

**Another Look at Equity and Enterprise Valuation
Based on Multiples**

Mingcheng Deng
University of Minnesota

Peter Easton
University of Notre Dame

Julian Yeo
Columbia Business School

April 2009

1. Introduction

Market price multiples (levered and/or unlevered) are commonly cited in the popular press as summary statistics for comparison of the market valuation of fundamental financial variables among a set of comparable firms. In practice, these multiples are widely used as a preliminary screening device to rank stocks. In cases where firm-specific detailed projections are difficult (for example, privately-held companies or when the proposed entity has yet to be created), these multiples serve as a substitute for comprehensive valuation. The widespread use of price-multiples stems, at least partially, from their ease of computation. However, these price-multiples often do not yield sensible estimates; in particular negative fundamental/financial measures are meaningless. Further, valuations of the same firm based on different price-multiples are often difficult to reconcile. Reliance on one measure to the exclusion of another likely ignores important value-relevant information.

Liu, Nissim and Thomas (2001) – hereafter LNT -- provides a comprehensive analysis of the absolute and relative performance of several multiples in explaining stock prices. Their focus is on a set of forward-looking multiples; hence they examine a subsample of stocks: (1) that have analyst following and are included in the I/B/E/S data base; (2) for which all (earnings-based, cash flow-based, book value-based, and sales-based) multiples are positive; (3) that are in an I/B/E/S industry sector which has at least four other firms; (4) with a market price greater than \$2 per share; and (5) that have at least 30 monthly return observations (not necessarily continuous) over a 60 month period ending at the valuation date. They find that forward earnings multiples out-perform current earnings multiples which, in turn, out-perform multiples based on cash flow, book

value, and sales. In addition, they find that the relative performance of these multiples is consistent across industries.

Since LNT requires analysts' earnings and growth forecasts and excludes firm-year observations with negative values for any value driver, their results are only representative of larger, profitable firms with analyst following. It is common to find firms reporting negative earnings, negative cash flows, and in some cases negative book values. The exclusion of firms with negative financial metrics implicitly assumes that the subject firms whose values are being examined only have non-negative distributions of these key/fundamental financial attributes. In other words, when a profitable firm is only compared with other profitable firms, the analysis, at least implicitly, assumes that this firm has an earnings distribution that is not within the population of firms that will make losses at some point.

We observe the common reference to book values and sales multiples for firms with negative financial metrics (for example, internet stocks, start-ups and growth firms). As Damodaran (2002) points out, sales and/or book value may do better when earnings are negative. In one sense, the sales multiple is obviously better inasmuch as it, at least, gives a positive valuation (though this does not necessarily mean a lower valuation error).

Since there is no empirical evidence documenting the usefulness of multiples for a sample of firms that are more representative of the general population of firms (firms with losses, smaller start up firms, etc.), we begin by examining the usefulness of multiples in enterprise valuation and in equity valuation. Our methodology for comparison of the various multiples follows LNT quite closely but our study differs in several important ways. First, we focus on multiples of current financial variables; we do

not consider forward earnings-based multiples. Removing the restriction that the firm is followed by I/B/E/S allows us to analyze a broader cross-section of stocks. Second, we include firms with negative fundamentals, including negative EBITDA, negative earnings and negative book values. Third, we use a different industry classification: where possible we group on 4-digit SIC code and, where this is not possible, we group on 3-digit SIC code. This industry classification allows us to analyze the usefulness of multiples at a more micro industry level. Fourth, we focus our analyses on the absolute mean and median valuations error instead of the inter-quartile range of errors as in LNT; the use of the inter-quartile range is not appropriate for multiples with skewed distributions (sales multiples, for example, are always non-negative resulting in a distribution that is less dispersed, clustered around zero, and right-skewed).

Although price multiples are often cited as a basis for valuation, there is rarely a reconciliation of conflicting valuations based on multiples of various firm fundamentals. A common criticism of the use of price multiples is the inability to reconcile different multiples. In an attempt to incorporate information from different multiples, LNT also examines short-cut intrinsic value measures incorporating book value and forward earnings based on the residual income model. They find that their intrinsic value measures perform considerably worse than forward earnings. Even though these measures contain more information than forward earnings, they attribute the worse performance to potential measurement error associated with the terminal value estimates required for the intrinsic value calculation.

Beatty, Riffe, and Thompson (1999) uses the price-scaled regressions to compare different linear combinations of value drivers. LNT also combines two or more value

drivers based on Beatty, Riffe, and Thompson (1999) and calculate the mean and median pricing errors. Little or no improvements are observed. Given that the extant literature is silent on how harmonic means are calculated when different multiples are combined, we extend the method developed in LNT to combine multiples and consider the change in the valuation error when we consider a combination of multiples rather than a single multiple.

LNT acknowledge that their results may not be generalizable to a broader cross-section of firms; we provide evidence that a number of their conclusions do not apply. We find that sales are not the worst valuation fundamental. In fact, we find that the mean absolute valuation errors for multiples based on sales are the lowest for both enterprise and market value multiples. When we compare book value and earnings as valuation fundamentals, we do not find earnings-based multiples outperform book value-based multiples. Overall, we find that book values (net operating assets as the fundamental for enterprise valuation and book value of equity as the fundamental for market valuation) outperform all other fundamentals. We attribute the difference between our results and those reported in LNT the fact that we do not restrict our samples to non-negative observations of the accounting fundamentals and to firms followed by I/B/E/S. Also, our focus is on current financial measures rather than forward looking measures. Our findings are consistent with conventional wisdom that negative value drivers do not yield sensible valuation estimates. When compared to fundamentals that generally do not have negative realizations (sales, most book value measures), financial fundamentals with negative values, on average, do not outperform those with non-negative financial fundamentals.

We show vast improvement in valuation errors when an average omitted variable (intercept) is incorporated in the calculation of our harmonic means. This, in turn, implies that the traditional method (without adjustment) of applying price multiples to obtain value estimates is inadequate. Our results show that, when combining fundamentals from different financial statements, pricing errors are significantly improved. More specifically, we observe the largest improvement in valuation errors when balance sheet fundamentals (net operating assets and book value of equity) are combined with fundamentals from the income statement (EBITDA).

Our results have implications for the use of multiples in investment decisions. We provide insights into the absolute and relative performance of different price multiples with a population of firms that are more representative of those in COMPUSTAT. In addition, we show: (1) that the shortcomings of negative value drivers can be mitigated by incorporating an average omitted variable (intercept) in the calculation of harmonic means; and (2) how different price multiples can be reconciled to provide incremental information by combining different multiples from different financial statements.

2. **Method**

2.1 *The Fundamentals*

Many valuation texts focus on enterprise value; that is, the value of the operations of the firm to its owners – debt and equity holders. This permits a comparison of firm values that are not affected by capital structure. For litigation purposes, enterprise values are often computed in order to compare firms with different capital structures. On the

other hand, from an investor's standpoint, the focus may be on the value of equity, consistent with the observation that analysts and the popular financial press often associates financial fundamentals with the market value of the equity. We consider the use of multiples to value both the operations of the firm (the enterprise value) and the value of stock-holder's equity in the firm.

We examine a set of fundamentals (sometimes referred to as value drivers) that are commonly used in practice. The fundamentals we consider as the basis for enterprise valuation are: (1) the top-line of the income statement – sales revenue; (2) free cash flow to debt and equity holders; (3) earnings before interest, taxes, depreciation and amortization, EBITDA;¹ and (4) net operating assets, NOA. The fundamentals we consider for equity valuation are: (1) sales revenue; (2) earnings before extraordinary items; (3) EBITDA; and (4) book value of equity.

2.2 *Valuation using Price Multiples*

LNT shows that the performance of price multiples improves when these multiples are calculated using the harmonic mean rather than the simple mean or median. Like LNT, we consider multiples calculated as the harmonic mean. We calculate valuation errors for the subject firm (always calculated out-of-sample) as the difference between the actual price and the predicted price divided by the actual price.

LNT derives a method that relaxes the assumption that prices are directly proportional to the valuation fundamental. We begin with an outline of the LNT method and then we extend it to permit a valuation based on a combination of two fundamentals. This extension is important in the context of our study given that we are interested in the

¹ EBITDA is often used as a rough approximation for cash flow from operating activities.

possibility that valuation errors may be improved by combining fundamentals in the valuation.

2.2.1 The LNT method

LNT begins with the assumption that the price of firm i in year t is proportional to a fundamental x_{it} and that there is a possibility of a non-zero average price when the fundamental is equal to zero:

$$p_{it} = \alpha_t + \beta_t x_{it} + \varepsilon_{it} \quad (1)$$

where β_t is the multiple on the fundamental. Following Beatty, Riffe, and Thompson (1999), Easton and Sommers (2002), and LNT we divide both sides of equation (1) by price:

$$1 = \alpha_t \frac{1}{p_{it}} + \beta_t \frac{x_{it}}{p_{it}} + \frac{\varepsilon_{it}}{p_{it}} \quad (2)$$

LNT derive the formula for α_t and β_t such that the variance of $\frac{\varepsilon_{it}}{p_{it}}$ is minimized and the expected value of $\frac{\varepsilon_{it}}{p_{it}}$ is zero. The derivation is as follows:

$$\text{Min } \text{var}\left(1 - \alpha_t \frac{1}{p_{it}} - \beta_t \frac{x_{it}}{p_{it}}\right)$$

subject to:

$$\sum \left(1 - \alpha_t \frac{1}{p_{it}} - \beta_t \frac{x_{it}}{p_{it}}\right) = 0$$

Note that:

$$\text{var}\left(1 - \alpha_t \frac{1}{p_{it}} - \beta_t \frac{x_{it}}{p_{it}}\right) = \sum \left(1 - \alpha_t \frac{1}{p_{it}} - \beta_t \frac{x_{it}}{p_{it}}\right)^2$$

Because:

$$E\left(1 - \alpha_t \frac{1}{P_{it}} - \beta_t \frac{x_{it}}{P_{it}}\right) = 0.$$

For ease of exposition, let $\frac{1}{P_{it}} = m_{it}$ and $\frac{x_{it}}{P_{it}} = n_{it}$, so that the minimization problem is

$$\text{Min } \sum (1 - \alpha m - \beta n)^2$$

subject to:

$$\sum (1 - \alpha m - \beta n) = 0.$$

This problem can be solved by forming the Lagrangian:

$$L = \sum (1 - \alpha m - \beta n)^2 - \lambda \sum (1 - \alpha m - \beta n)$$

Taking derivatives respect to α, β , and λ yields:

$$\frac{\partial L}{\partial \alpha} = 2 \sum (1 - \alpha m - \beta n) \cdot m - \lambda \sum m = 0 \quad (3)$$

$$= \sum m - \alpha \sum (m)^2 - \beta \sum mn - \frac{\lambda}{2} \sum m = 0$$

$$\frac{\partial L}{\partial \beta} = 2 \sum (1 - \alpha m - \beta n) \cdot n - \lambda \sum n = 0 \quad (4)$$

$$= \sum n - \alpha \sum mn - \beta \sum (n)^2 - \frac{\lambda}{2} \sum n = 0$$

$$\frac{\partial L}{\partial \lambda} = \sum (1 - \alpha m - \beta n) = 0 \quad (5)$$

$$= \sum -\alpha \sum m - \beta \sum n = 0$$

Assuming that there are N samples in the population (that is $\sum = N$) and solving for three equations simultaneously, we have:

$$\alpha_t = \frac{N(\sum m \sum n^2 - \sum n \sum mn)}{(\sum n)^2 \sum m^2 + (\sum m)^2 \sum n^2 - 2 \sum m \sum n \sum mn} \quad (6)$$

$$\beta_t = \frac{N(\sum n \sum m^2 - \sum m \sum mn)}{(\sum n)^2 \sum m^2 + (\sum m)^2 \sum n^2 - 2 \sum m \sum n \sum mn} \quad (7)$$

Given:

$$E[x] = \frac{\sum x}{n}$$

$$var = E[x^2] - (E[x])^2$$

$$cov = E[x \cdot y] - E[x] \cdot E[y]$$

we have:

$$\beta_t = \frac{E[n]var(m) - cov(m, n)E[m]}{E[m]^2 var(n) + E[n]^2 var(m) - 2E[m]E[n]cov(m, n)}$$

Substituting $\frac{1}{p_{it}} = m$ and $\frac{x_{it}}{p_{it}} = n$ leads to:

$$\beta_t = \frac{E[\frac{x_{it}}{p_{it}}]var(\frac{1}{p_{it}}) - cov(\frac{1}{p_{it}}, \frac{x_{it}}{p_{it}})E[\frac{1}{p_{it}}]}{E[\frac{1}{p_{it}}]^2 var(\frac{x_{it}}{p_{it}}) + E[\frac{x_{it}}{p_{it}}]^2 var(\frac{1}{p_{it}}) - 2E[\frac{1}{p_{it}}]E[\frac{x_{it}}{p_{it}}]cov(\frac{1}{p_{it}}, \frac{x_{it}}{p_{it}})}$$

2.2.2 Extending LNT to two multiples without an intercept

Let the price of firm i in year t be proportional to two fundamentals x_{it} and y_{it} and permit the possibility of a non-zero average price when the fundamental is equal to

zero:

$$p_{it} = \alpha_t x_{it} + \beta_t y_{it} + \varepsilon_{it} \quad (8)$$

where α_t and β_t are the multiples on the fundamentals. Following LNT, we divide both sides of equation (8) by price:

$$1 = \alpha_t \frac{x_{it}}{p_{it}} + \beta_t \frac{y_{it}}{p_{it}} + \frac{\varepsilon_{it}}{p_{it}} \quad (9)$$

As in LNT, we minimize the variance of $\frac{\varepsilon_{it}}{p_{it}}$, subject to the restriction that the expected

value of $\frac{\varepsilon_{it}}{p_{it}}$ is zero (subscript understood):

$$\text{Min} \sum (1 - \alpha \frac{x_{it}}{p_{it}} - \beta \frac{y_{it}}{p_{it}})^2$$

subject to:

$$\sum (1 - \alpha \frac{x_{it}}{p_{it}} - \beta \frac{y_{it}}{p_{it}}) = 0.$$

The solutions have the same form as (6) and (7), that is:

$$\beta_t = \frac{E[n] \text{var}(m) - \text{cov}(m, n) E[m]}{E[m]^2 \text{var}(n) + E[n]^2 \text{var}(m) - 2E[m]E[n] \text{cov}(m, n)}.$$

where $m = \frac{x_{it}}{p_{it}}$ and $n = \frac{y_{it}}{p_{it}}$. Substituting into (7), we have:

$$\beta_t = \frac{E[\frac{y_{it}}{p_{it}}] \text{var}(\frac{x_{it}}{p_{it}}) - \text{cov}(\frac{x_{it}}{p_{it}}, \frac{y_{it}}{p_{it}}) E[\frac{x_{it}}{p_{it}}]}{E[\frac{x_{it}}{p_{it}}]^2 \text{var}(\frac{y_{it}}{p_{it}}) + E[\frac{y_{it}}{p_{it}}]^2 \text{var}(\frac{x_{it}}{p_{it}}) - 2E[\frac{x_{it}}{p_{it}}]E[\frac{y_{it}}{p_{it}}] \text{cov}(\frac{x_{it}}{p_{it}}, \frac{y_{it}}{p_{it}})}.$$

2.2.3 Extending LNT to two multiples with an intercept

Assume that the price of firm i in year t is proportional to two fundamentals x_{it} and y_{it} and that there is a possibility of a non-zero average price when the fundamentals are equal to zero:

$$p_{it} = \alpha_t + \beta_t x_{it} + \rho_t y_{it} + \varepsilon_{it} \quad (10)$$

Following LNT, we divide both sides of equation (10) by price:

$$1 = \alpha_t \frac{1}{p_{it}} + \beta_t \frac{x_{it}}{p_{it}} + \gamma_t \frac{y_{it}}{p_{it}} + \frac{\varepsilon_{it}}{p_{it}} \quad (11)$$

so that the minimization problem (with subscription understood) is:

$$\text{Min } \sum (1 - \alpha m - \beta n - \gamma q)^2$$

subject to:

$$\sum (1 - \alpha m - \beta n - \gamma q) = 0.$$

where $m = \frac{1}{p_{it}}$, $n = \frac{x_{it}}{p_{it}}$, and $q = \frac{y_{it}}{p_{it}}$. This problem can be solved by forming the

Lagrangian:

$$L = \sum (1 - \alpha m - \beta n - \gamma q)^2 - \lambda \sum (1 - \alpha m - \beta n - \gamma q)$$

Taking derivatives respect to α, β, γ and λ yields:

$$\frac{\partial L}{\partial \alpha} = 2 \sum (1 - \alpha m - \beta n - \gamma q) \cdot m - \lambda \sum m = 0 \quad (12)$$

$$= \sum m - \alpha \sum (m)^2 - \beta \sum mn - \gamma \sum mq - \frac{\lambda}{2} \sum m = 0$$

$$\frac{\partial L}{\partial \beta} = 2 \sum (1 - \alpha m - \beta n - \gamma q) \cdot n - \lambda \sum n = 0 \quad (13)$$

$$= \sum n - \alpha \sum mn - \beta \sum (n)^2 - \gamma \sum nq - \frac{\lambda}{2} \sum n = 0$$

$$\frac{\partial L}{\partial \gamma} = 2 \sum (1 - \alpha m - \beta n - \gamma q) \cdot q - \lambda \sum q = 0 \quad (14)$$

$$= \sum q - \alpha \sum mq - \beta \sum nq - \gamma \sum q^2 - \frac{\lambda}{2} \sum q = 0$$

$$\frac{\partial L}{\partial \lambda} = \sum (1 - \alpha m - \beta n - \gamma q) = 0 \quad (15)$$

$$= \sum -\alpha \sum m - \beta \sum n - \gamma \sum q = 0$$

Solving the four equations simultaneously, we have:

$$\beta_t = \frac{1}{\Delta} \left[\mu_n \sigma_{mq}^2 - \mu_m \sigma_{mq} \sigma_{nq} + \sigma_{mn} (-\mu_m \sigma_q + \mu_q \sigma_{mq}) + \sigma_m (\mu_q \sigma_{nq} - \mu_n \sigma_q) \right], \quad (16)$$

$$\gamma_t = \frac{1}{\Delta} \left[\mu_q \sigma_{mn}^2 - \sigma_{mn} (\mu_n \sigma_{mq} + \mu_m \sigma_{nq}) + \mu_n \sigma_m \sigma_{nq} + \sigma_n (\mu_m \sigma_{mq} - \mu_q \sigma_m) \right], \quad (17)$$

$$\alpha_t = \frac{1}{\mu_m} \left[1 - \beta \mu_n - \gamma \mu_q \right], \quad (18)$$

where :

$$\begin{aligned} \Delta \equiv & \mu_q^2 \sigma_{mn}^2 + \mu_n^2 \sigma_{mq}^2 + \mu_m^2 \sigma_{nq}^2 + 2\mu_n \mu_q \sigma_m \sigma_{nq} - \mu_q^2 \sigma_m \sigma_n + 2\mu_m \sigma_{mq} (\mu_q \sigma_n - \mu_n \sigma_{nq}) \\ & - \sigma_q (\mu_n^2 \sigma_m + \mu_m^2 \sigma_n) - 2\sigma_{mn} (\mu_n \mu_q \sigma_{mq} + \mu_m \mu_q \sigma_{nq} - \mu_m \mu_n \sigma_q), \end{aligned}$$

and $\mu_m = E[m]$, $\sigma_m = var(m)$, and $\sigma_{mq} = cov(m, q)$. We can apply equations (16) to (18)

to estimate the case of two multiples with an intercept.

3. Sample and Data

We collect all COMPUSTAT firm-year observations from 1963 to 2006 and prices from CRSP for all firms traded on NYSE, AMEX and NASDAQ. We require firm-year observations to have complete data for the following items: sales, EBITDA, earnings before extraordinary items, net operating assets, common stockholders' equity, free cash flow, market capitalization, and enterprise value (see the Appendix for detailed descriptions of these variables).² Lastly, we require share price three months after the fiscal year end to be greater than or equal to \$1.

² For our multiples with enterprise value as the deflator, we restrict our analysis to observations with positive enterprise value (firms with net financial assets larger than common stockholders' equity have negative enterprise values).

When formulating our groups of comparable firms, we first sort firms according to their four-digit Standard Industry Classification (SIC) code and size (market capitalization). We start by forming groups of 21 firms (including target and comparable firms) for each four-digit year industry-size matched combination.³ We are able to find 2,209 groups of firms (46,389 firm-year observations) using the four-digit SIC code. For the remaining firms for which we are unable to form groups of 21 firms, we pool observations within the same three-digit SIC code and, again, we sort firms by size. We then repeat our procedure by forming groups of 21 firms. We form an additional 1,177 groups of firms (24,717 firm-year observations) in this manner. The resulting sample includes 71,106 firm-year observations from 1963 to 2004.⁴

Within the groups of 21 firms, we have our target firm and 20 comparable firms. In practice, extreme price-multiples are excluded in computing valuation errors. In an attempt to emulate this idea, we remove the firms with the highest and lowest price multiple within the set of 20 comparable firms. Thus our analyses are based on 18 comparable firms and a final sample of 64,334 firm-year observations.⁵

³ LNT cites Kim and Ritter (1999) claiming that SIC codes frequently misclassify firms, they use the industry classification by I/B/E/S (based loosely on SIC codes with adjustments). They use the intermediate Industry classification (Sector, Industry, and Group) as Sector is too broad, and Group is too narrow to allow for the inclusion of sufficient comparable firms (at least 5). We cannot use IBES classifications like LNT as this would pose a restriction and bias on our sample size. We also consider alternative industry classification (as in Fama and French) and find similar results to those reported using 4 and 3 digit SIC.

⁴ LNT's sample before excluding any negative price multiples includes 26,613 firms.

⁵ Although not tabulated, we also repeat our analysis using group of 11 firms. The empirical results using 11 firms provide stronger support for our conclusions.

4. Results

Table 1 outlines the descriptive statistics for the various price multiples. Our sample size (71,106) is about 2.5 times larger than LNT (26,613). This is mostly due to LNT's focus on firms with forward information, hence restricting their sample to firms with I/B/E/S analyst earnings per share forecasts. Given our focus is on multiples of current financial fundamentals, we have a broader cross-section of companies. In addition, we have a longer sample period covering firm-year observations from 1963 to 2004 (in contrast to LNT's sample period of 1982 to 1999).

LNT notes that most distributions of their price multiples for firms with I/B/E/S following contain very few negative values (with the exception of free cash flows). In their sample, about 10 percent of firms reported losses.⁶ This is in stark contrast to our sample in which half of the firms reported losses; about 25 percent of the firms in our sample reported negative EBITDA, whereas less than 5 percent of LNT's sample firms had negative EBITDA.

The results of our first stage analysis are reported in Table 2. We report the distribution of absolute pricing errors for multiples based on the various fundamentals. These valuation errors are calculated without incorporating the intercept in the price-fundamental relation, which is traditionally how multiple analyses are done in practice. The valuation error for the subject firm (always calculated out-of-sample) is the difference between the actual price and the predicted price divided by the actual price. We report two measures of central tendency (mean and median) and four non-parametric distribution measures (frequency of absolute percentage error less than % percent,

⁶ LNT excludes all of these negative observations from their analyses.

frequency less than 10 percent, frequency less than 25 percent, and frequency less than 100 percent).

Since LNT eliminates all negative observations, their valuation errors are skewed to the left with a median that is greater than the mean. The skewness is particularly prominent for multiples based on sales and cash flows and less noticeable for those based on forward earnings. Rather than inferring from the median or the mean, LNT focuses on the inter-quartile range in assessing the performance of different price multiples.

The use of the inter-quartile range is not appropriate in our analysis. Since we include firms with negative fundamentals, price multiples based on fundamentals that are non-negative (such as sales) would yield smaller inter-quartile ranges for pricing errors. We examine the performance of our price multiples by focusing on the mean and median absolute valuation errors.

In Table 2, Panel A, we report the absolute valuation errors based on harmonic means of firms from the same industry. In Table 2, Panel B, we report the absolute valuation errors based on medians of firms from the same industry. The mean absolute valuation errors for the analyses based on the median are higher than for multiples based on the harmonic mean. This is consistent with LNT's findings that performance improves when multiples are computed using the harmonic mean relative to median ratio of price multiples for comparable firms. The median absolute equity valuation errors are lower for multiples based on the median than for those based on the harmonic mean whereas the median absolute enterprise valuation errors are higher for multiples based on the median than for those based on harmonic means. Given the absolute performance of

different price multiples is similar under both approaches, we proceed by tabulating only the valuation errors calculated using the harmonic means of firms from the same industry.

Examination of the mean and median valuation errors indicates the following ranking. Mean absolute valuation errors for Enterprise Value are lowest when Sales is the valuation fundamental, closely followed by NOA. EBITDA ranks third while FCF ranks last with the highest valuation errors. Mean absolute valuation errors for Market Value are lowest when Sales or Book Value are the fundamental and valuation errors are much higher when EBITDA or Net Income is the valuation fundamental. The ranking based on median absolute valuation errors is similar with the exception that Enterprise valuation errors based on NOA multiples are lower than errors based on Sales multiples and Market valuation errors are lower when Book value is the fundamental than when Sales is the fundamental valuation variable.

Our results are in sharp contrast to those reported in LNT in the following ways. First, valuation errors based on sales are not the largest when compared with errors based on other fundamentals. Second, when comparing book value and earnings, we do not find that earnings metrics outperform book value metrics. This is due to the fact that only a small proportion of the observations have negative NOA and book value, while a significant number of our sample firms report losses and negative EBITDA. This result may, in part, may also be due to earnings being more volatile from one period to another. Third, we find that, for market value multiples, EBITDA has lower valuation errors than Net Income. Again, we attribute this to the fact that negative multiples do not yield sensible value estimates; twice as many firms report negative net income as compared with negative EBITDA.

To provide further insights into the relative performance of these multiples, for each of the 3,386 industry-year observations, we rank the multiples by their median absolute out-of-sample valuation errors based on harmonic means (without intercept). Lower ranks imply lower valuation errors; that is, rank 1 is assigned to multiples with the lowest valuation errors; while rank 4 is assigned to multiples with the highest valuation errors.

For our enterprise value multiples, we find that NOA has rank 1 for 46.66 percent of our industry year observations; while sales ranks 1 for 30.86 percent of our industry year observations. FCF ranks last in 74.66 percent of our industry year observations. The mean or median rank score shows that NOA ranks the best, followed by Sales, then EBITDA and lastly FCF.

For our market value multiples, book value ranks highest for 57.00 percent of our industry year observations; while Sales ranks highest for 19.26 percent of our industry year observations. The mean and median rank score indicates that, when comparing the relative ranking of these multiples, Book Value ranks the best, followed by sales, EBITDA and Net Income last. Visual depictions of the relative ranking of multiples across our industry year observations are provided in Figure A.

In Table 4, we show the results of incorporating the average effect of omitted variables by allowing for an intercept in the price-fundamental relation. When compared to the valuation errors reported in Table 3, the range of these valuation errors is narrower. Our results show improvement in the absolute performance of all the price-multiples. We find that the poor performing multiples in our previous analysis have the highest reduction in valuation errors (EBITDA, and FCF for enterprise value multiples; EBITDA

and net income for market value multiples). Our results indirectly imply that some (on average) adjustments are required in applying price multiple analysis to yield sensible value estimates, especially for value drivers such as earnings and FCF.

The improvements in absolute performance of the price multiples are not uniform. The rank ordering of these value drivers also changes after incorporating the average effect of omitted variables. NOA and book value remain as the best fundamentals for both Enterprise Value and Market Value. Depending on whether we focus on the median or the mean absolute valuation error, EBITDA and Sales are ranked either second or third for enterprise and market value multiples.

Our results are consistent with LNT inasmuch as allowing for an intercept improves performance mainly for poorly performing multiples; however, the relative performance of the price multiples does not change. We observe vast improvement in valuation errors for our EBITDA, Net Income and FCF multiples. Nevertheless, FCF and net income remain the poorest valuation fundamentals for Enterprise Value and Market Value multiples, respectively.

We repeat the same analysis for each of 3,386 industry-year observations. We assign the ranking of 1 to 4 for each price multiple at each industry-year level. The relative ranking is narrower. Book values (NOA and book value of equity) have the highest frequency of being ranked first (44 percent for enterprise value multiples and 39 percent for market value multiples). FCF has the highest frequency of being ranked last (69 percent) for enterprise value. In contrast to conventional wisdom about earnings, Net Income ranks last for 40 percent of our industry-year observations. The mean and

median rank of these multiples are similar to those reported in Table 4, with the exception that EBITDA now outperforms Sales after including average effects in the valuations.

Price multiples are often cited without much reconciliation being made between conflicting multiples. One common criticism of the use of price multiples is the inability to reconcile different multiples. In an attempt to incorporate information from different multiples, LNT also examines short-cut intrinsic value measures incorporating book value and forward earnings based on the residual income model. They find that their intrinsic value measures perform considerably worse than forward earnings. Even though these measures contain more information than forward earnings, they attribute the poorer performance to potential measurement error associated with the terminal value estimates required for the intrinsic value calculation.

LNT also combines two or more value drivers using price-scaled regressions to compare different linear combinations of value drivers based on Beatty, Riffe, and Thompson (1999). Little or no improvement is observed. In addition, LNT combines P/B and P/E ratios using the conditional P/E (P/B) that is appropriate given the forecast of earnings growth (forecasted book profitability) of the subject firm. They estimate the relation between forward P/E and forecast earnings growth (P/B with forecast ROE) for each industry-year; then they obtain the P/E (P/B) corresponding to the earnings growth (forecast ROE) for the firm being valued. Again, they find little or no improvement in performance over the unconditional P/E and P/B multiples.

Given that the extant literature is silent on how harmonic means can be calculated when different multiples can be combined, we extend the method developed in LNT to combine multiples and consider the change in the valuation error when we consider a

combination of multiples rather than a single multiple. Table 6 shows the distribution of absolute pricing errors when combining two multiples from different financial statements. This in turn provides an insight into the incremental information when price multiples from different financial statements are combined.

The mean and median valuation errors for our highest-ranked value driver -- NOA (book value) is 0.695 (0.664) at the mean and 0.478 (0.484) at the median. We observe improvement in the absolute performance of our NOA price multiples when they are combined with information on other statements. The biggest improvement in valuation errors is when NOA is combined with EBITDA for enterprise value multiples and Book Value is combined with EBITDA for Market Value multiples. Similar results are reported when an intercept is incorporated in combining the two value drivers.

An argument for the use of sales multiples is that they yield positive, and therefore possibly more meaningful, valuations when other fundamentals are negative. Combining multiples allows us to examine the extent to which negative fundamentals may, indeed, be incrementally useful (over sales alone) in valuation. Table 7 reports the results of combining sales and income (net income EBITDA) multiples. Adding the income multiples reduces the valuation error for the entire sample. For Enterprise Value multiples without incorporating the intercepts, Panel A of Table 7 shows that the median valuation error drops from 0.519 when Sales alone is being used to 0.408 when EBITDA is combined with Sales. Similarly for Market Value multiples, the median valuation errors when Sales is being used is 0.650. When Sales is combined with EBITDA, the median valuation error is lower at 0.458; while when Sales is combined with NI, the valuation error is at 0.468. We also observe improvements when EBITDA and/or Net

Income are combined with Sales in our analysis with intercepts. However, the difference in valuation errors when an additional multiple is incorporated is smaller compared to the difference in valuation errors when no intercepts are incorporated.

More importantly, our results show that when sales is being combined with EBITDA and/or Net Income, improvements in valuation errors are observed regardless of whether the subject firms have positive or negative EBITDA. For example, when intercepts are incorporated in our analysis of Enterprise Value multiples, the valuation errors are reduced from 0.462 for sales multiples to 0.372 when Sales and EBITDA are combined for the set of subject firms with positive EBITDA. For the set of subject firms with negative EBITDA, we also observe improvements from 0.719 when Sales alone is being used to 0.618 when Sales is combined with EBITDA.

5. Conclusions

The extant literature on the usefulness of multiples in explaining stock prices focuses mostly on a set of observations that have non-negative analysts' earnings forecasts. We document the usefulness of multiples in enterprise valuation and in equity valuation for a sample of firms that are more representative of the general population of firms (firms with losses, smaller start-up firms, etc). Our focus is on multiples of current financial variables. We do not consider forward earnings-based multiples as removing this restriction allow us to analyze a broader cross-section of stocks. We do not exclude firms with negative fundamentals. We use an industry classification at a more micro industry level; where possible we group on 4-digit SIC code and where this is not possible, we group on 3-digit SIC code. We focus our analyses on the absolute mean and

median valuation error instead of the inter-quartile. The use of inter-quartile range is not appropriate as some multiples have skewed distributions (for example, sales multiples are always non-negative), these multiples will naturally have a distribution that is less dispersed.

Our results are in sharp contrast to those reported in LNT (2002) in the following ways. First, valuation errors based on sales are not the largest when compared with errors based on other fundamentals. Second, when comparing book value and earnings, we do not find that earnings metrics outperform book value metrics. We attribute our results to the fact that only a small proportion of the observations have negative NOA and book value, while a significant number of our sample firms report losses and negative EBITDA. Our results may also in part driven by the fact that current earnings metrics (rather than forward earnings) are more volatile from one period to another. Third, we find that for market value multiples, EBITDA has lower valuation errors than Net Income. Again, we attribute this to the fact that negative multiples do not yield sensible value estimates. Twice as many firms report negative net income compared with negative EBITDA in our sample.

The widespread use of price-multiples stems, in part, from their ease of computation. There are two common criticisms for the use of multiples. First, these price-multiples often do not yield sensible estimates with negative multiples. We show vast improvement in valuation errors when an average omitted variable (intercept) is incorporated in the calculation of harmonic means. Our results, in turn, imply that the traditional method (without adjustment) of applying price multiples to obtain value estimates is inadequate.

Another common criticism of the use of multiples is that valuations of the same firm based on different price-multiples are difficult to reconcile. We extend the extant literature by demonstrating how harmonic means can be calculated when different multiples are combined, while incorporating intercepts in our analysis. This in turn enables us to examine the change in valuation errors when a combination of multiples is used instead of just a single multiple. Our results show valuation errors are significantly improved when combining fundamentals from different financial statements. Our results show that largest improvement in valuation errors when balance sheet fundamentals (net operating assets and book value of equity) are combined with fundamentals from the income statement (EBITDA).

References

- Alford, A. (1992). "The Effect of the Set of Comparable Firms on the Accuracy of the Price-earnings Valuation Method." *Journal of Accounting Research* 30: 94-108.
- Beatty, R. P., Riffe, S. M., and R. Thompson. (1999). "The Method of Comparables and Tax Court Valuations of Private Firms: An Empirical Investigation." *Accounting Horizon* 13: 177-199.
- Damodaran, A. "Investment Valuation: Tools and Techniques for Determining the Value of any Asset." 2nd edition. Wiley Finance, 2002.
- Easton, P., and G. Sommers. (2002). "Scale and the Scale Effect in Market-Based Accounting Research." *Journal of Business, Finance, and Accounting*: 25-56.
- Kim, M., and J. Ritter. (1999). "Valuing IPOs." *Journal of Financial Economics* 53: 409-437.
- Liu, J., Nissim, D., and J. Thomas. (2002). "Equity Valuation using Multiples." *Journal of Accounting Research* 40(1): 135 – 172.
- Nissim, D., and S. Penman. (2001). "Ratio Analysis and Equity Valuation: From Research to Practice." *Review of Accounting Studies* 6,1: 109-154.
- Penman, S. "Financial Statement Analysis and Security Valuation." 3rd edition. Irwin: McGraw-Hill, 2006.
- Trueman, B., Wong, M.H.F., and X. Zhang. (2000). "The Eyeballs Have It: Searching for the Value in Internet Stocks." *Journal of Accounting Research* 38: 137-162.

Table 1
Distribution of Price Multiples

Panel A: Our Sample with n=71,106

	Mean	Median	SD	p1	p5	p10	p25	p75	p90	p95	p99
NOA/EV	1.211	0.515	82.77	-0.151	0.005	0.045	0.200	0.874	1.204	1.543	1.211
EBITDA/EV	0.055	0.103	19.19	-0.753	-0.184	-0.076	0.031	0.171	0.268	0.374	0.055
FCF/EV	-0.410	-0.011	94.15	-1.379	-0.438	-0.264	-0.103	0.050	0.142	0.260	-0.410
Sales/EV	2.009	0.717	60.80	0.000	0.036	0.107	0.299	1.544	2.842	4.174	2.009
BookValue/P	1.312	0.478	28.34	-0.278	0.051	0.113	0.250	0.819	1.305	1.762	1.312
NetIncome/P	0.004	0.040	7.06	-1.251	-0.332	-0.158	-0.016	0.081	0.138	0.188	0.004
Sales/P	3.455	0.810	3.36	0.000	0.032	0.099	0.311	1.895	4.111	6.696	3.455
EBITDA/P	0.392	0.113	71.60	-0.513	-0.160	-0.070	0.030	0.232	0.415	0.602	0.392

*Panel B: LNT's Sample with n=26,613***

Sales/EV	0.939	0.708	0.788	0.086	0.169	0.234	0.396	1.234	1.925	2.495	3.981
EBITDA/EV	0.113	0.110	0.060	-0.031	0.026	0.044	0.075	0.147	0.187	0.215	0.276
BookValue/P	0.549	0.489	0.336	0.050	0.131	0.184	0.308	0.707	0.985	1.180	1.620
FCF/P	-0.025	0.002	0.252	-1.008	-0.379	-0.218	-0.069	0.050	0.131	0.234	0.648
NetIncome/P	0.050	0.056	0.073	-0.249	-0.043	0.005	0.033	0.080	0.108	0.130	0.178
Sales/P	1.419	0.988	1.416	0.098	0.215	0.313	0.552	1.773	2.991	4.080	7.112

** LNT excludes any negative price multiples in their analysis, their final sample n=19,879

**Table 2: Distribution of Absolute Pricing Errors for Multiples
Using 4-digit and 3 digit SIC, Sample Size: 64,334 - 1963-2006*****

	Mean	Median	Frequency <5%	Frequency <10%	Frequency<25%	Frequency <100%
<i>Panel A: Multiples based on harmonic means of firms from the same industry</i>						
Multiples of EV						
NOA	0.694	0.478	4.7%	10.8%	27.9%	88.6%
EBITDA	2.088	0.597	3.0%	8.2%	23.7%	68.8%
FCF	4.856	1.427	0.3%	1.1%	5.1%	35.3%
Sales	0.583	0.519	2.3%	6.7%	22.3%	92.5%
Multiples of MV						
BV	0.664	0.484	3.0%	8.2%	25.5%	90.9%
EBITDA	2.097	0.665	2.4%	6.7%	20.0%	68.8%
NIBE	3.312	0.987	2.1%	5.1%	14.8%	50.5%
Sales	0.650	0.597	1.7%	5.1%	17.7%	91.4%
<i>Panel B: Multiples based on medians of firms from the same industry</i>						
Multiples of EV						
NOA	1.216	0.456	8.0%	15.0%	31.7%	81.7%
EBITDA	1.274	2.334	6.6%	12.9%	29.6%	68.3%
FCF	7.151	2.074	1.7%	3.1%	7.8%	29.8%
Sales	1.097	0.483	5.9%	11.6%	28.0%	82.8%
Multiples of MV						
BV	0.824	0.429	6.7%	13.4%	31.4%	85.1%
EBITDA	2.374	0.574	5.6%	11.1%	26.3%	66.1%
NIBE	3.582	0.749	5.4%	10.5%	23.9%	57.2%
Sales	1.382	0.537	5.0%	9.9%	24.7%	79.7%

Table 3 Relative ranking of multiples across industry.

For each 3,386 industry year observations, multiples are ranked by **absolute** out-of-sample pricing errors based on harmonic means **without intercept**. Lower ranks imply lower pricing errors (better performance). Chi-squares are reported in parentheses. * denotes significance at 0.01 level.

Panel A: Multiples of EV

		Multiple of EV			
		NOA	EBITDA	FCF	Sales
Ranks	1	1580 (635.58*)	688 (29.68)	73 (706.80*)	1045 (46.55*)
	2	1133 (96.97*)	777 (5.71)	116 (630.40*)	1360 (311.50*)
	3	540 (110.98*)	1213 (158.68*)	669 (37.22)	964 (16.31)
	4	133 (601.40*)	708 (22.66)	2528 (3340.16*)	17 (812.84*)
Mean Rank		1.77	2.57	3.67	1.99
Median Rank		2	3	4	2

Panel B: Multiples of MV

		Multiple of MV			
		BV	EBITDA	NIBE	Sales
Ranks	1	1930 (1386.85*)	482 (156.95*)	322 (324.99*)	652 (44.69)
	2	835 (0.16)	834 (0.18)	335 (309.08*)	1382 (338.76*)
	3	458 (178.30*)	1399 (360.61*)	811 (1.49)	718 (19.51)
	4	163 (551.89*)	671 (36.39)	1918 (1356.31*)	634 (53.34*)
Mean Rank		1.66	2.67	3.28	2.39
Median Rank		1	3	4	2

Figure 1: Relative Ranking of multiples across industries
 Multiples are ranked based on **absolute** out-of-sample pricing errors based on harmonic means **without intercept**. Lower ranks imply lower pricing errors (better performance).

Figure 1b: Multiples of EV

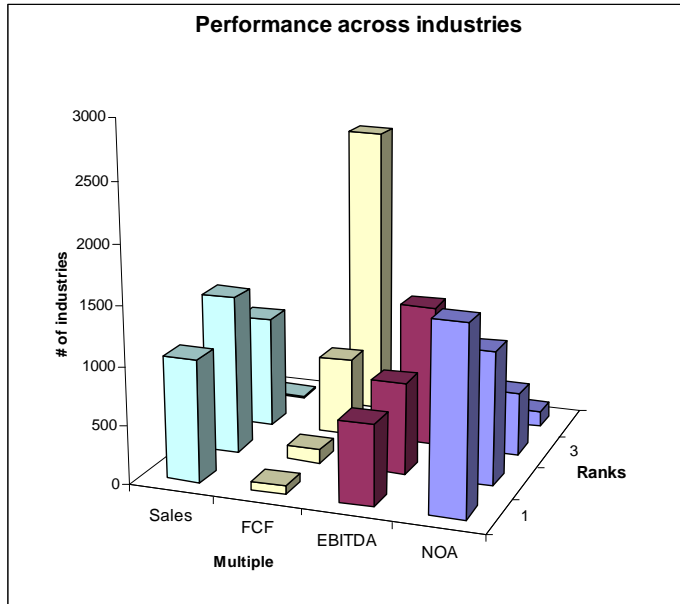
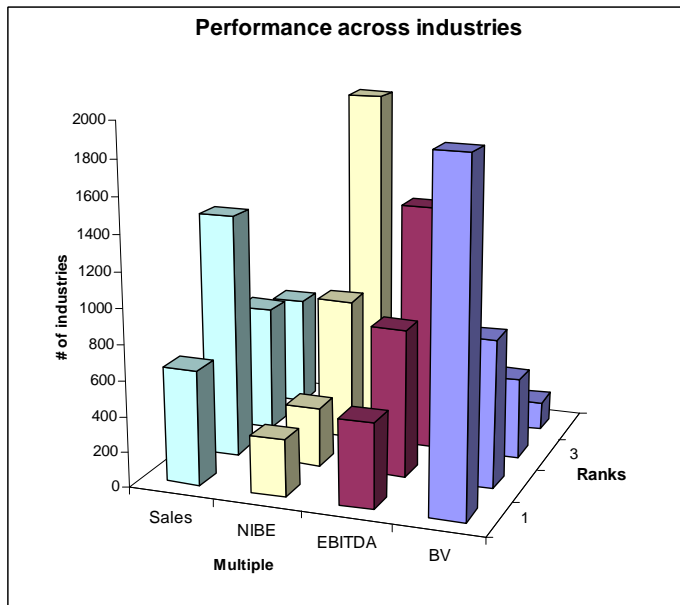


Figure 1b: Multiples of MV



**Table 4: Distribution of Absolute Pricing Errors for Multiples (with intercepts)
Using 4-digit and 3 digit SIC, Sample Size: 64,334 - 1963-2006**

	Mean	Median	Frequency <5%	Frequency <10%	Frequency<25%	Frequency <100%
<i>Multiples based on harmonic means of firms from the same industry</i>						
<i>Multiples of EV</i>						
NOA	0.453	0.343	6.1%	14.4%	37.8%	94.3%
EBITDA	0.505	0.377	4.4%	11.6%	33.8%	91.9%
FCF	0.700	0.573	2.1%	6.1%	20.3%	83.9%
Sales	0.512	0.406	3.8%	9.9%	30.2%	93.1%
<i>Multiples of MV</i>						
BV	0.441	0.344	5.2%	13.4%	37.5%	94.9%
EBITDA	0.467	0.364	4.9%	12.5%	35.2%	93.8%
NIBE	0.531	0.406	4.6%	11.7%	32.5%	90.7%
Sales	0.486	0.394	4.3%	11.1%	32.2%	94.1%

Table 5 Relative ranking of multiples across industry (with intercept)

For each 3,386 industry year, multiples are ranked by **absolute** out-of-sample pricing errors based on harmonic means **with intercept**. Lower ranks imply lower pricing errors (better performance). Chi-squares are reported in parentheses.

Panel A: Multiples of EV

		Multiple of EV			
		NOA	EBITDA	FCF	Sales
Ranks	1	1501 (506.05*)	1017 (34.34)	284 (373.78*)	584 (81.40*)
	2	960 (15.22)	1021 (35.97)	326 (320.05*)	1079 (63.86*)
	3	688 (29.68)	930 (8.24)	437 (198.10*)	1331 (277.31*)
	4	237 (438.85*)	418 (216.91*)	2339 (2631.49*)	392 (244.03*)
Mean Rank		1.90	2.22	3.43	2.45
Median					
Rank		2	2	4	3

Panel B: Multiples of MV

		Multiple of MV			
		BV	EBITDA	NIBE	Sales
Ranks	1	1330 (276.16*)	801 (2.45)	724 (17.73)	531 (117.59*)
	2	859 (0.18)	1107 (80.17*)	621 (60.07*)	799 (2.67)
	3	746 (11.93)	988 (23.65)	672 (35.97)	980 (21.05)
	4	451 (184.78*)	490 (150.14*)	1369 (322.51*)	1076 (62.22*)
Mean Rank		2.09	2.34	2.79	2.77
Median					
Rank		2	2	3	3

Figure 2: Relative Ranking of multiples across industries

Multiples are ranked based on **absolute** out-of-sample pricing errors based on harmonic means **with intercept**. Lower ranks imply lower pricing errors (better performance).

Figure 2a: Multiples of EV

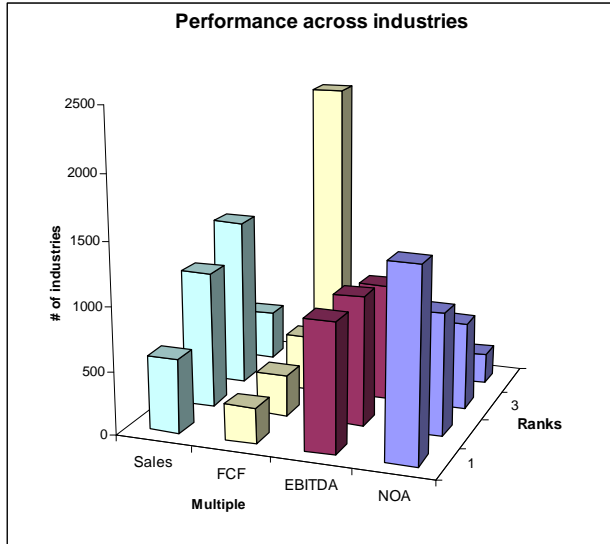


Figure 2b: Multiples of MV

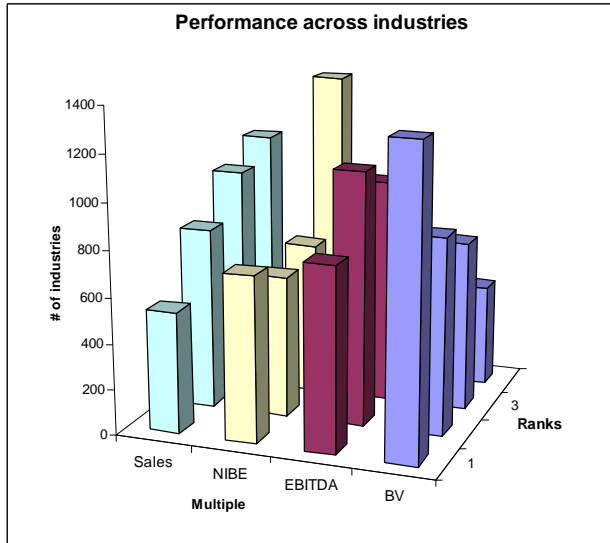


Table 6: Distribution of Absolute Pricing Errors combining two multiples using 4-digit and 3 digit SIC, Sample Size: 64,334; 1963-2006

Multiple A	Multiple B	Mean	Median	Frequency <5%	Frequency <10%	Frequency <25%	Frequency <100%
<i>Panel A: absolute pricing errors (without intercept)</i>							
<i>Multiples of EV</i>							
NOA		0.694	0.478	4.7%	10.8%	27.9%	88.6%
NOA	EBITDA	0.508	0.374	9.1%	17.0%	36.5%	90.8%
NOA	FCF	0.550	0.416	8.1%	15.2%	33.1%	89.9%
NOA	Sales	0.526	0.400	8.4%	15.7%	34.3%	91.4%
<i>Multiples of MV</i>							
BV		0.664	0.484	3.0%	8.2%	25.5%	90.9%
BV	EBITDA	0.535	0.404	6.9%	13.8%	33.0%	90.9%
BV	NIBE	0.551	0.408	7.0%	14.0%	32.8%	90.6%
BV	Sales	0.549	0.423	6.5%	13.0%	31.6%	91.1%
<i>Panel B: absolute pricing errors (with intercept)</i>							
<i>Multiples of EV</i>							
NOA		0.453	0.343	6.1%	14.4%	37.8%	94.3%
NOA	EBITDA	0.398	0.282	11.1%	20.9%	45.4%	95.1%
NOA	FCF	0.422	0.306	10.0%	19.1%	42.5%	94.5%
NOA	Sales	0.419	0.303	10.3%	19.5%	42.9%	94.6%
<i>Multiples of MV</i>							
BV		0.441	0.344	5.2%	13.4%	37.5%	94.9%
BV	EBITDA	0.392	0.291	9.6%	19.2%	44.2%	95.4%
BV	NIBE	0.395	0.293	9.7%	19.3%	44.0%	95.3%
BV	Sales	0.404	0.303	9.2%	18.4%	42.7%	95.2%

Table 7: Distribution of Absolute Pricing Errors combining sales multiples with other multiples using 4-digit and 3 digit SIC, Sample Size: 64,334; 1963-2006

Multiple A	Multiple B	Mean	Median	Frequency <5%	Frequency <10%	Frequency <25%	Frequency <100%
<i>Panel A: All Firms (without intercepts)</i>							
<i>Multiples of EV</i>							
Sales		0.583	0.519	2.3%	6.7%	22.3%	92.5%
Sales	EBITDA	0.540	0.408	6.8%	13.7%	32.9%	90.9%
<i>Multiples of MV</i>							
Sales		0.650	0.597	1.7%	5.1%	17.7%	91.4%
Sales	EBITDA	0.596	0.458	5.8%	11.9%	28.9%	88.8%
Sales	NI	0.613	0.468	6.0%	12.0%	28.7%	88.7%
<i>Panel B: All Firms (with intercepts)</i>							
<i>Multiples of EV</i>							
Sales		0.512	0.406	3.8%	9.9%	30.2%	93.1%
Sales	EBITDA	0.449	0.325	8.7%	17.0%	39.9%	93.7%
<i>Multiples of MV</i>							
Sales		0.441	0.344	5.2%	13.4%	37.5%	94.9%
Sales	EBITDA	0.423	0.317	8.7%	17.6%	41.1%	94.5%
Sales	NI	0.425	0.319	8.9%	17.5%	41.1%	94.6%
<i>Panel C: Target firms with positive EBITDA (n=57,674)</i>							
<i>Multiples of EV</i>							
Sales		0.539	0.462	2.4%	7.0%	22.9%	83.6%
Sales	EBITDA	0.500	0.372	6.9%	13.9%	33.2%	84.9%
<i>Multiples of MV</i>							
Sales		0.512	0.406	3.8%	9.9%	30.2%	93.1%
Sales	EBITDA	0.449	0.325	8.7%	17.0%	39.9%	93.7%
Sales	NI	0.474	0.358	8.3%	31.2%	85.1%	91.3%
<i>Target firms with negative EBITDA (n=13,432)</i>							
<i>Multiples of EV</i>							
Sales		0.767	0.719	0.8%	2.4%	8.6%	84.0%
Sales	EBITDA	0.728	0.618	2.9%	6.0%	15.4%	70.9%
<i>Multiples of MV</i>							
Sales		0.441	0.344	5.2%	13.4%	37.5%	94.9%
Sales	EBITDA	0.423	0.317	8.7%	17.6%	41.1%	94.5%
Sales	NI	0.425	0.319	8.9%	17.5%	41.1%	94.6%

Appendix A - Variable measurement

We follow Nissim and Penman (2001) in our variable measurement.

Price (P):	Share price from CRSP, as of 3 months after fiscal year end
Market Capitalization (MC)	$P \times \text{shares outstanding}$ (#25)
Enterprise Value (EV):	Market Capitalization + Net Financial Obligations
Net Financial Obligations (NFO):	Financial Liabilities - Financial Assets
Financial Liabilities (FL):	Debt in current liabilities (#34) + long-term debt (#9) + preferred stock (#130) - preferred treasury stock (#227) + preferred dividends in arrears (#242) + minority interest (#38).
Financial Assets (FA):	Cash and short-term investments (#1) plus investments and advances (#32)
Sales (SALES):	Sales (#12)
Core Operating Income (COI):	Comprehensive Net Income + Net Financial Expense – Unusual Operating Income
Comprehensive Net Income (CNI):	Net income (#172) - preferred dividends (#19) + the change in marketable securities adjustment (change in #238) + the change in cumulative translation adjustment (change in #230).
Net Financial Expense (NFE)	After tax interest expense ($\#15 \times (1 - \text{marginal tax rate})$) + preferred dividends (#19) - after tax interest income ($\#62 \times (1 - \text{marginal tax rate})$).
Unusual Operating Income (UOI):	After tax non-operating income (expense) excluding interest and equity in earnings ($(\#190 - \#55) \times (1 - \text{marginal tax rate})$) + after tax special items ($\#17 \times (1 - \text{marginal tax rate})$) + extraordinary items & discontinued operations (#48) + cumulative translation adjustment (#230) and - lag cumulative translation adjustment (lag #230).
EBITDA:	Earnings before interest, taxes, depreciation and amortization = EBITDA (#13)
Net Operating Assets (NOA):	Common Stockholders' Equity + Net financial Obligations
Common Stockholders' Equity (CSE):	Common equity (#60) + preferred treasury stock (#227) - preferred dividends in arrears (#242).
Cash Flows from operations (CFO):	EBITDA – interest expense (#15) – tax expense (#16) – net change in working capital (calculated as change in current assets (#4) – change in cash and cash equivalents (#1) – change in current liabilities (#5) + change in debt included in current liabilities (#34). Data items 15, 16, 1 or 34 are set to zero if missing.
Free Cash Flow (FCF)	CFO – capital expenditures (#128) + acquisitions (#129) + increase in investment (#113) – sale of PP&E (#107) minus sale of investment (#109).

Marginal Tax Rate

The top statutory federal tax rate plus 2% average state tax rate. The top federal statutory corporate tax rates were 46% in 1979–1986, 40% in 1987, 34% in 1988–1992 and 35% after 1993.

