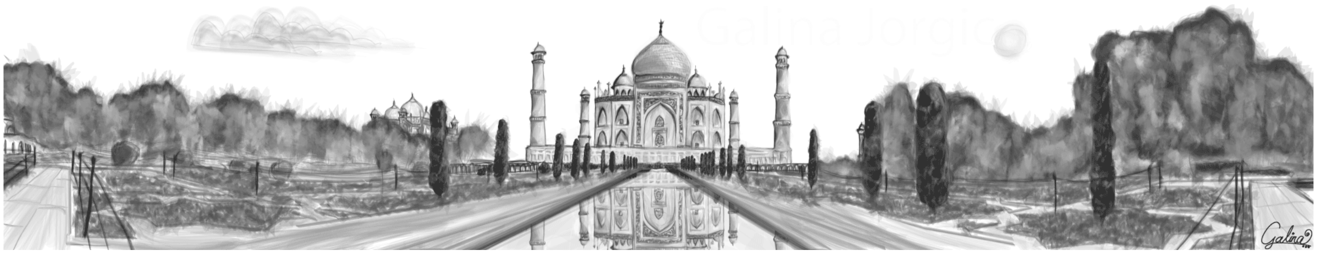
Engineering Mechanics: Statics
Lecture Series



ST10: Shear and Moment Equations
Statically Determinate Beams

This document is a written version of video lecture ST10, which can be found online at the web addresses listed below

Educative Technologies, LLC

Lab101.Space

https://www.youtube.com/c/drstructure

2018

Statics– ST10

Shear and Moment Equations for Statically Determinate Beams

In Lecture ST09 we talked about shear and bending moment in beams and how to calculate them at specific points. For example, consider the simply supported beam shown in Figure 1.

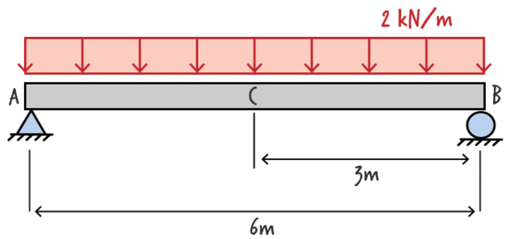


Figure 1: A simply supported beam

To determine the shear force and bending moment at point C, we start by calculating the beam's support reactions. In this case, they are shown in Figure 2.

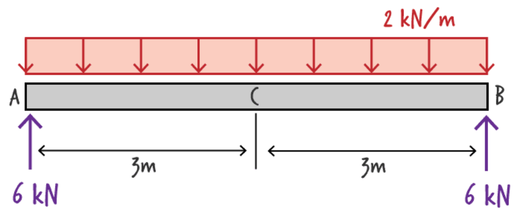


Figure 2: Support reactions for the simply supported beam

We then cut the beam at the point of interest, which is currently point C. Our next step is to write and solve the equilibrium equations for the left segment of the beam, as depicted in Figure 3.

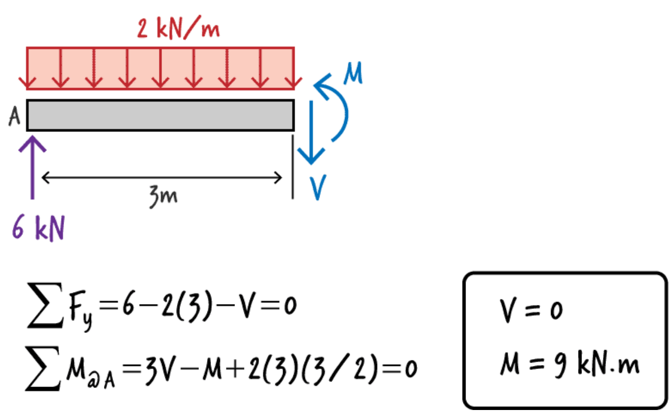


Figure 3: Shear and bending moment at point C

Generally, since the shear and bending moment may not be constant along the length of a typical beam, we often need to express these internal forces mathematically or graphically. Doing this allows us to be able to determine their critical (maximum and minimum) locations and values.

Consider the steel and glass building shown in Figure 4.

Figure 4: A steel and glass building

We are going to draw the shear and moment diagrams for three beams belonging to the skeleton of the structure. Two of the beams are in Figure 4, and are colored red. The other beam, located on the side of the building, is also shown in the color red and is visible in Figure 5.

Figure 5: Side view of the steel and glass building

In this lecture, we are going to assume that all three beams are simply supported.

Let's start our analysis with the long beam on the second floor. The roof panel, made of reinforced concrete, rests completely on two transversal beams labeled AB and CD (see Figure 6).

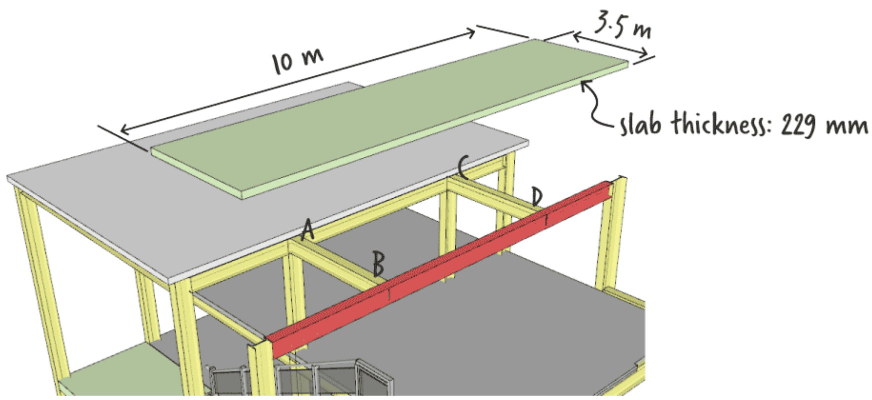


Figure 6: The long beam on the second floor of the building

Given the specific weight of concrete, $24\ kN/m^3$, we can calculate the intensity of the distributed load per beam, as shown in Figure 7.

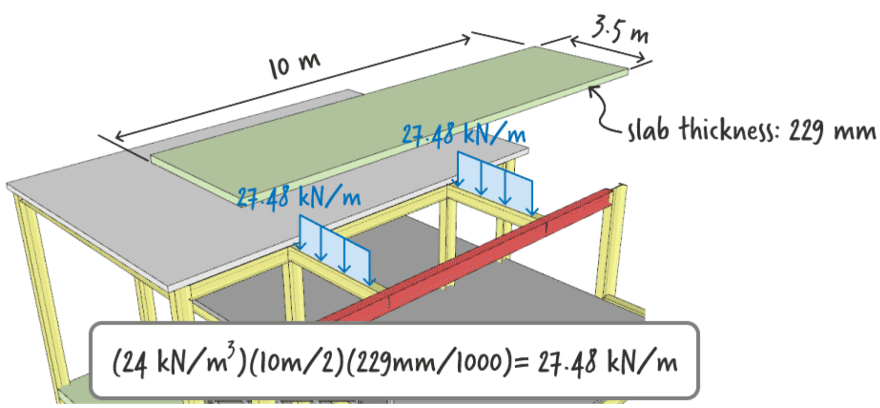


Figure 7: Slab load distributed to the two transversal beams

The distributed load on each transversal beam transfers to the red beam as a concentrated load. The magnitude of the concentrated load is equal to the intensity of the distributed load times half the length of the transversal beam. This calculation, shown in Equation [1], yields a concentrated load magnitude of 48 kN.

$$\text{Concentrated Load} = (27.48\ kN/m)(3.5m)/2 \approx 48\ kN \qquad [1]$$

Note that the red beam is going to be subjected to two concentrated loads since there are two transversal beams supported by it (see Figure 8).

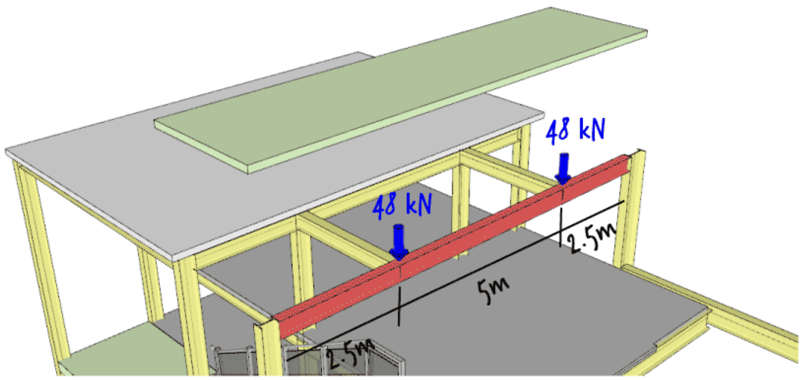
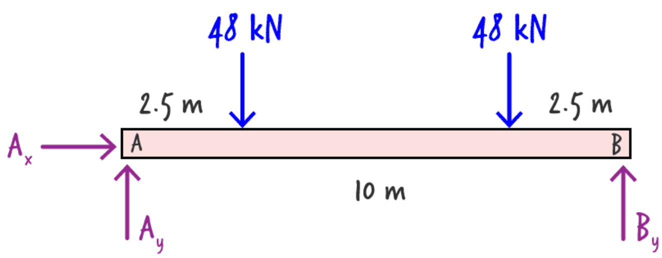


Figure 8: The long beam subjected to two concentrated loads

To come up with the shear and moment equations for this beam, we first need to calculate the support reactions. The free-body diagram for the beam, the static equilibrium equations, and the resulting support reactions are shown in Figure 9.



$$\sum F_x = A_x = 0$$

$$\sum F_y = A_y + B_y - 48 - 48 = 0$$

$$\sum M_{@A} = 2.5(48) + 7.5(48) - 10B_y = 0$$

$$\boxed{\begin{aligned} A_x &= 0 \\ A_y &= 48\text{ kN} \\ B_y &= 48\text{ kN} \end{aligned}}$$

Figure 9: Calculating support reactions for the long beam

Note how the two concentrated loads divide the beam into three segments (see Figure 10).

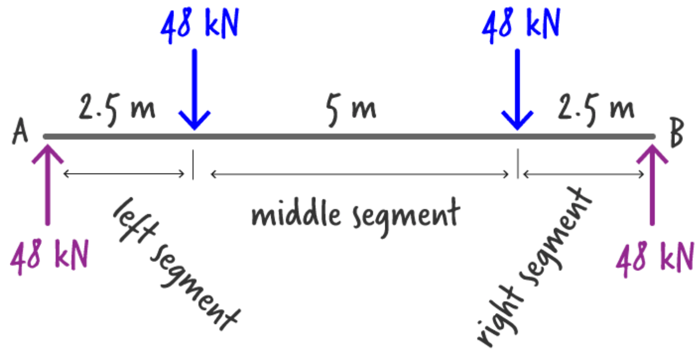


Figure 10: The segmentation of the long beam into three parts

Consequently, we need to write one shear equation and one moment equation for each segment.

For the left segment, we are going to cut the beam at some arbitrary point between the left end of the beam and the point of application of the left-most concentrated load. We label the length of the segment x, as depicted in Figure 11.

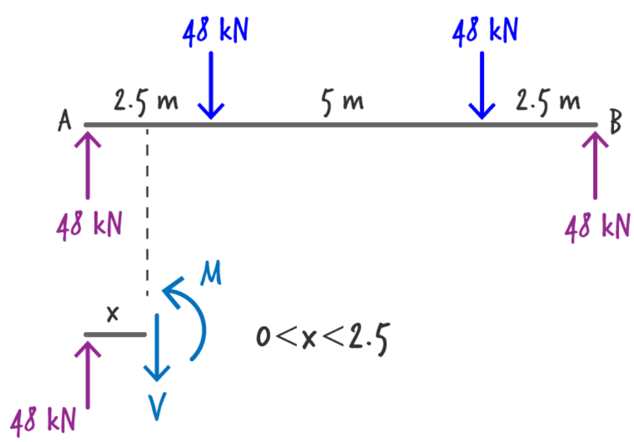


Figure 11: Diagram for expressing the internal forces for the left beam segment

The static equilibrium equations for the free-body diagram can be written as follows:

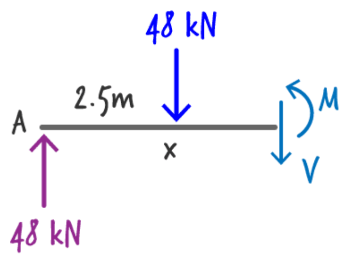
$$\sum F_y = 48 - V = 0 \quad [2]$$

$$\sum M_{@A} = (x)(V) - M = 0 \quad [3]$$

Solving the above equations for shear (V) and moment (M), we get the following equations:

$$V(x)=48\ kN \tag{4}$$
$$M(x)=48x\ kN.m \tag{5}$$

Now let's turn our attention to the middle segment of the beam. If we cut the beam shown in Figure 10 at some arbitrary point between the two concentrated loads, we get the following free-body diagram shown in Figure 12.



$2.5<x<7.5$

Figure 12: Diagram for expressing the internal forces for the middle beam segment

Here too, we are going to show shear and moment at the cut point as V and M, respectively. The two equilibrium equations for the free-body diagram are as follows:

$$\sum F_y=48-48-V=0 \tag{6}$$
$$\sum M_{@A}=2.5(48)+(x)(V)-M=0 \tag{7}$$

Solving the above equations for shear and moment, we get the necessary equations for the middle segment of the beam. They are as follows:

$$V(x)=0 \tag{8}$$
$$M(x)=120\ kN.m \tag{9}$$

As the above equations indicate, shear in the entire middle segment is zero, while moment remains constant, with a magnitude of 120 kN.m.

Finally, to write the shear and moment equations for the right segment of the beam, we can cut it at an arbitrary point between the right-most concentrated load and the right support, as shown in Figure 13.

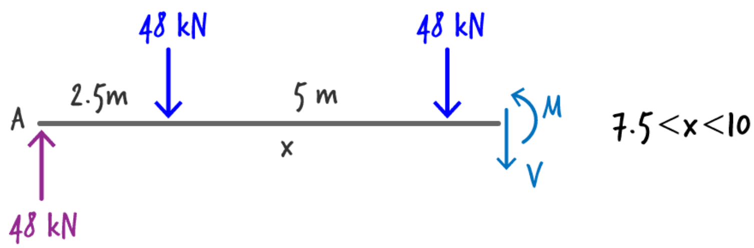


Figure 13: Diagram for expressing the internal forces for the right beam segment

The above free-body diagram results in the following equilibrium equations:

$$\sum F_y = 48 - 48 - 48 - V = 0 \qquad [10]$$

$$\sum M_{@A} = 2.5(48) + 7.5(48) + (x)(V) - M = 0 \qquad [11]$$

Solving them for V and M, we get the following equations:

$$V(x) = -48 \text{ kN} \qquad [12]$$

$$M(x) = 480 - 48x \text{ kN.m} \qquad [13]$$

In summary, given that the concentrated loads divide the beam into three segments, we need to express shear and moment using piecewise continuous functions, as shown below.

$$V(x) = \begin{cases} 48 \text{ kN} & 0 < x < 2.5 \\ 0 & 2.5 < x < 7.5 \\ -48 \text{ kN} & 7.5 < x < 10 \end{cases} \qquad [14]$$

$$M(x) = \begin{cases} 48x \text{ kN.m} & 0 \le x \le 2.5 \\ 120 \text{ kN.m} & 2.5 \le x \le 7.5 \\ 480 - 48x \text{ kN.m} & 7.5 \le x \le 10 \end{cases} \qquad [15]$$

The next beam we wish to deal with is the one supporting a concrete slab in front of the building (see Figure 4). The beam carries half the load of the slab. Because the slab rests on the beam, we are going to treat the load as a uniformly distributed load (see Figure 14).

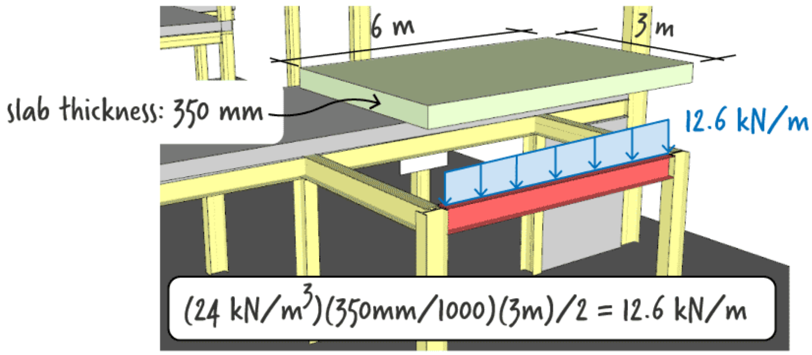
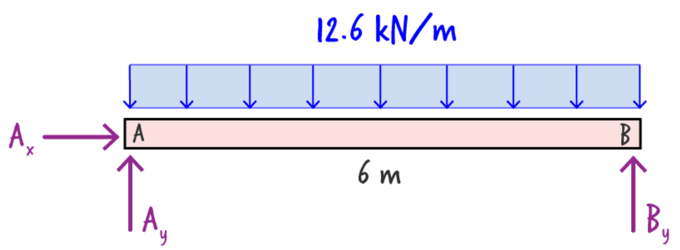


Figure 14: The uniformly distributed load applied to the short beam

The beam's free-body diagram, the needed equilibrium equations, and the calculated support reactions are shown in Figure 15.



$$\sum F_x = A_x = 0 \qquad A_x = 0$$
$$\sum F_y = A_y + B_y - (6)(12.6) = 0 \qquad A_y = 37.8 \text{ kN}$$
$$\sum M_{@A} = (12.6)(6)(3) - 6B_y = 0 \qquad B_y = 37.8 \text{ kN}$$

Figure 15: Calculating support reactions for the short beam

Since there is no point of load discontinuity within the beam, we can express the shear equation, or moment equation, using one equation only. To do so, we cut the beam at an arbitrary point and draw the needed free-body diagram, as shown in Figure 16.

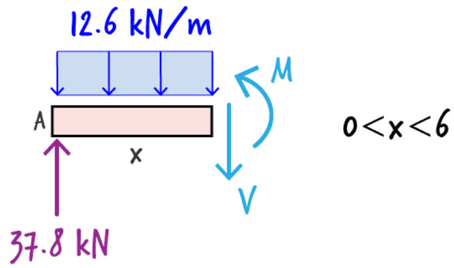


Figure 16: Diagram for expressing the internal forces for the short beam

The equilibrium equations for the free-body diagram are given below.

$$\sum F_y = 37.8 - 12.6x - V = 0 \tag{16}$$

$$\sum M_{@A} = (12.6)(x)(x/2) + (x)(V) - M = 0 \tag{17}$$

These equations, when solved for V and M, give us the shear and moment equations for the entire beam. They are as follows:

$$V(x) = 37.8 - 12.8x \text{ kN} \quad 0 < x < 6 \tag{18}$$

$$M(x) = 37.8x - 6.3x^2 \text{ kN.m} \quad 0 \le x \le 6 \tag{19}$$

Finally, let's turn our attention to the side beam on the first floor of the building shown in Figure 5. The beam is subjected to two loads: a concentrated load due to the weight of the staircase and a distributed load due to the weight of the slab (see Figure 17).

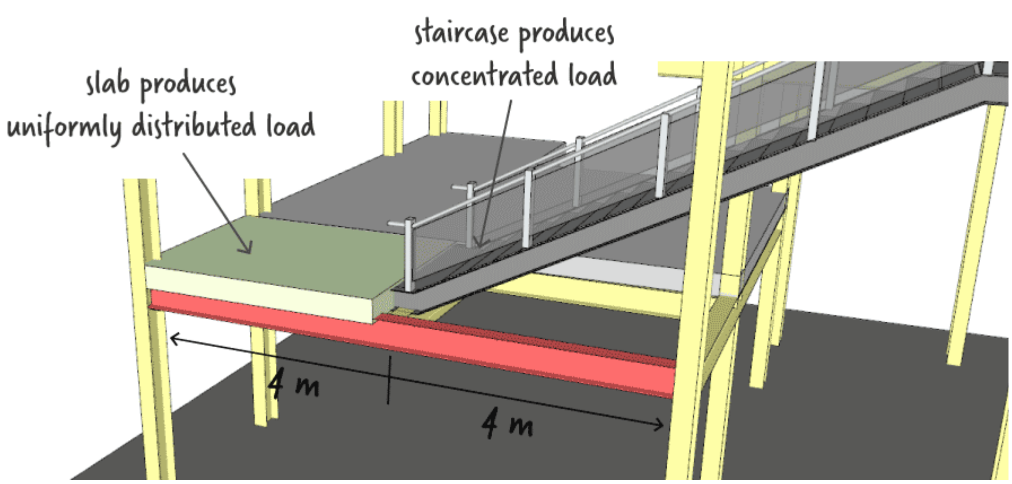


Figure 17: Slab and staircase loads on the side beam

Here, we assume the concentrated load exerted on the red beam due to the weight of the staircase is 5 kN, and the distributed load due to the weight of the slab is 8.4 kN.m, as shown in Figure 18.

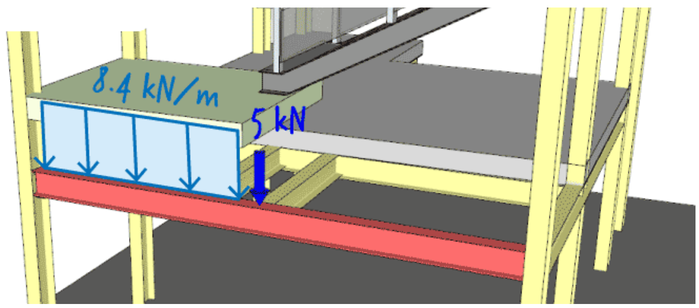
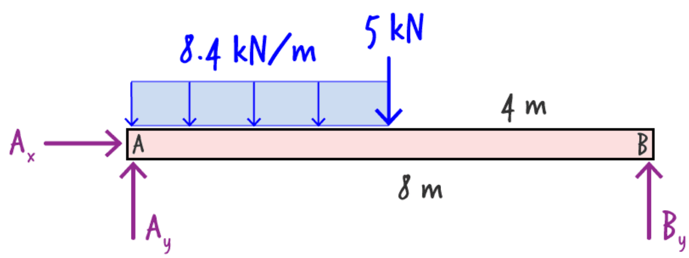


Figure 18: Distributed and concentrated loads on the side beam

To write the shear and moment equations for the beam, we start by calculating its support reactions (see Figure 19).



$$\sum F_x = A_x = 0 \qquad A_x = 0$$

$$\sum F_y = A_y + B_y - 5 - 8.4(4) = 0 \qquad A_y = 27.7\ \text{kN}$$

$$\sum M_{@A} = 8.4(4)(2) + 5(4) - 8B_y = 0 \qquad B_y = 10.9\ \text{kN}$$

Figure 19: Calculating support reactions for the side beam

In this case, the loads divide the beam into two segments. Therefore, we need to write a pair of equations for each segment. For the left segment, we cut the beam at distance x from point A and draw the needed free-body diagram, as shown in Figure 20.

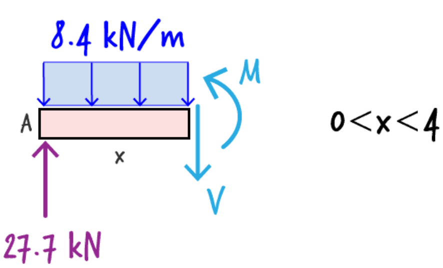


Figure 20: Diagram for expressing the internal forces for the left segment of the side beam

We then write the necessary equilibrium equations as follows:

$$\sum F_y = 27.7 - 8.4x - V = 0 \quad [20]$$

$$\sum M_{@A} = 8.4(x)(x/2) + (x)(V) - M = 0 \quad [21]$$

When Equations [20] and [21] are solved for V and M, we get the following shear and moment equations:

$$V = 27.7 - 8.4x \text{ kN} \quad [22]$$

$$M = 27.7x - 4.2x^2 \text{ kN.m} \quad [23]$$

For the shear and moment equations for the right segment of the beam, we need to cut it somewhere between the point of application of the concentrated load and the right support, as shown in Figure 21.

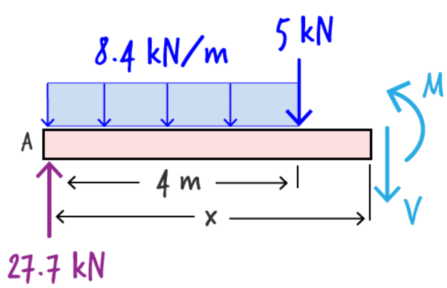


Figure 21: Diagram for expressing the internal forces for the right segment of the side beam

The two equilibrium equations for the free-body diagram shown above are:

$$\sum F_y = 27.7 - 8.4(4) - 5 - V = 0 \quad [24]$$

$$\sum M_{@A} = 8.4(4)(4/2) + (5)(4) + (x)(V) - M = 0 \quad [25]$$

Solving them for V and M, we get:

$$V = -10.9 \text{ kN} \quad [22]$$
$$M = 87.2 - 10.9x \text{ kN.m} \quad [23]$$

In summary, the shear and moment equations for the side beam can be written as follows:

$$V = \begin{cases} 27.7 - 8.4x \text{ kN} & 0 < x < 4 \\ -10.9 \text{ kN} & 4 < x < 8 \end{cases} \quad [22]$$

$$M = \begin{cases} 27.7x - 4.2x^2 \text{ kN.m} & 0 \leq x \leq 4 \\ 87.2 - 10.9x \text{ kN.m} & 4 \leq x \leq 8 \end{cases} \quad [23]$$

Exercise problems

Formulate the shear and moment equations for each of the beams shown below.