Engineering Mechanics

Engineering Mechanics

It is a branch of applied sciences that describes and predicts the state of rest or of uniform motion of ***bodies*** under the action of ***forces.***

Engineering Mechanics deals with the application of principles of mechanics and different laws in a systematic manner.

Engineering Mechanics

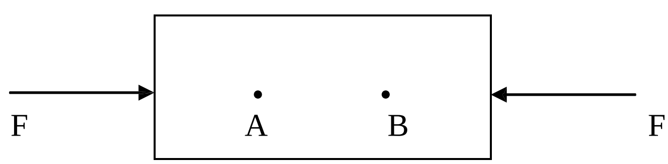
- Mechanics of Solids
  - Mechanics of Rigid bodies
    - ***Statics***
    - Dynamics
  - Mechanics of Deformable bodies
- Mechanics of Fluids

Concepts of: Physical quantity, Scalar quantity, and Vector quantity

Particle: A particle is a body of infinitely small volume and the entire mass of the body is assumed to be concentrated at a point.

Rigid body: It is one, which does not alter its shape, or size or the distance between any two points on the body does not change on the application of external forces.

Deformable body: It is one, which alters its shape, or size or the distance between any two points on the body changes on the application of external forces.



In the above example, the body considered is rigid as long as the distance between the points A and B remains the same before and after application of forces, or else it is considered as a deformable body.

Force: According to Newton‘s I law, force is defined as an action or agent, which changes or tends to change the state of rest or of uniform motion of a body in a straight line.

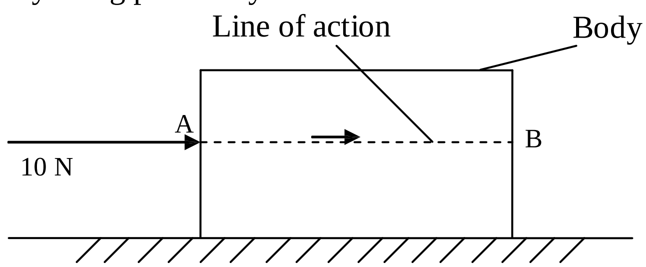
Units of force: The gravitational (MKS) unit of force is the kilogram force and is denoted as ‗kgf‘. The absolute (SI) unit of force is the Newton and is denoted as ‗N‘.

Note: 1 kgf = ‗g‘ N (But $g = 9.81 m/s^2$) Therefore 1 kgf = 9.81 N or ≅ 10 N.

Characteristics of a force

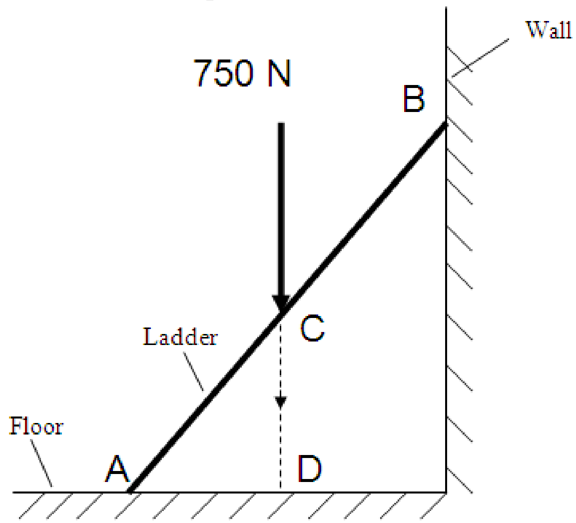
These are ones, which help in understanding a force completely, representing a force and also distinguishing one force from one another.
A force is a vector quantity. It has four important characteristics, which can be listed as follows.
1) **Magnitude**: It can be denoted as 10 kgf or 100 N.
2) **Point of application**: It indicates the point on the body on which the force acts.
3) **Line of action**: The arrowhead placed on the line representing the direction represents it.
4) **Direction**: It is represented by a co-ordinate or cardinal system.

Ex.1: Consider a body being pushed by a force of 10 N as shown in figure below.



The characteristics of the force acting on the body are
1) Magnitude is 10 N.
2) Point of application is A.
3) Line of action is A to B or AB.
4) Direction is horizontally to right.

Ex.2: Consider a ladder AB resting on a floor and leaning against a wall, on which a person weighing 750 N stands on the ladder at a point C on the ladder.

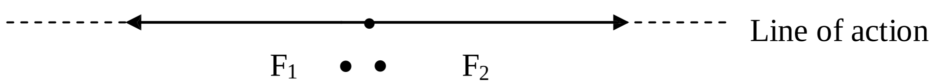


The characteristics of the force acting on the ladder are
1) Magnitude is 750 N.
2) Point of application is C.
3) Line of action is C to D or CD.
4) Direction is vertically downward.

<u>Idealization or assumptions in Mechanics</u>: In applying the principles of mechanics to practical problems, a number of ideal conditions are assumed. They are as follows.
1) A body consists of continuous distribution of matter.
2) The body considered is perfectly rigid.
3) A particle has mass but not size.
4) A force acts through a very small point.

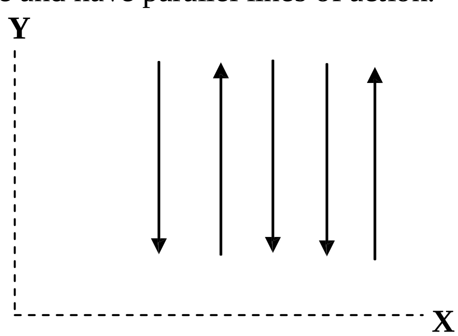
<u>Classification of force systems</u>: Depending upon their relative positions, points of applications and lines of actions, the different force systems can be classified as follows.

1) **<u>Collinear forces</u>**: It is a force system, in which all the forces have the same line of action.



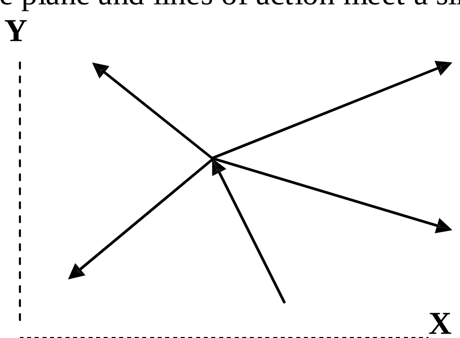
Ex.: Forces in a rope in a tug of war.

2) **<u>Coplanar parallel forces</u>**: It is a force system, in which all the forces are lying in the same plane and have parallel lines of action.



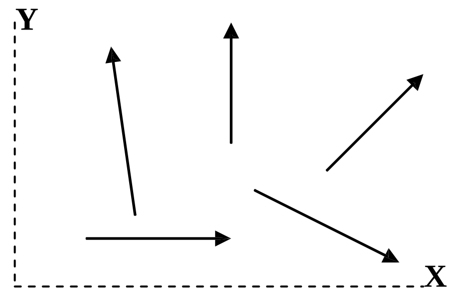
Ex.: The forces or loads and the support reactions in case of beams.

<u>3) Coplanar Concurrent forces:</u> It is a force system, in which all the forces are lying in the same plane and lines of action meet a single point.



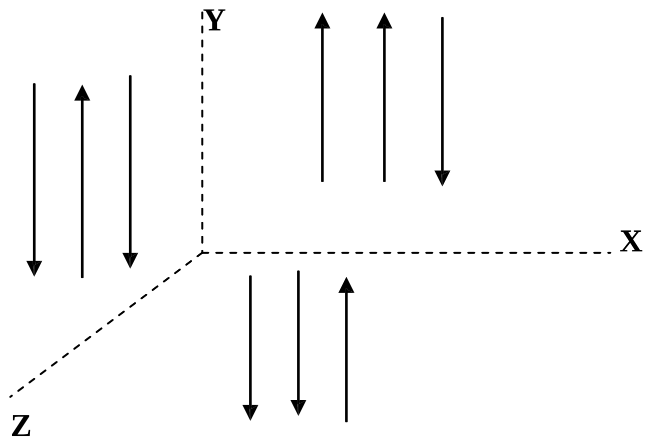
Ex.: The forces in the rope and pulley arrangement.

4) **<u>Coplanar non-concurrent forces:</u>** It is a force system, in which all the forces are lying in the same plane but lines of action do not meet a single point.



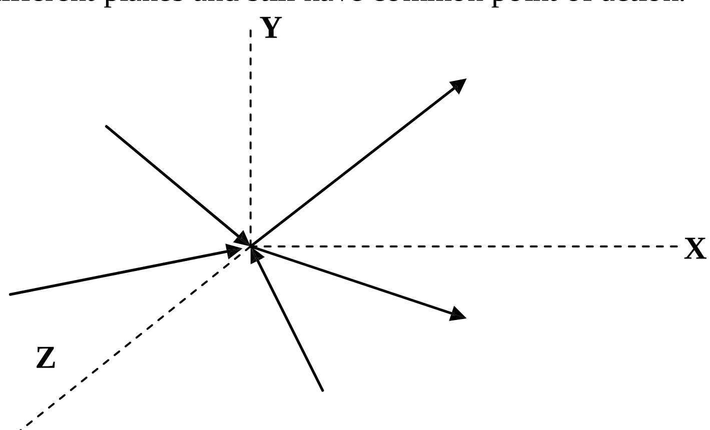
Ex.: Forces on a ladder and reactions from floor and wall, when a ladder rests on a floor and leans against a wall.

5) **<u>Non- coplanar parallel forces</u>**: It is a force system, in which all the forces are lying in the different planes and still have parallel lines of action.



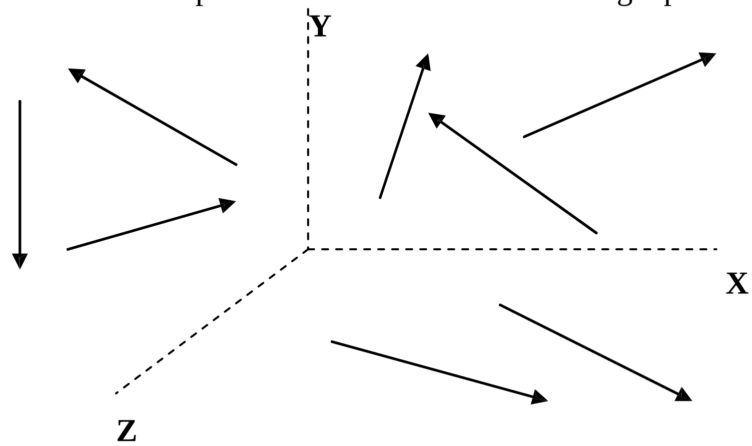
Ex: The forces acting and the reactions at the points of contact of bench with floor in a classroom.

6) <u>Non- coplanar concurrent forces</u> It is a force system, in which all the forces are lying in the different planes and still have common point of action.



Ex.: The forces acting on a tripod when a camera is mounted on a tripod.

7) **Non- coplanar non-concurrent forces**: It is a force system, in which all the forces are lying in the different planes and also do not meet a single point.



Ex.: Forces acting on a building frame.

Fundamental Laws in Mechanics

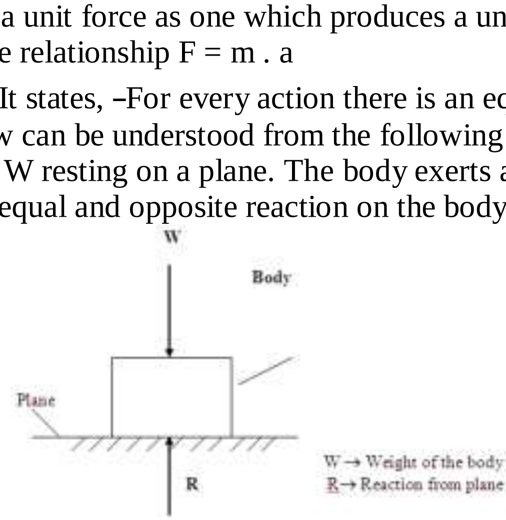
Following are considered as the fundamental laws in Mechanics.

1) Newton‘s I law
2) Newton‘s II law
3) Newton‘s III law
4) Principle or Law of transmissibility of forces
5) Parallelogram law of forces.

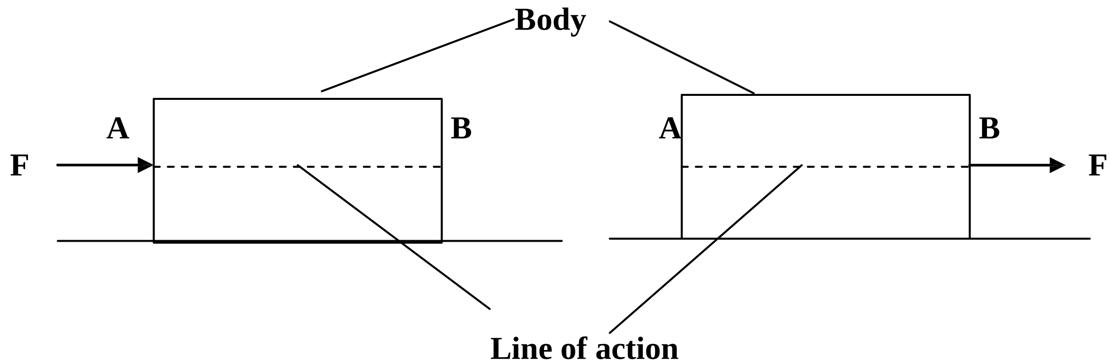
1) Newt on ' s I law: It states, –Every body continues in its state of rest or of uniform motion in a straight line, unless it is compelled to do so by force acting on it.‖
This law helps in defining a force.

2) Newt on ' s II law: It states, –The rate of change of momentum is directly proportional to the
applied force and takes place in the direction of the impressed force.‖
This law helps in defining a unit force as one which produces a unit acceleration in a body of unit mass, thus deriving the relationship F = m . a

3) Newt on ' s III law: It states, –For every action there is an equal and opposite reaction.‖
The significance of this law can be understood from the following figure.
Consider a body weighing W resting on a plane. The body exerts a force W on the plane and in turn the plane exerts an equal and opposite reaction on the body.



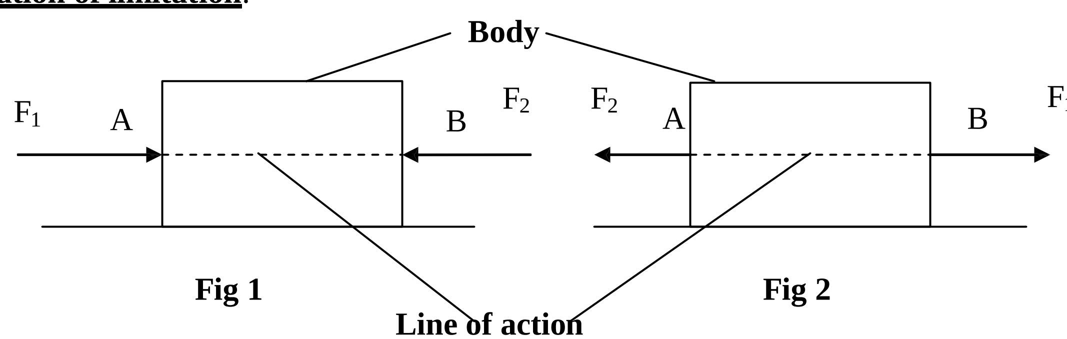
4) **<u>Principle or Law of transmissibility of forces</u>:** It states, **–The state of rest or of Uniform motion of a <u>rigid body</u> is unaltered if the point of application of the force is Transmitted to any other point along the line of action of the force.”**



From the above two figures we see that the effect of the force F on the body remains the same when the force is transmitted through any other point on the line of action of the force.

This law has a limitation that it is applicable to rigid bodies only.

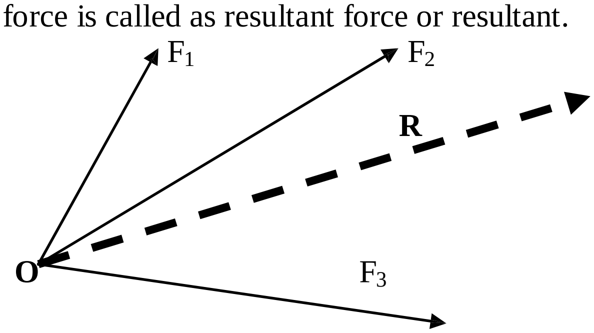
<u>Explanation of limitation</u>:



In the example if the body considered is deformable, we see that the effect of the two forces on the body are not the same when they are shifted by principle of transmissibility. In the first case the body tends to compress and in the second case it tends to elongate. Thus principle of transmissibility is not applicable to deformable bodies or it is applicable to rigid bodies only.

<u>Resultant Force:</u>

Whenever a number of forces are acting on a body, it is possible to find a single force, which can produce the same effect as that produced by the given forces acting together. Such a single force is called as resultant force or resultant.

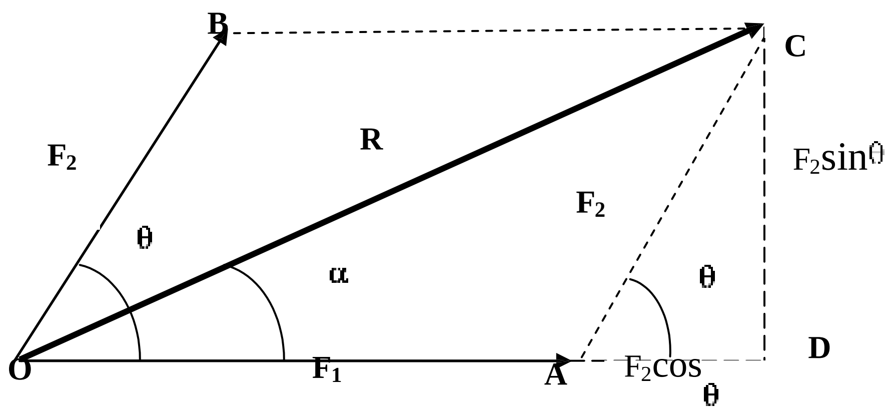


In the above figure R can be called as the resultant of the given forces F_1, F_2 and F_3.

The process of determining the resultant force of a given force system is known as **Composition of forces.**

The resultant force of a given force system can be determining by **Graphical** and **Analytical** methods. In **analytica**l methods two different principles namely: **Parallelogram law** of forces and **Method of Resolution** of forces are adopted.

Parallelogram law of forces: This law is applicable to determine the resultant of two coplanar concurrent forces only. This law states *–If two forces acting at a point are represented both in magnitude and direction by the two adjacent sides of a parallelogram, then the resultant of the two forces is represented both in magnitude and direction by the diagonal of the parallelogram passing through the same point."*



Let F_1 and F_2 be two forces acting at a point O and θ be the angle between them. Let OA and OB represent forces F_1 and F_2 respectively both in magnitude and direction. The resultant R of F1 and F_2 can be obtained by completing a parallelogram with OA and OB as the adjacent sides of the parallelogram. The diagonal OC of the parallelogram represents the resultant R both magnitude and direction.

From the figure $OC = \sqrt{OD^2 + CD^2}$

$$= \sqrt{(OA + AD)^2 + CD^2}$$

$$= \sqrt{(F_1 + F_2 \cos\theta)^2 + (F_2 \sin\theta)^2}$$

$$\text{i.e } R = \sqrt{F_1^2 + F_2^2 + 2.F_1.F_2.\cos\theta} \quad \text{---------> 1}$$

Let α be the inclination of the resultant with the direction of the F1, then

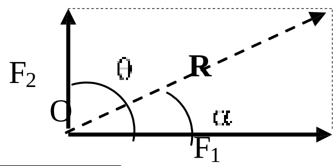
$$\alpha = \tan^{-1}\left[\frac{F_2 \sin\theta}{F_1 + F_2.\cos\theta}\right] \quad \text{-------> 2}$$

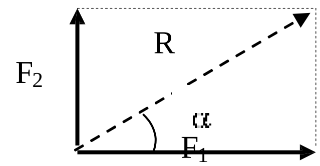
Equation 1 gives the magnitude of the resultant and Equation 2 gives the direction of the resultant.

<u>Different cases of parallelogram law:</u>

For different values of θ, we can have different cases such as follows:

<u>Case 1: When $\theta = 90^0$:</u>

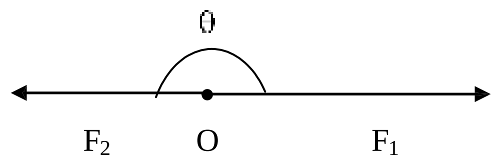




$R = \sqrt{F_1^2 + F_2^2}$

$\alpha = \tan^{-1}\left[\dfrac{F_2}{F_1}\right]$

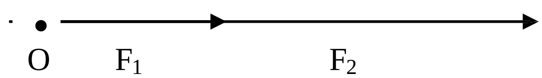
<u>Case 2: When $\theta = 180^0$:</u>



$R = [\ F_1 - F_2\ ]$

$\alpha = 0^0$

<u>Case 3: When $\theta = 0^0$:</u>



$R = [\ F_1 + F_2\ ]$

$\alpha = 0^0$

Example *Determine the magnitude of the resultant of the two forces of magnitude 12 N and 9 N acting at a point if the angle between the two forces is 30°.*

Solution:

$$P = 12 \text{ N}$$
$$Q = 9 \text{ N}$$
$$\theta = 30$$
$$\text{Resultant } R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$
$$= \sqrt{(12)^2 + (9)^2 + 2 \times 12 \times 9 \cos 30}$$
$$R = 20.29 \text{ N}$$
$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$
$$= \frac{9\sin 30}{12 + 9\cos 30}$$
$$\tan\alpha = 0.2273$$
$$\boldsymbol{\alpha = 12.81.}$$

Example *Find the magnitude of two equal forces acting at a point with an angle of 60° between them, if the resultant is equal to $30\sqrt{3}$ N.*

Solution:

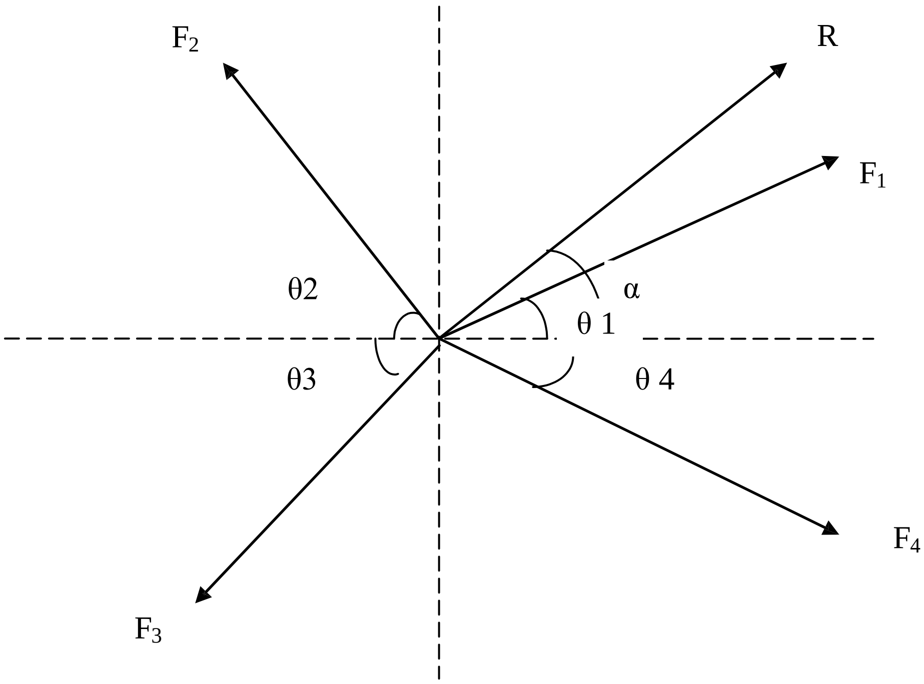
$$P = Q = F$$
$$R = 30\sqrt{3}$$
$$\theta = 60°$$
$$R = \sqrt{F_1{}^2 + F_2{}^2 + 2F_1F_2\cos\theta}$$
$$30\sqrt{3} = \sqrt{2F^2 + 2F^2\cos 60}$$
$$= F\sqrt{2(1 + \cos 60)}$$
$$F = \frac{30\sqrt{3}}{\sqrt{3}}$$
$$\mathbf{F = 30\ N.}$$

MODULE 2

Analysis of Force Systems-Concurrent & Non Concurrent System

Introduction

If two or more forces are acting in a single plane and passing through a single point , such a force system is known as a



coplanar concurrent force system

Let F1, F2, F3 , F4 represent a coplanar concurrent force system. It is required to determine the resultant of this force system.

It can be done by first resolving or splitting each force into its component forces in each direction are then algebraically added to get the sum of component forces.

These two sums are then combines using parallelogram law to get the resultant of the force systems.

In the ∑fig, let fx_1, fx_2, fx_3, fx_4 be the components of Fx_1, Fx_2, Fx_3, Fx_4 be the forces in the X-direction.

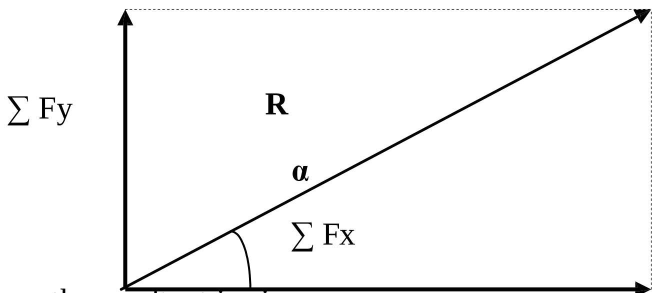
Let ∑ Fx be the algebraic sum of component forces in an x-direction

$$\sum Fx = fx_1 + fx_2 + fx_3 + fx_4$$

Similarly,

$$\sum Fy = fy_1 + fy_2 + fy_3 + fy_4$$

By parallelogram law,



The magnitude os the resultant is given as

$$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$$

The direction of resultant can be obtained if the angle α made by the resultant with x direction is determined here,

$$\alpha = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$$

The steps to solve the problems in the coplanar concurrent force system are, therefore as follows.

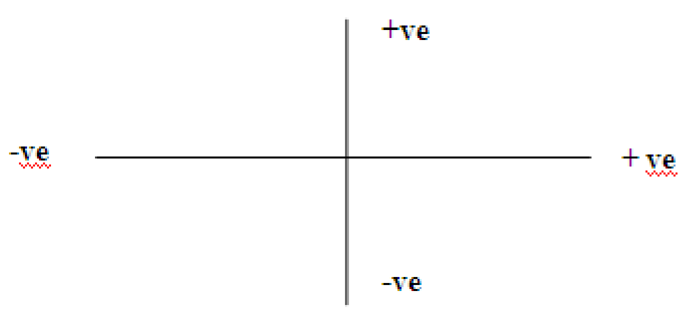
1. Calculate the algebraic sum of all the forces acting in the x-direction (ie. $\sum Fx$) and also in the y- direction (ie. $\sum Fy$)
2. Determine the direction of the resultant using the formula

$$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$$

3. Determine the direction of the resultant using the formula

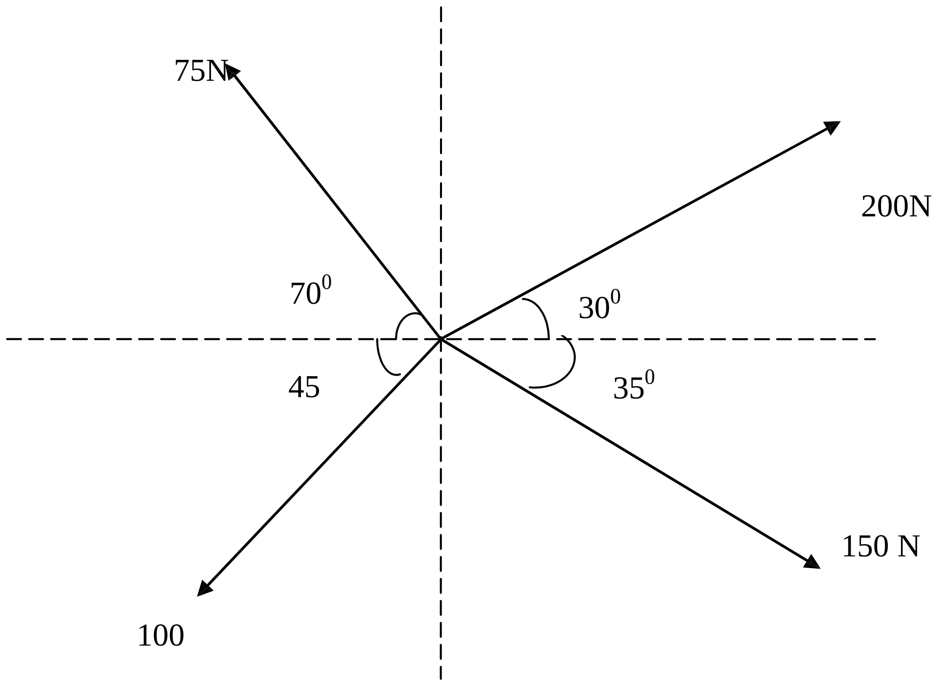
$$\alpha = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$$

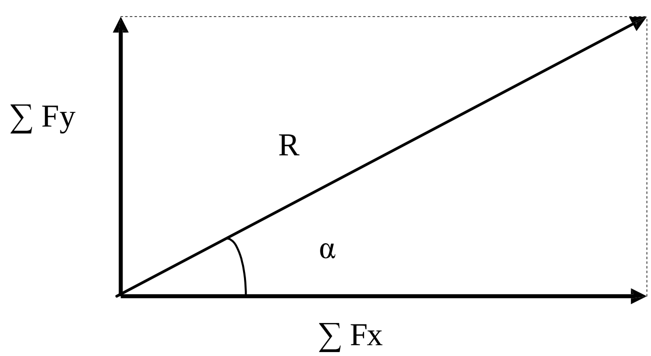
Sign Conventions:



Problems

1. Determine the magnitude & direction of the resultant of the coplanar concurrent force system shown in figure below.





Let R be the given resultant force system

Let α be the angle made by the resultant with x- direction.

The magnitude of the resultant is given as

$$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2} \quad \text{and} \quad \alpha = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$$

$\sum Fx = 200\cos 30^0 - 75\cos70^0 - 100\cos45^0 + 150\cos 35^0$

$\sum Fx = 199.7N$

$\sum Fy = 200\sin 30^0 + 75\sin70^0 - 100\sin45^0 - 150\sin 35^0$

$\sum Fy = 13.72\ N$

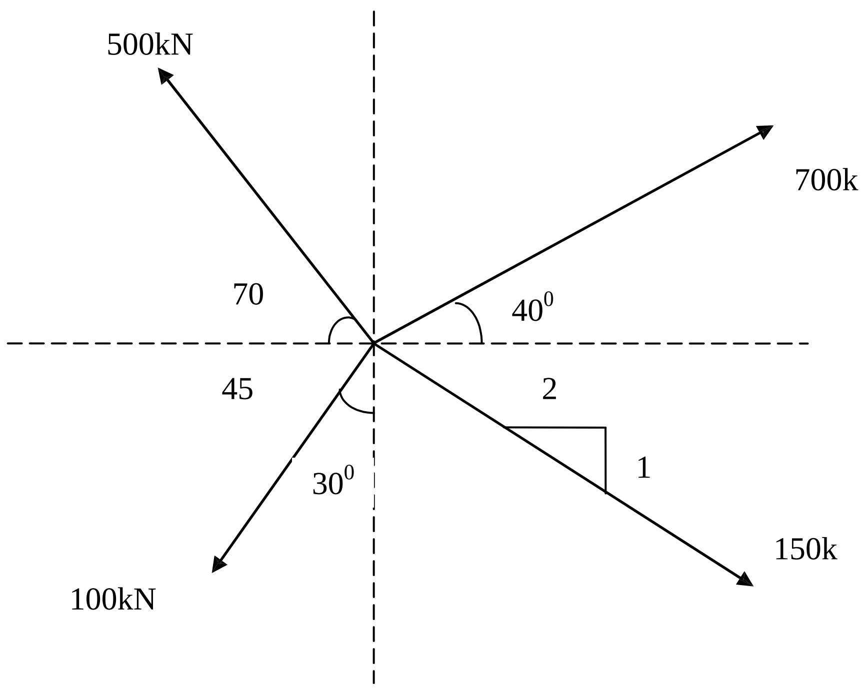
$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$

$R = 200.21N$

$\alpha = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$

$\alpha = \tan^{-1}(13.72/\ 199.72) = 3.93^0$

2. Determine the resultant of the concurrent force system shown in figure.



Let R be the given resultant force system

Let α be the angle made by the resultant with x- direction.

The magnitude of the resultant is given as

$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$ and $\alpha = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$

$\sum Fx = 700\cos 40^0 - 500\cos70^0 - 800\cos60^0 + 200\cos 26.56^0$

$\sum Fx = 144.11$ kN

$\sum Fy = 700\sin 40^0 + 500\sin70^0 - 800\sin60^0 - 200\sin 26.56^0$

$\sum Fy = 137.55$kN

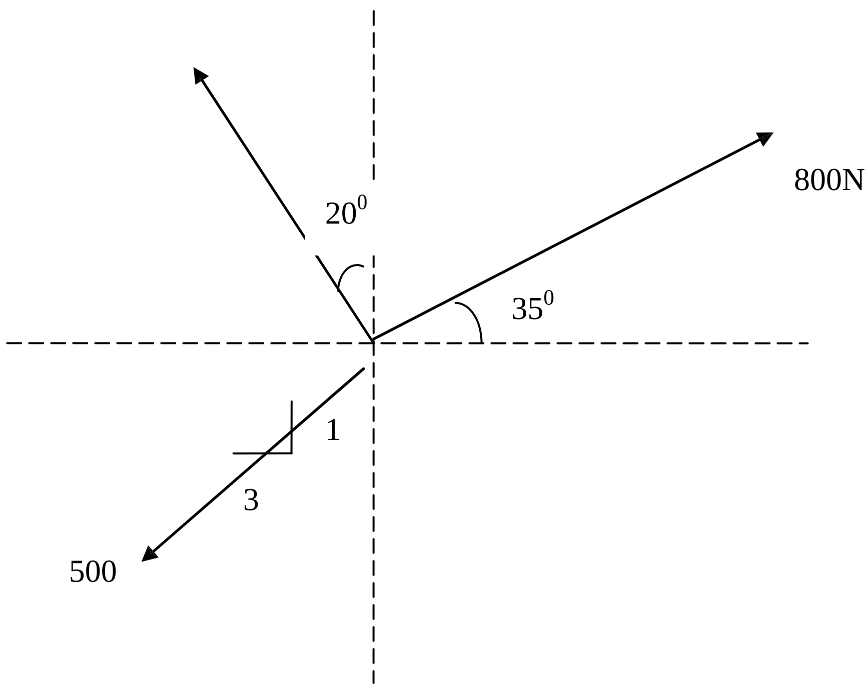
$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$

R= 199.21N

$\alpha = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$

$\alpha = \tan^{-1}(137.55/ 144.11) = 43.66$

3. Determine the resultant of a coplanar concurrent force system shown in figure below



Let R be the given resultant force system

Let α be the angle made by the resultant with x- direction.

The magnitude of the resultant is given as

$$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2} \quad \text{and} \quad \alpha = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$$

$\sum Fx = 800\cos 35^0 - 100\cos 70^0 + 500\cos 60^0 + 0$

$\sum Fx = 1095.48$ N

$\sum Fy = 800\sin 35^0 + 100\sin 70^0 + 500\sin 60^0 - 600$

$\sum Fy = 110.90$ N

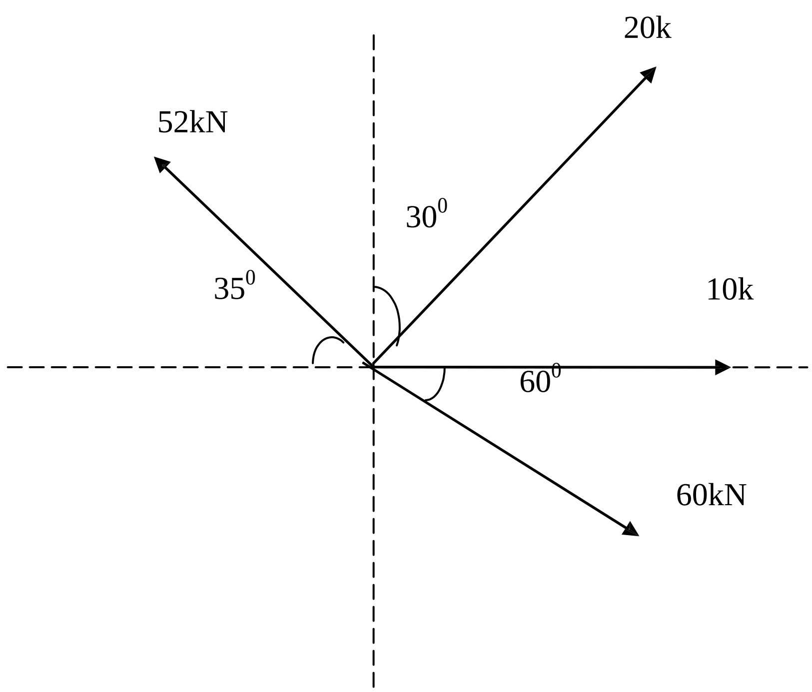
$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$

$R = 1101.08$ N

$\alpha = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$

$\alpha = \tan^{-1}(110.90/ 1095.48) = 5.78^0$

4. The Magnitude and direction of the resultant of the resultant of the coplanar concurrent force system shown in figure.



Let R be the given resultant force system

Let α be the angle made by the resultant with x- direction.

The magnitude of the resultant is given as

$$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2} \quad \text{and} \quad \alpha = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$$

$\sum Fx = 20\cos 60^0 - 52\cos 30^0 + 60\cos 60^0 + 10$

$\sum Fx = 7.404$ kN

$\sum Fy = 20 \sin 60^0 + 52\sin 30^0 - 60\sin 60^0 + 0$

$\sum Fy = -8.641$ kN

$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$

R= 11.379 N

$\alpha = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$

$\alpha = \tan^{-1}(-8.641 / 7.404) = -49.40$

5. Determine the Magnitude and direction of the resultant of the resultant of the coplanar concurrent force system shown in figure.

$\theta_1 = \tan^{-1}(1/2) = 26.57$

$\theta_2 = 53.13$

Let R be the given resultant force system

Let α be the angle made by the resultant with x- direction.

The magnitude of the resultant is given as

$$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2} \quad \text{and} \quad \alpha = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$$

$\sum Fx = 200\cos 26.57^0 - 400\cos 53.13^0 - 50\cos 60^0 + 100\cos 50^0$

$\sum Fx = -21.844$ kN

$\sum Fy = 200 \sin 26.57^0 + 400\sin 53.13^0 - 50\sin 60^0 - 100 \sin 50^0$

$\sum Fy = 289.552$ kN

$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$

R= 290.374 N

$\alpha = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$

$\alpha = \tan^{-1}(289.552 / 21.844) = -85.68$

6. A hook is acted upon by 3 forces as shown in figure. Determine the resultant force on the hook.

Let R be the given resultant force system

$\sum Fx =?$

$\sum Fy =?$

Let α be the angle made by the resultant with x- direction.

The magnitude of the resultant is given as

$$R= \sqrt{(\sum Fx)^2 +(\sum Fy)^2} \quad \text{and} \quad \alpha = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$$

$\sum Fx = 80 \cos 25^0 + 50\cos 50^0 + 10\cos 45^0$

$\sum Fx = 111.71$ kN

$\sum Fy = 80 \sin 25^0 + 50\sin 50^0 - 10\cos 45^0$

$\sum Fy = 65.04$ kN

$R= \sqrt{(\sum Fx)^2 +(\sum Fy)^2}$

$R= 129.26$ N

$\alpha = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$

$\alpha = \tan^{-1}(65.04 /111.71) = 30.20$

7. Two forces are acting on a structure at apoint‗O‖ ,asshowninfig.Determinethe resultant force on the structure.

Let R be the given resultant force system

Let α be the angle made by the resultant with x- direction.

In Δ^{le} AOC

$\cos 60^0 = AC/ 6$

$AC = 6\cos 60^0$

AC =3 m, BC= 6m

In Δ^{le} AOC

$\sin 60^0 = OC/ 6$

$OC = 6\sin 60^0$

OC = 5.196m

In Δ^{le} OBC,

$\theta = \tan^{-1}(OC/BC)$

$= \tan^{-1}(5.19 / 6) = 40.89^0$

$\sum Fx = 800 - 600 \cos 40.89^0$

$\sum Fx = 346.41$ N

$\sum Fy = 0 - 600 \sin 40.89^0$

$\sum Fy = - 392.76$ kN

$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$

$R = 523.7$ N

$\alpha = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$

$\alpha = \tan^{-1}(-392.76 / 346.41) = 48.58$

Note:

From the above two figures, we can write

$\sum Fx = R \cos \alpha$

i.e The algebraic sum of all horizontal component forces is equal to the horizontal component of the resultant.

$\sum Fy = R \sin \alpha$

i.e The algebraic sum of all vertical component forces is equal to the vertical component of the resultant.

8. Two forces of magnitude 500N & 100N are acting at a point as shown in fig below. Determine the magnitude & Direction of third unknown force, such that the resultant of all the three forces has a magnitude of 1000N, making an angle of 45^0 as shown.

Let F_3 be the required third unknown force, which makes angle θ_3 with x- axis as shown

$F_3 = ?$ $\quad \theta_3 = ?$

We know that

$R \cos \alpha = \sum Fx$

$1000 \cos 45^0 = 500 \cos 30^0 + 1000 \cos 60^0 + F_3 \cos \theta_3$

$F_3 \cos \theta_3 = -225.906$N ------------- (1)

$R \sin \alpha = \sum Fy$

$1000\sin 45^0 = 500\sin 30^0 + 1000\sin 60^0 + F_3 \sin \theta_3$

$F_3 \sin \theta_3 = -408.91N$ ---------------- (2)

Dividing the Equation (2) by (1)

i.e. $F_3 \sin \theta_3 / F_3 \cos \theta_3 = -408.91 / -225.906$

$\tan \theta_3 = 1.810$

$\theta_3 = \tan^{-1}(1.810)$

$= 61.08$

From (1)

$F_3 \cos \theta_3 = -225.906N$

$F_3 = -225.906 / \cos 61.08 = -467.14$ N

Here , we have –ve values from both $F_3 \cos \theta_3$ and $F_3 \sin \theta_3$ (X & Y components of force F3). Thus the current direction for force F3 is represented as follows.

9. Two forces of magnitude 500N and 100N are acting at a point as shown in fig below. Determine the magnitude & direction of a 3rd unknown force such that the resultant of all the three forces has a magnitude of 1000N, making an angle of 450 & lying in the second quadrant.

$F_3=?$, $\theta_3=?$

Let F_3 be a required third unknown force making an angle θ_3 with the x- axis to satisfy the given condition.

Let us assume F_3 to act as shown in fig.

We known that

$R \cos \alpha = \sum Fx$

$-1000\cos 45^0 = 500\cos 30^0 + 100\cos 60^0 + F_3 \cos \theta_3$

$F_3 \cos \theta_3 = -1190.119N$ ------------- (1)

$R \sin \alpha = \sum Fy$

$1000\sin 45^0 = 500\sin 30^0 + 100\sin 60^0 + F_3 \sin \theta_3$

$F_3 \sin \theta_3 = 370.50N$ ---------------- (2)

Dividing the Equation (2) by (1)

i.e. $F_3 \sin \theta_3 / F_3 \cos \theta_3 = 370.50 / -1190.119$

$\tan \theta_3 = 0.3113$

$\theta_3 = \tan^{-1}(0.3113)$

$= 17.29$

From (2)

$F_3 \sin \theta_3 = 370.50N$

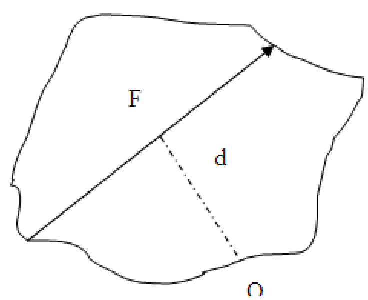
$F_3 = 370.50/\sin 17.29 = 1246.63N$

COMPOSITION OF COPLANAR NONCONCURRENT FORCE SYSTEM

If two or more forces are acting in a single plane, but not passing through the single point, such a force system is known as coplanar non concurrent force system.

Moment of Force:

It is defined as the rotational effect caused by a force on a body. Mathematically Moment is defined as the product of the magnitude of the force and perpendicular distance of the point from the line of action of the force from the point.

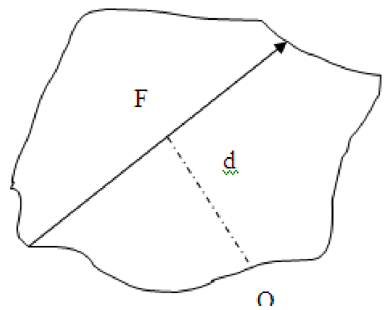


Let –F‖ be a force acting in a plane. Let‖ O‖ be a point or particle in the same plane. Let –d ‖ be the perpendicular distance of the line of action of the force from the point –O‖ . Thus the moment of the force about the point –O‖ is given as

Mo= F x d

Moment or rotational effect of a force is a physical quantity dependent on the units for force and distance. Hence the units for moment can be –Nm‖ or –KNm‖ or – N mm‖ etc.

The moment produced by a force about differences points in a plane is different. This can be understood from the following figures.



Let – F‖ be a force in a plane and O_1, O_2, and O_3 be different points in the same plane
Let moment of the force – F‖ about point O_1 is Mo,
$Mo_1 = F \times d_1$
Let moment of the force – F‖ about point O_2 is Mo,
$Mo_2 = F \times d_2$

Let moment of the force – F‖ about point O_3 is Mo,
Mo= 0x F
The given force produces a clockwise moment about point O1 and anticlockwise moment about O_2 . A clockwise moment ( ) is treated as positive and an anticlockwise moment ( ) is treated as negative.

Note; The points O_1, O_2, O_3 about which the moments are calculated can also be called as moment centre.

Couple

Two forces of same magnitude separated by a definite distance, (acting parallely) in aopposite direction are said to form a couple.
A couple has a tendency to rotate a body or can produce a moment about the body. As such the moment due to a couple is also denoted as M.
Let us consider a point O about which a couple acts. Let S be the distance separating the couple. Let d1 & d2 be the perpendicular distance of the lines of action of the forces from the point o.

Thus the magnitude of the moment due to the couple is given a s
Mo = (Fx d1) + (F x d2)

Mo = F x d
i.e The magnitude of a moment due to a couple is the product of force constituting the couple & the distance separating the couple . Hence the units for magnitude of a couple can be N m, kN m , N mm etc.

Varignon's principle of moments:

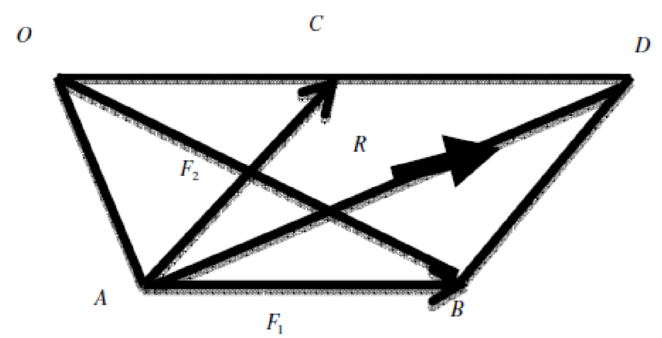
If a number of coplanar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point.
PROOF:
For example, consider only two forces F_1 and F_2 represented in magnitude and direction by AB and AC as shown in figure below.

Let O be the point, about which the moments are taken. Construct the parallelogram ABCD and complete the construction as shown in fig.

By the parallelogram law of forces, the diagonal AD represents, in magnitude and Direction, the resultant of two forces F_1 and F_2, let R be the resultant force.
By geometrical representation of moments
the moment of force about O=2 Area of triangle AOB
the moment of force about O=2 Area of triangle AOC
the moment of force about O=2 Area of triangle AOD

But,
Area of triangle AOD=Area of triangle AOC + Area of triangle ACD
Also, Area of triangle ACD=Area of triangle ADB=Area of triangle AOB
Area of triangle AOD=Area of triangle AOC + Area of triangle AOB

Multiplying throughout by 2, we obtain
2 Area of triangle AOD =2 Area of triangle AOC+2 Area of triangle AOB
i.e., Moment of force R about O=Moment of force F_1 about O + Moment of force F_2 about O
Similarly, this principle can be extended for any number of forces.



By using the principles of resolution composition & moment it is possible to determine Analytically the resultant for coplanar non-concurrent system of forces.
The procedure is as follows:

1. Select a Suitable Cartesian System for the given problem.

2. Resolve the forces in the Cartesian System
3. Compute $\sum fxi$ and $\sum fyi$
4. Compute the moments of resolved components about any point taken as the moment Centre O. Hence find $\sum M0$

$$R = \sqrt{\left(\Sigma f x_i\right)^2 + \left(\Sigma f y_i\right)^2}$$

$$\alpha_R = \tan^{-1}\left(\frac{\sum f y_i}{\sum f x_i}\right)$$

5. Compute moment arm $d_R = \left|\frac{\sum M_o}{R}\right|$

6. Also compute x- intercept as $x_R = \left|\frac{\sum M_o}{\sum f_{x_i}}\right|$

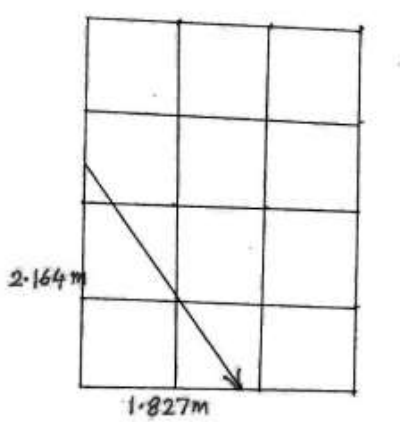
7. And Y intercept as $y_R = \left|\frac{\sum M_o}{\sum f_{x_i}}\right|$

Problems

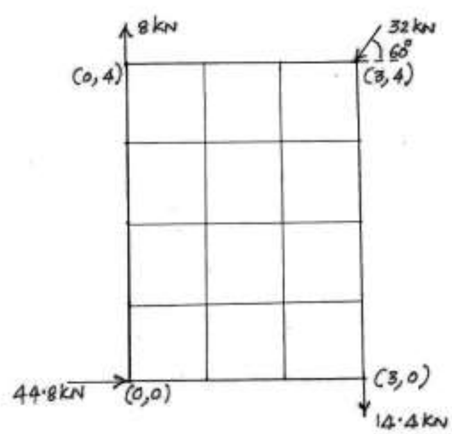
Example 1: Compute the resultant for the system of forces shown in Fig 2 and hence compute the Equilibriant.

$$\sum f_{x_i} = 44.8 - 32\cos 60^\circ$$
$$= 28.8 \text{ KN}$$

$$\sum f_{y_i} = 8 - 14.4 - 32\sin 60^\circ$$
$$= -34.11 \text{ KN}$$
$$R = 44.6 \text{ KN}$$
$$\alpha_R = 49.83^\circ$$
$$\circlearrowleft + \sum M_o = -14.4(3) + 32\cos 60^o (4) - 32\sin 60^o (3)$$
$$= -62.34 KNM$$
$$d_R = \frac{62.34}{44.64} = 1.396 \text{ m}$$
$$x_R = \frac{62.34}{34.11} = 1.827 \text{ m}$$
$$y_R = \frac{62.34}{28.8} = 2.164 \text{ m}$$



Fig· 2(a) Example 1



Fig·2 Example 1

Example 2: Find the Equilibriant for the rigid bar shown in Fig 3 when it is subjected to forces.

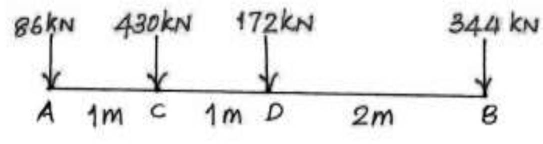


Fig · 3 Example 2

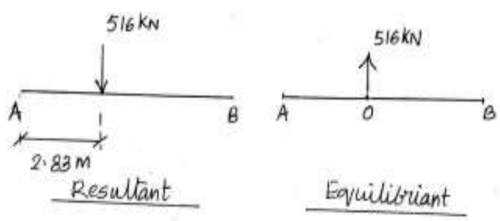


Fig · 3(a) Example 2

$$\circlearrowleft + \sum M_A = -430(1) + 172(2) - 344(4)$$
$$= -1462 \text{ KNM}$$

- Resultant and Equilibriant

$$\sum f_{x_i} = 0$$
$$\sum f_{y_i} = -516KN$$
$$\alpha_R = 90^o;$$

Example 3: A bar AB of length 3.6 m and of negligible weight is acted upon by a vertical force F1 = 336kN and a horizontal force F2 = 168kN shown in Fig 4. The ends of the bar are in contact with a smooth vertical wall and smooth incline. Find the equilibrium position of the bar by computing the angle θ.

$$\tan\alpha = {0.9}/{1.2}$$
$$\alpha = 36.87^o$$

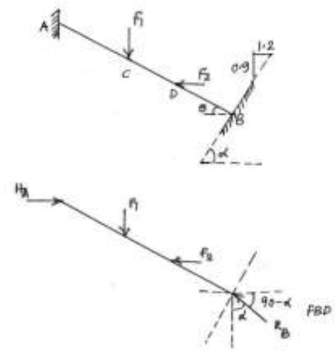


Fig 4 Example 3

$$\sum f_{x_i} = 0$$
$$H_A - F_2 - R_B \cos 53.13^o = 0 \ldots\ldots\ldots\ldots(1)$$
$$\sum f_{y_i} = 0$$
$$R_B \sin 53.13^o - F_1 = 0$$
$$R_B = 420KN;$$

- Eq. 1 gives HA=420 KN

$$\varsigma + \sum M_B = 0;$$
$$-H_A(3.6\sin\theta) + 336(2.1\cos\theta) - 168\,(1.2\sin\theta) = 0$$
$$-1310.4\sin\theta + 705.6\cos\theta = 0$$
$$\tan\theta = 0.538$$
$$\theta = 28.3^o$$

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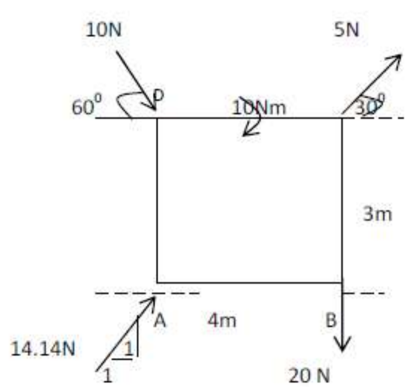
2. Determine the resultant of the force system acting on the plate. As shown in figure given below with respect to AB and AD.



$\sum Fx = 5\cos30^0 + 10\cos60^0 + 14.14\cos45^0$

$= 19.33N$

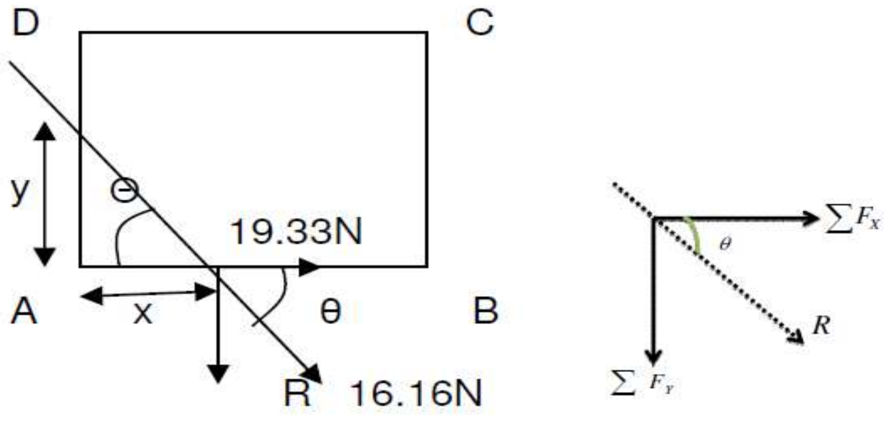
$\sum Fy = 5\sin 30^0 - 10\sin 60^0 + 14.14\sin 45^0$

$= -16.16N$

$R = \sqrt{(\sum Fx^2 + \sum Fy^2)} = 25.2N$

$\theta = Tan^{-1}(\sum Fy / \sum Fx)$

$\theta = Tan^{-1}(16.16/19.33) = 39.89^0$



Tracing moments of forces about A and applying varignon's principle of moments we get
$+16.16X = 20x4 + 5\cos 30^0x3 - 5\sin 30^0x4 + 10 + 10\cos 60^0x3$

$x = 107.99/16.16 = 6.683m$

Also $\tan 39.89 = y/6.83$
$y = 5.586m.$

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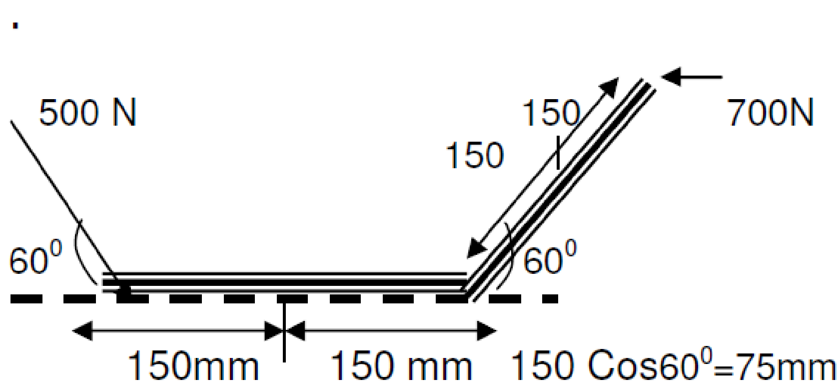
3. The system of forces acting on a crank is shown in figure below. Determine the magnitude , direction and the point of application of the resultant force.



$\sum Fx = 500\cos 60^0 - 700$

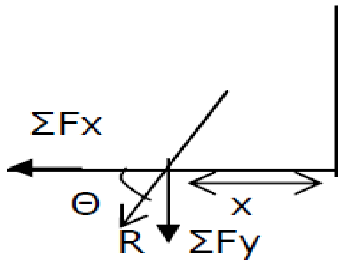
$= 450N$

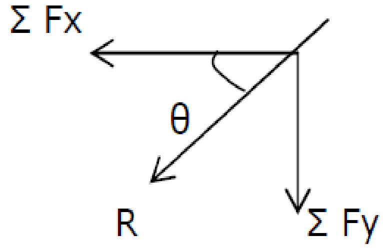
$\sum Fy = 500\sin 60^0$

$= -26.33N$

$R = \sqrt{(\sum Fx^2 + \sum Fy^2)} = (-450)^2 + (-2633)^2$

R = 267.19N (Magnitude)





$\theta = Tan^{-1}(\sum Fy / \sum Fx)$

$\theta = Tan^{-1}(2633/450)$

$\theta = 80.30^0$ (Direction)

Tracing moments of forces about O and applying varignon's principle of moments we get

$-2633x\ x = -500x \sin 60^0 x300 - 1000x150 + 1200x150\cos 60^0 - 700x300\sin 60^0$

x = -371769.15/-2633

x= 141.20mm from O towards left (position).

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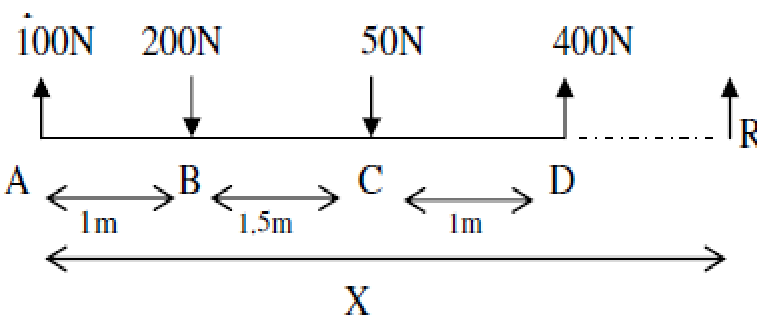
4. For the system of parallel forces shown below, determine the magnitude of the resultant and also its position from A .



$\sum Fy = +100 - 200 - 50 + 400$

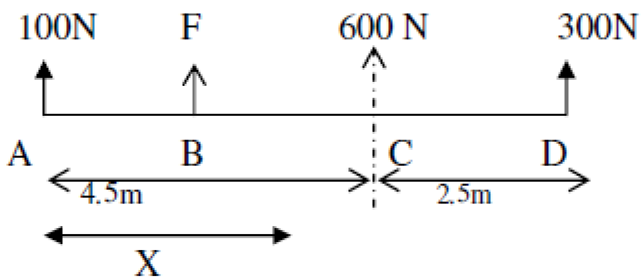
$= +250N$

ie. $R = \sum Fy = 250N$ ( )

Since $\sum Fx = 0$

Taking moments of forces about A and applying varignon's principle of moments
-250 x = -400 x 3.5 + 50 x 2.5 + 200 x 1 – 100 x 0
X = -1075/ -250 = 4.3m

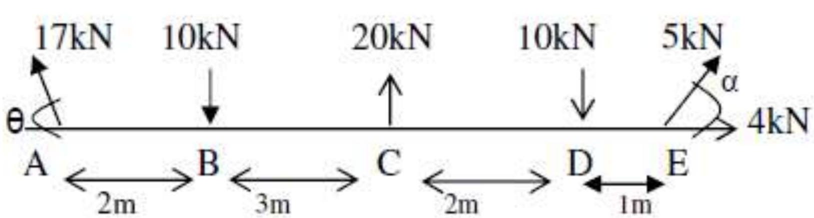
5. The three like parallel forces 100 N,F and 300 N are acting as shown in figure below. If the resultant R=600 N and is acting at a distance of 4.5 m from A ,find the magnitude of force F and position of F with respect to A



Let x be the distance from A to the point of application of force F
Here R = ∑ Fy
600=100+F+300
F = 200 N

Taking moments of forces about A and applying varignon's principle of moments,
We get
600 x 4.5 = 300 x 7 + F x
200 x = 600 x 4.5 -300 x 7.
X = 600/200 = 3m from A

6. A beam is subjected to forces as shown in the figure given below. Find the magnitude, direction and the position of the resultant force.



Given tan θ= 15/8 sin θ = 15/ 17 cos θ = 8/17
tanα= 3/4 sin α = 3/5 cos α = 4/5
∑ Fx = 4 +5 cos α – 17 cos θ
= 4 + 5 x 4/5 – 17 x 8/17
∑ Fx = 0

$\sum Fy = 5 \sin \alpha - 10 + 20 - 10 + 17 \sin \theta$

$\quad = 5 \times 3/5 - 10 + 20 - 10 + 17 \times 15/17$

$\sum Fy = 18$ kN (↑)

Resultant force $R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2} = \sqrt{0+18^2}$

$R = 18$ kN(↑)

Let x = distance from A to the point of application R

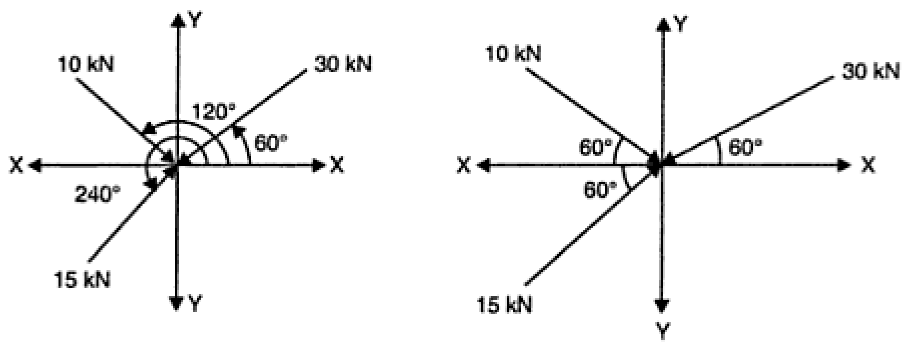
Taking moments of forces about A and applying Varignon's theorem of moments

$-18x = -5 \times \sin \alpha \times 8 + 10 \times 7 - 20 \times 5 + 10 \times 2$

$\quad = -3 \times 8 + 10 \times 7 - 20 \times 5 + 10 \times 2$

$X = -34/-18 = 1.89$m from A (towards left)

Example *Three forces of magnitude 30 kN, 10 kN and 15 kN are acting at a point O. The angles made by 30 kN force, 10 kN force and 15 kN force with x-axis are 60°, 120° and 240° respectively. Determine the magnitude and direction of the resultant force.*



F.B.D

Solution: $\Sigma H = -30 \cos 60 + 10 \cos 60 + 15 \cos 60$

$= -2.5$ kN

$\Sigma V = -30 \sin 60 - 10 \sin 60 + 15 \sin 60$

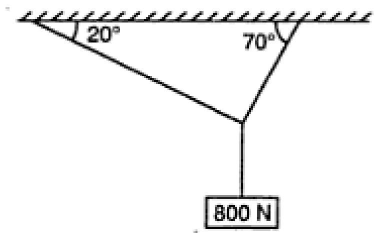
$= -21.65$ kN

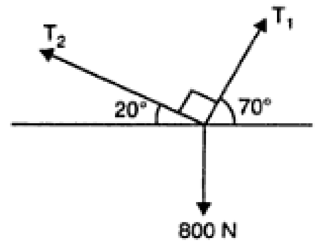
$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(-2.5)^2 + (21.65)^2}$$

$= 21.79$ kN

$$\tan \alpha = \frac{\Sigma V}{\Sigma H} = \frac{-21.65}{-2.5} = 83°41'.$$

Example *A weight of 800 N is suspended by two chains as shown in figure. Determine the tensions in each chain.*





F.B.D.

Solution:

$$\Sigma H = 0$$

$$-T_2 \cos 20 + T_1 \cos 70 = 0$$

$$T_1 \cos 70 = T_2 \cos 20$$

$$T_2 = \frac{T_1 \cos 70}{\cos 20}$$

$$T_2 = T_1\,(0.364) \qquad \ldots(i)$$

$$\Sigma V = 0$$

$$T_2 \sin 20 + T_1 \sin 70 = 800$$

$$\sin 20\; T_1\,(0.364) + T_1 \sin 70 = 800$$

$$1.3\; T_1 = 800$$

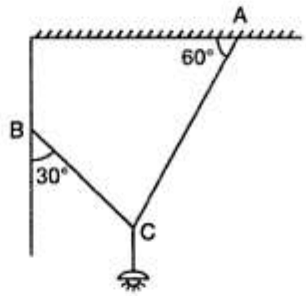
$$T_1 = 751.75 \text{ N}$$

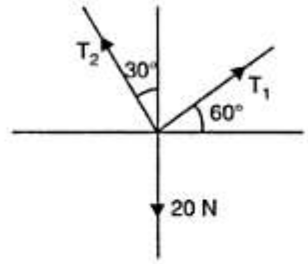
From (*i*)

$$T_2 = 751.75\,(0.364)$$

$$\mathbf{T_2 = 273.64\ N.}$$

Example *An electric light fixture weighing 20 N hangs from a point C, by two strings AC and BC. AC is inclined at 60° to the horizontal and BC at 30° to the vertical as shown in Fig. Determine the forces in the strings AC and BC.*





F.B.D.

Solution:

$$\Sigma H = 0$$

$$-T_2 \sin 30 + T_1 \cos 60 = 0$$

$$T_2 \sin 30 = T_1 \cos 60$$

$$T_2 = T_1 \frac{\cos 60}{\sin 30}$$

$$T_2 = T_1 \frac{0.5}{0.5}$$

$$T_2 = T_1$$

$$\Sigma V = 0$$

$$T_2 \cos 30 + T_1 \sin 60 = 20$$

$$T_1 \cos 30 + T_1 \sin 60 = 20$$

$$1.73\, T_1 = 20$$

$$T_1 = \frac{20}{1.73} = \mathbf{11.547\ N}$$

$$T_1 = T_2 = 11.547\ \text{N}$$

Example *Two forces of magnitude 15 N and 12 N are acting at a point. If the angle between the two forces is 60°, determine the resultant of the forces in magnitude and direction.*

Solution:

$$P = 15\ \text{N}$$

$$Q = 12\ \text{N}$$

$$\theta = 60^\circ$$

Resultant ⇒

$$R = \sqrt{(15)^2 + (12)^2 + 2 \times 15 \times 12 \times \cos 60}$$

$$= 23.43\ \text{N}$$

Direction ⇒

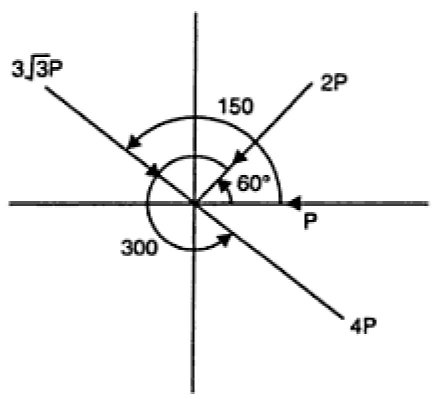
$$\tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta}$$

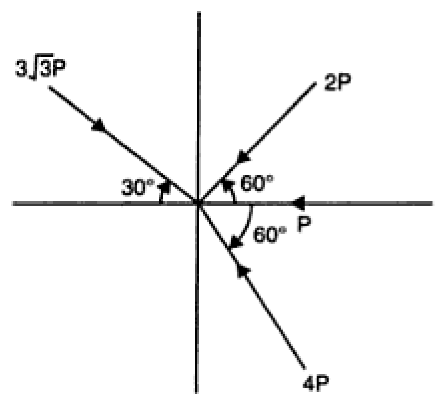
$$= \frac{12 \sin 60}{15 + 12 \cos 60}$$

$$= 0.495$$

$$\alpha = \mathbf{26.31.}$$

Example *Four forces of magnitude P, 2P, $3\sqrt{3}$ P and 4P are acting at a point O. The angles made by these forces with x-axis are 0°, 60°, 150° and 300° respectively. Find the magnitude and direction of the resultant force.*





Solution: $\Sigma H = -P - 2P \cos 60 + 3\sqrt{3}\ P \cos 30 - 4P \cos 60$

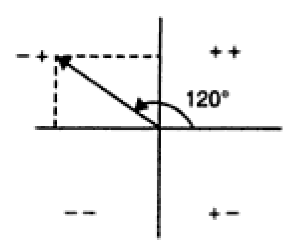
$$\Sigma H = \frac{P}{2} = 0.5\,P$$

$$\Sigma V = -2P \sin 60 - 3\sqrt{3}\ P \sin 30 + 4P \sin 60^\circ$$

$$= -0.87\,P$$

$$R = P\sqrt{(0.5)^2 + (-0.87)^2}$$

R = PN

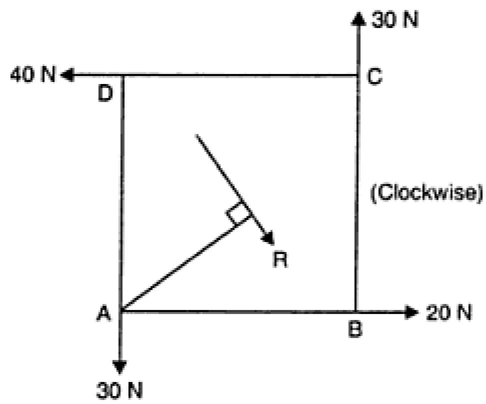


$$\tan\phi = \frac{\Sigma V}{\Sigma H} = \frac{-0.87}{0.5} = 1.74$$

$$\phi = 60.11$$

$$\phi = 180 - 60 = 120$$

Example *Four forces of magnitude 20 N, 30 N, 40 N and 50 N are acting respectively along four sides of a square taken in order. Determine the magnitude, direction and position of the resultant force.*



Solution:

$$\Sigma H = 20 - 40 = -20$$

$$\Sigma V = 30 - 50 = -20$$

$$R = \sqrt{(-20)^2 + (-20)^2}$$

$$R = 28.28 \text{ N}$$

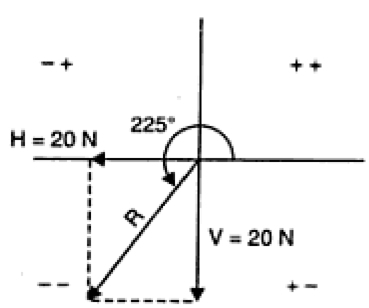
Direction of the resultant

$$\tan\phi = \frac{\Sigma V}{\Sigma H} = \frac{-20}{-20}$$

$$\tan\phi = 1$$

$$\phi = 45°$$

Since ΣH and ΣV are –ve ϕ has between 180° and 270 *i.e.*, $\phi = 180 + 45 = 225$.



Position of the resultant force :

The position of the resultant force is obtained by equating the clockwise moments and anticlockwise moment about A.

Let x = perpendicular distance between A and line of action of the resultant force

a = side of the square ABCD

Taking moments about A

$$-50 \times 0 - 20 \times 0 - 30 \times a - 40 \times a = R \times \text{Perpendicular distance of } R \text{ from } A$$

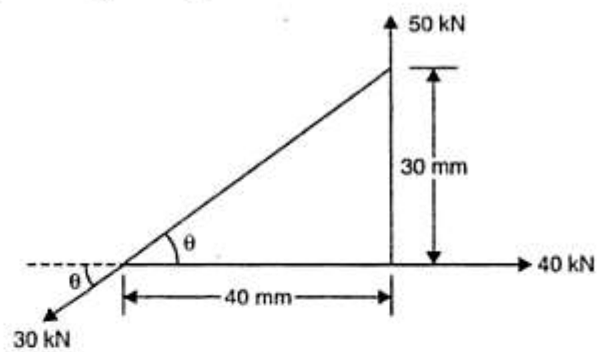
$$-30\,a - 40\,a = 28.28 \times x$$

$$-70\,a = 28.\,28 \times x$$

$$x = -\frac{70\,a}{28.28}$$

$$x = -2.47\,a \text{ (anticlockwise).}$$

Example *A triangle ABC has its sides AB = 40 mm along positive x-axis and side BC = 30 mm along positive y-axis. Three forces of 40 kN, 50 kN and 30 kN act along the sides AB, BC and CA respectively. Determine the resultant of such a system of forces.*



Solution:

From figure $\tan\theta = \frac{30}{40} = 0.75$

$$\theta = \tan^{-1} 0.75$$

$$\theta = 36°.87'$$

$$\Sigma H = 40 - 30\cos\theta$$

$$= 40 - 30\cos 36°.87'$$

$$\Sigma H = 16 \text{ kN}$$

$$\Sigma V = 50 - 30\sin\theta$$

$$= 50 - \sin 36°.87' \times 30$$

$$\Sigma V = 32 \text{ kN}$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(16)^2 + (32)^2}$$

R = 35.78 kN.

Example *The four coplanar forces are acting at a point as shown in figure. Determine the resultant and direction of the resultant.*

Solution: $\Sigma H = 1000 + 2000\cos 60 - 5196\cos 30 + 4000\cos 60$

$$= -499.87 \text{ N}$$

$$\Sigma V = 2000\sin 60 + 5196\sin 30 - 4000\sin 60$$

$$= 865.95 \text{ N}$$