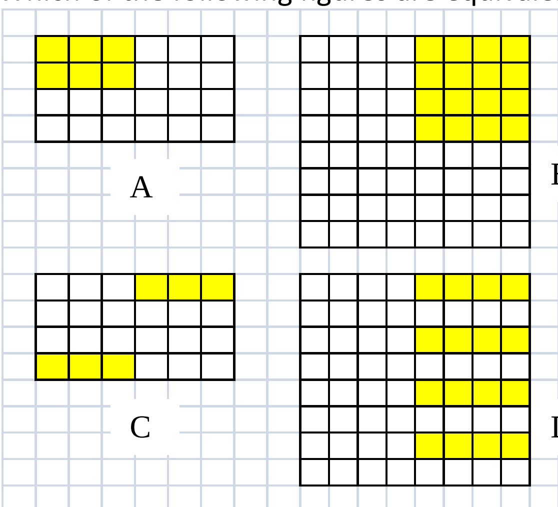
Equivalent Fractions

Student Probe

Which of the following figures are equivalent to ¼?



Answers and Misconceptions

All four figures are equivalent to ¼:

Figure A:

- Visualize (looks like ¼); students can divide it vertically and horizontally
- Numeric; students might recognize the pieces as 6 pieces out of 24 (6/24=1/4)

Figure B:

- Visualize (looks like ¼); students can divide it vertically and horizontally
- Numeric; students might recognize the pieces as 16 pieces out of 64 (16/64=1/4)

Figure C:

- Visualize; students can manipulate the pieces around to create the same shape as figure A
- Numeric; students might recognize the pieces as 6 pieces out of 24 (6/24=1/4)

Figure D:

- Visualize; students can manipulate the pieces around to create the same shape as figure B
- Numeric; students might recognize the pieces as 16 pieces out of 64 (16/64=1/4)

Students who do not see A & B as equivalent are struggling with the idea “the whole makes a difference in defining the fraction”.

At a Glance

What: Understand equivalent fractions using visual models

Common Core Standards: CC.4.NF.1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Mathematical Practices:
Reason abstractly and quantitatively.
Attend to precision.
Look for and make use of structure.

Who: Students who do not understand that fractional amounts can be represented in various ways.

Grade Level: 4

Prerequisite Vocabulary: rectangle, dimensions

Prerequisite Skills: naming fractional parts of a whole

Delivery Format: individual, small group

Lesson Length: 15-30 minutes

Materials, Resources, Technology: color tiles, grid paper, colored pencils

Student Worksheets: None

Students who do not see A & C or B & D as equivalent are struggling with the idea "fractional parts of an area must have the same size but not necessarily the same shape".

Lesson Description

The lesson is intended to help students develop an understanding of fraction equivalence using visual models with attention to how the number and size of the parts differ. Students will sketch two rectangles with different dimensions on grid paper to represent the same fractional amounts. At the end of the lesson students will respond to questions that address the idea of equivalence and how changing the whole affects the numbers used in representing the fraction.

Rationale

Students often fail to understand that a fractional amount can be represented in various ways depending upon the number of pieces and the size of the whole. The relationship between the denominator and the size of the pieces is critical to students' abilities to effectively problem-solve when given contextual situations involving fractional amounts. Students often think that as the denominator gets bigger the amount gets larger. This misconception often keeps students from generating accurate models of equivalent fraction ideas. The models serve as the foundation for developing the conceptual level of knowledge required for applying the algorithms used for computation. When students are able to understand the relationship between the whole, the size of the pieces, and the numerator and denominator, they will be able to progress into more complex fractional ideas and algorithms.

Preparation

Provide students with copies of grid paper and colored pencils to sketch and shade their rectangles. Have color tiles available for students who may need them. Make student copies and provide students with a copy of the formative assessment after instruction has been given.

Lesson

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
1. Tom is preparing to plant his spring gardens. He plans on having two vegetable gardens this year. He would like the two gardens to have the same fractional part of vegetables. Tom would like to plant ¼ of each garden with tomatoes, ½ of each garden with corn, and the rest of each garden with beans. Tom would like some help deciding where to plant each vegetable. He has one problem: "He wants both gardens to be rectangular in shape; but the two gardens are different sizes (dimensions)". Use the grid paper provided to sketch two possible garden plans. (The two gardens can have any dimensions; as long as they are different.)	Check to see if students are having trouble getting started. Check to make sure students have used whole numbers for their dimensions. Look for students who created two rectangles that have the same dimensions but are rotated to look differently (7x5) and a (5x7) Verify that students have sketched two different rectangular gardens and accurately divided it into the appropriate size pieces for the different vegetables.	If students are having trouble generating the rectangular gardens, suggest that students use color tiles to explore the forming of the rectangles before recording on grid paper. If whole numbers were not used redirect them to whole number options that will work. *Note: Students will have to divide squares in half to accurately draw and label the garden.* Ask, "Are the two rectangles the same or different?" Why? Ask, "Can we move (transform) the two rectangles so that they look the same?" Ask, "How do you know that each of your sections (for vegetables) is accurately drawn and shaded?" *Note: Students need to realize that the number of individual grid squares must be the same for each of the ¼ pieces they have.*
2. For Garden A, how many grid squares are labeled to be planted with beans?	Verify students have accurately counted and labeled their section of the garden planted with beans.	

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
3. Did everyone get the same amount of squares for Garden A? Is this okay, can people have different amounts and still plant the garden to meet Tom's requirements?	Students should share their squares counted for Garden A that were planted with beans. Students should be discussing that the "whole" causes the number of squares to be different even though the fractional amounts are the same.	If students are not making the connection between the whole and the amount of squares, provide students with simple examples to clarify. Example: Ask, "Would you want half of a King Size "snickers bar" or half of a mini "snickers bar?" Why? (expect students to say because you get more of the big candy bar)
4. What fractional part of Garden A is planted with beans?	Students should be discussing the numerator and denominator in terms of squares shaded (labeled) and total squares in the garden. Some students may say ¼ because of the ability to visualize the ¼ pieces in their drawing. Through discussion of how the students got their fractional names students need to understand that fractional amounts can be named differently but still have the same amount.	If students are not naming the part appropriately, provide students with explicit instruction on how to record a fraction (how many you see that are shaded or labeled over how many of that same size it would take to fill the entire "whole"). If students are not making the connection, further probing is required. Ask students who think their fractions are not equivalent about the processes used to write the fraction name to identify the pieces labeled.

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
5. Repeat the same questioning processes for Garden B. (Steps 2-4)		
6. Compare the size of Garden A with Garden B. How does this affect the fractional part for beans labeled for each garden?		
7. Ask students to reflect and write down what mathematical ideas they learned or discovered during the lesson.	*Topics might include:* Fraction pieces must be the same size but not necessarily the same shape. Understand that the whole makes a difference in defining the fraction. Understand that as the denominator increases the size of the pieces decreases.	

Variations

Once students are comfortable using ½ and ¼, transition them to different numbers such as 1/3, 2/3, 1/5, 2/5, 4/5, etc.…

This can be used as a way to scaffold instruction based on the complexity of the fractions used.

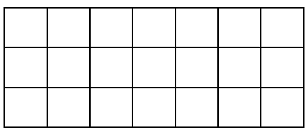
Formative Assessment

Using the two rectangles below:

1) Shade 1/3 of each of the rectangles below.
2) Write a fractional name for the shaded portion **(other than 1/3).**
3) How are the shaded portions from each rectangle **the same?**
4) How are the shaded portions from each rectangle **different?**
5) Draw a rectangle with different dimensions from Rectangles A & B and shade 2/3 of the area.

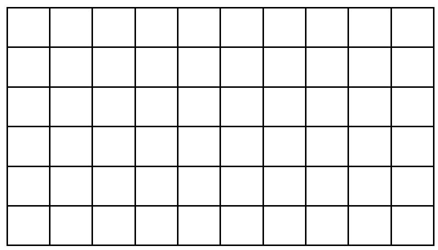
<u>Questions 1 and 2</u>

Rectangle A



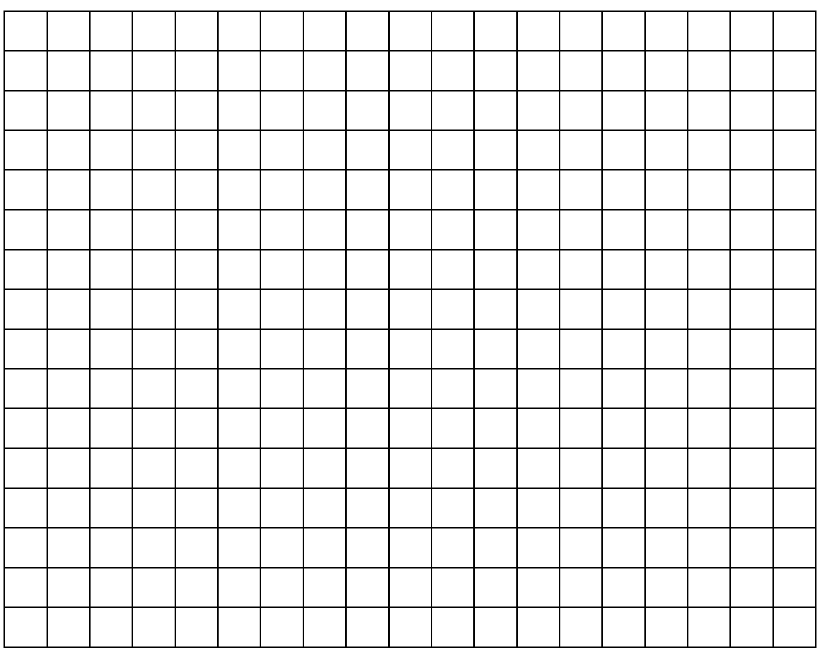
Fractional Name ______________

Rectangle B



Fractional Name ______________

<u>Use the Grid Below to respond to Question 5</u>



References

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