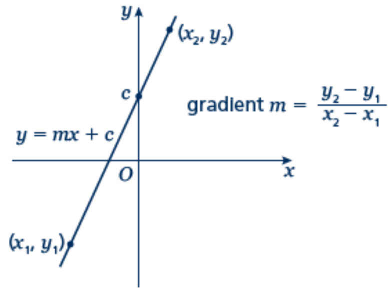
Equations of parallel lines

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- A straight line has the equation $y = mx + c$, where m is the gradient and c is the y-intercept (where $x = 0$).
- The equation of a straight line can be written in the form $ax + by + c = 0$, where a, b and c are integers.
- When given the coordinates (x_1, y_1) and (x_2, y_2) of two points on a line the gradient is calculated using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$



Example 1 A straight line has gradient $-\frac{1}{2}$ and y-intercept 3.
Write the equation of the line in the form $ax + by + c = 0$.

$m = -\frac{1}{2}$ and $c = 3$ So $y = -\frac{1}{2}x + 3$ $\frac{1}{2}x + y - 3 = 0$ $x + 2y - 6 = 0$	**1** A straight line has equation $y = mx + c$. Substitute the gradient and y-intercept given in the question into this equation. **2** Rearrange the equation so all the terms are on one side and 0 is on the other side. **3** Multiply both sides by 2 to eliminate the denominator.

Example 2 Find the gradient and the y-intercept of the line with the equation $3y - 2x + 4 = 0$.

$3y - 2x + 4 = 0$ $3y = 2x - 4$ $y = \frac{2}{3}x - \frac{4}{3}$ Gradient $= m = \frac{2}{3}$ y-intercept $= c = -\frac{4}{3}$	**1** Make y the subject of the equation. **2** Divide all the terms by three to get the equation in the form $y = \ldots$ **3** In the form $y = mx + c$, the gradient is m and the y-intercept is c.

Example 3 Find the equation of the line which passes through the point (5, 13) and has gradient 3.

$m = 3$ $y = 3x + c$	**1** Substitute the gradient given in the question into the equation of a straight line $y = mx + c$.
$13 = 3 \times 5 + c$	**2** Substitute the coordinates $x = 5$ and $y = 13$ into the equation.
$13 = 15 + c$ $c = -2$	**3** Simplify and solve the equation.
$y = 3x - 2$	**4** Substitute $c = -2$ into the equation $y = 3x + c$

Example 4 Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

$x_1 = 2$, $x_2 = 8$, $y_1 = 4$ and $y_2 = 7$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-4}{8-2} = \frac{3}{6} = \frac{1}{2}$	**1** Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line.
$y = \frac{1}{2}x + c$	**2** Substitute the gradient into the equation of a straight line $y = mx + c$.
$4 = \frac{1}{2} \times 2 + c$	**3** Substitute the coordinates of either point into the equation.
$c = 3$	**4** Simplify and solve the equation.
$y = \frac{1}{2}x + 3$	**5** Substitute $c = 3$ into the equation $y = \frac{1}{2}x + c$

Practice question

1

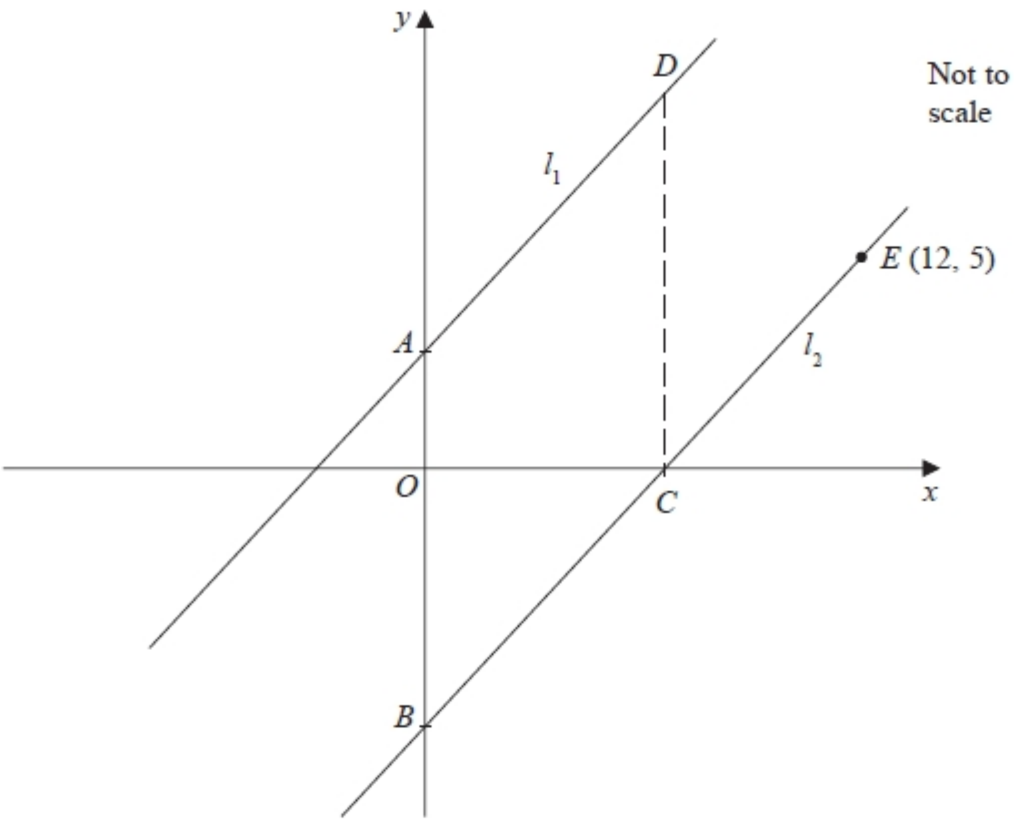


Figure 2

Figure 2 shows the straight line l_1 with equation $4y = 5x + 12$

(a) State the gradient of l_1

The line l_2 is parallel to l_1 and passes through the point E (12, 5), as shown in Figure 2.

(b) Find the equation of l_2. Write your answer in the form $y = mx + c$, where m and c are constants to be determined.

The line l_2 cuts the x-axis at the point C and the y-axis at the point B.

(c) Find the coordinates of

(i) the point B,
(ii) the point C.

The line l_1 cuts the y-axis at the point A.

The point D lies on l_1 such that $ABCD$ is a parallelogram, as shown in Figure 2.

(d) Find the area of $ABCD$.

2 The straight line $\mathbf{L}_1$ passes through the points A and B with coordinates $(-4, 4)$ and $(2, 1)$, respectively.

(a) Find the equation of $\mathbf{L}_1$ in the form $ax + by + c = 0$

The line $\mathbf{L}_2$ is parallel to the line $\mathbf{L}_1$ and passes through the point C with coordinates $(-8, 3)$.

(b) Find the equation of $\mathbf{L}_2$ in the form $ax + by + c = 0$

The line $\mathbf{L}_3$ is perpendicular to the line $\mathbf{L}_1$ and passes through the origin.

(c) Find an equation of $\mathbf{L}_3$

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Answer

1 (a) $\frac{5}{4}$

(b) $y = \frac{5}{4}x + c$

$y = \frac{5}{4}x - 10$

(c) $B = 0, -10$

$C = 8, 0$

2 (a) $x + 2y - 4 = 0$ (b) $x + 2y + 2 = 0$ (c) $y = 2x$