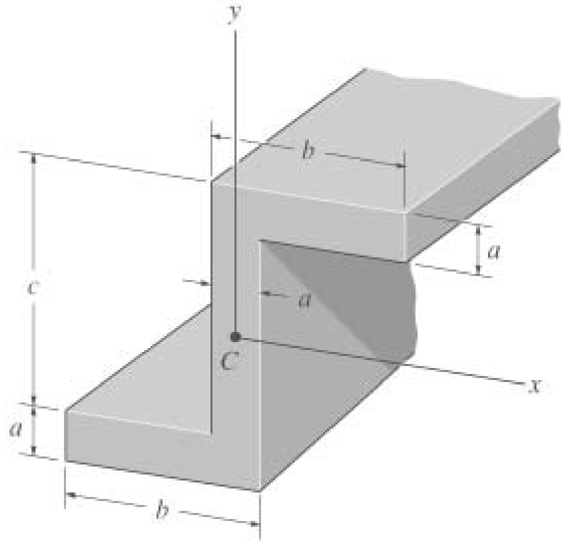
$I_{xy} = 0$

$$I_{xy} = \int_{-a}^{a} x\,\frac{b + b\frac{x^2}{a^2}}{2}\left(b - b\frac{x^2}{a^2}\right)dx \qquad I_{xy} = 0.00\ \text{m}^4$$

---

**Problem 10-67**

Determine the product of inertia for the cross-sectional area with respect to the *x* and *y* axes that have their origin located at the centroid *C*.




Given:

$a = 20\ \text{mm}$

$b = 80\ \text{mm}$

$c = 100\ \text{mm}$

Solution:

$$I_{xy} = 2b\,a\frac{c}{2}\left(\frac{b}{2} - \frac{a}{2}\right)$$

$$I_{xy} = 4800000.00\,\text{mm}^4$$

---

**Problem 10-68**

Determine the product of inertia for the beam's cross-sectional area with respect to the *x* and *y* axes.

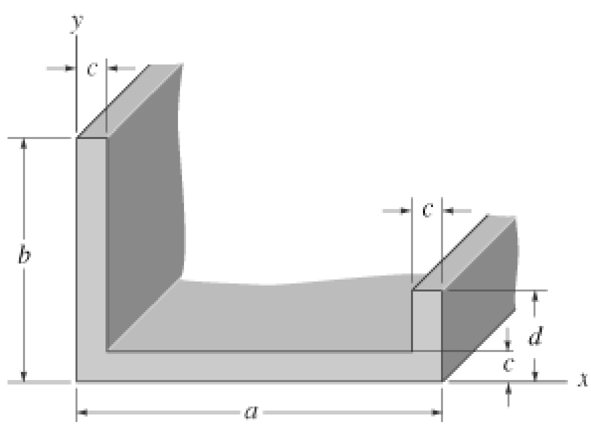
Given:

$a = 12\ \text{in}$

$b = 8\ \text{in}$

$c = 1\ \text{in}$

$d = 3\ \text{in}$



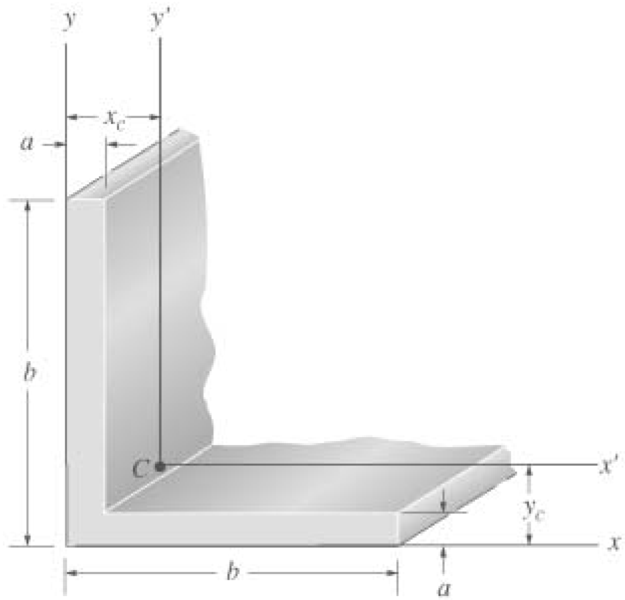

Solution:

$$I_{xy} = \left(\frac{c}{2}\right)\left(\frac{b}{2}\right)c\,b + \left(\frac{a}{2}\right)\left(\frac{c}{2}\right)(a - 2c)c + d\,c\left(a - \frac{c}{2}\right)\left(\frac{d}{2}\right) \qquad I_{xy} = 97.75\,\text{in}^4$$

---

**Problem 10-69**

Determine the location $(x_c, y_c)$ of the centroid *C* of the angle's cross-sectional area, and then compute the product of inertia with respect to the $x'$ and $y'$ axes.




Given:

$a = 18$ mm

$b = 150$ mm

Solution:

$$x_c = \frac{\left(\frac{a}{2}\right)a\,b + a(b-a)\left(\frac{a+b}{2}\right)}{a\,b + a(b-a)}$$

$x_c = 44.1\,\text{mm}$

$$y_c = \frac{\left(\frac{b}{2}\right)a\,b + \left(\frac{a}{2}\right)a(b-a)}{a\,b + a(b-a)}$$

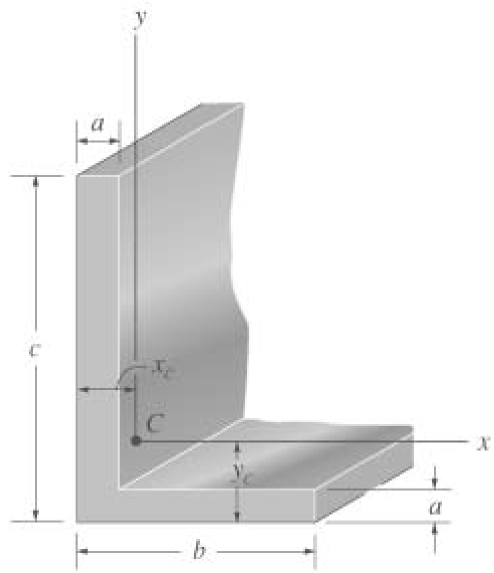
$y_c = 44.1\,\text{mm}$

$$I_{x'y'} = a\,b \cdot -\left(x_c - \frac{a}{2}\right)\left(\frac{b}{2} - y_c\right) + a(b-a) \cdot -\left(y_c - \frac{a}{2}\right)\left(\frac{b}{2} + \frac{a}{2} - x_c\right)$$

$I_{x'y'} = -6.26 \times 10^6\,\text{mm}^4$

**Problem 10-70**

Determine the product of inertia of the beam's cross-sectional area with respect to the *x* and *y* axes that have their origin located at the centroid *C*.




Given:

$a = 5\ \text{mm}$

$b = 30\ \text{mm}$

$c = 50\ \text{mm}$

Solution:

$$x_c = \frac{a(b-a)\left(\frac{a+b}{2}\right) + c\,a\left(\frac{a}{2}\right)}{a(b-a) + a\,c}$$

$x_c = 7.50\,\text{mm}$

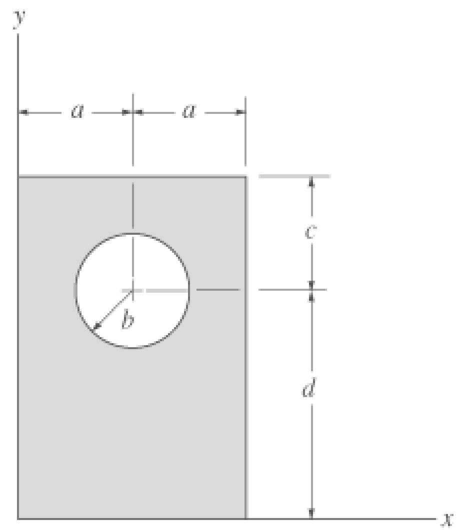
$$y_c = \frac{a(b-a)\left(\frac{a}{2}\right) + c\,a\left(\frac{c}{2}\right)}{a(b-a) + c\,a}$$

$y_c = 17.50\,\text{mm}$

$$I_{xy} = (b-a)a\left(\frac{a}{2} - y_c\right)\left(\frac{a+b}{2} - x_c\right) + a\,c\left(\frac{a}{2} - x_c\right)\left(\frac{c}{2} - y_c\right)$$

$I_{xy} = -28.1 \times 10^3\,\text{mm}^4$

**Problem 10-71**




Determine the product of inertia for the shaded area with respect to the $x$ and $y$ axes.

Given:

$a = 2\text{ in}$

$b = 1\text{ in}$

$c = 2\text{ in}$

$d = 4\text{ in}$

Solution:

$$l_{xy} = 2a(c+d)a\left(\frac{c+d}{2}\right) - \pi b^2 a\, d$$

$$l_{xy} = 119\,\text{in}^4$$

---

**Problem 10-72**

Determine the product of inertia for the beam's cross-sectional area with respect to the $x$ and $y$ axes that have their origin located at the centroid $C$.

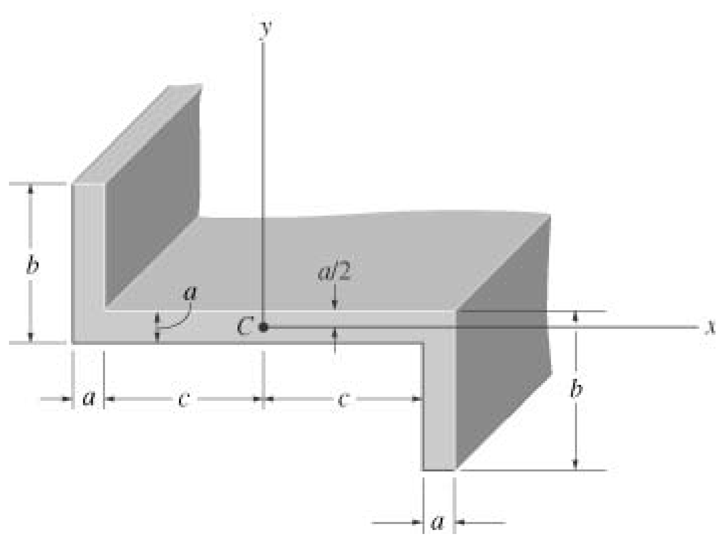
Given:

$a = 1\text{ in}$ $\quad b = 5\text{ in}$ $\quad c = 5\text{ in}$

Solution:

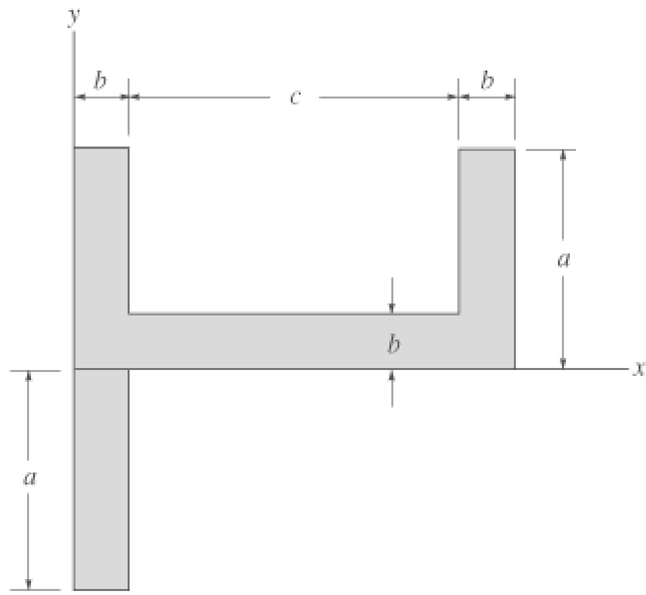
$$I_{xy} = 2b\,a\left(\frac{a}{2} - \frac{b}{2}\right)\left(c + \frac{a}{2}\right)$$

$$I_{xy} = -110\text{ in}^4$$



**Problem 10-73**

Determine the product of inertia for the cross-sec-tional area with respect to the *x* and *y* axes.




Given:

$a = 4\ \text{in}$

$b = 1\ \text{in}$

$c = 6\ \text{in}$

Solution:

$$I_{xy} = b\,a\left(\frac{a}{2}\right)\left(c + \frac{3b}{2}\right) + c\,b\left(b + \frac{c}{2}\right)\left(\frac{b}{2}\right) \qquad I_{xy} = 72\ \text{in}^4$$

**Problem 10-74**

Determine the product of inertia for the beam's cross-sectional area with respect to the *u* and *v* axes.

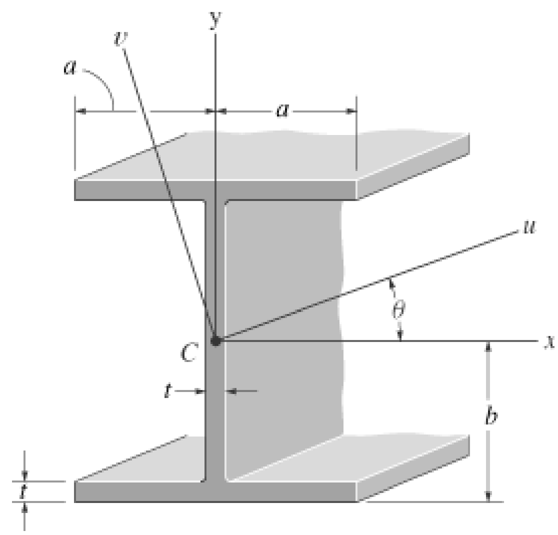
Given:

$a = 150$ mm

$b = 200$ mm

$t = 20$ mm

$\theta = 20$ deg




Solution:

Moments of inertia $I_x$ and $I_y$:

$$I_x = \frac{1}{12}2a(2b)^3 - \frac{1}{12}(2a - t)(2b - 2t)^3 \qquad I_x = 511.36 \times 10^6 \text{ mm}^4$$

$$I_y = \frac{2}{12}t(2a)^3 + \frac{2}{12}(b - t)t^3 \qquad I_y = 90240000.00 \text{ mm}^4$$

The section is symmetric about both $x$ and $y$ axes; therefore $I_{xy} = 0$.

$$I_{xy} = 0\text{mm}^4$$

$$I_{uv} = \left(\frac{I_x - I_y}{2}\right)\sin(2\theta) + I_{xy}\cos(2\theta) \qquad I_{uv} = 135 \times 10^6 \text{ mm}^4$$

---

**Problem 10-75**

Determine the moments of inertia $I_u$ and $I_v$ and the product of inertia $I_{uv}$ for the rectangular area.The $u$ and $v$ axes pass through the centroid $C$.

Given:

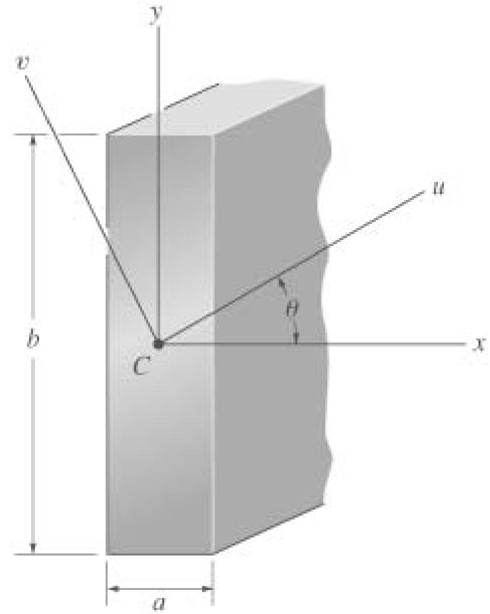
$a = 40$ mm

$b = 160$ mm

$\theta = 30$ deg

Solution:

$I_x = \frac{1}{12} a b^3 \qquad I_y = \frac{1}{12} b a^3 \qquad I_{xy} = 0 \text{ mm}^4$




$$I_u = \frac{I_x + I_y}{2} + \left(\frac{I_x - I_y}{2}\right)\cos(2\theta) - I_{xy}\sin(2\theta)$$

$I_u = 10.5 \times 10^6 \text{mm}^4$

$$I_v = \left(\frac{I_x + I_y}{2}\right) - \left(\frac{I_x - I_y}{2}\right)\cos(2\theta) - I_{xy}\sin(2\theta)$$

$I_v = 4.05 \times 10^6 \text{mm}^4$

$$I_{uv} = \left(\frac{I_x - I_y}{2}\right)\sin(2\theta) + I_{xy}\cos(2\theta)$$

$I_{uv} = 5.54 \times 10^6 \text{mm}^4$

---

**Problem 10-76**

Determine the distance $y_c$ to the centroid of the area and then calculate the moments of inertia $I_u$ and $I_v$ for the channel`s cross-sectional area. The $u$ and $v$ axes have their origin at the centroid $C$. For the calculation, assume all corners to be square.

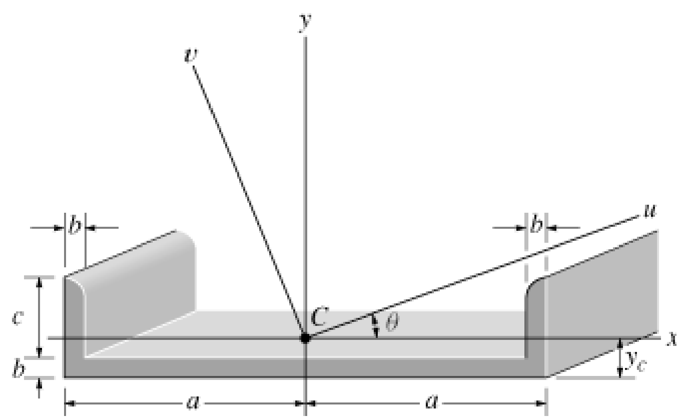
Given:

$a = 150 \text{ mm}$

$b = 10 \text{ mm}$

$c = 50 \text{ mm}$

$\theta = 20 \text{ deg}$



Solution:

$$y_c = \frac{2a\,b\frac{b}{2} + 2c\,b\left(b + \frac{c}{2}\right)}{2a\,b + 2c\,b}$$

$y_c = 12.50\ \text{mm}$

$$I_x = \frac{1}{12}2a\,b^3 + 2a\,b\left(y_c - \frac{b}{2}\right)^2 + 2\left[\frac{1}{12}b\,c^3 + b\,c\left(b + \frac{c}{2} - y_c\right)^2\right]$$

$I_x = 908.3 \times 10^3\ \text{mm}^4$

$$I_y = \frac{1}{12}b\,(2a)^3 + 2\left[\frac{1}{12}c\,b^3 + c\,b\left(a - \frac{b}{2}\right)^2\right]$$

$I_y = 43.53 \times 10^6\ \text{mm}^4$

(By symmetry) $I_{xy} = 0\ \text{mm}^4$

$$I_u = \left(\frac{I_x + I_y}{2}\right) + \left(\frac{I_x - I_y}{2}\right)\cos(2\theta) - I_{xy}\sin(2\theta)$$

$I_u = 5.89 \times 10^6\ \text{mm}^4$

$$I_v = \left(\frac{I_x + I_y}{2}\right) - \left(\frac{I_x - I_y}{2}\right)\cos(2\theta) + I_{xy}\sin(2\theta)$$

$I_v = 38.5 \times 10^6\ \text{mm}^4$

---

**Problem 10-77**

Determine the moments of inertia for the shaded area with respect to the *u* and *v* axes.

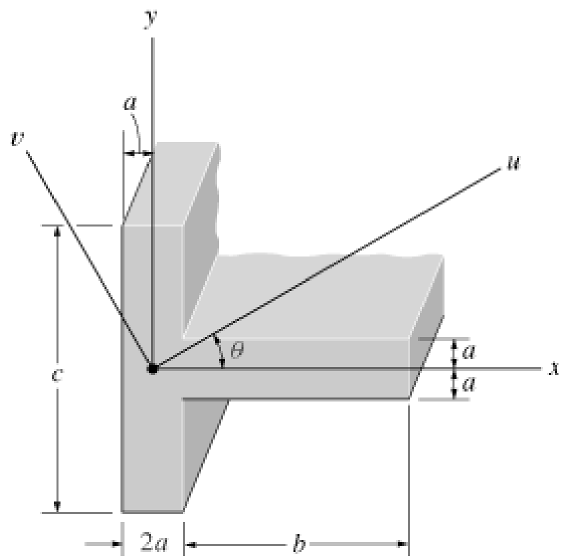
Given:

$a = 0.5\ \text{in}$

$b = 4\ \text{in}$

$c = 5\ \text{in}$

$\theta = 30\ \text{deg}$



Solution:

*Moment and Product of Inertia about x and y Axes*: Since the shaded area is symmetrical about the *x* axis,

$I_{xy} = 0 \text{ in}^4$

$$I_x = \frac{1}{12}2ac^3 + \frac{1}{12}b(2a)^3$$ $I_x = 10.75 \text{ in}^4$

$$I_y = \frac{1}{12}2ab^3 + 2ab\left(a + \frac{b}{2}\right)^2 + \frac{1}{12}c(2a)^3$$ $I_y = 30.75 \text{ in}^4$

*Moment of Inertia about the Inclined u and v Axes*

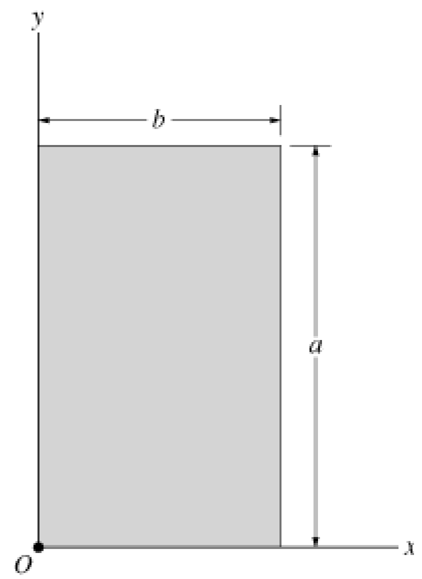
$$I_u = \left(\frac{I_x + I_y}{2}\right) + \left(\frac{I_x - I_y}{2}\right)\cos(2\theta) - I_{xy}\sin(2\theta)$$ $I_u = 15.75 \text{ in}^4$

$$I_v = \left(\frac{I_x + I_y}{2}\right) - \left(\frac{I_x - I_y}{2}\right)\cos(2\theta) + I_{xy}\sin(2\theta)$$ $I_v = 25.75 \text{ in}^4$

---

**Problem 10-78**

Determine the directions of the principal axes with origin located at point *O*, and the principal moments of inertia for the rectangular area about these axes.




Given:

$a = 6 \text{ in}$

$b = 3 \text{ in}$

Solution:

$$I_x = \frac{1}{3}ba^3$$ $I_x = 216 \text{ in}^4$

$$I_y = \frac{1}{3}ab^3$$ $I_y = 54 \text{ in}^4$

$$I_{xy} = \frac{a}{2}\frac{b}{2}ab$$ $I_{xy} = 81 \text{ in}^4$

$$\tan(2\theta) = \frac{-2I_{xy}}{I_x - I_y} \qquad \theta = \frac{1}{2}\operatorname{atan}\left(2\frac{I_{xy}}{-I_x + I_y}\right) \qquad \theta = -22.5\,\text{deg}$$

$$I_{max} = \frac{I_x + I_y}{2} + \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} \qquad I_{max} = 250\,\text{in}^4$$

$$I_{min} = \frac{I_x + I_y}{2} - \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} \qquad I_{min} = 20.4\,\text{in}^4$$

---

**Problem 10-79**

Determine the moments of inertia $I_u$, $I_v$ and the product of inertia $I_{uv}$ for the beam's cross-sectional area.

Given:

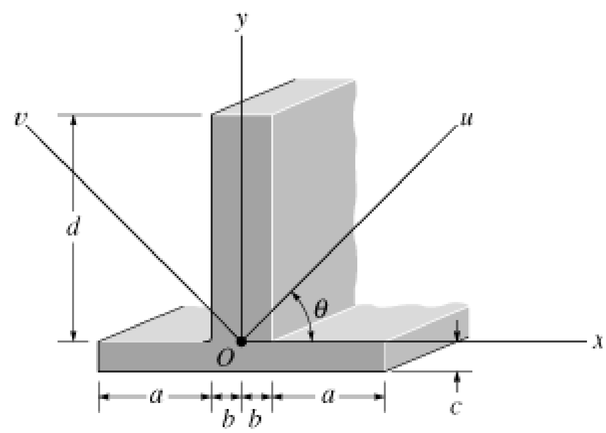
$\theta = 45\ \text{deg}$

$a = 8\ \text{in}$

$b = 2\ \text{in}$

$c = 2\ \text{in}$

$d = 16\ \text{in}$




Solution:

$$I_x = \frac{2}{3}(a+b)\,c^3 + \frac{1}{12}2b\,d^3 + 2b\,d\left(\frac{d}{2}\right)^2 \qquad I_x = 5.515 \times 10^3\ \text{in}^4$$

$$I_y = \frac{1}{12}[2(a+b)]^3 c + \frac{1}{12}(2b)^3 d \qquad I_y = 1.419 \times 10^3\ \text{in}^4$$

$$I_{xy} = 0\ \text{in}^4$$

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos(2\theta) - I_{xy}\sin(2\theta) \qquad I_u = 3.47 \times 10^3\ \text{in}^4$$

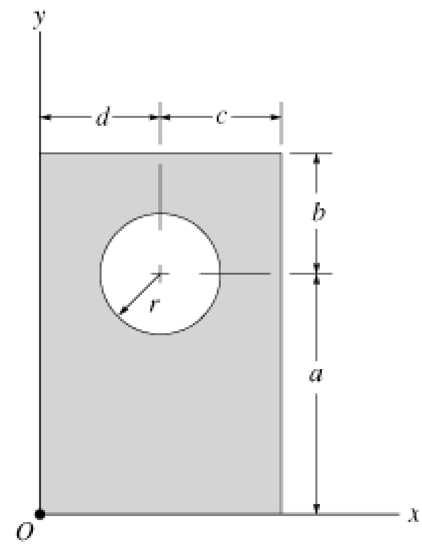
$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos(2\theta) + I_{xy}\sin(2\theta) \qquad I_v = 3.47 \times 10^3\ \text{in}^4$$

$$I_{uv} = \frac{I_x - I_y}{2}\sin(2\theta) + I_{xy}\cos(2\theta)$$ $I_{uv} = 2.05 \times 10^3\ \text{in}^4$

---

**Problem 10-80**

Determine the directions of the principal axes with origin located at point *O*, and the principal moments of inertia for the area about these axes.




Given:

$a = 4\ \text{in}$

$b = 2\ \text{in}$

$c = 2\ \text{in}$

$d = 2\ \text{in}$

$r = 1\ \text{in}$

Solution:

$$I_x = \frac{1}{3}(c+d)(a+b)^3 - \left(\frac{\pi r^4}{4} + \pi r^2 a^2\right)$$ $I_x = 236.95\ \text{in}^4$

$$I_y = \frac{1}{3}(a+b)(c+d)^3 - \left(\frac{\pi r^4}{4} + \pi r^2 d^2\right)$$ $I_y = 114.65\ \text{in}^4$

$$I_{xy} = \left(\frac{a+b}{2}\right)\left(\frac{d+c}{2}\right)(a+b)(d+c) - d\,a\,\pi r^2$$ $I_{xy} = 118.87\ \text{in}^4$

$$\tan(2\ \theta_p) = \frac{-I_{xy}}{\frac{I_x - I_y}{2}} \qquad \theta_p = \frac{1}{2}\operatorname{atan}\left(2\,\frac{I_{xy}}{-I_x + I_y}\right)$$ $\theta_p = -31.39\ \text{deg}$

$$\theta_{p1} = \theta_p$$ $\theta_{p1} = -31.39\ \text{deg}$

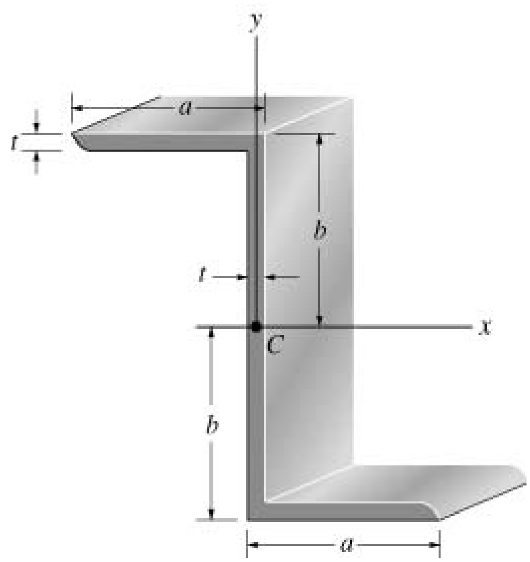
$$\theta_{p2} = 90\ \text{deg} + \theta_{p1}$$ $\theta_{p2} = 58.61\ \text{deg}$

$$I_{max} = \frac{I_x + I_y}{2} + \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^{\ 2}}$$ $I_{max} = 309\ \text{in}^4$

$$I_{min} = \frac{I_x + I_y}{2} - \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}{}^2}$$ $I_{min} = 42.1\ \text{in}^4$

**Problem 10-81**

Determine the principal moments of inertia for the beam's cross-sectional area about the principal axes that have their origin located at the centroid *C*. Use the equations developed in Section 10.7. For the calculation, assume all corners to be square.




Given: $a = 4\ \text{in}$ $b = 4\ \text{in}$ $t = \frac{3}{8}\ \text{in}$

Solution:

$$I_x = 2\left[\frac{1}{12}a\,t^3 + a\,t\left(b - \frac{t}{2}\right)^2\right] + \frac{1}{12}t\,(2b - 2t)^3$$

$I_x = 55.55\ \text{in}^4$

$$I_y = 2\left[\frac{1}{12}t\,(a - t)^3 + t(a - t)\left(\frac{a - t}{2} + \frac{t}{2}\right)^2\right] + \frac{1}{12}2b\,t^3$$ $I_y = 13.89\ \text{in}^4$

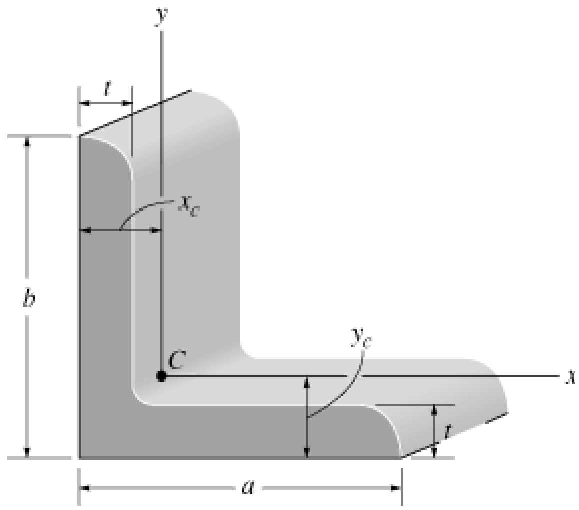
$$I_{xy} = -2\left[\frac{a - t}{2} + \left(\frac{t}{2}\right)\right]\left(b - \frac{t}{2}\right)t(a - t)$$ $I_{xy} = -20.73\ \text{in}^4$

$$I_{max} = \frac{I_x + I_y}{2} + \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}{}^2}$$ $I_{max} = 64.1\ \text{in}^4$

$$I_{min} = \frac{I_x + I_y}{2} - \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}{}^2}$$ $I_{min} = 5.33\ \text{in}^4$

**Problem 10-82**

Determine the principal moments of inertia for the angle's cross-sectional area with respect to a set of principal axes that have their origin located at the centroid *C*. Use the equation developed in Section 10.7. For the calculation, assume all corners to be square.




Given:

$a = 100$ mm

$b = 100$ mm

$t = 20$ mm

Solution:

$$x_c = \frac{t\,b\frac{t}{2} + (a-t)t\left(t + \frac{a-t}{2}\right)}{t\,b + (a-t)t} \qquad x_c = 32.22\ \text{mm}$$

$$y_c = \frac{t\,b\frac{b}{2} + (a-t)t\frac{t}{2}}{t\,b + (a-t)t} \qquad y_c = 32.22\ \text{mm}$$

$$I_x = \frac{1}{12}t^3(a-t) + t(a-t)\left(x_c - \frac{t}{2}\right)^2 + \frac{1}{12}t\,b^3 + t\,b\left(\frac{b}{2} - x_c\right)^2 \qquad I_x = 3.142 \times 10^6\ \text{mm}^4$$

$$I_y = \frac{1}{12}b\,t^3 + b\,t\left(x_c - \frac{t}{2}\right)^2 + \frac{1}{12}t(a-t)^3 + t(a-t)\left(t + \frac{a-t}{2} - x_c\right)^2 \qquad I_y = 3.142 \times 10^6\ \text{mm}^4$$

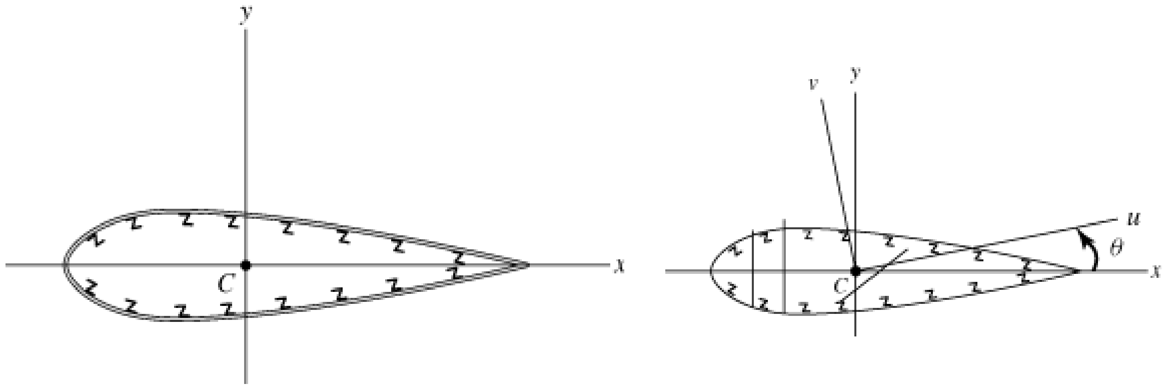
$$I_{xy} = -\left(x_c - \frac{t}{2}\right)\left(\frac{b}{2} - y_c\right)b\,t - \left(\frac{a-t}{2} + t - x_c\right)\left(y_c - \frac{t}{2}\right)(a-t)t \qquad I_{xy} = -1.778 \times 10^6\ \text{mm}^4$$

$$I_{max} = \left(\frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\right) - I_{xy} \qquad I_{max} = 4.92 \times 10^6\ \text{mm}^4$$

$$I_{min} = \left(\frac{I_x + I_y}{2}\right) + \left(\frac{I_x - I_y}{2}\right) + I_{xy} \qquad I_{min} = 2.22 \times 10^6\ \text{mm}^4$$

**Problem 10-83**

The area of the cross section of an airplane wing has the listed properties about the *x* and *y* axes passing through the centroid *C*. Determine the orientation of the principal axes and the principal moments of inertia.




Given: $I_x = 450\ \text{in}^4$ $I_y = 1730\ \text{in}^4$ $I_{xy} = 138\ \text{in}^4$

Solution:

$$\tan(2\theta) = \frac{-2I_{xy}}{I_x - I_y} \qquad \theta = \frac{1}{2}\,\text{atan}\left(2\frac{I_{xy}}{-I_x + I_y}\right)$$

$\theta = 6.08\ \text{deg}$

$$I_{max} = \frac{I_x + I_y}{2} + \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^{\,2}}$$

$I_{max} = 1745\ \text{in}^4$

$$I_{min} = \frac{I_x + I_y}{2} - \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^{\,2}}$$

$I_{min} = 435\ \text{in}^4$

---

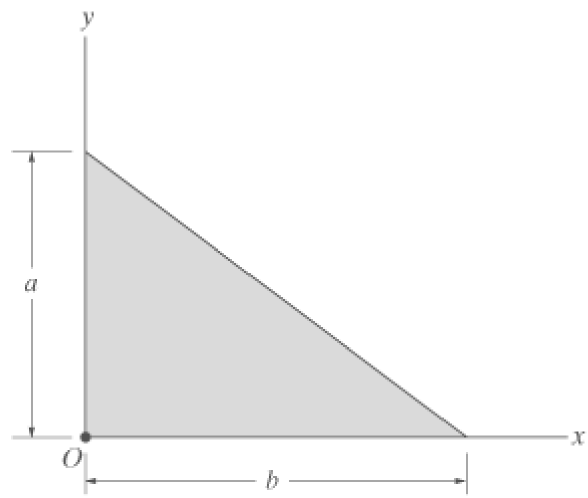
**Problem 10-84**

Using Mohr's circle, determine the principal moments of inertia for the triangular area and the orientation of the principal axes of inertia having an origin at point *O*.

Given:

$a = 30\ \text{mm}$

$b = 40\ \text{mm}$



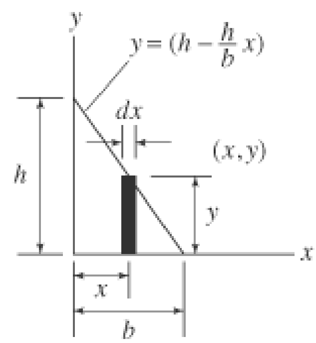

Solution:

Moment of inertia $I_x$ and $I_y$:

$$I_x = \frac{1}{12} b a^3 \qquad I_x = 90 \times 10^3 \, mm^4$$

$$I_y = \frac{1}{12} a b^3 \qquad I_y = 160 \times 10^3 \, mm^4$$

Product of inertia $I_{xy}$:

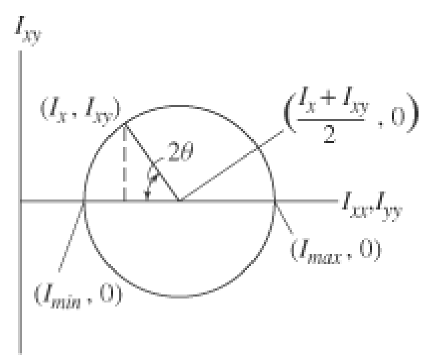



$$I_{xy} = \int_0^b \frac{x}{2}\left(a - \frac{a}{b}x\right)^2 dx \qquad I_{xy} = 60 \times 10^3 \, mm^4$$

Mohr's circle :

$$OA = \sqrt{\left(\frac{I_x + I_y}{2} - I_x\right)^2 + I_{xy}^2}$$

$$OA = 69.462 \times 10^3 \, mm^4$$




$$I_{max} = \left(\frac{I_x + I_y}{2} + OA\right)$$

$$I_{max} = 194.462 \times 10^3 \, mm^4$$

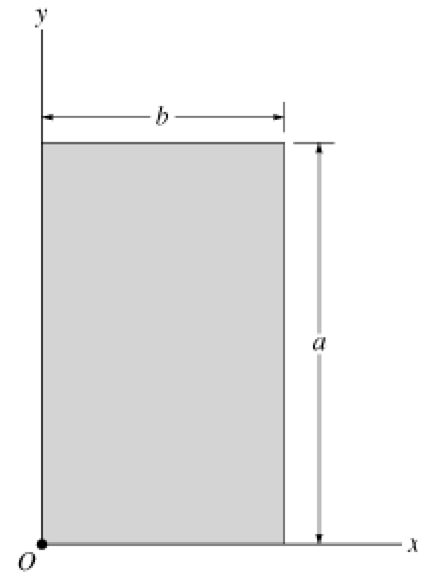
$$I_{min} = \left(\frac{I_x + I_y}{2} - OA\right)$$

$$I_{min} = 55.5 \times 10^3 \, mm^4$$

$$\tan(2\theta) = \frac{I_{xy}}{\frac{I_x + I_y}{2} - I_x} \qquad \theta = \frac{1}{2} \operatorname{atan}\left(2 \frac{I_{xy}}{-I_x + I_y}\right) \qquad \theta = 29.9 \, deg$$

**Problem 10-85**

Determine the directions of the principal axes with origin located at point *O*, and the principal moments of inertia for the rectangular area about these axes.
Solve using Mohr's circle.




Given:

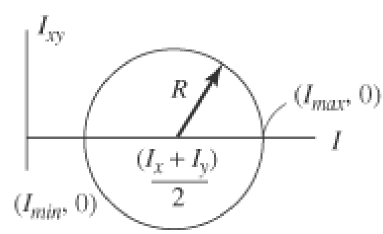
$a = 6 \text{ in}$

$b = 3 \text{ in}$

Solution:

$I_x = \frac{1}{3} b a^3$ $\qquad I_x = 216 \text{ in}^4$

$I_y = \frac{1}{3} a b^3$ $\qquad I_y = 54 \text{ in}^4$




$I_{xy} = \frac{a}{2}\frac{b}{2} a b$ $\qquad I_{xy} = 81 \text{ in}^4$

$R = \sqrt{\left[I_x - \left(\frac{I_x + I_y}{2}\right)\right]^2 + I_{xy}^2}$ $\qquad R = 114.55 \text{ in}^4$

$I_{max} = \frac{I_x + I_y}{2} + R$ $\qquad I_{max} = 250 \text{ in}^4$

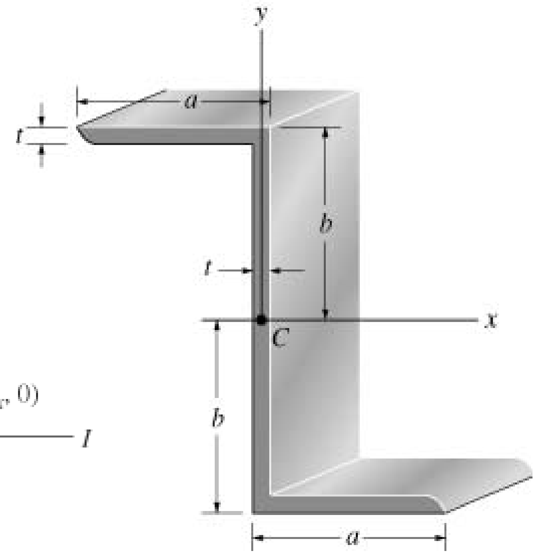
$I_{min} = \frac{I_x + I_y}{2} - R$ $\qquad I_{min} = 20.4 \text{ in}^4$

$\theta_{p1} = \frac{-1}{2} \operatorname{asin}\left(\frac{I_{xy}}{R}\right)$ $\qquad \theta_{p1} = -22.50 \text{ deg}$

$\theta_{p2} = \theta_{p1} + 90 \text{ deg}$ $\qquad \theta_{p2} = 67.50 \text{ deg}$

**Problem 10-86**

Determine the principal moments of inertia for the beam's cross-sectional area about the principal axes that have their origin located at the centroid *C*. For the calculation, assume all corners to be square. Solve using Mohr's circle.

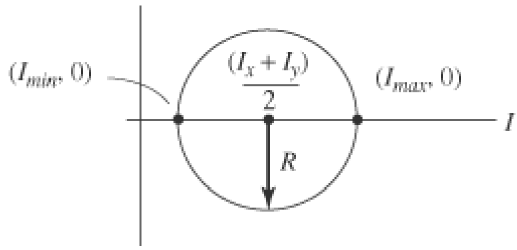



Given:

$a = 4\ \text{in}$

$b = 4\ \text{in}$

$t = \frac{3}{8}\ \text{in}$




Solution:

$$I_x = 2\left[\frac{1}{12}a\,t^3 + a\,t\left(b - \frac{t}{2}\right)^2\right] + \frac{1}{12}t\,(2b - 2t)^3 \qquad I_x = 55.55\ \text{in}^4$$

$$I_y = 2\left[\frac{1}{12}t\,(a - t)^3 + t(a - t)\left(\frac{a - t}{2} + \frac{t}{2}\right)^2\right] + \frac{1}{12}2b\,t^3 \qquad I_y = 13.89\ \text{in}^4$$

$$I_{xy} = -2\left[\frac{a - t}{2} + \left(\frac{t}{2}\right)\right]\left(b - \frac{t}{2}\right)t(a - t) \qquad I_{xy} = -20.73\ \text{in}^4$$

$$R = \sqrt{\left(I_x - \frac{I_x + I_y}{2}\right)^2 + I_{xy}^{\ 2}} \qquad R = 29.39\ \text{in}^4$$

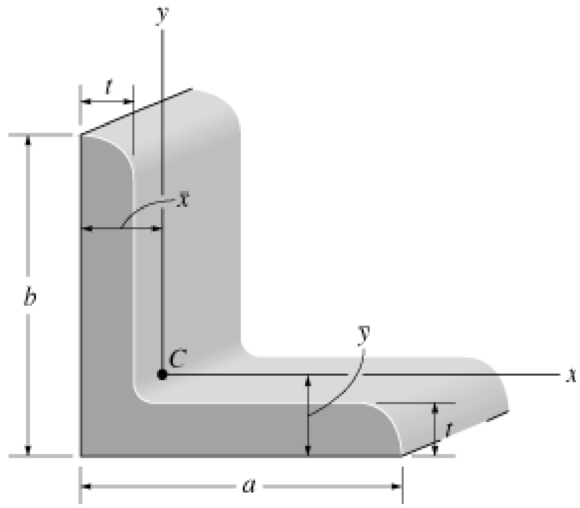
$$I_{max} = \frac{I_x + I_y}{2} + R \qquad I_{max} = 64.1\ \text{in}^4$$

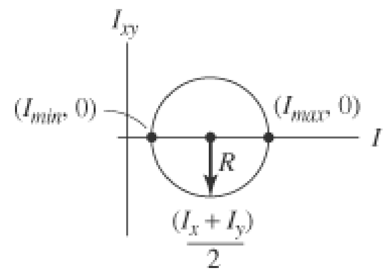
$$I_{min} = \frac{I_x + I_y}{2} - R \qquad I_{min} = 20.45\ \text{in}^4$$

---

**Problem 10-87**

Determine the principal moments of inertia for the angle's cross-sectional area with respect to a set of principal axes that have their origin located at the centroid *C*. For the calculation, assume all corners to be square. Solve using Mohr's ciricle.




Given: $a = 100\ \text{mm}$ $b = 100\ \text{mm}$ $t = 20\ \text{mm}$

Solution:

$$x_c = \frac{tb\left(\frac{t}{2}\right) + (a-t)t\left(t + \frac{a-t}{2}\right)}{tb + (a-t)t} \qquad x_c = 32.22\,\text{mm}$$

$$y_c = \frac{tb\left(\frac{b}{2}\right) + (a-t)t\left(\frac{t}{2}\right)}{tb + (a-t)t} \qquad y_c = 32.22\,\text{mm}$$

$$I_x = \frac{1}{12}t^3(a-t) + t(a-t)\left(x_c - \frac{t}{2}\right)^2 + \frac{1}{12}tb^3 + tb\left(\frac{b}{2} - x_c\right)^2 \qquad I_x = 3.142 \times 10^6\,\text{mm}^4$$

$$I_y = \frac{1}{12}bt^3 + bt\left(x_c - \frac{t}{2}\right)^2 + \frac{1}{12}t(a-t)^3 + t(a-t)\left(t + \frac{a-t}{2} - x_c\right)^2$$

$$I_y = 3.142 \times 10^6\,\text{mm}^4$$

$$I_{xy} = -\left(x_c - \frac{t}{2}\right)\left(\frac{b}{2} - y_c\right)bt - \left(\frac{a-t}{2} + t - x_c\right)\left(y_c - \frac{t}{2}\right)(a-t)t \qquad I_{xy} = -1.778 \times 10^6\,\text{mm}^4$$

$$R = \sqrt{\left(I_x - \frac{I_x + I_y}{2}\right)^2 + I_{xy}^2} \qquad R = 1.78 \times 10^6\,\text{mm}^4$$

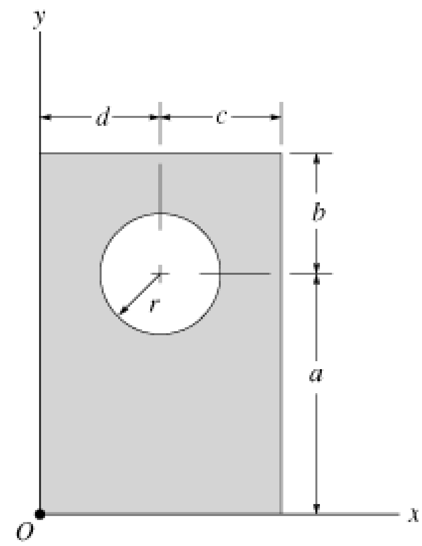
$$I_{max} = \frac{I_x + I_y}{2} + R \qquad I_{max} = 4.92 \times 10^6\ \text{mm}^4$$

$$I_{min} = \frac{I_x + I_y}{2} - R \qquad I_{min} = 1364444.44\ \text{mm}^4$$

---

**Problem 10-88**

Determine the directions of the principal axes with origin located at point *O*, and the principal moments of inertia for the area about these axes. Solve using Mohr's circle




Given:

$a = 4$ in

$b = 2$ in

$c = 2$ in

$d = 2$ in

$r = 1$ in

Solution:

$$I_x = \frac{1}{3}(c+d)(a+b)^3 - \left(\frac{\pi r^4}{4} + \pi r^2 a^2\right) \qquad I_x = 236.95\ \text{in}^4$$

$$I_y = \frac{1}{3}(a+b)(c+d)^3 - \left(\frac{\pi r^4}{4} + \pi r^2 d^2\right) \qquad I_y = 114.65\ \text{in}^4$$

$$I_{xy} = \left(\frac{a+b}{2}\right)\left(\frac{d+c}{2}\right)(a+b)(d+c) - d\,a\pi r^2 \qquad I_{xy} = 118.87\ \text{in}^4$$

$$R = \sqrt{\left[I_x - \left(\frac{I_x + I_y}{2}\right)\right]^2 + I_{xy}^2} \qquad R = 133.67\ \text{in}^4$$

$$I_{max} = \frac{I_x + I_y}{2} + R \qquad I_{max} = 309\ \text{in}^4$$

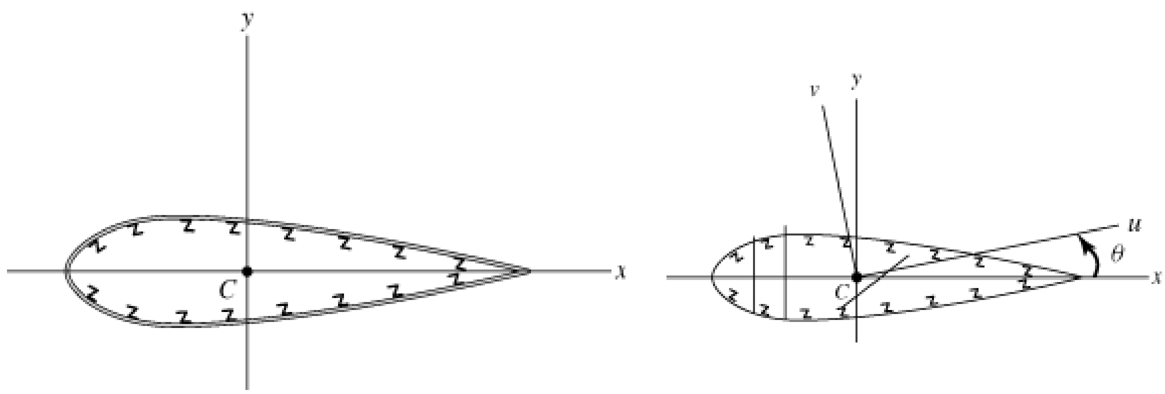
$$I_{min} = \frac{I_x + I_y}{2} - R \qquad I_{min} = 42.1\ \text{in}^4$$

$$\theta_{p1} = \frac{-1}{2}\operatorname{asin}\left(\frac{I_{xy}}{R}\right) \qquad \theta_{p1} = -31.39\ \text{deg}$$

$$\theta_{p2} = \theta_{p1} + \frac{\pi}{2} \qquad \theta_{p2} = 58.61\ \text{deg}$$

**Problem 10-89**

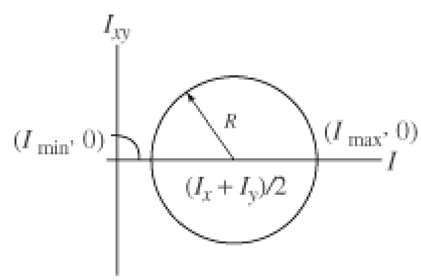
The area of the cross section of an airplane wing has the listed properties about the *x* and *y* axes passing through the centroid *C*. Determine the orientation of the principal axes and the principal moments of inertia. Solve using Mohr's circle.




Given: $I_x = 450\ \text{in}^4$

$I_y = 1730\ \text{in}^4$

$I_{xy} = 138\ \text{in}^4$




Solution:

$$R = \sqrt{\left[I_x - \left(\frac{I_x + I_y}{2}\right)\right]^2 + I_{xy}^2} \qquad R = 654.71\ \text{in}^4$$

$$I_{max} = \left(\frac{I_x + I_y}{2} + R\right) \qquad I_{max} = 1.74 \times 10^3\ \text{in}^4$$

$$I_{min} = \left(\frac{I_x + I_y}{2} - R\right) \qquad I_{min} = 435\ \text{in}^4$$

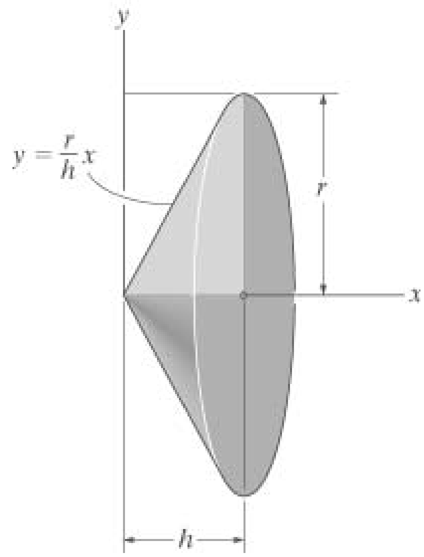
$$\theta_{p1} = \frac{1}{2}\operatorname{asin}\left(\frac{I_{xy}}{R}\right) \qquad \theta_{p1} = 6.08\,\text{deg}$$

$$\theta_{p2} = \theta_{p1} + 90\ \text{deg} \qquad \theta_{p2} = 96.08\,\text{deg}$$

---

**Problem 10-90**

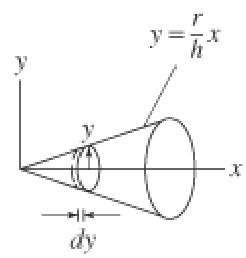
The right circular cone is formed by revolving the shaded area around the *x* axis. Determine the moment of inertia $l_x$ and express the result in terms of the total mass *m* of the cone. The cone has a constant density $\rho$.




Solution:

$$m = \int_0^h \rho\,\pi\left(\frac{r x}{h}\right)^2 dx = \frac{1}{3}\ h\ \rho\ \pi\ r^2$$

$$l_x = \frac{3m}{\pi h r^2}\int_0^h \frac{1}{2}\pi\left(\frac{r x}{h}\right)^4 dx = \frac{3}{10}\ m\ r^2$$

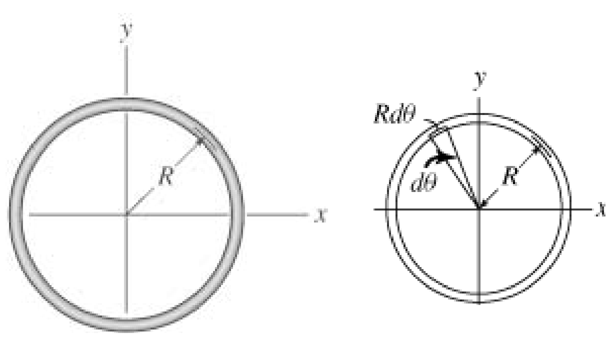



$$l_x = \frac{3}{10}\ m\ r^2$$

---

**Problem 10-91**

Determine the moment of inertia of the thin ring about the *z* axis. The ring has a mass *m*.



Solution:

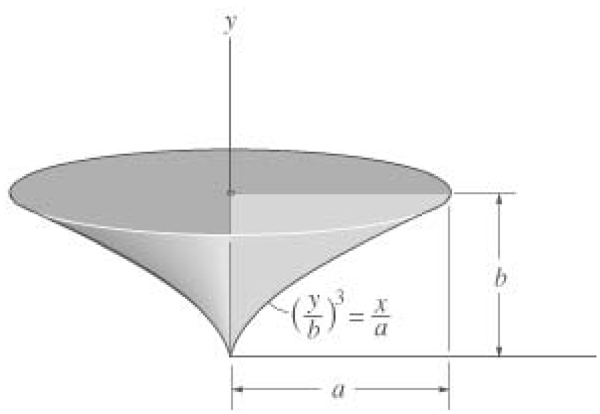
$$m = \rho 2\pi R \qquad \rho = \frac{m}{2\pi R}$$

$$I = \int_0^{2\pi} \left(\frac{m}{2\pi R}\right) R^2 R \, d\theta = m\, R^2 \qquad I = m\, R^2$$

---

**Problem 10-92**

The solid is formed by revolving the shaded area around the $y$ axis. Determine the radius of gyration $k_y$. The specific weight of the material is $\gamma$.




Given:

$$a = 3 \text{ in}$$

$$b = 3 \text{ in}$$

$$\gamma = 380 \frac{\text{lb}}{\text{ft}^3}$$

Solution:

$$m = \int_0^b \gamma \pi \left[a\left(\frac{y}{b}\right)^3\right]^2 dy \qquad m = 2.66 \text{ lb}$$

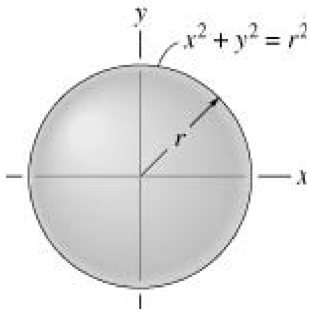
$$I_y = \int_0^b \gamma \pi \left[a\left(\frac{y}{b}\right)^3\right]^2 \frac{1}{2}\left[a\left(\frac{y}{b}\right)^3\right]^2 dy \qquad I_y = 6.46 \text{ lb}\cdot\text{in}^2$$

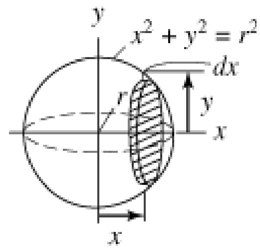
$$k_y = \sqrt{\frac{I_y}{m}} \qquad k_y = 1.56 \text{ in}$$

---

**Problem 10-93**

Determine the moment of inertia $I_x$ for the sphere and express the result in terms of the total mass $m$ of the sphere. The sphere has a constant density $\rho$.




Solution:

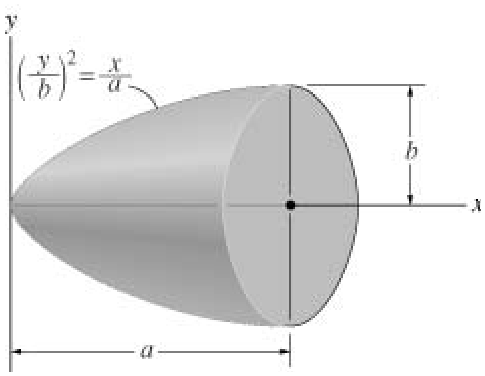
$$m = \rho\frac{4\pi r^3}{3} \qquad \rho = \frac{3m}{4\pi r^3}$$

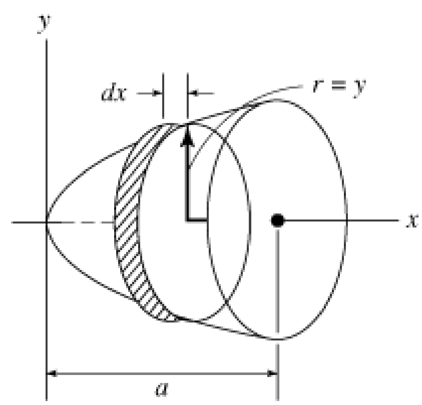
$$I_x = \int_{-r}^{r} \frac{1}{2}\left(\frac{3m}{4\pi r^3}\right)\pi\left(r^2 - x^2\right)\left(r^2 - x^2\right)\,dx = \frac{2}{5}\,m\,r^2 \qquad I_x = \frac{2}{5}\,m\,r^2$$

---

**Problem 10-94**

Determine the radius of gyration $k_x$ of the paraboloid. The density of the material is $\rho$.




Units Used: $\text{Mg} = 1000\text{ kg}$

Given: $\rho = 5\,\frac{\text{Mg}}{\text{m}^3}$ $\quad a = 200\text{ mm} \quad b = 100\text{ mm}$

Solution:

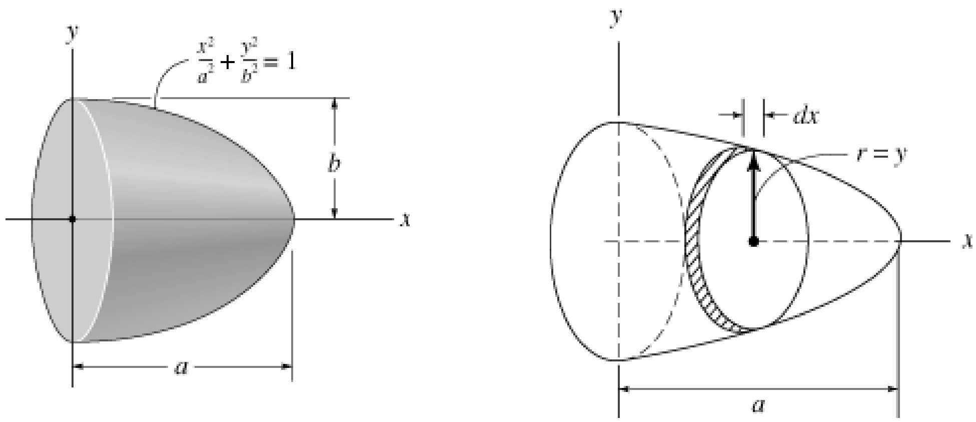
$$m_p = \int_0^a \rho\,\pi\left(\frac{b^2 x}{a}\right) dx \qquad m_p = 15.71\ \text{kg}$$

$$I_x = \int_0^a \frac{1}{2}\rho\,\pi\left(\frac{b^2 x}{a}\right)\left(\frac{b^2 x}{a}\right) dx \qquad I_x = 52.36 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$k_x = \sqrt{\frac{I_x}{m_p}} \qquad k_x = 57.7\ \text{mm}$$

**Problem 10-95**

Determine the moment of inertia of the semi-ellipsoid with respect to the *x* axis and express the result in terms of the mass *m* of the semiellipsoid. The material has a constant density $\rho$.




Solution:

$$m = \int_0^a \rho\,\pi b^2\left(1-\frac{x^2}{a^2}\right) dx = \frac{2}{3}\ a\ \rho\ \pi\ b^2 \qquad \rho = \frac{3m}{2\pi a b^2}$$

$$I_x = \int_0^a \frac{1}{2}\left(\frac{3m}{2\pi a b^2}\right)\pi b^2\left(1-\frac{x^2}{a^2}\right) b^2\left(1-\frac{x^2}{a^2}\right) dx = \frac{2}{5}\ m\ b^2 \qquad I_x = \frac{2}{5}\ m\ b^2$$

**Problem 10-96**

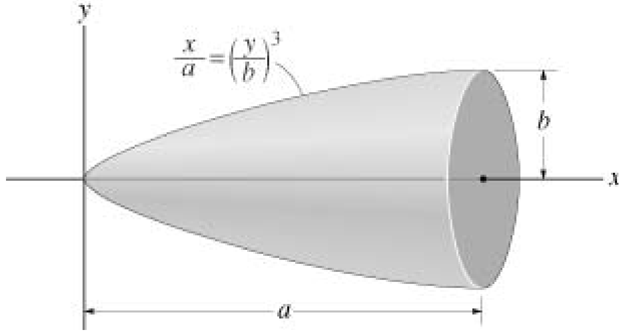
Determine the radius of gyration $k_x$ of the body. The specific weight of the material is $\gamma$.

Given:

$\gamma = 380 \frac{\text{lb}}{\text{ft}^3}$

$a = 8 \text{ in}$

$b = 2 \text{ in}$




Solution:

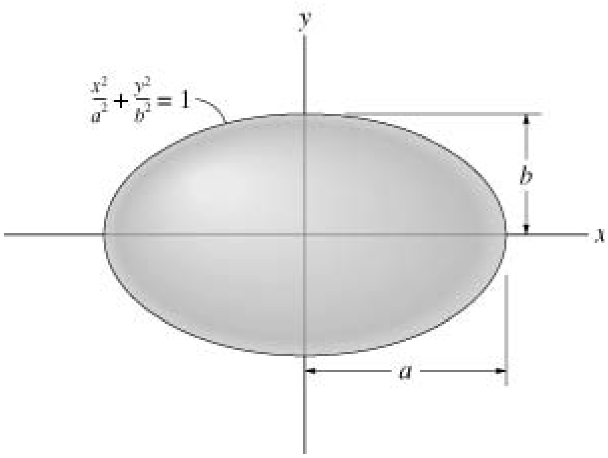
$$m_b = \int_0^a \gamma \pi b^2 \left(\frac{x}{a}\right)^{\frac{2}{3}} dx \qquad m_b = 13.26 \text{ lb}$$

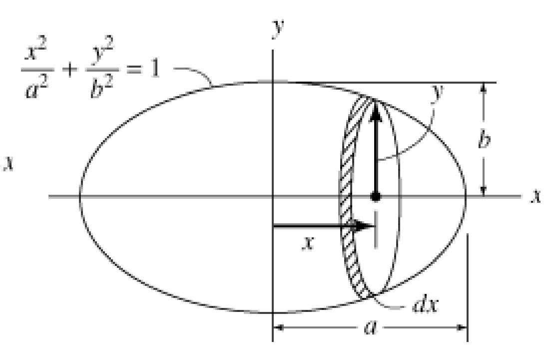
$$I_x = \int_0^a \frac{1}{2} \gamma \pi b^2 \left(\frac{x}{a}\right)^{\frac{2}{3}} b^2 \left(\frac{x}{a}\right)^{\frac{2}{3}} dx \qquad I_x = 0.59 \text{ slug} \cdot \text{in}^2$$

$$k_x = \sqrt{\frac{I_x}{m_b}} \qquad k_x = 1.20 \text{ in}$$

**Problem 10-97**

Determine the moment of inertia for the ellipsoid with respect to the $x$ axis and express the result in terms of the mass $m$ of the ellipsoid. The material has a constant density $\rho$.



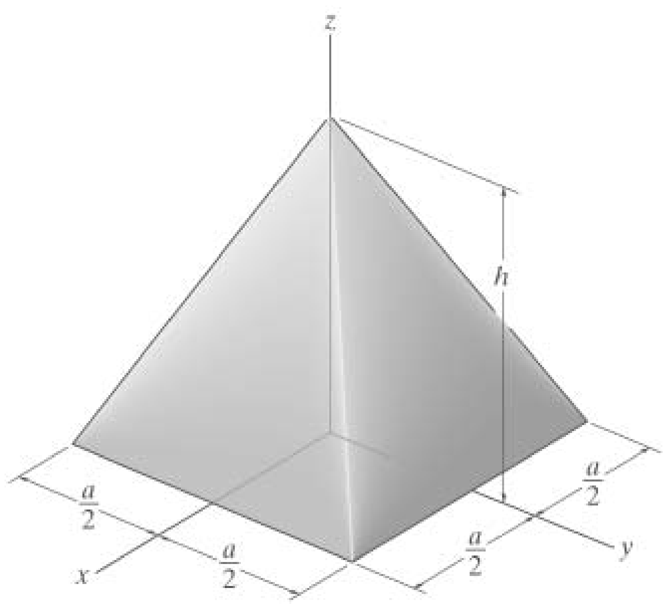
Solution:

$$m = \int_{-a}^{a} \rho\, \pi b^2 \left(1 - \frac{x^2}{a^2}\right) dx = \frac{4}{3} a \rho\, \pi b^2 \qquad \rho = \frac{3m}{4\pi a\, b^2}$$

$$I_x = \int_{-a}^{a} \frac{1}{2} \frac{3m}{4\pi a\, b^2} \pi b^2 \left(1 - \frac{x^2}{a^2}\right) b^2 \left(1 - \frac{x^2}{a^2}\right) dx = \frac{2}{5} m\, b^2 \qquad I_x = \frac{2}{5} m\, b^2$$

**Problem 10-98**

Determine the moment of inertia of the homogeneous pyramid of mass *m* with respect to the *z* axis. The density of the material is $\rho$. *Suggestion:* Use a rectangular plate element having a volume of $dV = (2x)(2y)\ dz$.




Solution:

$$V = \int_{0}^{h} \left[a\left(1 - \frac{z}{h}\right)\right]^2 dz = \frac{1}{3} h\, a^2 \qquad \rho = \frac{m}{V} = \frac{3m}{a^2 h}$$

$$I_z = \frac{3m}{a^2 h} \int_{0}^{h} \frac{1}{6} \left[a\left(1 - \frac{z}{h}\right)\right]^4 dz = \frac{1}{10} m\, a^2 \qquad I_z = \frac{1}{10} m\, a^2$$

**Problem 10-99**

The concrete shape is formed by rotating the shaded area about the $y$ axis. Determine the moment of inertia $I_y$. The specific weight of concrete is $\gamma$.

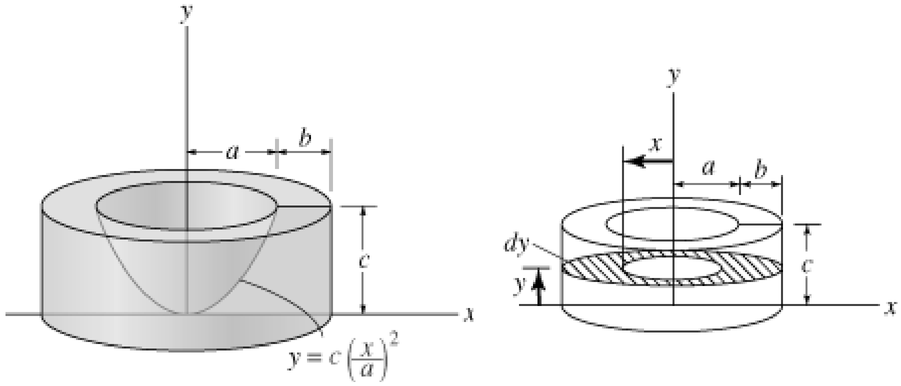
Given:

$\gamma = 150 \frac{\text{lb}}{\text{ft}^3}$

$a = 6 \text{ in}$

$b = 4 \text{ in}$

$c = 8 \text{ in}$




Solution:

$$I_y = \frac{1}{2}\gamma\pi(a+b)^2 c(a+b)^2 - \int_0^c \frac{1}{2}\gamma\left(\pi\frac{a^2 y}{c}\right)\frac{a^2 y}{c}\,dy \qquad I_y = 2.25\,\text{slug}\cdot\text{ft}^2$$

---

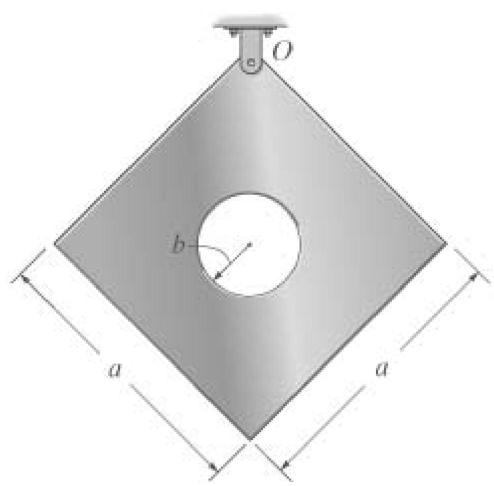
**Problem 10-100**

Determine the moment of inertia of the thin plate about an axis perpendicular to the page and passing through the pin at $O$. The plate has a hole in its center. Its thickness is $c$, and the material has a density of $\rho$

Given:

$a = 1.40 \text{ m}$ $\quad c = 50 \text{ mm}$

$b = 150 \text{ mm}$ $\quad \rho = 50 \frac{\text{kg}}{\text{m}^3}$




Solution:

$$I_G = \frac{1}{12}\rho a^2 c\left(a^2 + a^2\right) - \frac{1}{2}\rho\pi b^2 c b^2$$

$$I_G = 1.60\,\text{kg}\cdot\text{m}^2$$

$$I_0 = I_G + m d^2$$

$$m = \rho a^2 c - \rho \pi b^2 c$$

$$m = 4.7233 \text{ kg}$$

$$I_0 = I_G + m(a \sin(45 \text{ deg}))^2$$

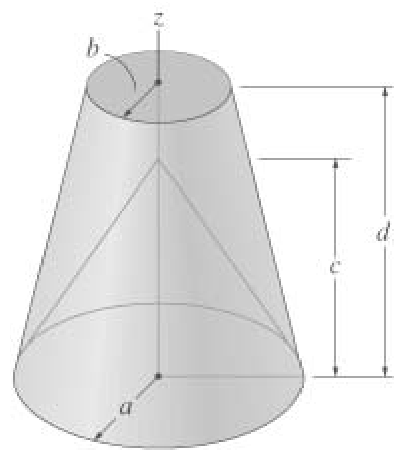
$$I_0 = 6.23 \text{ kg}\cdot\text{m}^2$$

**Problem 10-101**

Determine the moment of inertia $I_z$ of the frustum of the cone which has a conical depression. The material has a density $\rho$.

Given:

$$\rho = 200 \frac{\text{kg}}{\text{m}^3}$$




$$a = 0.4 \text{ m}$$

$$b = 0.2 \text{ m}$$

$$c = 0.6 \text{ m}$$

$$d = 0.8 \text{ m}$$

Solution:

$$h = \frac{d\,a}{a - b}$$

$$I_z = \frac{3}{10}\left[\rho\left(\frac{1}{3}\pi a^2 h\right)\right]a^2 - \frac{3}{10}\left[\rho\left(\frac{1}{3}\pi a^2 c\right)\right]a^2 - \frac{3}{10}\left[\rho\left[\frac{1}{3}\pi b^2 (h-d)\right]\right]b^2$$

$$I_z = 1.53 \text{ kg}\cdot\text{m}^2$$

**Problem 10-102**

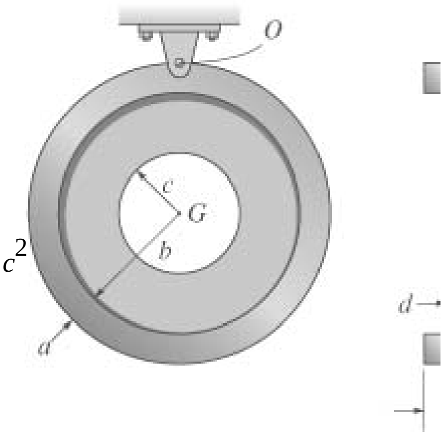
Determine the moment of inertia for the assembly about an axis which is perpendicular to the page and passes through the center of mass $G$. The material has a specific weight $\gamma$.

Given:

$$a = 0.5 \text{ ft} \qquad d = 0.25 \text{ ft}$$

$b = 2\ \text{ft}$ $e = 1\ \text{ft}$

$c = 1\ \text{ft}$ $\gamma = 90\ \dfrac{\text{lb}}{\text{ft}^3}$




Solution:

$$I_G = \frac{1}{2}\gamma\pi(a+b)^2 e(a+b)^2 - \frac{1}{2}\gamma\pi b^2(e-d)\,b^2 - \frac{1}{2}\gamma\pi c^2 d\,c^2$$

$I_G = 118\ \text{slug}\cdot\text{ft}^2$

**Problem 10-103**

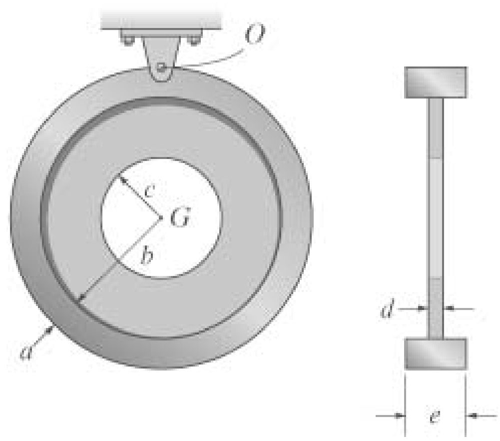
Determine the moment of inertia for the assembly about an axis which is perpendicular to the page and passes through point $O$. The material has a specific weight $\gamma$.

Given:

$a = 0.5\ \text{ft}$ $d = 0.25\ \text{ft}$

$b = 2\ \text{ft}$ $e = 1\ \text{ft}$

$c = 1\ \text{ft}$ $\gamma = 90\ \dfrac{\text{lb}}{\text{ft}^3}$




Solution:

$$I_G = \frac{1}{2}\gamma\pi(a+b)^2 e(a+b)^2 - \frac{1}{2}\gamma\pi b^2(e-d)\,b^2 - \frac{1}{2}\gamma\pi c^2 d\,c^2$$

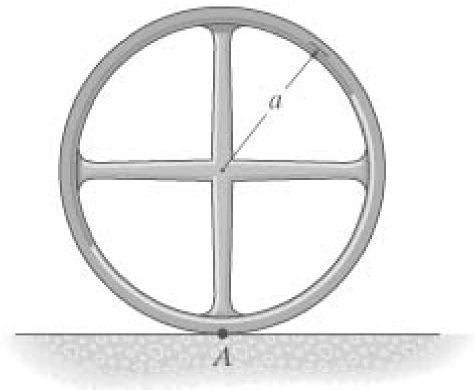
$I_G = 118\ \text{slug}\cdot\text{ft}^2$

$M = \gamma\pi(a+b)^2 e - \gamma\pi b^2(e-d) - \gamma\pi c^2 d$ $\quad M = 848.23\ \text{lb}$

$I_O = I_G + M(a+b)^2$ $\quad I_O = 283\ \text{slug}\cdot\text{ft}^2$

**Problem 10-104**

The wheel consists of a thin ring having a mass $M_1$ and four spokes made from slender rods, each having a mass $M_2$. Determine the wheel's moment of inertia about an axis perpendicular to the page and passing through point $A$.




Given:

$M_1 = 10\ \text{kg}$

$M_2 = 2\ \text{kg}$

$a = 500\ \text{mm}$

Solution:

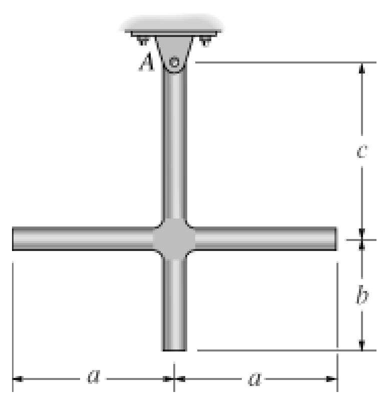
$$I_G = M_1 a^2 + 4\frac{1}{3}M_2 a^2$$

$$I_A = I_G + (M_1 + 4M_2)a^2$$

$$I_A = 7.67\ \text{kg}\cdot\text{m}^2$$

**Problem 10-105**

The slender rods have a weight density $\gamma$. Determine the moment of inertia for the assembly about an axis perpendicular to the page and passing through point $A$.




Given:

$\gamma = 3\ \frac{\text{lb}}{\text{ft}}$

$a = 1.5\ \text{ft}$

$b = 1\ \text{ft}$

$c = 2\ \text{ft}$

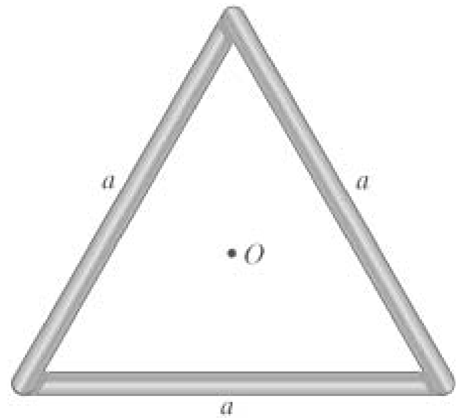
Solution:

$$I = \frac{1}{3}\gamma(b+c)(b+c)^2 + \frac{1}{12}\gamma 2a(2a)^2 + \gamma 2a\,c^2$$

$$I = 2.17\ \text{slug}\cdot\text{ft}^2$$

**Problem 10-106**

Each of the three rods has a mass $m$. Determine the moment of inertia for the assembly about an axis which is perpendicular to the page and passes through the center point $O$.




Solution:

$$I_O = 3\left[\frac{1}{12}m a^2 + m\left(\frac{a \sin(60 \text{ deg})}{3}\right)^2\right]$$

$$I_O = \frac{1}{2}m a^2$$

**Problem 10-107**

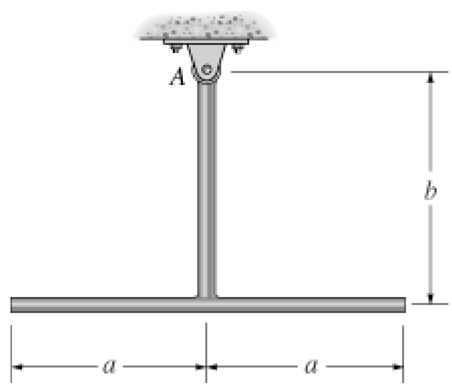
The slender rods have weight density $\gamma$. Determine the moment of inertia for the assembly about an axis perpendicular to the page and passing through point $A$

Given:

$$\gamma = 3 \frac{\text{lb}}{\text{ft}}$$

$$a = 1.5 \text{ ft}$$

$$b = 2 \text{ ft}$$




Solution:

$$I_A = \frac{1}{3}\gamma b b^2 + \frac{1}{12}\gamma 2a (2a)^2 + \gamma(2a) b^2$$

$$I_A = 1.58 \text{ slug} \cdot \text{ft}^2$$

**Problem 10-108**

The pendulum consists of a plate having weight $W_p$ and a slender rod having weight $W_r$. Determine the radius of gyration of the pendulum about an axis perpendicular to the page and passing through point $O$.

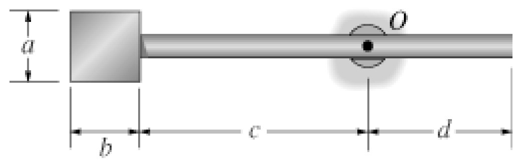
Given:

$W_p = 12\ \text{lb}$ $a = 1\ \text{ft}$

$W_r = 4\ \text{lb}$ $b = 1\ \text{ft}$

$c = 3\ \text{ft}$

$d = 2\ \text{ft}$




Solution:

$$I_O = \frac{1}{12} W_r (c+d)^2 + W_r\left(\frac{c+d}{2} - c\right)^2 + \frac{1}{12} W_p\left(a^2 + b^2\right) + W_p\left(c + \frac{b}{2}\right)^2$$

$$k_O = \sqrt{\frac{I_O}{W_p + W_r}} \qquad k_O = 3.15\,\text{ft}$$

---

**Problem 10-109**

Determine the moment of inertia for the overhung crank about the $x$ axis. The material is steel having density $\rho$.

Units Used:

$\text{Mg} = 1000\ \text{kg}$

Given:

$\rho = 7.85\ \dfrac{\text{Mg}}{\text{m}^3}$

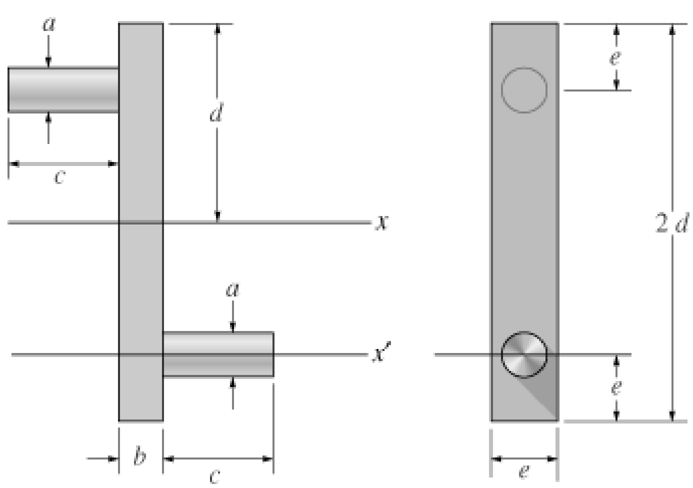
$a = 20\ \text{mm}$

$b = 20\ \text{mm}$

$c = 50\ \text{mm}$

$d = 90\ \text{mm}$

$e = 30\ \text{mm}$




Solution:

$m = \rho\,\pi\left(\dfrac{a}{2}\right)^2 c$ $\qquad m = 0.12\ \text{kg}$

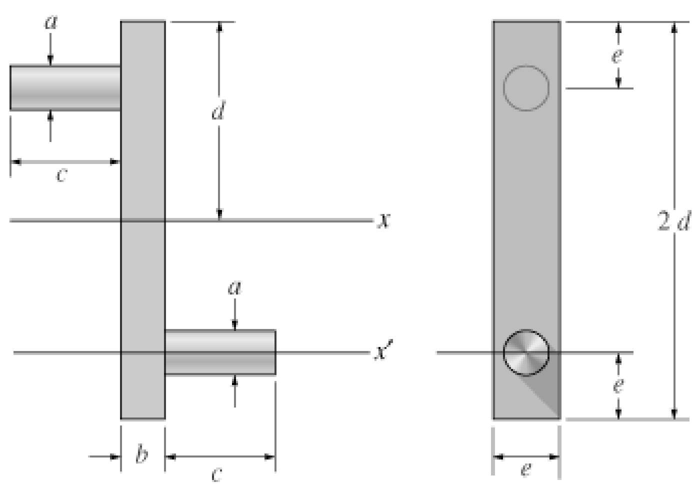
$M = \rho 2 d b e$ $\qquad M = 0.85\ \text{kg}$

$$I_x = 2\left[\frac{1}{2}m\left(\frac{a}{2}\right)^2 + m(d-e)^2\right] + \frac{1}{12}M\left[(2d)^2 + e^2\right]$$

$$I_x = 3.25 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

**Problem 10-110**

Determine the moment of inertia for the overhung crank about the $x'$ axis. The material is steel having density $\rho$.




Units used:

$$\text{Mg} = 1000\ \text{kg}$$

Given:

$$\rho = 7.85\ \frac{\text{Mg}}{\text{m}^3}$$

$$a = 20\ \text{mm}$$

$$b = 20\ \text{mm}$$

$$c = 50\ \text{mm}$$

$$d = 90\ \text{mm}$$

$$e = 30\ \text{mm}$$

Solution:

$$m = \rho\pi\left(\frac{a}{2}\right)^2 c \qquad m = 0.12\ \text{kg}$$

$$M = \rho 2d b e \qquad M = 0.85\ \text{kg}$$

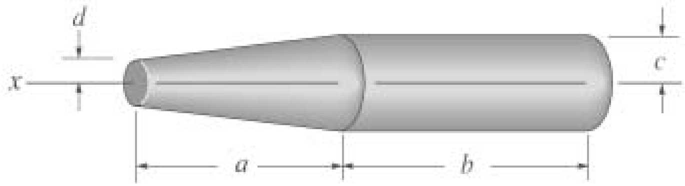
$$I_x = 2\left[\frac{1}{2}m\left(\frac{a}{2}\right)^2 + m(d-e)^2\right] + \frac{1}{12}M\left[(2d)^2 + e^2\right]$$

$$I_{x'} = I_x + (M + 2m)(d-e)^2 \qquad I_{x'} = 7.19 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

**Problem 10-111**

Determine the moment of inertia for the solid steel assembly about the $x$ axis. Steel has a specific weight $\gamma_{st}$.

Given:




$a = 2\ \text{ft}$

$b = 3\ \text{ft}$

$c = 0.5\ \text{ft}$

$d = 0.25\ \text{ft}$

$\gamma_{st} = 490\ \frac{\text{lb}}{\text{ft}^3}$

Solution:

$$h = \frac{c\,a}{c-d}$$

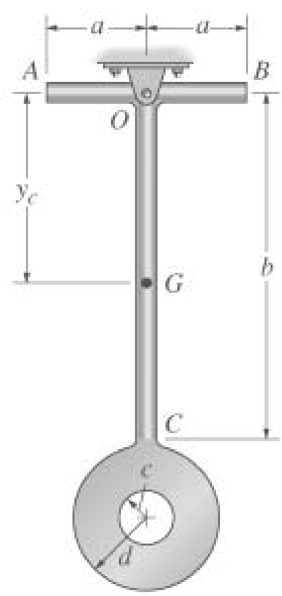
$$I_x = \gamma_{st}\left[\pi c^2 b\left(\frac{c^2}{2}\right) + \frac{\pi}{3}c^2 h\left(\frac{3c^2}{10}\right) - \frac{\pi}{3}d^2(h-a)\left(\frac{3d^2}{10}\right)\right]$$

$$I_x = 5.64\ \text{slug}\cdot\text{ft}^2$$

---

**Problem 10-112**

The pendulum consists of two slender rods $AB$ and $OC$ which have a mass density $\rho_r$. The thin plate has a mass density $\rho_p$. Determine the location $y_c$ of the center of mass $G$ of the pendulum, then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through $G$.




Given:

$\rho_r = 3\ \frac{\text{kg}}{\text{m}}$

$\rho_s = 12\ \frac{\text{kg}}{\text{m}^2}$

$a = 0.4\ \text{m}$

$b = 1.5\ \text{m}$

$c = 0.1 \text{ m}$

$d = 0.3 \text{ m}$

Solution:

$$y_c = \frac{b\rho_r \frac{b}{2} + \pi d^2 \rho_s (b+d) - \pi c^2 \rho_s (b+d)}{b\rho_r + \pi d^2 \rho_s - \pi c^2 \rho_s + \rho_r 2a}$$

$y_c = 0.888 \text{ m}$

$$I_G = \frac{1}{12} 2a\rho_r (2a)^2 + 2a\rho_r y_c^2 + \frac{1}{12} b\rho_r b^2 \ ...$$
$$+ b\rho_r \left(\frac{b}{2} - y_c\right)^2 + \frac{1}{2}\pi d^2 \rho_s d^2 + \pi d^2 \rho_s (b + d - y_c)^2 \ ...$$
$$+ \frac{1}{2}\pi c^2 \rho_s c^2 - \pi c^2 \rho_s (b + d - y_c)^2$$

$I_G = 5.61 \text{ kg}\cdot\text{m}^2$

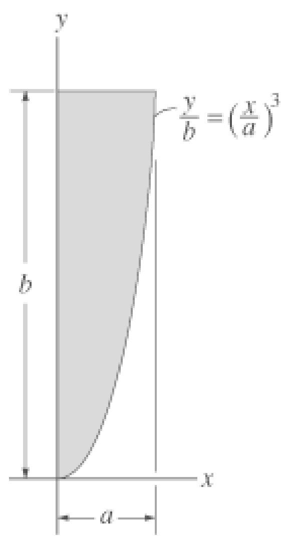
**Problem 10-113**

Determine the moment of inertia for the shaded area about the $x$ axis.

Given:

$a = 2 \text{ in}$

$b = 8 \text{ in}$




Solution:

$$I_x = \int_0^b y^2 a \left(\frac{y}{b}\right)^{\frac{1}{3}} \, dy$$

$I_x = 307 \text{ in}^4$

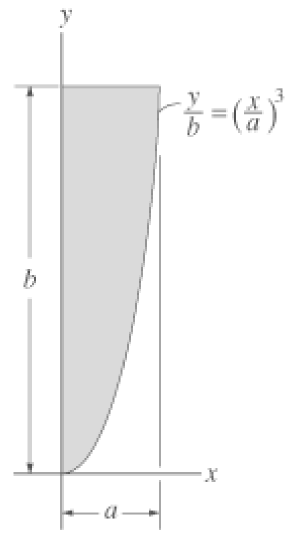
**Problem 10-114**

Determine the moment of inertia for the shaded area about the $y$ axis.

Given:

$a = 2\ \text{in}$

$b = 8\ \text{in}$



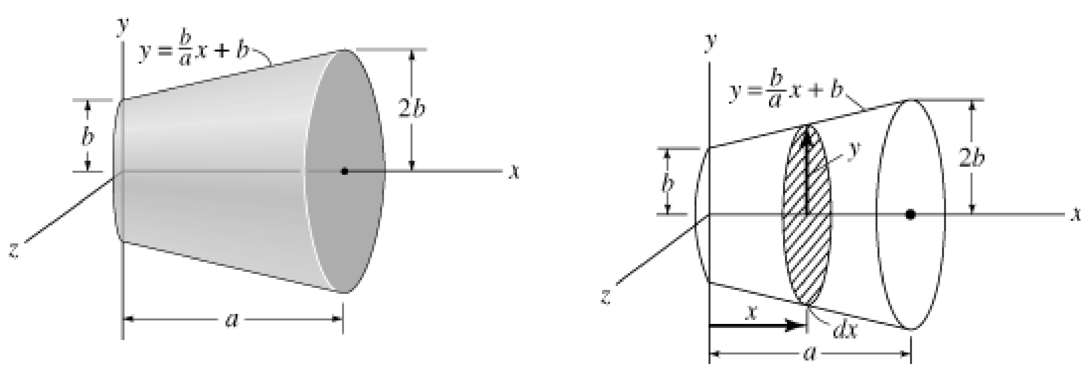

Solution:

$$I_y = \int_0^a x^2\left[b - b\left(\frac{x}{a}\right)^3\right] dx$$

$$I_y = 10.67\ \text{in}^4$$

**Problem 10-115**

Determine the mass moment of inertia $I_x$ of the body and express the result in terms of the total mass $m$ of the body. The density is constant.




Solution:

$$m = \int_0^a \rho\,\pi\left(\frac{b\,x}{a} + b\right)^2 dx = \frac{7}{3}\,a\,\rho\,\pi b^2 \qquad \rho = \frac{3m}{7\pi a\,b^2}$$

$$I_x = \int_0^a \frac{1}{2}\left(\frac{3m}{7\pi a b^2}\right)\pi\left(\frac{b x}{a}+b\right)^2\left(\frac{b x}{a}+b\right)^2 \mathrm{d}x = \frac{93}{70} m b^2$$

$$I_x = \frac{93}{70} m b^2$$

---

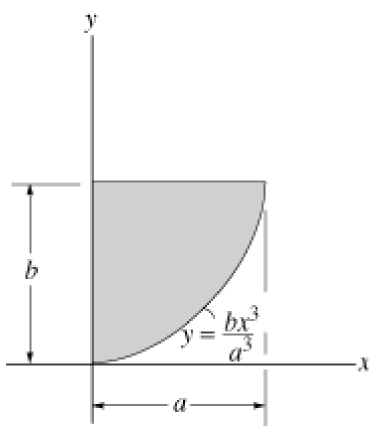
**Problem 10-116**

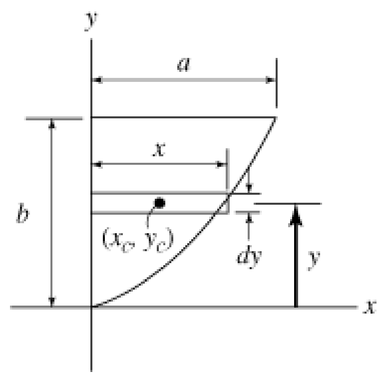
Determine the product of inertia for the shaded area with respect to the $x$ and $y$ axes.

Given:

$a = 1$ m

$b = 1$ m




Solution:

$$I_{xy} = \int_0^b \frac{1}{2} y a \left(\frac{y}{b}\right)^{\frac{1}{3}} a \left(\frac{y}{b}\right)^{\frac{1}{3}} \mathrm{d}y \qquad I_{xy} = 0.1875 \text{ m}^4$$

---

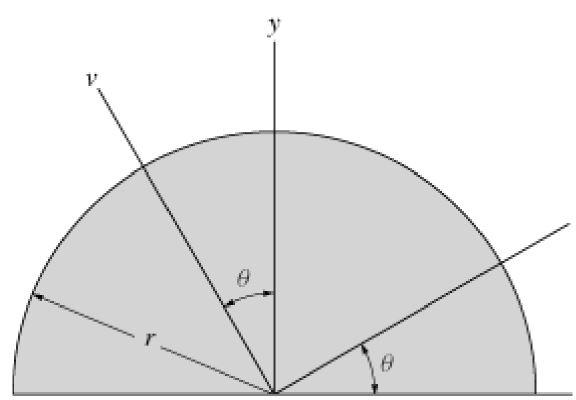
**Problem 10-117**

Determine the area moments of inertia $I_u$ and $I_v$ and the product of inertia $I_{uv}$ for the semicircular area.

Given:

$r = 60$ mm

$\theta = 30$ deg



Solution:

$$I_x = \frac{\pi r^4}{8} \qquad I_y = I_x$$

$$I_{xy} = 0 \text{ mm}^4$$

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos(2\theta) - I_{xy}\sin(2\theta) \qquad I_u = 5.09 \times 10^6 \text{ mm}^4$$

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos(2\theta) - I_{xy}\sin(2\theta) \qquad I_v = 5.09 \times 10^6 \text{ mm}^4$$

$$I_{uv} = \frac{I_x - I_y}{2}\sin(2\theta) + I_{xy}\cos(2\theta) \qquad I_{uv} = 0 \text{ m}^4$$

---

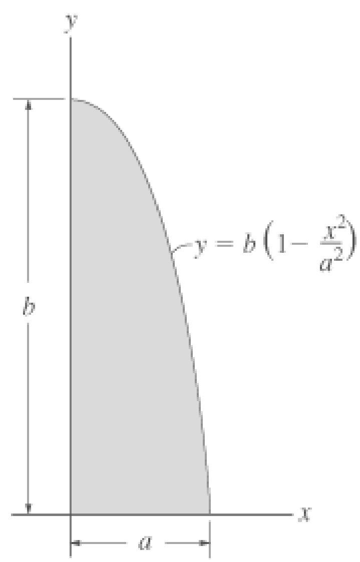
**Problem 10-118**

Determine the moment of inertia for the shaded area about the $x$ axis.

Given:

$a = 3 \text{ in}$

$b = 9 \text{ in}$




Solution:

$$I_x = \int_0^b y^2 a\sqrt{1 - \frac{y}{b}}\, dy$$

$I_x = 333 \text{ in}^4$

---

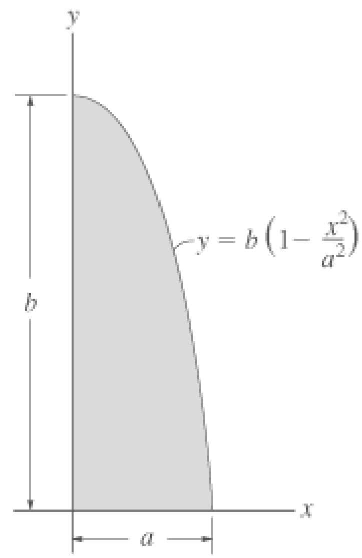
**Problem 10-119**

Determine the moment of inertia for the shaded area about the $y$ axis.

Given:

$a = 3\ \text{in}$

$b = 9\ \text{in}$




Solution:

$$I_y = \int_0^a x^2 b\left(1 - \frac{x^2}{a^2}\right) dx$$

$I_y = 32.4\ \text{in}^4$

---

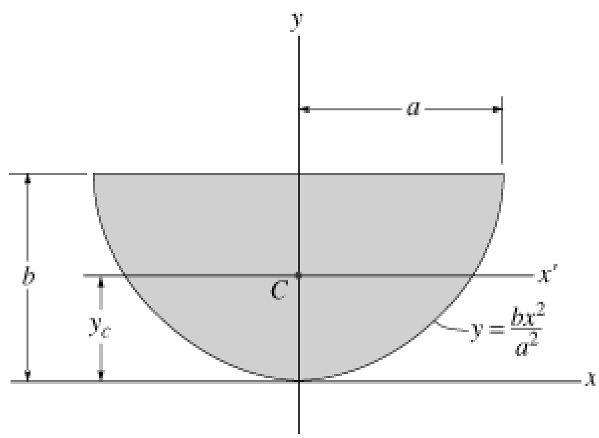
**Problem 10-120**

Determine the area moment of inertia of the area about the $x$ axis. Then, using the parallel-axis theorem, find the area moment of inertia about the $x'$ axis that passes through the centroid $C$ of the area.

Given:

$a = 200\ \text{mm}$

$b = 200\ \text{mm}$




Solution:

$$I_x = \int_0^b y^2 2a\sqrt{\frac{y}{b}}\ dy \qquad I_x = 914 \times 10^6\ \text{mm}^4$$

Find the area and the distance to the centroid

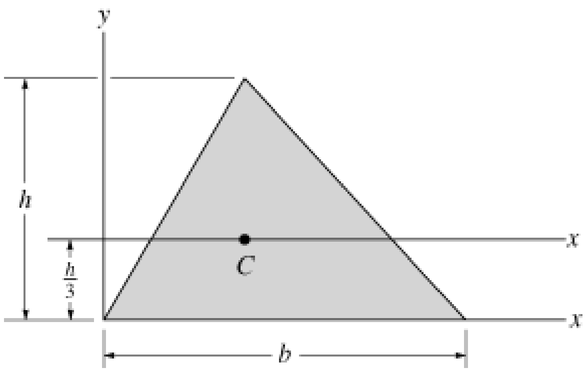
$$A = \int_0^b 2a\sqrt{\frac{y}{b}}\ dy \qquad A = 53.3 \times 10^3\ \text{mm}^2$$

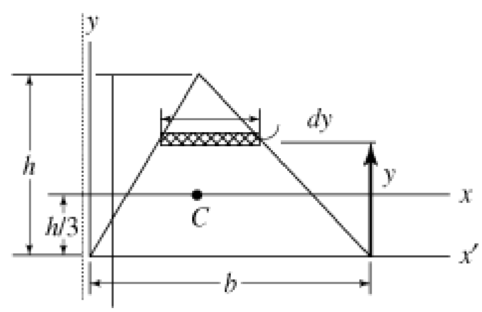
$$y_c = \frac{1}{A}\int_0^b y2a\sqrt{\frac{y}{b}}\,dy \qquad y_c = 120.0\,\text{mm}$$

$$I_{x'} = I_x - A\,y_c^{\,2} \qquad I_{x'} = 146 \times 10^6\,\text{mm}^4$$

**Problem 10-121**

Determine the area moment of inertia for the triangular area about (a) the $x$ axis, and (b) the centroidal $x'$ axis.




Solution:

$$I_x = \int_0^h y^2\,\frac{b}{h}(h-y)\,dy = \frac{1}{12}\cdot h^3\cdot b \qquad I_x = \frac{1}{12}\,b\,h^3$$

$$I_{x'} = \frac{b\,h^3}{12} - \frac{1}{2}b\,h\left(\frac{h}{3}\right)^2 = \frac{1}{36}\cdot h^3\cdot b \qquad I_{x'} = \frac{1}{36}\,b\,h^3$$

**Problem 10-122**

Determine the product of inertia of the shaded area with respect to the $x$ and $y$ axes.

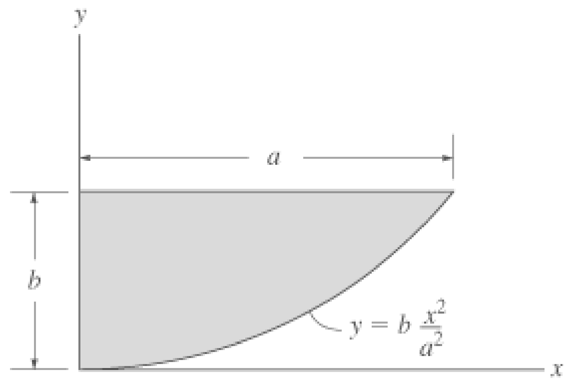
Given:

$a = 2$ in

$b = 1$ in

Solution:

$$I_{xy} = \int_0^b \frac{a}{2}\sqrt{\frac{y}{b}}\,y\,a\sqrt{\frac{y}{b}}\,dy$$




$$I_{xy} = 0.667\ \text{in}^4$$