

# Equations and Inequalities

1

## Unit Overview

Investigating patterns is a good foundation for studying Algebra 1. You will begin this unit by analyzing, describing, and generalizing patterns using tables, expressions, graphs, and words. You will then write and solve equations and inequalities in mathematical and real-world problems.

## Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

## Academic Vocabulary

- consecutive

## Math Terms

- sequence
- common difference
- expression
- variable
- equilateral
- equation
- solution
- formula
- literal equation
- graph of an inequality
- solution of an inequality
- compound inequality
- conjunction
- disjunction
- absolute value
- absolute value notation
- absolute value equation
- absolute value inequality

## ESSENTIAL QUESTIONS



How can you represent patterns from everyday life by using tables, expressions, and graphs?



How can you write and solve equations and inequalities?

## EMBEDDED ASSESSMENTS

These assessments, following Activities 2 and 4, will give you an opportunity to demonstrate what you have learned about patterns, equations, and inequalities.

### Embedded Assessment 1:

Patterns and Equations p. 33

### Embedded Assessment 2:

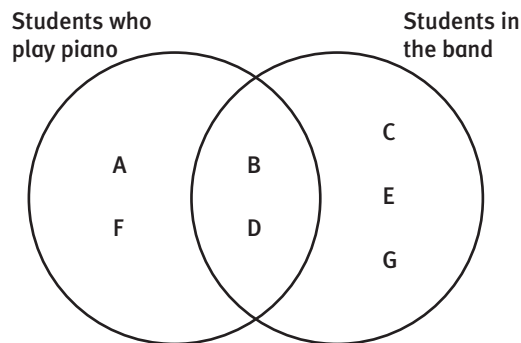
Inequalities and Absolute Value p. 61

# Getting Ready

Write your answers on notebook paper.

Show your work.

- What is  $\frac{2}{3} + \frac{4}{5}$ ?
- What condition must be met before you can add or subtract fractions?
- Jennifer is checking Megan's homework. They disagree on the answer to this problem:  $4^2 \times 2^2$ . Jennifer says the product is 32 and Megan says it is 64. Who has the correct answer? Explain how she arrived at that correct product.
- A piece of lumber  $2\frac{1}{4}$  feet long is to be cut into 3 equal pieces. How long will each piece of cut wood be? Give the measurement in feet and in inches.
- Arrange the following expressions in order of their value from least to greatest.
  - $4 - 6$
  - $-4 + 6$
  - $-4 - 6$
- Which expression has the greater value? Justify your answer.
  - $-8 + 3$
  - $-8 \times 3$
- Which of the following are equal to 14.95?
  - $2.3 \times 6.5$
  - $21.45 - 6.5$
  - $8.32 + 6.63$
- Which equation has the least solution?
  - $x + 5 = 13$
  - $-6x = -30$
  - $\frac{x}{4} = 18$
  - $x - 2 = -11$
- Combine like terms in the following expressions.
  - $10x - 4x$
  - $-15n + 3n$
  - $7.5y + 1.6y - 2$
  - $m + 4 - 2m$
- The Venn diagram below provides a visual representation of the students in Mr. Griffin's class who participate in music programs after school. What does the diagram tell you about the musical involvement of Student B and Student G? Explain how you reached your conclusion.



# Investigating Patterns

## Cross-Country Adventures

### Lesson 1-1 Numeric and Graphic Representations of Data

#### ACTIVITY 1

#### Learning Targets:

- Identify patterns in data.
- Use tables, graphs, and expressions to model situations.
- Use expressions to make predictions.

**SUGGESTED LEARNING STRATEGIES:** Sharing and Responding, Create Representations, Discussion Groups, Look for a Pattern, Interactive Word Wall

Mizing spent his summer vacation traveling cross-country with his family. Their first stop was Yellowstone National Park in Wyoming and Montana. Yellowstone is famous for its geysers, especially one commonly referred to as Old Faithful. A geyser is a spring that erupts intermittently, forcing a fountain of water and steam from a hole in the ground. Old Faithful can have particularly long and fairly predictable eruptions. As a matter of fact, park rangers have observed the geyser over many years and have developed patterns they use to predict the timing of the next eruption.

Park rangers have recorded the information in the table below.

Length of Eruption (in minutes)	Approximate Time Until Next Eruption (in minutes)
1	46
2	58
3	70
4	82

1. Describe any patterns you see in the table.
2. Why might it be important for park rangers to be able to predict the timing of Old Faithful's eruptions?
3. If an eruption lasts 8 minutes, about how long must park visitors wait to see the next eruption? Explain your reasoning using the patterns you identified in the table.

#### My Notes

#### CONNECT TO HISTORY

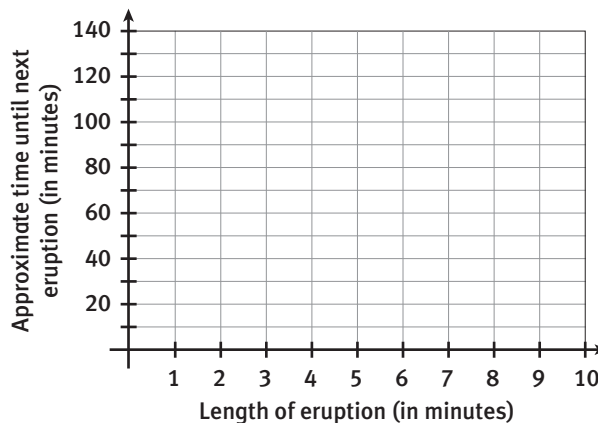
Yellowstone National Park was the first National Park. The park was established by Congress on March 1, 1872. President Woodrow Wilson signed the act creating the National Park Service on August 25, 1916.

#### DISCUSSION GROUP TIPS

- Work with your peers to set rules for:
- discussions and decision-making
  - clear goals and deadlines
  - individual roles as needed

**My Notes**

4. Graph the data from the table on the grid below.



5. **Reason quantitatively.** Mizing and his family arrived at Old Faithful to find a sign indicating they had just missed an eruption and that it would be approximately 2 hours before the next one. How long was the eruption they missed? Explain how you determined your answer.

**MATH TERMS**

A **sequence** is a list of numbers, and each number is called a *term* of the sequence. For example: 2, 4, 6, 8, ... and 2, 5, 10, 17, ... are sequences.

Patterns can be written as **sequences**.

- 6. Using the table or graph above, write the approximate times until the next Old Faithful eruption as a sequence.
- 7. How would you describe this sequence of numbers?

## Lesson 1-1

### Numeric and Graphic Representations of Data

#### ACTIVITY 1

continued

In the table below, 5 and 8 are **consecutive** terms. Some sequences have a **common difference** between consecutive terms. The common difference between the terms in the table below is 3.

Sequence: 5, 8, 11, 14...

Term number	Term
1	5
2	8
3	11
4	14

- Identify two consecutive terms in the sequence of next eruption times that you created in Item 6.
- The sequence of next eruption times has a common difference. Identify the common difference.
- Each term in the sequence above can be written using the first term and repeated addition of the common difference. For example, the first term is 5, the second term is  $5 + 3$ , and the third term can be expressed as  $5 + 3 + 3$  or  $5 + 2(3)$ . Similarly, the terms in the sequence of next eruption times can also be written using repeated addition of the common difference.
  - Write the approximate waiting time for the next eruption after eruptions lasting 4 and 5 minutes using repeated addition of the common difference.
  - Model with mathematics.** Let  $n$  represent the number of minutes an eruption lasts. Write an **expression** using the **variable**  $n$  that could be used to determine the waiting time until the next eruption.
  - Check the accuracy of your expression by evaluating it when  $n = 2$ .
  - Use your expression to determine the number of minutes a visitor to the park must wait to see another eruption of Old Faithful after a 12-minute eruption.

My Notes

#### ACADEMIC VOCABULARY

**Consecutive** refers to items that follow each other in order.

#### MATH TIP

A common difference is also called a **constant difference**.

#### MATH TERMS

An **expression** may consist of numbers, variables, and operations. A **variable** is a letter or symbol used to represent an unknown quantity.

## My Notes

## Check Your Understanding

SB-Mobile charges \$20 for each gigabyte of data used on any of its smartphone plans.

11. Copy and complete the table showing the charges for data based on the number of gigabytes used.

Number of Gigabytes Used	Total Data Charge
1	
2	
3	
4	
5	

12. Graph the data from the table. Be sure to label your axes.
13. Write a sequence to represent the total price of a data plan.
14. The sequence you wrote in Item 13 has a common difference. Identify the common difference.
15. Let  $n$  represent the number of gigabytes used. Write an expression that can be used to determine the total data charge for the phone plan.
16. Use your expression to calculate the total data charge if 10 gigabytes of data are used.

**LESSON 1-1 PRACTICE**

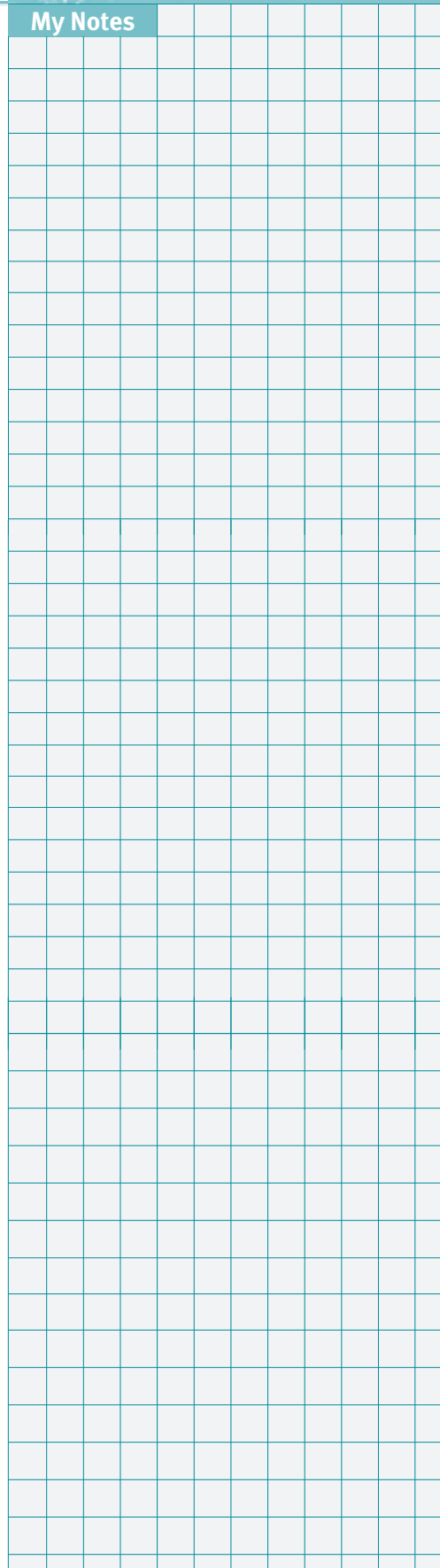
Travis owns stock in the SBO Company. After the first year of ownership the stock is worth \$45 per share. Travis estimates that the value of a share will increase by \$2.80 per year.

- 17.** Copy and complete the table showing the value of the stock over the course of several years.

Year	Share Value
1	\$45
2	
3	
4	
5	

- 18.** Write a sequence to show the increase in the stock value over the course of several years.
- 19. Make use of structure.** The sequence you wrote in Item 18 has a common difference. Identify the common difference.
- 20.** Let  $n$  represent the number of years that have passed. Write an expression that can be used to determine the value of one share of SBO stock.
- 21.** Use your expression to calculate the value of one share of stock after 20 years.

My Notes



**My Notes**

**CONNECT TO HISTORY**

Mesa Verde National Park was created by President Theodore Roosevelt in 1906 as the first National Park designated to preserve “the works of man.” The park protects nearly 5000 known archeological sites and 600 cliff dwellings, offering a look into the lives of the ancestral Pueblo people who lived there from 600–1300 AD.

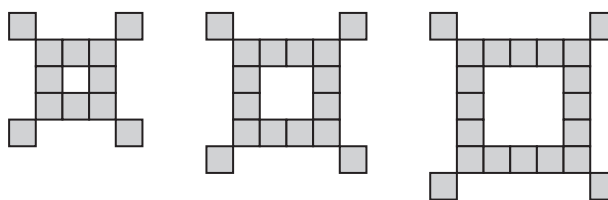
**Learning Targets:**

- Use patterns to write expressions.
- Use tables, graphs, and expressions to model situations.

**SUGGESTED LEARNING STRATEGIES:** Look for a Pattern, Create Representations, Think-Pair-Share, Discussion Groups, Sharing and Responding

Mizing and his family also visited Mesa Verde National Park in Colorado. As Mizing investigated the artifacts on display from the ancestral Pueblo people who once called the area home, Mizing began to notice that the patterns used to decorate pottery, baskets, and textiles were geometric.

Mizing found a pattern similar to the one below particularly interesting.



**1. Reason abstractly.** Draw the next two figures in the pattern.

**2. Create a table showing the relationship between the figure number and the number of small squares in each figure.**

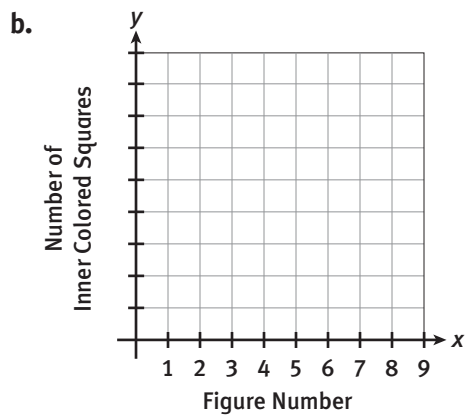
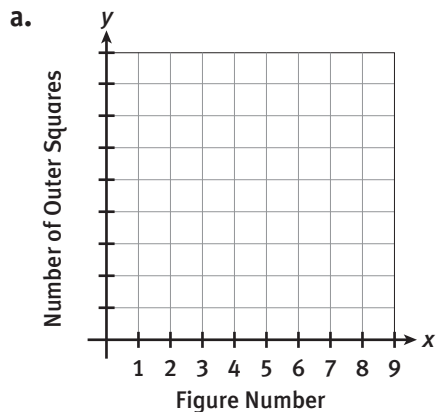
Figure Number	Number of Squares





**My Notes**

8. Describe any numerical patterns you see in the table.
  
9. Write the numbers of inner colored squares as a sequence.
  
10. Does the sequence of numbers of inner colored squares have a common difference? If so, identify it. If not, explain.
  
11. **Model with mathematics.** Graph the data from the table on the appropriate grid. Be sure to label an appropriate scale on the  $y$ -axis.



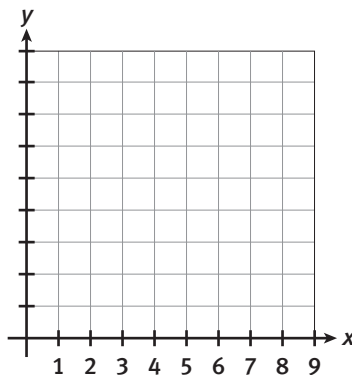


**My Notes**

18. Mizing found that he could determine the perimeter of any figure in the pattern using the expression  $2n + 1$ . Use Mizing's expression to calculate the perimeters of the next three figures in the pattern. Use the table below to record your calculations.

Figure Number	Perimeter (cm)

19. Create a sequence to represent the perimeters of the figures in the pattern. Does the sequence have a common difference? If so, identify it. If not, explain.
20. Represent the relationship between the figures in the pattern and their perimeters as a graph. Be sure to label your axes and the scale on the  $y$ -axis.





**ACTIVITY 1**

*continued*

**ACTIVITY 1 PRACTICE**

Write your answers on notebook paper.  
Show your work.

**Lesson 1-1**

1	3	3	3
2	6	12	3
3	9	27	3
4	12	48	3

- Describe any patterns you notice in the table of values.
- Write the second column of the table as a sequence. Identify the common difference.
- Create a graph to show the relationship between the first two columns of the table.
- Write an expression to represent the relationship between the first two columns of the table.
- Explain how you could use your expression to determine the value that would appear in the 25th row of the second column.
- Draw the next three figures in the pattern.
- Determine the perimeter of each of the first six figures. Assume each side of each pentagon measures 1 inch.
- What is the perimeter of the 10th figure? Justify your response.
- Use the variable  $n$  to write an expression that could be used to determine the perimeter of any figure in the pattern.
- Use your expression to calculate the perimeter of the 50th figure. Be sure to include units.
- Evaluate  $3x - 12$  for  $x = 5$ .
- Evaluate  $45 + 15(n - 1)$  for  $n = 6$ .

**MATHEMATICAL PRACTICES**

**Construct Viable Arguments and Critique the Reasoning of Others**

- Figure 1 of a mosaic pattern contains one tile and figure 2 contains four tiles. Nancy and Richard were asked to predict the number of tiles in figure 3. Nancy wrote 9, and Richard wrote 7. Who is correct? Explain your reasoning.

**Lesson 1-2**

Use the visual pattern below for Items 6–10.



Figure 1

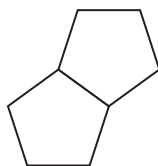


Figure 2

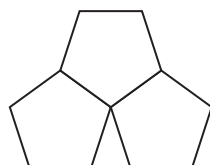


Figure 3

# Solving Equations

## What's My Number?

### Lesson 2-1 Writing and Solving Equations

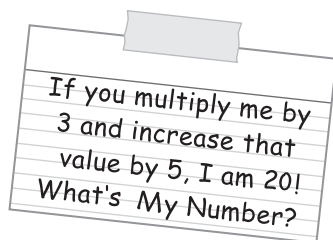
#### Learning Targets:

- Use the algebraic method to solve an equation.
- Write and solve an equation to model a real-world situation.

**SUGGESTED LEARNING STRATEGIES:** Guess and Check, Create Representations, Discussion Groups, Identify a Subtask, Note Taking

Let's play "What's My Number?"

1. **Make sense of problems.** Determine the number, and explain how you came up with the solution.



2. You may have used an **equation** to answer Item 1. Write an equation that could be used to represent the problem in Item 1.

3. For each equation, tell whether the given value of  $x$  is a **solution**. Explain.

a.  $2 + 4(x - 1) = 22$ ;  $x = 6$

b.  $12 - \frac{x}{4} = 8$ ;  $x = 24$

c.  $3.8 + 6x = 8.6$ ;  $x = 0.9$

One way to solve an equation containing a variable is to use the algebraic method. This method is also called the *symbolic method* or *solving equations using symbols*.

#### My Notes

#### DISCUSSION GROUP TIPS

Work with your peers to set rules for:

- discussions and decision-making
- clear goals and deadlines
- individual roles as needed

#### MATH TERMS

An **equation** is a mathematical statement that shows that two expressions are equal. A **solution** is any value that makes an equation true when substituted for the variable.

**My Notes**

**MATH TIP**

The **Properties of Equality** state that you can perform the same operation on both sides of an equation without affecting the solution.

**Addition Property of Equality**  
If  $a = b$ , then  $a + c = b + c$ .

**Subtraction Property of Equality**  
If  $a = b$ , then  $a - c = b - c$ .

**Multiplication Property of Equality**  
If  $a = b$ , then  $ac = bc$ .

**Division Property of Equality**  
If  $a = b$ , then  $a \div c = b \div c$ ,  
when  $c \neq 0$ .

Other important properties are:

**Symmetric Property of Equality**  
If  $a = b$ , then  $b = a$ .

**Commutative Property of Addition/Multiplication**  
 $a + b = b + a$   
 $ab = ba$

**Associative Property of Addition/Multiplication**  
 $(a + b) + c = a + (b + c)$   
 $(ab)c = a(bc)$

**Distributive Property**  
 $a(b + c) = ab + ac$

**Example A**

Solve the equation  $3x + 90 + 2x = 360$  using the algebraic method, showing each step. List a property or provide an explanation for each step. Check your solution.

$$3x + 90 + 2x = 360$$

Original equation

$$3x + 2x + 90 = 360$$

Commutative Property of Addition

$$5x + 90 = 360$$

Combine like terms.

$$5x + 90 - 90 = 360 - 90$$

Subtraction Property of Equality

$$5x = 270$$

Combine like terms.

$$\frac{5x}{5} = \frac{270}{5}$$

Division Property of Equality

$$x = 54$$

Simplify.

Check by substitution:

$$3(54) + 90 + 2(54) = 360$$

$$162 + 90 + 108 = 360$$

$$360 = 360$$

**Solution:**  $x = 54$

**Try These A**

**a.** Solve the “What’s My Number?” problem using the equation you wrote in Item 2 and the algebraic method.

Solve each equation using the algebraic method, showing each step. List a property or provide an explanation for each step.

**b.**  $-5x - 6 = 1$

**c.**  $12d + 2 - 3d = 5$

**d.**  $7.4p - 9.2p = -2.6 + 5.3$

**e.**  $\frac{x}{4} + 7 = 12$

**f.**  $20x - 3 + 5x = 22$

**g.**  $8 = \frac{3}{2}w - 12 + 8$

**Check Your Understanding**

**4.** Is  $x = 4$  a solution of  $2(x - 3) + 7 = 23$ ? How do you know?

**5.** Which property of equality would you use to solve the equation  $\frac{x}{5} = 13$ ? Explain your answer.



## Lesson 2-1

### Writing and Solving Equations

## ACTIVITY 2

continued

Julio has 5 more dollars than Dan. Altogether, Julio and Dan have 19 dollars. How much money does each young man have?

- Let  $d$  represent the amount of money, in dollars, that Dan has. Use  $d$  to write an expression that represents the amount of money that Julio has.
- Write an equation to represent the problem situation.
- In the space below, solve your equation from Item 7, showing each step. State a property or provide an explanation for each step. Check your solution.
- Interpret your solution to Item 8 within the context of the problem.
- Verify the reasonableness of your solution by checking that your answer to Item 9 matches the information given in the original problem situation at the top of the page.

My Notes

### MATH TIP

When using the Addition or Subtraction Property of Equality, you may find it helpful to add or subtract vertically on both sides of the equation.

$$\begin{array}{r} 2d + 5 = 19 \\ \underline{-5 \quad -5} \\ 2d \quad = 14 \end{array}$$



**Learning Targets:**

- Write and solve an equation to model a real-world situation.
- Interpret parts of an expression in terms of its context.

**SUGGESTED LEARNING STRATEGIES:** Close Reading, KWL Chart, Create Representations, Discussion Groups, Construct an Argument

The Future Engineers of America Club (FEA) wants to raise money for a field trip to the science museum. The club members will hold an engineering contest to raise money. They are deciding between two different contests, the Straw Bridge contest and the Card Tower contest.

The Straw Bridge contest will cost the club \$5.50 per competitor plus \$34.60 in extra expenses. The Card Tower contest will cost \$4.25 per competitor plus \$64.60 in extra expenses.

To help decide which contest to host, club members want to determine how many competitors they would need for the costs of the two contests to be the same.

1. Write an equation that sets the costs of the two contests equal.
2. Solve the equation from Item 1 by using the algebraic method, showing each step. List a property of equality or provide an explanation for each step.

Equations	Properties/Explanations
	Original equation
	Multiplication Property of Equality: Multiply each side by 100.

**My Notes**

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

## My Notes

3. **Model with mathematics.** Interpret the meaning of the value of  $x$  in the context of the problem.
4. The FEA club estimates they will have more than 30 competitors in their contest. Make a recommendation to the club explaining which contest would be the better choice and why.
5. The FEA club will charge each competitor \$10 to enter the engineering contest. Write an expression for the club's revenue if  $x$  competitors enter the contest.
6. a. Write an equation to find the break-even point for the fundraiser using the contest you recommended to the FEA club.
- b. Solve the equation. State a property of equality or provide an explanation for each step. How many competitors does the club need to break even?
7. How much profit will the FEA club earn from 32 competitors if they use the contest you recommended?

**CONNECT TO BUSINESS**

*Revenue* is the amount of money made selling a product or service.  
*Profit* is earnings after costs are subtracted from the revenue.

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

The *break-even point* occurs when revenue equals cost.

$$\text{Revenue} = \text{Cost}$$

## Lesson 2-2

### Equations with Variables on Both Sides

## ACTIVITY 2

continued

8. The Future Engineers of America Club treasurer was going back through the fundraising records. On Monday, the club made revenue of \$140 selling contest tickets at \$10 each. One person sold 8 tickets, but the other person selling that day forgot to write down how many she sold. Write and solve an equation to determine the number of tickets the other person sold.

### Check Your Understanding

9. How can you use the Multiplication Property of Equality to rewrite the equation  $0.6x + 4.8 = 7.2$  so that the numbers in the problem are integers and not decimals?
10. When writing an expression or equation to represent a real-world situation, why is it important to be able to describe what each part of the expression or equation represents?

## LESSON 2-2 PRACTICE

On-the-Go Phone Company has two monthly plans for their customers. The EZ Pay Plan costs \$0.15 per minute. The 40 to Go Plan costs \$40 per month plus \$0.05 per minute.

11. a. Write an expression that represents the monthly bill for  $x$  minutes on the EZ Pay Plan.  
b. Write an expression that represents the monthly bill for  $x$  minutes on the 40 to Go Plan.
12. Write an equation to represent the point at which the monthly bills for the two plans are equal.
13. Solve the equation, showing each step. List a property of equality or provide an explanation for each step.
14. Interpret the solution of the equation within the context of the problem.
15. **Construct viable arguments.** Which plan should you choose if you want only 200 minutes per month? Justify your response.

My Notes

**My Notes**

**MATH TIP**

If an equation includes the product of a number and an expression in parentheses, you can simplify by applying the Distributive Property.

Distributive Property:  
 $a(b + c) = ab + ac$

**MATH TIP**

You can eliminate the fractions in an equation by multiplying both sides of the equation by the least common denominator of the fractions.

**Learning Targets:**

- Solve complex equations with variables on both sides and justify each step in the solution process.
- Write and solve an equation to model a real-world situation.

**SUGGESTED LEARNING STRATEGIES:** Create Representations, Construct an Argument, Think-Pair-Share, Create a Plan

Some equations require multiple steps to solve them efficiently.

1. The equation  $3x - 2(x + 3) = 5 - 2x$  is solved in the table below. Complete the table by stating a property or providing an explanation for each step.

Equations	Properties/Explanations
$3x - 2(x + 3) = 5 - 2x$	
$3x - 2x - 6 = 5 - 2x$	
$x - 6 = 5 - 2x$	
$x + 2x - 6 = 5 - 2x + 2x$	
$3x - 6 = 5$	
$3x - 6 + 6 = 5 + 6$	
$3x = 11$	
$\frac{3x}{3} = \frac{11}{3}$	
$x = 3\frac{2}{3}$	

**Solution:**  $x = 3\frac{2}{3}$

2. Solve the following equations. State a property of equality or provide an explanation for each step.

a.  $5x + 8 = 3x - 3$

b.  $2(4y + 3) = 16$

c.  $\frac{2}{3}p + \frac{1}{5} = \frac{4}{5}$

d.  $\frac{3}{4}a - \frac{1}{6} = \frac{2}{3}a + \frac{1}{4}$

## Lesson 2-3

### Solving More Complex Equations

## ACTIVITY 2

continued

- 3. Model with mathematics.** Bags of maple granola cost \$2 more than bags of apple granola. The owner of a restaurant ordered 6 bags of maple granola and 5 bags of apple granola. The total cost of the order was \$56.
- Let  $m$  represent the cost of a bag of maple granola. Write an expression for the cost of 6 bags of maple granola.
  - Use the variable  $m$  to write an expression for the cost of a bag of apple granola.
  - Write an expression for the cost of 5 bags of apple granola.
  - Write an equation to show that the cost of 6 bags of maple granola and 5 bags of apple granola was \$56.
  - Solve your equation to find the cost per bag of each type of granola.

### Check Your Understanding

- Suppose you are asked to solve the equation  $\frac{3}{4}x - \frac{2}{3} = \frac{1}{6}x$ .
  - What number could you multiply both sides of the equation by so that the numbers in the problem are integers and not fractions?
  - What property allows you to do this?
- Explain how the Commutative Property of Addition could help you solve the equation  $-6x + 10 + 8x = 12 - 4x$ .

My Notes

## My Notes

## LESSON 2-3 PRACTICE

Solve the following equations, and explain each step.

6.  $6x + 3 = 5x + 10$

7.  $6 + 0.10x = 0.15x + 8$

8.  $5 - 4x = 6 + 2x$

9.  $9 - 2x = 7x$

10.  $2(x - 4) + 2x = -6x - 2$

11.  $\frac{1}{2}x - 3 = \frac{3}{2}x + 4$

12. Ben bought 4 pairs of jeans and a T-shirt that cost \$8. He had a coupon for \$3 off the price of each pair of jeans and spent a total of \$92 before tax.

- Define a variable and write an equation to represent the situation.
- What was the price of each pair of jeans?

13. **Use appropriate tools strategically.** A student solved the equation  $3(2x - 6) = 4x + 8$  and found the solution to be  $x = 2.6$ . How could you use estimation and mental math to show that the student's solution is incorrect?



**Learning Targets:**

- Identify equations that have no solution.
- Identify equations that have infinitely many solutions.

**SUGGESTED LEARNING STRATEGIES:** Marking the Text, Think-Pair-Share, Predict and Confirm, Construct an Argument, Sharing and Responding

Remember that a solution of an equation with one variable is a value of the variable that makes the equation true.

- The set of numbers  $\left\{\frac{1}{2}, 3, 6, 17, 0, 11\right\}$  contains possible solutions to the following equations. Determine which of these numbers are solutions to each of the following equations.
  - $9x + 5 = 4(x + 2) + 5x$
  - $7x - 10 = 3x + 14$
  - $3x - 12 = 3(x + 1) - 15$

An equation has *no solution* if there is no value of the variable that will create a true mathematical statement. An equation has *infinitely many solutions* if there are an unlimited number of values of the variable that will create a true mathematical statement.

- Laura, Nia, and Leo solved the following three equations as shown. Identify each of the equations as having *one solution*, *no solution*, or *infinitely many solutions*. Justify your responses.

<u>Laura</u>	<u>Nia</u>	<u>Leo</u>
$-(5x + 3) - 4x = 8 + 6x$	$3(x + 4) - 10 = 3x + 2$	$2(4x + 3) - 3x = 17 + 5x$
$-5x - 3 - 4x = 8 + 6x$	$3x + 12 - 10 = 3x + 2$	$8x + 6 - 3x = 17 + 5x$
$-9x - 3 = 8 + 6x$	$3x + 2 = 3x + 2$	$5x + 6 = 17 + 5x$
$\underline{+ 9x}$	$\underline{- 3x}$	$\underline{- 5x}$
$-3 = 8 + 15x$	$2 = 2$	$6 = 17$
$\underline{- 8}$		
$-11 = 15x$		
$\underline{- 11}$		
$\underline{15} = \underline{15x}$		
$\underline{- 11}$		
$\underline{15} = x$		

**My Notes**

**MATH TIP**

An equation is true when both sides of the equation have the same value. Otherwise, the equation is false.

$2 + 3 = 5$  is a true mathematical statement because  $2 + 3$  has the same value as 5.

$2 + 3 = 6$  is a false mathematical statement because  $2 + 3$  does not have the same value as 6.

## My Notes

## Check Your Understanding

Determine solutions to each of the following equations.

3.  $3(2z + 4) = 6(5z + 2)$

4.  $3(x + 1) + 1 + 2x = 2(2x + 2) + x$

5.  $8b + 3 - 10b = -2(b - 2) + 3$

Some equations are true for all values of the variable. This type of equation has all real numbers as solutions. This means the equation has *infinitely many solutions*. Other equations are false for all values of the variable. This type of equation has *no solutions*.

6. Which of the following equations has no solutions and which has all real numbers as solutions? Explain your reasoning.

a.  $3x + 5 = 3x$

b.  $4r - 2 = 4r - 2$

7. **Critique the reasoning of others.** A student claims that the equation  $2x + 6 = 4x + 4$  has no solutions because when you substitute 0 for  $x$ , the left side has a value of 6 and the right side has a value of 4. Is the student's reasoning correct? Explain.

8. Explain why the equation  $x + 3 = x + 2$  has no solution.

9. **Reason quantitatively.** For what value of  $a$  does the equation  $3x + 5 = ax + 5$  have infinitely many solutions? Explain.

10. Consider the equation  $nx - 4 = 6$ .

- a. What are the solutions if the value of  $n$  is 0? Explain.

- b. What if the value of  $n$  is 2? What is the solution of the equation? How do you know?

## Lesson 2-4

### Equations with No Solution or Infinitely Many Solutions

## ACTIVITY 2

continued

### Check Your Understanding

**Make use of structure.** Create an equation that will have each of the following as its solution.

11. One solution
12. No solution
13. Infinitely many solutions
14. A solution of zero

### LESSON 2-4 PRACTICE

Solve each equation. If an equation has no solutions or if an equation has infinitely many solutions, explain how you know.

15.  $3x - x - 5 = 2(x + 2) - 9$
16.  $7x - 3x + 7 = 3(x - 4) + 20$
17.  $-2(x - 2) - 4x = 3(x + 1) - 9x$
18.  $5(x + 2) - 3 = 3x - 8x + 7$
19.  $4(x + 3) - 4 = 8x + 10 - 4x$
20.  $3(x + 2) + 4x - 5 = 7(x + 1) - 6$
21. **Construct viable arguments.** Justify your response for each of the following.
  - a. Write an equation with no solutions that has the expression  $3x + 6$  on the left side of the equal sign. Demonstrate that your equation has no solutions.
  - b. Write an equation with infinitely many solutions that has the expression  $3x + 6$  on the left side of the equal sign. Demonstrate that your equation has infinitely many solutions.

My Notes

## My Notes

## MATH TERMS

A **literal equation** has more than one variable, and the equation can be solved for a specific variable.

Formulas are examples of literal equations. A **formula** is an equation written using symbols that describes the relationship between different quantities.

## Learning Targets:

- Solve literal equations for a specified variable.
- Use a formula that has been solved for a specified variable to determine an unknown quantity.

**SUGGESTED LEARNING STRATEGIES:** Identify a Subtask, Close Reading, Work Backward, Create Representations, Discussion Groups

A **formula** describes how two or more quantities are related. Formulas are important in many disciplines; geometry, physics, economics, sports, and medicine are just a few examples of fields in which formulas are widely used.

A formula is an example of a **literal equation**. A literal equation contains more than one variable. Literal equations and formulas can be solved for a specific variable using the same procedures as equations containing one variable.

## Example A

Solve the equation  $4x + b = 12$  for  $x$ .

**Step 1:** Isolate the term that contains  $x$  by subtracting  $b$  from both sides.

$$\begin{array}{ll} 4x + b = 12 & \text{Original equation} \\ 4x + b - b = 12 - b & \text{Subtraction Property of} \\ & \text{Equality} \\ 4x = 12 - b & \text{Combine like terms.} \end{array}$$

**Step 2:** Isolate  $x$  by dividing both sides by 4.

$$\begin{array}{ll} \frac{4x}{4} = \frac{12 - b}{4} & \text{Division Property of Equality} \\ x = \frac{12 - b}{4} & \text{Simplify.} \end{array}$$

**Solution:**  $x = \frac{12 - b}{4}$ , or  $x = 3 - \frac{b}{4}$

## Try These A

Solve each equation for  $x$ .

a.  $ax + 7 = 3$

b.  $cx - 10 = -5$

c.  $-3x + d = -9$

## Lesson 2-5

### Solving Literal Equations for a Variable

## ACTIVITY 2

continued

### Check Your Understanding

- Is the equation  $2x + 4 = 5x - 6$  a literal equation? Explain.
- Describe the similarities and differences between solving an equation containing one variable and solving a literal equation for a variable.

### Example B

The equation  $v = v_0 + at$  gives the velocity in meters per second of an object after  $t$  seconds, where  $v_0$  is the object's initial velocity in meters per second and  $a$  is its acceleration in meters per second squared.

- Solve the equation for  $a$ .
- Determine the acceleration for an object whose velocity after 15 seconds is 25 meters per second and whose initial velocity was 15 meters per second.

a.	$v = v_0 + at$	Original equation
	$v - v_0 = v_0 - v_0 + at$	Subtraction Property of Equality: Subtract $v_0$ from both sides.
	$v - v_0 = at$	Combine like terms.
	$\frac{v - v_0}{t} = \frac{at}{t}$	Division Property of Equality: Divide both sides by $t$ .
	$\frac{v - v_0}{t} = a$	Simplify.
	$a = \frac{v - v_0}{t}$	Symmetric Property of Equality

- To determine the acceleration for an object whose velocity after 15 seconds is 25 meters per second and whose initial velocity was 15 meters per second, substitute 25 for  $v$ , 15 for  $v_0$ , and 15 for  $t$ .

$$a = \frac{25 - 15}{15} = \frac{10}{15} = \frac{2}{3} \text{ m/s}^2$$

### Try These B

The equation  $t = 13p + 108$  can be used to estimate the cooking time  $t$  in minutes for a stuffed turkey that weighs  $p$  pounds. Solve the equation for  $p$ . Then find the weight of a turkey that requires 285 minutes to cook.

### My Notes

### CONNECT TO PHYSICS

An object's *velocity* is its speed in a particular direction. Its *acceleration* is the rate of change in velocity. If an object has a positive acceleration, it is speeding up; if it has a negative acceleration, it is slowing down.

### READING MATH

Sometimes a variable may include a subscript. A subscript is a small number or letter written to the lower right of a variable. For example, the variable  $v_0$  has the subscript 0. The subscript 0 is often read "nought." (*Nought* is another word for 0.) A variable with a subscript of 0 usually indicates an initial value. So,  $v_0$  indicates the initial value of the velocity, or the velocity when the time  $t = 0$ .

My Notes

3. Reason abstractly. Solve for the indicated variable in each formula.

Name	Formula	Solve for
Distance	$d = rt$ , where $d$ is the distance an object travels, $r$ is the average rate of speed, and $t$ is the time traveled	$r$
Pressure	$p = \frac{F}{A}$ , where $p$ is the pressure on a surface, $F$ is the force applied, and $A$ is the area of the surface	$F$
Kinetic energy	$k = \frac{1}{2}mv^2$ , where $k$ is the kinetic energy of an object, $m$ is its mass, and $v$ is its velocity	$m$
Gravitational energy	$U = mgh$ , where $U$ is the gravitational energy of an object, $m$ is its mass, $g$ is the acceleration due to gravity, and $h$ is the object's height	$h$
Boyle's Law	$p_1V_1 = p_2V_2$ , where $p_1$ and $V_1$ are the initial pressure and volume of a gas and $p_2$ and $V_2$ are the final pressure and volume of the gas when the temperature is kept constant	$V_2$

Check Your Understanding

- Solve the equation  $w + i = \frac{s}{c}$  for  $c$ .
- Why do you think being able to solve a literal equation for a variable would be useful in certain situations?

LESSON 2-5 PRACTICE

Solve each equation for the indicated variable.

6.  $W = Fd$ , for  $d$

7.  $P = \frac{W}{t}$ , for  $W$

8.  $P = \frac{W}{t}$ , for  $t$

9.  $ak - r = on$ , for  $k$

10. Reason quantitatively. In baseball, the equation  $E = \frac{9R}{I}$  gives a pitcher's earned run average  $E$ , where  $R$  is the number of earned runs the player allowed and  $I$  is the number of innings pitched.
- Solve the equation for  $I$ . State a property or provide an explanation for each step.
  - Last season, a pitcher had an earned run average of 2.80 and allowed 70 earned runs. How many innings did the pitcher pitch last season?

## ACTIVITY 2 PRACTICE

Write your answers on notebook paper.

Show your work.

## Lesson 2-1

1. Which of the following shows the Addition Property of Equality?

A.  $\frac{4x}{4} = \frac{10}{4}$   
 $x = 2.5$

B.  $16x - 2 + 2 = 18 + 2$   
 $16x = 20$

C.  $8x + 2x = 10x$

D.  $4(3x + 1) = 8$   
 $12x + 4 = 8$

Solve each equation.

2.  $3z - 2 = 13$

3.  $-5y - 10 = -60$

4.  $4x - 5 + 2x = -2$

5.  $\frac{x-8}{3} = 3$

6. Which equation has the solution  $x = -4$ ?

A.  $2(x - 4) = 16$

B.  $3x + 6 - 2x = -2$

C.  $\frac{1}{4}x + 4 = 3$

D.  $(x + 6)3 = -6$

7. Is 7 a solution of  $5x - 3 = 12$ ? Justify your answer.

8. The perimeter of triangle  $ABC$  is 54. The triangle has side lengths  $AB = 3x$ ,  $BC = 4x$ , and  $AC = 5x$ . Find the length of each side.

A group of 19 students want to see the show at the planetarium. Tickets cost \$11 for each student who is a member of the planetarium's frequent visitor program and \$13 for each student who is not a member. The total cost of the students' tickets is \$209. Use this information for Items 9–16.

9. Let  $x$  represent the number of students in the group who are members of the frequent visitor program. Write an expression in terms of  $x$  for the number of students in the group who are not members.
10. Write an equation to determine the value of  $x$ . Explain what each part of your equation represents.
11. Solve the equation you wrote in Item 10. List a property of equality or provide an explanation for each step.
12. How many students in the group are members of the frequent visitor program? How many are not members? Explain how you know.

## Lesson 2-2

13. Joining the frequent visitor program at the planetarium costs \$5 per year. Write an equation that can be used to determine  $n$ , the number of visits per year for which the cost of being a member of the frequent visitor program is equal to the cost of not being a member.
14. Solve your equation from Item 13. List a property of equality or provide an explanation for each step.
15. Explain the meaning of the solution of the equation.
16. Nash plans to visit the planetarium twice in the next year. Should he join the frequent visitor program? Explain.

For Items 17–24, solve the equations, and explain each step.

17.  $8x + 5 = 3x + 15$

18.  $3x + 11 = 2x - 5$

19.  $6x - 9 = 8x + 11$

20.  $0.5x - 3.5 = 0.2x - 0.5$

## Lesson 2-3

21.  $6 - 2(x + 6) = 3x + 4$

22.  $3x + 2(x - 1) = 9x + 4$

23.  $5(x - 2) + x = 6(x + 3) - 4x$

24.  $2 - 3(4 - x) = 5(2 - x) + 4x$

## ACTIVITY 2

continued

25. Which equation has the greatest solution?
- A.  $0.4x - 2.5 = 1.3x + 4.7$   
 B.  $2.4(x - 6) = -5.6x + 8$   
 C.  $\frac{3}{4}x + 5 = \frac{1}{2}x + 9$   
 D.  $\frac{1}{3}x + \frac{2}{3} = \frac{1}{4}(x + 3)$
26. Provide a reason for each step in solving the equation shown below.

$$\begin{aligned} 2(x - 1) - 3(x + 2) &= 8 - 4x \\ 2x - 2 - 3x - 6 &= 8 - 4x \\ -1x - 8 &= 8 - 4x \\ -1x + 4x - 8 &= 8 - 4x + 4x \\ 3x - 8 &= 8 \\ 3x - 8 + 8 &= 8 + 8 \\ 3x &= 16 \\ \frac{3x}{3} &= \frac{16}{3} \\ x &= 5\frac{1}{3} \end{aligned}$$

27. Tyrell exercised this week both by walking and by biking. He walked at a rate of 4 mi/h and biked at a rate of 12 mi/h. The total distance he covered both walking and biking was 36 miles, and Tyrell spent one more hour walking than biking.
- a. Define a variable and write an equation for this situation.  
 b. How many hours did Tyrell spend on each activity?

## Lesson 2-4

Solve each equation. If an equation has no solutions, or if an equation has infinitely many solutions, explain how you know.

28.  $3(x + 4) - 4 = 2(x + 4) + x$   
 29.  $4(2x + 6) = 5(x + 5) + 2$   
 30.  $6x - 8x + 5 = -2(x + 2) + 7$   
 31.  $0.3x + 1.8 = 0.4(x + 5) - 0.2$   
 32.  $6(x + 2) + 2x - 4 = 8(x - 2) + 8$   
 33.  $\frac{1}{2}(x + 1) + \frac{1}{4} = \frac{2}{3}x - \frac{1}{6}x + \frac{3}{4}$

## Lesson 2-5

34. Which shows the equation  $c = a\left(\frac{w}{150}\right)$  correctly solved for the variable  $w$ ?
- A.  $w = a(150c)$   
 B.  $w = a\left(\frac{c}{150}\right)$   
 C.  $w = \frac{150}{ac}$   
 D.  $w = \frac{150c}{a}$

Solve each equation for the indicated variable.

35.  $w = gm$ , for  $m$   
 36.  $Q = \frac{1}{2}P + 15$ , for  $P$   
 37.  $I = \frac{V}{R}$ , for  $R$   
 38.  $y = mx + b$ , for  $m$   
 39. The equation  $f = d + e + t$  can be used to find an athlete's final score  $f$  in an Olympic trampoline event, where  $d$  is the difficulty score,  $e$  is the execution score, and  $t$  is the time of flight score.
- a. Solve the equation for  $t$ .  
 b. An athlete's final score is 55.675. His difficulty score is 14.6, and his execution score is 24.9. What is the athlete's time of flight score?

## MATHEMATICAL PRACTICES

## Construct Viable Arguments and Critique the Reasoning of Others

40. A student solved the equation  $2(x - 4) - 4x = -6x + 9x + 4$  as shown below. Did the student solve the equation correctly? If so, list a property or explanation for each step. If not, solve the equation correctly, and list a property or explanation for each step.

$$\begin{aligned} 2(x - 4) - 4x &= -6x + 9x + 4 \\ 2x - 8 - 4x &= -6x + 9x + 4 \\ -2x - 8 &= 3x + 4 \\ -2x + 2x - 8 &= 3x + 2x + 4 \\ -8 &= 5x + 4 \\ -8 - 4 &= 5x + 4 - 4 \\ -12 &= 5x \\ -60 &= x \end{aligned}$$



As part of a social studies class project on economics, Annette and Jeff are researching the benefits of membership in an online music club

1. Yearly membership with the online music club costs \$48. Members pay \$0.99 per song to download music. Nonmembers may download songs for \$1.29 each.
  - a. Copy and complete the tables below to represent the yearly cost to download songs for members and nonmembers.

Members	
Number of Songs	Total Cost
0	
1	
2	
3	
4	

Nonmembers	
Number of Songs	Total Cost
0	
1	
2	
3	
4	

- b. Describe any patterns you notice in the tables.
  - c. Represent the total cost of songs purchased for members and nonmembers as sequences. Tell whether each sequence has a common difference, and if so, identify it.
  - d. Use the variable  $n$  to write expressions for the total cost of downloading  $n$  songs for members and for nonmembers.
  - e. Use your expressions to determine the total cost of 8 songs for members and for nonmembers.
2. To determine whether becoming a member of the online music club is cost effective, Annette and Jeff must know at what point the costs for members and nonmembers are equal.
  - a. Write an equation to represent the point at which the total cost of downloading songs as a club member is equal to the total cost of downloading songs as a nonmember. Then solve your equation and interpret your solution within the context of the problem.
  - b. Assume you download 4 songs per week. Would it be beneficial for you to become a member of the online music club? Justify your response. (*Remember: There are 52 weeks in a year.*)
3. Members pay a \$48 membership fee each year. The literal equation  $c = 48y + 0.99n$  represents the total cost  $c$  for a person who is a member of the music club for  $y$  years and who downloads  $n$  songs. Solve the equation for  $y$ .

4. The class is also working on creating family budgets. A sample monthly mortgage payment  $M$  can be represented by the equation  $3250 - M = 2(M - 500) + 1500$ . Jeff and Annette each solved the equation, but they disagree on the solution. Decide who is correct. For the correct solution, justify each step by writing a property or an explanation. For the incorrect solution, identify the error in the solution process.

**Annette**

$$\begin{aligned}
 3250 - M &= 1500 + 2(M - 500) \\
 3250 - M &= 1500 + 2M - 1000 \\
 3250 - M &= 2M + 1500 - 1000 \\
 3250 - M &= 2M - 500 \\
 3250 - M + M &= 2M + M - 500 \\
 3250 &= 3M - 500 \\
 3250 + 500 &= 3M - 500 + 500 \\
 \frac{3750}{3} &= \frac{3M}{3} \\
 1250 &= M \\
 M &= \$1250
 \end{aligned}$$

**Jeff**

$$\begin{aligned}
 3250 - M &= 1500 + 2(M - 500) \\
 3250 - M &= 1500 + 2M - 1000 \\
 3250 - M &= 1500 - 1000 + 2M \\
 3250 - M &= 500 + 2M \\
 3250 - M + M &= 500 + 2M + M \\
 3250 &= 500 + 3M \\
 3250 - 500 &= 500 - 500 + 3M \\
 2750 &= 3M \\
 \frac{2750}{3} &= \frac{3M}{3} \\
 916.67 &= M \\
 M &= \$916.67
 \end{aligned}$$

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates the following characteristics:			
<b>Mathematics Knowledge and Thinking</b> (Items 1a–e, 2a, 3, 4)	<ul style="list-style-type: none"> <li>Fluent use of patterns, sequences, and tables to write expressions and equations</li> <li>Accuracy in solving a literal equation</li> </ul>	<ul style="list-style-type: none"> <li>Adequate understanding of how to use patterns, sequences, and tables to write expressions and equations</li> <li>Correct solution of a literal equation</li> </ul>	<ul style="list-style-type: none"> <li>Partial understanding of how to use patterns, sequences, and tables to write expressions and equations</li> <li>Partially solved literal equation</li> </ul>	<ul style="list-style-type: none"> <li>Inaccurate or incomplete understanding of how to use patterns, sequences, and tables to write expressions and equations</li> <li>No attempt to solve a literal equation</li> </ul>
<b>Problem Solving</b> (Items 1e, 2a, 2b, 4)	<ul style="list-style-type: none"> <li>Appropriate and efficient strategy that results in a correct answer</li> <li>Correct identification of an error in a solution process</li> </ul>	<ul style="list-style-type: none"> <li>Strategy that may include unnecessary steps but results in a correct answer.</li> <li>Correct identification of an error, but with an incorrect reason given</li> </ul>	<ul style="list-style-type: none"> <li>Strategy that results in some incorrect answers</li> <li>Correct identification of an error with no reason given</li> </ul>	<ul style="list-style-type: none"> <li>No clear strategy when solving problems</li> <li>No identification of an error in a solution process</li> </ul>
<b>Mathematical Modeling / Representations</b> (Items 1a, 1d, 2a)	<ul style="list-style-type: none"> <li>Clear and accurate creation of a table to describe a real-world scenario</li> <li>Effective understanding of how to write expressions and equations to represent a real-world scenario</li> </ul>	<ul style="list-style-type: none"> <li>Little difficulty creating a table to describe a real-world scenario</li> <li>Functional understanding of how to write expressions and equations to represent a real-world scenario</li> </ul>	<ul style="list-style-type: none"> <li>Partially accurate table to describe a real-world scenario</li> <li>Partial understanding of how to write expressions and equations to represent a real-world scenario</li> </ul>	<ul style="list-style-type: none"> <li>Inaccurate or incomplete table to describe a real-world scenario</li> <li>Little or no understanding of how to write expressions and equations to represent a real-world scenario</li> </ul>
<b>Reasoning and Communication</b> (Items 1b, 2b, 4)	<ul style="list-style-type: none"> <li>Precise use of appropriate math terms and language to describe patterns and to justify each step in the solution of an equation</li> <li>Clear and accurate conclusion drawn from an equation</li> </ul>	<ul style="list-style-type: none"> <li>Adequate description of patterns and justification of each step in the solution of an equation</li> <li>Reasonable conclusion drawn from an equation</li> </ul>	<ul style="list-style-type: none"> <li>Confusing description of patterns and/or justification of the steps in the solution of an equation</li> <li>Partially correct conclusion drawn from an equation</li> </ul>	<ul style="list-style-type: none"> <li>Incomplete or inaccurate description of patterns and/or justification of the steps in the solution of an equation</li> <li>Incomplete or inaccurate conclusion drawn from an equation</li> </ul>

# Solving Inequalities

## Physical Fitness Zones

### Lesson 3-1 Inequalities and Their Solutions


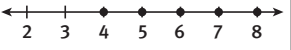
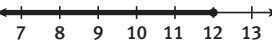
#### Learning Targets:

- Understand what is meant by a solution of an inequality.
- Graph solutions of inequalities on a number line.

**SUGGESTED LEARNING STRATEGIES:** Levels of Questions, Think-Pair-Share, Interactive Word Wall, Construct an Argument, Quickwrite

Spartan Middle School students participate in Physical Education testing each semester. In order to pass, 12- and 13-year-old girls have to do at least 7 push-ups and 4 modified pull-ups. They also have to run one mile in 12 minutes or less.

You can use an inequality to express the passing marks in each test.

	Push-Ups, $p$	Modified Pull-Ups, $m$	One-Mile Run, $r$
Verbal	At least 7 push-ups	At least 4 pull-ups	12 minutes or less
Inequality	$p \geq 7$	$m \geq 4$	$r \leq 12$
Graph			

1. Why do you think the graphs of push-ups and pull-ups are dotted but the graph of the mile run is a solid ray?
2. **Reason quantitatively.** Jamie ran one mile in 12 minutes 15 seconds, did 8 push-ups, and did 4 modified pull-ups. Did she pass the test? Explain.
3. Karen did 7 push-ups.
  - a. Is this a passing number of push-ups? Which words in the verbal description indicate this? Explain.
  - b. How is this represented in the inequality  $p \geq 7$ ?

#### My Notes

#### MATH TERMS

The **graph of an inequality** in one variable is all the points on a number line that make the inequality true.

#### WRITING MATH

Other phrases that are equivalent to "at least" are "no less than" and "no fewer than." These phrases are also represented by the inequality symbol  $\geq$ .

The **solution of an inequality** in one variable is the set of numbers that make the inequality true. To verify a solution of an inequality, substitute the value into the inequality and simplify to see if the result is a true statement.

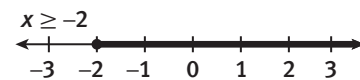
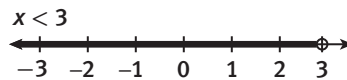
**My Notes**

4. Use the table below to figure out which  $x$ -values are solutions to the equation and which ones are solutions to the inequality. Show your work in the rows of the table.

$x$ -values	Solution to the equation? $2x + 3 = 5$	Solution to the inequality? $2x + 3 > 5$
-1		
0		
1		
2		
8.5		

5. How many solutions are there to the equation  $2x + 3 = 5$ ? Explain.
6. Which numbers in the table are solutions to the inequality  $2x + 3 > 5$ ? Are these the only solutions to the inequality? Explain.
7. Would 1 be a solution to the inequality  $2x + 3 \geq 5$ ? Explain.

Here are the number line graphs of two different inequalities.



8. Use the graphic organizer to compare and contrast the two inequalities and graphs that are shown above.

Similarities	Differences

## Lesson 3-1

### Inequalities and Their Solutions

## ACTIVITY 3

continued

9. Think about why the graphs are different.
- Why is one of the graphs showing a solid ray going to the left and the other graph showing a solid ray going to the right?
  - Why does one graph have an open circle and the other graph a filled-in circle?

### Check Your Understanding

10. Write an inequality to represent each statement.
- $x$  is less than 12.
  - $m$  is no greater than 35.
  - Your height  $h$  must be at least 42 inches for you to ride a theme park ride.
  - A child's age  $a$  can be at most 12 for the child to order from the children's menu.
11. How are the graphs of  $x > -4$  and  $x \geq -4$  alike and how are they different?

### LESSON 3-1 PRACTICE

Graph each inequality on a number line.

- $x < -2$
- $x \geq 5$
- $x < 4$
- $x > -2$
- $x \geq \frac{1}{2}$
- Write a real-world statement that could be represented by the inequality  $x \leq 6$ .
- Attend to precision.** Consider the inequalities  $x \leq -3$  and  $x \geq -3$ .
  - Graph  $x \leq -3$  and  $x \geq -3$  on the same number line.
  - Describe any overlap in the two graphs.
  - Describe the combined graphs.

My Notes

### WRITING MATH

An open circle represents  $<$  or  $>$  inequalities, and a filled-in circle represents  $\leq$  or  $\geq$  inequalities.

## My Notes

**Learning Targets:**

- Write inequalities to represent real-world situations.
- Solve multi-step inequalities.

**SUGGESTED LEARNING STRATEGIES:** Create Representations, Guess and Check, Look for a Pattern, Think-Pair-Share, Identify a Subtask

- 1. Make sense of problems.** Chloe and Charlie are taking a trip to the pet store to buy some things for their new puppy. They know that they need a bag of food that costs \$7, and they also want to buy some new toys for the puppy. They find a bargain barrel containing toys that cost \$2 each.
  - a.** Write an expression for the amount of money they will spend if the number of toys they buy is  $t$ .
  - b.** Chloe has \$30 and Charlie has one-third of this amount with him. Use this information and the expression you wrote in Part (a) to write an inequality for finding the number of toys they can buy.

There are different methods for solving the inequality you wrote in the previous question. Chloe suggests that they guess and check to find the number of new toys that they could buy.

- 2.** Use Chloe's suggestion to find the number of new puppy toys that Chloe and Charlie can buy with their combined money.

Charlie remembered that they could use algebra to solve inequalities. He imagined that the inequality symbol was an equal sign. Then he used equation-solving steps to solve the inequality.

- 3.** Use Charlie's method to solve the inequality you wrote in Item 1b.

## Lesson 3-2

### Solving Inequalities

My Notes

4. Did you get the same answer using Charlie’s method as you did using Chloe’s method? Explain.

### Check Your Understanding

5. How would you graph the solution to Charlie and Chloe’s inequality?
6. Jaden solved an inequality as shown below. Describe and correct any errors in his work.

$$\begin{aligned}
 3x + 5 + 6x &> 23 \\
 9x + 5 &> 23 \\
 9x &> 28 \\
 x &> 7\frac{1}{9}
 \end{aligned}$$

Chloe liked the fact that Charlie’s method for solving inequalities did not involve guess and check, so she asked him to show her the method for the inequality  $-2x - 4 > 8$ .

Charlie showed Chloe the work below to solve  $-2x - 4 > 8$ .

$$\begin{aligned}
 -2x - 4 &> 8 \\
 -2x - 4 + 4 &> 8 + 4 \\
 -2x &> 12 \\
 \frac{-2x}{-2} &> \frac{12}{-2} \\
 x &> -6
 \end{aligned}$$

When Chloe went back to check the solution by substituting a value for  $x$  back into the original inequality, she found that something was wrong.

7. Confirm or disprove Chloe’s conclusion by substituting values for  $x$  into the original inequality.

**My Notes**

Chloe tried the problem again but used a few different steps.

$$\begin{aligned}
 -2x - 4 &> 8 \\
 -2x + 2x - 4 &> 8 + 2x \\
 -4 &> 8 + 2x \\
 -4 - 8 &> 8 - 8 + 2x \\
 -12 &> 2x \\
 \frac{-12}{2} &> \frac{2x}{2} \\
 -6 &> x
 \end{aligned}$$

Chloe concluded that  $x < -6$ .

- 8. Is Chloe's conclusion correct? Explain.
  
- 9. Explain what Chloe did to solve the inequality.

Charlie looked back at his work. He said that he could easily fix his work by simply switching the inequality sign.

- 10. **Critique the reasoning of others.** What do you think about Charlie's plan? Explain.

Although all of these methods worked, Charlie and Chloe wanted to know why they were working.

Here is an experiment to discover what went wrong with Charlie's first method. Look at what happens when you multiply or divide by a negative number.

Directions	Numbers	Inequality
Pick two different numbers.	2 and 4	$2 < 4$
Multiply both numbers by 3.	$2(3)$ and $4(3)$	$6 < 12$
Multiply both numbers by $-3$ .	$2(-3)$ and $4(-3)$	$-6 > -12$
Divide both numbers by 2.	$2 \div 2$ and $4 \div 2$	$1 < 2$
Divide both numbers by $-2$ .	$2 \div (-2)$ and $4 \div (-2)$	$-1 > -2$

- 11. Try this experiment again with two different numbers. Record your results in the *My Notes* section of this page. Compare your results to the rest of your class.



## Lesson 3-2

### Solving Inequalities

## ACTIVITY 3

continued

**12. Express regularity in repeated reasoning.** What happens each time you multiply each side of an inequality by a negative number? What happens each time you divide each side of an inequality by a negative number?

**13.** How does this affect how you solve an inequality?

### Example A

Solve and graph:  $-5x + 8 \leq -2x + 23$

**Step 1:** Subtract 8 from both sides.

$$\begin{aligned} -5x + 8 - 8 &\leq -2x + 23 - 8 \\ -5x &\leq -2x + 15 \end{aligned}$$

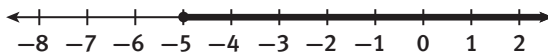
**Step 2:** Add  $2x$  to both sides.

$$\begin{aligned} -5x + 2x &\leq -2x + 2x + 15 \\ -3x &\leq 15 \end{aligned}$$

**Step 3:** Divide both sides by  $-3$ .  
Remember to reverse the inequality sign.

$$\begin{aligned} \frac{-3x}{-3} &\geq \frac{15}{-3} \\ x &\geq -5 \end{aligned}$$

**Solution:**  $x \geq -5$



### Try These A

Solve and graph each inequality.

a.  $3 - 4x \leq 11$

b.  $6 - 3(x + 2) > 15$

c.  $2(x + 5) < 8(x - 3)$

My Notes

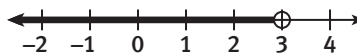
### MATH TIP

Substitute some sample answers back into the original inequality to check your work.

## My Notes

## Check Your Understanding

14. Write two different inequalities that have the solution graphed on the number line below.



15. Explain why you reverse the inequality sign when you multiply or divide both sides of an inequality by a negative number.

## MATH TIP

Remember to substitute some sample answers back into the original inequality to check your work.

## LESSON 3-2 PRACTICE

Solve and graph each inequality.

16.  $5 < 3x + 8$
17.  $5 < -3x + 8$
18.  $3x - 8 + 4x > 6$
19.  $-5x + 2 \geq -8x$
20.  $4 - 2(x + 1) < 18$
21.  $-6x - 3 \leq -4x + 1$
22.  $3(x + 7) \geq 2(2x + 8)$
23.  $-2x - 3 + 8 < -3(3x + 5)$
24. **Model with mathematics.** Riley and Rhoda plan to buy several bags of dog food and a dog collar. Each bag of dog food costs \$7, and the dog collar costs \$5.
- Use the information above to write an expression for the amount of money they will spend if they buy  $b$  bags of food.
  - Riley and Rhoda have \$30. Use this information and the expression you wrote for Part (a) to write an inequality for finding the number of bags of food they could buy.
  - Solve the inequality and graph the solutions. Check your answer in the original situation.
25. In Example A,  $-5x + 8 \leq -2x + 23$  was solved by dividing each side of the inequality by  $-3$  in the last step. Is there another way to solve this inequality so that you can avoid dividing by a negative number? Explain.

**Learning Targets:**

- Graph compound inequalities.
- Solve compound inequalities.

**SUGGESTED LEARNING STRATEGIES:** Vocabulary Organizer, Look for a Pattern, Create Representations, Think-Pair-Share, Note Taking

**Compound inequalities** are two inequalities joined by the word *and* or by the word *or*. Inequalities joined by the word *and* are called **conjunctions**. Inequalities joined by the word *or* are **disjunctions**. You can represent compound inequalities using words, symbols, or graphs.

1. Complete the table. The first two rows have been done for you.

Verbal Description	Some Possible Solutions	Inequality	Graph
all numbers from 3 to 8, inclusive	3.5, 4, $4\frac{1}{3}$ , 5, 6, 7.9, 8	$x \geq 3$ and $x \leq 8$	
all numbers less than 5 or greater than 10	-2, 0, 3, 4, 4.8, $10\frac{3}{4}$ , 11	$x < 5$ or $x > 10$	
all numbers greater than -1 and less than or equal to 4			
all numbers less than or equal to 3 or greater than 6			

2. Use the graphic organizer below to compare and contrast the graphs for conjunctions and disjunctions.

Similarities	Differences

My Notes

My Notes

**Example A**

Spartan Middle School distributes this chart to students each year to show what students must be able to do to pass the fitness test.

Age	Mile Run (min:sec)		Push-Ups		Modified Pull-Ups	
	Boys	Girls	Boys	Girls	Boys	Girls
12	8:00–10:30	9:00–12:00	10–20	7–15	7–20	4–13
13	7:30–10:00	9:00–12:00	12–25	7–15	8–22	4–13

Write and graph a compound inequality that describes the push-up range for 12-year-old boys.

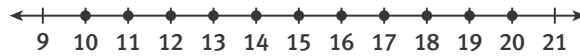
**Step 1:** Choose a variable.

Let  $p$  represent the number of push-ups for 12-year-old boys.

**Step 2:** Determine the range and write an inequality.

The range is whole numbers  $p$  such that  $p \geq 10$  and  $p \leq 20$ .

**Solution:** The compound inequality is  $10 \leq p \leq 20$ .



**WRITING MATH**

The inequality  $p \geq 10$  is equivalent to the inequality  $10 \leq p$ . So the compound inequality “ $p \geq 10$  and  $p \leq 20$ ” can also be written as “ $10 \leq p \leq 20$ .”

**CONNECT TO AP**

In upper-level mathematics classes, inequalities are expressed in interval notation. The interval notation for  $x > 3$  is  $(3, \infty)$ .

**Try These A**

Write and graph a compound inequality for each range or score.

- the push-up range for 13-year-old boys
- the pull-up range for 13-year-old girls
- the mile run range for 12-year-old girls
- the mile run range for 13-year-old boys
- a score outside the healthy fitness zone for girls’ push-ups

**3. Attend to precision.** Why are individual points used in the graphs for Example A and some of the graphs in Try These A?

## Lesson 3-3

### Compound Inequalities

## ACTIVITY 3

continued

The solution of the conjunction will be *the solutions that are common to both parts*.

### Example B

Solve and graph the conjunction:  $3 < 3x - 6 < 8$

**Step 1:** Break the compound inequality into two parts.

$$3 < 3x - 6 \text{ and } 3x - 6 < 8$$

**Step 2:** Solve and graph  $3 < 3x - 6$ .

$$3 < 3x - 6$$

$$3 + 6 < 3x - 6 + 6$$

$$9 < 3x$$

$$3 < x \text{ or } x > 3$$



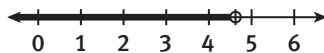
**Step 3:** Solve and graph  $3x - 6 < 8$ .

$$3x - 6 < 8$$

$$3x - 6 + 6 < 8 + 6$$

$$3x < 14$$

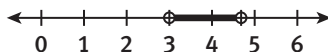
$$x < 4\frac{2}{3}$$



**Step 4:** Determine what is common to the solutions of each part.

In the inequalities and graphs in Steps 2 and 3, the points between 3 and  $4\frac{2}{3}$  are in common.

**Solution:**  $3 < x < 4\frac{2}{3}$



### Try These B

Solve and graph each conjunction.

a.  $-1 < 3x + 5 < 6$

b.  $2 < \frac{x}{3} - 5 < 6$

c.  $3 < 2(x + 2) - 7 \leq 13$

d.  $-2 < 3(x + 6) < 18$

My Notes

### MATH TIP

Remember to substitute some sample answers back into the original inequality to check your work.

My Notes

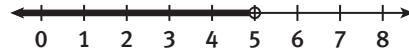
The solution of a disjunction will be *all the solutions from both its parts*.

**Example C**

Solve and graph the compound inequality:  $2x - 3 < 7$  or  $4x - 4 \geq 20$ .

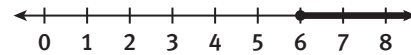
**Step 1:** Solve and graph  $2x - 3 < 7$ .

$$\begin{aligned} 2x - 3 + 3 &< 7 + 3 \\ 2x &< 10 \\ x &< 5 \end{aligned}$$



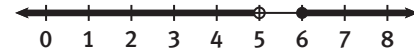
**Step 2:** Solve and graph  $4x - 4 \geq 20$ .

$$\begin{aligned} 4x - 4 &\geq 20 \\ 4x - 4 + 4 &\geq 20 + 4 \\ 4x &\geq 24 \\ x &\geq 6 \end{aligned}$$



**Step 3:** Combine the solutions.

**Solution:**  $x < 5$  or  $x \geq 6$



**Try These C**

Solve and graph each compound inequality.

a.  $5x + 1 > 11$  or  $x - 1 < -4$

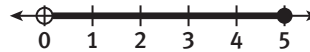
b.  $-5x > 20$  or  $x - 2 \geq -7$

**MATH TIP**

Remember to substitute some sample answers back into the original inequality to check your work.

**Check Your Understanding**

4. The solutions of a conjunction are graphed below. What is the inequality?



5. Describe the difference in the graph of the conjunction " $x > 2$  and  $x < 10$ " and the graph of the disjunction " $x > 2$  or  $x > 10$ ."

**LESSON 3-3 PRACTICE**

6. **Reason quantitatively.** A Category 2 hurricane has wind speeds of at least 96 miles per hour and at most 110 miles per hour. Write the wind speed of a Category 2 hurricane as two inequalities joined by the word *or* or *and*.

**Solve and graph each compound inequality on a number line.**

7.  $-2x + 3 < 8$  and  $3(x + 4) - 11 < 10$

8.  $-3x + 5 > -1$  or  $2(x + 4) > 14$

9. Write a real-world statement that could be represented by the compound inequality  $\$7.50 < p \leq \$18.50$ .

**ACTIVITY 3 PRACTICE**

Write your answers on notebook paper.

Show your work.

**Lesson 3-1**

- Describe the similarities and differences in the solutions of  $2x - 7 = 15$  and  $2x - 7 \leq 15$ .
- For the equation  $-3x + 2 = 8$  and the inequality  $-3x + 2 > 8$ , which  $x$ -values indicated below are solutions of the equation and which are solutions of the inequality?
  - 3
  - 2
  - 1
  - 0
- Describe the graph of  $x > -2$ .
- Describe the graph of  $x \leq -2$ .
- Graph  $x < 1\frac{1}{2}$  on a number line.
- Graph  $x \geq -3$  on a number line.

**Lesson 3-2**

- Mayumi plans to buy pencils and a notebook at the school store. A pencil costs \$0.15, and a notebook costs \$1.59. Mayumi has \$5.00. Which inequality could she use to find the number of pencils she can buy?
  - $5.00 < 0.15x + 1.59$
  - $5.00 \geq 0.15x + 1.59$
  - $5.00 < 0.15x - 1.59$
  - $5.00 \geq 0.15x - 1.59$
- Which values of  $a$  and  $b$  disprove the statement below?

If  $a > b$ , then  $a^2 > b^2$ .

- $a = 2, b = 0$
- $a = 4, b = 1$
- $a = 3, b = -5$
- $a = \frac{3}{4}, b = \frac{1}{2}$

Solve the inequality and graph the solutions on a number line. Check your answers.

- $5x - 4 > -4$
- $8 > 6 + \frac{2}{5}x$
- $5 - 3x \leq 8$
- $3x - 4 \geq 6x + 11$
- $x - 2 \geq -8x + 16$
- $3x - 7 < 2(2x - 1)$
- $2\left(\frac{1}{2}x - 4\right) < -(x - 5)$
- $\frac{2x - 11}{3} \geq -x - 2$

Write an inequality that requires more than one step to solve and that has the given solution.

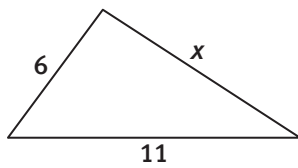
- $x < -3$
- $x > 1$
- Roy is attending his cousin's graduation ceremony in another town. Roy has already driven 30 miles and the ceremony starts in 2 hours.
  - Let  $r$  represent Roy's driving speed in miles per hour. Write an expression to show the total distance Roy will have driven in 2 hours.
  - The graduation is 150 miles from Roy's home. Use this information and your expression from Part (a) to write an inequality showing the possible speeds Roy could drive to make it to the ceremony on time.
  - Solve your inequality and graph the solutions. What does your solution mean in the context of the problem?

**Lesson 3-3**

Use the table for Items 20 and 21.

Average High and Low Monthly Temperatures		
	January	August
Austin, TX	41–62°F	75–97°F
Columbus, OH	20–36°F	63–83°F

20. Write a compound inequality for the range of temperatures for Austin, TX, in August.
21. Write a compound inequality for the range of temperatures for Columbus, OH, in January.
22. The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.



- a. For the triangle shown, Anna said that any value of  $x$  greater than 5 is possible. Explain Anna's error.
  - b. Write a compound inequality that represents all possible values of  $x$ .
23. Find a value for  $n$  so that the compound inequality  $-n < x < n$  has no solutions.

Solve each compound inequality and graph the solutions on a number line. Check your answers.

24.  $-2 < 3x + 4 < 31$
25.  $1 < 5x - 9 < 6$
26.  $-2x + 7 > 1$  and  $4x + 3 \geq -13$
27.  $0 \leq \frac{6-x}{9}$  and  $-2x \leq -10$
28.  $5x - 2 < -7$  or  $2x + 1 > 5$
29.  $7x - 2 < -30$  or  $4x + 5 > 13$
30.  $\frac{1}{2} < \frac{2x-9}{2} \leq 3$
31.  $3 \leq 2(x+4) - 3 < 15$
32.  $-2(x+2) - 7 > 9$  or  $3(x+3) > -6$

**MATHEMATICAL PRACTICES**

**Construct Viable Arguments and Critique the Reasoning of Others**

33. The inequality  $x + 5 < x + 4$  has no solutions. Explain why.



## Student Distances

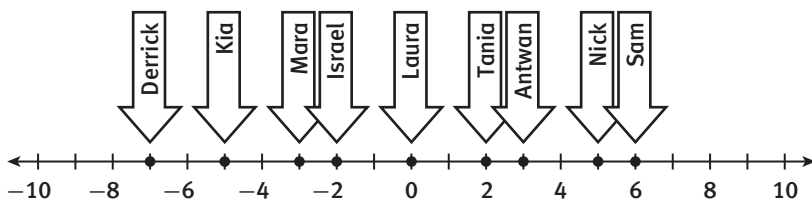
### Lesson 4-1 Absolute Value Equations

#### Learning Targets:

- Understand what is meant by a solution of an absolute value equation.
- Solve absolute value equations.

**SUGGESTED LEARNING STRATEGIES:** Paraphrasing, Create Representations, Think-Pair-Share, Note Taking, Identify a Subtask

Ms. Patel is preparing the school marching band for the homecoming show. She has the first row of band members stand in positions along a number line on the floor of the band room. The students' positions match the points on a number line as shown.



1. Use the number line to write each student's distance from 0 next to their name. For example, Tania is 2 units away from 0. Israel's distance from 0 is also 2 units even though he is at  $-2$ .

Derrick	Laura
Kia	Israel
Mara	Antwan
Tania	Nick
Sam	

The **absolute value** of a number is the distance from 0 to the number on a number line. Using **absolute value notation**, Mara's distance is  $|-3|$  and Antwan's distance is  $|3|$ . Since Mara and Antwan are each 3 units from 0,  $|-3| = 3$  and  $|3| = 3$ .

2. **Attend to precision.** Write each person's distance from 0 using absolute value notation.

**Absolute value equations** can represent distances on a number line.

3. The locations of the two students who are 5 units away from 0 are the solutions of the absolute value equation  $|x| = 5$ . Which two students represent the solutions to the equation  $|x| = 5$ ?

My Notes

#### READING MATH

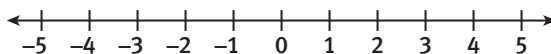
Read  $|-3|$  as "the absolute value of negative three."

#### MATH TERMS

An **absolute value equation** is an equation involving the absolute value of a variable expression.

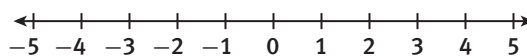
**My Notes**

4. You can create a graph on a number line to represent the solutions of an absolute value equation. Graph the solutions of the equation  $|x| = 5$  on the number line below. Then use the graph to help you explain why it makes sense that the equation  $|x| = 5$  has two solutions.



Absolute value equations can also represent distances between two points on a number line.

5. In the student line, which two people are 4 units away from 1? Mark their location on the number line below.



The equation  $|x| = 4$  represents the numbers located 4 units away from 0. So the equation  $|x| = 4$  can also be written as  $|x - 0| = 4$ , which shows the distance (4) away from the point 0. In Item 5, you were looking for the numbers located 4 units away from 1. So you can write the absolute value equation  $|x - 1| = 4$  to represent that situation.

6. What are two possible values for  $x - 1$  given that  $|x - 1| = 4$ ? Explain.
7. Use the two values you found in Item 6 to write two equations showing what  $x - 1$  could equal.
8. Solve each of the two equations that you wrote in Item 7.

The solutions in Item 8 represent the two points on the number line that are 4 units from 1.

9. How do the solutions relate to your answer for Item 5?

## Lesson 4-1

### Absolute Value Equations

## ACTIVITY 4

continued

10. Draw a number line to show the answer to each question. Then write an absolute value equation to represent the points described.

a. Which two points are 2 units away from 0?

b. Which two points are 5 units away from  $-2$ ?

c. Which two points are 3 units away from 4?

11. Solve these absolute value equations.

a.  $|x| = 10$

b.  $|x| = -3$

c.  $|x + 2| = 7$

d.  $|2x - 1| = 5$

In the equation  $|x - 3| = 7$ , the 7 indicates that the distance between  $x$  and 3 is 7 units. There are two points that are 7 units from 3. These can be found by solving the equations  $x - 3 = 7$  and  $x - 3 = -7$ . Rewriting an absolute value equation as two equations allows you to solve the absolute value equation using algebra.

### Example A

Solve the equation  $|x - 3| = 7$ .

**Step 1:** Rewrite the equation as two equations that do not have absolute value symbols.

$$x - 3 = 7$$

$$x - 3 = -7$$

**Step 2:** Solve  $x - 3 = 7$ .

$$x - 3 + 3 = 7 + 3$$

$$x = 10$$

**Step 3:** Solve  $x - 3 = -7$ .

$$x - 3 + 3 = -7 + 3$$

$$x = -4$$

**Solution:** 10,  $-4$

My Notes

### MATH TIP

If there are no values of  $x$  that make an equation true, the equation has **no solution**.

## My Notes

## Try These A

Solve each absolute value equation.

a.  $|x - 5| = 1$

b.  $|x + 5| = 2$

c.  $|2x + 3| = 11$

d.  $|3x - 4| = 8$

You may need to first isolate the absolute value expression to solve an absolute value equation.

## Example B

Solve the equation  $6|x + 2| = 18$ .

**Step 1:** Divide both sides of the equation by 6 so that the absolute value expression is alone on one side of the equation.

$$\frac{6|x + 2|}{6} = \frac{18}{6}$$

$$|x + 2| = 3$$

**Step 2:** Rewrite the equation as two equations that do not have absolute value symbols.

$$x + 2 = 3 \text{ and } x + 2 = -3$$

**Step 3:** Solve  $x + 2 = 3$ .

$$x + 2 - 2 = 3 - 2$$

$$x = 1$$

**Step 4:** Solve  $x + 2 = -3$ .

$$x + 2 - 2 = -3 - 2$$

$$x = -5$$

**Solution:** 1, -5

## Try These B

Solve each absolute value equation.

a.  $3|x - 1| = 12$

b.  $|x| - 14 = 6$

c.  $|x + 4| + 5 = 8$

d.  $3|x + 6| - 7 = 20$

## MATH TIP

Check your answers by substituting the solutions in the original equation.

$$6|1 + 2| = 18$$

$$6|3| = 18$$

$$6(3) = 18$$

$$18 = 18$$

$$6|-5 + 2| = 18$$

$$6|-3| = 18$$

$$6(3) = 18$$

$$18 = 18$$



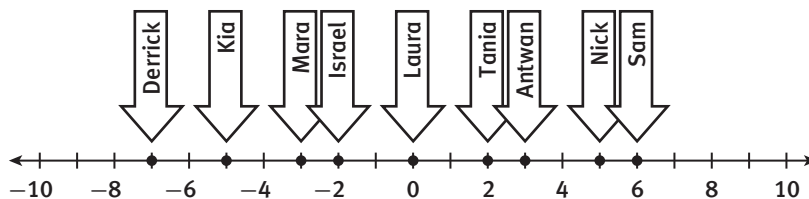
My Notes

**Learning Targets:**

- Solve absolute value inequalities.
- Graph solutions of absolute value inequalities.

**SUGGESTED LEARNING STRATEGIES:** Role Play, Visualization, Create Representations, Guess and Check, Think-Pair-Share

Here is the marching band line-up once again.



1. Which people in the line up are 3 or fewer units from 0?
2. Show the portion of the number line that includes numbers that are 3 or fewer units from 0.

The graph you created in Item 2 can be represented with an **absolute value inequality**. The inequality  $|x| \leq 3$  represents the numbers on a number line that are 3 or fewer units from 0.

3. Circle the numbers below that are solutions of  $|x| \leq 3$ . Explain why you chose those numbers.

-1    3    0.5    4    -3.1

4. **Reason abstractly.** How many solutions does the inequality  $|x| \leq 3$  have?
5. If you were to write a compound inequality for the graph of  $|x| \leq 3$  that you sketched in Item 2, would it be a conjunction (“and” inequality) or a disjunction (“or” inequality)? Explain.

## Lesson 4-2

### Absolute Value Inequalities

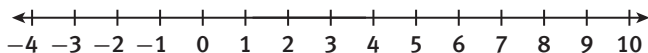
## ACTIVITY 4

*continued*

My Notes

6. Write a compound inequality to represent the solutions to  $|x| \leq 3$ .

7. What numbers are more than 4 units away from 3 on a number line?  
Show the answer to this question on the number line.



The absolute value inequality  $|x - 3| > 4$  represents the situation in Item 7. A “greater than” symbol indicates that the distances are greater than 4.

8. Circle the numbers below that are solutions to the inequality  $|x - 3| > 4$ .  
Explain why you chose those numbers.

7.1    7    0    -2    6.9    100

9. **Construct viable arguments.** If you were to write a compound inequality for the graph of  $|x - 3| > 4$  that you sketched in Item 7, would it be a conjunction or disjunction? Explain.
10. To solve  $|x - 3| > 4$  for  $x$ , you need to write the absolute value inequality as a compound inequality.
- Based on the graph from Item 7, the expression  $x - 3$  is either greater than 4 or less than  $-4$ . Write this statement as a compound inequality.
  - Solve each of the inequalities you wrote in Part (a). Graph the solution.

**My Notes**

**11.** Make a graph that represents the answer to each question. Then write an absolute value inequality that has the solutions that are graphed. Finally, write each absolute value inequality as a compound inequality.

**a.** What numbers are less than 2 units from 0?

**b.** What numbers are 4 or more units away from 0?

**c.** What numbers are 4 or fewer units away from -2?

**12.** Describe the absolute value inequalities  $|x| < 3$  and  $|x| > 3$  as conjunctions or disjunctions and justify your choice in each case.

You can solve absolute value inequalities algebraically.

**Example A**

Solve the inequality  $|2x| + 3 > 9$ . Graph the solutions.

**Step 1:** Subtract 3 from both sides of the inequality so that the absolute value expression is alone on one side.

$$\begin{aligned} |2x| + 3 - 3 &> 9 - 3 \\ |2x| &> 6 \end{aligned}$$

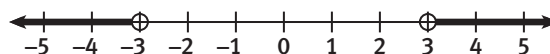
**Step 2:** Rewrite the equation as a compound inequality. Determine if the relationship is “and” or “or.”

$$2x > 6 \text{ or } 2x < -6$$

**Step 3:** Solve the inequalities.

$$\begin{aligned} 2x > 6 \text{ or } 2x < -6 \\ x > 3 \text{ or } x < -3 \end{aligned}$$

**Solution:**  $x > 3$  or  $x < -3$





## Lesson 4-2

### Absolute Value Inequalities

## ACTIVITY 4

*continued*

### Try These A

Solve each absolute value inequality and graph the solutions.

a.  $|2x - 7| > 3$

b.  $|3x + 8| < 5$

c.  $|4x| - 3 \geq 5$

d.  $2|x| + 7 \leq 11$

e.  $-3|x - 9| \geq -21$

f.  $-5|2x - 8| < -20$

My Notes

### MATH TIP

Remember that when multiplying or dividing each side of an inequality by a negative number, you must reverse the inequality symbol.

## My Notes

## Check Your Understanding

13. Describe the similarities and differences between solving an absolute value equation and solving an absolute value inequality.

## LESSON 4-2 PRACTICE

14. Graph the following and then write an absolute value inequality that represents each question.
- What numbers are 5 units or more away from  $-2$  on a number line?
  - What numbers are 5 units or fewer from  $-2$  on a number line?

Solve each inequality and graph the solutions.

15.  $|x - 4| \leq 2$

16.  $|x - 5| > 3$

17.  $\left|\frac{2}{3}x + 5\right| > 4$

18.  $\left|\frac{3x-5}{2}\right| \leq 7$

19.  $|x| - 3 \leq 4$

20.  $-3|x| < -12$

21.  $6 \leq |2x - 9|$

22.  $-5|x + 12| > -35$

23. **Critique the reasoning of others.** Isabelle was asked to write an absolute value inequality to represent the numbers that are less than 1 unit away from 7 on a number line. Isabelle wrote  $|x - 1| < 7$ . Explain and correct Isabelle's error.

## ACTIVITY 4 PRACTICE

Write your answers on notebook paper.

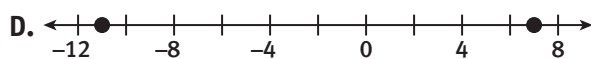
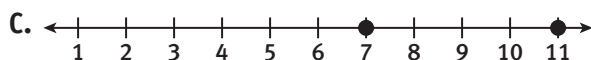
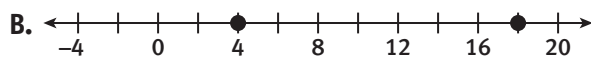
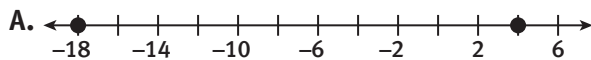
Show your work.

### Lesson 4-1

- Use a number line to show the numbers that are 3 units from  $-1$  on a number line. Then write an absolute value equation to describe the graph.

- Explain why  $|x| = -5$  does not have a solution.

- Which graph shows the solutions of  $|x - 11| = 7$ ?



- Suppose that  $x$  is negative and  $|x| = n$ . What conclusion can you draw?

- $x$  and  $n$  are opposites.
- $x$  and  $n$  are equal.
- $x$  is greater than  $n$ .
- None of the above

For Items 5–17, solve each absolute value equation.

Check your answers.

- $|x| = 7$
- $|x - 2| = -2$
- $|x - (-2)| = 5$
- $|3(x - 1)| = 15$
- $\left|\frac{2}{5}x\right| = 4$
- $|2(x - 3)| = 10$
- $|3(x - 2)| = x$
- $|4(x + 2)| + 9 = 15$

13.  $|-2x + 3| = 7$

14.  $|-3(x - 7)| = 21$

15.  $-5|x - 2| = -20$

16.  $-3|x + 5| + 7 = 4$

17.  $\left|\frac{2x - 5}{7}\right| = 3$

### Lesson 4-2

For Items 18–21, graph the solutions. Then write an absolute value inequality that represents each question.

- What numbers are more than 3 units from  $-1$  on a number line?
- What numbers are less than 3 units from  $-1$  on a number line?
- What numbers are 5 or fewer units away from 3?
- What numbers are 3 or more units away from 5?

For Items 22–25, graph the solutions of each absolute value inequality and write compound inequalities for the solutions.

22.  $|x| > 3$

23.  $|x| < 3$

24.  $|x - 4| \geq 7$

25.  $|x - 4| \leq 7$

26. Which describes the solutions of  $|6x - 3| > 21$ ?

- all numbers greater than 4
- all numbers greater than  $-3$  and greater than 4
- all numbers between  $-3$  and 4
- all numbers less than  $-3$  and greater than 4

27. Without solving, match each absolute value equation or inequality with its number of solutions. Justify your answers.

$|x - 7| < -2$       one solution

$|x| = 0$       no solutions

$|x + 1| > -5$       infinitely many solutions

28. The solutions to which absolute value inequality are shown in the graph below?



- A.  $|x + 1| < 1$   
 B.  $|x + 1| > 1$   
 C.  $|x - 1| < 1$   
 D.  $|x - 1| > 1$
29. Create a graphic organizer that compares and contrasts the following equation and inequalities.

$$|x - 3| = 6$$

$$|x - 3| \leq 6$$

$$|x| - 3 > 6$$

For Items 30–40, solve each absolute value inequality and graph the solutions.

30.  $|x - 2| > 3$   
 31.  $|x - 5| < 2$   
 32.  $|2x + 7| \geq 5$   
 33.  $|3x + 2| \leq 11$   
 34.  $\left| \frac{5x - 3}{2} \right| < 6$

35.  $|4(x - 1)| > 16$   
 36.  $|x - 7| + 3 < 2$   
 37.  $|x + 5| - 2 < 3$   
 38.  $|2(x + 1)| - 7 \leq 1$   
 39.  $\left| \frac{3x - 1}{4} \right| \geq 5$   
 40.  $-2|3x - 4| \leq -6$   
 41. Marty said that all absolute value inequalities that contain the symbol  $<$  are conjunctions. Give a counterexample to show that Marty's statement is incorrect.

**MATHEMATICAL PRACTICES**

**Model with Mathematics**

42. According to some medical websites, normal body temperature can be as much as one degree above or below  $98.6^\circ\text{F}$ . Write a compound inequality that shows the range of normal body temperatures,  $t$ . Then write an absolute value inequality that shows the same information. Explain how you wrote the absolute value inequality, and include a number line graph in your explanation.

The table below shows ranges for the daily calorie needs of 15-year-old males according to the United States Department of Agriculture. Use the table for Items 1 and 2.

Daily Calories for 15-Year-Old Males	
Sedentary	no more than 2200 calories
Moderately Active	2400 to 2800 calories
Highly Active	2800 to 3200 calories

- Write and graph an inequality for the daily number of calories that are recommended for a sedentary 15-year-old male.
- Use the information for a moderately active 15-year-old male.
  - Draw a graph on a number line for the daily calorie requirements.
  - Write a compound inequality for the graph.

It is recommended that teenagers between 14 and 18 years old consume at least 46 grams of protein per day. The table shows the amounts of protein present in various foods. Use the table for Items 3 and 4.

Food	Amount of Protein
Milk	8 grams per cup
Chicken	7 grams per ounce
Beans	16 grams per cup
Yogurt	11 grams per cup

- Darwin is 15 years old. So far today, he has consumed a total of 25 grams of protein. He plans to eat chicken at dinner.
  - Let  $c$  represent the number of ounces of chicken Darwin eats at dinner. Write an inequality to show how much chicken Darwin can eat and meet the minimum requirement for daily protein.
  - Solve your inequality from Part (a) and graph the solutions. How many ounces of chicken must Darwin eat?
- Describe at least two other foods or combinations of foods from the table that Darwin could eat at dinner and meet the minimum requirement for daily protein. Justify your answers.
- Darwin also keeps track of his target heart rate when he is exercising. The range for the number of heart beats per minute  $R$  for someone his age is  $|R - 136| \leq 20$ . Determine the solutions to the inequality and graph them on a number line.

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
<b>Math Knowledge and Thinking</b> (Items 1, 2a, 2b, 3a, 3b, 5)	<ul style="list-style-type: none"> <li>• Clear and accurate understanding of how to solve and graph inequalities, including compound and absolute value inequalities</li> </ul>	<ul style="list-style-type: none"> <li>• Largely correct understanding of how to solve and graph inequalities, including compound and absolute value inequalities</li> </ul>	<ul style="list-style-type: none"> <li>• Partial understanding of how to solve and graph inequalities, including compound and absolute value inequalities</li> </ul>	<ul style="list-style-type: none"> <li>• Inaccurate or incomplete understanding of how to solve and graph inequalities, including compound and absolute value inequalities</li> </ul>
<b>Problem Solving</b> (Items 3b, 4, 5)	<ul style="list-style-type: none"> <li>• Appropriate and efficient strategy that results in a correct answer</li> </ul>	<ul style="list-style-type: none"> <li>• Strategy that may include unnecessary steps but results in a correct answer</li> </ul>	<ul style="list-style-type: none"> <li>• Strategy that results in some incorrect answers</li> </ul>	<ul style="list-style-type: none"> <li>• No clear strategy when solving problems</li> </ul>
<b>Mathematical Modeling / Representations</b> (Items 1, 2a, 2b, 3a, 5)	<ul style="list-style-type: none"> <li>• Clear and accurate understanding of how to write and graph inequalities, including compound and absolute value inequalities, to represent real-world data or a real-world scenario</li> </ul>	<ul style="list-style-type: none"> <li>• Largely correct understanding of how to write and graph inequalities, including compound and absolute value inequalities, to represent real-world data or a real-world scenario</li> </ul>	<ul style="list-style-type: none"> <li>• Partial understanding of how to write and graph inequalities, including compound and absolute value inequalities, to represent real-world data or a real-world scenario</li> </ul>	<ul style="list-style-type: none"> <li>• Little or no understanding of how to write and graph inequalities, including compound and absolute value inequalities, to represent real-world data or a real-world scenario</li> </ul>
<b>Reasoning and Communication</b> (Items 3b, 4)	<ul style="list-style-type: none"> <li>• Clear and accurate conclusions drawn from an inequality and a table of data</li> </ul>	<ul style="list-style-type: none"> <li>• Reasonable conclusions drawn from an inequality and a table of data</li> </ul>	<ul style="list-style-type: none"> <li>• Partially correct conclusions drawn from an inequality and a table of data</li> </ul>	<ul style="list-style-type: none"> <li>• Incomplete or inaccurate conclusions drawn from an inequality and a table of data</li> </ul>