Entanglement, holography, and strange metals

Gordon Research Conference
Correlated Electron Systems
Mount Holyoke, June 27, 2012

Lecture at the 100th anniversary Solvay conference,
Theory of the Quantum World
arXiv:1203.4565

sachdev.physics.harvard.edu

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Max Metlitski

Liza Huijse

Brian Swingle

Debanjan Chowdhury

Matthias Punk

Erez Berg

Junhyun Lee

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Max Metlitski

Liza Huijse

Brian Swingle

Debanjan Chowdhury

Posters

Matthias Punk

Erez Berg

Junhyun Lee

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“Complex entangled” states of quantum matter, *not* adiabatically connected to independent particle states

Gapped quantum matter

Spin liquids, quantum Hall states

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Strange metals, Bose metals

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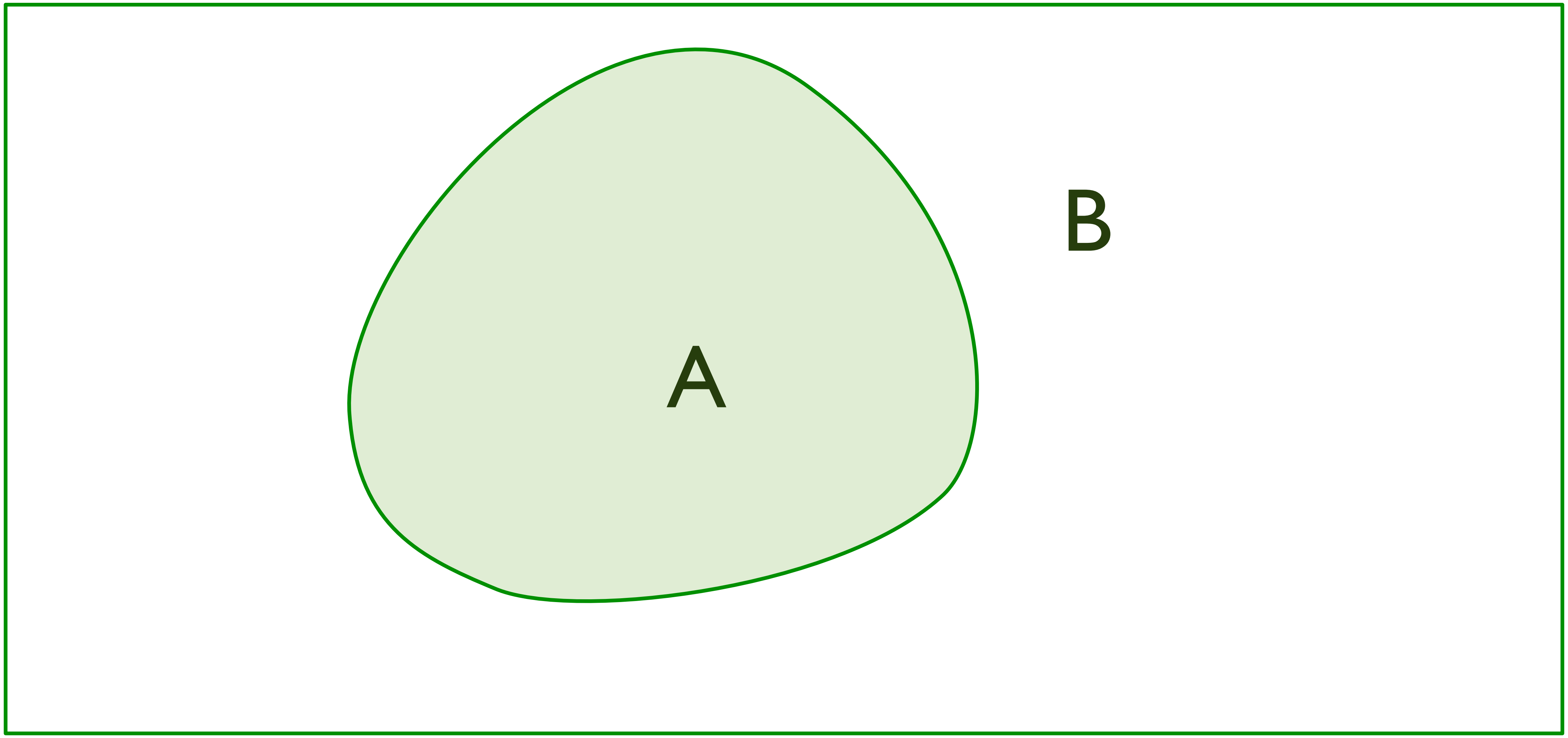
Entanglement entropy



$$|\Psi\rangle \quad \Rightarrow \quad \text{Ground state of entire system,}$$
$$\rho = |\Psi\rangle\langle\Psi|$$

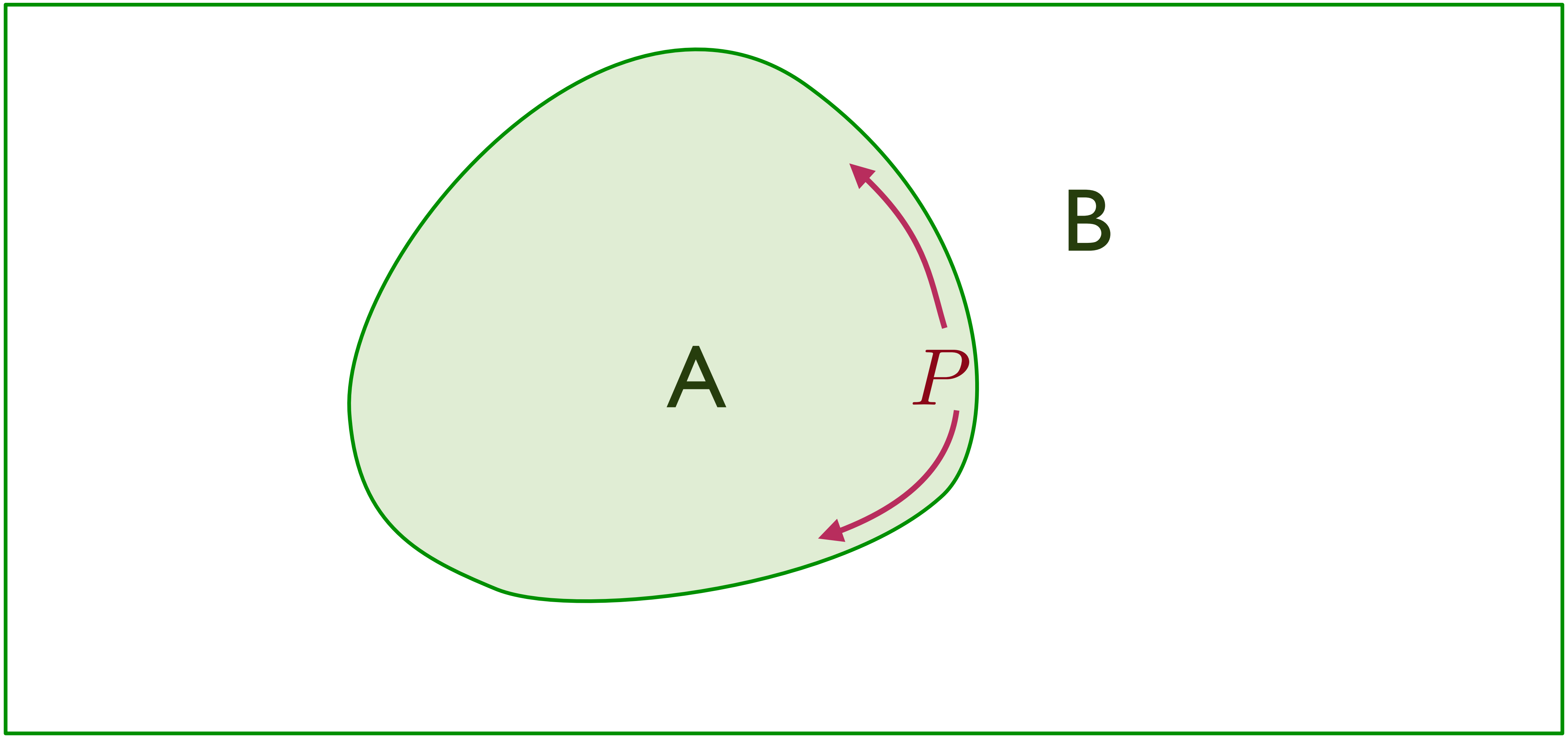
$$\rho_A = \text{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_E = -\text{Tr}\left(\rho_A \ln \rho_A\right)$$

M. Srednicki, Phys. Rev. Lett. **71**, 666 (1993)

Gapped quantum matter

Entanglement entropy of a band insulator

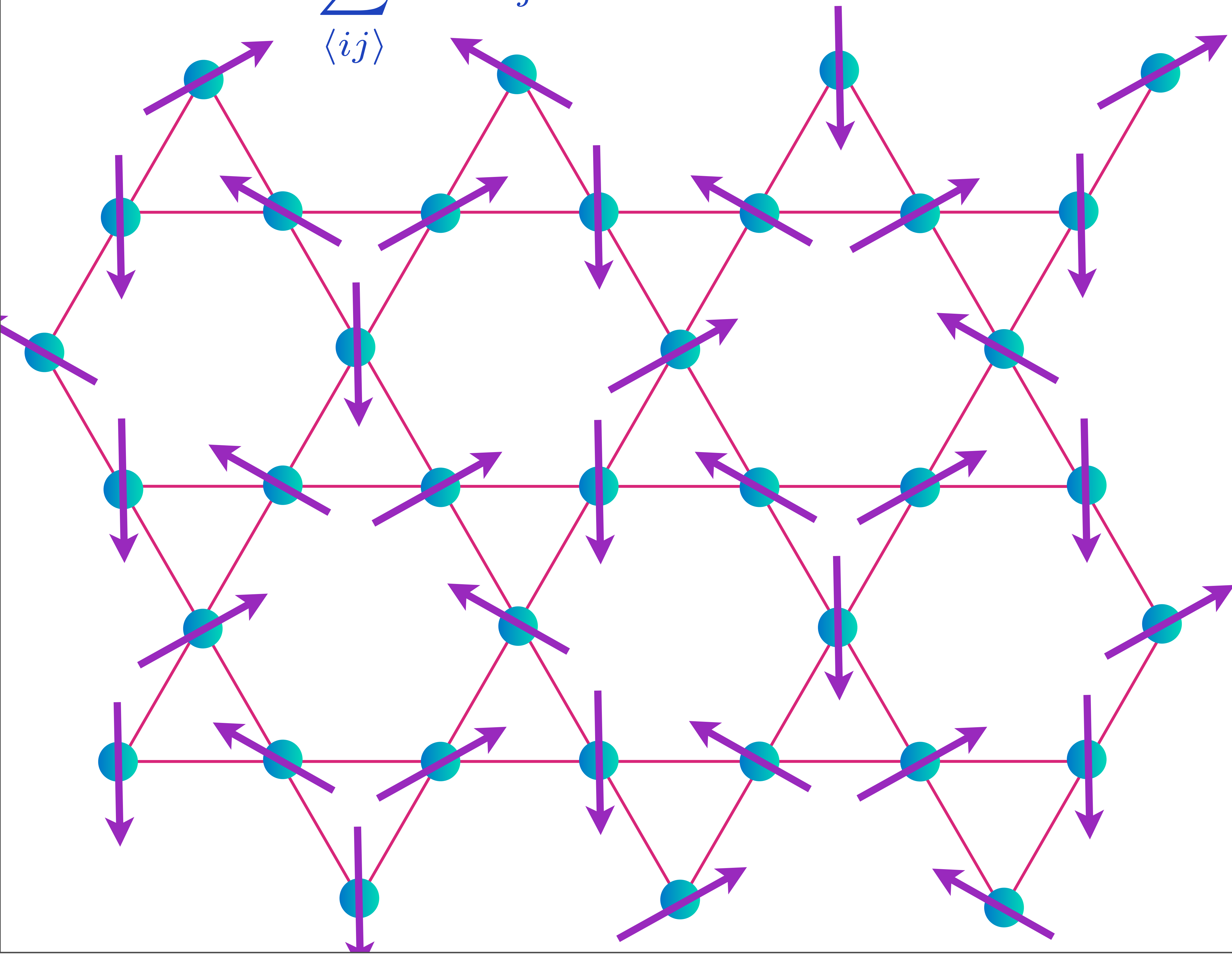


$$S_E = aP - b\exp(-cP)$$

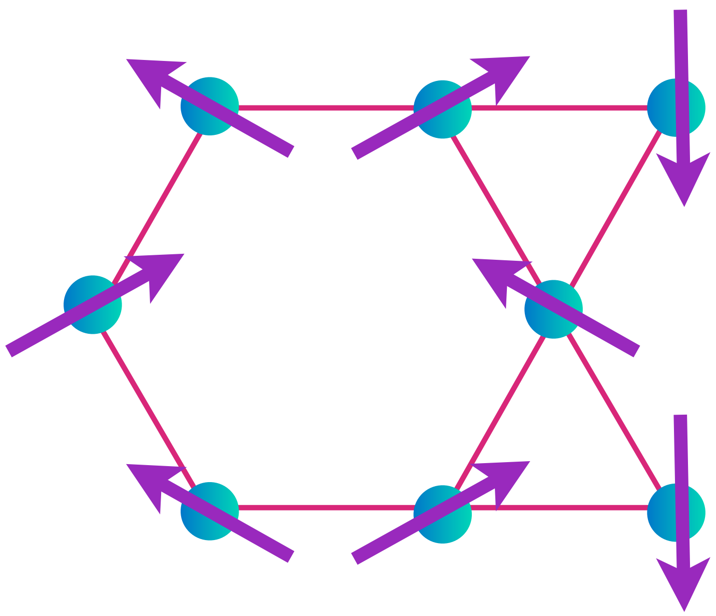
where P is the surface area (perimeter) of the boundary between A and B.

Kagome antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Kagome antiferromagnet: Z_2 spin liquid



non-collinear Néel state

Entangled quantum state:
A stable "$\mathbb{Z}_2$ spin liquid".
The excitations carry 'electric' and 'magnetic' charges of an emergent $\mathbb{Z}_2$ gauge field.

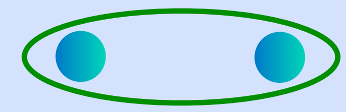
s_c $\qquad$ s

S. Sachdev, *Phys. Rev. B* **45**, 12377 (1992)
Y. Huh, M. Punk, and S. Sachdev, *Phys. Rev.* B **84**, 094419 (2011)

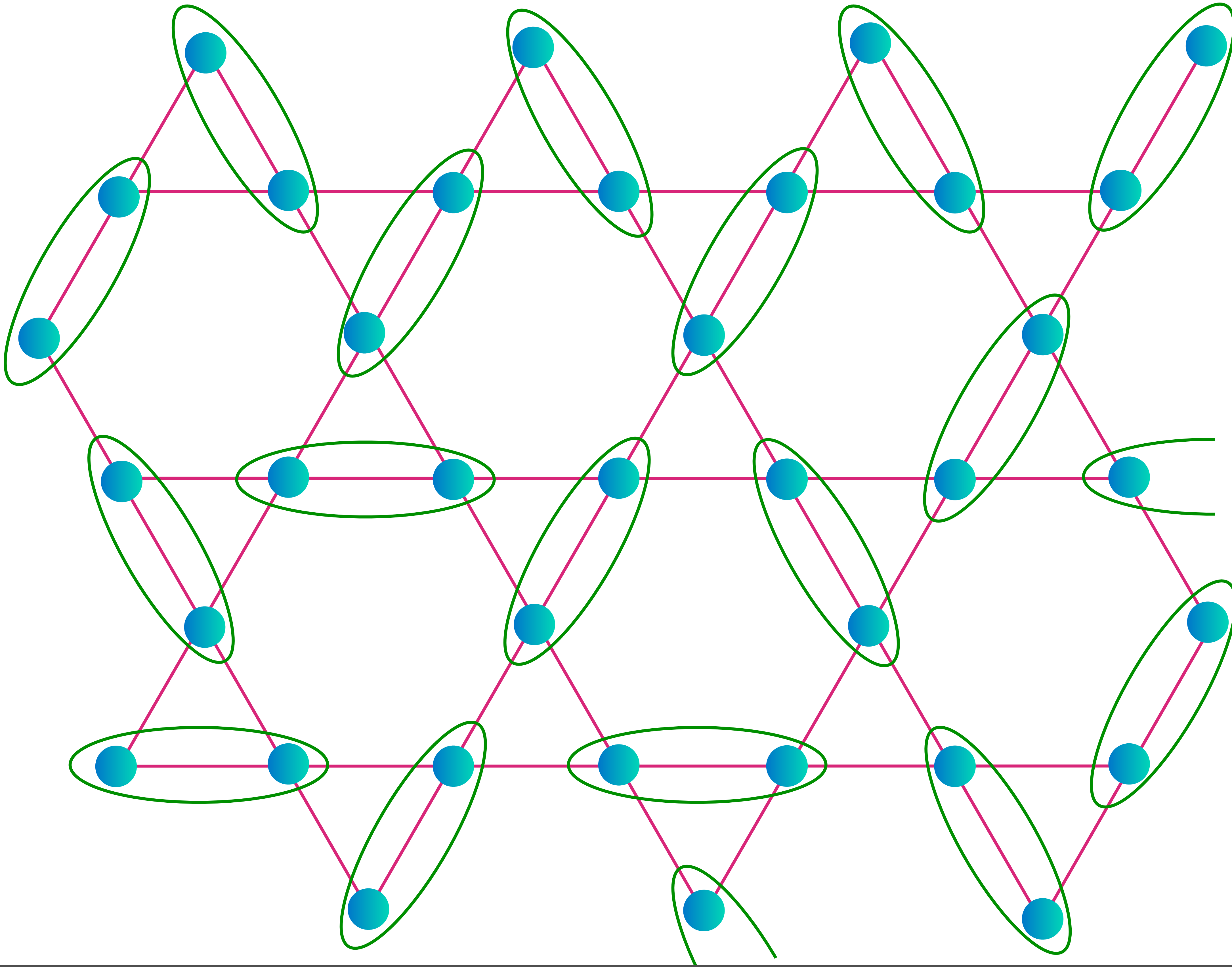
The Z_2 spin liquid was introduced in
N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991),
X.-G. Wen, *Phys. Rev. B* **44**, 2664 (1991)

Kagome antiferromagnet: *RVB*

Pick a reference configuration



$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

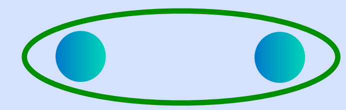


P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

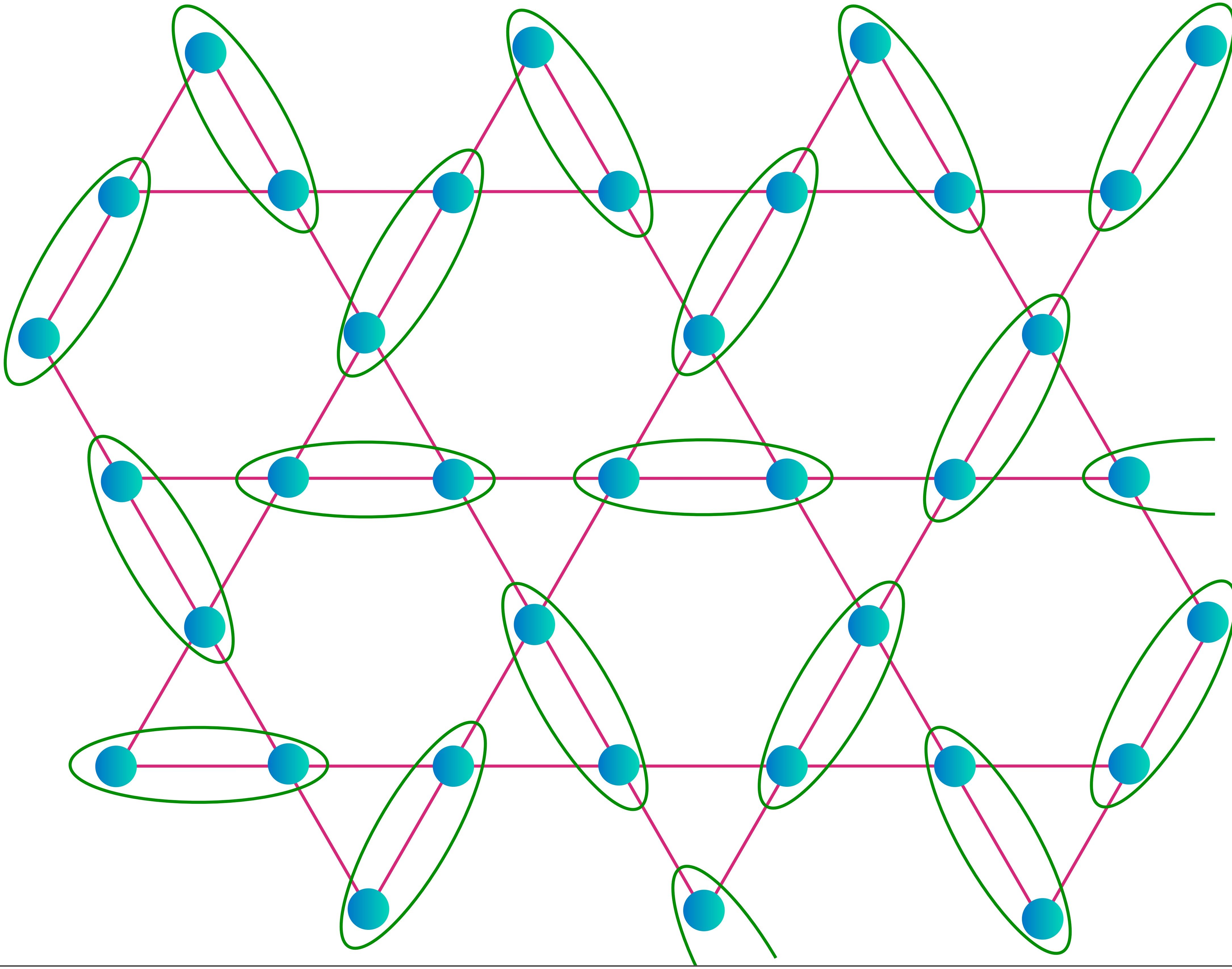
D. Rokhsar and S. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988).

Kagome antiferromagnet: *RVB*

A nearby configuration



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

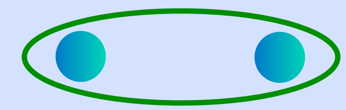


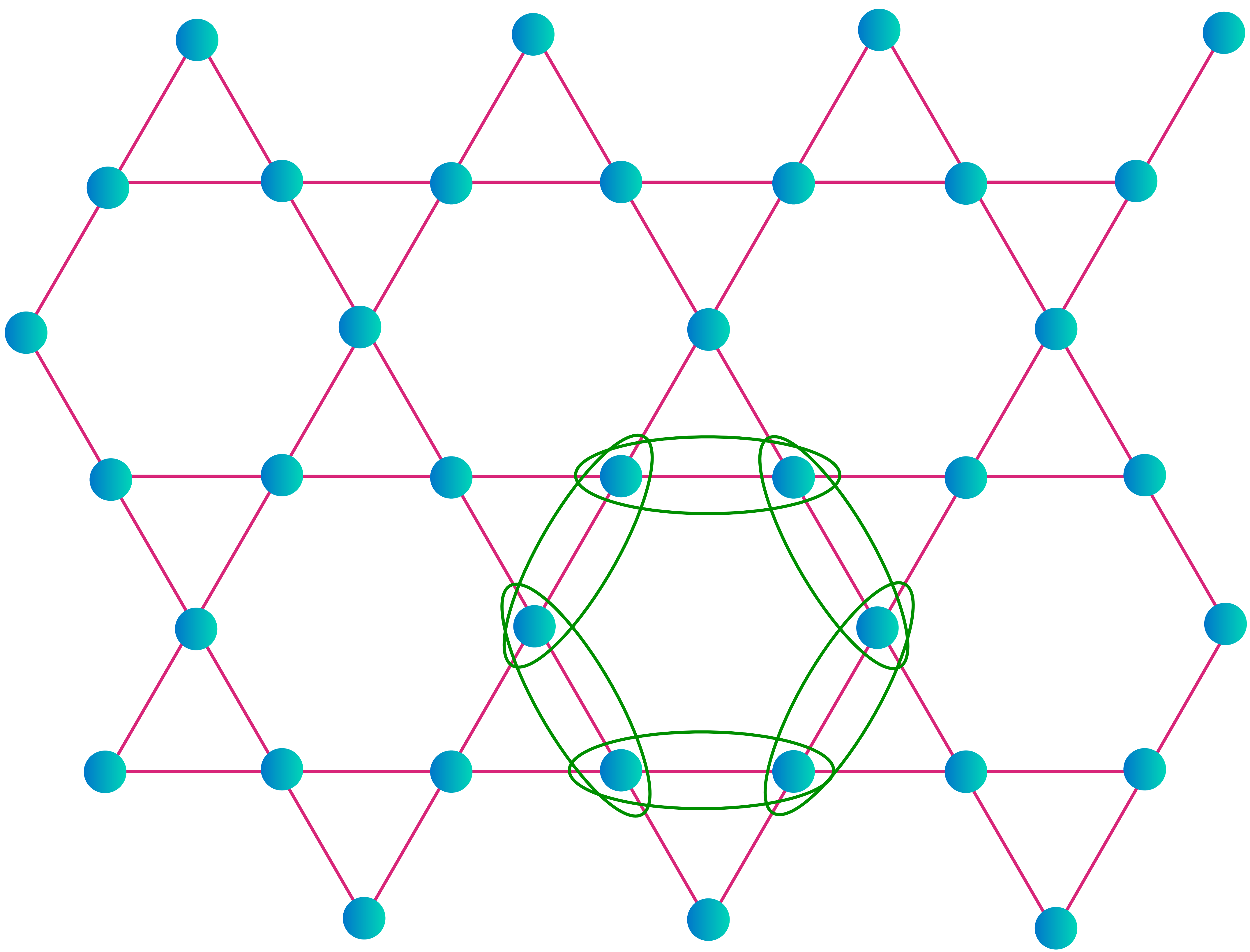
P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

D. Rokhsar and S. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988).

Kagome antiferromagnet: *RVB*

Difference: a closed loop

 $= \frac{1}{\sqrt{2}} \left(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$



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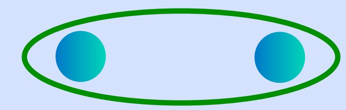
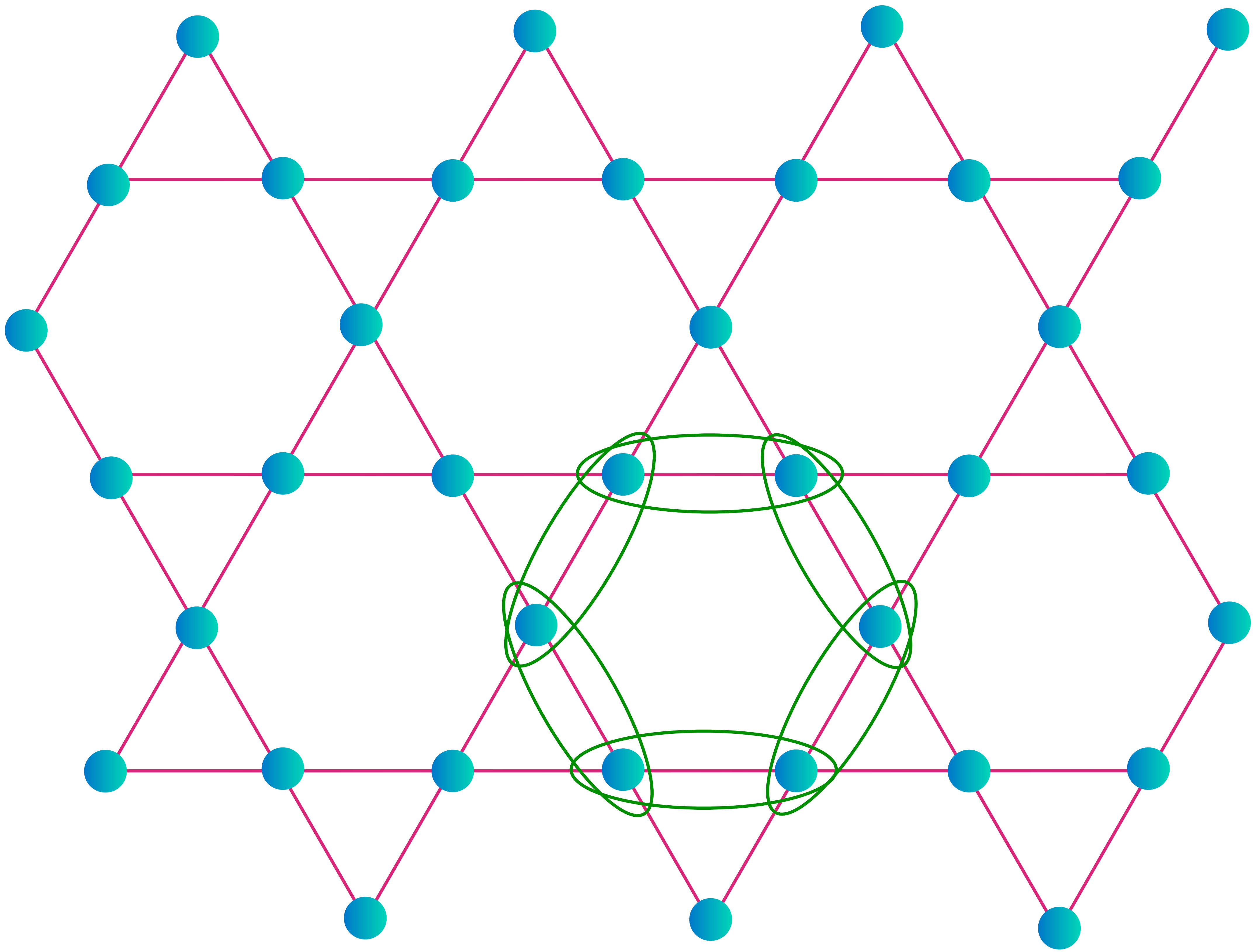
Kagome antiferromagnet: *RVB*

Ground state: sum over closed loops



$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$



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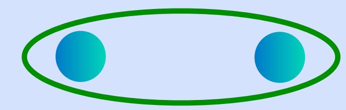
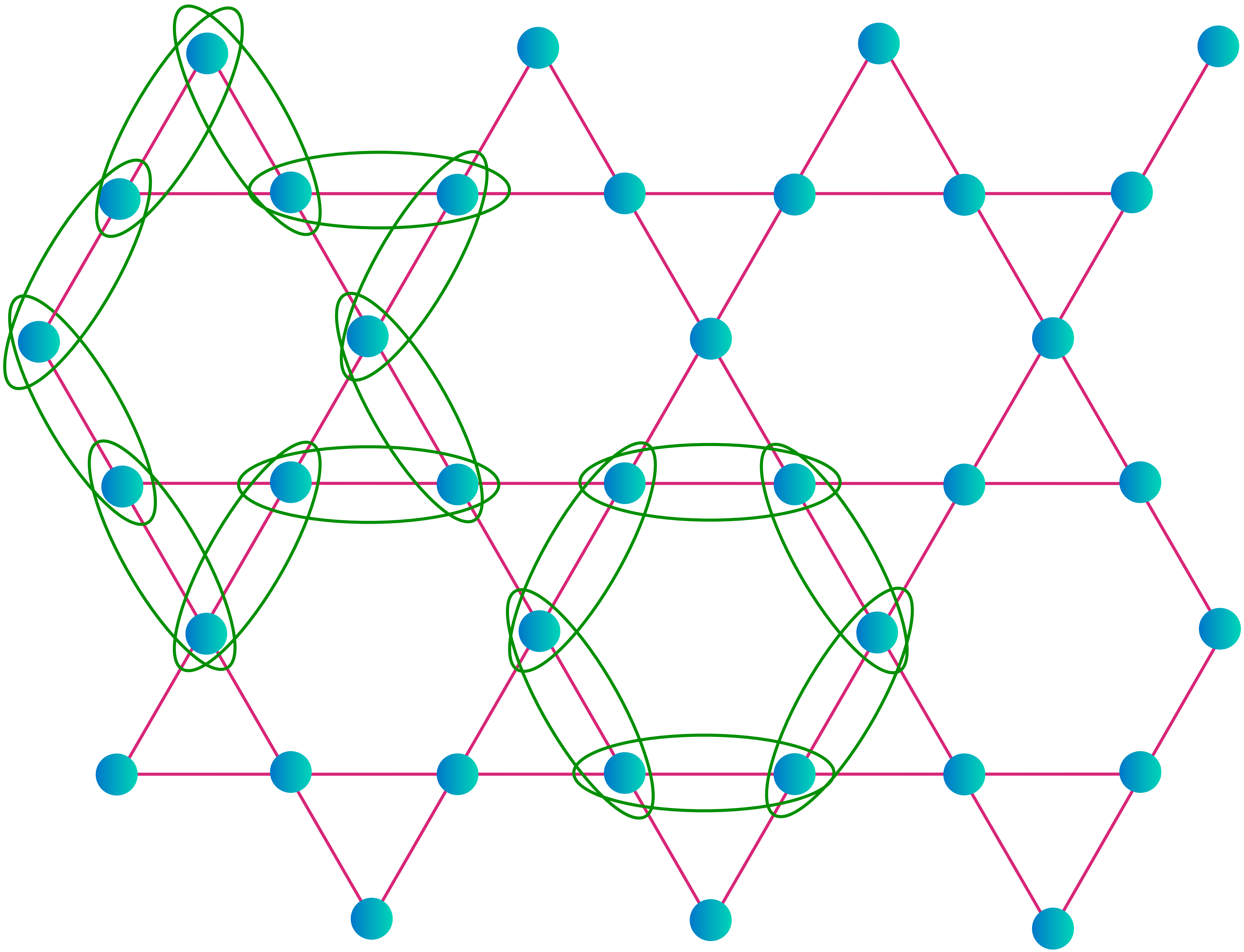
 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$



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Kagome antiferromagnet: *RVB*

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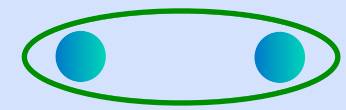
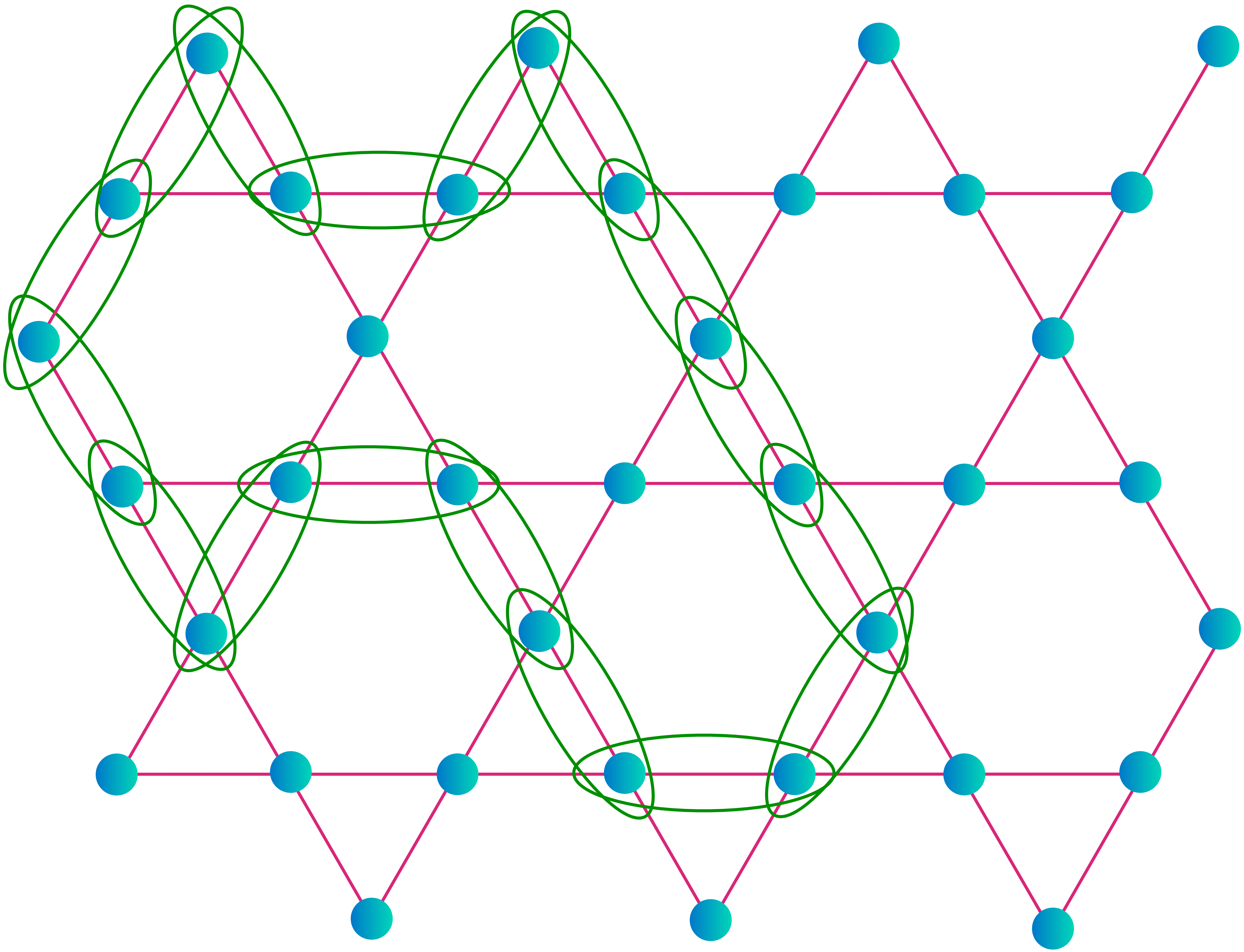
 $= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$



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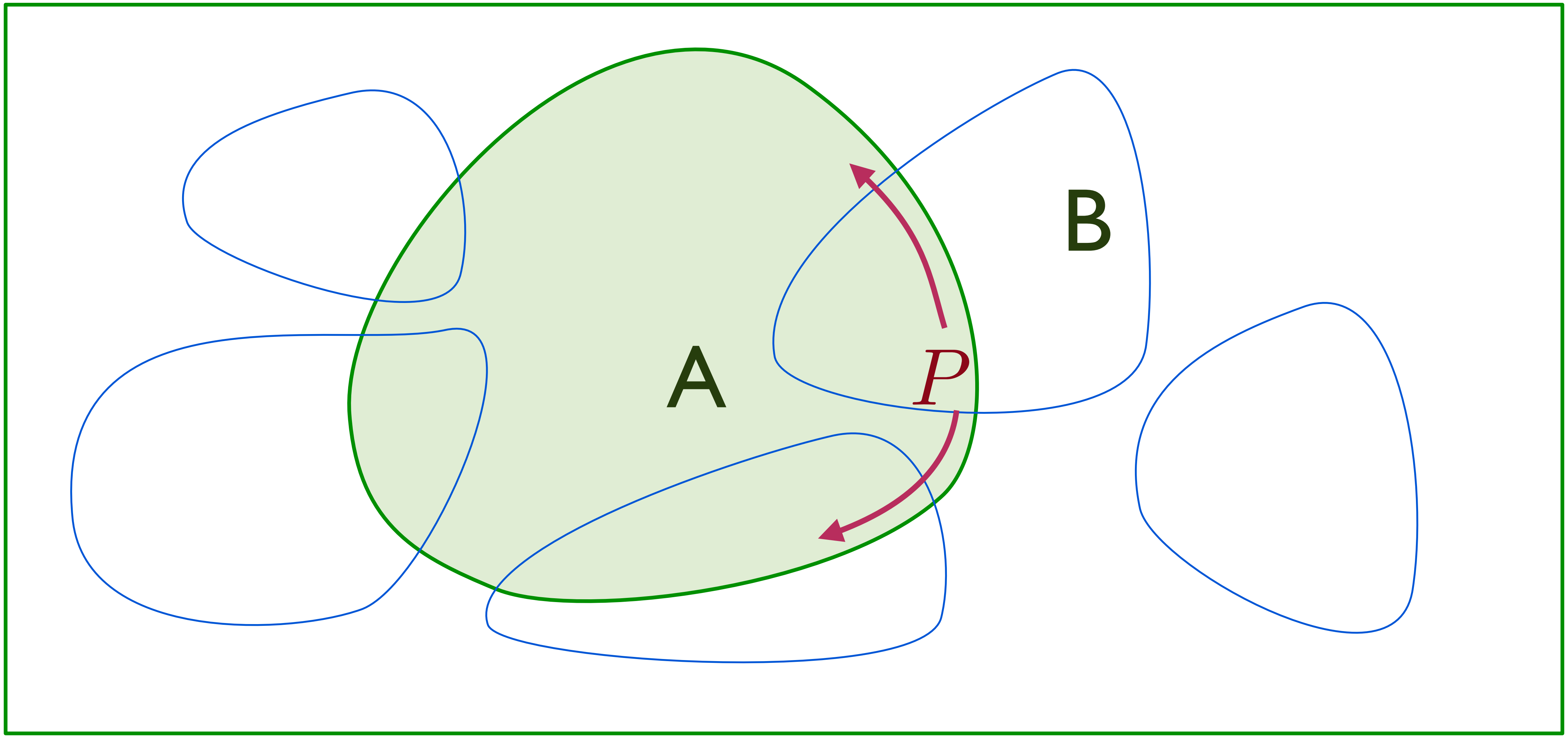
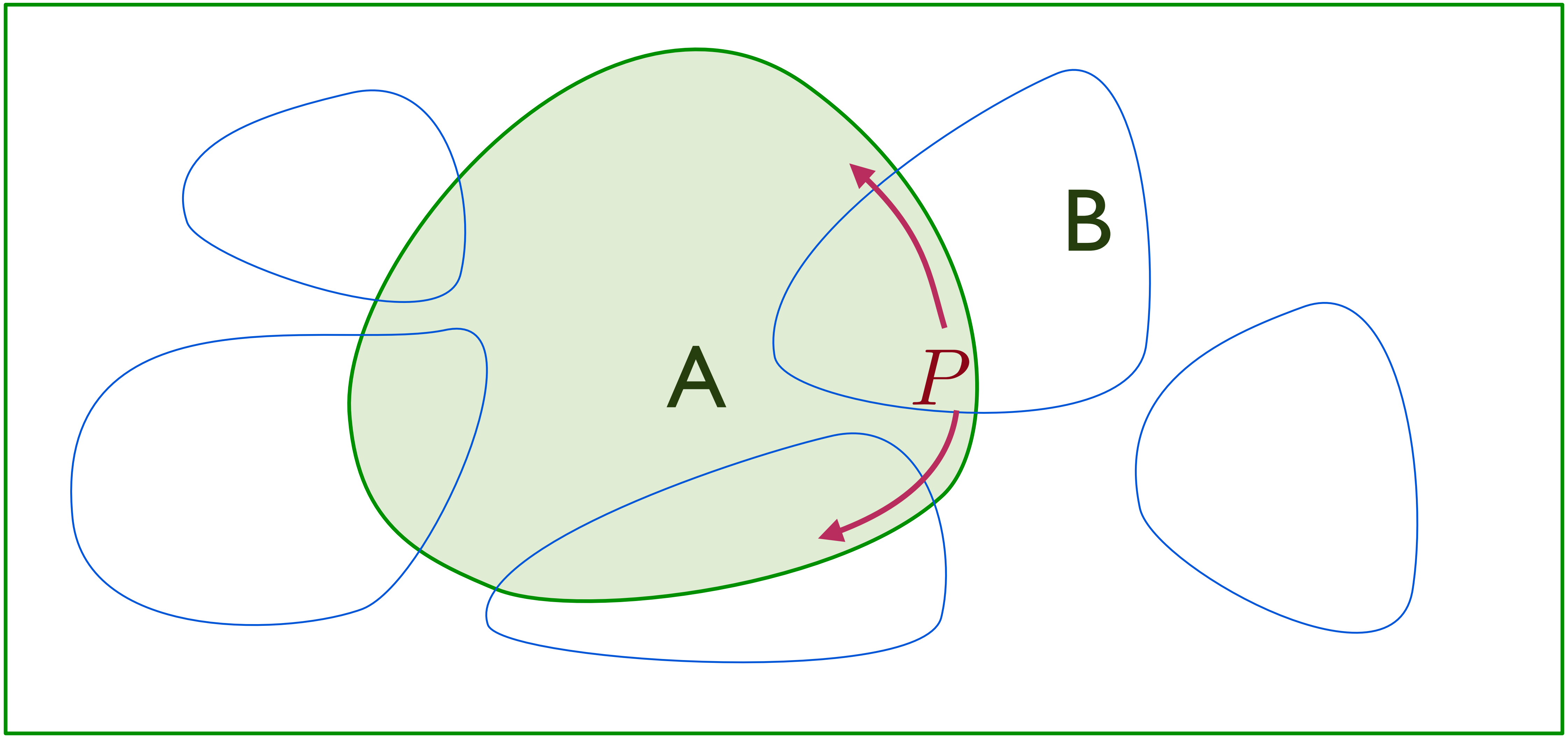
D. Rokhsar and S. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988).

Entanglement in the Z_2 spin liquid



Sum over closed loops: only an even number of links cross the boundary between A and B

Entanglement in the Z_2 spin liquid

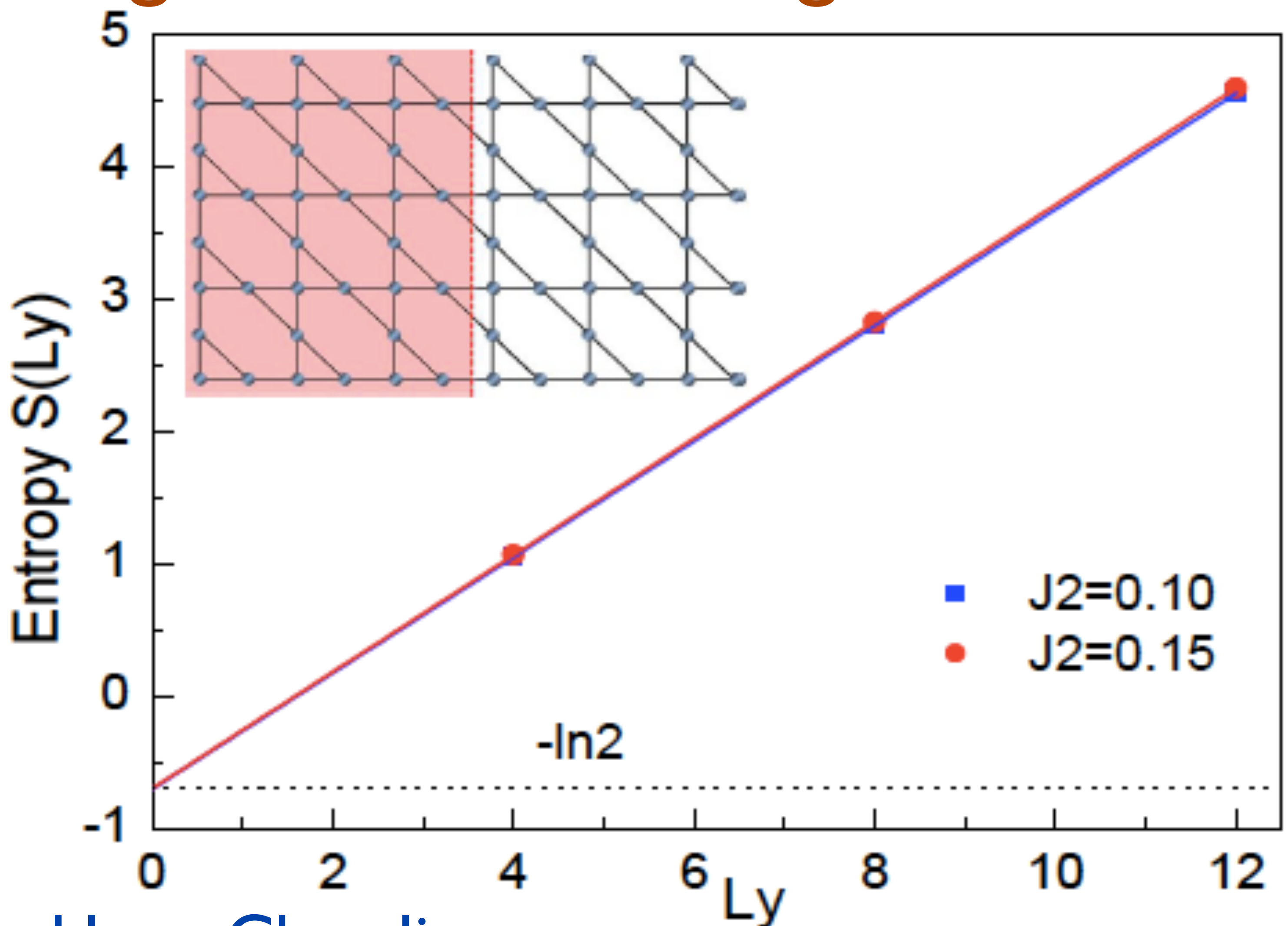


$$S_E = aP - \ln(2)$$

where P is the surface area (perimeter) of the boundary between A and B.

A. Hamma, R. Ionicioiu, and P. Zanardi, Phys. Rev. A **71**, 022315 (2005)
M. Levin and X.-G. Wen, Phys. Rev. Lett. **96**, 110405 (2006); A. Kitaev and J. Preskill, Phys. Rev. Lett. **96**, 110404 (2006)
Y. Zhang, T. Grover, and A. Vishwanath, Phys. Rev. B **84**, 075128 (2011)

Kagome antiferromagnet

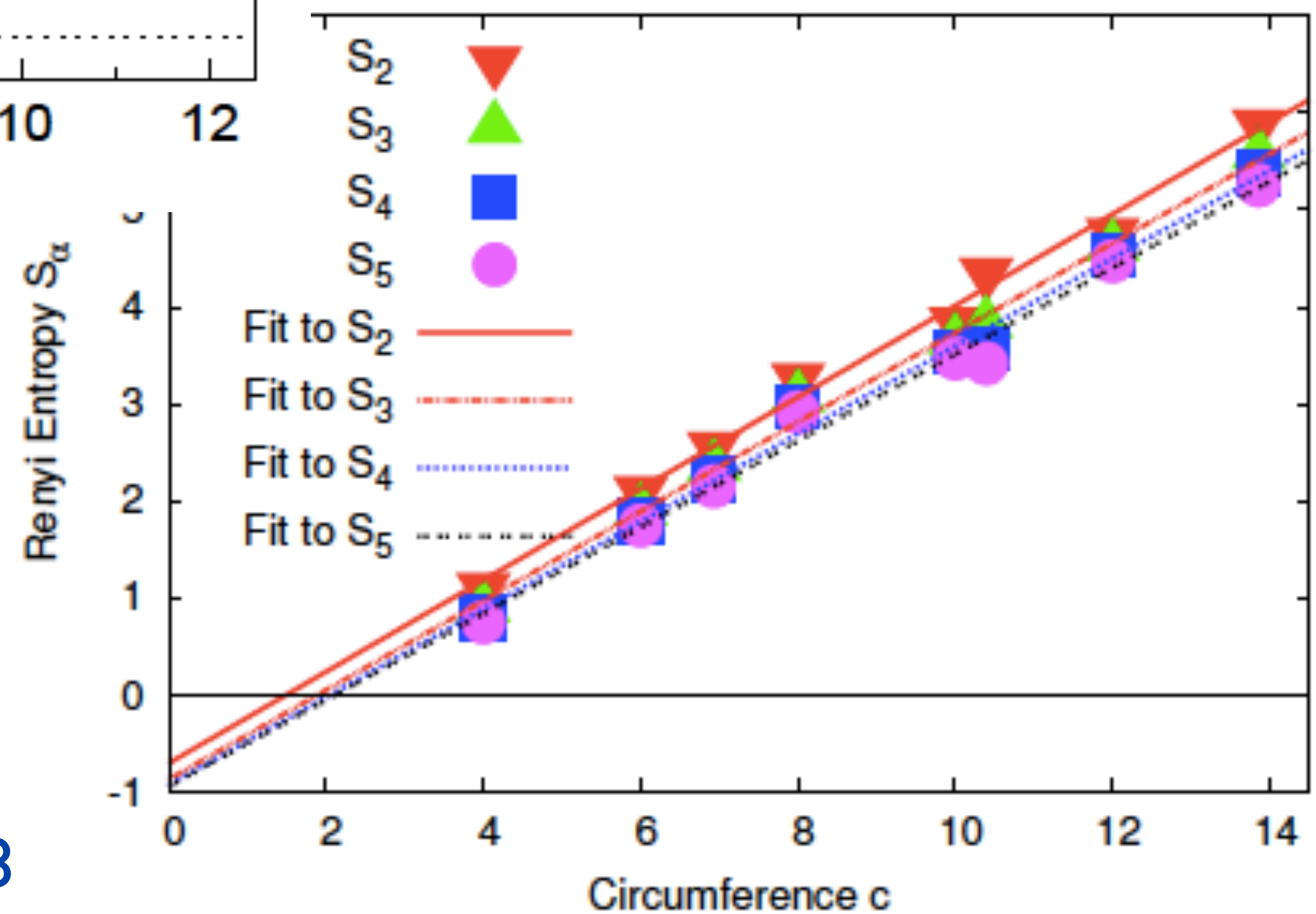


Strong numerical evidence for a Z_2 spin liquid

Simeng Yan, D. A. Huse, and S. R. White, *Science* **332**, 1173 (2011).

Hong-Chen Jiang, Z. Wang, and L. Balents, arXiv:1205.4289

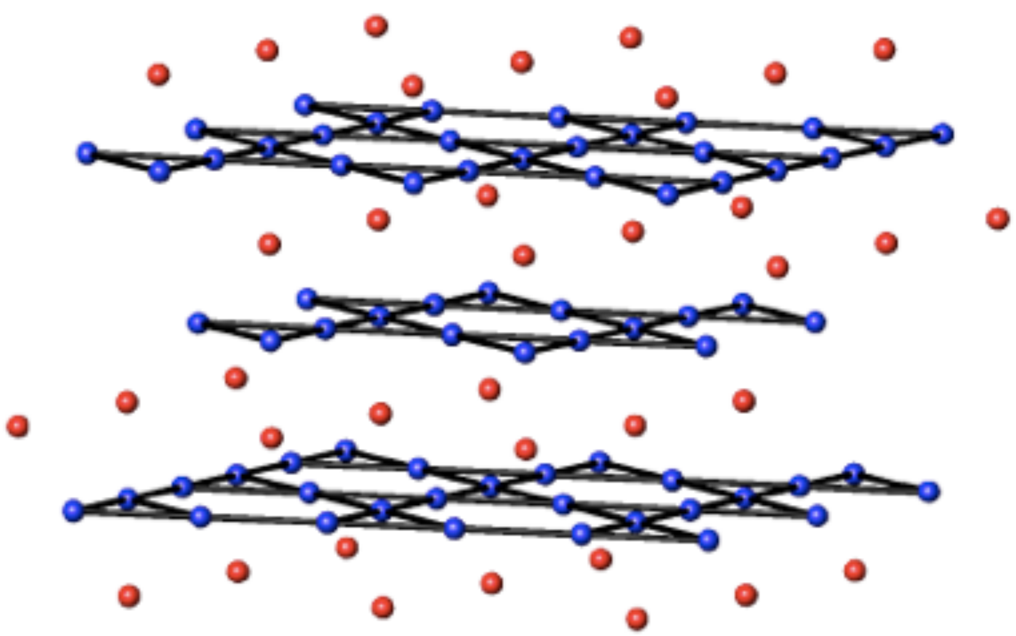
S. Depenbrock, I. P. McCulloch, and U. Schollwoeck, arXiv:1205.4858

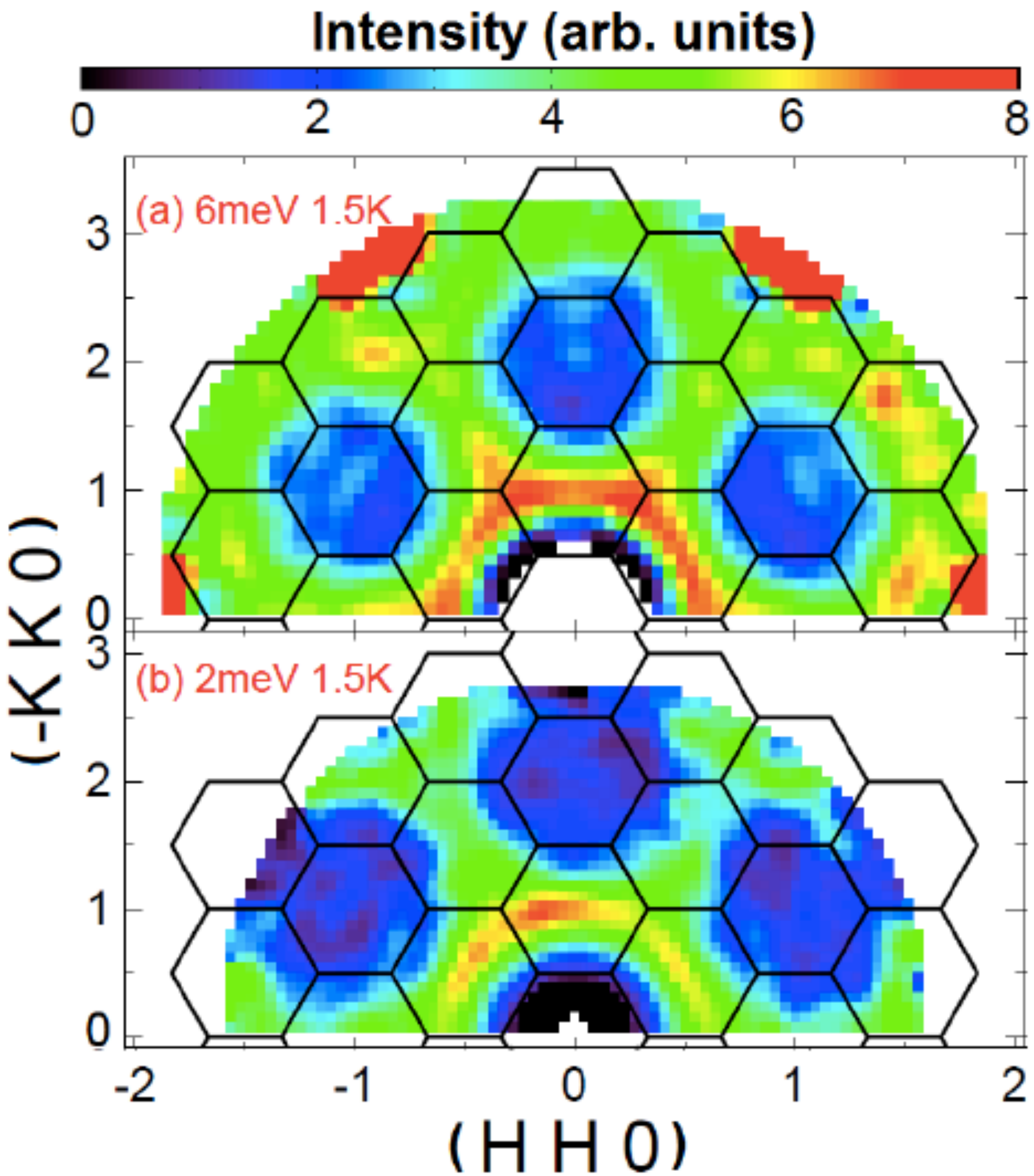


Kagome antiferromagnet

Evidence for spinons
Young Lee,
APS meeting, March 2012
http://meetings.aps.org/link/BAPS.2012.MAR.H8.5

$ZnCu_3(OH)_6Cl_2$ (also called Herbertsmithite)

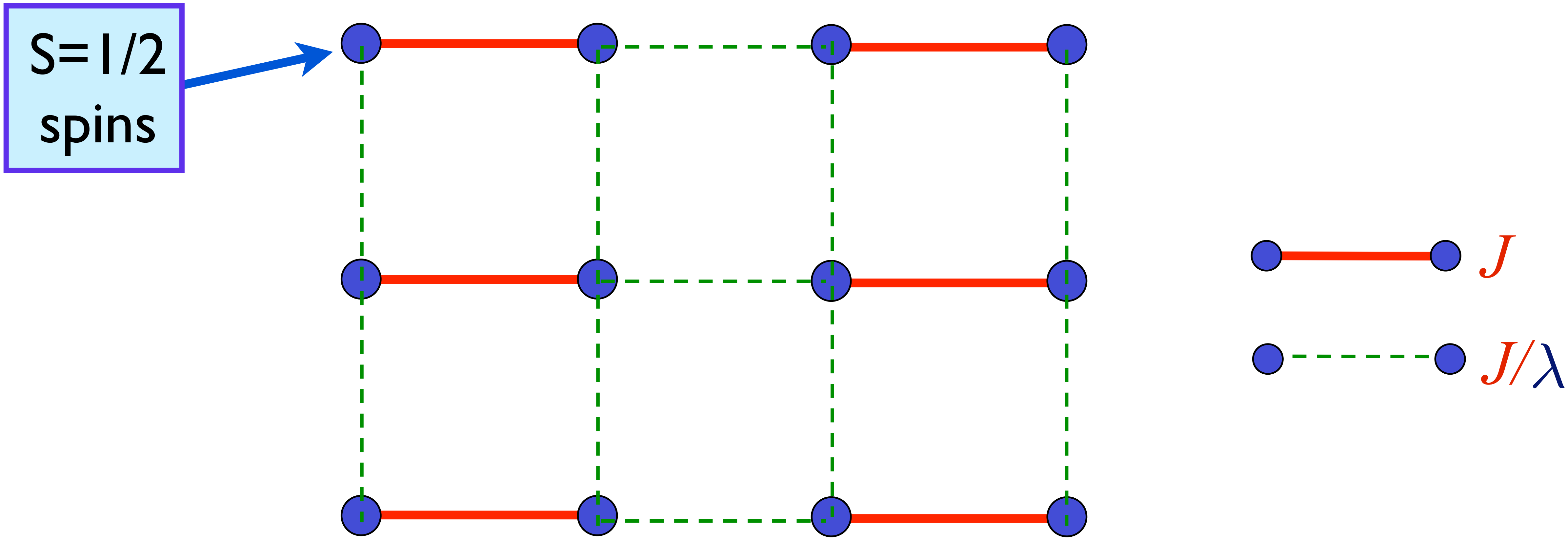




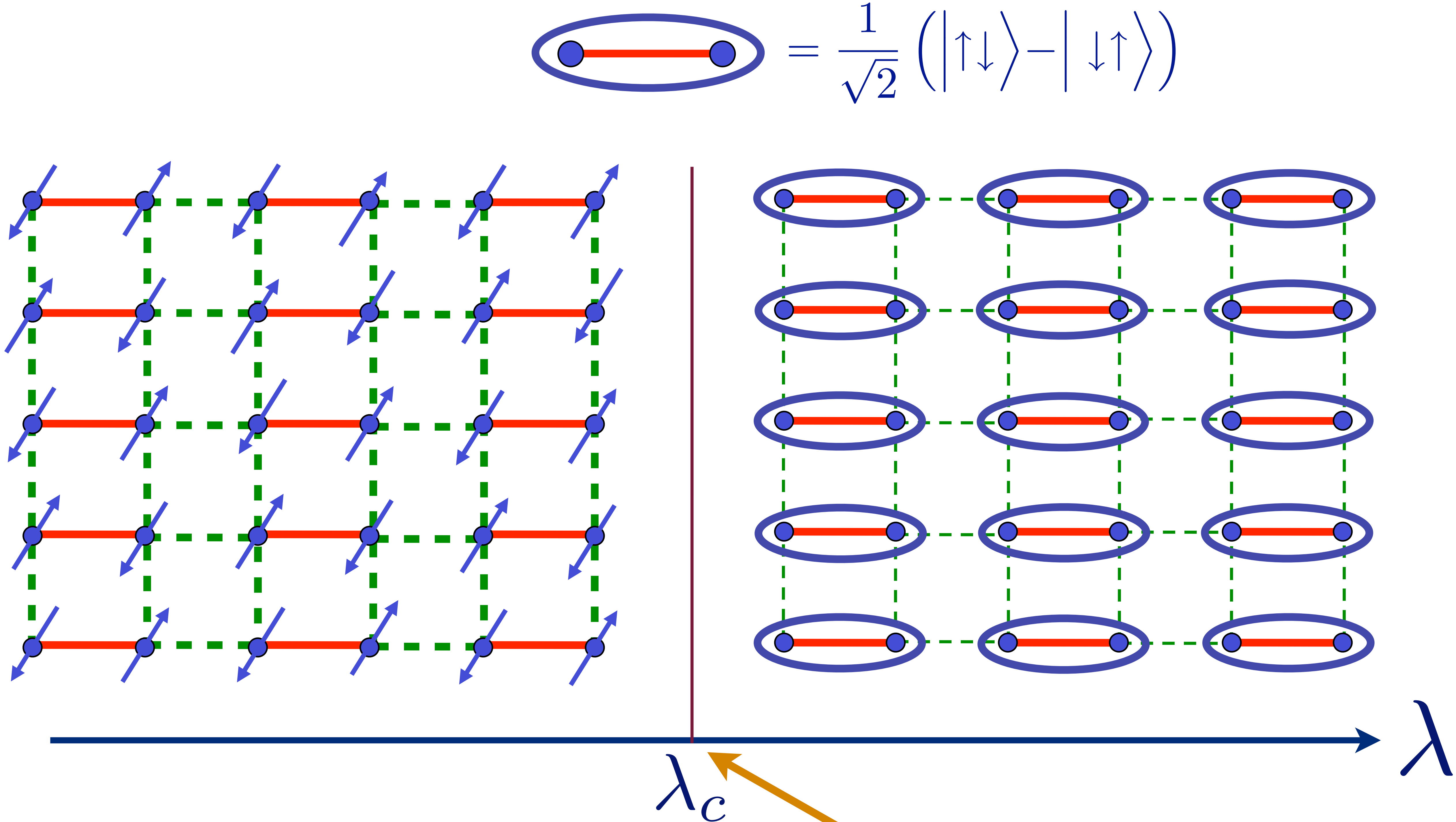
Conformal quantum matter

Coupled dimer antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Examine ground state as a function of λ

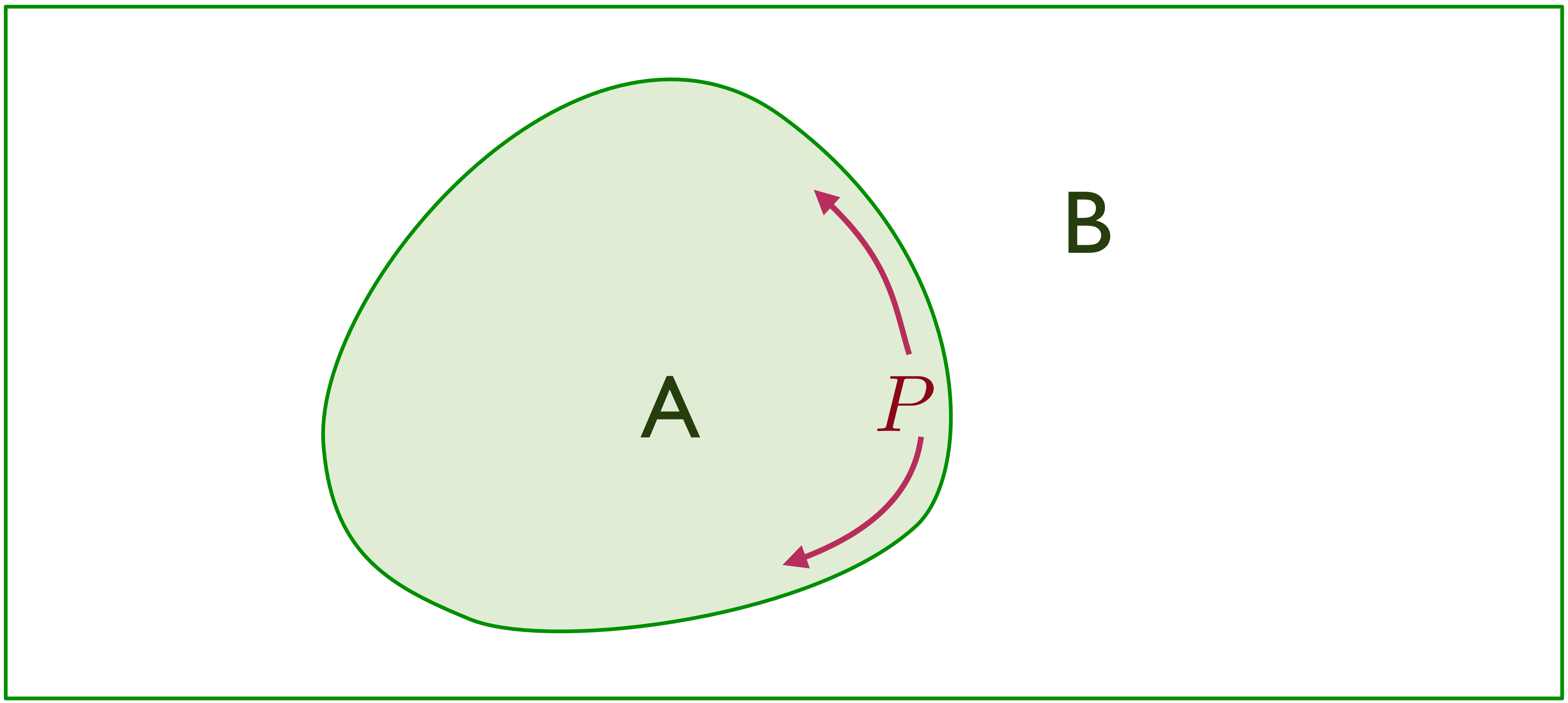


Quantum critical point described by a CFT3 (O(3) Wilson-Fisher)

S. Chakravarty, B.I. Halperin, and D.R. Nelson, Phys. Rev. Lett. **60**, 1057 (1988).
N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989).
A. W. Sandvik and D. J. Scalapino, Phys. Rev. Lett. **72**, 2777 (1994).

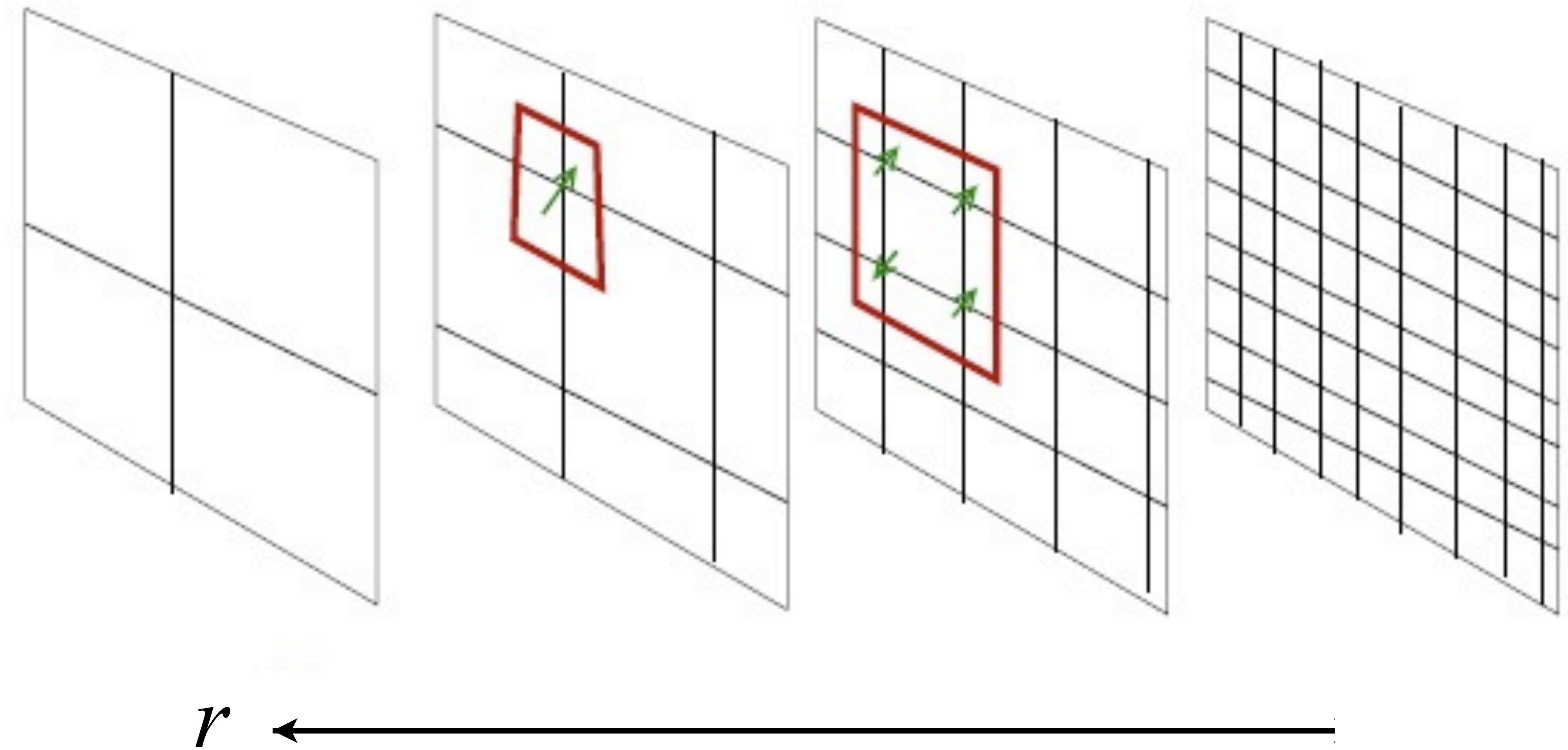
Entanglement at the quantum critical point

- Entanglement entropy obeys $S_E = aP - \gamma$, where γ is a shape-dependent universal number associated with the CFT3.



M.A. Metlitski, C.A. Fuertes, and S. Sachdev, Phys. Rev. B 80, 115122 (2009)
B. Hsu, M. Mulligan, E. Fradkin, and Eun-Ah Kim, Phys. Rev. B 79, 115421 (2009)
H. Casini, M. Huerta, and R. Myers, JHEP 1105:036, (2011)
I. Klebanov, S. Pufu, and B. Safdi, arXiv:1105.4598

Holography



Key idea: $\Rightarrow$ Implement r as an extra dimension, and map to a local theory in $d + 2$ spacetime dimensions.

For a relativistic CFT in d spatial dimensions, the metric in the holographic space is uniquely fixed by demanding the following scale transformaion $(i = 1 \ldots d)$

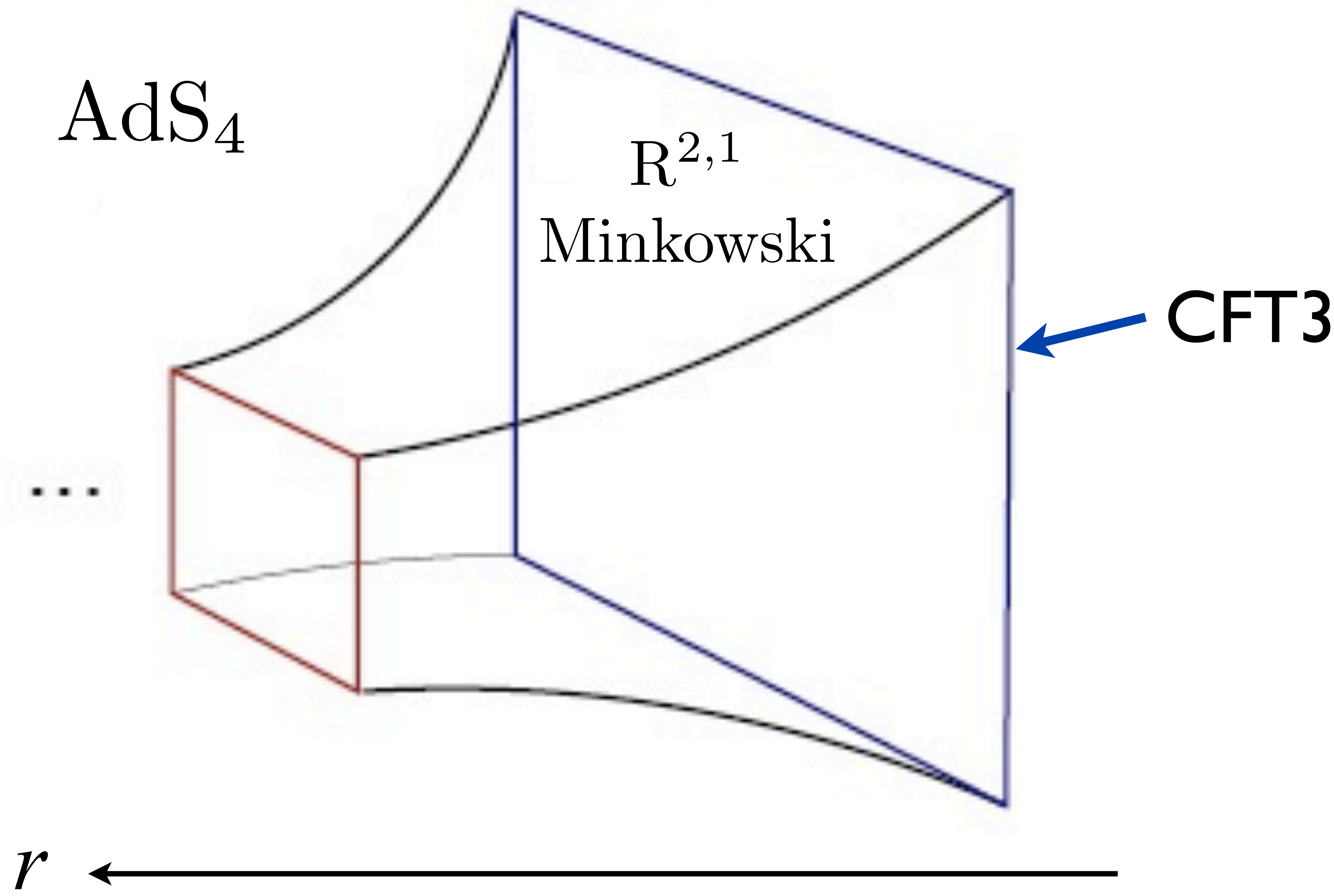
$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

This gives the unique metric

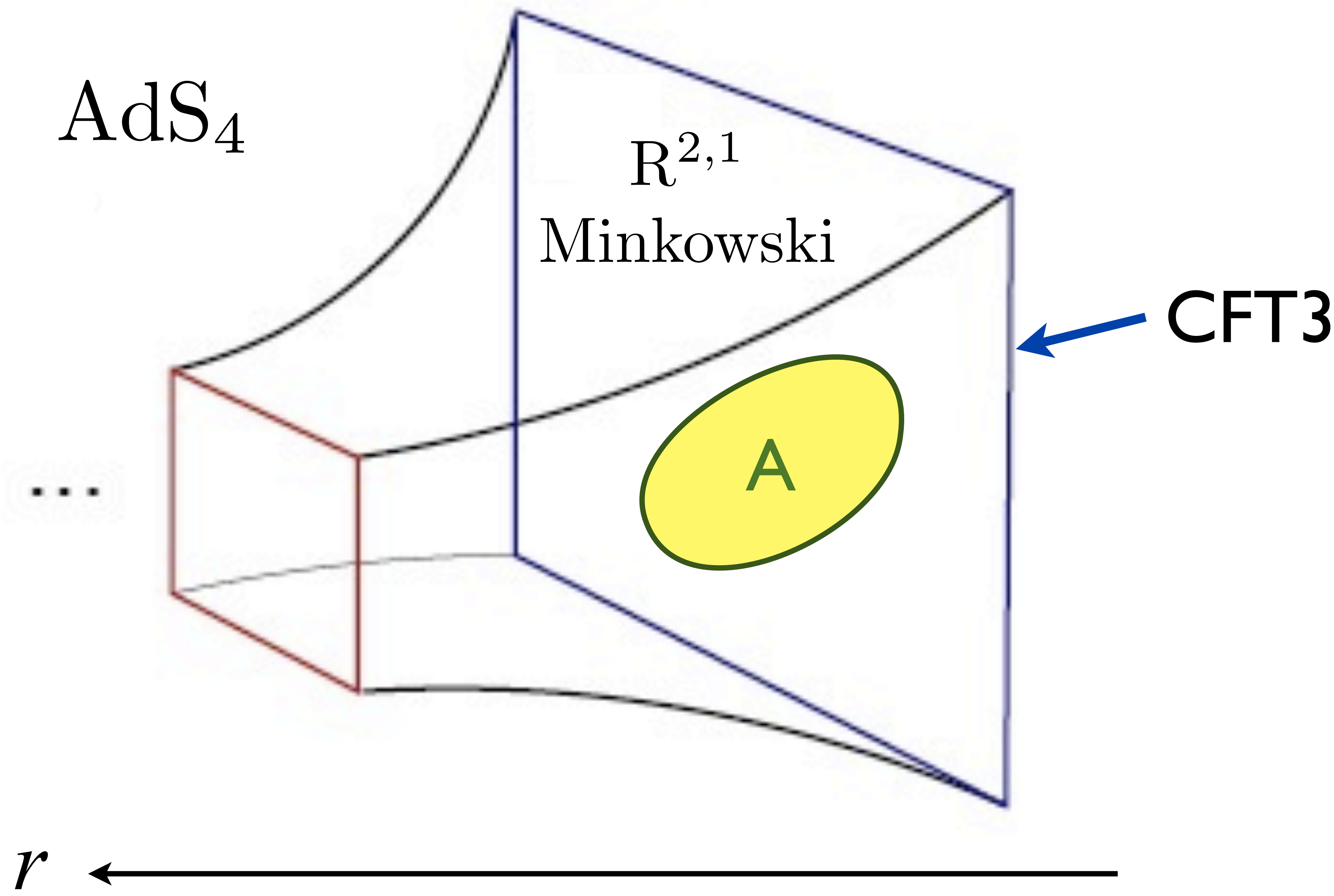
$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx_i^2\right)$$

Reparametrization invariance in r has been used to the prefactor of dx_i^2 equal to $1/r^2$. This fixes $r \to \zeta r$ under the scale transformation. This is the metric of the space AdS_{d+2}.

AdS/CFT correspondence



AdS/CFT correspondence



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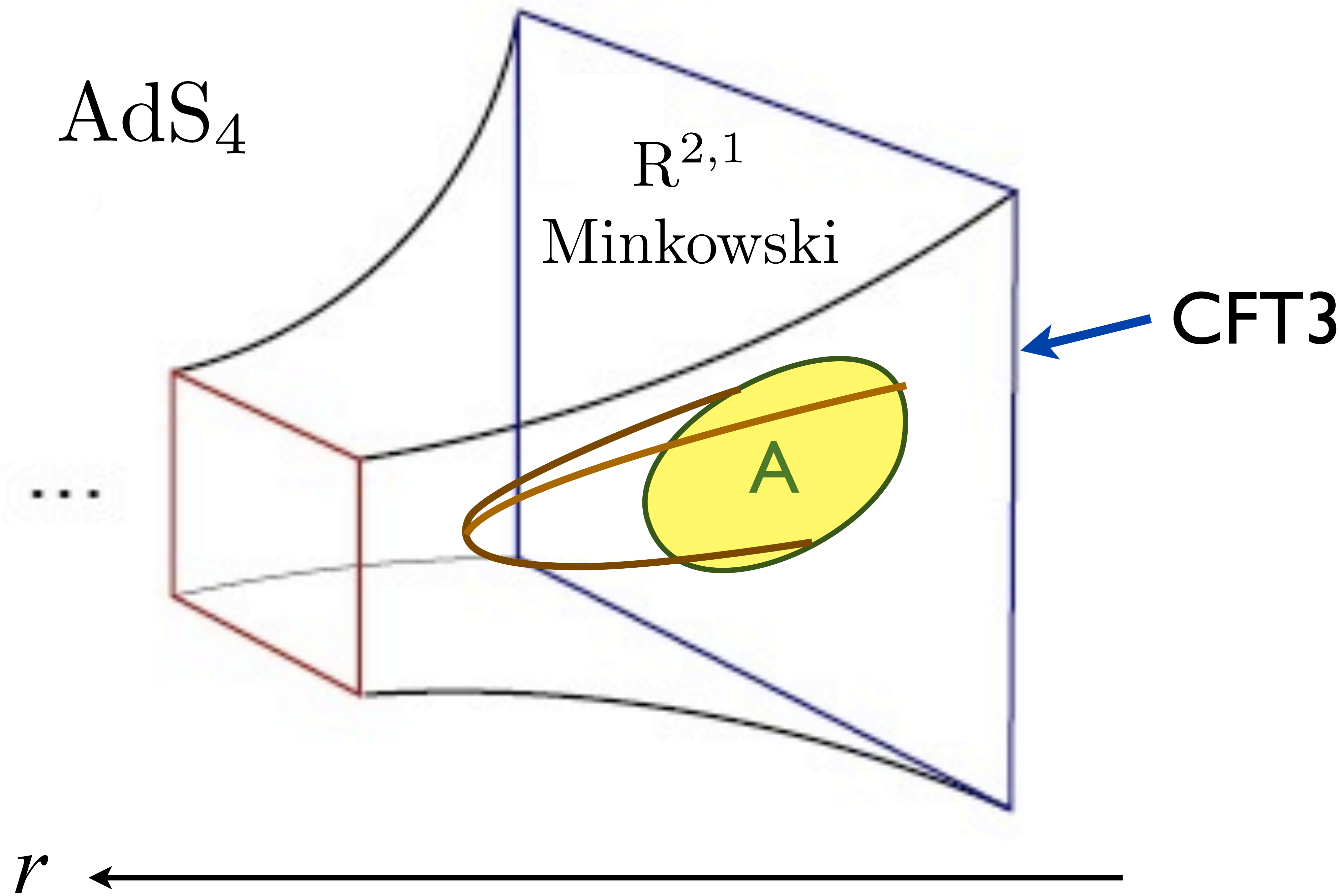
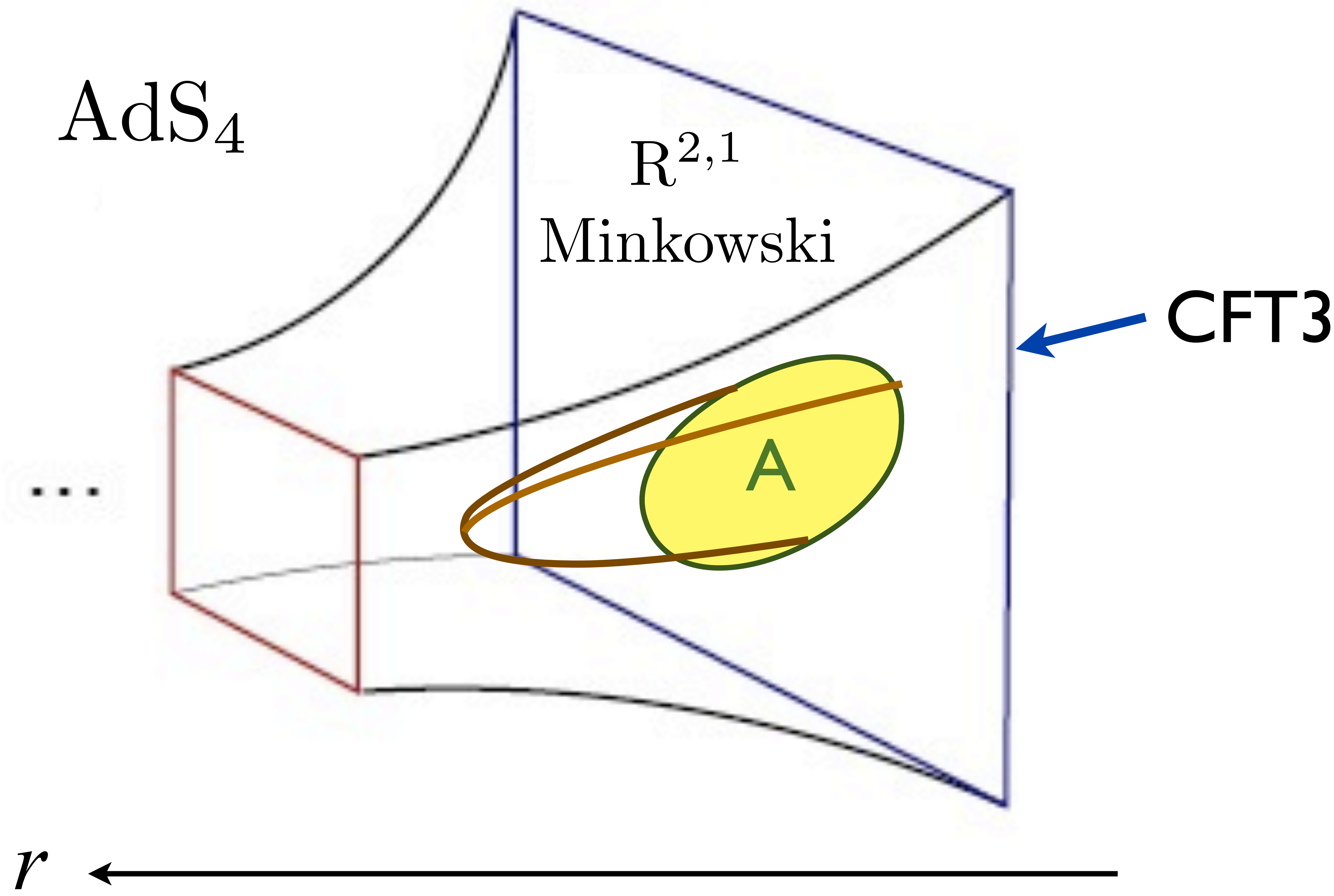


Associate entanglement entropy with an observer in the enclosed spacetime region, who cannot observe "outside" : *i.e.* the region is surrounded by an imaginary horizon.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

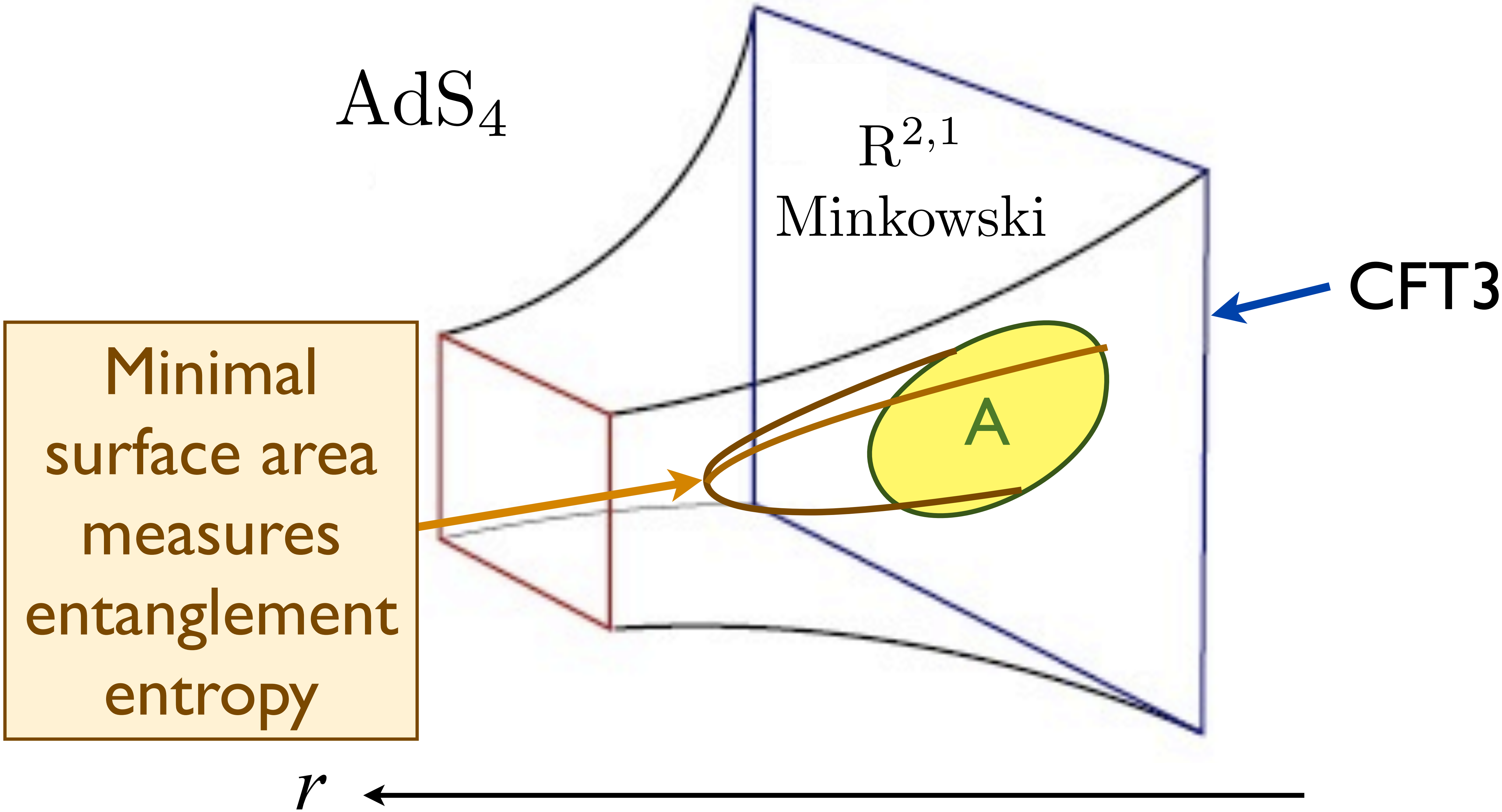
AdS/CFT correspondence



The entropy of this region is bounded by its surface area
(Bekenstein-Hawking-'t Hooft-Susskind)

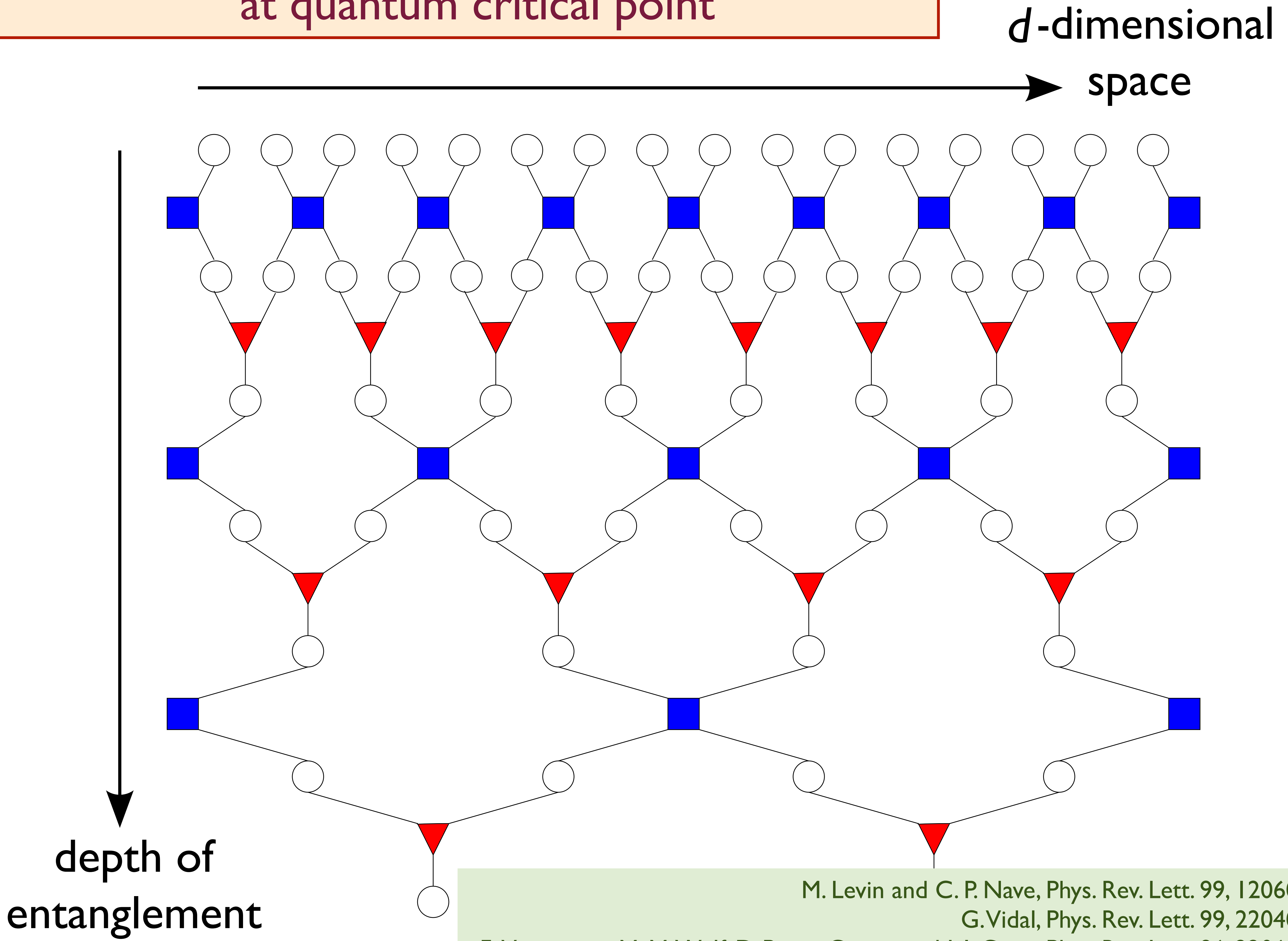
S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

AdS/CFT correspondence



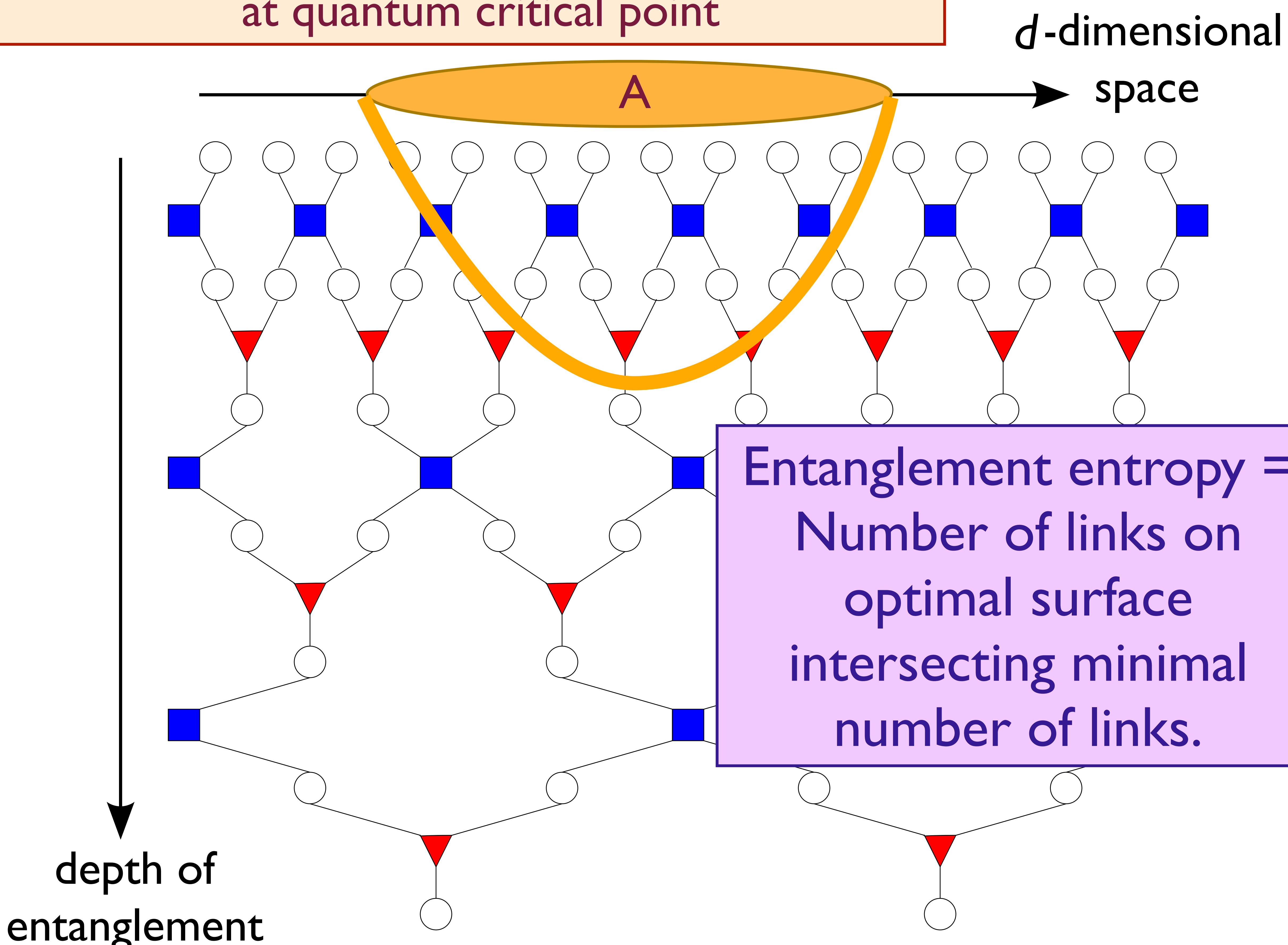
S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

Tensor network representation of entanglement at quantum critical point



M. Levin and C. P. Nave, Phys. Rev. Lett. 99, 120601 (2007)
G. Vidal, Phys. Rev. Lett. 99, 220405 (2007)
F. Verstraete, M. M. Wolf, D. Perez-Garcia, and J. I. Cirac, Phys. Rev. Lett. 96, 220601 (2006)

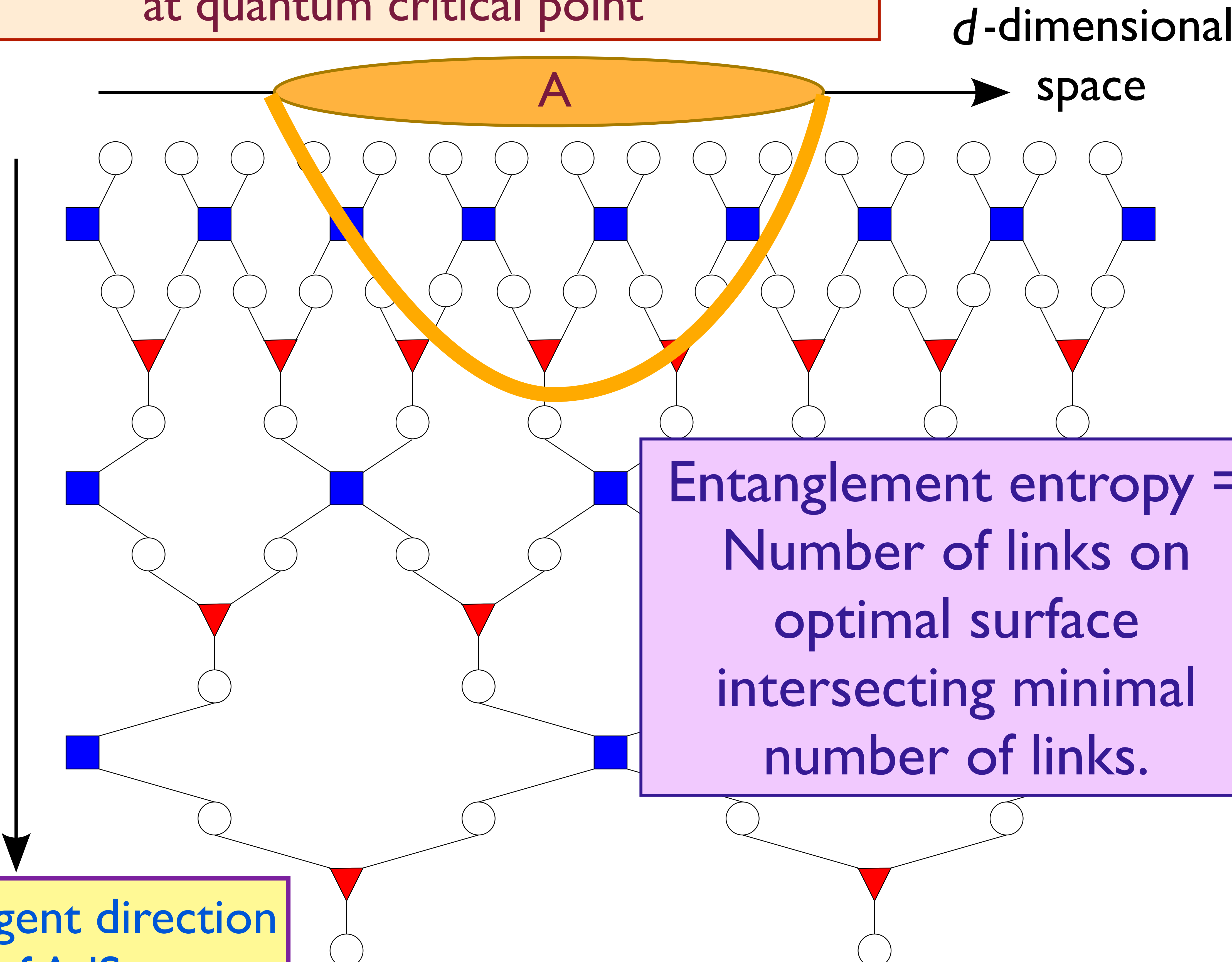
Tensor network representation of entanglement at quantum critical point



Entanglement entropy = Number of links on optimal surface intersecting minimal number of links.

Brian Swingle, arXiv:0905.1317

Tensor network representation of entanglement at quantum critical point

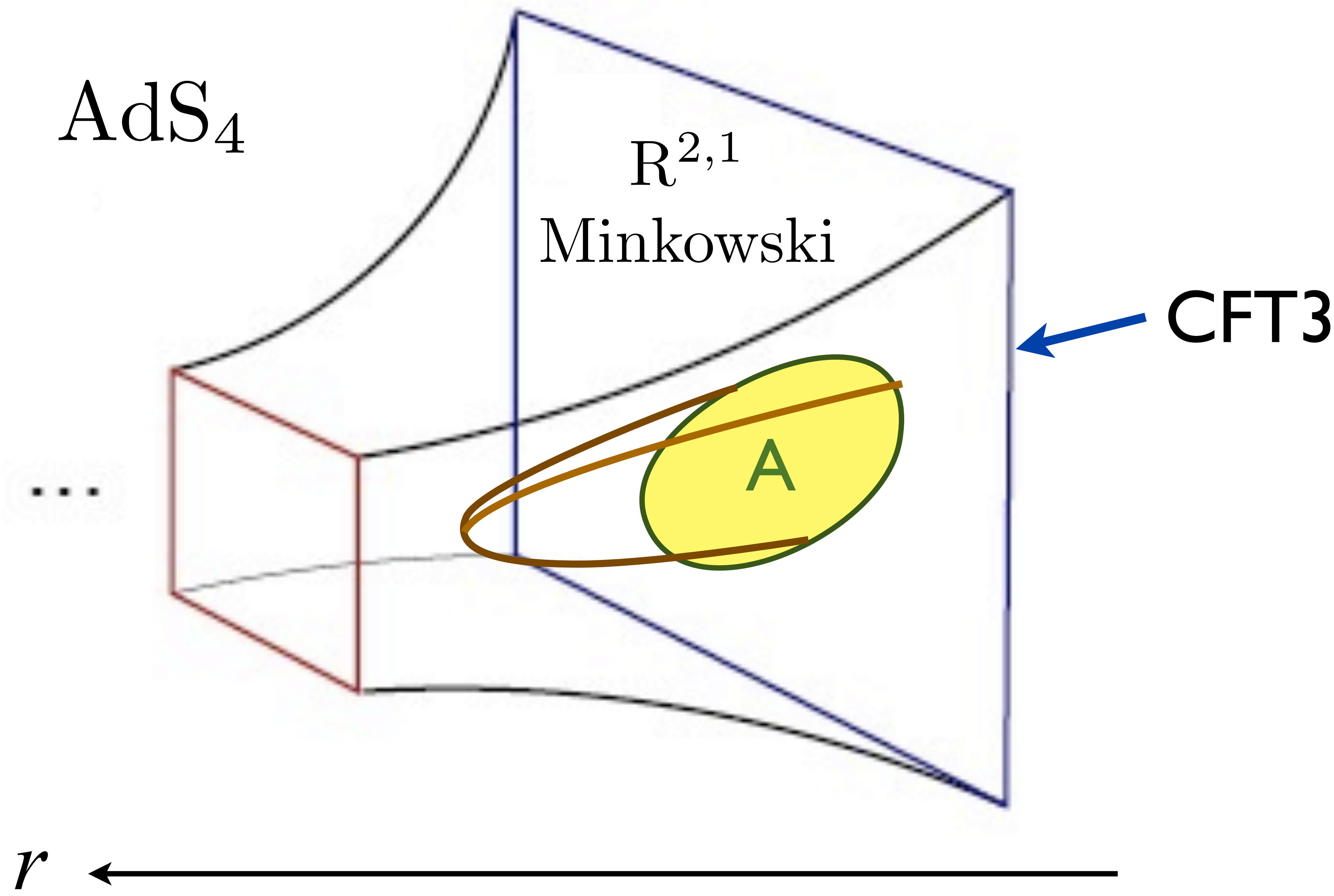


Entanglement entropy = Number of links on optimal surface intersecting minimal number of links.

Emergent direction of AdS_{d+2}

Brian Swingle, arXiv:0905.1317

AdS/CFT correspondence



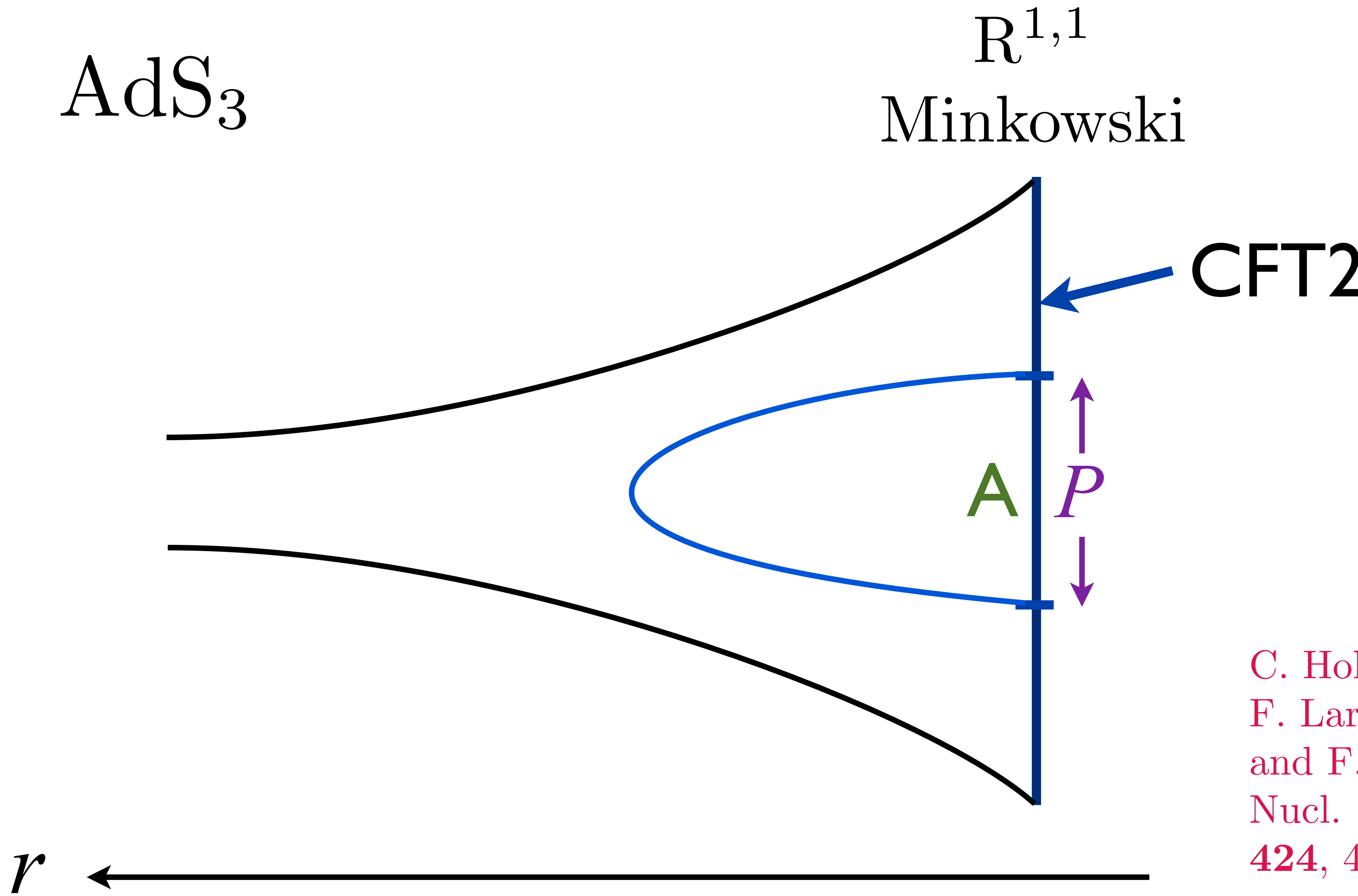
- Computation of minimal surface area yields

$$S_E = aP - \gamma,$$

where γ is a shape-dependent universal number.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

AdS/CFT correspondence



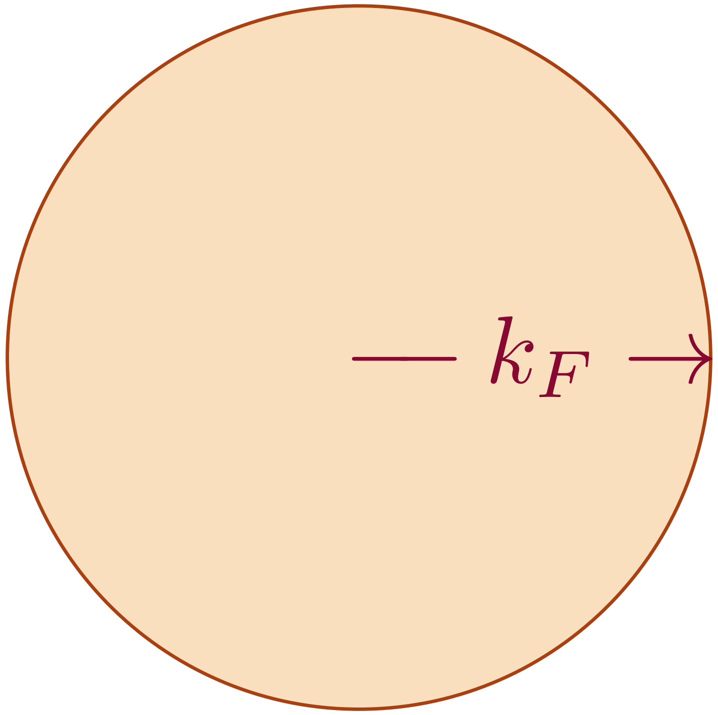
C. Holzhey, F. Larsen and F. Wilczek, Nucl. Phys. B **424**, 443 (1994).

- Computation of minimal surface area, or direct computation in CFT2, yield $S_E = (c/6)\ln P$, where c is the central charge.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

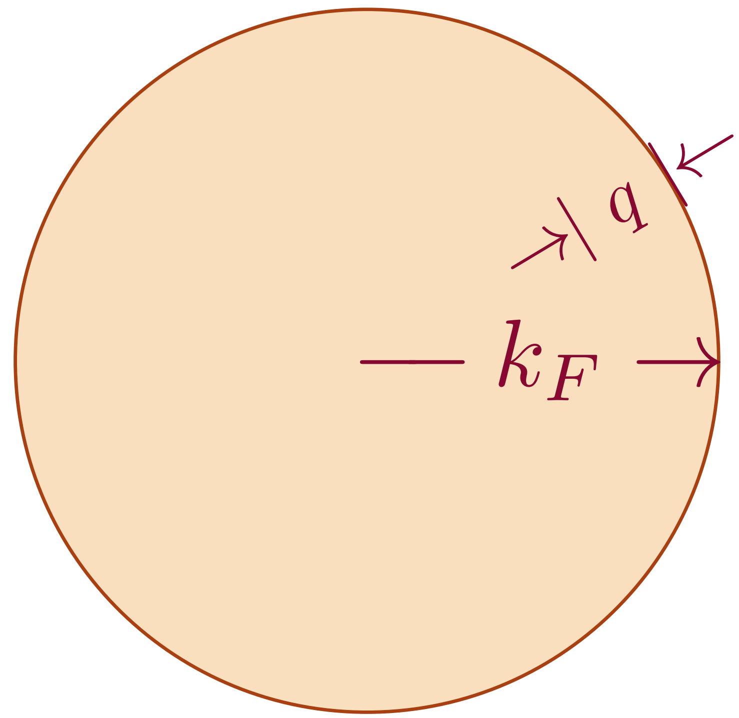
Compressible quantum matter

The Fermi liquid



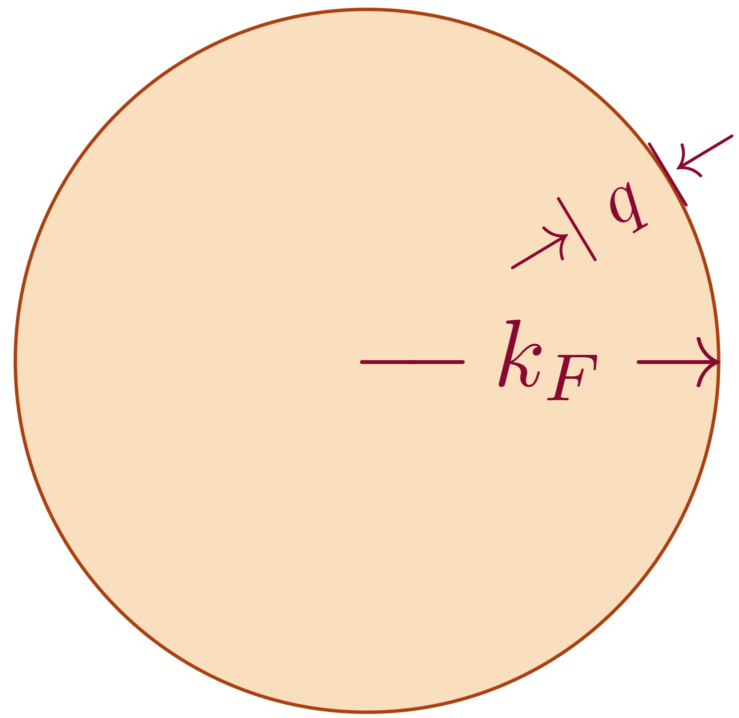
- Fermi wavevector obeys the Luttinger relation $k_F^d \sim \mathcal{Q}$, the fermion density

The Fermi liquid



- Fermi wavevector obeys the Luttinger relation $k_F^d \sim \mathcal{Q}$, the fermion density
- Sharp particle and hole of excitations near the Fermi surface with energy $\omega \sim |q|^z$, with dynamic exponent $z = 1$.

The Fermi liquid

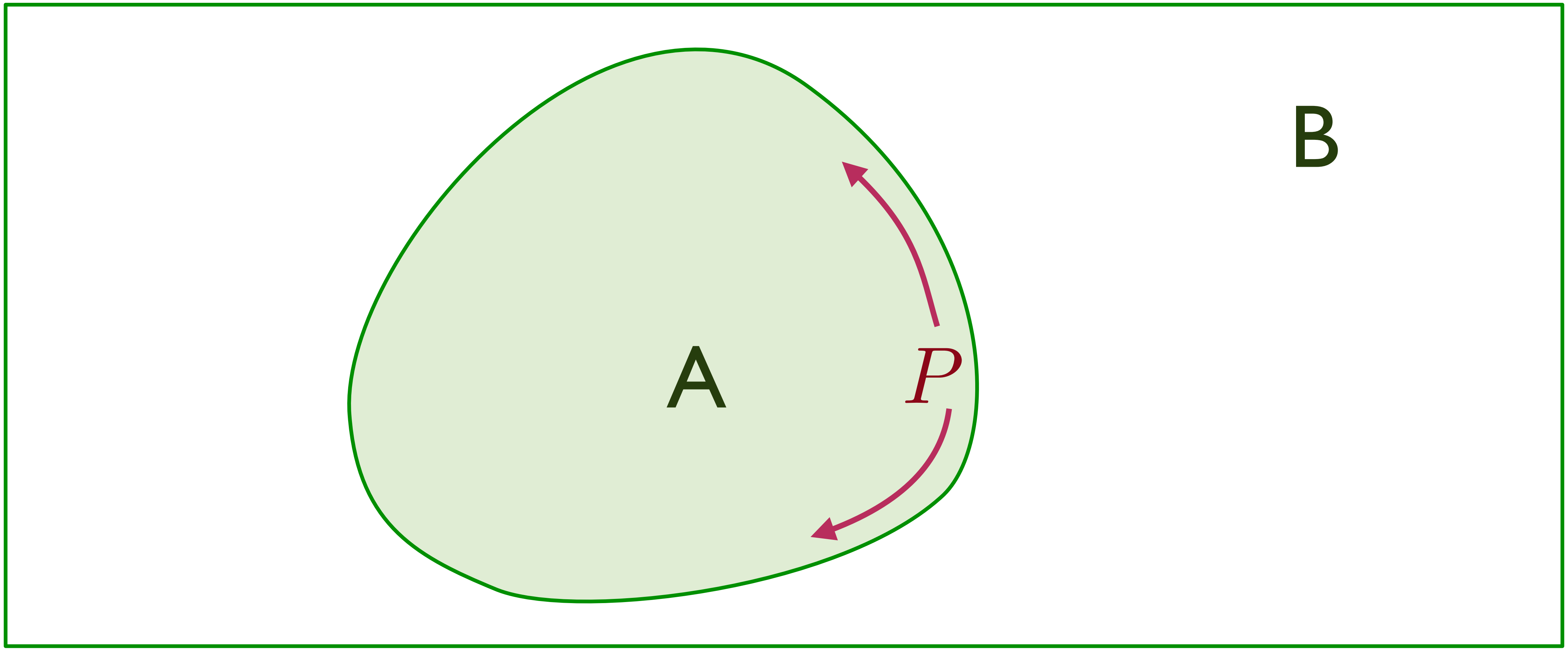


- Fermi wavevector obeys the Luttinger relation $k_F^d \sim \mathcal{Q}$, the fermion density

- Sharp particle and hole of excitations near the Fermi surface with energy $\omega \sim |q|^z$, with dynamic exponent $z = 1$.

- The phase space density of fermions is effectively one-dimensional, so the entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

Entanglement entropy of Fermi surfaces



Logarithmic violation of "area law": $S_E = \frac{1}{12}(k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

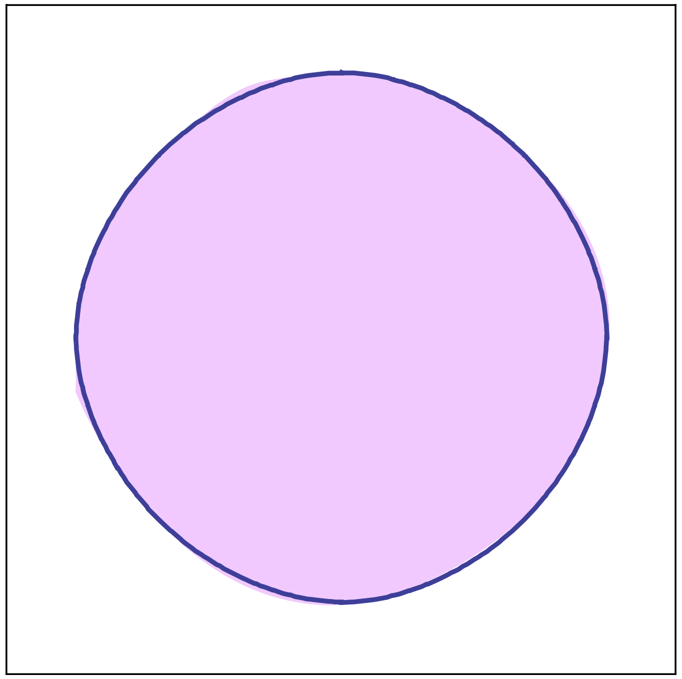
To obtain a "non-Fermi liquid" we start with a Fermi liquid, and couple it (as strongly as possible) to a low-energy bosonic mode at

(A) a non-zero wavevector, of order the reciprocal lattice vector

(B) zero wavevector

(A) Fermi liquid+antiferromagnetism

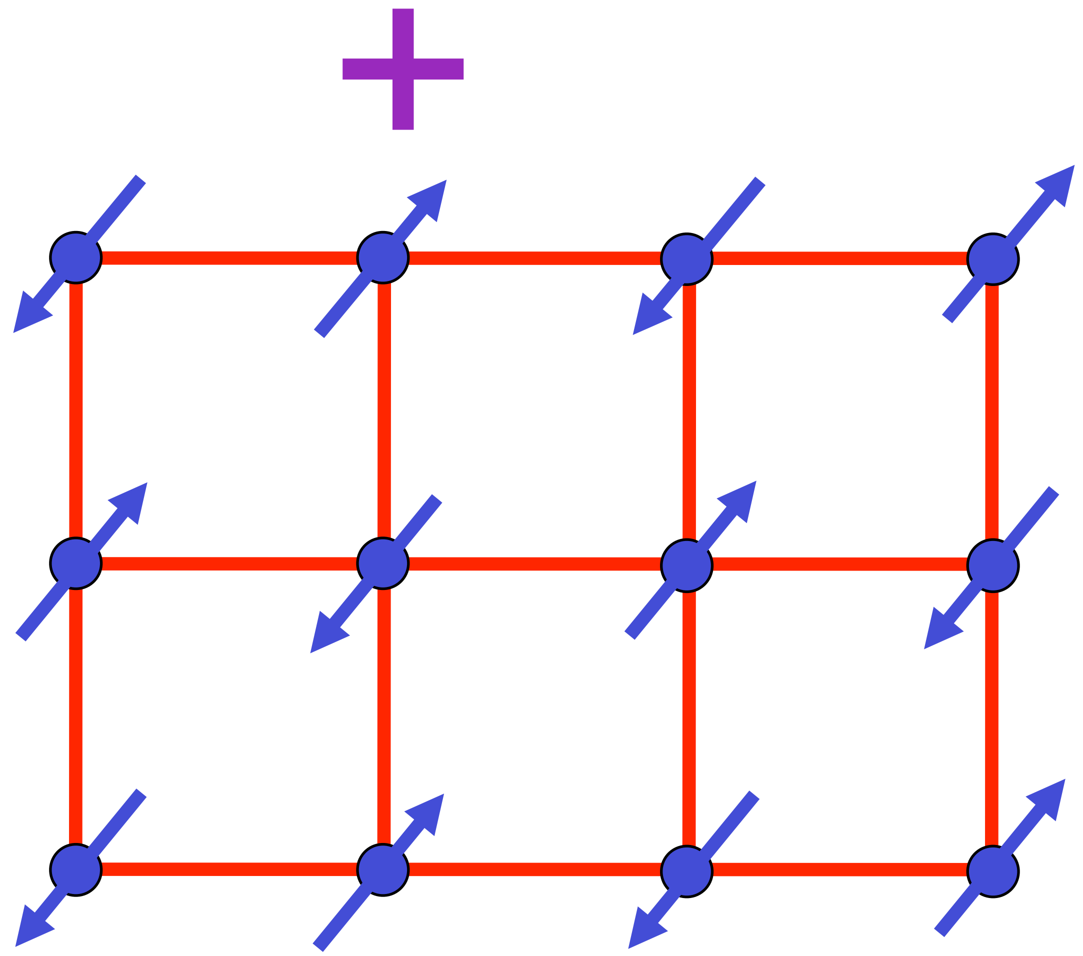
Metal with "large" Fermi surface



The electron spin polarization obeys

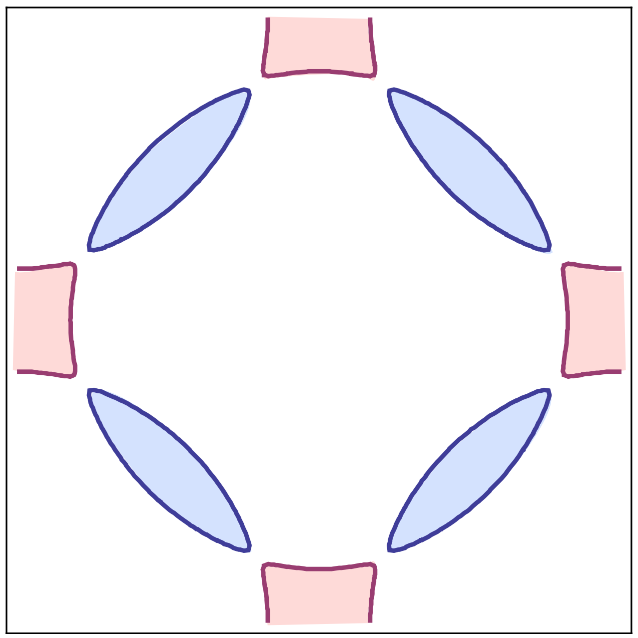
$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.



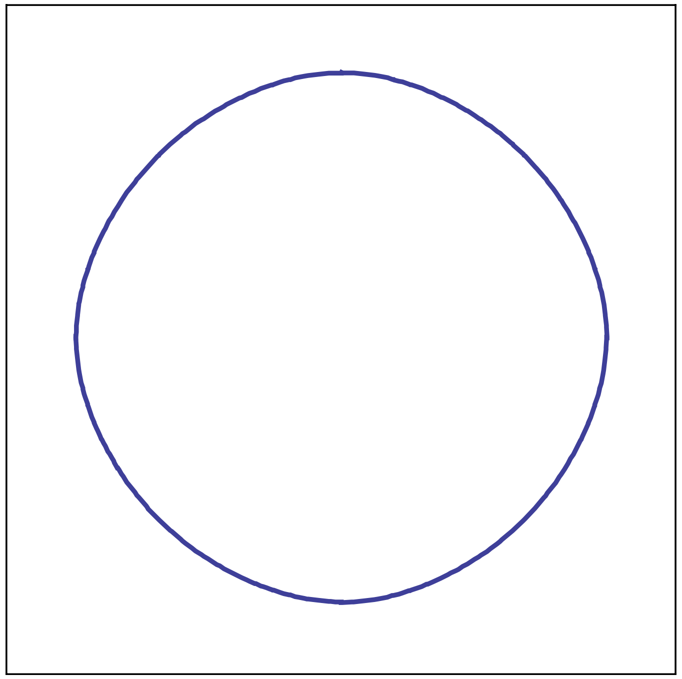
(A) Fermi liquid+antiferromagnetism

Quantum critical point of Fermi surface reconstruction



$\langle \vec{\varphi} \rangle \neq 0$

Metal with electron and hole pockets



$\langle \vec{\varphi} \rangle = 0$

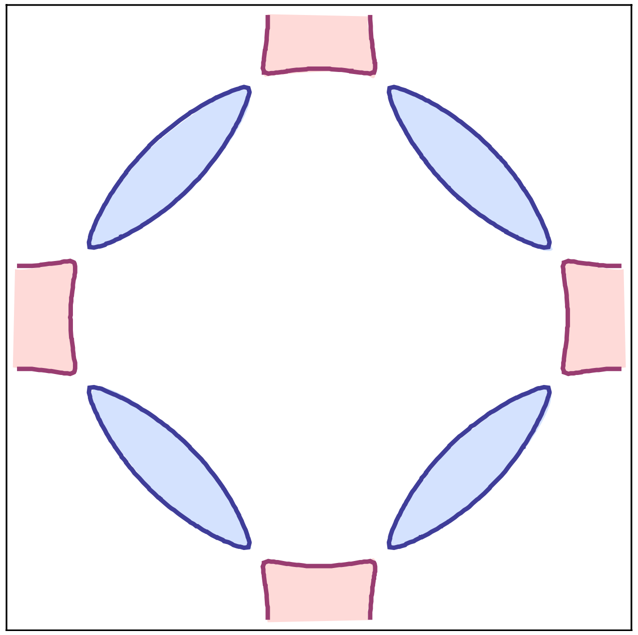
Metal with "large" Fermi surface

← Increasing interaction

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

(A) Fermi liquid+antiferromagnetism

Quantum critical point of Fermi surface reconstruction



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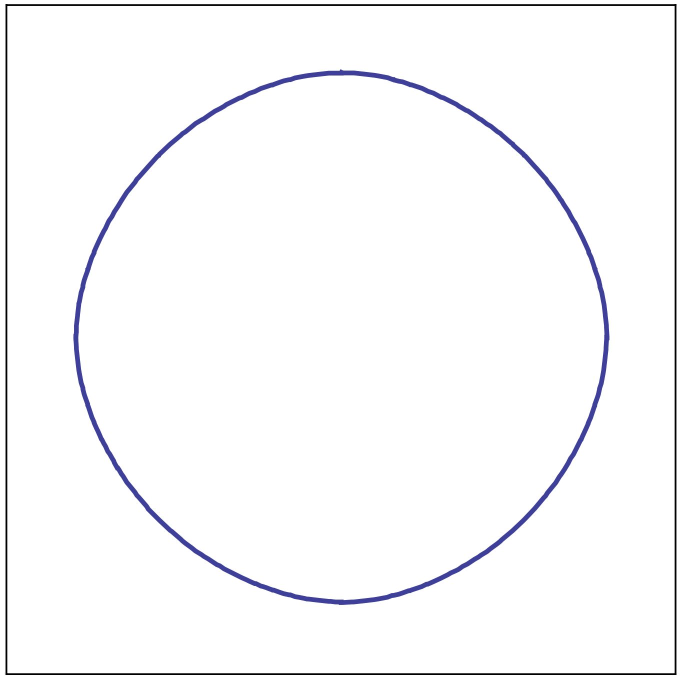

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Erez Berg

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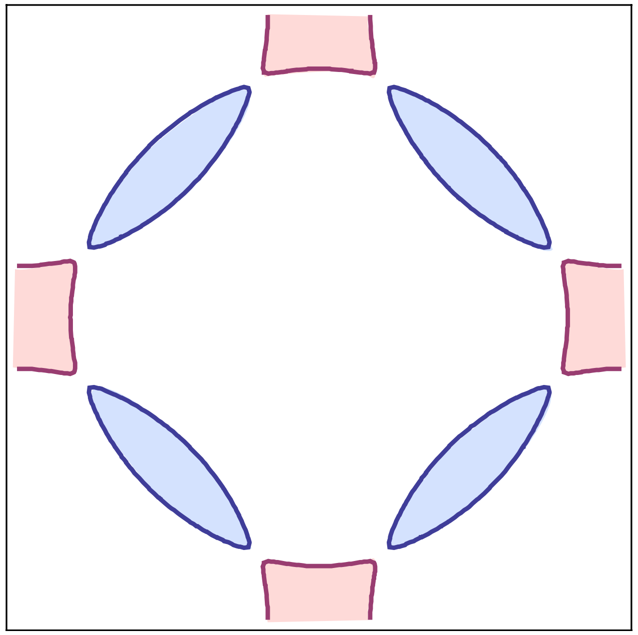
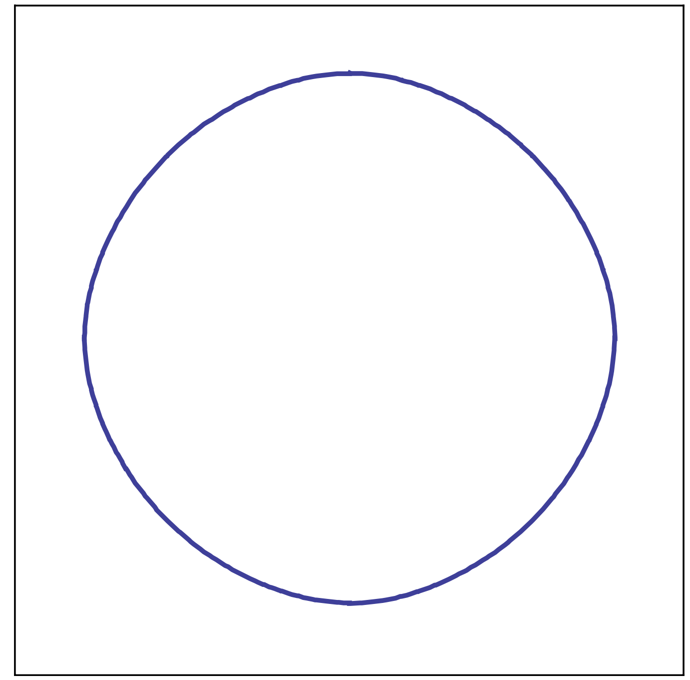




Erez Berg

$\langle \vec{\varphi} \rangle \neq 0$

$\langle \vec{\varphi} \rangle = 0$

Quantum Monte Carlo without the sign problem !

Results showing Fermi surface reconstruction, antiferromagnetic criticality, and spin-singlet *d*-wave superconductivity

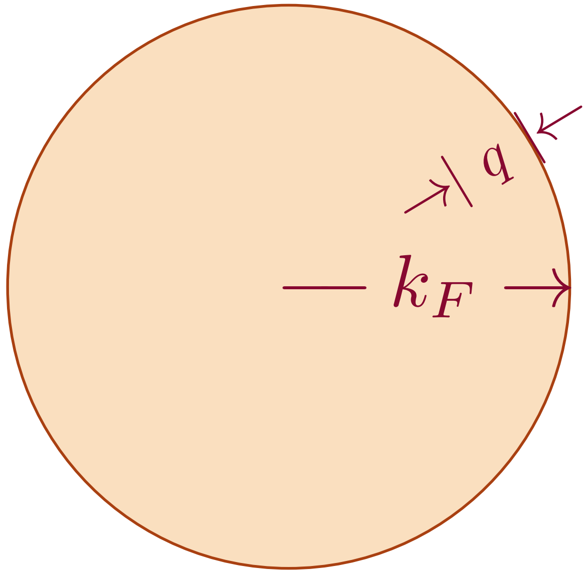
E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

(B) Strange metals

To obtain a compressible state which is not a Fermi liquid, take a Fermi surface in $d = 2$, and couple it to *any* gapless bosonic field, ϕ, which has low energy excitations near $\mathbf{q} = 0$. The field ϕ could represent

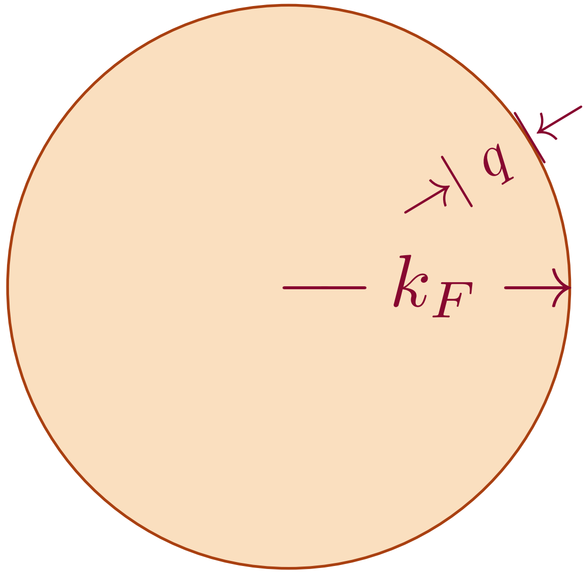
- quantum critical point of ferromagnetic ordering
- qcp of breaking of point-group symmetry (Ising-nematic order)
- qcp of breaking of time-reversal symmetry
- qcp of circulating currents
- transverse component of an Abelian or non-Abelian gauge field.
- ...

The Fermi liquid



- $k_F^d \sim \mathcal{Q}$, the fermion density
- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.
- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.
- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

The Fermi liquid



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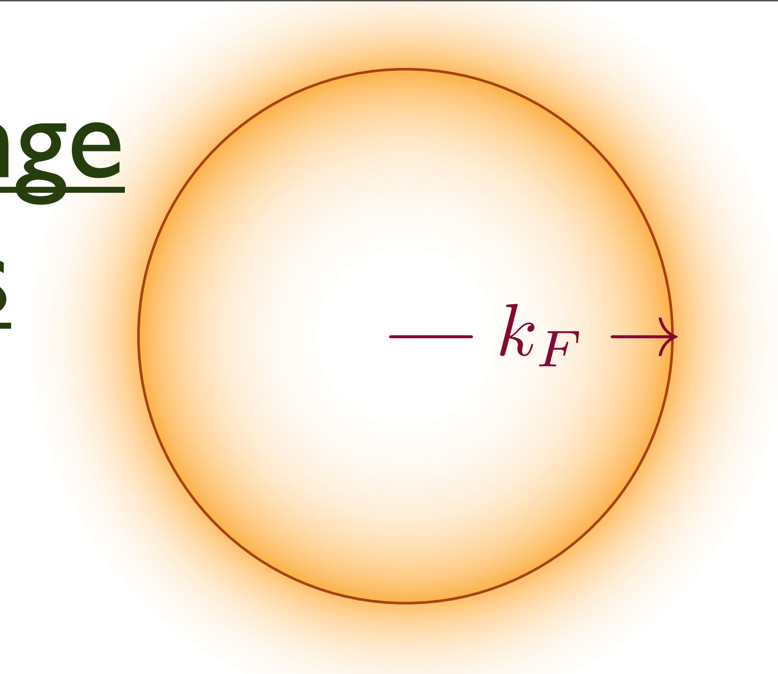

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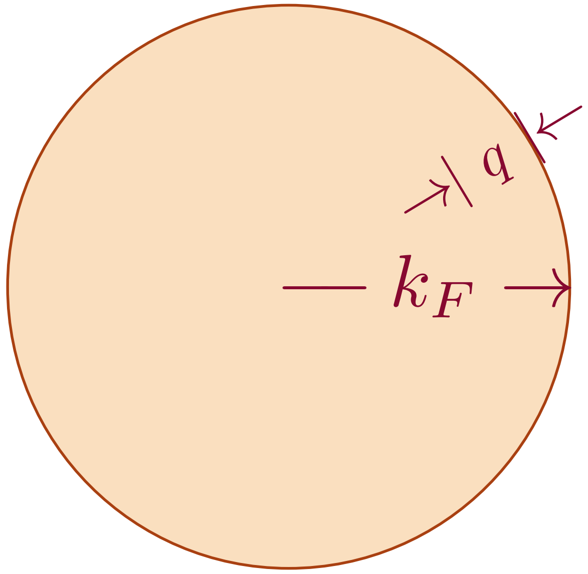
The Fermi liquid



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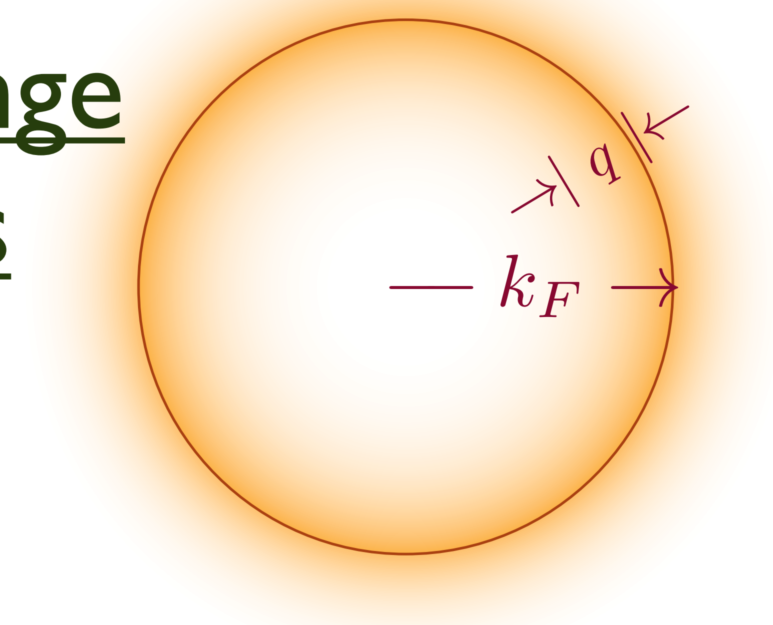
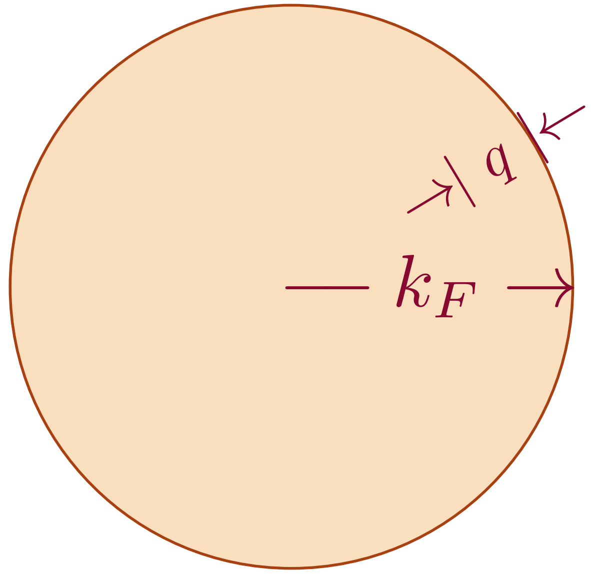


- $k_F^d \sim \mathcal{Q}$.
- Diffuse fermionic excitations with $z = 3/2$ to three loops.

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

The Fermi liquid



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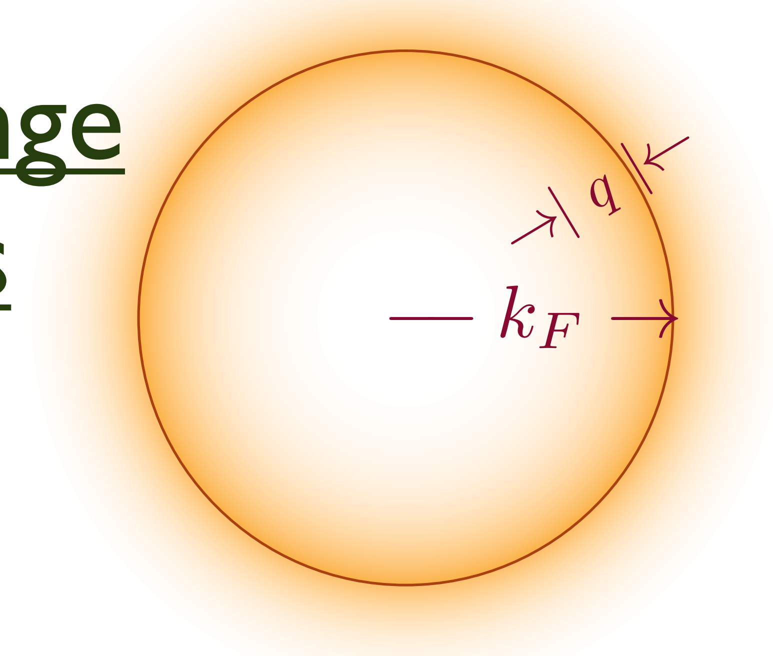
(B) Strange metals



- $k_F^d \sim \mathcal{Q}$.
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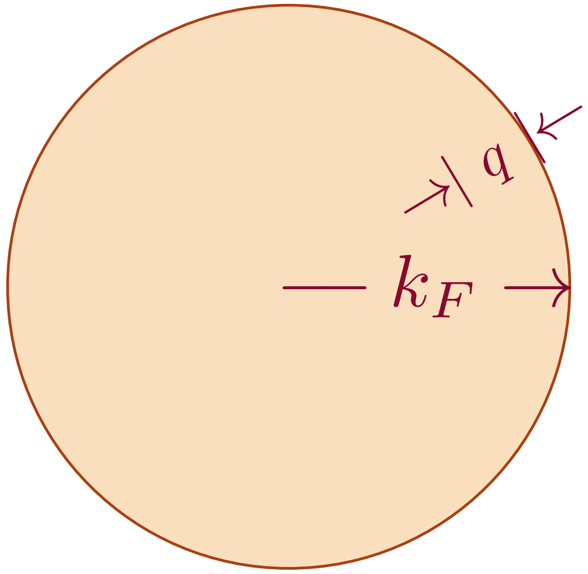
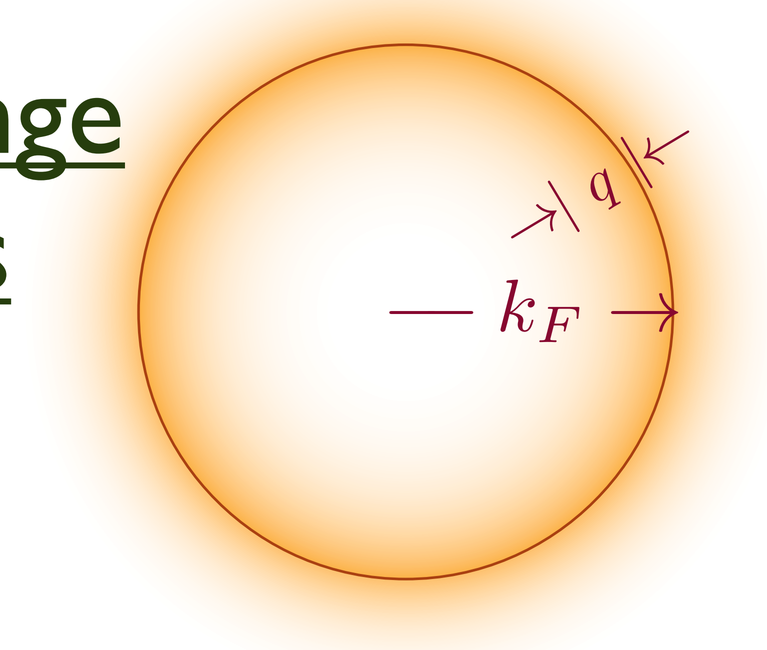
- $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$.

Monday, July 2, 12

# The Fermi liquid	# (B) Strange metals
	
• $k_F^d \sim \mathcal{Q}$, the fermion density	• $k_F^d \sim \mathcal{Q}$.
• Sharp fermionic excitations near Fermi surface with $\omega \sim \lvert q \rvert^z$, and $z = 1$.	• Diffuse fermionic excitations with $z = 3/2$ to three loops.
• Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.	• $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$.
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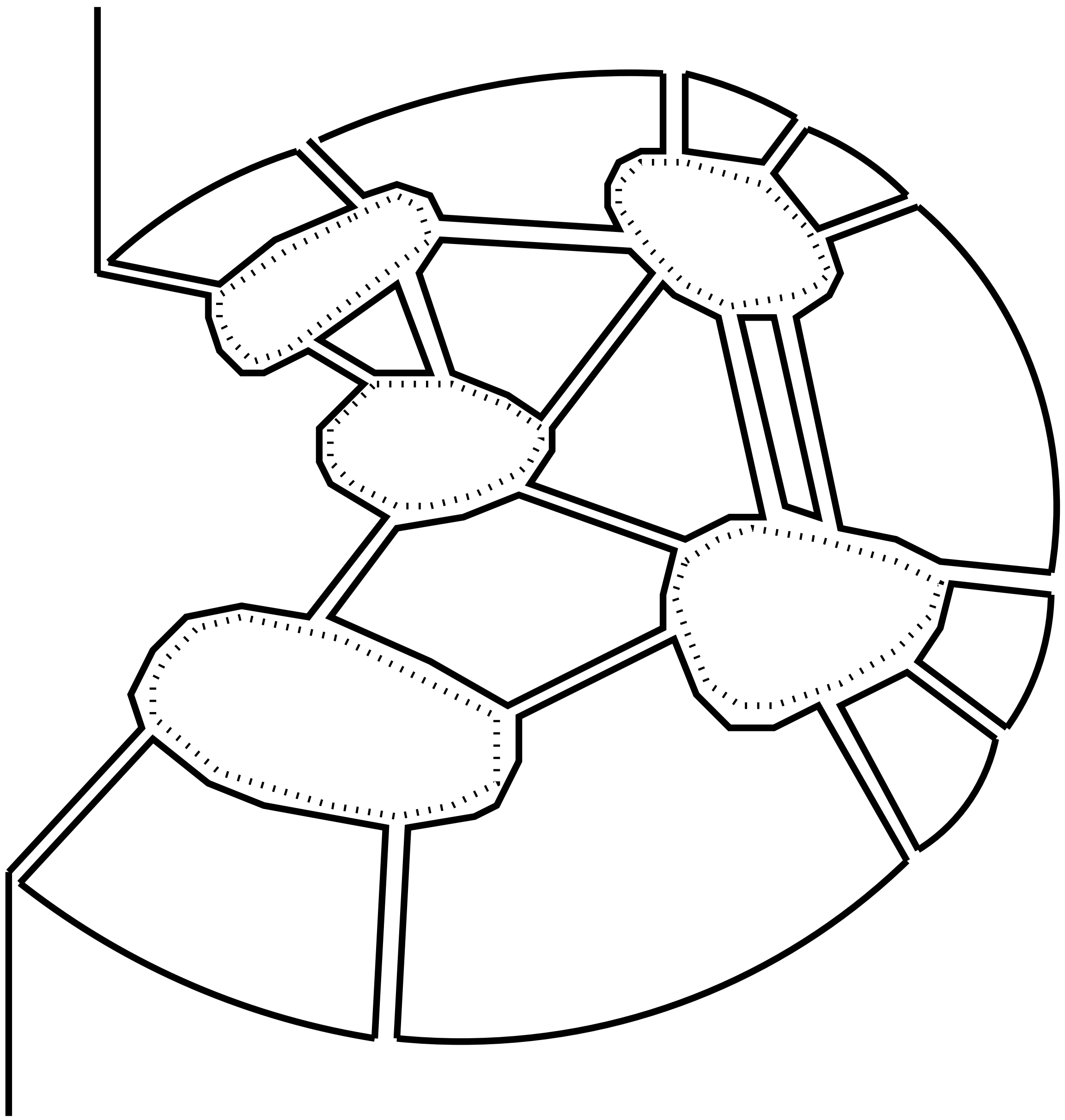
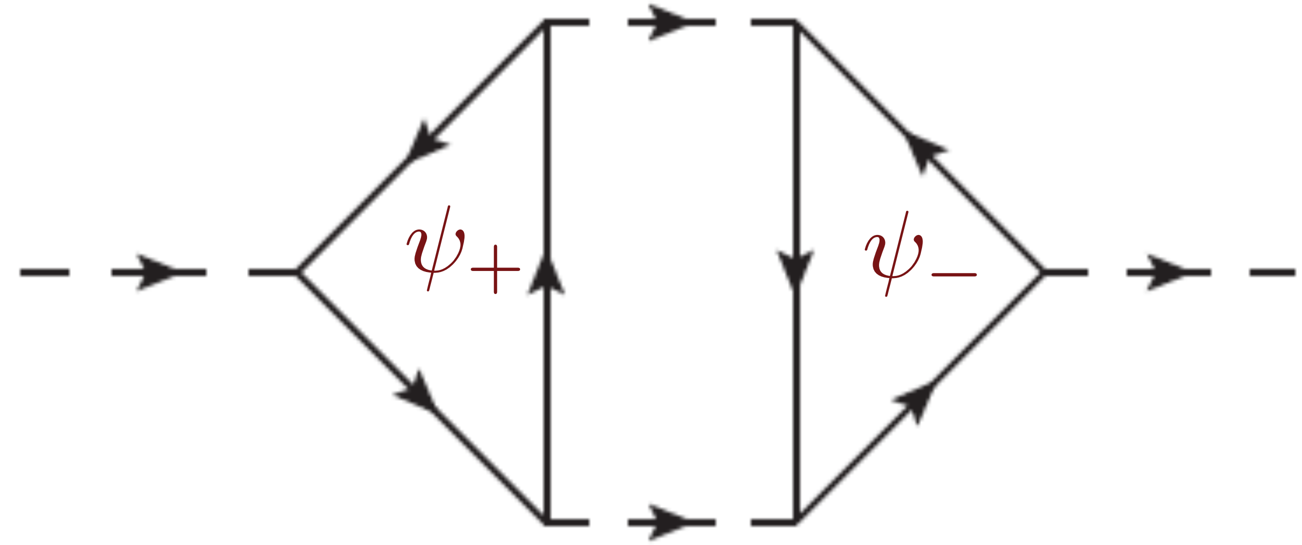
Y. Zhang, T. Grover, and A. Vishwanath, *Phys. Rev. Lett.* **107**, 067202 (2011)

Computations in the $1/N$ expansion





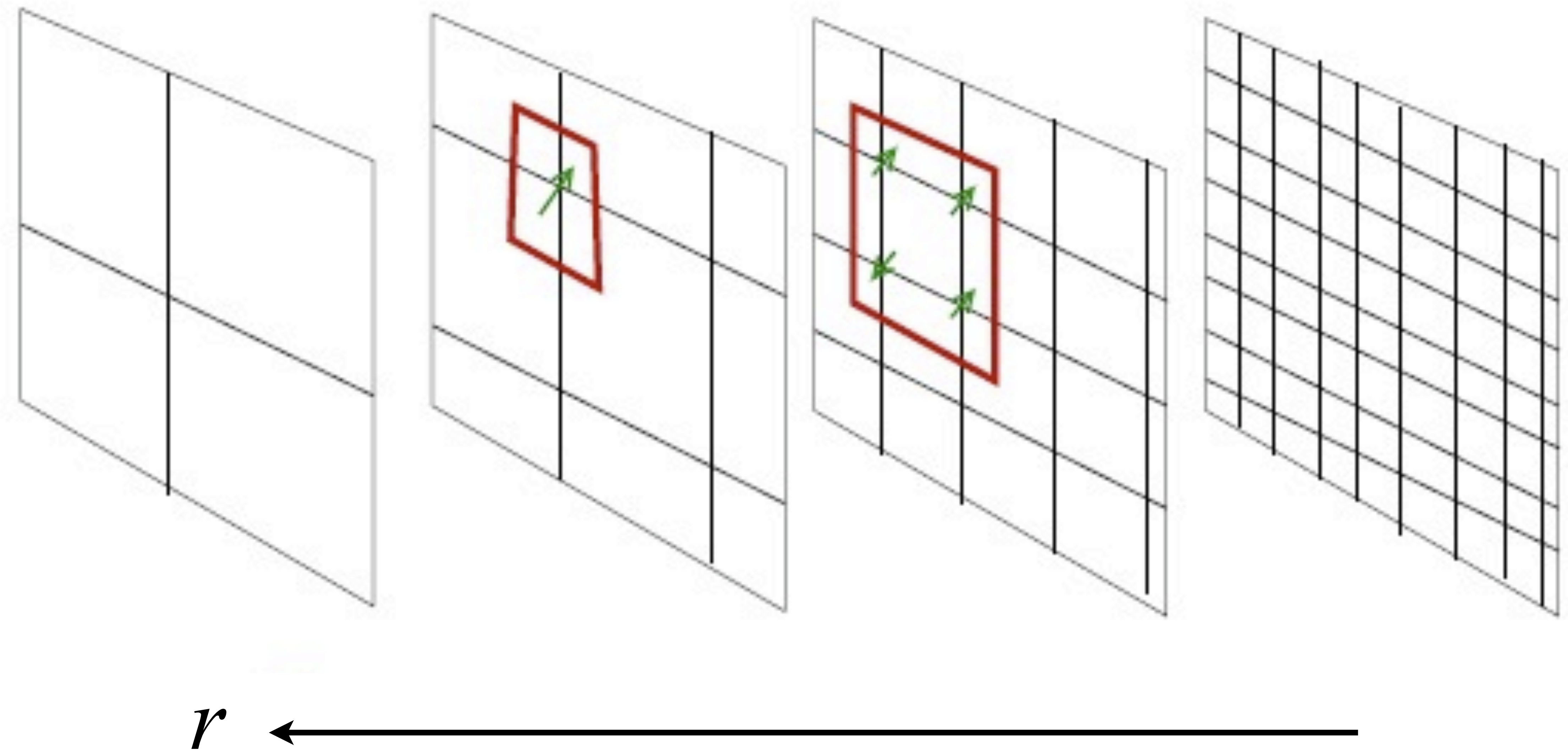
Graph mixing antipodal arcs is $\mathcal{O}\left(N^{3/2}\right)$ (instead of $\mathcal{O}\left(N\right)$), violating genus expansion

All planar graphs of fermions on an arc of the Fermi surface are as important as the leading term

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Sung-Sik Lee, *Physical Review* B **80**, 165102 (2009)

Holography



Consider the metric which transforms under rescaling as

$$\begin{aligned} x_i &\rightarrow \zeta\, x_i \\ t &\rightarrow \zeta^z\, t \\ ds &\rightarrow \zeta^{\theta/d}\, ds. \end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for "relativistic" quantum critical points).

θ is the violation of hyperscaling exponent.
The most general choice of such a metric is

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

We have used reparametrization invariance in r to choose so that it scales as $r \rightarrow \zeta^{(d-\theta)/d} r$.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

At $T > 0$, there is a *horizon*, and computation of its Bekenstein-Hawking entropy shows

$$S \sim T^{(d-\theta)/z}.$$

So θ is indeed the violation of hyperscaling exponent as claimed. For a compressible quantum state we should therefore *choose* $\theta = d - 1$.
No additional choices will be made, and all subsequent results are consequences of the assumption of the existence of a holographic dual.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of strange metals

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

$$\theta = d - 1$$

Application of the Ryu-Takayanagi minimal area formula to a dual Einstein-Maxwell-dilaton theory yields

$$S_E \sim \mathcal{Q}^{(d-1)/d} P \ln P$$

which a co-efficient *independent* of UV details and of the shape of the entangling region. These properties are just as expected for a circular Fermi surface with $\mathcal{Q} \sim k_F^d$.

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of strange metals

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

$$\theta = d - 1$$

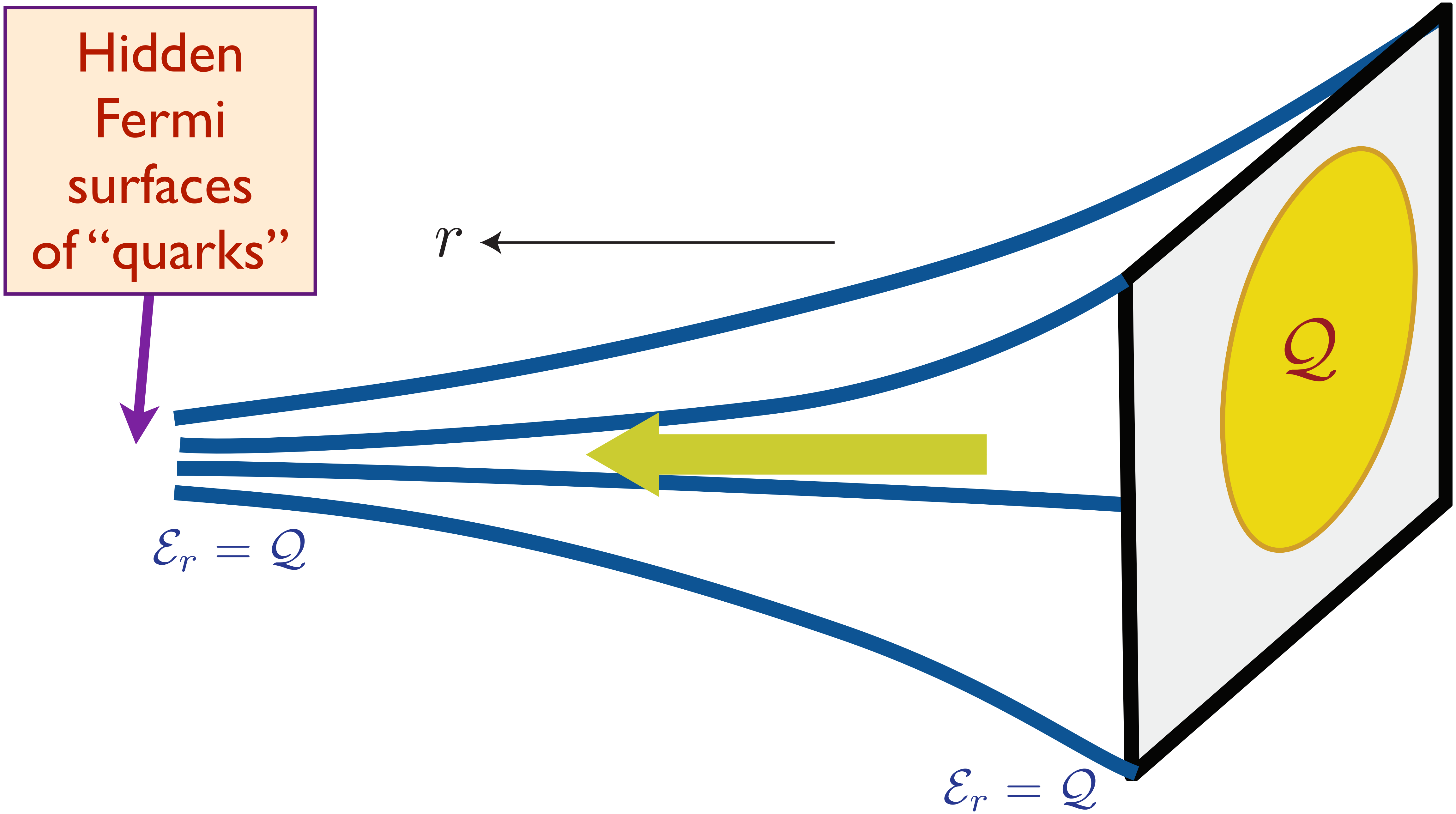
The null energy condition (stability condition for gravity) yields a new inequality

$$z \geq 1 + \frac{\theta}{d}$$

In $d = 2$, this implies $z \geq 3/2$. So the lower bound is precisely the value obtained from the field theory.

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of strange metals



Gauss Law and the "attractor" mechanism
$\Leftrightarrow$ Luttinger theorem on the boundary

Conclusions

Gapped quantum matter

- Numerical and experimental observation of a spin liquid on the kagome lattice. Likely a Z_2 spin liquid.

Conclusions

Conformal quantum matter

- Numerical and experimental observation in coupled-dimer antiferromagnets, and at the superfluid-insulator transition of bosons in optical lattices.

Conclusions

Compressible quantum matter

- Field theory of a non-Fermi liquid obtained by coupling a Fermi surface to a gapless scalar field with low energy excitations near zero wavevector. Obtained promising holographic dual of this theory.