Programme 6 Worksheet 1: Palindromic Pursuits

Take a number.	87
Reverse it.	78
Add the two numbers.	165
Reverse the answer.	561
Add the two numbers.	726
Reverse the answer.	627
Add the two numbers.	1353
Reverse the answer.	3531
Add the two numbers.	4884

The answer is a *palindrome*. It reads the same in both directions.

1. Investigate what happens when you carry out the above process with the following starting numbers:

83	243	34	38
68	472	123	341
35	79	41	

   Choose some numbers of your own; reverse the digits and add. Continue until a palindrome appears.

2. Record the number of steps that each number takes to produce a palindrome. Can you describe what is happening?

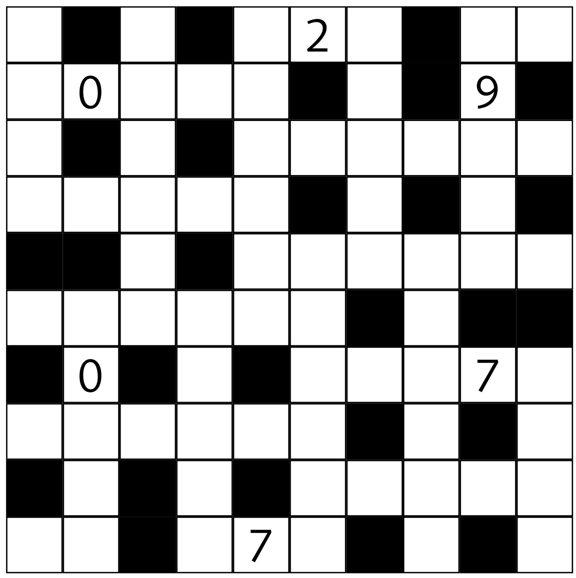
3. If you consider just two-digit numbers, how many would you have to investigate altogether? Which numbers less than 100 give a palindrome in just one step? Can you explain your results? What about numbers with more than two digits?

4. Some dates, like 18/11/81, are palindromic. How many palindromic dates can you find from the 1990s?

5. Work out 11^2, 11^3, 11^4 .... What do you notice? What about powers of 101, 111, 1001, 1111, 10101, 11111...?

Challenge

The answer to each clue in this crossword is a palindrome. A few digits have been placed to start you off. Try and complete the crossword. A calculator may be useful.



$6247^2 – 6244^2$

$(1497 + 1382) \times 24$

$(12^3 – 10^2) \times 402$

$(7^3 – 7) \times 14^2$

$(6778^2 – 6775^2) \times 2$

76668×11

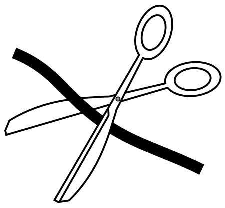
$45^3 + 3724$

$(697^2 – 414^2) \times 2$

$2938^2 – 2784^2$

Programme 6 Worksheet 2: Cuts Me Up

1. If you cut a piece of string once you will have 2 pieces.



If you cut the string again you will have 3 pieces.

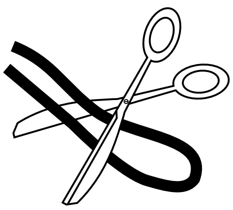
What happens if you continue cutting the string? Complete this table:

number of cuts (*n*)	0	1	2	3	4	5	6
number of pieces (*p*)	1	2	3				

How many pieces would you get if you made 100 cuts?

Can you describe the connection between the number of cuts and the number of pieces of string?

2. Investigate what happens if you fold the string once before you cut.
   You can record your results in the table below.

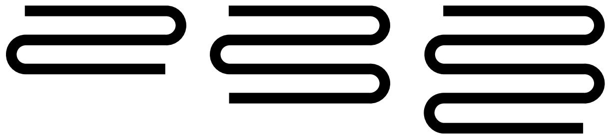


number of cuts (*n*)	0	1	2	3	4	5	6
number of pieces (*p*)	1	3	5				

What is the relation between *n* and *p* this time?

What if you made 100 cuts?

3. Investigate and describe the connection between the number of cuts and the number of pieces for strings folded like these:



Use tables to record your results and to search for patterns. Try to find the relation between *n* and *p* in each case.

4. You can also loop the string around one blade of the scissors before making a cut. Try cutting with different numbers of loops and record your results in this table.

number of loops (*l*)	0	1	2	3	4	5	6
number of pieces (*p*)	2	3	4				

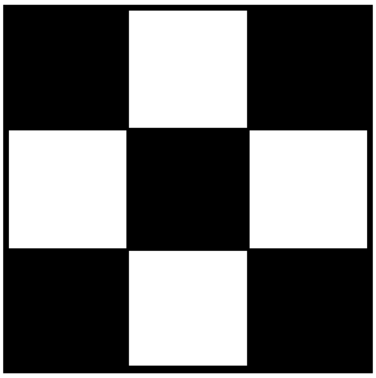
Can you describe the relationship between *l* and *p*?

Programme 6 Worksheet 3: Chessboard Challenge

How many squares are there on a chessboard? No – not 64!

You could start by looking at a smaller board.

For example, this 3 x 3 board has 14 squares.



9 1x1 squares
4 2x2 squares
1 3x3 square

14 squares in total

Check that you can find these squares.

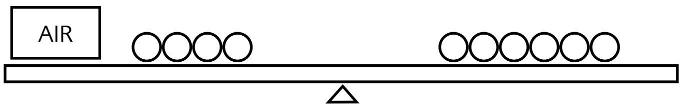
Now investigate boards of other sizes.

When you have found how many squares there are on the 8 x 8 board, try to explain how you could work out the number of squares on a square board of any size.

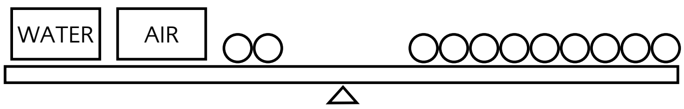
Programme 7 Worksheet 1: Balancing Act

The Balance of Nature must be maintained with vigilance. It is always recalibrated at New Year and at midsummer and the values of the elements are changed for security purposes. The Wizard is responsible for divining the true values to be used and ensuring that the instructions for setting the balance are correct. Stability in the Garden of Reason depends on solving the balances to find the elements.

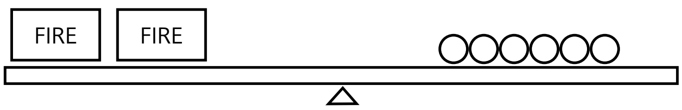
1. Last New Year the balances looked like this. One kilogram is shown by each circle.



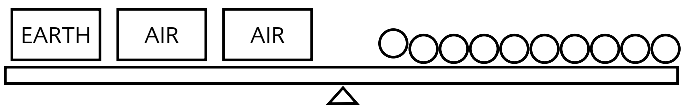
A + 4 = 6



W + A + 2 = 9



2F = 6



E + 2A = 10

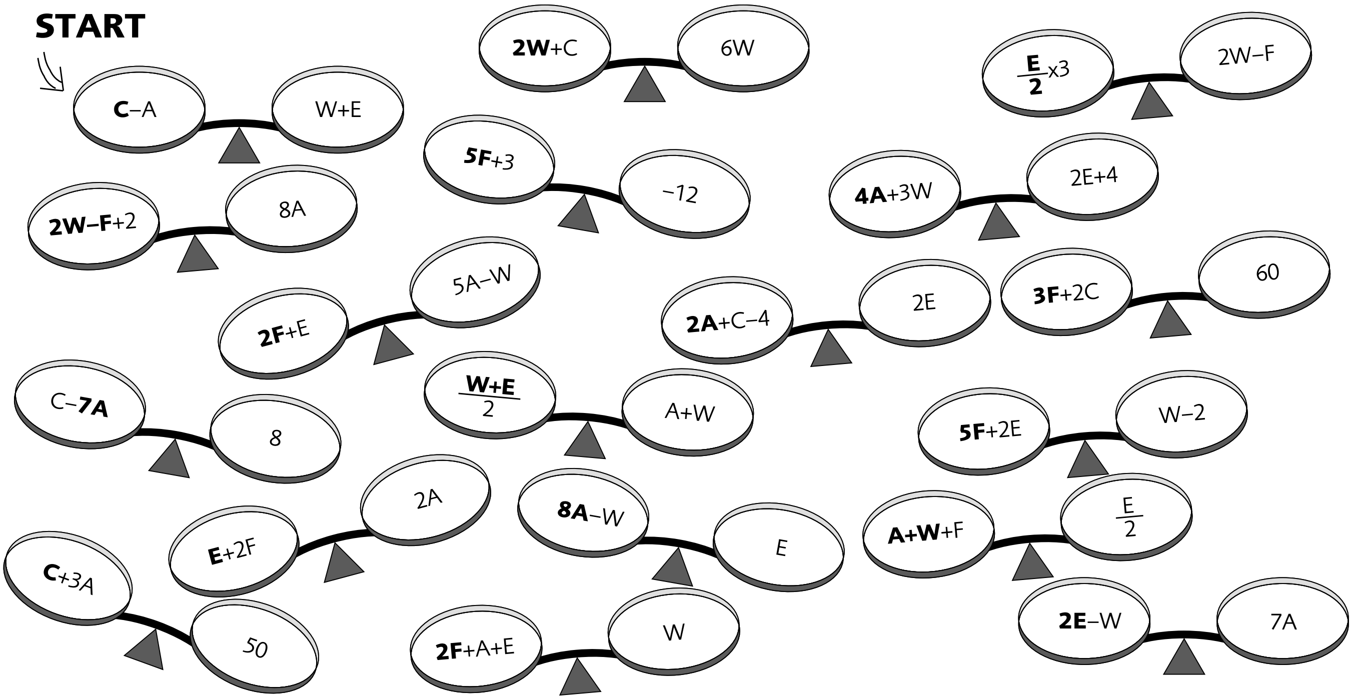
Work out the value of each element.

2. The Wizard wrote a scroll with instructions for the Guard and Lisa to follow in case the balance was disturbed again and he wasn't around. It contained these four statements:
   Earth + (2 x Air) + Fire = 11
   3 x Water = 12kg
   Water + Air + 6kg = 5kg
   (3 x Fire) + Water = 7kg

Can you show how to solve these to find the value the Wizard has chosen for each element?

3. The fifth element – Custard – is yet to be set! Can you write down some possible equations for Custard to go with the other values found in the programme? They were: Air = 3, Water = 9, Fire = –2, Earth = 25.

Programme 7 Worksheet 2: Amazing Balances



The Wizard had to attend a Wizard's Convention at midsummer. and couldn't be in the Garden of Reason to recalibrate the Balance of Nature. He asked the Guard to do it, but (conscious as always of security) he only told the Guard the value of Custard. He hid information to work out the remaining values in a maze of balances. He also put some false balances in to add to the confusion! The Guard found it all too much and went to look for Lisa!

The Wizard told the Guard that Custard was 36.

Find the correct values for all the elements.
To work through the maze:

- Move from the right-hand pan of the balance labelled 'START', by finding another balance that has the same expression written in bold on the left-hand pan.
- See what is written on the right of this balance and find the next balance with this expression on the left in bold.
- Continue until you reach a dead end.
- The final balance should contain enough information for you to find the value of an element.

You may find it useful to write down the equations represented by the balances as you go along.

When you reach the last balance use the equations you have found on your route to work out the values of all the elements and to check them.

Programme 7 Worksheet 3: Wizard Training

Discover how to amaze and baffle your friends and family. Follow the instructions in the examples below to find techniques that will demonstrate your intellectual prowess and superior skills!

Try these puzzles:

1. Think of a number.
   Add 5.
   Take away 3.
   Take away the number you first thought of.

   What happens?

Try another start number.

Will the answer always be the same?
Can you prove it?

2. Think of a number.
   Add 3.
   Double it.
   Divide by 2.
   Take away the starting number.

   What is the answer?

Try to explain what is happening.

3. Think of a number.
   Add 10.
   Double it.
   Take away 12.
   Multiply by 3.
   Add 6.
   Divide by 6.
   Subtract the number you started with.

Try different starting numbers.

Use algebra to show what is happening.

4. Think of a number.
   Add 3.
   Multiply by 10.
   Subtract 20.
   Divide by 5.
   Take away twice the number you first thought of.

Can you prove that the answer will always be the same?

5. Think of a number.
   Multiply by 2.
   Add 5.
   Multiply by 5.
   Subtract 25.
   Divide by 10.

What is happening?

Try to explain how this puzzle works.
Can you prove it?

Make up your own 'think of a number' puzzle and check using algebra that it will always work. Now memorise the steps and go and stun people with your mind-reading powers!

Challenge

Write down your house number.
Multiply it by 2.
Add 5.
Multiply by 50.
Add your age in years.
Add 365.
Subtract 615.

What is your answer?

Try to see how it works.

Programme 8 Worksheet 1: None Shall Pass!

The Guard has more puzzles like the one he set Lisa, just in case there are any more visitors to the garden who want to pass through the doors. He is particularly keen to keep out the Ravening Hordes! See if you can find the three numbers in each of the puzzles:

1.
- The number on the second door is 3 more than the number on the first door.
- The number on the third door is 7 more than the number on the second door.
- The sum of the three numbers is 70.

2.
- The number on the second door is 5 more than the number on the first door.
- The number on the third door is 5 more than the number on the second door.
- The sum of the three numbers is 40.

3.
- The number on the second door is double the number on the first door.
- The number on the third door is 3 times the number on the first door.
- The three numbers add up to 93.

4.
- The three numbers are consecutive whole numbers.
- The product of the three numbers is 1320.

5.
- The number on the second door is twice the number on the first door.
- The number on the third door is twice the number on the second door.
- The product of the three numbers is 1000.

Make up your own puzzle like these.
Swap puzzles with a partner and try to solve each other's.

Programme 8 Worksheet 2: Try, Try and Try Again

The number 19 can be expressed as the sum of a pair of numbers:

19 = 17 + 2
19 = 1 + 18
19 = 8.4 + 10.6

If the two numbers are then multiplied together they give different products:

17 x 2 = 34
1 x 18 = 18
8.4 x 10.6 = 89.04

Find the pair of numbers that gives the greatest product.

Investigate the greatest product that can be found when other numbers are split into pairs.

Can you generalise your results?

What if the number is split into 3 parts? Can you find the split that gives the greatest product?

Extend your investigation to 4, 5… *n* parts.

Try to generalise your findings.

Programme 8 Worksheet 3: Pi in the Sky

Find out more about the number π.

What is it?

How can you find a value for it?

How can it be used?

What methods are used to calculate it?

How accurately has it been calculated?

What did earlier civilisations know about this number?

Why is it so fascinating?

You could create a set of references to information about π in books, newspapers or journals, or you could use this list of web links:

http://www.pipage.fsnet.co.uk/index.html
A UK site aimed at GCSE students and featuring a π fan club.

http://www.links2go.com/topic/Constants
A list of 'π' links.

http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Pi_through_the_ages.html
Background to the history of calculating π, and links to other resources.

http://www.joyofpi.com/index.htm
The website to accompany the book *The Joy of Pi.*

http://users.aol.com/s6sj7gt/cadenza.htm
An example of mnemonic writing based on the digits of π, claiming to be the world's longest.

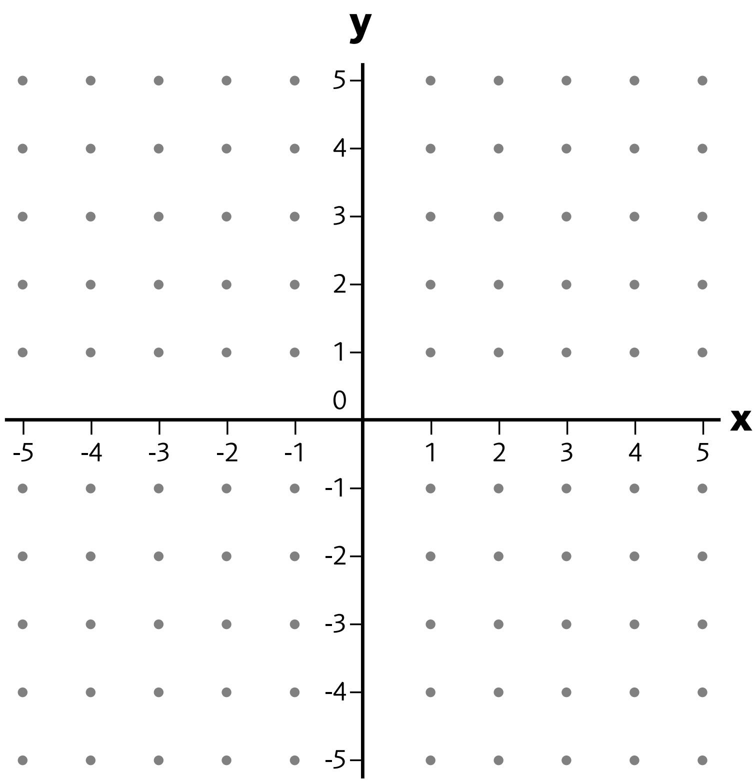
Include some details about the content of the materials you have chosen and, if appropriate, a description of how they could be used by other students.

Make a display of interesting, informative or amusing material and facts to do with π.

Programme 9 Worksheet 1: Restorative Vegetable Soup

The Wizard is making his restorative soup. He claims it will increase intelligence, boost energy levels and act as a general tonic. He sends Lisa and the Guard to the vegetable plot to collect some unusual ingredients. As always nothing is straightforward. They have to identify the vegetables on the grid, using the Wizard's notes, given below. Use the graph to draw the lines that will help you work out the coordinates of the points where the vegetables have been left.

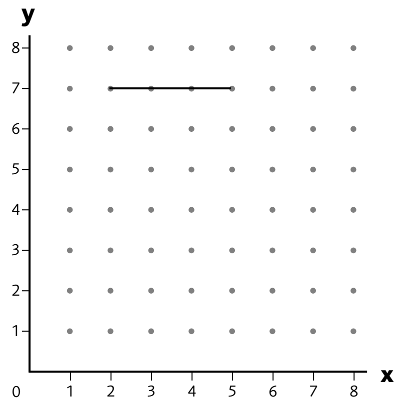
Here is the Wizard's list of ingredients:



- one artichoke at the origin
- curly kale at the point where $x = 3$ meets $y = -4$
- a bunch of chervil at the point where $y = 2$ meets $x = -3$
- an aubergine at the point where $y = 4$ crosses the y-axis
- a marrow at the point where $x = 3$ and $y = x$ intersect
- 5 asparagus spears, one at each of the points on $y = -2$ where the value of x is a positive whole number
- one head of chicory at the point where $y = 2x - 1$ crosses the y-axis
- a root of celeriac at the point of intersection of $y = 2x - 1$ and $y = x$
- sorrel leaves where $y = -x$ crosses $x = 3$
- 3 zucchini at the vertices of the triangle formed by the intersection of the lines $y = 2$, $y = -x$ and $y = x + 2$

Programme 9 Worksheet 2: Line Them Up

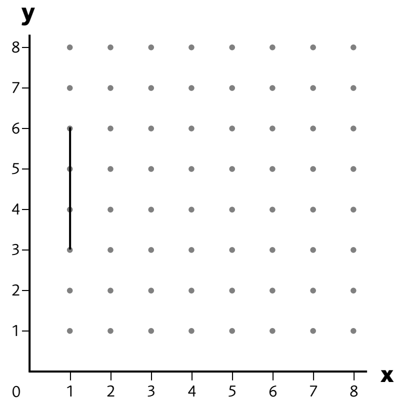
You are going to play noughts and crosses on the grids provided.



Play with a partner and take turns to place an O or an X on a grid point.

Describe using coordinates the position where you wish to place your mark.

The winner is the first to get four points in a line.

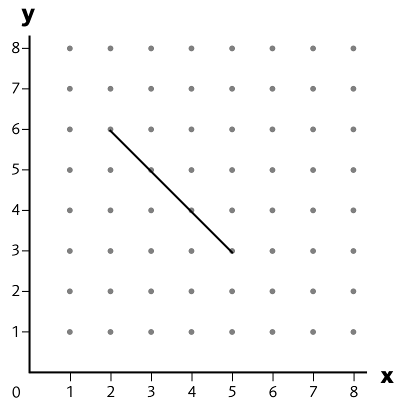


Play a few games and see what happens.

What do you notice about the coordinates of the points on horizontal winning lines? What about vertical or diagonal winning lines?

Challenge

What if you add the rule that you can only claim a win if you can correctly give the equation of your winning line?

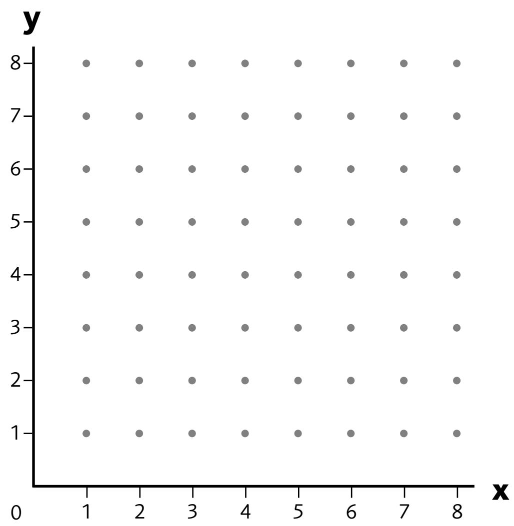


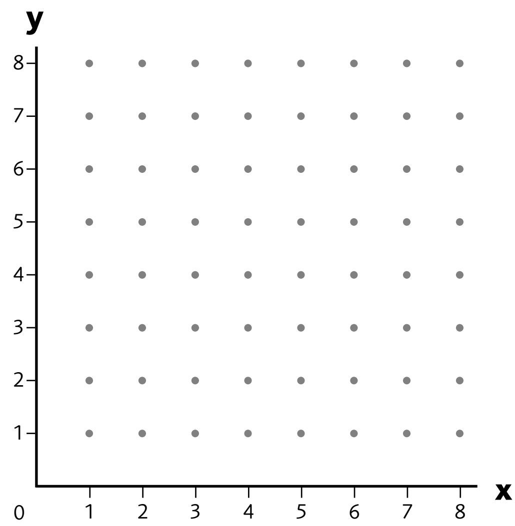
What if you continue until the grid is full?
The winner is the player with the most winning lines.

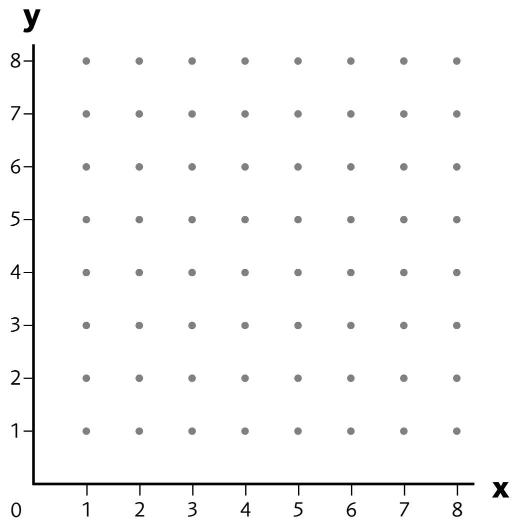
How many winning lines are possible on the grid?

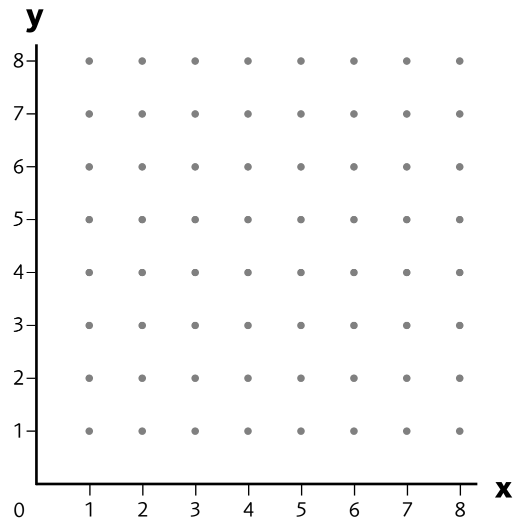
What are their equations?

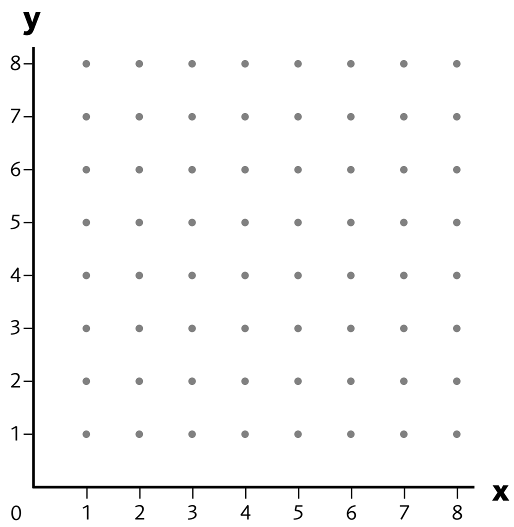
Programme 9 Worksheet 2: Line Them Up

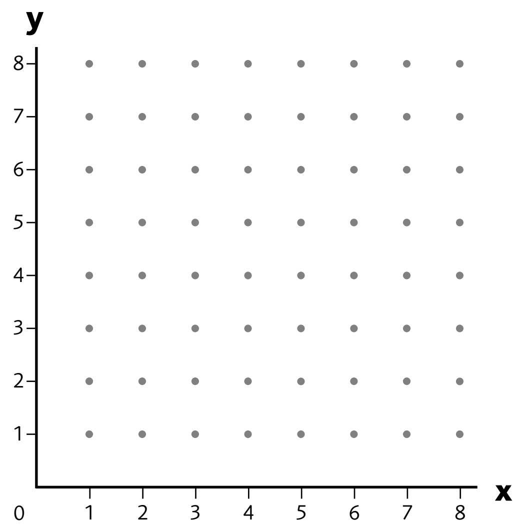




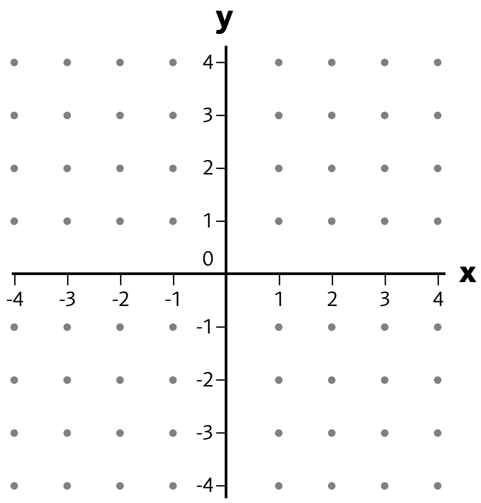


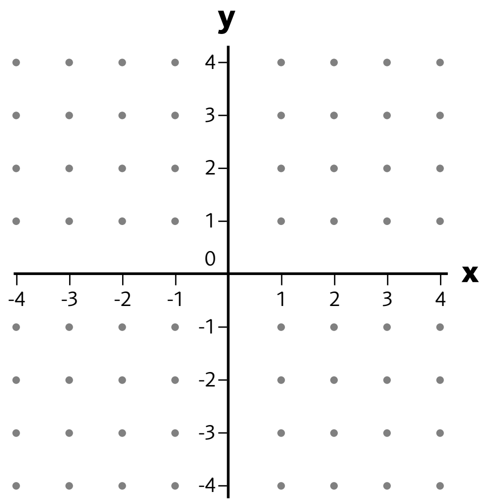


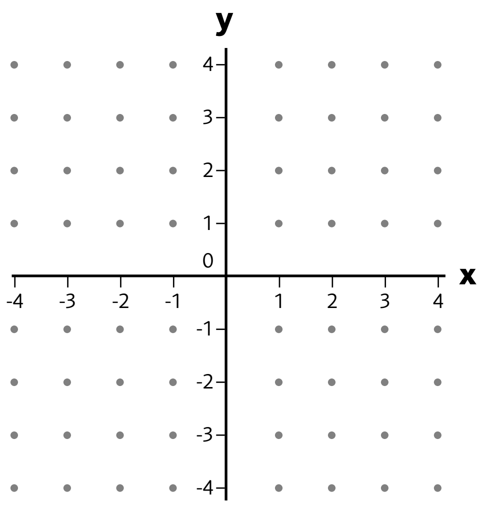


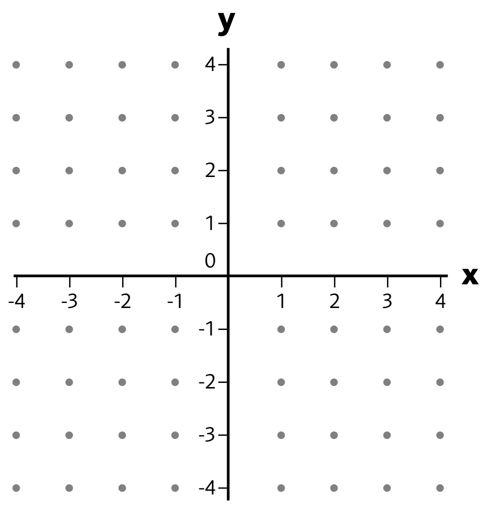


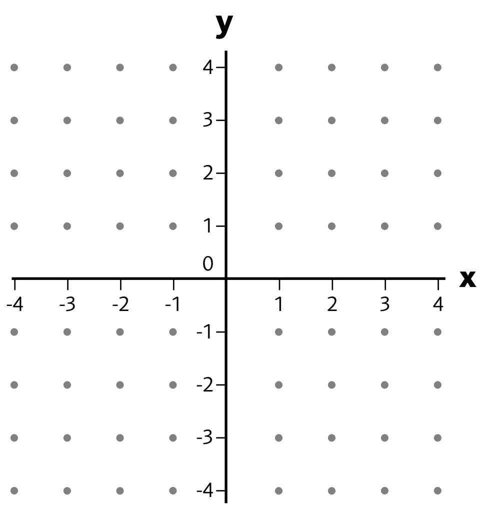
Programme 9 Worksheet 2: Line Them Up

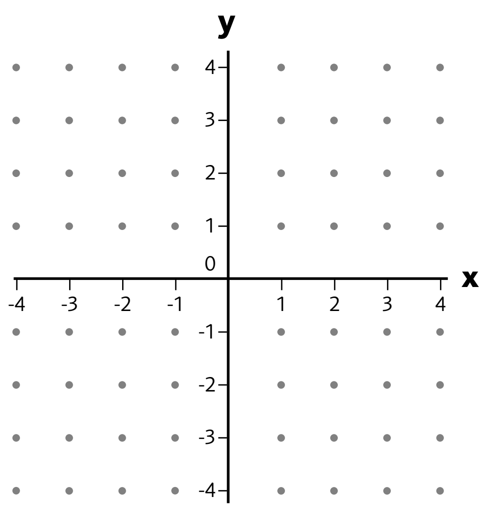








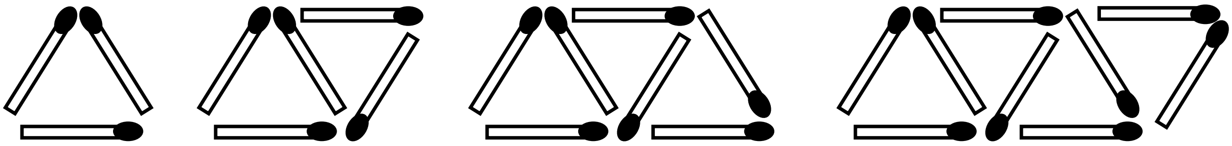




Programme 9 Worksheet 3: Plotting Patterns

Here is a sequence of patterns made from matchsticks.

Patterns 1 2 3 4



Matches 3 5 7 9

Here is a table showing the pattern number (*x*) and the number of matches used to make it (*y*).

pattern number (*x*)	1	2	3	4	5	6	7	8	9	10
number of matches (*y*)	3	5	7	9						

Complete the table.

Try to describe the number patterns that you notice.

Plot each pair of values as points on a graph.

Describe the graph as fully as you can.

Consider the difference between successive pairs of values of *y*. How does this relate to the graph you have plotted?

Investigate other sets of matchstick patterns that 'grow' according to a fixed rule.
Make tables and plot points as above.

How are the graphs of these values related to the differences that can be found in the tables?

How is the point where the graph meets the *y*-axis related to your patterns in each case?

Try to find a general rule, for each set of patterns you have made, that allows you to calculate the number of matches (*y*) for a given pattern number (*x*). How is your rule related to the graph you have plotted?

Programme 10 Worksheet 1: Plentiful Peppers

A Lisa investigated what happened when she put just one pepper in the first machine and changed the number on the front. She worked out that the machine added 3 to the number on the front and multiplied the result by the number of peppers.
She could have fixed the number on the front and put in different numbers of peppers instead.
See what happens by completing these tables of values for the first machine.

1. The number on the front of the machine is **2**.

number of peppers put in	1	2	3	4	5	6
number of peppers coming out			15			

2. The number on the front of the machine is **1**.

number of peppers put in	1	2	3	4	5	6
number of peppers coming out					16	

3. The number on the front of the machine is **0**.

number of peppers put in	1	2	3	4	5	6
number of peppers coming out		3				

4. The number on the front of the machine is **–1**.

number of peppers put in	1	2	3	4	5	6
number of peppers coming out						

5. The number on the front of the machine is **–2**.

number of peppers put in	1	2	3	4	5	6
number of peppers coming out						

Draw a graph of the values in each table. What do you notice? What happens if the number on the front of the machine is –3? What about other values?

B Lisa found that the second machine added the number of peppers she put in to the number on the front and then added 2.

Draw and complete tables (like the ones above) that show what happens when you fix the number on the front of the second machine and put in different numbers of peppers.

Draw a graph of the values in your tables and describe what you find out.
What front numbers might give Lisa a strange result?

Programme 10 Worksheet 2: Fully Functional

Here are some tables showing inputs and outputs from different function machines that the Wizard has ordered. These are a bit more sophisticated than the early models that Lisa experimented with. Some of them perform more than one operation to give the output.

The machines are still being tested. Unfortunately, in each set of outputs there is one wrong value.

Find a function that fits each pattern of inputs and outputs, and describe it.

Find the wrong value and correct it.

1.

input	output
2	5
4	8
0	0
3	6
1	2

2.

input	output
3	0
–1	–4
2	–1
1	–5
0	–3

3.

input	output
–3	–11
1	–1
0	–2
–2	–8
2	4

4.

input	output
–5	–11
1	8
–4	–8
–2	–2
0	4

5.

input	output
–5	–3
–2	3
2	10
1	9
–1	5

6.

input	output
–4	12
3	5
1	7
–2	10
2	9

7.

input	output
–4	0
2	3
3	3.5
–1	1.5
5	6.5

8.

input	output
–7	6.5
–2	–4
3	–1.5
–1	–2.5
4	–1

9.

input	output
6	14
–2	–3.5
3	6.5
–1	–1.5
1	2.5

10.

input	output
3	4
–1	2
–3	1
0	3
5	5

11.

input	output
5	26
4	17
2	5
1	2
3	15

12.

input	output
2	12
0	0
3	29
1	3
4	48

Challenge

Can you use a spreadsheet to help you discover the functions?

Make up some functions of your own, and give your partner sets of inputs and outputs to for them to work out your function.

Programme 10 Worksheet 3: The Ghost in the Machine

Work in small groups of three to five.

One person is going to act as a function machine and compute outputs from numbers input by the rest of the group.

Decide who is going to be the machine first. This person decides on a rule that will be applied to all inputs.

The object of the game is to guess the rule by giving the machine inputs (x) and being told the resulting outputs (y).

Before you can win the game by stating the rule, you must give a number (x) and correctly predict the result (y) that the machine would produce.

Play the game and see what happens.

Play again with a different person as the machine.

Think about the best ways to monitor and record your results.

Can you use graphs to help you make your predictions?

Can you use algebra to describe what the machine is doing?

Challenge

What happens if you have two inputs (x and y) that the machine combines in some way to produce an output (z)?

What happens if two players are machines and one machine's result is fed into the other? (You could ask the first machine to give the second a piece of paper with their output written on it ready for processing).