## Experiment 1

## Equipotential and Electric Field Mapping

### 1.1 Objectives

1. Determine the lines of constant electric potential for two simple configurations of oppositely charged conductors.
2. Determine the electric field from lines of constant electric potential.
3. Set up an elementary circuit.
4. Measure the voltage in a circuit with a multimeter.

### 1.2 Introduction

We are surrounded by electric fields in our daily life. Electricity plays an essential role in the global economy but we cannot sense small amounts directly and sensing large amounts of it is a shocking experience (literally). For small amounts of electric fields, we use tools to measure them. One tool is a multimeter, which can be set to measure many different aspects of electricity.

### 1.3 Key Concepts

In case you don't remember your Physics II lecture material, you'll need to refer to the chapters in your textbook on Electrostatics and Electromagnetic Fields. Alternatively, you can find a summary on-line at HyperPhysics ${ }^{1}$. Search for the keywords: Coulomb's Law, Electric Field, Voltage, Work

Key concepts can be a part of a quiz and you'll really need to know them in order to succeed in this lab. Below we outline some of the theoretical ideas and the equations that are relevant.

To explore the relationship between electric charges, the electric fields they produce, and equipotential lines, you can play with a simulation from the University of Colorado. ${ }^{2}$

### 1.4 Theory

## Force between 2 point charges

To understand where the concept of an electric field comes from, let's start with one of the simplest electric systems: two point charges, $q_{1}$ and $q_{2}$, separated by a distance $r$. The force that they exert on each other, $F_{E}$, is given by Coulomb's Law,

$$
\begin{equation*}
F_{E}=\frac{1}{4 \pi \epsilon_{0}} \frac{q_{1} q_{2}}{r^{2}} . \tag{1.1}
\end{equation*}
$$

If the charges are electrons, then $q_{1}=q_{2}=-e$, where $e$ is the elementary charge. The force on one of the electrons is thus

$$
\begin{equation*}
F_{E}=\frac{1}{4 \pi \epsilon_{0}} \frac{e^{2}}{r^{2}}, \tag{1.2}
\end{equation*}
$$

Like charges (meaning they have the same sign) repel and opposite charges attract, so these electrons will repel each other. If we want to hold these two electrons at that constant distance r , we must balance the repelling forces by exerting an equal force inward, a bit like a spring.

[^0]
## Electric potential

If we push them together, we must act against the force of these charges. Let's figure out how much energy we have to expend (how much work we must do) to get them closer to one another. According to the work-energy theorem, if one component in a closed system pushes or pulls on other components, those other components gain or lose energy. In this case, the energy of our pushing is "stored" as potential energy in the 2-electron system. Thus, the force between objects is related to the potential energy they have. Notice that if we push the electrons to be half as distant from one another ( $\left.r^{\prime}=\frac{1}{2} r\right)$, then the force to keep them there is 4 times as much.

To find the potential energy from knowing the force, recall that

$$
\begin{equation*}
W=F \Delta s \tag{1.3}
\end{equation*}
$$

That is, work is equal to the force $F$ exerted times the distance $\Delta s$ over which that force was exerted. This formula is pretty easy when the force is constant, like if you were pushing a sled at a constant velocity across a uniform surface (with constant friction). The force that you exert at any location is the same. On the other hand, Eq. 1.1 says that the force between two charges changes with distance. That makes the work equation, Eq. 1.3, very difficult to use without applying math that goes beyond the scope of this course. ${ }^{3}$

If we do the fancy math involved, we find that the potential energy $U$ of a charge $q_{1}$ due to a charge $q_{2}$ that is $r$ away is

$$
\begin{equation*}
U=\frac{1}{4 \pi \epsilon_{0}} \frac{q_{1} q_{2}}{r} \tag{1.4}
\end{equation*}
$$

Now we have the electric potential energy, the energy that an object has due to its position relative to a system of electric charges.

You may think we're already done - we set out to find the electric potential, right? Well, it turns out that "electric potential energy" is not the same as "electric potential". Physicists want to know what the potential field is of a charge $q_{1}$ without worrying about the amount of charge on $q_{2}$. So they define the electric potential $V$, to be the electric potential energy divided by $q_{2}$, the charge that is feeling the force:

[^1]\[

$$
\begin{align*}
V & =\frac{U}{q_{2}} \\
V & =\frac{1}{4 \pi \epsilon_{0}} \frac{q_{1} q_{2}}{r} \frac{1}{q_{2}}  \tag{1.5}\\
V & =\frac{1}{4 \pi \epsilon_{0}} \frac{q_{1}}{r}
\end{align*}
$$
\]

Notice that the electric potential of a charge is independent of the charge used to measure it. So now if someone asks you what the electric potential is of a charge, you can answer them without needing to know what they're measuring it with. In other words, instead of saying "the electric potential energy of $q_{2}$ due to $q_{1}$ ", you can say "the electric potential due to $q_{1}$ ". Note that the electric potential is also sometimes called "voltage". If you make a map of electric potential values at different points, you can then draw equipotential lines that connect all the points that have the same potential (or voltage).

## Electric field

Now we want to place a charge in an electric potential field to find out how much it is pushed and in what direction. To do this, visualize the potential field as a hill. The greater the potential, the higher the elevation. If we put a ball on the side of the hill it will roll downward, from a higher to lower potential. Similarly, a positive charge will act the same way. If you walk along the side of the hill, without going up or down the hill, it is analogous to walking along an equipotential line. The steeper the hill, or the greater the change in potential over a certain displacement, the faster the ball (charge) will accelerate. This change in electric potential over a certain displacement is called the electric field. In the form of a formula, the magnitude of the electric field $E$ is given by

$$
\begin{equation*}
|E|=\frac{|\Delta V|}{\Delta s} \tag{1.6}
\end{equation*}
$$

where $\Delta V$ is the change in the potential (or voltage) and $\Delta s$ is the distance.
Finally, to visualize the direction that the charge is pushed, we need only to draw a line that follows the "hill" from high to low potential. That line is called an electric field line. Notice that since the equipotential
lines run along the side of the hill, and the electric field lines run down the hill, they are always perpendicular to one another. Note that both lines can and often do curve; at each point in space, the electric field line at that point is at a right angle to the equipotential line at that point. Electric field lines never cross and they point in the direction going from a higher voltage to a lower voltage.

### 1.5 In today's lab

In this experiment we will measure the electric potential in the vicinity of two different charge distributions and use that measurement to find the electric field.

Each charge distribution will consist of two metal objects mounted on conductive paper. We consider metals to be ideal conductors (same electric potential everywhere on the metal). If we wave the potential probe in the air between the conductors, we would not read any potential. This is because air is a very effective insulator and so electrons don't flow through it. On the other hand, the conductive paper is partly conductive, allowing us to travel partway down the "potential hill" or "voltage escalator". Thus, the electric potential depends on the location of the point on the paper.

### 1.6 Equipment

- DC power supply (Fig. 1.1)
- Digital multimeter (Fig. 1.2)
- Conducting paper with charge distributions (Fig. 1.3)
- 2 grid sheets (graph paper) with 1 cm squares
- 3 banana-to-banana wires ("banana" is the term for a kind of plug on the end of a wire)
- 1 banana-to-probe wire (the probe is a plastic handle with a sharp metal point sticking out of it)


Figure 1.1: DC power supply


Figure 1.2: Digital multimeter


Figure 1.3: Conductors mounted on semi-conductive paper.

### 1.7 Procedure

## Safety Tips

- When plugging or unplugging wires, first turn off all electronics that are connected or will become connected to the circuit.
- When using the potential probe, do not press hard enough on the paper to make a dent or hole.


## Two parallel conducting plates

The first distribution will consist of two parallel bars (or plates), seen in Fig. $1.3(\mathrm{a})$. This can be thought of as a 2D model of a parallel plate capacitor. A schematic of the parallel plate experiment is shown in Fig. 1.4. In this configuration, one of the two plates will be electrically connected to the positive terminal of the power supply and held at a constant positive electric potential. The other plate will be electrically connected to the negative terminal, or the "ground", of the power supply. All electric potentials are measured with respect to this electric potential (i.e. $V_{\text {ground }}=0 \mathrm{~V}$ ). The ground terminal of the power supply will also be connected to the ground, or COM port of the multimeter. The probe used to measure the electric potential is connected to the "V" port of the multimeter.

## 1. Setup:

a) Sketch the two conducting plates on a grid sheet provided by your lab instructor. Each lab partner will turn in their own copy of the grid sheet.
b) Use a banana-to-banana wire to connect one of the parallel plates to the positive (red) terminal of the power supply.
c) Use a second banana-to-banana wire to connect the second plate to the negative (black) terminal of the power supply.
d) Connect the COM port of the digital multimeter to the negative terminal of the power supply using a third banana-to-banana wire.


Figure 1.4: Schematic of experimental setup for parallel plates.
e) Connect the electric potential probe to the "V" port of the digital multimeter.
Your setup should now look something like Fig. 1.5.
f) Adjust the power supply to an electric potential of 10 V .
g) Set the digital multimeter to measure the direct current (DC) electric potential. The dial should point to the V that has 2 straight lines (one solid and one dashed) over it so you are measuring the DC voltage. Press the "RANGE" button on the multimeter until the meter reads the electric potential to the nearest tenth of a volt.
2. Measure electric potential. Use the electric potential probe to find at least six points having the same electric potential in the region between the plates (region A in Fig. 1.6) and record these points on your grid sheet. Then, find four points extending beyond the region of the plates having that same electric potential (two points in each region B shown in Fig. 1.6).


Figure 1.5: Photo of completed setup for the parallel plates.


Figure 1.6: Regions near the parallel plates. Note that region A is between the two plates and region $B$ is beyond the ends of the plates.
3. Draw equipotential line. Use a pen to draw a smooth curve/line that intersects all of the points found in Step 2. Make sure to label the line with its potential value.
4. Repeat Steps 2 and 3 for at least two additional electric potentials. (Your final graph should have at least 3 lines of electric potential drawn on it.)
5. Measure the distance $s_{\text {plates }}$ between the parallel plates and assign a reasonable uncertainty to this measurement. Use this measured distance and the 10.0 V electric potential difference between the plates to calculate the electric field between the plates and its uncertainty (Question 1).
6. Measure the distance $s_{\text {lines }}$ between two of your lines of equipotential; calculate the difference in electric potential between these two lines; calculate the electric field and its uncertainty using these measured values (Question 2).
7. Draw electric field lines.
a) Draw and label at least eight electric field lines. You should include at least 2 field lines near the edges of the parallel plates in region B.
b) Indicate the direction of the electric field with an arrow on each of your electric field lines.
8. Answer questions 1-4 which can be found at the end of this write-up.

## Ring and point

The second charge distribution will be a point source at the center of a ring. A schematic of this arrangement is shown in Fig. 1.7.

## 1. Setup:

a) Sketch the ring and point on another grid sheet provided by your lab instructor. To help draw the ring, there is a spare ring in the lab that you can trace.


Figure 1.7: Schematic of experimental setup for point charge in ring.
b) Use a banana-to-banana wire to connect the point at the center of the ring to the positive (red) terminal of the power supply.
c) Use a second banana-to-banana wire to connect the ring to the negative (black) terminal of the power supply.
d) Connect the COM port of the digital multimeter to the negative terminal of the power supply using a third banana-to-banana wire.
e) Connect the electric potential probe to the "V" port of the digital multimeter.
f) Adjust the power supply to an electric potential of 10 V .
g) Set the digital multimeter to measure DC electric potential (voltage) and press the "RANGE" button on the multimeter until the meter reads electric potential to the nearest tenth of a volt.

## 1. Equipotential and Electric Field Mapping

2. Measure electric potential. Use the electric potential probe to find at least eight points having the same electric potential inside the ring. Record these points on your grid paper.
3. Draw equipotential line. Use a pen to draw a smooth curve/line that intersects all of the points found in Step 2.
4. Repeat Steps 2 and 3 for at least two additional electric potentials. (Your final graph should have at least 3 lines of electric potential drawn on it.)
5. Draw and label the electric field lines (at least eight of them). Indicate the direction of the electric field with an arrow on each of your electric field lines.
6. Answer questions 5-6 which can be found at the end of this write-up.

### 1.8 Questions

1. Use the electric potential difference between the two parallel plates and Eq. 1.6 to calculate the magnitude of the electric field between the plates. In addition, calculate the uncertainty in the electric field. See Appendix A to reference how to propagate uncertainty through a division of two values, or you can use the formula given here: $\delta E=E\left(\left|\frac{\delta V}{V}\right|+\left|\frac{\delta s}{s}\right|\right)$. Show all work and make sure to include units!

Electric potential between the plates: $V \pm \delta V=$

Distance separating the plates: $s_{\text {plates }} \pm \delta s_{\text {plates }}=$

Your calculated magnitude of the electric field: $E \pm \delta E=$
2. Use the electric potential difference between two of your equipotential lines (between the parallel plates and not near the edge of the plates), the distance separating these two lines and Eq. 1.6 to calculate the magnitude of the electric field between the plates. In addition, calculate the uncertainty in the electric field. Do not use the plates themselves as the equipotentials in this question and show your work.

Electric potential between the equipotentials: $V \pm \delta V=$

Distance separating the equipotentials: $s_{\text {lines }} \pm \delta s_{\text {lines }}=$

Your calculated magnitude of the electric field: $E \pm \delta E=$

## 1. Equipotential and Electric Field Mapping

3. Compare your results for the electric field obtained in Questions 1 and 2 using uncertainties. (If you have forgotten how to compare two values using their uncertainties, see Appendix A ). Are your results consistent? If they are not consistent, give an explanation for why that might be.
4. Given your answer to Question 3, how does the electric field inside the plates change with position (look at region A, not region B, of Fig. 1.6)? Compare your results to what you would expect in the ideal case of parallel-plates (i.e. a capacitor).
1.8. Questions
5. Is the metal ring itself an equipotential? This should be verified by measurement. What did you do to make this measurement?

## 1. Equipotential and Electric Field Mapping

6. Now think about the region of space outside the circular ring in the second part of the procedure.
a) Predict the magnitude of the electric field in the region outside the circular ring. Justify this prediction.
b) Test your prediction experimentally and describe what you did. Was your prediction accurate?

[^0]:    ${ }^{1}$ http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html
    ${ }^{2}$ http://phet.colorado.edu/sims/charges-and-fields/charges-and-fields_ en.html

[^1]:    ${ }^{3}$ If you know how to do integration, the general work equation is $W=\int_{a}^{b} F \mathrm{~d} s$, where $\mathrm{d} s$ is the differential length. You may find an extra factor of 2 compared to Eq. 1.4, because this energy is divided between the two charges.

