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# ESSAIS SUR LA MACROÉCONOMIE DES IMPERFECTIONS SUR LE MARCHÉ DU CAPITAL

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# ESSAYS IN THE MACROECONOMICS OF CAPITAL MARKET IMPERFECTIONS

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**Résumé**

Cette thèse comporte trois volets dont le thème commun est la présence d'imperfections sur le marché du capital.

Le premier, en supposant que les firmes doivent financer une part de leur coûts de recrutement sur des marchés financiers imparfaits, réconcilie le modèle de furetage et d'appariement sur le marché du travail de Mortensen-Pissarides avec les données macroéconomiques. En particulier, les postes vacants et la tension sur le marché du travail sont à la fois très volatiles et leur ajustement suite à des chocs de productivité est progressif. Lorsque la prime sur le financement externe se comprime durant une expansion, et de manière progressive dû à l'accumulation de liquidités par les firmes, l'incitation à recruter pour un bénéfice espéré donné d'un nouvel employé est plus forte. Ceci génère un mécanisme de propagation suffisamment puissant pour reconcilier le modèle avec les données. Une extension à des séparations d'emploi endogènes préserve le mécanisme de propagation du modèle tout en lui permettant d'être cohérent avec certaines propriétés des flux de travailleurs sur le cycle.

Le second documente en premier l'existence de phénomènes de congestion dans l'allocation du capital physique similaires à ce qui est observé sur le marché du travail, et étudie dans un modèle d'équilibre général quantitatif ces effets pour la propagation de chocs technologiques et, donc, pour l'étude des fluctuations conjoncturelles. La calibration du modèle sur les flux de capitaux mesurés au niveau des firmes mène à la conclusion que ces effets sont négligeables. L'introduction de liquidation du capital des firmes faisant banqueroute ne change rien à cette conclusion car les flux concernés par cette réallocation de capital sont trop petits.

Le dernier volet de cette thèse s'écarte du cadre d'une économie fermée. Une caractéristique de l'investissement direct étranger que la théorie économique a du mal à réconcilier est le fait que, en période d'expansion, les flux d'investissement rentrant dans une économie d'accueil et les flux d'investissement de cette même économie vers l'étranger augmentent ensembles. En imposant des frictions dans l'allocation du capital à des établissements à l'étranger, avec la possibilité de fermer ces établissements pour réallouer ailleurs le capital qui y était engagé, un modèle dynamique à deux pays devient cohérent avec l'observation empirique sur les flux d'investissements directs.

**Mots clefs**: Imperfections sur le marché du capital et du travail, cycle conjoncturel

**Abstract**

The first chapter shows that the propagation properties of the standard search and matching model of equilibrium unemployment are significantly altered when vacancy costs require some external financing on frictional credit markets. Agency problems on credit markets lead to higher costs of vacancies. When the former are counter-cyclical, this greatly increases the elasticity of vacancies to productivity through two distinct channels: (i) a cost channel - lowered unit costs during an upturn as credit constraints are relaxed increase the incentive to post vacancies; (ii) a wage channel - the improved bargaining position of firms afforded by the lowered cost of vacancies limits of the upward pressure of market tightness on wages. As a result, the model can match the observed volatility of unemployment, vacancies and labor market tightness. Moreover, the progressive easing of financing constraints to innovations generates persistence in the response of market tightness and vacancies, a robust feature of the data and shortcoming of the standard model. Extending the model to allow for endogenous job separation improves its ability to match gross labor flows statistics while preserving its propagation properties.

The second chapter documents the existence of time-varying congestion in the (re)allocation of physical capital akin to what is observed on labor markets. It then builds a model with search frictions for the allocation of physical capital in order to investigate its implications for the business cycle. While the model is in principle capable of generating substantial internal propagation to small exogenous shocks, the quantitative effects are modest once it is calibrated to fit firm-level capital flows. The model is then extended to credit market frictions that lead to countercyclical default as in the data. Although countercyclical default directly affects capital reallocation, even in this extended model, search frictions in physical capital markets play only a small role for business cycle fluctuations.

The final chapter models flows of foreign direct investment (FDI) in a two country, two sector DSGE framework. The allocation of capital to production capacity abroad is subject to a search-and-matching friction with endogenous capital reallocation, capturing the additional cost and time involved in adjusting production capacity abroad. The model is calibrated on observed gross inflows and outflows of FDI and leads to dynamics of net foreign direct investment consistent with the empirical evidence documented in this chapter: inward and outward net flows of FDI are positively correlated whereas a standard International Real Business Cycle model has the prediction of a negative correlation. Moreover, the model solves the aggregate investment quantity puzzle as it generates cross-country correlations in-line with the data.

**Key words**: Imperfections in capital and labor markets, business cycles

# Introduction générale

Cette thèse est composée de trois études dans lesquelles des imperfections sur le marché du capital affectent la dynamique cyclique d'agrégats macroéconomiques. La méthode commune est celle des modèles dynamiques d'équilibre général stochastique, une méthodologie ayant connu une grande progression depuis ses débuts dans les travaux pionniers de Kydland et Prescott (1982) et l'étude de modèles du cycle conjoncturel réel. La première étude aborde une problématique particulière aux modèles de chômage d'équilibre basés sur des frictions d'appariement: ces modèles sont incapables d'être cohérents, simultanéement, avec l'observation que les variables pour lesquelles le modèle a des prédictions sont à la fois très volatiles et persistantes. Il apparaît qu'un financement externe sur des marchés du crédit imparfait des coûts de recrutement peut solutionner ce problème double. Lorsque que la prime sur les fonds externes varie inversement avec le cycle, ceci augmente l'incitation pour les firmes à créer des emplois durant une période d'expansion économique. La diminution progressive de la dépendance sur les fonds externes via l'accumulation de liquidités fait en sorte que ce phénomène est persistant. Le second est une investigation dans un modèle d'équilibre général quantitatif des effets de congestion dans l'allocation du capital physique pour la propagation de chocs technologiques et, donc, pour l'étude des fluctuations conjoncturelles. La calibration du modèle sur les flux de capitaux mesurés au niveau des firmes mène à la conclusion que ces effets sont négligeables. L'introduction de liquidation du capital des firmes faisant bankroute ne change rien à cette conclusion car les flux concernés par cette réallocation de capital sont trop petits. Le dernier volet de cette thèse s'écarte du cadre d'une économie fermée. Une caractéristique de l'investissement direct étranger que la théorie économique a du mal à réconcilier est le fait que, en période d'expansion, les flux d'investissement rentrant dans l'économie d'accueil et les flux d'investissement de cette même économie vers l'étranger augmentent ensembles. En imposant des frictions dans l'allocation du capital à des établissements à l'étranger, avec la possibilité de fermer ces établissements pour réallouer ailleurs le capital qui y était engagé, un modèle dynamique à deux pays devient cohérent avec l'observation empirique sur les flux dinvestissement direct.

Les conditions d'accès au crédit influencent la création, l'expansion et, en générale, la dynamique des entreprises (Hubbard, 1998, Stein, 2002). Alors que beaucoup d'attention à été portée sur la relation entre le coût du financement et les nouveaux investissements en capital physique (e.g., Bernanke et Gertler, 1989, Kiyotaki et Moore, 1997), il y a un intérêt plus récent dans le rapport entre les termes du crédit et la création d'emplois. En particulier, Acemoglu (2001) et Wasmer et Weil (2004) démontrent que des imperfections sur le marché du crédit peuvent occasionner des taux de chômage d'équilibre plus élvevés. Le première chapitre de cette thèse examine le lien entre le coût du crédit et les fluctuations cycliques du chômage.[1]

Les modèles de chômage d'équilibre du type furetage et appariement de Mortensen et Pissarides (1994), ayant eu du succès dans l'analyse du marché du travail en équilibre stationnaire, souffrent de deux grandes faiblesses lors de l'étude des fluctuations cycliques sur le marché du travail. Le premier est un manque d'amplification de variations à la productivité du travail. Les variables centrales au modèle, telles que le taux de postes vacants, le taux de chômage et le ratio des deux, la tension sur le marché du travail, sont très volatiles au cours du cycle conjoncturel, un fait que le modèle standard a de la difficulté à reproduire. Deuxièmement, les données révèlent que la tension sur le marché du travail ne s'ajuste que progressivement aux chocs de productivité, qu'il y a beaucoup de persistence sur le marché du travail, alors que le modèle implique que l'ajustment le plus important est contemporain au choc.

Ce type de modèle est régit par deux conditions principales, dont une condition de création d'emploi. Cette dernière égalise pour les firmes le coût moyen de combler un poste au benéfice espéré d'un nouvel employé. Si les firmes doivent financer une fraction de leurs coûts de recrutement sur des marchés financiers imparfaits, ces fonds externes comportent alors une prime de risque qui augmente le coût moyen de recrutement. Par contre, si cette prime est contre-cyclique, c'est-à-dire qu'elle se comprime durant une expansion, elle aura l'effet de limiter la hausse du coût moyen de recrutement venant des effets de congestion sur le marché du travail et, pour un bénéfice espéré d'un nouvel employé, les firmes auront un incitation plus forte à créer des emplois. Qui plus est, le relâchement des contraintes de financement est progressif de par l'accumulation de liquidités par les firmes, réduisant leur dépendance sur le financement externe. Ainsi, ce mécanisme est capable de générer à la fois de l'amplification et de la persistence pour

[1] Il est important de noter l'accélérateur financier venant de l'interaction entre l'état du marché du travail et du crédit est identifier dans le travail de Wasmer et Weil (2004).

rendre le modèle cohérent avec les données.

La deuxième implication du modèle, dérivée de l'hypothèse sur le mécanisme de détermination des salaires, est que la prime contre-cyclique sur les fonds externes occasionne un degré de rigidité salariale. Sous une généralisation de la règle de Nash, le salaire est croissant en le coût d'opportunité de la relation de travail pour la firme. Cette dernière consiste en le coût de quitter la négotiation avec un travailleur donné pour fureter sur le marché du travail, un coût qui dépend de la congestion sur le marché du travail et du coût en ressources du processus de recrutement. Dans le cas présent, ce coût de recrutement dépend des termes sur les fonds externes qui s'amèliorent durant une expansion. Ceci limite la hausse des salaires durant une expansion et est une source d'amplification additionnelle des variations de la productivité du travail au court du cycle.

Une extension à une endogénéisation du taux de séparation permet au modèle d'être cohérent avec le comportement des flux de travailleurs tout en préservant le résultat principal de propagation. En particulier, alors que l'hypothèse d'un taux de séparation constant implique des flux bruts de pertes d'emplois pro-cycliques, en contradiction nette avec les données. L'extension à un taux de séparation endogène génére des flux de pertes d'emplois contre-cycliques et volatils, tel que dans les données.

La contribution générale de cette étude est la considération que la dynamique des coûts de recrutement est importante dans la compréhension de la dynamique cyclique du marché du travail, et que les imperfections sur le marché du crédit sont source crédible et quantitativement significative de cette dynamique. Par ailleurs, le mécanisme en jeux opère par une augmentation de la rigidité du coût marginal de la production au niveau de la firme, une composante que nous savons importante pour la dynamique de la nouvelle courbe de Phillips. A la lumière de ceci, il y sans doute une avenue à explorer dans la transmission de la politique monétaire via la création d'emploi et le marché du travail.

Le deuxième chapitre de cette thèse s'inspire des nombreux travaux empiriques récents ayant mis à la lumière d'importantes réallocations de capital physique au-delà de l'accumulation se faisant via l'investissement dans de nouvelles unités, mesure de l'investissement habituellement utilisée dans les comptes nationaux (Ramey et Shapiro 2001, Eisfeldt et Rampini 2006 et 2007). Selon les recherches d'Eisfeldt et Rampini (2006 et 2007), les flux bruts d'investissement sont de l'ordre de 20% du stock de capital existant, soit plus du double des nouvelles dépenses en immobilisations fixes. De plus, il apparaît que la réallocation de capital usagé est une composante significative, de l'ordre de 24%, de ces flux d'investissement.

Parallèment à ceci, ces mêmes études, ainsi que d'autres, notent des coûts, ou des frictions, dans l'allocation du capital physique. D'une part, Eisfeldt et Rampini (2006) constatent que la réallocation est plus importante durant les périodes d'expansion économique versus les contractions, alors que c'est justement durant ces dernières que les bénéfices à la réallocation sont les plus importantes. Dans la même veine, des enquêtes au niveau des firmes révelent une large distribution dans les taux d'investissement à un moment donné dans le temps, certaines firmes n'étant engagées dans aucune dépense d'investissement alors que d'autres vivent des périodes de pique d'investissement (i.e., des taux d'investissement très élevés). D'autre part, il semble que les coûts impliqués lors de la réallocation de capital physique sont imposants. Pour illustrer le cas, Ramey et Shapiro (1998) se penchent sur une étude de cas dans l'industrie de l'aéronautique. Cette industrie est caractérisée par un haut degré de spécificité du capital. Ainsi les pièces et équipements se revendent avec une escompte moyenne de 28% du coût de remplacement. Ce phénomène rappelle ce que Shleifer et Vishny (1992) caractétisèrent d'illiquidité des actifs, voulant que les actifs fixes de firmes dans un secteur en déclin sont vendus avec un rabais d'autant plus imporant que les acheteurs potentiels vivent également une conjoncture difficile.

Ceci étant dit, le second chapitre explore les conséquences pour les modèles d'analyse du cycle conjoncturel de l'inclusion des faits stylisés décrits plus haut quant à la réallocation du capital physique. Plus précisement, il étudie les propriétés de propagation de chocs exogènes induits par des imperfections dans la réallocation du capital physique du type de furetage et appariement.

La réponse à cette première question est, une fois le modèle calibré sur les flux d'investissments bruts observés, que les implications quantitatives ne sont pas très grandes. La raison principale de ce résultat est la taille modeste des flux concernés qui sont insuffisants pour affecter de manière importante la dynamique du stock de capital agrégé et, par l'entremise, la production agrégée.

Ce constat mène à l'extension suivante: quel mécanisme viendrait augmenter la séparation de capital de son emploi courant à un taux variant inversement avec le cycle économique. Un candidat serait des imperfections sur les marchés du crédit. Effectivement, le nombre de défauts sur paiement d'intérêts ou le nombre de mises en faillite sont des phénomènes clairement contra-cyclique, offrant possiblement le mécanisme nécassaire à l'amplification de chocs exogènes. Malheureusement, en se basant sur les quantités de capital physique concernées par ces évènements de la base de Compustat, une base détaillée de toutes les firmes cotées aux États-Unis, on en vient encore à la conclusion que les flux concernés sont bien trop faibles pour affecter les conclusions quantitatives d'un modèle de cycle conjoncturel.

Le dernier volet de cette thèse cherche à expliquer une obervation au sujet des flux d'investissements directs que la théoirie classique ne peut réconcilier. Ce fait est la corrélation contemporaine positive entre flux d'investissements directs entrants dans une économie d'accueil, et les flux d'investissements directs de cette même économie vers l'étranger. En d'autres termes, les périodes durant lesquelles un pays accueille plus d'investissement est également une période où le pays investit plus à l'étranger. Un modèle standard de cycle conjoncturel international prédit justement une corrélation négative entre ces flux pour motif de lissage de la consommation des ménages.

L'enjeu est d'envergure alors que les économies sont de plus en plus intégrées, et les flux d'investissements directs sont un vecteur d'intégration important. Dans le cas d'une économie fortement intégrée comme le canada, ces flux sont loin d'être négligeables étant de l'ordre de 20% de l'investissement agrégé au cours des 50 dernières années. Mais l'importance de ces flux ne se restreint pas seulement au cas du Canada. L'investissment direct étranger à pris au cours des 15 dernières années une ampleur similaire dans les pays de l'Union Européenne, se situant dans une fourchette entre 10 et 20 % de l'investissement agregé selon le pays.

Ce projet reprend l'idée de Gordon et Bovenberg (1996) selon laquelle les firmes étrangères sont à un désavantage par rapport aux firmes domestiques dans l'établissement et la gestion d'une entreprise dans l'économie d'accueil. Alors que ces auteurs introduisent ce concept par un coût proportionel à la production de la firme étrangère, ici la différence sera dans l'allocation du capital physique qui présentera des difficultés pour les entreprises s'établissant à l'étranger. Concrètement, les firmes étrangères doivent payer un coût par projet d'investissement, et ce projet ne se réalise qu'une fois le capital nécessaire localisé. De plus, à tout moment une proportion du capital physique établie à l'étranger peut-être retiré pour une réallocation soit vers l'économie d'origine, soit vers un autre établissement à l'étranger.

Les effets de congestion dans l'allocation du capital à l'étranger permettent de répliquer la corrélation positive en flux d'investissment directs mentionnée plus tôt et de la manière suivante. Une période d'expansion économique, en présentant des rendements sur le capital plus élevés, attire les investissements directs étrangers. La même raison entraîne une baisse des nouveaux investissements de cette économie en expansion vers l'étranger. Dans un modèle classique, ceux-ci sont les seuls mécanismes présents et il en découle une corrélation négative entre flux d'investissement direct entrant dans l'économie en expansion et les flux d'investissements de cette économie vers l'extérieur. Par contre, cette même baisse de capital disponible pour investissement à l'étranger entraîne une hausse dans la probabilité pour les unités demeurantes d'être allouées à l'étranger. Ceci limite la baisse initiale des investissements à l'étranger *réalisés* de l'économie en expansion et génère la corrélation positive observée dans les données. Qui plus est, lorsque les décisions de réallocation du capital physique à l'étranger sont endogènes, la période d'expansion dans l'économie d'origine augmente le coût d'opportunité de réallouer le capital en place à l'étranger. De ce phénomène il résulte une baisse de la réallocation de capital à l'étranger qui vient limiter en plus la baisse des nouveaux investissements directs à l'étranger. Le solde fait en sorte que la corrélation entre flux d'investissements directs est positive.

# Chapter 1

# Credit, Vacancies and Unemployment Fluctuations

**Abstract**

The propagation properties of the standard search and matching model of equilibrium unemployment are significantly altered when vacancy costs require some external financing on frictional credit markets. Agency problems lead to higher costs of vacancies. When the former are counter-cyclical, this greatly increases the elasticity of vacancy postings to productivity through two distinct channels: (i) a cost channel - lowered unit costs during an upturn as credit constraints are relaxed increase the incentive to post vacancies; (ii) a wage channel - the improved bargaining position of firms afforded by the lowered cost of vacancies limits of the upward pressure of market tightness on wages. As a result, the model can match the observed volatility of unemployment, vacancies and labor market tightness. Moreover, the progressive easing of financing constraints to innovations generates persistence in the response of market tightness and vacancies, a robust feature of the data and shortcoming of the standard model. Extending the model to allow for endogenous job separation improves its ability to match gross labor flow statistics while preserving its propagation properties.

## 1.1 Introduction

The standard Mortensen and Pissarides (1994) search and matching model of equilibrium unemployment has been argued in many places to be inconsistent with key business cycle facts (e.g. Shimer, 2005, Fujita and Ramey, 2007). In particular it cannot explain the high volatilities of unemployment, vacancies and market tightness, nor the persistence in the adjustment of these variables to exogenous shocks. Subsequent research has focused on whether the lack internal propagation, both in terms of amplification and persistence, stems from the structure of the model itself (e.g., Shimer 2004, Fujita and Ramey, 2007) or whether it is a question of setting an appropriate calibration (e.g., Hagedorn and Manovskii, 2008).

Firms in these models must expend resources to fill job vacancies, a time consuming process in the presence of search frictions on labor markets. Under Nash bargaining as a wage mechanism, wages absorb much of the change in the expected benefit to a new worker induced by fluctuations in labor productivity. As a result, Shimer (2005) argues that the incentives to post vacancies change little over the business cycle and, quite naturally, a first branch of research has focused on the dynamics of wages as a means of generating amplification of exogenous innovations. Such studies have either altered the particulars of the wage determination mechanism, or as Hagedorn and Manovskii (2008), followed an alternative calibration strategy that results in a rigid wage.[1] In order to address the second empirical shortcoming, the persistence in market adjustments, a second strand of research has focused on the structure of vacancy costs. Fujita and Ramey (2007), for example, develop a story about sunk costs to vacancy creation such that the strongest change in market tightness occurs several periods after the original shock. Their approach, however, does not generate any additional amplification.[2]

This paper extends the baseline equilibrium unemployment framework by assuming that external finance must be called upon to fund part of a firm's vacancy costs, and that agency problems cause credit markets to be frictional. While there exists a large body of evidence suggesting that credit market frictions play an important role for firm behavior, both empirical and theoretical work focusing on their implications for firm growth and investment decisions, recent work has developed on linking credit market imperfections to job creation.[3] Both Acemoglu (2001) and Wasmer and Weil (2004), for example, show how credit market imperfections can lead to higher equilibrium unemployment

[1]Examples of alternate wage determination include backward-looking social norms (Hall, 2003), staggered wage contracting (Gertler and Trigari, 2009) or information asymmetries over productivity (Menzio, 2006). In essence, the parametrization in Hagedorn and Manovskii (2008) of the value of non-market activities and the relative Nash bargaining weight ensures that the wage is highly inelastic to its time-varying components, i.e. labor productivity and the degree of market tightness.

[2]Fujita and Ramey (2007) argue that by combining their modeling of job vacancies with a Hagedorn and Manovskii (2008) calibration, their model can address both issues pertaining to the propagation of productivity shocks. Alternate approaches to modeling vacancy costs include Yashiv (2006) and Rotemberg (2006) in which the cost of vacancies is a declining function of the number of vacancies a firm posts, or Shao and Silo (2008) who consider a model of firm endogenous entry .

[3]Empirically, panel data studies find that small firms with more difficult access to credit, take on more debt, and have investment rates that are more sensitive to cash flows even after controlling for future profitability. See Hubbard (1998) and Stein (2002) for surveys.

by restricting firm entry.[4] Moreover, Acemoglu (2001) provides evidence that credit constrained industries have lower employment shares, while Rendon (2001) finds that labor demand is both restricted and more elastic at credit constrained firms.

Due to a problem of costly state verification in lending relationships, firms in the model write standard debt contracts, in the spirit of Gale and Hellwig (1985), to fund vacancies over internal funds or assets. The higher shadow cost of external over internal funds increases the unit cost of vacancies. However, the degree of agency costs is alleviated during economic upturns by increased profitability and as firms accumulate liquidity, opening two channels through which the elasticity of job vacancies to productivity is increased: (i) a cost channel, driving a time-varying wedge in the job creation condition in which lowered unit costs during an upturn, as constraints are eased, increase the incentive to post vacancies. Amplification arises by inducing a change in costs for a given expected profit from a filled vacancy; (ii) a wage channel - under Nash bargaining as a wage mechanism, the lowered cost of vacancies limits part of the upward pressure of market tightness on wages by improving the bargaining position of firms. This provides amplification by increasing the elasticity of expected profits from new hires to shifts in productivity, and hence the incentive to post vacancies.

This 'financial accelerator' is distinct from previous mechanisms to address the issue of propagation by addressing simultaneously the lack of amplification and persistence. First, amplification is a result of both a vacancy cost and wage channel. The former, which plays a dominant role, is a novel feature in which the key is a time varying cost of recruiting new workers due to the necessity to raise external funds on frictional credit markets.[5] The latter is distinct from previous work in that the source of wage rigidity is a consequence of frictional credit markets and not an inherent feature of the wage rule or a particular calibration of the model. Second, the progressive easing of financing constraints as firms accumulate assets induces persistence in the adjustments of labor market variables to productivity shocks. Whereas in standard search models, or models with increased wage rigidity for that matter, the largest response of market tightness is contemporaneous to the exogenous shock, the height of the response in this setting is reached several quarters after the innovation. Amplification and persistence here are inextricably linked.

The model's quantitative results, detailed in section 3, are set against a comparable

[4]Linking current costs to financial markets is also a features of bank loan models as in Chirstiano et al (2005), or commercial debt models as in Carlstrom and Fuerst (2000).

[5]Similar dynamics in the cost of recruiting arise in Yashiv (2006) and Rotemberg (2006) due to their assumption of increasing returns to job postings.

framework without credit frictions.[6] The propagation potential is significant, generating a highly pro-cyclical labor market tightness that comes close to replicating the volatility relative to output observed in the data (13.45 against 15.41 in the data and 4.76 in the standard model).[7] As a result, the relative volatility of unemployment, which is 6.82 in the data, rises to 4.92 in the presence of credit frictions compared to 1.70 in the standard model. Importantly, the model remains consistent with the empirical observation of a strong negative correlation between vacancies and the unemployment rate, or the Beveridge curve. The second significant implication is a sluggish response of vacancies and market tightness to a technological innovation. U.S. quarterly data display a high degree of persistence, measured as positive autocorrelations in the growth rate of market tightness of 0.67, 0.48 and 0.33 at the first, second and third lags respectively. The benchmark calibration leads to autocorrelations of 0.64, 0.30 and 0.13 at the first, second and third lags in the growth rate of market tightness, whereas a standard search model generates virtually no auto-correlation.[8]

The benchmark model allows only for exogenous separation of workers out of employment, resulting in an inability of the model to be consistent with observations on gross labor flows. Section 4 extends the model to allow for endogenous labor separation by introducing a job specific productivity shock observed at the beginning of each period. Jobs drawing a productivity below a certain threshold are terminated. However, contrary to Mortensen and Pissarides (1994), some of the separations are inefficient owing to restrictions on current losses that push the cut-off productivity above that for which the surplus of the job match is null. The main results regarding propagation are robust to this extension. Moreover, the model is largely consistent with the cyclical properties of gross labor flows, generating counter-cyclical gross hires and job losses, while preserving a Beveridge relationship between unemployment and vacancies.

This paper contributes to the growing literature on the quantitative ability of the job matching framework to explain labor market business cycle facts, and concurs with the conclusion drawn in Fujita and Ramey (2007) that the costs of creating new vacancies can play a significant role in accounting for the observed patterns in employment ad-

[6]The model is set in a DSGE framework as in Merz (1995) or Andolfatto (1996), extended to frictional credit markets in a manner similar to Carlstrom and Fuerst's (1997) work with the canonical real business cycle model.

[7]Second moments correspond to Hodrick-Prescott filtered data. Time series cover the period 1977:1 to 2005:4.

[8]This criticism is akin to that of Real Business Cycles (RBC) models advanced by Cogley and Nason (1995) in their inability to generated persistence in the the growth rate of output. This issue motivates Andolfatto's (1996) work on introducing search frictions on labor markets in an RBC framework, but it does not focus on the persistence of labor market variables.

justments. The originality here is that these costs evolve endogenously as a function of credit market conditions and can simultaneously address the lack of amplification and persistence to productivity shocks. While the macroeconomic consequences of credit market imperfections have generally focused on their consequences for capital investment, e.g. models of financial intermediation and agency costs by Bernanke and Gertler (1989) or Kiyotaki and Moore (1997), this paper finds that their implications for labor markets should not be overlooked.[9]

## 1.2 Model

The model is populated by two types of agents: firms that produce using labor and households who decide on optimal consumption and purchases of risk-free bonds. The allocation of labor from households to firms involves a costly and time-consuming matching process, following the now common approach of Mortensen and Pissarides (1994), adapted to a representative household framework as in Merz (1995) or Andolfatto (1996).[10] The additional assumption is that firms must seek external funds over accumulated liquidity in order to finance current vacancies, and that the lending relationship is subject to a credit market friction of the costly state verification type. The resulting debt contract is characterized by an optimal monitoring threshold and vacancy postings.

### 1.2.1 Labor markets and households

Firms post job vacancies $V_t$ to attract unemployed workers $U_t$ at a unit cost of $\gamma$. Jobs are filled via a constant returns to scale matching function taking vacancies and unemployed workers as arguments, $M(U_t, V_t)$. Define $\theta_t = \frac{V_t}{U_t}$ as labor market tightness from the point of view of the firm, or the v-u ratio. The matching probabilities are $\frac{M(U_t,V_t)}{V_t} = p(\theta_t)$ and $\frac{M(U_t,V_t)}{U_t} = f(\theta_t)$ for firms and workers respectively, with $\partial p(\theta_t)/\partial\theta_t < 0$ and $\partial f(\theta_t)/\partial\theta_t > 0$. Note that $f(\theta_t) = \theta_t p(\theta_t)$. Once matched, jobs are destroyed at the exogenous rate $\delta$ per period. Thus employment $N_t$ and unemployment $U_t$ evolve

[9]Two notable exceptions are Acemoglu (2001) and Wasmer and Weil (2004) cited earlier. This paper is closest in spirit to the latter which first identifies the financial accelerator at play when hiring is conditional on the availability of external funds. Both papers, however, are mainly concerned with steady state implications, not the dynamic propagation of shocks.

[10]For a formal treatment of the set-up, see Mortensen and Pissarides (1994). The introduction of labor search to quantitative business cycle research is owed to the contributions of Merz (1995) and Andolfatto (1996). Labor force participation choices are not considered here, individuals are either employed or unemployed. See Wasmer and Garibaldi (2005) or Haefke and Reite (2006) for models of labor market participation.

according to

$$N_{t+1} = (1-\delta)N_t + p(\theta_t)V_t \tag{1.1}$$

$$U_{t+1} = (1-f(\theta_t))U_t + \delta N_t \tag{1.2}$$

The representative household, given existing employment and unemployment, chooses optimal consumption and purchases of risk-free bonds, which pay a rate $r_t$ the following period, in order to maximize the value function:[11]

$$V(N_t, U_t, B_{t-1}) = \max_{C_t, B_t} \left[U(C_t) + \beta E_t V(N_{t+1}, U_{t+1}, B_t)\right],$$

subject to the budget constraint $W_t N_t + bU_t + (1+r_{t-1})B_{t-1} + \Pi_t = C_t + B_t + T_t$, and the laws of motion for matched labor (1.1) and unemployment (1.2). The government raises $T_t$ in taxes to fund unemployment benefits $U_t b$, while employed workers earn the wage $W_t$. $\Pi_t$ are firm profits rebated lump sum at the end of the period. Denoting the multiplier on the budget constraint by $\lambda$, the first order conditions are

$$(C_t) \ : \ U_C(C_t) = \lambda_t \tag{1.3}$$

$$(B_t) \ : \ \lambda_t = \beta E_t \lambda_{t+1}(1+r_t) \tag{1.4}$$

### 1.2.2 Financial contract and vacancy decisions

The informational assumptions are chosen to generate standard debt contracts, in the tradition of Gale and Hellwig (1985) and Williamson (1987), set in a quantitative macroeconomic framework as in Carlstrom and Fuerst (1997). The contracts are written on a competitive capital market (in the sense that there is a large number of insignificant lenders and firms) and lenders are assumed to hold sufficiently large and diversified portfolios to ensure perfect risk pooling, with the result that investors behave as if they were risk neutral. Repayment of the debt is assumed to occur within the period such that there is a unit opportunity cost to funds.[12] The competitive pressure ensures that each lender-firm pair will write a contract which maximizes the expected value of the firm subject to the constraint that the expected return to the lender covers the opportunity cost of funds.[13]

[11] As in Andolfatto (1996), each worker is a member of a household that offers perfect insurance against labor market outcomes and is involved in a passive search process.

[12] The present contract is written for intra-period loans while Bernanke et al (1998) consider inter-period contracts which take into account aggregate uncertainty.

[13] If the expected utility of the firm is not maximized subject to this constraint, some other investor can offer a contract which is more attractive to the firm and still make a profit, see Gale and Hellwig (1985).

Define firm period net revenues as $x\,(X - W)\,N$, where $X$ is the aggregate level of technology, $W$ is the wage rate and $x$ is a random variable, i.i.d. across firms and time, with positive support, cdf $H(x)$, pdf $h(x)$ and $E(x) = 1$.[14] The crucial assumption is that agents have asymmetric information over the realization of the random variable $x$. This state can only be observed by lenders at some cost proportional to realized net revenues, $0 < \mu < 1$.

The timing of events in each period is as follows. Assume that vacancy costs $\gamma V$ must be paid before production occurs. All agents observe the aggregate state $X$ and, given initial assets $A$, firms borrow $(\gamma V - A)$ from financial markets to pay for period vacancy postings.[15] Lenders and borrowers agree on a contract that specifies a cutoff productivity $\overline{x}$ such that if $x > \overline{x}$, the borrower pays $\overline{x}\,(X - W)\,N$ and keeps the equity $(x - \overline{x})\,(X - W)\,N$. If $x < \overline{x}$, the borrower receives nothing and the lender claims the residual net of monitoring costs.

Define the expected gross share of returns going to the lender as

$$\Gamma(\overline{x}) = \int_0^{\overline{x}} x dH(x) + \int_{\overline{x}}^{\infty} \overline{x} dH(x)$$

noting that $\Gamma'(\overline{x}) = 1 - H(\overline{x}) > 0$ and $\Gamma''(\overline{x}) = -h(\overline{x}) < 0$, and expected monitoring costs as

$$\mu G(\overline{x}) = \mu \int_0^{\overline{x}} x dH(x)$$

with $\mu G'(\overline{x}) = \mu \overline{x} h(\overline{x})$.[16] It is easy to see that the expected gross share to the lender will always be positive.[17] Given this set of definitions we can conveniently express the lender's participation constraint as $[\Gamma(\overline{x}) - \mu G(\overline{x})]\,(X - W)\,N = (\gamma V - A)$, which states that the returns net of monitoring costs must equal the value of the loan.

Given the assumptions on the functional forms, notably constant returns to scale in production and a linear monitoring technology, only the evolution of aggregate assets is needed to know the cost faced by firms on credit markets and all firms will choose

[14] Alternatively the firm's period net revenue could be expressed as $(xX - W)N$ with $x$ drawn from a positive support with lower bound $W$. Either formulation guarantees a positive payoff function ensuring that the problem is well defined. This is similar to the approach in Carlstrom and Fuerst (2000) which consists of assuming that firms sell their product at a time varying mark-up over costs.

[15] Bank loan models, as in Christiano, Eichenbaum and Evans (2005) for example, assume that all current costs, in their case the wage bill, must be financed by bank loans. The assumption of a fraction of vacancy cost needing external financing is consistent with evidence on firm financing, such as Devereux and Schiantarelli (1989) and sufficient to generate the results in this paper.

[16] The expected share of returns going to the borrower under the contract is $\Upsilon(\overline{x}) = \int_{\overline{x}}^{\infty} (x - \overline{x})\, dH(x)$. Note that $\Gamma(\overline{x}) + \Upsilon(\overline{x}) = 1$.

[17] To do so, take the limits $\lim_{\overline{x}\to 0} \Gamma(\overline{x}) = \int_0^{\infty} \overline{x} dH(x) = 0$, $\lim_{\overline{x}\to \infty} \Gamma(x) = \int_0^{\infty} x dH(x) = 1 > 0$ and recall that $\Gamma(\overline{x})$ is strictly increasing and concave in $\overline{x}$.

the same ratio of vacancies to assets (see Carlstrom and Fuerst, 1997). These evolve according to $A_{t+1} = \varsigma \left[1 - \Gamma(\overline{x}_t)\right] (X_t - W_t) N_t$, where the parameter $0 < \varsigma < 1$ ensures self-financing does not occur.[18] Rearranging as

$$A_{t+1} = \varsigma \left[ (X_t - W_t) N_t - \left( 1 + \frac{\mu G(\overline{x}_t) (X_t - W_t) N_t}{\gamma V_t - A_t} \right) (\gamma V_t - A_t) \right] \tag{1.5}$$

focuses on the premium associated with external funds, $\frac{\mu G(\overline{x}_t)(X_t - W_t)N_t}{\gamma V_t - A_t}$, which for any $\mu > 0$ is strictly positive.

We can now write the optimal incentive compatible contracting problem with non-stochastic monitoring and repayment within the period. Vacancy postings and the threshold $\overline{x}$ are chosen to maximize the expected gross return to the firm subject to the lender's participation constraint

$$\begin{aligned} J(N_t, A_t) &= \max_{V_t, \overline{x}_t} \left[1 - \Gamma(\overline{x}_t)\right] (X_t - W_t) N_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J(N_{t+1}, A_{t+1}) \\ & \quad \text{subject to } \left[\Gamma(\overline{x}_t) - \mu G(\overline{x}_t)\right] (X_t - W_t) N_t = (\gamma V_t - A_t), \end{aligned}$$

and the laws of motion for employment (1.1) and aggregate assets (1.5), where firms use the stochastic discount factor $\beta E_t \frac{\lambda_{t+1}}{\lambda_t}$.

### 1.2.3 Job creation under credit constraints

Denoting the multiplier on the lender's participation constraint by $\phi$, the optimality condition for vacancy postings describes a job creation condition

$$\frac{\gamma \phi_t}{p(\theta_t)} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_n(N_{t+1}, A_{t+1})$$

equating the average cost of a vacancy, $\frac{\gamma \phi_t}{p(\theta_t)}$, to the expected marginal value of an additional employed worker $\beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_n(N_{t+1}, A_{t+1})$.

In order to derive the marginal value of a worker to the firm, $J_n(N_t, A_t)$, differentiate the firm's value function with respect to $N$,

$$J_n(N_t, A_t) = \left[1 - \Gamma(\overline{x}_t)\right] (X_t - W_t) + \phi_t \left[\Gamma(\overline{x}_t) - \mu G(\overline{x}_t)\right] (X_t - W_t) + (1-\delta) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_n(N_{t+1}, A_{t+1})$$

[18] The assumption of some depletion in the stock of assets is needed to rule out eventual self-financing. Carlstrom and Fuerst (1997) assume that consumers and entrepreneurs have different time discount factors, while Bernanke, Gertler and Gilchrist (1999) assume that a fraction of the entrepreneurial population exits every period consuming their assets on the way out. It is assumed here that firms retain a fraction of their earnings toward next period's assets while rebating the remaining to households as profits.

The first term corresponds to the net return on an employee accruing to the firm under the debt contract. The second term captures the value an additional worker brings to the firm by relaxing the financing constraint in terms of an increased ability to reimburse the loan. The final term captures the value of the continued relationship. For the sake of simplifying the notation, call $\Omega(\overline{x}_t) \equiv 1 - \Gamma(\overline{x}_t) + \phi_t [\Gamma(\overline{x}_t) - \mu G(\overline{x}_t)]$. Combining the marginal value of a worker with the optimality condition for vacancies, and making use of the household bond Euler equation (1.4), yields the intertemporal condition for vacancy postings

$$\frac{\gamma\phi_t}{p(\theta_t)} = \frac{1}{1+r_t} E_t \left[\Omega(\overline{x}_{t+1}) (X_{t+1} - W_{t+1}) + (1-\delta)\frac{\gamma\phi_{t+1}}{p(\theta_{t+1})}\right] \tag{1.6}$$

At this stage it is useful to show how this setting with credit frictions compares with a standard labor search model. Consider first the credit constraint multiplier $\phi_t$ on the cost side of the job creation condition. From the first order condition for the cutoff productivity, the multiplier may be expressed as

$$\phi_t = \frac{\Gamma'(\overline{x}_t)}{[\Gamma'(\overline{x}_t) - \mu G'(\overline{x}_t)]} \tag{1.7}$$

In the absence of monitoring costs the threshold $\overline{x}$ tends to the lower bound of its support. It is straightforward to show that $\partial\phi_t/\partial\overline{x}_t > 0$, and that in the limit $\lim_{\overline{x}_t \to 0} \phi_t = 1$. That is, for any positive monitoring cost, the presence of credit frictions drives up the average cost of vacancy postings to $\frac{\gamma\phi_t}{p(\theta_t)}$, as opposed to $\frac{\gamma}{p(\theta_t)}$; where $\phi_t$ can be interpreted as the shadow cost of external over internal funds.

Second, one can show that $\lim_{\overline{x}_t \to 0} \Omega(\overline{x}_t) = 1$ , such that in the absence of monitoring costs the first order condition (1.6) collapses to the standard job creation condition in a stochastic discrete time setting:

$$\frac{\gamma}{p(\theta_t)} = \frac{1}{1+r_t} E_t \left[X_{t+1} - W_{t+1} + (1-\delta)\frac{\gamma}{p(\theta_{t+1})}\right] \tag{1.8}$$

The received argument for the lack of amplification of productivity shocks is easily understood by this job creation condition equating the average cost of a vacancy to the expected benefit of a new job (see Shimer, 2005, Hall, 2005). A sudden rise in productivity, increasing the profits to the firm of a job, increases the incentive to post vacancies. The same rise in productivity, however, leads to a rise in the wage reducing the profits to firms. For most applications of the Nash bargaining solution, the wage is highly elastic to productivity such that the profits from a job for the firm are relatively inelastic to productivity shocks and, as a consequence, so are vacancy postings. There is, however, a second, overlooked, dampening mechanism built into the job creation condition. The same event leading to a rise in the job finding hazard for workers, and their ability to negotiate higher wages, also corresponds to an increase in the congestion facing firms. In other words, each job vacancy faces a decreasing probability $p(\theta_t)$ of being filled in a given unit of time. This increase in the average cost of hiring a worker further restricts firm entry, limiting the propagation of productivity shocks.

The first response to this issue has been to induce greater wage rigidity by either changing the structure of the model, i.e. settling on different wage determination mechanisms (Hall, 2003, Gertler and Trigari, 2009, Menzio, 2006), or following a calibration strategy resulting in a wage less elastic to productivity (Hagedorn and Manovskii, 2008). Here, credit frictions have the potential to amplify productivity shocks in manner that is fundamentally different, operating through the cost side of the job creation condition. Recall that in the presence of credit frictions the average cost to filling a vacancy is $\frac{\gamma\phi_t}{p(\theta_t)}$, whereas in the standard model it is $\frac{\gamma}{p(\theta_t)}$. The multiplier on the lender's participation constraint, $\phi_t$, which, as a measure of the shadow cost of external over internal funding, indicates how binding credit constraints are, in effect drives a time-varying wedge on the cost side relative to the frictionless model. If these constraints are counter-cyclical, or $\phi_t$ decreases during an economic upturn, there is a downward push on the average cost of vacancies that increases the incentive for firms to post vacancies.[19]

### 1.2.4 Workers and wages

The model is fully described once the rule for wages is determined. In order to define the values of a job ($V_n$) and unemployment ($V_u$) to a worker, differentiate the household's value function with respect to $N$ and $U$:

$$\begin{aligned} V_n(N_t, U_t, B_{t-1}) &= \lambda_t W_t + \beta E_t\left[(1-\delta)V_n(N_{t+1}, U_{t+1}, B_t) + \delta V_u(N_{t+1}, U_{t+1}, B_t)\right] \\ V_u(N_t, U_t, B_{t-1}) &= \lambda_t b + \beta E_t\left[(1-f(\theta_t))V_u(N_{t+1}, U_{t+1}, B_t) + f(\theta_t)V_n(N_{t+1}, U_{t+1}, B_t)\right] \end{aligned}$$

The current value of a job corresponds to the wage measured in utils and the discounted expected values of next period's state, which with probability $(1-\delta)$ remains employment. The value of unemployment is derived from the value of non-market activities, $\lambda_t b$, and the discounted expected value of next period's state, which with probability $f(\theta_t)$ is employment.

[19] In this formulation these constraints are counter-cyclical as the profitability on the investment project, here the net return from labor, rises more quickly than the leverage taken on by borrowers during an expansion. For a detailed analysis of the conditions under which credit market frictions create a financial accelerator which destabilizes the economy, see House (2006).

Splitting the surplus of a worker-firm match, defined as $S(t) = J_n(t) + \frac{V_n(t) - V_u(t)}{\lambda_t}$, under a generalization of Nash bargaining, as in Pissarides (2000), yields the wage rule[20]

$$W_t = \eta\omega_t \left[\Omega(\overline{x}_t)X_t + \gamma\phi_t\theta_t\right] + (1-\eta)\omega_t b \tag{1.9}$$

where $\omega_t = 1/\left[1 + \eta(\Omega(\overline{x}_t) - 1)\right]$. As with the job creation condition, when monitoring costs tend to 0 the wage rule (1.9) collapses to

$$W_t = \eta \left[X_t + \gamma\theta_t\right] + (1-\eta)b \tag{1.10}$$

This is simply the usual the wage rule without credit frictions and leads to the following proposition

Proposition 1 - *The canonical Mortensen-Pissarides search and matching model of equilibrium unemployment is a special case of the present model with frictional credit markets when the cost of monitoring tends to zero.*

While we will discuss in the next section the steady state and quantitative implications for labor-market dynamics, one important aspect of the modified wage rule is worth stressing here. A principal force in the cyclical properties of the wage rule is the term $\gamma\phi_t\theta_t$ which, along with the value of non-market activities, captures the relative bargaining positions of workers and firms. During an upturn, market tightness rises making it more costly for firms to pull out of the wage negotiations to search for another worker (recall that a rise in $\theta$ implies a drop in the probability of meeting a worker $p(\theta)$). In the presence of credit market frictions, the cost of a vacancy $\gamma\phi_t$ actually decreases during good times as conditions on credit markets improve. The strengthened bargaining position of firms limits somewhat the upward pressure on wages stemming from the rise in market tightness. The end result is to induce some degree of wage rigidity which will contribute to amplifying productivity shocks in the manner outlined above.[21]

### 1.2.5 Closing the model

From the household's budget constraint it is straightforward to derive an aggregate resource constraint

$$Y_t \left[1 - \mu G(\overline{x}_t)\right] = C_t + \gamma V_t.$$

[20] Wages are negotiated at the beginning of the period once the aggregate state is observed but before the firm draws an idiosyncratic productivity. The wage is not a function of the idiosyncratic productivity, lest it reveal the firm's productivity draw to creditors, but will reflect the terms faced by the firm on credit markets. It is assumed that wages cannot be renegotiated ex-post. Details on the derivation of the wage are presented in the appendix.

[21] As a note, both $\omega_t$ and $\Omega(\overline{x}_t)$ are relatively inelastic to productivity and will contribute only marginally to fluctuations in wages.

where $Y_t = X_t N_t$; $\mu G(\overline{x}_t)$ are resources consumed in monitoring and $\gamma V_t$ are vacancy costs.

The equilibrium of the model is then characterized by equations (1.3) and (1.4) from household optimization, a job creation condition (1.6), optimality condition for the threshold $\overline{x}_t$ in (1.7), the definition of market tightness, the lender's participation constraint, a wage rule (1.9), the aggregate resource constraint and laws of motion for asset accumulation, aggregate employment and unemployment.

## 1.3 Propagation properties of financial and labor market frictions

Before discussing some of the steady state labor market implications of credit market frictions in this setting, the assumptions on functional forms and calibration are presented in detail. The model is then solved by computing the unique rational expectations solution for a log-linearization around the deterministic steady state, and the dynamics are evaluated with a series of unconditional second moments and impulse response functions. The performance of the model is assessed by simulating a standard labor search model as a basis for comparison and performing a series a sensitivity analysis to key parameters and aspects of the model.

### 1.3.1 Functional forms and calibration

Following much of the real business cycle literature, aggregate technology is assumed stationary and to evolve according to

$$\log X_t = \rho_X \log X_{t-1} + \varepsilon_t^X,$$

with $\varepsilon_t^X \sim (0, \sigma_X^2)$ and $0 < \rho_X < 1$. Staying within this literature, the relevant parameters are chosen as $\rho_X = 0.975$ and $\sigma_X = 0.0072$ (e.g., King and Rebelo, 1999).

For household preferences, period utility is defined as $u(C) = \log C$. The idiosyncratic shock $x$ is assumed to follow a log-normal distribution with mean $E(x) = 1$; i.e. $\log(x) \sim N(-\frac{\sigma_{\log(x)}^2}{2}; \sigma_{\log(x)}^2)$. Finally, following much of the labor search literature, the matching technology is a Cobb-Douglas $M(U, V) = \chi U^\epsilon V^{1-\epsilon}$, with $0 < \epsilon < 1$ and $\chi > 0$.

The model is calibrated to quarterly data. The discount factor $\beta = 0.992$ is set so as to match an average annual real yield on a risk less 3-month treasury bill of 3.3%. For parameters pertaining to financial factors, the quarterly default rate is set to 1%, in the range of values reported in both Carlstrom and Fuerst (1997) and Bernanke et al. (1999), and implies a standard deviation of the idiosyncratic productivity $\sigma_x$ of 0.12. The resource cost of monitoring is set to $\mu = 0.0375$ so as to match a 3% steady state premium on external funds, which corresponds to the mid range of the spread between AAA and BAA commercial paper and a 3-month treasury bill over the period 1977-2004.[22] This resource cost of monitoring is much lower than in Carlstrom and Fuerst (1997) or Bernanke et al. (1999) in which it is set at 0.25 and 0.12, respectively. Evidence in Devereux and Schiantarelli (1989) suggests that firms fund over two thirds of their current expenses with internal funds, which is used to pin down the value of the parameter $\varsigma$. However, it is important to stress that in this model the fraction of current costs funded externally is in fact $\frac{\gamma V - A}{\gamma V + WN}$, which for any calibration is a very small fraction (around 1 to 2%).[23] Other investigations, such as Christiano et al (2005) assume that all current costs, in their case the entire wage bill, must be financed through bank loans. The sensitivity of the results to the calibration of the credit market will be examined below.

Several authors have argued that the targeted steady state rate of unemployment should include more than the rate of workers counted as unemployed as the model does not account for non-participation. Krause and Lubik (2007), for example, choose an unemployment rate of 12%, above the average rate observed for the United States. The benchmark calibration, however, will target a 7% unemployment rate as in Gertler and Trigari (2009). The cost of job vacancies is set to $\gamma = 0.125$, in the range of values suggested by the studies of Baron (1997) and Baron (1985), as cited in Ramey (2008). The elasticity in the labor matching function, $\epsilon$, is set to 0.6, which lies below the value of 0.72 used in Shimer (2005) but well within the range of values identified by Petrongolo and Pissarides (2001) in their survey of the matching function. The bargaining weight of the household in the wage negotiation, $\eta$, is set to 0.5. This mid-point is chosen to strike a balance between the extremes advocated in Hagedorn and Manovskii (2008) and Shimer (2005).[24] Finally, the quarterly rate of job separation is set to 6%, corresponding to the evidence presented in Davis, Faberman and Haltiwanger (2006), and the value of $\chi$ is chosen to obtain a job filling rate of 0.6.

The benchmark calibration results a replacement rate $b/w$ of 0.81. It is well known

[22] The yields are for Moody's seasoned AAA and BAA corporate bonds.

[23] In related evidence, Buera and Shin (2008) suggest that firms fund over 50% of their capital expenditure with external funds.

[24] The former adopt an extremely low value of the bargaining parameter in order to generate a wage with a low elasticity to productivity. The latter sets the bargaining weight equal to the weight on unemployment in the matching function as under the 'Hosios (1990) rule' in order to ensure constrained efficiency of the decentralized solution.

that the properties of labor search models change dramatically as this ratio tends to unity, and setting a high value as advocated by Hagedorn and Manovskii (2008) has the unappealing implication that workers gain little utility from accepting a job (see Mortensen and Nagypal, 2007).[25] While there is no definitive value for the replacement rate, the present calibration tries to stay clear of such issues by straying closer to the value used in Elsby and Michaels (2008).

### 1.3.2 Steady state implications

Proposition 2 - *There exists a unique steady state equilibrium in which the rate of unemployment is strictly increasing in the resource cost of monitoring, $\mu$.*

Proof. The job creation condition in the presence of credit constraints can be used to express the wage as a decreasing function of market tightness

$$w = 1 - \left(\frac{1}{\beta} - (1-\delta)\right) \frac{\phi\gamma}{\Omega(\overline{x})p(\theta)}$$

where aggregate productivity has been normalized to 1. Relative the to case with perfect credit markets, the additional cost induced by the necessity of external funds implies a steeper curve by the factor $\frac{\phi}{\Omega(\overline{x})} > 1$.[26] Figure 1.1 plots in $(\theta, w)$ space the job creation curve for the model with (solid line) and without (dashed line) credit frictions. The wage rule in the presence of credit frictions, $w = \eta\omega(\Omega(\overline{x}) + \gamma\phi\theta) + (1-\eta)\omega b$, has a slope $\omega\eta\gamma\phi$ greater than in the absence of credit market friction by the factor $\omega\phi > 1$ capturing the greater opportunity cost of a match to the firm that workers can exploit and, conditional on $\omega\left(\eta\Omega(\overline{x}) + (1-\eta)b\right) < 1$, the intersection of the wage rule and job creation condition is unique.

Combined, the two labor market equilibrium conditions, the job creation and wage rule, pin down equilibrium market tightness $\theta$ as

$$\gamma\left(\frac{\phi}{\omega\Omega(\overline{x})}\frac{(r+\delta)}{\chi}\theta^{\epsilon} + \eta\phi\theta\right) = (1-\eta)\left[1-b\right]$$

[25] The strategy employed here is to pin down the value of non-market activity such as to match an observed unemployment rate. This approach avoids some of the controversy surrounding the value of this parameter. Hagedorn and Manovskii (2008) reconcile the standard search model with key labor market statistics by employing an elevated value of the replacement rate of 0.96. Rotemberg (2006) chooses a value of 0.9, while Elsby and Michaels (2008) set the rate at a lower 0.86.

[26] $\frac{\phi}{\Omega(\overline{x})}$ is strictly increasing in $\overline{x}$ and $\lim_{\overline{x}-0} \frac{\phi}{\Omega(\overline{x})} = 1$.

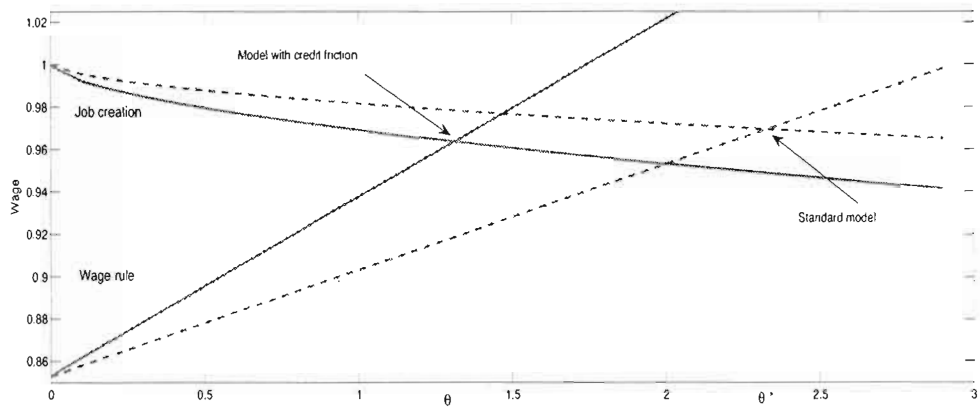


Figure 1.1: Steady state labor market equilibrium

which in the absence of credit friction is given by

$$\gamma\left(\frac{r+\delta}{\chi}\theta^{*\epsilon}+\eta\theta^{*}\right)=(1-\eta)\left[1-b\right]$$

where $\theta^*$ denotes equilibrium market tightness in the frictionless case. $\theta < \theta^*$ follows from the fact that $\phi > 1$ and $\frac{\phi}{\omega\Omega(\bar{x})} > 1$ for any strictly positive value of the monitoring cost $\mu$. To see the effect of an increase in $\mu$ on market tightness, consider first that $\frac{\partial\phi}{\partial\mu} > 0$, or that the measure of credit constraint is increasing in monitoring costs. Since it is also the case that $\frac{\partial\frac{\phi}{\omega\Omega(\bar{x})}}{\partial\mu} > 0$, an increase in monitoring costs leads to a decrease in equilibrium labor market tightness which, through the Beveridge relationship, implies a greater steady state rate of unemployment.[27] This insight is similar to that in Acemoglu (2001) and Wasmer and Weil (2004) in that credit frictions restrict firm entry on labor markets. Combined with a greater wage for every level of market tightness, credit frictions unambiguously lead to greater equilibrium unemployment.

### 1.3.3 Dynamic results

Several authors, as mentioned earlier, have noted the failure of the Mortensen-Pissarides framework to generate sufficient internal propagation of exogenous shocks to match key labor market statistics. Table 1.1 reports the Hodrick-Prescott filtered standard deviations relative to aggregate output of variables central to the labor market, along with their contemporaneous correlation with the cyclical component of aggregate output. The

[27] The effect on the equilibrium wage is ambiguous as higher recruiting costs both lowers job offers and affects the threat point in wage bargaining to the advantage of workers.

Table 1.1: Unconditional 2nd moments

| 1977:1-2005:4 | U.S. data | | Labor search | | Labor search with credit friction | |
|---|---|---|---|---|---|---|
| Variable: | a | b | a | b | a | b |
| $V$ | 8.83 | 0.89 | 3.70 | 0.93 | 9.39 | 0.98 |
| $\theta$ | 15.41 | 0.90 | 4.76 | 0.99 | 13.45 | 0.98 |
| $U$ | 6.83 | -0.88 | 1.70 | -0.77 | 4.92 | -0.79 |
| $N$ | 0.48 | 0.82 | 0.13 | 0.77 | 0. 37 | 0.79 |
| $\sigma(y)$ | 1.40 | | 1.04 | | 1.26 | |
| $corr(\Delta\theta, \Delta\theta_{-1})$ | 0.67 | | 0.03 | | 0.64 | |
| $corr(\Delta\theta, \Delta\theta_{-2})$ | 0.48 | | -0.01 | | 0.30 | |
| $corr(\Delta\theta, \Delta\theta_{-3})$ | 0.33 | | -0.01 | | 0.13 | |

a: Standard deviation relative to output; b: contemporaneous correlation with output. All moments, except market tightness growth, are Hodrick-Prescott filtered; Data sources: BLS, BEA.

first columns set the performance of the standard labor search model against moments from U.S. data and highlight its shortcomings in terms of amplification. The relative volatility of vacancies generated by the standard model is only 42% of that in the data. The dismal performance of the model extends to the measure of labor market tightness, which has a relative volatility of 15.41 in the data and 4.76 for the standard model. The performance in terms of unemployment or employment is hardly any better: the model generates a relative standard deviation for unemployment of 1.70 against a relative standard deviation of 6.82 in the data, or just 25% of the relative volatility observed in the data.

The second significant shortcoming regards the persistence in the adjustment to exogenous shocks. Evidence uncovered from reduced form VARs show that market tightness (and vacancies) have a sluggish response to productivity shocks, peaking several quarters after the innovation (see Fujita and Ramey, 2007). Another measure of this persistence, the auto-correlation in the growth rate of market tightness, is reported in the last three rows of Table 1.1. The data are characterized by a high degree of positive auto-correlation at the first three lags while the standard search model generates virtually no persistence.[28]

[28] This criticism resembles that addressed to RBC models regarding the persistence in the response of output to productivity shocks (see Cogley and Nason, 1995). The standard search model does generate some persistence in output growth, essentially because of the predetermined nature of employment, but still falls short of being consistent with the data. See also Andolfatto (1996).

#### 1.3.3.1 Vacancies and labor market tightness

We begin by examining, in Figure 1.2, the responses of vacancies and market tightness to a positive productivity shock in the standard (dashed line) and proposed (solid line) models. The introduction of credit frictions yields two improvements: first, the response is greatly amplified; second, the response is persistent, or the adjustment to the exogenous innovation is 'sluggish.' The unconditional second moments for the proposed model, presented in the last columns of Table 1.1, show that relative volatility of vacancies is large, at 9.39, and close to the value of 8.82 found in the data. The labor market tightness generated is also remarkably close to its empirical counterpart with a measure of relative volatility of 13.45, compared to 15.41 in the data. In term of persistence, vacancies, and market tightness, peak several quarters after the shock. More precisely, the model generates elevated positive autocorrelations in the growth rate of market tightness that are very close to the data at the first two lags, although decaying too rapidly at the third (see the last three rows of Table 1.1).

The large propagation potential of financial frictions results in a standard deviation of aggregate unemployment of 4.92, whereas the volatility of unemployment in the standard model is only 1.70 against 6.83 in the data. The standard deviation of aggregate output in the model with credit frictions, at 1.26, is closer to the value of 1.40 in the data. A standard labor search model generates a volatility of aggregate output barely beyond the impulse provided by the exogenous process with a standard deviation of 1.04. Figure 1.4, which plots the responses of output and unemployment to a positive productivity shock, illustrates the full impact of this financial accelerator on aggregate activity. Output continues to expand several quarters after the standard model has reached its peak and the strong flows of hiring lead to a deep drop in the unemployment rate.

Understanding the present results lies in the dynamics of the cost and wage channels of propagation outlined earlier. As both depend on the evolution of the shadow cost of external funds $\phi$, the first panel of Figure 1.3 plots the response of this measure of credit constraints following the same expansionary shock to productivity. While the constraint is relaxed on impact, the slow accumulation of assets pushes the constraint to its lowest several periods after the shock. The effect on the job creation condition through the cost of vacancies is strongest, therefore, several periods after the shock, as seen in Figure 1.2.

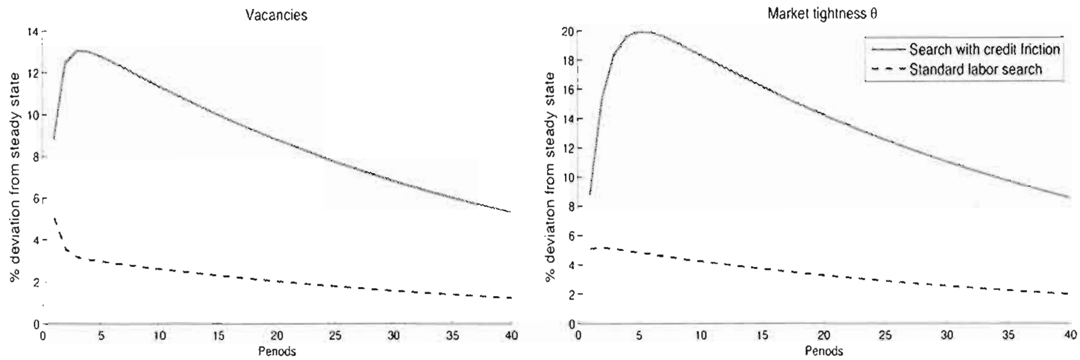


Figure 1.2: IRFs to a positive productivity shock, vacancies and market tightness

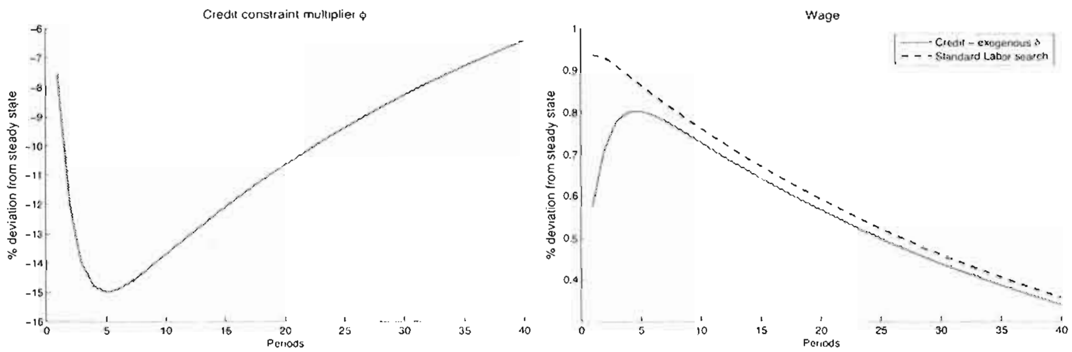


Figure 1.3: IRFs to a positive productivity shock, shadow cost of external funds and wage

The wage channel is illustrated in the second panel of Figure 1.3. Following an innovation to productivity, wages do not respond initially as strongly as in the standard model, and increase progressively for several quarters. This rigidity contributes to the elasticity of the initial response of market tightness and vacancies to a productivity shock, which is greater in the model with credit frictions (again, see the first panel of Figure 1.2). The ensuing rise in the wage, as market tightness continues to rise faster than $\phi$ decreases, counters some of the relaxing of the financing constraint for job creation. However, the continued rise in vacancies is testimony to the fact that the cost channel is largely dominant. The joint effect of these channels explains why the peak in market tightness is reached 6 quarters after the initial shock.

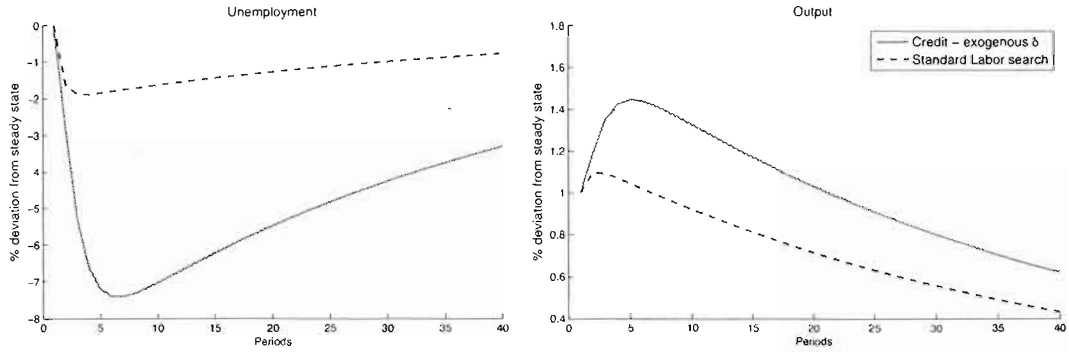


Figure 1.4: IRFs to a positive productivity shock, unemployment and output

#### 1.3.3.2 The shadow cost of external funds and robustness to the calibration of the credit market

The strength of these results relies on the degree of responsiveness of the shadow cost of external funds, $\phi$, to changes in aggregate productivity. At the height of the response to a positive innovation, this measure of financing constraints can drop by up to 15% relative to is steady state value, representing a high degree of volatility. While their is no empirical counterpart to verify directly the realism of such large changes, it is possible to compute the volatility implied on the premium on external funds, expressed as $\frac{\mu G(\overline{x}_t)(X_t - W_t)N_t}{\gamma V_t - A_t}$, and how it compares with the data. For example, the spread on AAA and BAA bonds have, respectively, standard deviations relative to output of 0.66 and 0.70, and contemporaneous correlations with output of -0.58 and -0.67. The benchmark calibration implies a premium that is slightly more volatile than for the highest quality commercial paper, with a relative volatility of 0.93 and a contemporaneous correlation with output of -0.98. The model, however, speaks more to the relation between the state of credit markets and fluctuations in the rate of unemployment. Along this dimension, the contemporaneous correlation of unemployment with the premium for the benchmark calibration is 0.82. This lies just above the correlations for the spreads in the data with unemployment, which is 0.62 on AAA bonds and 0.72 for BAA bonds.

To further ascertain the robustness of the results, the first columns of Table 1.2 present the effects of calibrating on either a 1% or a 4% premium by changing the resource cost of monitoring compared to the benchmark. A lower premium on external funds implies a reduced propagation of productivity shocks, the volatility of the v-u ratio dropping to 9.53 and the relative volatility of unemployment to 3.48. The inverse is observed when the premium on external finance is raised to 4%, the relative standard

Table 1.2: Robustness to credit market parameterization

| | Benchmark | | Premium | | | | $\frac{\gamma V - A}{\gamma V}$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | 1% | | 4% | | 50% | | 90% | |
| | a | b | a | b | a | b | a | b | a | b |
| $U$ | 4.92 | -0.79 | 3.48 | -0.75 | 5.50 | -0.81 | 2.75 | -0.69 | 6.08 | -0.86 |
| $V$ | 9.39 | 0.98 | 6.74 | 0.99 | 10.49 | 0.99 | 5.28 | 0.99 | 11.89 | 0.96 |
| $\theta$ | 13.45 | 0.98 | 9.53 | 0.98 | 15.03 | 0.98 | 7.5 | 0.95 | 16.68 | 0.99 |
| $\sigma(y)$ | 1.26 | | 1.14 | | 1.32 | | 1.08 | | 1.44 | |
| $corr(\Delta\theta, \Delta\theta_{-1})$ | 0.64 | | 0.49 | | 0.66 | | 0.42 | | 0.52 | |
| $corr(\Delta\theta, \Delta\theta_{-2})$ | 0.30 | | 0.25 | | 0.30 | | 0.29 | | 0.17 | |
| $corr(\Delta\theta, \Delta\theta_{-3})$ | 0.13 | | 0.12 | | 0.12 | | 0.19 | | 0.05 | |

a: Standard deviation relative to output; b: contemporaneous correlation with output.

All moments are Hodrick-Prescott filtered; Data sources: BLS, BEA.

deviation of $\theta$ rising to 15.03. With respect to the measure of persistence, the change in the premium affects mainly the auto-correlation at the first lag, a higher premium generating a greater coefficient. This occurs through a greater short run elasticity of the shadow cost of external funds to changes in productivity.

Next, the implications of changing the fraction of vacancy costs requiring external funds is examined in the last columns of Table 1.2. While this changes little the steady state premium on external funds, by affecting the elasticity of assets to aggregate shocks it changes significantly the dynamics of the shadow cost of external funds. For example, requiring 90% of vacancy cost to be funded externally increases the relative standard deviation of the v-u ratio to 16.68 and causes the short run auto-correlation in the growth rate of $\theta$ to decay more rapidly. When the fraction is reduced to 50%, there is little change to the persistence properties of the model but most of the result of amplification disappears. Further, reducing the cost of monitoring such that the premium on external funds funds tends to 0, along with reducing the requirement on external funds below 5% essentially reproduces the results of the standard model both in terms of volatility and persistence.

#### 1.3.3.3 Sensitivity to the calibration of the labor market and volatility of wages

This section first examines the sensitivity of the main results to changes in the calibration of labor market specific parameters. With results presented in Table 1.3, we look at the impact of variations in the unit cost of vacancies, the elasticity of the matching function with respect to unemployment and to different steady state rates of unemployment.

The dynamic properties of all labor search models are very sensitive to the value of unit search costs $\gamma$. When targeting a specific rate of unemployment, is implies quite different replacement ratios. For example, increasing its value from 0.125 to 0.25 reduces the replacement ratio to 63%. As a result the standard deviation of output drops from 1.26 to 1.09 while the relative volatility of labor market tightness declines to 7.80 from 13.45. The appropriate comparison however, in terms of evaluating the robustness of the results stemming from the inclusion of credit market frictions, is with those from the standard search model with $\gamma = 0.25$. Under such a scenario, the relative standard deviation of the v-u ratio generated is only 2.50, leading to the conclusion that the result of amplification is robust to variations in the unit cost of job vacancies. If the unit cost is reduced to 0.075, the standard deviation of market tightness increases to 18.62 in the model with credit friction and to 7.40 in the standard model. With regards to persistence, on the other hand, variations in the unit cost of vacancies have very little effect on the results presented for the benchmark calibration.

The elasticity of the matching function with respect to unemployment governs the manner in which movements in labor market tightness translate into changes in the hazard rates faced by workers and firms. A reduction in the elasticity with respect to unemployment, which increases the elasticity of the job finding rate to variations in market tightness, greatly increases the response of aggregate unemployment and output to exogenous innovations. For example, decreasing $\epsilon$ from 0.6 to 0.5 increases the standard deviation of output to 1.36 and the relative volatility of unemployment to 5.91. On the other hand, the incentives to post vacancies becomes less sensitive to the business cycle and the relative volatilities of both job vacancies and the v-u ratio decrease. The stronger movements in the rate of unemployment however, a stock variable in the model, tends to increase the persistence in the growth rate of market tightness at the first three lags, and most notably at the first. Increasing the value of the parameter $\epsilon$ to 0.75 has the inverse effects.

Finally, raising the target steady-state rate of unemployment, while increasing the volatility of aggregate output, has only a minor downward impact on the relative volatilities of vacancies, unemployment and the v-u ratio, and very little impact on the measure of persistence. The converse is true for a reduction in the target steady state rate of unemployment. Overall, the model behaves in its well-known directions (see Yashiv, 2006), and the main propagation mechanism outlined in the model is not altered by the calibration of the labor market.

The current model resulted in a certain degree of wage rigidity contributing amplification beyond that originating from the cost channel outlined above. The calibration

Table 1.3: Robustness to labor market parameterization

| | $\gamma$ | | | | $\epsilon$ | | | | $U$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.075 | | 0.25 | | 0.5 | | 0.75 | | 0.06 | | 0.10 | |
| | a | b | a | b | a | b | a | b | a | b | a | b |
| $U$ | 6.81 | -0.86 | 2.85 | -0.70 | 5.91 | -0.81 | 3.29 | -0.74 | 5.13 | -0.78 | 4.33 | -0.81 |
| $V$ | 12.99 | 0.98 | 5.45 | 0.99 | 8.10 | 0.97 | 11.66 | 0.99 | 9.27 | 0.99 | 9.7 | 0.96 |
| $\theta$ | 18.62 | 0.99 | 7.80 | 0.95 | 12.90 | 0.98 | 14.40 | 0.98 | 13.64 | 0.97 | 12.94 | 0.99 |
| $\sigma(y)$ | 1.51 | | 1.09 | | 1.36 | | 1.13 | | 1.21 | | 1.39 | |
| $corr(\Delta\theta, \Delta\theta_{-1})$ | 0.64 | | 0.63 | | 0.71 | | 0.51 | | 0.65 | | 0.58 | |
| $corr(\Delta\theta, \Delta\theta_{-2})$ | 0.30 | | 0.30 | | 0.34 | | 0.24 | | 0.30 | | 0.29 | |
| $corr(\Delta\theta, \Delta\theta_{-3})$ | 0.13 | | 0.13 | | 0.15 | | 0.10 | | 0.12 | | 0.14 | |

a: Standard deviation relative to output; b: contemporaneous correlation with output.

strategy employed by Hagedorn and Manovskii (2008), which they anchor on cyclical properties of an aggregate wage series, results in a standard deviation relative to output in the order of 0.20. Both Pissarides (2008) and Haefke et al. (2008), however, argue that the empirically relevant wage is that of new matches, or hires, which is characterized by near proportionality with productivity.[29] To situate in this context the degree of rigidity induced by the credit friction for the benchmark calibration, consider that it results in a standard deviation of wages of 0.57 whereas the standard labor search model generates a standard deviation of 0.86.[30] Therefore, it can be argued that the wage rigidity stemming from the interaction of labor market tightness and the shadow cost of external funds does not generate a high degree of wage rigidity, certainly not to the degree needed in Hagedorn and Manovskii (2008) to reconcile the standard model with the volatility of labor market variables observed in the data.

#### 1.3.3.4 Beveridge curve and cross-correlations

One concern for extensions to the standard framework is the violation of a robust empirical observation of a strong negative correlation between unemployment and vacancies, or the Beveridge curve.[31] Table 1.4 presents the contemporaneous cross-correlations of key labor market variables in the data and as generated by the models. In this respect

[29] For a surveys of wage time series and their properties, see Brandolini (1995) and Abraham and Haltiwanger (1995).

[30] The wage in all models is highly pro-cyclical in the sense of having a high degree of positive contemporaneous correlation with output.

[31] For instance, allowing for jobs to end endogenously by some efficient separation rule as in Mortensen and Pissarides (1994) leads to a counter-factual positive correlation between vacancies and the rate of unemployment.

Table 1.4: Labor market cross-correlations

| | U.S. data | | | | |
|---|---|---|---|---|---|
| | $U$ | $V$ | $\theta$ | $f(\theta)$ | $Y/N$ |
| $U$ | 1.00 | -0.89 | -0.97 | -0.95 | -0.41 |
| $V$ | - | 1 | 0.98 | 0.90 | 0.36 |
| $\theta$ | - | - | 1 | 0.95 | 0.40 |
| $q(\theta)$ | - | - | - | 1 | 0.40 |
| $Y/N$ | - | - | - | - | 1 |

| | Labor search | | | | | Labor search - Credit friction | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $U$ | $V$ | $\theta$ | $f(\theta)$ | $Y/N$ | $U$ | $V$ | $\theta$ | $f(\theta)$ | $Y/N$ |
| $U$ | 1.00 | -0.47 | -0.73 | -0.73 | -0.71 | 1.00 | -0.74 | -0.88 | -0.88 | -0.56 |
| $V$ | - | 1 | 0.95 | 0.95 | 0.96 | - | 1 | 0.97 | 0.97 | 0.96 |
| $\theta$ | - | - | 1 | 1 | 0.99 | - | - | 1 | 1 | 0.88 |
| $q(\theta)$ | - | - | - | 1 | 0.99 | - | - | - | 1 | 0.88 |
| $Y/N$ | - | - | - | - | 1 | - | - | - | - | 1 |

All moments are Hodrick-Prescott filtered; Data sources: BLS, BEA.

the proposed model is again an improvement on the standard model with a correlation between unemployment and vacancies of -0.74, two third of the way between the correlation in the data, -0.89, and the correlation generated by the standard model, -0.47.

The data are also characterized by a very strong negative correlation between the unemployment rate and the measure of labor market tightness, with a contemporaneous correlation of -0.97. The standard model generates a somewhat weak correlation -0.73. The presence of credit friction brings the correlation closer to the data at -0.88. By extension, the proposed model also improves on the correlation between the unemployment and job finding rates.

The proposed model is able to reduce the correlation between unemployment and labor productivity to -0.56, closer to a correlation of -0.41 in the data. This correlation is too strong in the standard labor search model, which generates a correlation of -0.71. This can be understood from the fact that the credit market imperfection, by amplifying movements in unemployment that peak several quarters after labor productivity, increases the disconnect between the two time series. Both models fall short, however, of being consistent with the correlations between labor productivity and vacancies or market tightness. These have a mild positive correlation in the data, around 0.4, whereas both models generate very high positive correlations.

## 1.4 Extension to endogenous job separation

The previous section assumed that all labor separations occurred at a constant, exogenous rate. However, endogenous job separations have been argued an important feature of equilibrium employment models both because time varying separations are a salient empirical observation and, by directly affecting the stock of matched job, offer an important mechanism for the propagation of exogenous shocks.[32] This section extends the basic set up to allow for an endogenous labor separation margin.

### 1.4.1 An endogenous job separation margin

Assume that each job within a firm draws an i.i.d. productivity $z$, where $z \in [0, \infty($ with cdf $H(z)$, pdf $h(z)$ and $E(z) = 1$, and that this job productivity is observed by both the worker and the firm before the idiosyncratic productivity $x$ is known. Firms and individual workers negotiate a wage conditional on the productivity of the job, $W(z)$, and a job drawing productivity $z < \overline{z}$ is not profitable and terminated.[33] Given frictional credit markets, this threshold is defined such that current net revenues are non negative, or $\overline{z}$ is such that $\overline{z}X - W(\overline{z}) = 0$. This job destruction margin differs for the efficient separation rule in Mortensen and Pissarides (1994) in which the value of the cut-off corresponds to that for which the job yields no surplus to either the worker or the firm. A separation rule that is efficient from the point of view of both parties involves a cut-off for which the losses in current revenue are equal to the expected value of the job in the future

$$z_t^* X_t - W_t(z_t^*) = -\beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_N(N_{t+1})$$

where $z_t^*$ is the job productivity threshold in the absence of credit market frictions. The restriction that firms cannot run current period losses implies that $\overline{z}_t > z_t^*$. In other words the cut-off productivity is higher in the presence of credit market frictions resulting in a higher rate of endogenous separations, and part of these separations will be inefficient.[34]

[32] Though Shimer (2007) and Hall (2005) argued against the job separation view of unemployment fluctuations, recent studies by Fujita and Ramey (2008) and Elsby, Michaels and Solon (2007) using the same CPS data have shown that separations play a significant part in accounting for variations in the rate of unemployment.

[33] Wages are not modeled as a function of the firm's idiosyncratic productivity as separations could be used by creditors as a source of information on the firm's productivity draw for the current period.

[34] The efficient threshold in the model with credit frictions would be $\widetilde{z}_t X_t - W_t(\widetilde{z}_t) = -\frac{\gamma \varrho_t}{p(\theta_t)\Omega(\overline{z}_t)}$, and all job destructions in between the job productivities $\widetilde{z}$ and $\overline{z}$ are inefficient. Adopting this threshold would, however, be inconsistent with the assumption of frictional credit markets.

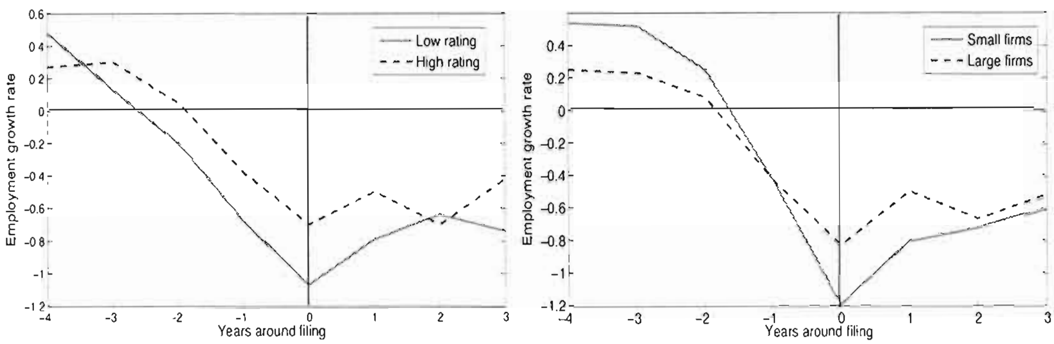


Figure 1.5: Employment growth during financial distress - by size and credit rating

In fact, there is evidence that more financially-constrained firms engage in stronger employment contractions during periods of financial distress. Using bankruptcy filings as an indicator of financial distress and size for as a proxy for access to credit markets, as in Gertler and Gilchrist (1994), the slowdown and contraction in employment growth at small firms is more pronounced than at larger, presumably less credit constrained, firms during the run-up to a bankruptcy filing (see the first panel of Figure 1.5).[35] A more direct measure of financial constraint is a firm's credit rating which dictates the terms of external financing. The contraction in employment at firms with a lower credit rating, as seen in the second panel of Figure 1.5, is much more severe than at high rating firms. Unfortunately this information cannot distinguish changes in employment due to a hiring freeze or a rise in job separations, due to layoffs or quits, as only information on the number of employees is provided.[36] Nonetheless, the work of Davis, Faberman and Haltiwanger (2006) indicates that an overwhelming majority of the change in employment at contracting firms is due to separations. This evidence provides support for an extension in which the rate of job separations as influenced by the degree of financial constraints.

With non-profitable jobs terminated before vacancies and debt contracts are determined, workers available for production are given by $\widetilde{N}_t = \int_{\bar{z}_t}^{\infty} N_t dH(z)$ , where $N_t$ is beginning of period employment. It is also assumed that some separations occur

[35] Firm level observations on employment are obtained from the Compustat data base and bankruptcy filing dates from the Bankruptcy Research Database. Small firms correspond to those with less than 1000 employees, large firms have over 4500 employees. The cut off for a low rating is CCC+, a good rating refers to BB+ or above.

[36] Indeed, it may be that current employees, observing private information concerning the productivity of the firm before outsiders, begin to search for other jobs on the current job. On the importance of job-to-job transitions see, for example, Fallick and Fleischmann (2004), Nagypal (2004) or Faberman and Nagypal (2007).

exogenously at the end of the period at the rate $\delta^x$ such that total job separation is $\delta_t = \delta^x + \int_0^{\overline{z}_t} dH(z)$.

The timing assumption affords the following two benefits. First, ex-ante all firms face identical problems and make identical choices such that the analysis continues in a representative firm framework.[37] Second, expected net revenues $\int_{\overline{z}}^{\infty} (zX - W(z))\, dH(z)\widetilde{N}$ are always positive, which leaves the expected gross shares of net revenue under the debt contract unchanged, and the optimal contracting problem is naturally expressed in a similar fashion with the appropriately modified participation constraint and laws of motion for employment and aggregate assets.[38] The optimality conditions for vacancy postings and the monitoring threshold retain the same form as earlier, and the job creation condition and wage rule are now

$$\frac{\gamma\phi_t}{p(\theta_t)} = \frac{1}{1+r_t} E_t \left[ \Omega(\overline{x}_{t+1}) \int_{\overline{z}_{t+1}}^{\infty} (zX_{t+1} - W_{t+1}(z))\, dH(z) + (1-\delta_{t+1}) \frac{\gamma\phi_{t+1}}{p(\theta_{t+1})} \right] \quad (1.11)$$

$$W_t(z) = \eta\omega_t \left[\Omega(\overline{x}_t) z X_t + \gamma\theta_t\phi_t\right] + (1-\eta)\omega_t b \quad (1.12)$$

### 1.4.2 Quantitative results

The idiosyncratic shock $z$ is assumed to follow a log-normal distribution with mean $E(z) = 1$; i.e. $\log(z) \sim N(-\frac{\sigma^2_{\log(z)}}{2}, \sigma^2_{\log(z)})$. The steady state endogenous job separation rate is set to 0.03, or 1/2 of total separations, which is in-line with evidence on the rate of layoffs in Davis, Faberman and Haltiwanger (2006). This results in a standard deviation of the job specific shocks of $\sigma_{log(z)} = 0.17$.

Endogenous job separations generate two opposing forces on the amplification of productivity shocks, with the second often overlooked in the literature. On the one hand, counter-cyclical movements in the rate of separation contribute to rapidly increasing employment and production during an economic upturn. In this sense endogenous separations provide a degree of amplification. On the other hand, this same mechanism puts a downward pressure on the expected benefit of new jobs or hires for firms, dampening their incentive to post vacancies. The results in Table 1.5 clearly show the impact of the two mechanisms outlined above. First, the volatility of aggregate output is amplified, rising from a standard deviation of 1.26 to 1.29. Note however that this amplification is

[37] Had the job productivity shock been drawn after the firm's idiosyncratic shock, the heterogeneity would lead to a multiplicity of separation conditions. It would then be necessary to follow a distribution of employment at each firm according to it's history.

[38] The contracting problem to this extension is fully set up in the appendix.

Table 1.5: Unconditional 2nd moments - extension to endogenous job separation

| 1977:1-2005:4 | U.S. data | | Labor search - Credit friction | | | | Labor Search - Standard | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Exogenous $\delta$ | | Endogenous $\delta$ | | Exogenous $\delta$ | | Endogenous $\delta$ | |
| Variable: | a | b | a | b | a | b | a | b | a | b |
| $V$ | 8.83 | 0.89 | 9.39 | 0.99 | 7.18 | 0.99 | 3.71 | 0.92 | 3.16 | -0.16 |
| $\theta$ | 15.41 | 0.90 | 13.45 | 0.98 | 11.44 | 0.95 | 4.76 | 0.99 | 4.03 | 0.99 |
| $U$ | 6.83 | -0.88 | 4.92 | -0.79 | 4.65 | -0.80 | 1.70 | -0.77 | 5.12 | -0.88 |
| $N$ | 0.48 | 0.82 | 0.37 | 0.79 | 0.35 | 0.80 | 0.13 | 0.77 | 0.39 | 0.88 |
| $\sigma(y)$ | 1.40 | | 1.26 | | 1.29 | | 1.03 | | 1.39 | |

a: Standard deviation relative to output; b: contemporaneous correlation with output.
All moments, but output growth, are Hodrick-Prescott filtered; Data sources: BLS, BEA.

less than for the standard model, which can be accounted for by a separation that is insufficiently volatile over the business cycle as compared to the extended standard model and the data. Second, the relative volatility of vacancies in the model with endogenous separation is lower than in the model with exogenous separations, dropping to 7.18. The same holds for the relative volatility of market tightness. Nonetheless the results are still a large improvement on the standard labor search model with either exogenous or endogenous separations. Results for the latter are reported in the last columns of Table 1.5. While output and unemployment are clearly more volatile, the relative volatility of market tightness is only 26% of that in the data.

#### 1.4.2.1 Endogenous separation and gross labor flows

Table 1.6 presents unconditional second moments for transition hazards and gross flows, in particular their H-P filtered standard deviations relative to output, and their contemporaneous correlation with output. While there is broad consensus concerning the strong procyclicality of the job finding hazard, which all models can replicate, the models with credit friction alone generates relative volatilities close to the data. While models of exogenous separations cannot match the dynamics of the separation rate in the data by construction, both extended models do a good job of matching the relative volatility and counter-cyclicality in the data although, as mentioned above, the model with credit frictions does not generate a sufficiently volatile rate of job separations.[39]

[39] I thank Shigeru Fujita and Garey Ramey for kindly sharing their data. The raw monthly series were first adjusted by a 12 month backward-looking moving average, as in Fujita and Ramey (2008). Quarterly series were then computed by averaging over monthly observations.

Table 1.6: Unconditional 2nd moments - labor market hazards and flows

| 1977:1-2005:4 | U.S. data | | Labor search - Credit friction | | | | Labor search - Standard | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | endogenous $\delta$ | | exogenous $\delta$ | | endogenous $\delta$ | | exogenous $\delta$ | |
| Hazard rates: | a | b | a | b | a | b | a | b | a | b |
| $f(\theta)$ | 4.83 | 0.6 | 4.58 | 0.95 | 5.38 | 0.98 | 1.61 | 0.99 | 1.90 | 0.99 |
| $\delta$ | 3.34 | -0.86 | 1.22 | -0.62 | - | - | 4.17 | -0.93 | - | - |
| Worker flows: | | | | | | | | | | |
| *Gross hires* | 2.02 | -0.53 | 1.93 | -0.45 | 2.53 | 0.55 | 3.98 | -0.73 | 1.34 | 0.44 |
| *Gross losses* | 2.98 | -0.86 | 2.02 | -0.82 | 0.37 | 0.79 | 3.93 | -0.91 | 0.13 | 0.77 |

a: Standard deviation relative to output; b: contemporaneous correlation with output. All moments are Hodrick-Prescott filtered; Data sources: BLS, BEA, Fujita and Ramey (2006).

The following decomposition is useful in understanding the predictions in terms of worker flows. The law of motion for aggregate unemployment separates all changes into flows of job losses and hires:

$$\begin{aligned} U_{t+1} - U_t &= \delta_t N_t - f(\theta_t) U_t \\ \Delta U_{t+1} &= Losses_t - Hires_t \end{aligned}$$

where gross job losses is denoted by $Losses_t = \delta_t N_t$ and gross hires are given by $Hires_t = f(\theta_t)U_t$. Gross hirings are the product of movements in the rate of unemployment and the job finding hazard. Since both move in opposite directions over the cycle. the model with endogenous separations is able to generate a negative correlation of gross hiring because the pool of unemployed is shrinking sufficiently to counter the effect of the rise in the job finding hazard. The assumption of constant job separation rates. on the other hand, leads to pro-cyclical gross hires (see Table 1.6), in contradiction of the strong negative correlation with the business cycle uncovered in the work of Blanchard and Diamond (1990) and Fujita and Ramey (2007).

It is no surprise that both models of constant separation rates are far outdone at matching the counter-cyclical gross job losses over the business cycle. The statistics on job losses for these models are simply those of aggregate employment as this is the only time-varying component of gross job losses, thus generating pro-cyclical gross job losses. For the current calibration gross job losses are almost as volatile as in the data in the extended model and are strongly counter-cyclical.

## 1.5 Conclusion

It has been argued that the standard model of equilibrium unemployment cannot generate sufficient propagation as productivity shocks, by inducing a rise in wages, have little effect on firm profits from a new employee and, hence, on the incentive to post new job vacancies. This paper has shown that when vacancies must be funded in part on frictional credit markets, agency problems can lead to higher, time-varying, unit costs that greatly increase the elasticity of vacancies to productivity. This propagation mechanism operates through two distinct channels: (i) a cost channel - lowered unit costs during an upturn as credit constraints are relaxed increase the incentive to post vacancies; (ii) a wage channel - the improved bargaining position of firms afforded by the lowered cost of vacancies limits part of the upward pressure of market tightness on wages. The quantitative exercise has shown that the cost channel is dominant in allowing the model to match the observed volatility of unemployment, vacancies and labor market tightness. Moreover, the progressive easing of financing constraints to innovations as firms accumulate assets generates persistence in the response of market tightness and vacancies, a robust feature of the data and shortcoming of the standard model. The paper thus concludes that the dynamics of vacancy creation costs are an essential element in understanding the cyclical behavior of job creation and the dynamics of the labor market. Extending the model to allow for endogenous job separations improved its ability to match gross labor flow statistics while preserving the propagation properties.

Two questions remain and warrant further investigation in subsequent research. First, how general these results are to the type of friction present on credit markets is an open question. This can, however, be partially addressed by considering that any friction which will generate a counter-cyclical cost of external funds will have the same qualitative implications. Second, if hiring is conditional on the state of credit markets, it may be that worker flows, as opposed to investment in new capital goods, are an alternative channel for the transmission of monetary policy shocks that affect the cost of credit. This avenue seems particularly promising as the propagation mechanism in the paper can be interpreted as increasing the rigidity of the firm's marginal cost to changes in production. Often referred in the New Keynesian literature as a greater degree of real rigidity, this property is known to be essential for the dynamics of inflation and for allowing any significant scope for monetary policy.

# Chapter 2

# Search in Physical Capital as a Propagation Mechanism

**Abstract**

This paper builds a model with search frictions for the allocation of physical capital and investigates its implications for the business cycle. While the model is in principle capable of generating substantial internal propagation to small exogenous shocks, the quantitative effects are modest once we calibrate the model to fit firm-level capital flows. The model is then extended to credit market frictions that lead to countercyclical default as in the data. Although countercyclical default directly affects capital reallocation, even in this extended model search frictions in physical capital markets play only a small role for business cycle fluctuations. [1]

## 2.1 Introduction

Physical capital is often specific to a certain task and/or fixed to a particular location. These specificities imply that physical capital markets are subject to potentially important allocation frictions. Most of the modern macro literature has ignored these market imperfections and examined instead the effects of aggregate investment constraints such as time-to-build delays (e.g. Kydland and Prescott, 1982) or convex adjustment costs (e.g. Cogley and Nason, 1995). The general conclusion from this literature is that in general equilibrium, such aggregate investment constraints have relatively small business cycle effects on their own. In this paper, we investigate whether the same holds true for market imperfections. In particular, we introduce search frictions for the allocation

[1] Written with A. Kurmann.

of physical capital into an otherwise standard real business cycle (RBC) model and ask whether these imperfections help generate more amplified and persistent responses to small exogenous shocks.

Our investigation is motivated by empirical evidence from industry- and firm-level data, discussed in detail in Section 2, that lead to three stylized observations. First, depending on the degree of specificity, a substantial amount of physical capital remains unmatched in any given period. Second, congestion in the physical capital market is countercyclical from the point of view of the supplier; i.e. the probability of (re-)allocating a given unit of capital to a firm increases in business cycle upturns and inversely decreases in downturns. Third, the distribution of investment rates across individual firms is wide, even in narrowly defined sectors and independent of aggregate conditions. The three observations suggest that physical capital markets are characterized by similar frictions to labor markets and thus, our modelization draws on the now widely employed search approach for the labor market, pioneered by Blanchard and Diamond (1990) and Mortensen and Pissarides (1994), and introduced into the DGE context by Merz (1995), Andolfatto (1996) and Den Haan, Ramey and Watson (2000).

The model we develop in Section 3 is populated by representative households and firms. Firms must post projects at a cost to search for available physical capital that is supplied endogenously by households.[2] The probability of a match varies with the state of the economy and depends on the ratio of available capital to the total number of posted projects. Once matched, households keep lending their capital to the same firm until separation, which is assumed to occur with exogenous probability in the baseline model. Once separated, the capital returns to the household for reallocation.

Under relatively weak conditions, the proposed search environment implies countercyclical congestion in physical capital markets, as in the data. This mechanism has potentially important aggregate consequences. In the wake of a positive technology shock, for example, the decrease in allocation frictions together with the presence of readily available unmatched capital means that the reaction of *productive* matched capital stocks and indirectly labor demand is more important than in the RBC benchmark. This effect continues over several periods after the shock and may lead to more amplified and persistent output dynamics.

To assess the quantitative importance of the search friction, Section 4 calibrates the model to fit long-run averages of firm-level capital flows using Compustat data and

[2] As opposed to most labor search models where the supply of available workers is fixed, we endogenize the supply of available capital for the model to be consistent with balanced growth properties of aggregate capital stocks.

compares its business cycle characteristics with the ones of the RBC benchmark. The main result is that capital flows in and out of production are not important enough for search frictions to have a significant impact. Only when we increase separation and reallocation to counterfactually large flows does the model generate more amplified and persistent output dynamics.

Based on this result, Section 5 extends the baseline model with credit market frictions. Following Townsend (1979), firms are subject to idiosyncratic productivity shocks that occur after all optimal decisions are taken and that households (the lenders) can observe only after incurring a monitoring cost. This costly state verification problem implies an optimal debt contract that results in endogenous capital separation through default. In particular, households monitor all loss-making firms and sever the lending relationship with those whose productivity level is below some threshold that makes refinancing more expensive than reallocating the capital to another firm.

The extension is motivated by the observation that different measures of financial distress and related capital sales / liquidations are countercyclical. Similar to Den Haan, Ramey and Watson's (2000) argument that countercyclical job destruction generates substantial internal propagation in labor search models, countercyclical capital separations in our model may magnify and prolong the effects of exogenous shocks as more (less) capital gets separated in downturns (upturns) and needs to go through a time-consuming reallocation process.[3] As an interesting by-product, the extended model also allows us to assess the importance of taking into account costly capital reallocation when quantifying the business cycle effects of credit frictions. In fact, existing DGE models with costly state verification such as Carlstrom and Fuerst (1997) or Bernanke, Gertler and Gilchrist (1999) only investigate the effects of net worth on investment and output but ignore the reallocation of capital from bankrupt firms. With the exception of a few special cases, these net worth effects alone have relatively small consequences for business cycle fluctuations. It is therefore interesting to see how the addition of costly capital reallocation changes this result.[4]

As the quantitative analysis reveals, the extended model indeed generates countercyclical capital separations as well as countercyclical risk premia, in line with the data. This latter result constitutes an improvement over the credit friction models of Carl-

[3] By contrast to Den Haan, Ramey and Watson (2000) where job destruction is an efficient outcome, capital separations in our model are the consequence of an information friction and thus socially inefficient. As we discuss in Section 5, this assumption is based on firm-level evidence indicating that capital separations due to (presumably efficient) sales and mergers are mildly procyclical rather than countercyclical.

[4] Section 5 provides more details about the business cycle effects of the net worth channel of credit frictions.

strom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999) where risk premia are either procyclical or acyclical.[5] The extended model also implies more volatile and persistent output fluctuations. Closer inspection reveals, however, that the increased internal propagation is mostly a general equilibrium effect brought about by a smaller (or even inverse) reaction of household consumption and thus labor supply to exogenous shocks. Once we calibrate the model to match the consumption dynamics in the data, the extended model implies only modest amplification and persistence. The conclusion of the paper thus remains that capital separation and reallocation flows on their own are too small for search frictions in physical capital markets to play an important role for business cycle fluctuations.

The results of our paper mostly concur with existing studies on the business cycle effects of physical capital specificities. Ramey and Shapiro (1998), for example, examine the aggregate effects of large military spending shocks in a world where moving capital from one sector to another is subject to a time-delay and a fixed cost. For certain specifications, they report some output amplification effects. However, these effects are based on unusually important sectoral shifts and the model is not analyzed in a full-blown DGE context. Boldrin, Christiano and Fisher (2001), in turn, consider a model with habit persistence and one-period inflexibilities for both labor and capital. While their focus is mostly on asset pricing implications, their model is capable of generating substantial persistence in output growth. However, this result seems to be due mostly to the imposed adjustment delay on hours worked. Finally, Veracierto (2002) examines the effects of investment irreversibilities and concludes that they do not matter for the business cycle.[6] The main contribution of our paper compared to these studies is that we focus more squarely on the *time-varying* nature of the market imperfections involved in the allocation of physical capital. First, we document that congestion in the physical capital market is countercyclical. Second, we introduce search frictions to capture the state-dependent nature of this congestion and show that it has interesting consequences in general equilibrium, mostly through its indirect effect on labor supply.[7] Third, we

[5]As we discuss in Section 5, the countercyclical risk premium is a direct consequence of the time-varying costs of incomplete contracting in a world with ex-post factor specificity that Willamson (1979) or more recently Caballero and Hammour (1996) term the *fundamental transformation problem.*

[6]A recent literature examines the role of nonconvexities in plant-level adjustment costs for aggregate investment dynamics, which can be considered as a combination of costs to both allocation of new capital and reallocation of used capital. See for example Kahn and Thomas (2006a) and the references therein. As in Veracierto (2002), these costs are found to have only small general equilibrium effects.

[7]Related to our model, Den Haan, Ramey, and Watson (2003) and Wasmer and Weil (2004) propose search frictions for the allocation of financing from lenders to firms. While relevant for new entrepreneurs and small firms, such frictions seem less obvious for large firms that account for the bulk of capital accumulation in the economy. Furthermore, their analysis is not carried out in a full-blown quantitative

are, to our knowledge, the first to explicitly calibrate a DGE model to gross capital flows from firm-level data. The relative unimportance of these capital flows (compared to, say, labor flows) is the main reason for our conclusion that search frictions in physical capital markets play only a modest role for business cycle fluctuations.

## 2.2 Empirical evidence

To motivate our extension of the RBC benchmark, this section first provides evidence on the time-varying nature of market imperfections in the allocation and reallocation of physical capital. Second, we review empirical studies on the wide distribution of investment rates across firms.

### 2.2.1 Allocation frictions for physical capital

Most physical capital is specific to a certain task and/or fixed to a particular location. The market imperfections brought about by these specificities are likely to imply substantial costs for the allocation and reallocation of physical capital. Similar to the labor market, one can think of these costs as search frictions that depend on the degree of specificity and potentially vary with business conditions. Unlike for the labor market where we observe aggregate unemployment and job advertisement rates, however, there is no comprehensive direct evidence on "unemployed" capital or unfilled investment projects.[8] Nevertheless, a substantial amount of indirect evidence exists that allows at least a partial characterization of the frictions involved.

We start by considering the market for leased non-residential property, which is one of the capital types most comparable to labor in the sense that similar to unemployment, vacant space is directly observable. Figure 2.1 shows the evolution of the average U.S. vacancy rate for industrial and office space in competitively leased multi-tenant buildings between 1988 and 2006. We obtained these data series from Torto Wheaton Richard Ellis, a large commercial real estate firm that surveys all major U.S. property markets on a quarterly basis.

Vacancies were at a record high at the end of the 1990-1992 recession, with the rate for office space approaching 20%. Vacancies then gradually decreased over the rest of the 1990s before jumping up again at the onset of the 2001 recession. On average, these vacancy rates are substantial (9.5% for industrial space and 14.5% for office space) and

DGE context.

[8] See Davis, Faberman and Haltiwanger (2006) for a recent survey of the relevant data for labor markets.

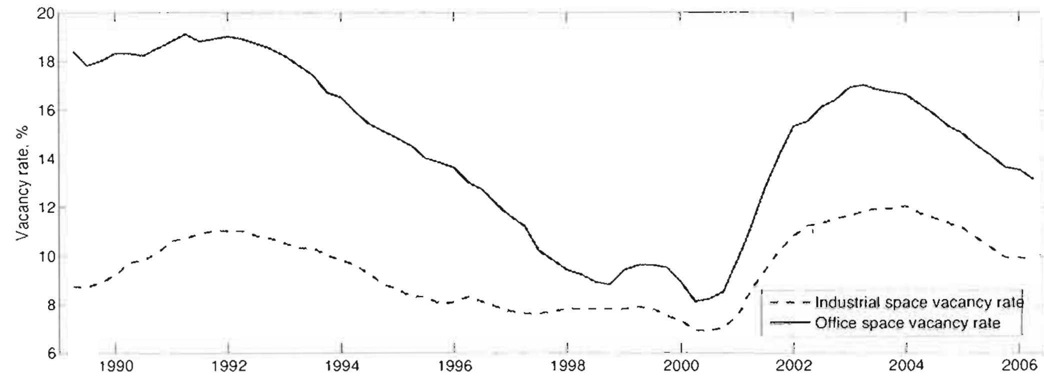


Figure 2.1: Vacancy rate for multi-tenant industrial and office space; average of 56 metropolitan U.S. markets. Source: Torto Wheaton Richard Ellis.

their time-varying nature suggests that congestion in the non-residential property market (from the point of view of the proprietor) varies inversely with the business cycle.[9]

Industrial and office space is, of course, a very specific type of capital because it is bound to a particular location and can hardly be converted for alternative usage. On the other end of the spectrum are newly finished, relatively mobile capital goods. Here, the BEA's Survey of Current Business (2000) allows us to observe detailed time series on inventories and output from capital goods producing industries. Using this information, we can compute the hazard rate $q_{it}$ with which a new unit of capital good $i$ is allocated as follows:

$$v_{it} = (1 - q_{it})(v_{it-1} + y_{it}),$$

with $v_{it}$ and $y_{it}$ denoting end-of-period $t$ inventories and output during period $t$ of capital good $i$, respectively. Table 2.1 reports the results for three large categories of finished capital goods over the sample 1977 to 1999.

As expected, the allocation rate for these capital goods is closer to unity (no friction) as production can be adjusted to accommodate demand and none of the capital types is bound to a specific location. Nevertheless, it is interesting to observe that industrial machinery – presumably a more specific capital good – takes on average longer to be allocated (i.e. a lower $q$) and congestion in that market reacts more inversely with the business cycle (i.e. the allocation rate $q$ is more procyclical).[10]

[9] Unfortunately, Torto Wheaton does not provide information on newly vacated space and, to our knowledge, none of the U.S. statistical agencies provides comparable data on the non-residential property market. Hence, we cannot compute hazard rates for the transition out of vacancy as it possible for the labor market where we have separate time series on newly unemployed individuals (e.g. Shimer, 2005).

[10] The traditional explanation for the existence of inventories relies on the assumption that production

Table 2.1: Allocation rates of finished capital goods

| | Average $q$ | $corr(q_t, gdp_t)$ |
|---|---|---|
| Industrial machinery and equipment | 0.70 | 0.40 |
| Motor vehicles and equipment | 0.83 | 0.16 |
| Electronic and other electric equipment | 0.90 | 0.16 |
| Average | 0.86 | 0.36 |

Notes: Second moments relate to Hodrick-Prescott filtered data

Aside from these direct measures, there is a host of indirect evidence about the importance and the countercyclical nature of the frictions in physical capital markets, especially where the reallocation of used capital is concerned. Eisfeldt and Rampini (2006), for example, use Compustat data to show that reallocation of used capital (measured as sales of plant, property and equipment plus acquisitions as a fraction of gross investment) is highly procyclical, with a Hodrick-Prescott filtered correlation coefficient with GDP of 0.64 for the sample 1971-2004.[11] By contrast, different measures of the benefits from reallocation (dispersion in firm level Tobin's $Q$, firm level investment rates, total factor productivity growth rates, and capacity utilization) are all countercyclical. If there were no reallocation frictions or if the degree of congestion in the used capital market was constant, we would expect most reallocations to take place when the benefits are greatest. Yet, exactly the opposite is the case.

Another piece of indirect evidence about reallocation frictions for used capital comes from a case study by Ramey and Shapiro (2001) who measure the resale value of equipment after the closure of three aeronautical plants. They find that other aerospace companies are overrepresented among buyers, and that even after taking into account age-related depreciation, the average resale value of equipment is only 28% of the replacement cost.[12] Although some of these losses may be due to unaccounted obsoleteness, Ramey and Shapiro's results suggest that the frictions involved in the reallocation of used capital are substantial. Otherwise, the used capital would not sell at such a large discount below its productive value.

is costly to adjust. As a result, firms use inventories to smooth production when faced with fluctuating sales (e.g. Blinder and Maccini, 1991). An alternative explanation relies on the existence of fixed delivery costs, inciting firms to hold inventory stocks. Firms thus make adjustments only when stocks are sufficiently far from their target (e.g. Kahn and Thomas, 2006b). Our argument of congestion differs from these explanations in the sense that we interpret the variation in hazard rates for inventory exit across goods as evidence of different degrees of market imperfections.

[11] Compustat collects a wide range of data, including information on physical capital, for all publicly traded firms in the U.S. We discuss this dataset in more detail in the calibration part of Section 4.

[12] Even for machine tools, which typically have a better resale value than specialized aerospace equipment, the resale value is only about 40% relative to the replacement cost.

Besides market imperfections in general, the specificity embodied in most physical capital can lead to an additional equilibrium effect that Shleifer and Vishny (1992) call *asset illiquidity* and that may explain part of the surprisingly low resale prices reported in Ramey and Shapiro's case study. Shleifer and Vishny argue that when firms sell assets or liquidate to meet financial constraints, the specific nature of capital means that the buyers who value these assets most are likely to be firms in the same industry. But financial distress often affects industries as a whole, which means that these buyers are likely to be financially constrained as well. As a result, the assets are sold at a steep discount within the same industry or to less constrained industry outsiders who have a lower valuation because the characteristics of the sold asset are suboptimal for their line of business or because they cannot value the asset appropriately.[13] Pulvino (1998) provides evidence about the countercyclical nature of asset illiquidity from the used aircraft market. Based on U.S. data of commercial aircraft transactions, Pulvino finds that financially constrained airlines sell air crafts at a 14% discount to the average market price, but that these discounts exist only in times when the airline industry is depressed and not when it is booming. Furthermore, aircraft leasing institutions pay a discount of 30% during industry recessions because they themselves value air crafts much lower than the actual airlines and because the risk associated with finding another lessee during recessions is much higher than in upturns.

A final, more aggregate piece of evidence about the frictions involved in the reallocation of physical capital comes from Becker et al. (2005) who use data from the Annual Capital Expenditure Survey (ACES). In existence since 1993, ACES is a representative dataset of U.S. firms that can be used to compute the capital stock of firms that disappear, either because they cease to be active or because they continue to operate under a different firm. The resulting series of total separated capital can then be compared with the following year's series of aggregate used capital expenditures. For the period 1993-1999, the resulting ratio of separated capital to used expenditures equals on average 64%, suggesting that reallocation frictions are substantial.[14]

In sum, the evidence presented here leads us to the following two stylized character-

[13] Ramey and Shapiro (2001) advance a telling example about a wind tunnel that was constructed to test aeronautical parts at high air speeds and that was leased out afterwards to test bicycle helmet designs.

[14] As other data sets on capital expenditures, ACES comes with several caveats. See Becker et al. (2005) for a detailed discussion. Also, the 64% absorption rate could be biased either upwards or downwards. On the one hand, expenditures in used capital include assets sold by continuing firms, which makes the effective absorption rate for separated capital from firm death even lower. On the other hand, some of the separated capital may be exported abroad in which case the effective absorption rate is higher.

izations of physical capital markets. First, allocation frictions for physical capital can be sizable depending on the degree of specificity of the capital good and whether it is new investment or a reallocation to another firm. Second, congestion in the physical capital market varies inversely with the business cycle; i.e. it is more costly and time-consuming to (re-)allocate physical capital to a firm in business cycle downturns than it is in upturns.

### 2.2.2 Distribution of investment rates across firms

Further evidence suggesting that the allocation of physical capital is probabilistic in nature comes from the well-documented wide distribution of investment rates across firms. Studies by Caballero, Engel and Haltiwanger (1995), Doms and Dunne (1998), Cooper, Haltiwanger and Power (1999) or Cooper and Haltiwanger (2005) show that investment at the plant level is characterized by a wide distribution. At any given point in time, there is a substantial mass of establishments with zero investment that coexists with establishments that have investment rates above 20% of their capital stock (i.e. investment spikes).[15]

Most of the literature has interpreted this large distribution of investment rates across establishments as the result of plant-specific productivity and non-convex adjustment costs that lead to (S,s) type investment rules (e.g. Khan and Thomas, 2006a and references therein). While this approach is certainly capable of rationalizing the observed data, the wide distribution of investment rates - even in narrowly defined sectors – affords another, potentially complementary explanation: one that focuses on *market imperfections* in the allocation of physical capital. In fact, there is plenty of circumstantial evidence suggesting that in expansionary periods, firms face sometimes substantial difficulties in securing a reliable supplier of capital goods.[16]

## 2.3 Model

As in the frictionless RBC benchmark, our model is populated by two agents: firms that produce using capital and labor; and households who decide on optimal consumption, leisure and investment in productive capital. But instead of instantaneous allocation, the matching of capital from households with firms involves a costly and time-consuming

[15]Becker et al. (2005) reconfirm these findings in their summary using plant-level data from the Annual Survey of Manufacturers (ASM).

[16]Interestingly, Statistics Canada collects information on intended capital purchases in one of their firm-level surveys that could be compared over time to actual expenditures. Unfortunately, this information is not publicly available at the moment.

search process. This search process is in principle very similar to the standard labor search environment (e.g. Andolfatto, 1996), with the exception that we endogenize the supply of available capital. This complication is necessary because depreciated capital needs to be replaced and, more importantly, because we want our model to be consistent with the stylized fact that output and capital grow on average at the same rate.

At the same time, our model retains a number of other simplifications that facilitate comparison with the RBC benchmark. First, there is no distinct sector for capital allocation. Instead, households directly act as capital lenders. Second, the same matching friction applies to the allocation of both new and used (i.e. previously separated) capital. This renders the analysis considerably easier as we do not need to keep track of different types of capital. Third, production is constant-returns-to-scale. Firms therefore choose the same optimal capital-labor ratio, independent of firm size, which allows us to abstract from firm heterogeneity.

### 2.3.1 Search and matching in the capital market

Capital is either in a productive state or in a liquid state. We define by $K_t$ the capital stock that enters the production function of a representative firm in period $t$. Liquid capital $L_t$, in turn, is made up of two components: used capital that has been separated previously from other firms and new capital made available by households.

To undertake investments, firms must post projects and search for liquid capital at cost $\kappa$ per project. We denote by $V_t$ the total number of posted projects in period $t$. Total capital additions to production in period $t$ is the result of a matching process $m(L_t, V_t)$, with $\partial m(\cdot)/\partial L_t > 0$ and $\partial m(\cdot)/\partial V_t > 0$. A firm's probability to find capital is therefore given by $p(\theta_t) = \frac{m(V_t, L_t)}{V_t}$ with $\partial p(\theta_t)/\partial \theta_t > 0$, where $\theta_t = \frac{L_t}{V_t}$ is a measure of congestion in the physical capital market from the household's point of view (i.e. the capital supplier). Likewise, the probability of liquid capital being matched to a firm equals $q(\theta_t) = \frac{m(V_t, L_t)}{L_t}$ with $\partial q(\theta_t)/\partial \theta_t < 0$.[17] Firms and households are assumed to be sufficiently small to take $p(\theta_t)$ and $q(\theta_t)$ as exogenous.

Capital matched to a firm in period $t-1$ enters production in period $t$. This match between firm and capital continues into period $t+1$ with probability $(1-s)$ and so on

[17] In addition, to ensure that $p(\theta_t)$ and $q(\theta_t)$ are between 0 and 1, we require that $m(L_t, V_t) \leq min[L_t, V_t]$

for the periods thereafter. Hence, the evolution of the capital stock is described by[18]

$$K_{t+1} = (1-\delta)(1-s)K_t + m(L_t, V_t). \tag{2.1}$$

With probability $s$, the match is terminated, in which case a fraction $\varphi$ net of depreciation $\delta$ of the capital is returned to the household; i.e. the household receives $\varphi(1-\delta)sK_t$. The remainder $(1-\varphi)(1-\delta)sK_t$ is a deadweight loss incurred during the separation process. Note that in this baseline formulation of our model, we keep the separation rate $s$ exogenous. In Section 5 below, we introduce credit market frictions to endogenize the separation rate.

### 2.3.2 Firms and households

At the beginning of each period, firms and households observe exogenous aggregate technology $X_t$. Given the existing capital stock $K_t$, the representative firm then posts new projects $V_t$ at unit cost $\kappa$ and hires labor $N_t$ to produce output $Y_t$ with constant-returns-to-scale technology

$$Y_t = f(X_t N_t, K_t), \tag{2.2}$$

with $f_N$, $f_K > 0$ and $f_{NN}$, $f_{KK} < 0$. The resulting profit maximization problem is described by

$$\begin{aligned} J(K_t) &= \max_{N_t, V_t} \left\{ f(X_t N_t, K_t) - \rho_t K_t - W_t N_t - \kappa V_t + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} J(K_{t+1}) \right\} \\ \text{s.t. } K_{t+1} &= (1-\delta)(1-s)K_t + p(\theta_t)V_t, \end{aligned}$$

where $\rho_t$ is the rental rate of capital; $W_t$ is the wage per unit of labor; and $\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}$ is the discount factor of future cash flows. This discount factor is a function of the marginal utility of consumption $\Lambda$ because the firm transfers all profits to the households. The firm takes $W_t$ and $\rho_t$ as exogenous. The exogeneity of $W_t$ is a direct consequence of our assumption of competitive labor markets. The exogeneity of $\rho_t$, in turn, implies that firms do not internalize the effects of their capital stock on the marginal productivity of capital and thus on the negotiation of $\rho_t$ (see below).

The first-order conditions of the optimization problem are

$$(N_t) : f_N(X_t N_t, K_t) = W_t \tag{2.3}$$

$$(V_t) : \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} J_K(K_{t+1}) = \frac{\kappa}{p(\theta_t)}. \tag{2.4}$$

[18] Since firm size is indeterminate, the separation rate $s$ describes either the probability that a firm disappears in a given period or the fraction of capital that gets separated from a given firm (aside from depreciation). In either case, the evolution of the aggregate capital stock is described by (2.1).

Equation (2.3) is the standard labor demand. Equation (2.4) states that the expected discounted marginal value of an additional unit of matched capital has to equal its average cost $\kappa/p(\theta_t)$, with the marginal value of an additional matched unit of capital $J_K(K)$ being defined as

$$J_K(K_t) = f_K(X_tN_t, K_t) - \rho_t + (1-\delta)(1-s)\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} J_K(K_{t+1}). \qquad (2.5)$$

This equation states that the value to the firm of an additional unit of capital is worth today's marginal product of capital net of the rental rate plus the expected future value net of depreciation in case the match is continued.

Households maximize the expected discounted flow of utility $u(C_t, 1-N_t)$ over consumption $C_t$, leisure $1 - N_t$ and the amount of liquid capital $L_t$ destined for matching with firms. Time spent working yields revenue $W_tN_t$, capital matched last period yields revenue $\rho_tK_t$, while unmatched capital is carried into the present period with zero net return. Formally, this problem is described by

$$\begin{aligned} V(U_t, K_t) = & \max_{C_t, N_t, L_t} [u(C_t, 1-N_t) + \beta E_t V(U_{t+1}, K_{t+1})] \\ & + \Lambda_t[W_tN_t + \rho_tK_t + \varphi(1-\delta)sK_t + U_t + D_t - C_t - L_t] \\ \text{s.t. } K_{t+1} = & (1-\delta)(1-s)K_t + q(\theta_t)L_t \end{aligned}$$

where $U_t = (1-q(\theta_{t-1}))L_{t-1}$ is the quantity of unmatched capital in the beginning of $t$; $D_t$ are firm profits transferred to households, and $\varphi(1-\delta)sK_t$ is the amount of separated capital returned into the budget constraint. Similar to the firm's optimization problem, we assume that the household considers $W_t$ and $\rho_t$ as exogenous.

The first-order conditions of the optimization problem are

$$(C_t) : u_C(C_t, 1-N_t) = \Lambda_t \qquad (2.6)$$

$$(N_t) : u_N(C_t, 1-N_t) = \Lambda_tW_t \qquad (2.7)$$

$$(L_t) : \beta E_t[V_U(U_{t+1}, K_{t+1})(1-q(\theta_t)) + V_K(U_{t+1}, K_{t+1})q(\theta_t)] = \Lambda_t \qquad (2.8)$$

The first two conditions are standard. The third condition states that the discounted expected utility of a marginal unit of liquid capital $L_t$ must equal the marginal utility of an additional unit of consumption. With probability $(1-q(\theta_t))$ liquid capital remains unmatched and is worth $V_U(U_{t+1}, K_{t+1})$ to the household, while with probability $q(\theta_t)$ it is matched with a project and turned into productive capital with marginal value $V_K(U_{t+1}, K_{t+1})$. From the above Bellman equation, we can derive these marginal values as

$$V_U(U_t, K_t) = \Lambda_t \qquad (2.9)$$

$$V_K(U_t, K_t) = \Lambda_t[\rho_t + \varphi(1-\delta)s] + (1-\delta)(1-s)\beta E_t V_K(U_{t+1}, K_{t+1}). \quad (2.10)$$

### 2.3.3 Rental rate of capital and equilibrium

To close the model, we follow much of the labor search literature and assume that once matched, households and firms determine the rental rate of capital by Nash bargaining over the surplus of the match. The relevant surplus is the sum of marginal benefits to each party: $S_t = J_K(K_t) + \frac{V_k(U_t,K_t)-V_U(U_t,K_t)}{\Lambda_t}$. Defining $\eta$ as the household's relative bargaining power, the household thus receives $\frac{V_k(U_t,K_t)-V_U(U_t,K_t)}{\Lambda_t} = \eta S_t$, while the firm's share is $J_K(K_t) = (1-\eta)S_t$. After some algebraic manipulations that are detailed in the appendix, we obtain the following expression for the rental rate

$$\rho_t = \eta\left[f_K(X_tN_t, K_t) + (1-\delta)(1-s)\frac{\kappa}{\theta_t}\right] + (1-\eta)[\delta + (1-\varphi)(1-\delta)s]. \qquad (2.11)$$

The first term in brackets is the maximum amount the firm is willing to pay per unit of capital. It equals the marginal product of capital plus the average cost that is saved by entering the proposed capital match rather than continuing to search. The second term in brackets is the household's opportunity cost of entering the proposed capital match, which equals the fraction not lost to depreciation when capital remains liquid, $\delta$, plus the deadweight loss in case the capital gets separated $(1-\varphi)(1-\delta)s$.

As mentioned before, the constant-returns-to-scale assumption for technology implies that all firms choose the same optimality conditions. The equilibrium of the economy is thus defined by the system of equations (B.1)-(B.18) and the definition of aggregate dividends $D_t = Y_t - W_tN_t - \rho_tK_t - \kappa V_t$ (see appendix for details). Dividends are positive because the search friction gives rise to a surplus for each unit of matched capital that the firm and household split as specified above.

### 2.3.4 Comparison with the RBC benchmark and qualitative considerations

In the following analysis, it will be useful to compare our capital search model with the RBC benchmark where capital can be allocated costlessly and instantaneously (see for example King and Rebelo, 2000). In fact, our model collapses to the RBC benchmark for the case where the cost of project postings $\kappa$ and the deadweight loss from separations $1-\varphi$ are both zero. Firms then post an infinity of projects and all capital is reallocated in the beginning of each period; i.e. $s = 1$, $q(\theta_t) = 1$ and $U_t = 0$. Under these assumptions, it can be shown that the repayment on liquidity equals the marginal product of capital: $\rho_t = f_K(X_tN_t, K_t)$.[19] Furthermore, choosing liquid capital $L_t$ amounts to di-

[19] The bargaining power $\eta$ is irrelevant in this case because perfect competition in the capital market draws the surplus between firms and lenders to zero.

rectly choosing the new stock of capital $K_{t+1}$. This implies a value of matched liquidity $V_K(U_t, K_t) = \Lambda_t[\rho_t + (1-\delta)]$, and the optimality condition for the choice of liquidity (i.e. new investment) reduces to the standard Euler equation: $\beta E_t \Lambda_{t+1}[\rho_{t+1} + (1-\delta)] = \Lambda_t$. Finally, by combining the household's budget constraint with the firm's first-order conditions and the capital accumulation equation, we recover the familiar national accounting identity of the RBC benchmark $Y_t = C_t + K_{t+1} - (1-\delta)K_t$.

The national accounting identity of our capital search model is quite different. Specifically, the household's budget constraint together with the definition of dividends yields

$$Y_t = C_t + [L_t + \kappa V_t] - [\varphi(1-\delta)sK_t + U_t]. \tag{2.12}$$

The first term in brackets on the right-hand side represents the total resources devoted to *gross investment* by households and firms. The second term in brackets denotes *idle capital* in the form of newly separated capital and unmatched capital from the previous period. The difference between the two quantities defines *net new investment*. Idle capital thus drives a wedge in the economy's resource constraint that increases the amount effectively made available to firms without affecting consumption. Akin to unemployment in models with labor market frictions, the presence of these additional resources may magnify and prolong the economy's reaction to shocks.

The second potential source of internal propagation in our model is the state-dependent nature of the search friction. In response to a persistent increase in aggregate productivity $X_t$, the marginal value of future matched capital increases. By virtue of conditions (2.4) and (2.12), firms and households thus find it optimal to increase $V_t$ and $L_t$, respectively. Which of the two responses is larger depends on the exact specification of the model and thus, we cannot say in general whether congestion in the physical capital market is procyclical or countercyclical. However, by combining (2.4) and (2.12) with the definition of the division of the surplus, we can show that the following proposition holds.

***Proposition 1*** - Congestion in the physical capital market – defined as the ratio of liquidity to project postings $\theta_t \equiv L_t/V_t$ – is increasing in the expected growth rate of the marginal utility of consumption.

**Proof:** see appendix. □

Under relatively weak conditions, this proposition implies that congestion is countercyclical, as evidenced in the data. For example, if preferences are additive and concave in consumption, $\theta_t$ is inversely related to consumption growth. Since consumption reacts gradually to persistent changes in aggregate productivity (e.g. Fig. 10 in King and Rebelo, 2000), congestion decreases in business cycle upturns and inversely increases in downturns. This countercyclical behavior of congestion has two effects. First, capital stocks react proportionally more after impact than if no search frictions were present. Second, the decrease in congestion implies that households devote a relatively smaller share of their resources to liquid capital and consume relatively more. As a result, the income effect on labor supply is larger and depresses the response of equilibrium hours on impact. But because the subsequent shift in labor demand is larger (as capital stocks accumulate faster), equilibrium hours may respond more in the periods after the shock. These effects together have the potential to generate amplified yet hump-shaped (i.e. persistent) responses of hours and output to technology shocks.

## 2.4 Quantitative evaluation

We explore the quantitative implications of search frictions in the allocation of capital by comparing the business cycle performance of our capital matching model to the RBC benchmark in terms of impulse response functions (IRFs) and unconditional second moments.

### 2.4.1 Shocks and functional forms

Following much of the RBC literature, we assume that the exogenous labor-augmenting shock $X_t$ has both a deterministic trend part $\bar{X}_t$ and a stochastic transitory part $A_t$. In particular $X_t \equiv A_t^{1/(1-\alpha)}\bar{X}_t$. The deterministic trend part evolves according to $\bar{X}_t = g\bar{X}_{t-1}$, with $g > 1$, and the stochastic transitory part evolves according to

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A,$$

with $\varepsilon_t^A \sim (0, \sigma_A^2)$.[20]

For household preferences, we follow King and Rebelo's (2000) baseline specification and define the family's period utility as $u(C, 1-N) = \log C + \frac{\omega}{1-\xi}(1-N)^{1-\xi}$. For production, we assume a Cobb-Douglas function with constant returns to scale of the form $f(XN, K) = A(\bar{X}N)^{1-\alpha}K^{\alpha}$ with $0 < \alpha < 1$. Finally, we follow much of the labor search literature and specify the matching technology as a Cobb-Douglas $m(V, L) = \chi V^{\epsilon}L^{1-\epsilon}$ with $0 < \epsilon < 1$. This constant returns to scale assumption implies that $p(\theta_t) = \theta_t q(\theta_t)$, which turns out to simplify the steady state computations in our model.

[20] The assumption of a deterministic trend in labor productivity implies that we need to normalize all aggregates by $\bar{X}_t$ so as to obtain a stationary system that we then simulate using the log-linear rational expectations solution algorithm of King and Watson (1998). We thank Bob King for providing us with the relevant Matlab code. Alternatively, we could have specified a stochastic technology shock that is difference stationary. Our results are robust to such an alternative specification of the shock process.

### 2.4.2 Calibration

We calibrate our model to U.S. quarterly data. For the parameters that are common with the RBC benchmark, we use calibrations that are standard in the literature (e.g. King and Rebelo, 2000). We set $g = 1.004$ and $\beta = 0.992$ so as to match an annual mean trend growth rate of 1.6% and an average annual real yield on a risk-less 3-month treasury bill of 4.95%. For the labor supply, we fix the parameter $\omega$ such that the average fraction of hours worked equals $n = 0.2$. Together with $\xi = 4$, this results in a Frisch elasticity of labor supply of 1. Furthermore, we set the share of capital in the production function to $\alpha = 1/3$, and the rate of depreciation of capital to $\delta = 0.025$. Finally, to calibrate the exogenous driving process for the temporary technology shock, we extract a Solow residual from the data and then subtract a linear trend with average growth rate $g$. Estimation of the above specified AR(1) process with this series yields $\rho_A = 0.979$ and $\sigma_A = 0.0072$.

For the non-standard parameters, we calibrate them to match long-run averages of *gross* aggregate capital flows. Unfortunately, the U.S. National Production and Income Accounts (NIPA) only measures investment flows of *new* capital goods and then infers aggregate capital stock as the sum of current and past investment flows less depreciation.[21] We thus need to look at firm-level data of capital flows. One of the first studies to do so is Ramey and Shapiro (1998) who use Compustat data to compute gross capital additions and subtractions of all publicly traded firms in the U.S.[22] For their full sample 1959-1995, Ramey and Shapiro thus find that annual gross flows of capital additions average 17.3% of depreciated capital stocks, with 70% of these flows coming from expenditures in new property, plant and equipment (PP&E), 25% from acquisitions of used capital, and the remaining 5% from entries of new firms. The aforementioned study by Eisfeldt and Rampini (2006) broadly confirms these findings. Based on a Compustat sample from 1971 to 2000, they find that reallocation of used capital makes up 24%

[21] In particular, new investment flows are measured as the total value of shipments from capital goods producing industries adjusted for imports and exports. See Becker et al. (2005) for a detailed discussion.

[22] Since Compustat covers publicly traded firms only, small and medium-size firms are likely to be underrepresented. It turns out, however, that as opposed to employment, most physical capital is concentrated in large publicly held firms. Compustat data should still therefore provide a useful approximation. If at all, the reported numbers underestimate the extent of capital reallocation because smaller unlisted firms are more likely to undergo major changes (merger/acquisition, bankruptcy, structural reorganization) and invest larger fractions in used capital. See Eisfeldt and Rampini (2007) for evidence. Also note that other firm-level surveys such as the Longitudinal Research Database (LRD) or ACES may be more representative of the economy than Compustat. At the same time, these surveys provide less detailed information on capital additions and subtractions, span over a smaller sample period and suffer from their own selection problems (e.g. Becker et al., 2005).

of gross investment and that the average annual gross investment rate equals 22% of depreciated capital stocks.[23]

A second useful piece of information from the Compustat dataset are the direct measures of separation flows. In Ramey and Shapiro's study, for example, total separations make up an annual average of 7.3% in terms of undepreciated capital and 4.8% in terms of depreciated capital. By themselves, these numbers are not very revealing because depreciation during the life-cycle of a capital unit is not captured by an actual outflow of capital. What is more interesting is the fraction of capital separations due to reasons other than depreciation. Here, Ramey and Shapiro report that 71% come from retirements – which we interpret as the final step of depreciation – 21% come from sales, and the remaining 9% come from exits due to mergers and bankruptcies. Hence, capital separations are an important phenomenon above and beyond deprecation, with about 30% of all separations being due to reallocations to new firms.

Based on this evidence, we choose a quarterly separation rate of $s = 0.01$. Together with $\delta = 0.025$, this calibration implies that 71% of all separations are due to depreciation and 29% are due to sales and firm exits / acquisitions, as in Ramey and Shapiro (1998). Furthermore, using the capital accumulation equation (B.1), we can derive that these calibrations imply a quarterly steady state *gross* investment rate of

$$\frac{m(V, L)}{K} = [g - (1 - \delta)(1 - s)] = 0.03875,$$

which corresponds to a yearly rate of 15.5%. This number lies somewhat below the Compustat evidence reported in Ramey and Shapiro (1998) and Eisfeldt and Rampini (2006). One has to keep in mind, however, that the gross investment rates in these two studies are likely to be exaggerated because part of the depreciation applied to capital stocks in Compustat represents accounting standards rather than actual decreases in the value-of-use. Finally, we set $\varphi = 0.95$ such that investment in used capital as a fraction of gross investment, $\varphi(1 - \delta)s$, coincides with the 24% reported by Eisfeldt and Rampini (2006).

Consider next the steady state probability of capital allocation $q$. On the one hand, we know from Section 2 that the hazard rate for different (relatively liquid) finished

[23] Apart from the different sampling period, one of the reasons for the difference in investment rates is that Eisfeldt and Rampini (2006) use book values while Ramey and Shapiro (1998) apply artificial price deflators to convert their capital measures to current costs that should reflect changes in productive value. Furthermore, Eisfeldt and Rampini (2006) measure reallocation indirectly as sales of PP&E *plus* acquisitions, while Ramey and Shapiro (1998) measure reallocation directly as all additions of used capital. Both count purchases of existing firms, however, arguing that mergers and acquisitions not only represent a change of ownership but often involve important modifications to the composition and use of existing capital. See Jovanovic and Rousseau (2004) for a similar argument.

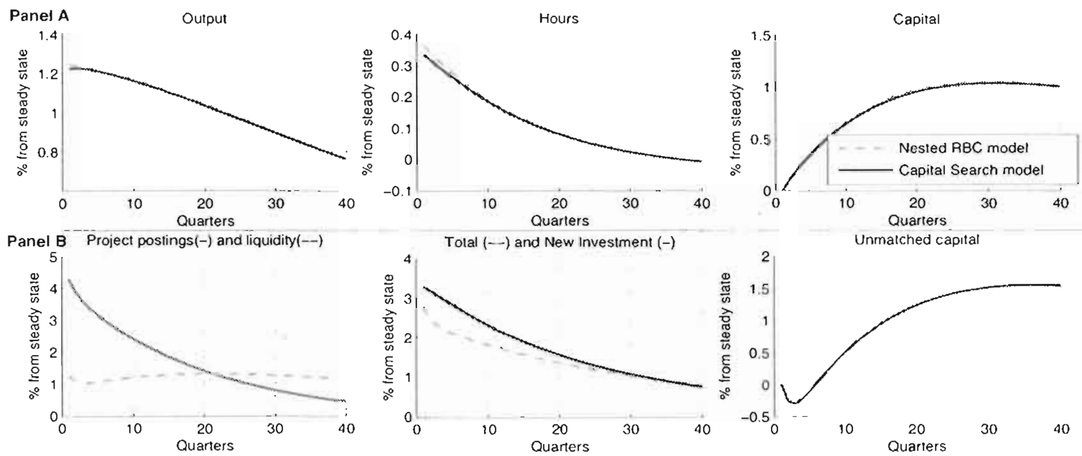


Figure 2.2: IRFs to a positive technology shock for baseline specification

capital goods averages $q = 0.86$. On the other hand, the vacancy rates for (less liquid) leased industrial and office space average 9.5% and 14.5% of total space, respectively. Defining the corresponding vacancy rate in our model as $U/(U+K) = (1-q)L/(U+K)$ and remembering that gross investment equals $m(V, L) = qL$, we can back out an average $q$. For the above gross rate of 0.03875, we obtain $q = 0.27$ if we use the vacancy rate of office space and $q = 0.19$ if we use the industrial vacancy rate. These numbers suggest that the average hazard rate is very different for different used and new capital goods. For the purpose of our model, we choose an average value of $q = 0.5$.

The remaining parameters to consider are the household's bargaining weight $\eta$ and the elasticity of the matching function $\epsilon$. It turns out that $\epsilon$ does not affect any of the steady state values. Furthermore, we have no particular long-run information to tie down $\eta$. In what follows, we set $\eta = 0.5$ and $\epsilon = 0.5$ and check afterwards whether the results are robust to alternative values.

### 2.4.3 Results

Panel A of Figure 2.2 plots the IRFs of output, productive capital and hours to a persistent, temporary technology shock for both our capital search model (solid lines) and the RBC benchmark (dotted lines). Panel B plots the IRFs of variables that are specific to our capital search model.

Consider first Panel B. In response to the technology shock, households devote more resources to liquidity and firms open up more vacancies. Hence, both total gross investment $m(L_t, V_t)$ and net new investment $I_t = [L_t + \kappa V_t] - [\varphi(1-\delta)sK_t + U_t]$ increase (since

$K_t$ and $U_t$ are predetermined). Furthermore, since preferences are additive and concave in consumption and the technology shock is persistent, congestion in the capital market $\theta_t = L_t/V_t$ decreases by proposition 1. For the first few periods after the shock, this decrease in congestion in the capital market leads to a proportionally more important response of capital stocks than in the RBC benchmark. Yet, as Panel A of Figure 2.2 shows, the difference is quantitatively negligible and its effect on output is dwarfed by the smaller response of hours. This latter result is due to the larger income effect on labor supply as the decrease in congestion lets the households devote more resources to consumption. Overall, output thus responds slightly less than in the RBC benchmark.

As we document in the appendix, the lack of internal propagation of the capital search model is robust to alternative calibrations of $q$, $\varphi$, $\epsilon$ and $\eta$.[24] The principal reason for this result is that capital separation and allocation flows implied by our calibration of $\delta$ and $s$ are too small for the countercyclical congestion mechanism to have a sizable effect. To illustrate this point, we resimulate the model with a much larger separation rate of $s = 0.15$. This would have the counterfactual implication that almost 70% of all capital leaves production in each year (including depreciation) and that average investment flows are equally important. We simply choose this calibration here for expositional purposes and to draw a comparison with Andolfatto (1996) who calibrates his labor search model to the same quarterly separation rate of $s = 0.15$.[25]

As Figure 2.3 shows, when separation and investment flows are much larger, the countercyclical congestion mechanism starts to matter. Panel B explains the origin of these changes. Liquid capital $L_t$ now hardly increases while the jump of project postings $V_t$ is almost as large as before. Hence, the drop in congestion is more important, which explains why capital stocks now respond almost twice as much in the periods following the shock than in the RBC benchmark. Furthermore, households devote a proportionally larger share to consumption on impact, which result is an amplified and humpshaped response of hours. The consequence is an amplified and more persistent reaction of output.

To sum up the quantitative evaluation, Table 2.2 compares the unconditional standard deviation of Hodrick-Prescott filtered output and autocorrelations of unfiltered

[24] Interestingly, an increase in the deadweight loss $1-\varphi$ slightly decreases the internal amplification of the model, thus replicating the result in Veracierto (2002, Table 1) that capital irreversibilities dampen rather than increase output fluctuations.

[25] For his calibration, Andolfatto (1996) finds that search frictions in the labor market yield significant output persistence in response to technology shocks. Den Haan, Ramey and Watson (2000) argue, however, that when the separation rate is calibrated to the more reasonable value of 10% per quarter, most of these effects disappear as long as separations are constant over the cycle (see their footnote 22). This is an interesting analogue to the point made here.

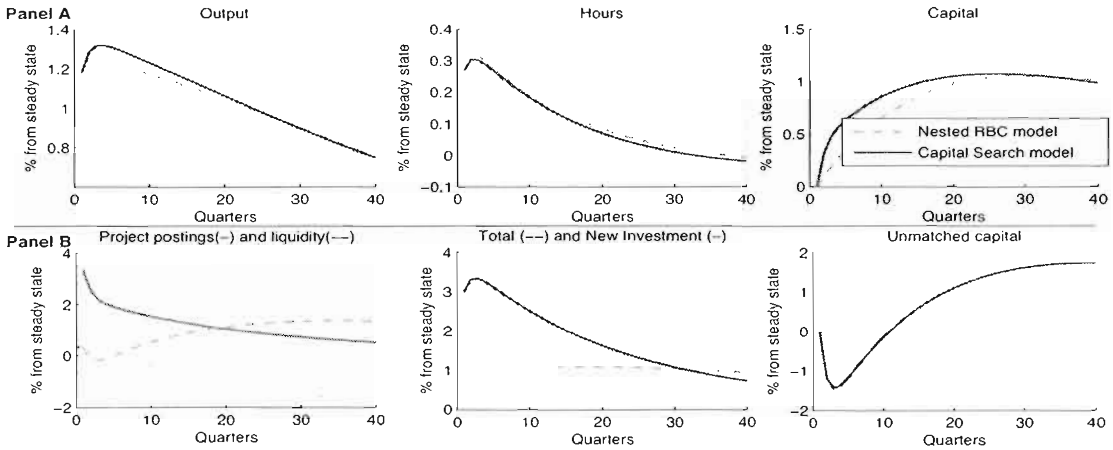


Figure 2.3: IRFs to a positive technology shock for counterfactually high separation rate

Table 2.2: Unconditional second moments of baseline capital search model

| | U.S. data | RBC benchmark | Capital search | |
|---|---|---|---|---|
| | | | $s = 0.01$ | $s = 0.15$ |
| $\sigma(y)$ | 1.66 | 1.17 | 1.16 | 1.22 |
| $corr(\Delta y, \Delta y_{-1})$ | 0.264 | 0.004 | 0.010 | 0.102 |
| $corr(\Delta y, \Delta y_{-2})$ | 0.227 | 0.003 | 0.005 | 0.035 |
| $corr(\Delta y, \Delta y_{-3})$ | 0.057 | 0.002 | 0.003 | 0.010 |

Notes: Standard deviation of output is H-P filtered; autocorrelations of growth rates are unfiltered. U.S. data are from DRI Economics for 1953:2 - 2001:4 (see appendix for details).

output growth of our capital search model with U.S. data and the RBC benchmark.

As discussed in King and Rebelo (2000), the benchmark RBC model is incapable of generating sizable amplification of the exogenous technology shock and remains below the standard deviation reported in the data despite the counterfactually large fluctuations in the exogenous technology shock. Likewise, as Cogley and Nason (1995) document, the RBC model fails to generate the sizable positive autocorrelation of output growth over several quarters that we observe in the data. Our capital search model – when appropriately calibrated – fails equally to generate internal amplification and persistence. The principal reason is that separation and reallocation flows are too small for the countercyclical congestion of our model to have sizable effects. In this sense, the proposed search friction for capital allocation has similarly negligible general equilibrium effects compared with models with adjustment costs on investment (e.g. Cogley and Nason, 1995 or more recently Khan and Thomas, 2006a) or time-to-build delays (e.g.

Kydland and Prescott, 1982) even though the qualitative implications of our model are quite different.

## 2.5 Endogenous capital separations due to credit frictions

Different empirical measures suggest that credit frictions and thus capital separations due to financial distress are countercyclical. Covas and Den Haan (2006), for example, document that default rates for U.S corporate bonds peak at the end of recessions. Likewise, we find that current liabilities of business failures taken from DRI (mnemonic: fail) are countercyclical.[26] Parallel to Den Haan, Ramey and Watson's (2000) argument that countercyclical job destruction implies substantial propagation in a labor search model, this suggests that extending our baseline model with credit frictions such as to generate countercyclical capital separations may, in fact, help our capital search model to generate more important business cycle effects.

As a by-product, the extension also allows us to assess the role of costly capital reallocation for the business cycle effects of credit frictions. In fact, existing DGE models with credit frictions such as Carlstrom and Fuerst (1997, CF henceforth) or Bernanke, Gertler and Gilchrist (1999, BGG henceforth) exclusively focus on the effects of net worth on investment and output. But since factors of production in these models can be moved costlessely from one firm to another, they abstract by definition from the effects of capital reallocation due to financial distress.

### 2.5.1 Model extension

As in CF and BGG, we introduce credit frictions through a costly state verification (CSV) mechanism originally proposed by Townsend (1979). Firms are subject to an idiosyncratic productivity shock that households (the capital lenders) can only observe after paying a monitoring cost. This assumption of asymmetric information implies that in the absence of monitoring, the firm would always want to underreport its productivity so as to avoid payment of the previously agreed upon rental rate. Households solve this agency problem with a debt contract that specifies monitoring and default if the idiosyncratic productivity level of the firm falls below some optimal threshold.

While we follow the same CSV approach, our model differs from CF and BGG in three important details. First, the optimal default threshold in our model is below the

[26] The H-P filtered contemporaneous correlation of Covas and Den Haan's (2006) default rate with real GDP is -0.33 for the period 1971-2004, and -0.77 for the period 1986-2004. The H-P filtered correlation coefficient of our liabilities series with real GDP is -0.33 for the sample 1948-1998 and -0.27 for the sample 1980-1998.

one in CF and BGG because capital reallocation is costly in our model while in CF and BGG, it is not. Second, we assume, as in the baseline mode, that firms transfer all of their profits to households at the end of the period. Hence, net worth – the channel through which credit frictions affect investment in CF and BGG – is absent. Third, we retain the assumption that the rental rate is determined so as to split the surplus of the lending relationship. CF and BGG assume instead that the lending market is perfectly competitive and thus, all of the surplus goes to the firm.

The specifics of the extended model are as follows. The representative firm's technology becomes

$$Y_t = a_t f(X_t N_t, K_t), \tag{2.13}$$

where $f(X_t N_t, K_t)$ describes the same constant-returns-to-scale function as before, and $a_t$ denotes the realization of the idiosyncratic productivity shock. Contrary to the aggregate shock $X_t$, which is known to all participants at the beginning of the period, the shock $a_t$ occurs after all optimal decisions have taken place and is only observed by the firm. As in CF and BGG, we assume that $a_t$ is independently and identically distributed over time and follows a lognormal distribution $\log(a) \sim N(-\frac{\sigma^2_{\log(a)}}{2}; \sigma^2_{\log(a)})$ so as to ensure $a_t \in [0, \infty]$ and $E(a) = 1$.[27]

To deal with the asymmetric information about firm productivity, households and firms negotiate the rental rate $\rho_t$ per unit of matched capital *prior to* the realization of $a_t$. If the firm makes positive profits (i.e. if $a_t \geq \bar{a}_t$ where $\bar{a}_t$ is such that $\bar{a}_t f$ $(X_t N_t, K_t) - W_t N_t - \rho_t K_t - \kappa V_t = 0$), the firm pays $\rho_t K_t$, the household refrains from monitoring and the capital match continues. If, on the other hand, $a_t < \bar{a}_t$ the firm is unable to pay the negotiated capital rental because we assume that the wage bill $W_t N_t$ and the cost of posting vacancies $\kappa V_t$ need to be covered first in order for the firm to continue operating next period. In this situation, the household pays the monitoring cost to verify the firm's production and decides on the continuation of the capital match. If $a_t$ is above some optimal threshold $\underline{a}_t$ that we derive below, the household takes all of the firm's production and covers for the totality of $W_t N_t$ and $\kappa V_t$ so as to continue the capital match. If instead $a_t$ is below the threshold $\underline{a}_t$, the household separates the match and takes back its capital stock without receiving or paying anything. In this case, the firm is liquidated and the difference between production and the cost of $W_t N_t$ and $\kappa V_t$ is picked up by an insurance that is funded with the dividends from profit-making firms.[28]

[27] The assumption that $a_t$ is independently and identically distributed in conjunction with constant-returns-to-scale technology simplifies the analysis as we do not need to consider the history of shocks incurred by each firm. Firm size thus remains irrelevant, which is why our notation continues to abstract from firm subscripts.

[28] See the appendix for the details on this insurance. Suffice to say here that we implicitly assume

Given these assumptions, endogenous separations $s_t^e$ due to financial distress are defined as

$$s_t^e = H(\underline{a}_t),$$

where $H(a)$ denotes the cumulative density of $a$. Aside from this endogenous part, we still allow for exogenous (constant) separations that we denote by $s^x$. Hence, the total separation rate is defined as

$$s_t = s^x + s_t^e. \tag{2.14}$$

Furthermore, the household's expected gross revenue from matched capital equals

$$\begin{aligned} R_t^K &= \int_{\bar{a}_t}^{\infty} \rho_t K_t dH(a) + \int_{\underline{a}_t}^{\bar{a}_t} \left[ af(X_t N_t, K_t) - W_t N_t - \kappa V_t \right] dH(a) \qquad (2.15) \\ &\quad -\tau \int_0^{\bar{a}_t} \left[ af(X_t N_t, K_t) \right] dH(a) + (1-\delta)\varphi_t s_t K_t. \end{aligned}$$

The first two terms denote net revenues from continuing relationships. The third term denotes the expected total monitoring cost paid by the household, which we assume to be a fixed proportion $\tau > 0$ of the defaulting firms' output. The fourth term corresponds to the value of separated capital returned to the household's budget constraint. In this last term, we assume that the recovery rate of separated capital $\varphi_t$ is time-varying and more specifically, that it is a convex function of total endogenous capital separations; i.e. $\varphi_t = \varsigma(s_t^e)$ with $\varsigma'(\bullet) < 0$ and $\varsigma''(\bullet) < 0$. Two considerations motivate this choice. First, we want to capture industry-specific asset illiquidity as proposed by Shleifer and Vishny (1992) that are otherwise absent in our representative agent model (see the discussion in Section 2). Second, the additional flexibility afforded by this function allows us to match the business cycle dynamics of endogenous capital separations due to financial distress.

Consider now the household's optimal choice of $\underline{a}_t$. It is the level of $a_t$ below which refinancing a firm is more expensive than severing the lending relationship and incurring the cost of reallocating the capital to another firm. More formally, we can derive it from the household's optimization problem as (see the appendix for a detailed description)

$$\Lambda_t(1-\delta)\varphi_t K_t = \Lambda_t \left[ \underline{a}_t f(X_t N_t, K_t) - W_t N_t - \kappa V_t \right] + (1-\delta) K_t \beta E_t V_K(U_{t+1}, K_{t+1}). \tag{2.16}$$

The left-hand side is the marginal value (in utility terms) of separating and returning the capital unit into the budget constraint for reallocation, where we assume that the

that firms or capital lenders on their own cannot contract a similar insurance on their own to prevent the firm from disappearing.

representative household takes $\varphi_t$ as exogenous. The right-hand side is the marginal revenue from matched capital plus the marginal value of continuing the match into the future.[29]

Conditional on selecting a debt contract, the proposed monitoring and separation scheme is optimal for both parties. The firm would not gain anything from reporting output below what it actually produced because in case of monitoring, it will lose all of its output anyway. Likewise, the household would not gain anything from negotiating a higher or lower auditing cutoff $\bar{a}_t$ or a separation threshold $\underline{a}_t$, by definition of the utility-maximizing condition in (2.16).

Since any revenues associated with productivity shocks below $\bar{a}_t$ are absorbed either by the capital lender (in case of continuation of the capital match) or by insurance (in case of capital separation), firms now maximize only over the positive portion of revenue net of current costs; i.e. $\int_{\bar{a}_t}^{\infty}\left[af(X_tN_t,K_t)-\rho_tK_t-W_tN_t-\kappa V_t\right]dH(a)$. As the appendix details, the first-order conditions resulting from this objective function would imply substantial over-hiring of labor relative to the RBC benchmark and thus an unrealistically high labor share. We correct this implication by assuming, in addition, that the representative firm in the extended model applies a constant markup $1/\psi \geq 1$ on its optimal decision problem.[30]

To close the model, we assume as before that the rental rate is determined by Nash bargaining over the surplus of the capital relationship. This rental rate is now conditional on the optimal $\underline{a}_t$ (see appendix)

$$\begin{aligned}\rho_t &= \eta\left[\bar{\mu}_t\psi f_K(X_tN_t,K_t)+(1-\delta)(1-s_t)\frac{\kappa}{\theta_t}[1-H(\bar{a}_t)]\right]+(1-\eta)\left[\delta+(1-\delta)(1-\varphi_t)s_t\right]\\ &\quad+\left[\rho_tH(\bar{a}_t)-(1-\eta)(\underline{\mu}_t-\bar{\mu}_t-\tau(1-\bar{\mu}_t))f_K(X_tN_t,K_t)\right], \qquad (2.17)\end{aligned}$$

where $\underline{\mu}_t=\int_{\underline{a}_t}^{\infty}adH(a)$ and $\bar{\mu}_t=\int_{\bar{a}_t}^{\infty}adH(a)$ denote partial expectations. Compared to the case with exogenous separation, the first term in brackets is altered to reflect the marginal product of capital and the saved search costs actually accruing to the firm. The third term in brackets represents the risk premium that arises because households

[29] It can be shown that $\underline{a}_tf(X_tN_t,K_t)<W_tN_t+\kappa V_t$; i.e. the household is willing to refinance distressed firms up to a certain point so as to continue the capital match. This is because walking away from a relationship to reallocate capital with another firm is costly in the sense that separated capital yields zero return in the next period and comes with the risk that rematching takes time. By contrast, lenders in the CF and BGG models never refinance since liquidating a defaulting firm and reallocating the capital is costless.

[30] As proposed by Blanchard and Kiyotaki (1987), such a markup could result from a situation where otherwise identical firms produce imperfectly substitutable goods such that each firm faces a downward-sloping demand in its relative price.

do not receive the full contractual payment $\rho_t$ (or even need to reinject money) and pay monitoring costs when the firm's idiosyncratic shock drops below $\bar{a}_t$.[31]

### 2.5.2 Calibration

To compare the extended model with the baseline model in which all separations are constant, we keep the common parameters unchanged in a first time; i.e. $q = 0.5$, $s = 0.01$, $\eta = 0.5$, and $\epsilon = 0.5$. Further below, we perform robustness checks with respect to alternative calibrations. The additional parameters requiring calibration are the markup of price over marginal cost, $1/\psi$, the fraction of output expended on monitoring, $\tau$, the fraction of capital separations due to financial distress, $s^e/s$, and the elasticity $(\partial\varphi/\partial s^e)/(s^e/\varphi)$ around steady state.[32]

The crucial dimensions we want to match with our calibration are the relative importance and business cycle dynamics of capital separations due to financial distress. Since the aforementioned studies on firm-level capital flows do not report such details, we compute the relevant series ourselves from Compustat data (see the appendix for a detailed description of the data). Specifically, we treat the following categories as capital separations due to financial distress: (i) exits due to liquidation (chapter 7); (ii) sales during the years (-1 0 1 2) around bankruptcy filings (chapter 11); and (iii) sales during the years (-1 0 1 2) around drops of more than 2 credit ratings in long term debt. Compustat provides information on the reasons of exit for disappearing firms as well as information about debt ratings of continuing firms. To identify firm bankruptcies, we link the Compustat database with information on chapter 11 filings from the Bankruptcy Research Database.[33] Total separations (defined as sales and exits) and retirements, in turn, are computed as in Ramey and Shapiro (1998).[34] Table 2.3 provides the thus computed averages for the sample 1980-1993.[35]

[31] Broadly speaking, this risk premium is the consequence of incomplete contracting in a world with ex-post factor specificity that Williamson (1979) and more recently Caballero and Hammour (1996) term the *fundamental transformation problem.* The general equilibrium consequence is reduced flexibility of separation decisions and, in turn, a slower capital accumulation process.

[32] Since we loglinearize the model, the other functional characteristics of $\varphi = g(s^e)$ are irrelevant.

[33] The Bankruptcy Research Database (BRD) is compiled by Lynn M. LoPucki from UCLA Law. Of the 751 reported cases of bankruptcy filings by large publicly traded firms since October 1979, we were able to match 623 firms with the unique firm identifiers used by Compustat (mnemonic: gvkey).

[34] Ramey and Shapiro (1998) count as total exits the ones related to mergers and liquidations but do not count exits due to privatizations, leveraged buyouts and other reasons.

[35] We start the sample only in 1980 because, as Davis, Haltiwanger, Jarmin and Miranda (2006) document, the proportion of medium-size and smaller firms listed publicly increased importantly in the early 1980s. This makes the Compustat sample more representative – especially with regards exits due to financial distress. The end date 1993 is chosen because thereafter, firms no longer provide accurate numbers for retirements. As mentioned before, Compustat data should be more representative for

Table 2.3: Capital separations

| | Retirements | Sales (S) | Exits (E) | S & E Total | S & E due to Fin. Distress |
|---|---|---|---|---|---|
| Fraction of PP&E | 4.94% | 1.31% | 1.11% | 2.42% | 0.15% |
| Correlation with output | 0.30 | 0.48 | 0.37 | 0.15 | -0.31 |
| Standard dev rel output | 21.82 | 4.82 | 99.5 | 39.52 | 2.46 |

Notes: Standard deviations and correlation coefficients apply to H-P filtered series; Data source from Compustat 1980-1993 (see appendix for details).

In line with Ramey and Shapiro (1998), retirements make up roughly two thirds of all separations while sales and exits make up about one third.[36] Sales and exits due to financial distress make up only 6% of total capital separations (and only 4.6% for the 1980-2004 period), which amounts to 0.15% of average capital stocks. The series is countercyclical, in line with the aforementioned evidence on the cyclicality of financial distress, and about two and a half times as volatile as output. To roughly match these characteristics, we calibrate $s^e/s = 0.05$ and set $(\partial\varphi/\partial s^e)/(s^e/\varphi)$ such that the relative volatility of $s^e$ in the model coincides with the one in the data.

For the other two additional parameters, we choose $1/\psi = 1/0.8 = 25\%$ and set the monitoring cost parameter to $\tau = 0.05$.[37]. The resulting long-run ratios of interest are the following: the consumption-output ratio equals 73.13%, which is in line with King and Rebelo (2000); the labor share equals 74%, which corresponds to estimates reported by Gollin (2002); the average annualized risk premium equals 3.56%, which lies in-between the spread of the post-war average Aaa corporate bond yield over the 3-month Treasury bill (1.87%) and the post-war average equity risk premium for the U.S. (7.58%); and profits (dividends) relative to output equal 8.9%, which is somewhat too high compared to the evidence reported in Basu and Fernald (1997).[38]

Before continuing, we return to Table 2.3 to consider the overall behavior of sales and exits. Both series are procyclical and especially exits are highly volatile relative

capital than for employment because physical capital is concentrated in large firms, most of which are publicly traded (e.g. Eisfeldt and Rampini, 2007).

[36]As discussed before, the total numbers are small because depreciation during the life-cycle of a capital unit is not matched by an actual outflow of capital.

[37]A great deal of controversy surrounds the costs related to bankruptcy. In our model, this cost should only entail the direct costs related to monitoring and reorganization. We therefore set it to a value that is well below estimates of direct and indirect costs of bankruptcy that seem to lie between 20 and 35% of output. See Carlstrom and Fuerst (1997) for a discussion. As robustness checks in the appendix reveal, the value of $\tau$ has little influence on the dynamics of our model.

[38]Other values of interest implied by our calibration but for which we do not have any empirical counterparts are: an average cost of posting vacancies relative to output equal to $v\kappa/y = 2.22\%$, and a standard deviation of the idiosyncratic productivity shock equal to $\sigma_o = 0.33$.

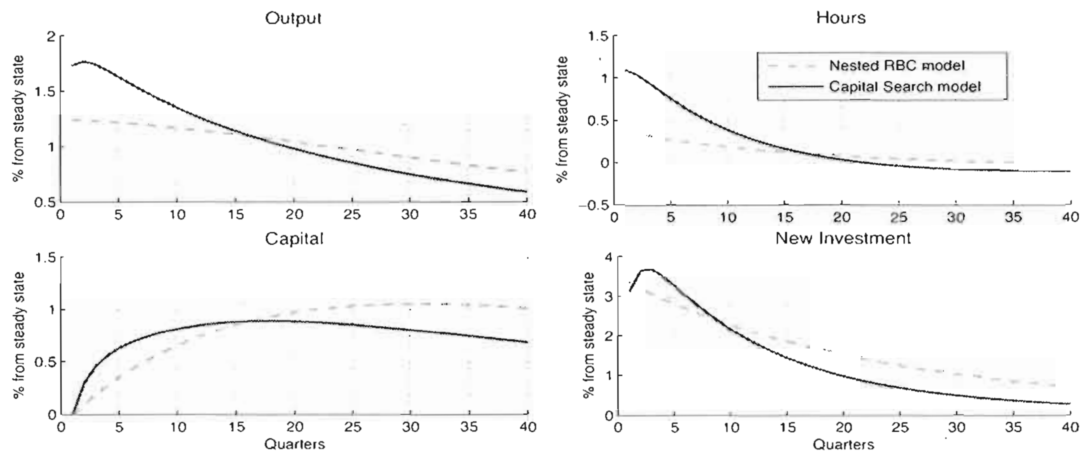


Figure 2.4: IRFs to a positive technology shock for the extended model

to output. This latter result is due to the large variations in mergers and acquisitions (M&A) that account for most of capital separations in the Compustat data.[39] Somewhat counterfactually, we omit these variations in our extended model and instead assume this part of capital separations to be constant. The reason for this omission is two-fold. First, as the below quantitative analysis shows, even small countercyclical capital separations due to credit frictions can have substantial effects in general equilibrium. Second, the procyclical nature of sales and M&A is likely to be the result of reallocation towards more efficient firms in the wake of technological change (e.g. Jovanovic and Rousseau, 2004). Our representative agent framework is designed, by definition, to quantify the effects of search frictions on their own but does not allow us to consider reallocation costs in conjunction with persistent productivity differences. As we discuss in the conclusion of the paper, this is an interesting avenue for future research.

### 2.5.3 Quantitative evaluation

As in Section 4, we start our quantitative evaluation by considering IRFs to a persistent but temporary technology shock. As is immediately apparent from Figure 2.4, the extended capital matching model (solid lines) generates a substantially amplified response of output and hours compared to the RBC benchmark (dotted lines).

The amplification has its origin in the state-dependent nature of the credit friction. To illustrate this, Figure 2.5 displays the IRFs of the variables related to changes in

[39]The procyclicality of M&A is consistent with evidence reported in Maksimovic and Phillips (2001). They use LRD data and find that change in ownership of large manufacturing plants is highly procyclical.

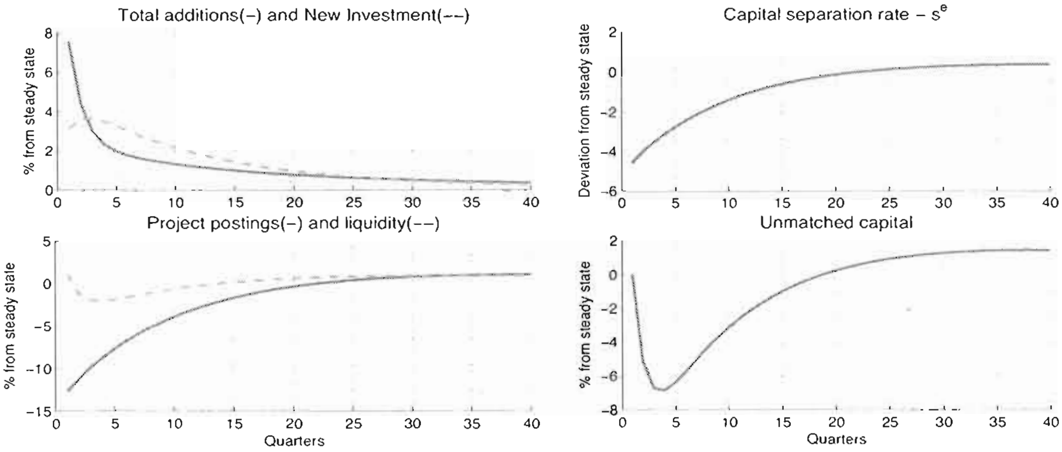


Figure 2.5: IRFs to a positive technology shock for the extended model

the stock of capital entering the production function. The positive technology shock shifts the firms' productivity distribution to the right, which means that bankruptcies and thus capital separations drop (top-right panel). Hence, less capital is separated from production and returned to the household's budget constraint for time-consuming rematching. This explains why productive capital stocks react more strongly than in the RBC benchmark.

As an indirect effect of the drop in capital separations, households now find it optimal to allocate more resources to new investment than in the baseline model with constant separations. Compared to the RBC model, consumption thus reacts less on impact, which results in a smaller income effect on labor supply. In addition, the more important reaction of productive capital implies that the marginal product of labor and thus labor demand increases more rapidly in the periods after the shock than in the RBC model. The conjunction of these two general equilibrium effects leads to a substantially larger response of equilibrium hours and, as the ensuing analysis reveals, this is what explains most of the increased internal amplification of output relative to the RBC benchmark.

Table 2.4 presents prominent unconditional second moments for U.S. postwar data, the RBC benchmark, the baseline capital search model with exogenous separations as well as the extended capital search model with endogenous separation. For this last case, we report two cases: one for $\epsilon = 0.5$, as used so far, and one for $\epsilon = 0.25$. As we will see, the calibration of this parameter now has important implications.

Table 2.4: Second moments for baseline calibration

| | U.S data | | RBC benchmark | | Capital search | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Exogenous | | Endogenous | | | |
| | | | | | | | $\epsilon = 0.5$ | | $\epsilon = 0.25$ | |
| | a | b | a | b | a | b | a | b | a | b |
| $c$ | 0.58 | 0.69 | 0.45 | 0.96 | 0.48 | 0.97 | 0.31 | -0.38 | 0.33 | 0.85 |
| $n$ | 0.95 | 0.87 | 0.29 | 0.97 | 0.27 | 0.98 | 0.63 | 0.97 | 0.40 | 0.97 |
| $k$ | - | - | 0.28 | 0.10 | 0.28 | 0.14 | 0.30 | 0.44 | 0.25 | 0.10 |
| $i$ | 2.89 | 0.87 | 2.68 | 0.99 | 2.68 | 0.99 | 2.14 | 0.99 | 2.53 | 0.99 |
| $s^e$ | 2.46 | -0.96 | - | - | - | - | 2.46 | -0.96 | 2.46 | -0.96 |
| $premium$ | 0.54 | -0.59 | 0.004 | 0.003 | 0.003 | 0.98 | 0.10 | -0.97 | 0.03 | -0.97 |
| $\sigma(y)$ | 1.66 | | 1.17 | | 1.16 | | 1.71 | | 1.28 | |

Notes: (a) Standard deviation relative to output; (b) contemporaneous correlation with output. All moments are Hodrick-Prescott filtered. Data source from DRI Basic Economics 1953:2-2001:4 (see appendix for details).

Consider first the case where $\epsilon = 0.5$. As indicated by the IRFs, this version of the extended model generates substantial amplification of output relative to the RBC benchmark. As for persistence, however, the model still fails to generate the marked positive autocorrelation of output growth that we see in the data (see Table 2.5 below). The increase in internal amplification is rooted in the general equilibrium effects on labor supply and labor demand that result in more volatile dynamics of hours. Interestingly, both the zero profit threshold $\bar{a}_t$ and the separation threshold $\underline{a}_t$ are countercyclical, which implies, in turn, that the model generates a countercyclical risk premium. Although the fluctuations of this premium are not as volatile in the data. this result is a significant success of our extended model over the RBC benchmark as well as over standard credit friction models without costly capital reallocation (see below).

Closer inspection of Table 4 reveals that the more volatile dynamics of equilibrium hours come at the cost of countercyclical consumption. which is clearly at odds with the data. In fact, the negative income effect brought about by the drop in capital separations is so strong that households choose to decrease their consumption on impact. These consumption dynamics hinge crucially on the elasticity $\epsilon$ that links the matching probability $q(\theta_t)$ to the congestion measure $\theta_t$.[40] For $\epsilon = 0.5$, the response of $q(\theta_t)$ is relatively large. We thus recalibrate $\epsilon = 0.25$ so as to roughly match the consumption dynamics in the data. The last column of Table 4 reports the results. Consumption is

[40] Recall from the first order condition (B.12) that the expected return from liquid capital is an average of the marginal values of matched and unmatched capital, weighted by the matching probability $q(\theta)$. A stronger cyclical response of $q(\theta)$ means the average return to liquid capital rises more quickly in an upturn.

now procyclical and almost as volatile as in the data. The consequence of this adjustment is a much smaller income effect on labor supply, which reduces the standard deviation of output to 1.28 – a value just slightly above the RBC benchmark.

This exercise makes clear that the interplay between time-consuming capital (re-)allocation and countercyclical capital separation leads to amplification by affecting the response of hours supplied by households. Exogenous shocks not only affect the factor productivity as in the RBC benchmark, but also the *stock* of productive capital and the amount of resources that need to go through the time-consuming allocation process. The time-varying capital separation rate limits the income effect of rising returns to capital, thus inducing households to shift more resources away from consumption towards investment and supplying more hours. However, once we calibrate the model to yield reasonable consumption dynamics, we find that these effects are modest and result only in a small increase in internal amplification.

### 2.5.4 Volatility of separations and robustness to alternative calibrations

As highlighted by the above results, a crucial ingredient for the marked internal propagation of our extended model is the income effect on labor supply whereby households withhold current consumption to finance capital investments. The following robustness checks assess to what extent alternative calibrations affect the performance of the model. In all of these exercises, we keep $\epsilon = 0.25$ so as to roughly match the consumption dynamics in the data and adjust the elasticity $(\partial\varphi/\partial s^e)/(s^e/\varphi)$ such as to keep the relative volatility of $s^e$ consistent with the Compustat data. Table 2.5 reports the results.

Table 2.5: Sensitivity of model performance to alternative calibrations

| | Baseline | Mean allocation rate | | Bargaining power | | Separation rates | |
|---|---|---|---|---|---|---|---|
| | calibration | $q(\theta) = 0.25$ | $q(\theta) = 0.75$ | $\eta = 0.45$ | $\eta = 0.75$ | $s^e/s = 0.01$ | $s = 0.02$ |
| $\sigma(y)$ | 1.28 | 1.29 | 1.28 | 1.28 | 1.26 | 1.29 | 1.62 |
| $\sigma(s^e)/\sigma(y)$ | 2.46 | 2.46 | 2.46 | 2.46 | 2.46 | 2.46 | 2.46 |
| $corr(s^e, y)$ | -0.96 | -0.97 | -0.96 | -0.96 | -0.96 | -0.96 | -0.97 |
| $corr(\Delta y, \Delta y_{-1})$ | -0.004 | -0.017 | 0.010 | -0.004 | -0.002 | -0.004 | 0.004 |
| $corr(\Delta y, \Delta y_{-2})$ | -0.004 | -0.013 | -0.002 | -0.005 | -0.003 | -0.004 | -0.01 |
| $corr(\Delta y, \Delta y_{-3})$ | -0.005 | -0.010 | -0.004 | -0.005 | -0.004 | -0.005 | -0.016 |

Notes: Standard deviations and cross-correlations are Hodrick-Prescott filtered.
Autocorrelations of growth rates are unfiltered.

Changes in $q(\theta)$, $\eta$ and $s^e/s$ (keeping $s = 0.01$) have essentially no impact on the dynamics of the model.[41] This result would even hold if we didn't adjust $(\partial\varphi/\partial s^e)/(s^e/\varphi)$ so as to keep $\sigma(s^e)/\sigma(y) = 2.46$. The reason for this robustness is that income effects on labor supply remain small when $\epsilon = 0.25$ and capital separations on their own are too insignificant to affect output significantly.

The dynamics of the model are more sensitive to changes in the average separation rate $s$. For example, when we calibrate $s = 0.02$ per quarter (keeping $s^e/s$ at 0.05), the standard deviation of output rises to $\sigma(y) = 1.62$. The mechanism for this increase in amplification is the same than before. The larger average $s$ implies that the drop in separated capital after a positive technology shock is more important and thus, households divert more resources away from consumption in order to achieve the desired amount of liquid capital. The resulting negative income effect increases the volatility of hours, thus leading to an amplified output response. As before, however, this effect is accompanied by a negative correlation of consumption with output. If we correct this counterfactual implication by lowering $\epsilon$ even more, the amplification of output is reduced substantially.

Finally, it is interesting to note that there are several calibrations for which the extended capital search model generates both important amplification and persistence effects. For example, if we set the elasticity $(\partial\varphi/\partial s^e)/(s^e/\varphi) = 0$ (i.e. $\varphi$ is constant) and $\epsilon = 0.5$, we obtain $\sigma(y) = 1.52$, $corr(\Delta y, \Delta y_{-1}) = 0.28$ and $corr(\Delta y, \Delta y_{-2}) = 0.08$ without counterfactual consumption dynamics (see appendix for details). This marked improvement in internal propagation is due to an overly volatile endogenous separation rate (more than a 1000 times as volatile than output). This illustrates that the *combination* of search frictions for physical capital and countercyclical capital separations due to credit frictions leads at least in principle to more important business cycle fluctuations. The issue is simply that the flows of physical capital in and out of production are not large and not volatile enough for these effects to play a substantial role.

## 2.6 Conclusion

In this paper, we examined the business cycle consequences of search frictions for the allocation of physical capital. The investigation is motivated by firm- and industry-level evidence on market imperfections in the allocation of physical capital. Despite the fundamentally different nature of physical capital and labor, we argue that the market imperfections involved in the allocation of these two factors are quite similar. We thus

[41] For the given calibration, there is no rational expectations solution to the model for values of $\eta$ below 0.45.

consider our paper as a first step towards analyzing capital allocation with the same type of search frictions that have proven fruitful for our understanding of labor markets. By the same token, we propose a complementary view to existing models of investment that focus on aggregate adjustment costs and building delays in a world with perfect markets.

The capital search model that we develop generates countercyclical congestion in physical capital markets, in line with the data. Our analysis in a modern DGE context suggests, however, that for reasonable calibrations, the internal propagation effects of these search frictions are modest. The main reason for this lack of internal propagation is quantitative: separation and reallocation flows of physical capital are too small for the search friction to play a significant role. This conclusion remains intact when we extend the model with credit market frictions that result in countercyclical capital separations. While the combination of countercyclical separations and imperfect capital (re-) allocation increases internal propagation, almost all of these effects stem from a general equilibrium income effect that these frictions have on labor supply. Once we tie down the model to generate consumption dynamics in line with the data, we find that capital separations due to financial distress are simply not important and volatile enough for them to generate significant internal propagation.

Our results provide an interesting contrast to Den Haan, Ramey and Watson (2000) who show that the introduction of countercyclical job destruction in a labor search model substantially magnifies and prolongs the business cycle effects of small shocks. This difference in results is mainly due to the fact that labor is twice as important an input to production as capital and that job destructions fluctuate on average much more over the business cycle than capital separations. Furthermore, job destructions overall are countercyclical while for capital separations, only the part linked to financial distress is countercyclical. This part makes up only a small fraction of all capital reallocations, which explains why its impact is so limited.

The comparison suggest that capital reallocations due to sales and M&A are a more important source of internal propagation. From our firm-level data, we know that most capital reallocations occur through these two channels and are substantially more volatile than capital reallocations due to financial distress. The problem is that sales and M&A are procyclical rather than countercyclical and thus, they would not generate more important business cycle dynamics in the proposed representative agent framework. At the same time, Jovanovic and Rousseau (2004) argue that sales and M&A of capital are often the consequence of reorganization in the aftermath of embodied technological progress. Hence, combining embodied technological progress in a heterogeneous firm framework with search frictions for the reorganization of physical capital could entail important internal propagation effects as it takes time for firms and sectors to reallocate factors of production to their most productive use.[42]

[42] Andolfatto and MacDonald (2006) propose a similar idea for the labor market to explain jobless recoveries.

# Chapter 3

# Endogenous Flows of Foreign Direct Investment and International Real Business Cycles

**Abstract**

This paper models flows of foreign direct investment (FDI) in a two country, two sector DSGE framework. The allocation of capital to production capacity abroad is subject to a search-and-matching friction with endogenous capital reallocation. The model is calibrated to observed gross inflows and outflows of FDI and leads to dynamics of net foreign direct investment consistent with the empirical evidence documented in this paper: inward and outward net flows of FDI are positively correlated whereas a standard International Real Business Cycle model has the prediction of a negative correlation. Moreover, the model solves the aggregate investment quantity puzzle as it generates cross-country correlations in-line with the data.

## 3.1 Introduction

Generally foreign investment is welcomed for bringing new capital to an economy and increasing productivity through the arrival of new technologies. This has also been the main focus of the theoretical and empirical literatures concerned with foreign direct investment. Little attention has been paid, however, to the short and medium run behavior of foreign-controlled firms and, in general, to their importance in understanding the business cycle of open economies. This seems somewhat surprising, as a commonly used measure of the rate at which foreigners gain control over a domestic economy, flows of foreign direct investment (FDI), are large and very volatile. In Canada, for example, foreign-owned firms generate up to one third of employment and control over a fifth of all assets, a share that has been stable over the last four decades.[1]

The bulk of FDI, among developed countries, involves the replication of production capacity abroad, or what is known as horizontal FDI, and in particular for the purpose of serving the host market (Brainard 1993, 1997). What is less well known, and is documented in detail for the case of Canada in Section 2, is that both net inflows into, and outflows from, a host economy of FDI by foreigners increase during an upturn. Moreover, business cycle fluctuations in net FDI in Canada and net Canadian investment abroad are positively correlated. Thus periods of increased net inflows into an expanding economy are also periods of increased investment abroad by that same economy. The classic international real business cycle, however, generates a negative correlation between these flows.

The approach taken by this paper, in Section 3, is to model flows of horizontal foreign direct investment in a two-country, two-sector model,[2] in which the allocation of capital to production abroad is subject to a friction of the search and matching type: bringing to fruition a new investment project abroad is costly and time consuming and, once in place, faces an endogenous termination probability. The model therefore provides a theoretical framework with endogenous gross inflows and outflows of foreign direct investment in which congestion effects on foreign investment markets impact the response of investment patterns to changes in productivity.

Several considerations motivate the modeling strategy adopted here. First, as argued by Gordon and Bovenberg (1996), due to a lack of knowledge of the domestic economy foreign firms are at a disadvantage in setting up and running a firm. While these authors capture this idea by assuming that output at foreign firms is reduced by some fixed proportion, the search and matching framework yields two distinct sources of costs. First, foreign firms expend more than domestic firms in bringing a new investment project to fruition.[3] Second, the probabilistic nature of the matching process captures

[1]These figures are for the manufacturing sector, see Baldwin and Dhaliwal (2001) and Baldwin and Gellatly (2005). The importance of FDI does not limit itself to the case of Canada. For example, the ratio of FDI to domestic investment in the US has risen from 6% in the 1970s to 15% in the 2000s. Lipsey (2000) reports ratios above 10% for many industrialized country over the period 1970 to 1995.

[2]This paper models horizontal FDI, treated in the trade theory literature by papers such as Markusen (1984), Markusen and Venables (2000), and Helpman, Melitz and Yeaple (2004). By opposition. vertical FDI refers to the "geographic distribution of production globally in response to the opportunities afforded by different markets." Models of the first category center around the "proximity-concentration" trade-off, while the second are models of factor proportions. See Markusen (2004) for a good overview of the multi-national firm literature.

[3]Proximity-concentration models of FDI, e.g. Helpman, Melitz and Yeaple (2004), generally assume

the fact that foreigners incur the cost of more time in setting up a new production facility or acquiring information about a risky investment project as in Gopinath (2004). [4]

Quantitatively, the model generates the high cyclical volatility of net FDI flows, and the positive correlation of net foreign direct investment inflows and outflows observed in the data. By contrast, a standard IRBC model with investment adjustment costs predicts a negative correlation, and lower volatilities of FDI flows. As Section 4 elaborates in assessing the quantitative implications of the model, the allocation friction is central to explaining the positive correlation of net inflows and net outflows of FDI. Following a positive technology shock in the host economy, whether in a standard IRBC model with investment adjustment costs or a search in FDI model with endogenous reallocation, flows of net inward FDI increase on impact. By simple arbitrage, gross flows of FDI from the host economy abroad decrease on impact, generating a negative correlation in the standard IRBC model. However, in the proposed model this same drop in the pool of capital goods available for allocation abroad increases the allocation probability for the capital owner in the short run, thus mitigating the drop in new allocations abroad and producing the positive correlation between inward and outward flows observed in the data.[5]

Extending the model to allow for endogenous capital reallocation, resulting from the introduction of match-specific idiosyncratic productivity shocks, raises the positive correlation between inward and outward net FDI even further. Drawing from the labor market literature, starting with Mortensen and Pissarides (1994), reallocation occurs for realizations of the idiosyncratic shock that yield a negative surplus to the relationship. Following an expansionary technology innovation in the host economy, the opportunity cost to the foreign capital lender of maintaining a unit of capital locked in with a foreign affiliate increases. This is because foreign affiliates increase their project initiations faster than foreign capital owners increase the pool of liquid capital available for allocation to production abroad (i.e. to the expanding host economy), increasing the probability for a given foreign capital lender of finding an appropriate investment project. Thus the rate of capital reallocation from foreign affiliates increases with the host's business cycle.

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a fix cost to setting up operations abroad, above the the cost of entering the domestic market. As will be discussed below, foreign affiliates pay a cost per investment project initiation. Although allocation of capital to domestic firms will be frictionless, this is only a special case of the search environment when the initiation cost is nil.

[4] This setup was explored extensively in a closed economy setting for the allocation of physical capital by Kurmann and Petrosky-Nadeau (2009). Gopinath (2004) models the difficulty in acquiring the information on investment projects in emerging projects as a time consuming search process.

[5] This is because, in this context, new allocations are a function of both the pool of capital available and the allocation probability, or congestion on the foreign capital market.

This change in the gross flow of capital from the expanding economy's direct investment abroad further counters the drop in gross outflows.

This paper is related to the growing literature on international real business cycles, dating back to the seminal contribution of Backus, Kehoe and Kydland (1992), and to the transmission channels of international business cycles. One measure of the international transmission of business cycles, the cross-country correlation of aggregate variables, poses a problem for standard IRBC models known as the quantity problem.[6] That is, the ordering of output and consumption cross correlations in the model is opposite to that in the data. While many papers have made contributions to reducing or solving this problem, few address another quantity problem involving aggregate hours and investment.[7] A result of focusing on net flows of FDI is that the model solves the investment quantity puzzle. That is, contrary to other international real business cycle model, the presence of congestions in foreign investment markets generates a positive cross-country correlation of aggregate investment consistent with the data.

## 3.2 Flows of FDI and Canada - U.S. business cycles

This section reviews evidence on the cyclical characteristics of FDI flows outlined in the introduction. While the Canadian economy is of particular interest for this study because of the large and historically stable share of economic activity originating in the foreign sector, it is increasingly significant for other industrialized economies as they further integrate. Flows of foreign direct investment into Canada, and flows of Canadian direct investment abroad, concerning overwhelmingly the United States, the focus is placed on the similarities and interdependence of both countries.

### 3.2.1 Canadian and U.S. business cycles

Despite a large difference in absolute size, in per capita terms the Canadian and U.S. economies are remarkably similar. The evolution of hours worked (indexed), real output, investment and consumption per capita in both countries, over the period 1976-2006, have but for a few episodes followed each other closely.[8] One example is the output per capita gap between the U.S. and Canada appearing during the 1990s, which also shows up as a gap in average hours worked.

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[6]See Backus, Kehoe and Kydland (1995).

[7]See Crucini (2008) for an extensive survey.

[8]See appendix C for data and technical details. The time series for the mentioned variables are plotted in Figure 1 of said appendix.

Table 3.1: Business cycle moments for Canada and the U.S.

| 1976:1 - 2005:4 | Canadian data | | U.S. data | | Cross-country |
|---|---|---|---|---|---|
| variable: | a | b | a | b | correlations |
| *Consumption* | 0.80 | 0.87 | 0.47 | 0.74 | 0.60 |
| *Hours* | 0.80 | 0.83 | 0.95 | 0.87 | 0.63 |
| *Investment* | 3.11 | 0.69 | 3.24 | 0.82 | 0.45 |
| *Output* | 1.53 | | 1.42 | | 0.75 |

a: Standard deviation relative to output;

b: Contemporaneous correlation with output;

All moments are Hodrick-Prescott filtered.

While aggregate trends have been similar, Table 3.1 examines differences in cyclical fluctuations of prominent macroèconomic variables, measured as 2nd moments for Hodrick-Prescott filtered quarterly data over 1976:1 - 2005:4. The Canadian and U.S. economies display approximately the same business cycle characteristics of these variables, although there is evidence of less aggregate consumption smoothing in Canada, seen as the larger relative volatility of consumption.[9]

One indicator of business cycle synchronization, the cross-country contemporaneous correlation of prominent macroeconomic variables, is reported in the last column of Table 3.1. In their extensive study of international business cycles, Ambler, Cardia and Zimmermann (2004) find much lower, although positive, cross country correlations than those for the Canada - U.S. pair, suggesting a higher than average degree of integration of both economies.[10] While both theoretical and empirical work have often followed trade as a vector of synchronization, the increasingly important channel of flows of foreign direct investment is explored in the next subsection.[11]

[9]See also Baxter and Crucini (1995), Ambler, Cardia and Zimmermann (2004) for similar observations.

[10]Ambler, Cardia and Zimmermann (2004) consider a sample of 20 industrialized countries, and all pairwise cross-country correlations, on quarterly data over the period 1960:1-2000:4. Average output, consumption, investment and hours cross-correlations are, respectively, 0.22, 0.14, 0.18, 0.26.

[11]Sales by multinational firms have outpaced the expansion of trade in manufactures over the last decades. See Markusen (2004). Kose and Yi (2001) explore and discuss the limitations of the trade approach to solving the quantity puzzles. Ambler, Cardia and Zimmermann (2002) explore the potential of a two country multi-sector model with trade in intermediate goods in addressing the same issue. Other avenues have been explored, such as variable capital utilization in Baxter and Farr (2005), or trade in capital goods in Boileau (1999). Iacoviello and Minetti (2006) explore the implications of imperfect cross-border credit relations for output cross-correlations. See also Schmitt-Grohé (1998) for an evaluation of various mechanisms.

### 3.2.2 Flows of FDI and foreign-controlled firms in Canada

There are essentially two ways in which foreigners can access a domestic economy: (i) by establishing a branch or new business; (ii) through mergers and acquisitions of domestic firms. A commonly used measure of the rate at which foreigners access a domestic economy, flows of foreign direct investment, can further be categorized as either 'horizontal' or 'vertical'. As described by Markusen (2004), horizontal FDI refers to the replication of capacity abroad, and vertical FDI to the division of the production process globally in order to exploit the benefits offered by different markets. As Brainard (1997) documents and argues, the majority of FDI between developed countries is horizontal. In addition, the large majority of foreign affiliate sales are destined to the host market.[12] There remains, however, a debate over the principle mode of accessing an economy, although Helpman, Melitz and Yeaple (2004) argue that it occurs mainly through 'greenfield' investment.[13]

In order to assess the extent and effect of foreign control over the national economy, in 1962 the Canadian government passed the Corporations Returns Act (CRA), requiring firms doing business in Canada to report financial and ownership data. Of the 40000 reporting firms in 2004, foreign controlled corporations accounted for 30.7% of total operating revenues and 28.5% of all assets held in Canada,[14] shares that have historically remained stable (see Figure 3.1).[15] The United States plays a central role in the foreign control of the Canadian economy, generating 62.6% of the operating revenues of foreign controlled corporations. The closest behind are the United Kingdom and Germany with, respectively, 7% and 6.5% of operating revenues.

By industrial sector, foreign control is most important in oil and gas, manufacturing and mining, and significant in wholesale trade, utilities, and transportation and ware-

[12]As reported in Brainard (1993), approximately 92% a foreign affiliate production in the United States is destined for the host market.

[13]By 'greenfield' investment, one refers to the establishment of a branch or new business. The position taken by Helpman et al. (2004) differs from that of Graham and Krugman (1995) according to whom the evidence is less clear and leans rather towards a larger role for mergers and acquisitions. While this paper will follow Helpman et al. (2004), it worth noting a recent contribution by Nocke and Yeaple (2007). These authors investigate the theoretical determinants of FDI by M&A or greenfield investment.

[14]'The notion of control encompasses both direct and effective control. Direct control is defined as a person, group or corporation holding, directly or indirectly, more than 50% of the voting equity. Effective control implies control through methods other than ownership of the majority voting equity, such as when more than 50% of the directors of a corporation are also directors of another corporation. A corporation is foreign controlled when either direct or effective control is held by a person, group or corporation not resident in Canada.' For additional information , see 'Corporations Returns Act. 2004,' catalogue no. 61-220. Statistics Canada, vol XI E, p. 3.

[15]This figure is reproduced from catalogue no. 61-220. Statistics Canada, vol XI E.

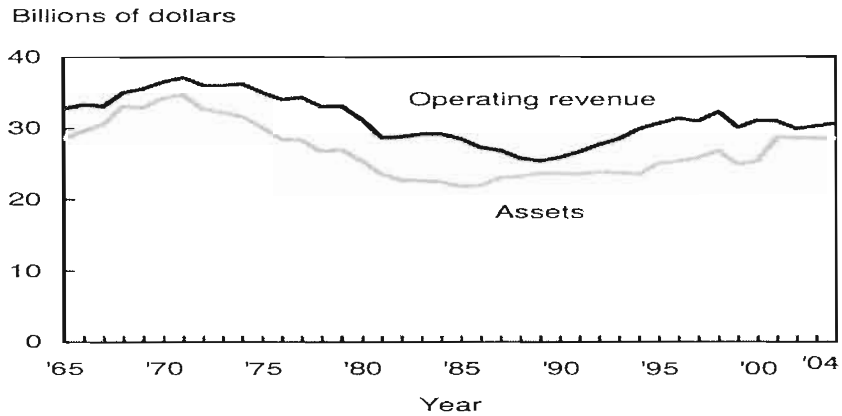


Figure 3.1: Share of assets and operating revenue under foreign control. Source: Statistics Canada

housing.[16] Manufacturing stands out as a sector with a large share of employment and high degree of foreign control, involving nearly one fifth of employment and where just over half of the revenues and assets are under foreign control. In fact, Baldwin and Gellatly (2005) estimate the share of manufacturing employment originating in the foreign sector to be 30% of total sectoral employment. Together, sectors with more than 20% foreign control, in terms of assets, involve 55% of employment. Although these are not the ideal measures of aggregate activity generated in the foreign sector, they give a sense of the importance of foreign controlled firms for aggregate outcomes.

#### 3.2.2.1 Flows of foreign direct investment

Flows of foreign direct investment into Canada (receipts) and flow of Canadian direct investment abroad (payments) from the Canadian Balance of Payments are large, historically around 20% of aggregate Canadian investment. The source and destination of these flows is overwhelmingly the U.S., generating a share of 44% of receipts and destination for 58% of payments. Except for a brief period in the early 1990s, payments have always exceeded receipts, leading to a persistent deficit offset only by Canada's historically positive trade balance (see Figure 3.2).

The business cycle component of net flows of FDI into Canada and flows of Canadian direct investments abroad, along with their cross-correlation are presented in Table 3.2.[17] Both flows are highly volatile, with H.-P. filtered standard deviations relative to

[16]See Table 2 of the appendix. The same figure also reports for these industries their share of total employment.

[17]It is important to stress that flows of portfolio investment are excluded, keeping only flows of direct

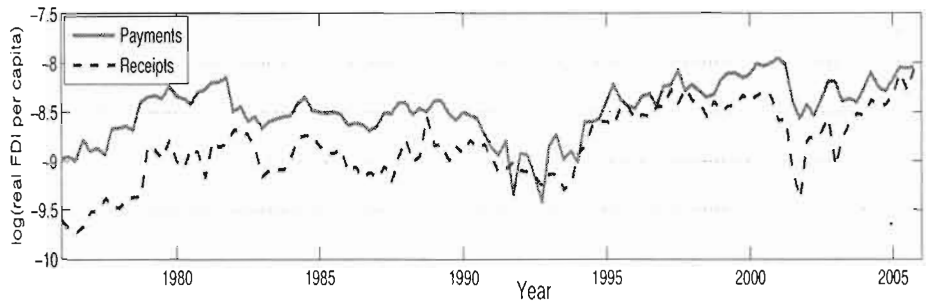


Figure 3.2: Flows of foreign direct investment receipts and payments, Canadian Balance of Payments.

Table 3.2: The business cycle of foreign direct investment.

| 1976:1 - 2005:4 | Net Foreign Direct Inv. in Canada | Net Canadian Direct Inv. Abroad |
|---|---|---|
| std. dev rel output | 14.19 | 8.8 |
| contemp.corr. with output | 0.36 | 0.40 |
| Ratio of inward to outward net FDI flow standard deviations | | 0.62 |
| Contemporaneous correlation, inward-outward net flows. | | 0.27 |

2nd moments were computed for Hodrick-Prescott filtered data. Data source: Statistics Canada

output of 14.19 and 8.8, respectively. By comparison, the relative volatility of aggregate investment is of the order of 3.11. Both net inflows and net outflows, that is net FDI in Canada and net Canadian Direct Investment Abroad move strongly with the Canadian business cycle, with respective contemporaneous correlations with Canadian GDP of 0.36 and 0.40. Moreover, Table 3.2 also reveals that net inward and net outward flows are positively correlated. That is, periods of increased net inflows of FDI into Canada are also accompanied by increased net Canadian direct investments abroad. This fact has not received much attention; equilibrium models tend to predict that capital would simultaneously flow into high productivity and out of low productivity countries.

## 3.3 IRBC with search in FDI and endogenous reallocation

The model develops a framework with *net* inflows and *net* outflows of foreign direct investment in a two country, two sector DSGE model, where *gross* investment flows in both directions evolve endogenously with the business cycle. Each country is populated

investment.

by domestic and foreign firms and a representative household. For simplicity the model abstracts from trade in consumption goods. Households decide on optimal consumption, an aggregate of goods produced by both types of firms, and allocation of investment goods to firms located at home or abroad. In order to initiate a new investment project abroad, foreign affiliates must disburse a flow cost $\kappa$. This cost is paid until the project is brought to fruition, a time-consuming task abstracted as a search and matching process with investment goods available for allocation abroad. No such friction applies to changing production capacity at domestic plants located in the home economy.[18] Thus domestic firms rent capital on spot markets while foreign affiliates choose the amount of new projects to initiate. Firms, domestic and foreign, hire labor on competitive domestic markets.

As a matter of notation, the first country is referred to as the 'Home' country and the second as the 'Foreign' country. Throughout, variables relating to the Foreign economy will be distinguished by an asterisk. For example, $k^{fdi}$ denotes the stock of capital held by foreigners in the 'Home' economy while $k^{fdi*}$ denotes the stock of capital held by foreigners in the 'Foreign' country, i.e. held by residents of the Home country. We begin by describing the friction to allocating physical capital abroad, domestic and foreign firms, and then examine the problem faced by the representative household in the Home economy, the problem in the Foreign economy being symmetrical. Endogenous capital reallocation is introduced before closing the model.

### 3.3.1 Undertaking a foreign direct investment

In order to form a unit of capital abroad, a new project, $v$, must be initiated at a cost of $\kappa$ by a foreign affiliate.[19] Meanwhile, a pool of liquid capital, $l$, must be made available to be allocated abroad once the right location has been found. This process of matching new projects and liquid capital is abstracted by a constant returns to scale matching technology $m(v, l)$. Denoting $\theta = \frac{l}{v}$ as a relative measure of capital liquidity, the probability for a given project initiation of becoming a productive unit of capital in the current period is given by $\frac{m(v,l)}{v} = m(1, \theta) = p(\theta)$ , with $\partial p(\theta)/\partial \theta > 0$. The

[18] In fact, the frictionless capital market is a special case of the search environment with $\kappa = 0$. This extreme assumption of no friction to allocating investment goods to domestic firms at home is made for simplicity. As long as allocating investment goods abroad is relatively more costly than the allocation at home, the results go through.

[19] This cost is reminiscent of Gordon and Bovenberg (1996) who assume that foreign investors, due to a lack of knowledge of the domestic economy, are at a disadvantage in setting up and running a firm. They capture this idea by assuming that output by foreign firms is reduced by some fixed proportion. Gopinath (2004) assumes that investors in emerging markets must disburse a cost to acquire information on investment projects while the length of the acquisition period is subject to search frictions.

equivalent probability for liquid capital is just $\frac{m(v,l)}{l} = m(1/\theta, 1) = q(\theta)$, $\partial q(\theta)/\partial\theta < 0$.

Once in place, a particular unit of foreign capital faces a probability (the determination of which is discussed later) $s_t$ of being terminated. When this occurs the unit of capital returns to the pool of liquid capital, net of depreciation, for reallocation. As a result, the total amount of liquid capital available for allocation abroad in the current period is defined as

$$l_t = i_t^{fdi} + (1-\delta)s_t k_t^{fdi} + u_t, \tag{3.1}$$

where $u_t = (1-q(\theta_{t-1}))l_{t-1}$ is unmatched liquid capital from the previous period carried forward with no net return, and $i_t^{fdi}$ are new investment goods added to the pool of liquid capital.

These assumptions result in the following law of motion for the stock of foreign capital in the Home economy

$$k_{t+1}^{fdi} = (1-\delta)(1-s_t)k_t^{fdi} + m(v_t, l_t). \tag{3.2}$$

For ease of comparison with the Balance of Payments, it is useful to rewrite the law of motion as $k_{t+1}^{fdi} = (1-\delta)k_t^{fdi} + m(v_t, l_t) - (1-\delta)s_t k_t^{fdi}$. The expression $m(v_t, l_t)$ corresponds to gross inflows of foreign direct investment while $(1-\delta)s_t k_t^{fdi}$ corresponds to gross outflows of foreign direct investment, the difference being net flows of inward FDI. The Home economy's direct investment abroad is likewise decomposed into gross outflows $m(v_t^*, l_t^*)$ and gross inflows $(1-\delta)s_t^* k_t^{fdi*}$(i.e., returning from the Foreign country).

### 3.3.2 Domestic and foreign producers

Domestic and foreign firms produce intermediate goods aggregated into a final homogeneous consumption good by an Armington (1969) aggregator $y_t = G\left(y_t^d, y_t^{fdi}\right) = \left[\phi(y_t^d)^\nu + (1-\phi)(y_t^{fdi})^\nu\right]^{\frac{1}{\nu}}$, with elasticity of substitution $\psi = 1/(1-\nu)$ and relative shares determined by the parameter $\phi$. The relative price of the foreign firm's good is then simply $p_t^{fdi} = G_2\left(y_t^d, y_t^{fdi}\right)$ and that of the domestic firm's good $p_t^d = G_1\left(y_t^d, y_t^{fdi}\right)$.

Domestic firms produce with technology $y_t^d = A_t(n_t^d)^{1-\alpha}(k_t^d)^\alpha$, hiring both factors of production from households on competitive markets. Optimization yields the following two first order conditions:

$$(n_t^d) \quad : \quad w_t^d = (1-\alpha)\frac{p_t^d y_t^d}{n_t^d}; \tag{3.3}$$

$$(k_t^d) \quad : \quad r_t^d = \alpha\frac{p_t^d y_t^d}{k_t^d}; \tag{3.4}$$

where $w_t^d$ and $r_t^d$ are, respectively, the remunerations of a unit of labor and a unit of capital at domestic firms.

Foreign firms in the Home economy hire domestic labor, $n^{fdi}$, and make capital adjustment decisions by choosing the number of new projects to initiate, $v$, with the production technology $y_t^{fdi} = A_t(n_t^{fdi})^{1-\alpha}(k_t^{fdi})^{\alpha}$. This yields the following dynamic program:

$$\begin{aligned} J(k_t^{fdi}) &= \max_{n_t^{fdi}, v_t} \left[ p_t^{fdi} y_t^{fdi} - w_t^{fdi} n_t^{fdi} - r_t^{fdi} k_t^{fdi} - \kappa v_t + \beta E_t \frac{\lambda_{t+1}^*}{\lambda_t^*} J(k_{t+1}^{fdi}) \right] \\ & \text{subject to } k_{t+1}^{fdi} = (1-\delta)(1-s_t)k_t^{fdi} + p(\theta_t)v_t \\ & \text{and } y_t^{fdi} = A_t(n_t^{fdi})^{1-\alpha}(k_t^{fdi})^{\alpha} \end{aligned}$$

where $w_t^{fdi}$ and $r_t^{fdi}$ are, respectively, the remunerations of labor and capital at foreign firms. The foreign affiliate uses the stochastic discount rate $\beta E_t \frac{\lambda_{t+1}^*}{\lambda_t^*}$ as all profits are transferred to the foreign household. Optimization yields the following two first order conditions:

$$\begin{aligned} (n_t^{fdi}) &: \quad w_t^{fdi} = (1-\alpha)\frac{p_t^{fdi} y_t^{fdi}}{n_t^{fdi}}; \\ (v_t) &: \quad \frac{\kappa}{p(\theta_t)} = \beta E_t \frac{\lambda_{t+1}^*}{\lambda_t^*} J_{k^{fdi}}(k_{t+1}^{fdi}); \end{aligned} \tag{3.5}$$

where $J_{k^{fdi}}(k_{t+1}^{fdi})$ is the marginal value of an additional unit of capital to the firm. While the first condition is quite standard, some interpretation of the optimality condition for project initiations is in order. This states that, at the margin, the discounted expected return to an additional unit of capital must be equal to the average cost of setting it up, $\frac{\kappa}{p(\theta_t)}$. As such, this may be interpreted as a "project creation" condition akin to the job creation condition in labor search and matching models. Differentiating the firm's value function, the marginal value is defined as

$$J_{k^{fdi}}(k_t^{fdi}) = \alpha \frac{p_t^{fdi} y_t^{fdi}}{k_t^{fdi}} - r_t^{fdi} + (1-\delta)(1-s_t)\beta E_t \frac{\lambda_{t+1}^*}{\lambda_t^*} J_{k^{fdi}}(k_{t+1}^{fdi}).$$

In combination with the first order condition for project initiations, this yields the forward looking condition

$$\frac{\kappa}{p(\theta_t)} = \beta E_t \frac{\lambda_{t+1}^*}{\lambda_t^*} \left\{ \alpha \frac{p_{t+1}^{fdi} y_{t+1}^{fdi}}{k_{t+1}^{fdi}} - r_{t+1}^{fdi} + (1-\delta)(1-s_{t+1}) \frac{\kappa}{p(\theta_{t+1})} \right\}. \tag{3.6}$$

### 3.3.3 Domestic households

Households choose a level of aggregate consumption of the final homogeneous good, hours to supply to both domestic and foreign employers, $n_t^d$ and $n_t^{fdi}$ respectively, and have two capital investment options: investing in firms at home, $i_t^d$, or investing in capacity abroad, $i_t^{fdi*}$. In addition, there are convex costs to producing new investment goods (domestic and foreign).[20] The resulting dynamic program for the representative household is thus

$$V(k_t^d, k_t^{fdi*}, u_t^*) = \max_{c_t, n_t^d, n_t^{fdi}, i_t, i_t^{fdi*}} \left[u(c_t, 1-n_t) + \beta E_t V(k_{t+1}^d, k_{t+1}^{fdi*}, u_{t+1}^*)\right]$$

$$\text{subject to} \quad w_t^d n_t^d + w_t^{fdi} n_t^{fdi} + r_t^d k_t^d + r_t^{fdi*} k_t^{fdi*} + \Pi_t^* = c_t + q_t^d i_t^d + q_t^{fdi*} i_t^{fdi*}$$

$$\text{and} \quad k_{t+1}^{fdi*} = (1-\delta)(1-s_t^*)k_t^{fdi*} + q(\theta_t^*)l_t^*$$

where $n_t = n_t^d + n_t^{fdi}$, $i_t^d = k_{t+1}^d - (1-\delta)k_t^d$, and $q_t^d$ and $q_t^{fdi*}$ are, respectively, the cost of new investment goods destined for plants at home and abroad. Under the assumption that the cost of adjusting physical capital is governed by the function $\Phi(\frac{i_t^j}{k_t^j})$, as in Hayashi (1982), this price is given by $q_t^j = \left[\Phi'(\frac{i_t^j}{k_t^j})\right]^{-1}$, for $j = d, fdi^*$, with $\Phi'(\bullet) > 0$ and $\Phi''(\bullet) < 0$, and such that in steady state $q = 1$. New investment goods destined for foreign direct investment are defined as $i_t^{fdi*} = l_t^* - (1-\delta)s_t^* k_t^{fdi*} - u_t^*$, where $(1-\delta)s_t^* k_t^{fdi*}$ is capital recouped from terminated operations abroad net of depreciation, and $u_t^*$ are units of investment goods not yet allocated. Again, $l_t^*$ is therefore the total amount of investment goods available for allocation to production abroad.

Denoting the multiplier on the budget constraint by $\lambda_t$, the optimality conditions are

$$(c_t) \;:\; u_c(c_t, 1-n_t) = \lambda_t \tag{3.7}$$

$$(n_t^d) \;:\; u_{n^d}(c_t, 1-n_t) = \lambda_t w_t^d \tag{3.8}$$

$$(n_t^{fdi}) \;:\; u_{n^{fdi}}(c_t, 1-n_t) = \lambda_t w_t^{fdi} \tag{3.9}$$

$$(i_{t+1}^d) \;:\; \lambda_t q_t^d = \beta E_t \lambda_{t+1}\left[r_{t+1}^d + q_{t+1}^d(1-\delta)\right] \tag{3.10}$$

$$(i_t^{fdi*}) \;:\; \lambda_t q_t^{fdi*} = \beta E_t[q(\theta_t^*)V_{k^{fdi*}}(k_{t+1}^d, k_{t+1}^{fdi*}, u_{t+1}^*) + (1-q(\theta_t^*))V_{u^*}(k_{t+1}^d, k_{t+1}^{fdi*}, u_{t+1}^*)] \tag{3.11}$$

The Euler equation for allocation of investment goods to domestic firms, equation (3.10), has the usual interpretation of equating the opportunity cost of the investment, in

[20] It is well known (see Backus, Kehoe and Kydland (1992), Baxter and Crucini (1995)) that without an adjustment cost to the production of new capital goods the volatility of new investment would be much too large in this setting.

terms of current period forgone consumption, to the expected return net of depreciation. The Euler equation governing foreign investment decisions, equation (3.11), has a similar interpretation. The expected return, however, is an average of the marginal values of matched ($V_{k^{fdi*}}(k^d_{t+1}, k^{fdi*}_{t+1}, u^*_{t+1})$) and unmatched ($V_{u^*}(k^d_{t+1}, k^{fdi*}_{t+1}, u^*_{t+1})$) capital, weighted by the matching probability $q(\theta^*_t)$ . The marginal values of allocated and non-allocated investment goods are given by

$$\begin{aligned} V_{u^*}(k^d_t, k^{fdi*}_t, u^*_t) &= \lambda_t q^{fdi*}_t; \\ V_{k^{fdi*}}(k^d_t, k^{fdi*}_t, u^*_t) &= \lambda_t \left[ r^{fdi*}_t + q^{fdi*}_t(1-\delta)s^*_t \right] + (1-\delta)(1-s^*_t)\beta E_t V_{k^{fdi*}}(k^d_{t+1}, k^{fdi*}_{t+1}, u^*_{t+1}). \end{aligned}$$

Since unmatched liquid capital yields no net return, its marginal value is simply the opportunity cost of funds. The marginal value of matched capital consists of the earnings on the unit, $r^{fdi*}_t$, and the value of capital separated for reallocation net of depreciation. The last term captures the continuation value if reallocation does not occur.

### 3.3.4 Repayment on foreign capital

Each unit of capital allocated abroad generates a surplus for the foreign affiliate and the capital lender. The repayment on capital allocated abroad is determined by Nash bargaining over the total surplus generated by the relationship, defined as $S_t = J(k^{fdi}_t) + \frac{V_{k^{fdi}}(k^{d*}_t, k^{fdi}_t, u_t) - V_u(k^{d*}_t, k^{fdi}_t, u_t)}{\lambda^*_t}$. This results in the following repayment rule:[21]

$$r^{fdi}_t = \eta\alpha \frac{p^{fdi}_t y^{fdi}_t}{k^{fdi}_t} + (1-\eta)q^{fdi}_t\delta + \eta(1-\delta)(1-s_t)\frac{\kappa}{\theta_t}. \tag{3.12}$$

By the first term, the repayment is increasing in the marginal product of capital. The second term captures the loss of value due to physical depreciation, measured by the price of investment goods, the cost of which is split according to the lender's bargaining weight $\eta$. The long-term nature of the relationship is captured by the final term. It represents the initiation costs saved by the firm in the continued operation of the unit of capital. By changing the relative threat point of the firm in negotiations, a rise in $\kappa$ puts upward pressure on the repayment.

### 3.3.5 Endogenous reallocation and profits

As in Mortensen and Pissarides (1994) for the labor market, the existence of a random idiosyncratic productivity to the match is assumed, the realization of which occurs after

[21] The appendix provides details on the derivation of this repayment rule.

production decisions are made and factor price equilibria are established. Denote this realization $a_t > 0$, where $a$ is independently distributed over time with probability density $h(a)$, cumulative density $H(a)$ and mean $E(a) = 1$, and follows a log normal distribution $\log(a) \sim N(-\frac{\sigma^2_{\log(a)}}{2}, \sigma^2_{\log(a)})$. The surplus generated by the relationship between a foreign affiliate and the capital lender (i.e. the household) is then an increasing function of this shock, $S(a_t)$. Once the shock is observed, both parties discontinue the match for realizations of $a_t < \underline{a}_t$ where $\underline{a}_t$ is define as $S(\underline{a}_t) = 0$. Using a result of Nash bargaining, the separation threshold is defined by [22]

$$r_t^{fdi} - \underline{a}_t \alpha \frac{p_t^{fdi} y_t^{fdi}}{k_t^{fdi}} - (1-\delta)(1-s_t)\frac{\kappa}{p(\theta_t)} = 0 \tag{3.13}$$

In effect, the match is discontinued if the realized marginal product of capital $a_t \alpha \frac{p_t^{fdi} y_t^{fdi}}{k_t^{fdi}}$ plus the search cost saved by maintaining the current unit of capital is inferior to the negotiated repayment. An increase in the average search cost $\frac{\kappa}{p(\theta_t)}$, for example, by increasing the opportunity cost of exiting the match, lowers the separation probability.

Finally, an insurance mechanism funded out of profits from continuing relationships is assumed in order to insure that ex-post the household receives the full ex-ante return to foreign capital, and that the full wage bill and costs of project initiations are covered.[23] Thus aggregate profits returned to the household are

$$\begin{aligned} \Pi_t &= \int_{\underline{a}_t}^{\infty} \left[ a p_t^{fdi} y_t^{fdi} - w_t^{fdi} n_t^{fdi} - r_t^{fdi} k_t^{fdi} - \kappa v_t \right] dH(a) \\ &\quad + \int_0^{\underline{a}_t} \left[ a p_t^{fdi} y_t^{fdi} - w_t^{fdi} n_t^{fdi} - r_t^{fdi} k_t^{fdi} - \kappa v_t \right] dH(a) && (3.14) \\ \Pi_t &= p_t^{fdi} y_t^{fdi} - w_t^{fdi} n_t^{fdi} - r_t^{fdi} k_t^{fdi} - \kappa v_t. && (3.15) \end{aligned}$$

Equilibrium is defined by the system of equations (3.1)-(3.15), production technologies for domestic, foreign firms and the final consumption good, and the definition for the separation rate, $s_t = H(\underline{a}_t)$, in both the Home and Foreign countries.

## 3.4 Quantitative results

The model is solved for the rational expectations equilibrium of the log-linear system of equations with the algorithm developed by King and Watson (1998). The quantitative implications are evaluated through impulse responses and unconditional second

[22] See appendix for details.

[23] Gordon and Bovenberg (1996) use a similar assumption about the realization of an idiosyncratic productivity shock, and use the law of large numbers to argue that there is no aggregate uncertainty. Here, an insurance funded out of aggregate profits is used to address the issue.

moments. The results for flows of FDI are discussed first, before looking at aggregate variables and cross-country correlations. In all instances the results are contrasted with those for a standard IRBC with investment adjustment costs, and a model with exogenous reallocation.[24]

### 3.4.1 Shocks and calibration

#### 3.4.1.1 Extraction of a Solow Residual.

The underlying exogenous processes for technology, as in Backus Kehoe and Kydland (1992), is assumed stationary and to follow a VAR(1) process with possible cross-country spill-overs. Parameter estimates are obtained by extracting Solow residuals for the Canadian and U.S. economies and then estimating the following bivariate VAR(1):[25]

$$\begin{bmatrix} A_t^{can} \\ A_t^{us} \end{bmatrix} = \begin{bmatrix} \rho_c & \rho_{c,us} \\ \rho_{us,c} & \rho_{us} \end{bmatrix} \begin{bmatrix} A_{t-1}^{can} \\ A_{t-1}^{us} \end{bmatrix} + \begin{bmatrix} e_t^{can} \\ e_t^{us} \end{bmatrix}$$

The results of the estimation are presented below for the period 1976:1 - 2005:4. As is usual in this sort of estimation, the persistence parameter is very high. Also, as can be gleaned from the covariance matrix, Canadian and U.S. innovations to the exogenous process for technology are positively correlated. In a subsection below, the sensitivity of the quantitative results to the specification of the exogenous process for technology will be examined.

$$\begin{bmatrix} \rho_c & \rho_{c,us} \\ \rho_{us.c} & \rho_{us} \end{bmatrix} = \begin{bmatrix} 0.9747 & 0.034 \\ -0.0174 & 0.9264 \end{bmatrix} \quad \text{Residuals Covariance matrix}: \begin{bmatrix} 0.079 & 0.021 \\ 0.021 & 0.05 \end{bmatrix}$$

#### 3.4.1.2 Calibration

The discount factor is set to $\beta = 0.99$ which corresponds to an average annual real yield on a risk-less bond of 4.1%. Preferences are separable in consumption and leisure, and take the form $u(c_t, 1-n_t) = \log(c_t) + \frac{\varpi(1-n_t)^{1-\xi}}{1-\xi}$. The parameter $\varpi$ is fixed such that the average fraction of total hours worked equals $n = 0.2$. Together with $\xi = 4$ this results in a Frisch elasticity of labor supply of 1. Furthermore, the share domestic goods in the production of the final good, $\phi$, is set such that the steady state equilibrium hours worked

[24] The details these models are presented in the appendix. It is important to note that the first corresponds to the search model without allocation frictions, i.e. a model with $\kappa = 0$.

[25] Disembodied productivity is measured as a residual for a Cobb-Douglas production function: $\log(A_t) = \log(y_t) - \alpha \log(n_t) - (1-\alpha)\log(k_t)$. The quarterly series of an aggregate capital stock for both economies is estimated using the perpetual inventory method.

in the foreign sector are 1/4 of total hours, which is in the range of employment shares reported earlier. The share of capital in the production function is set to $\alpha = 1/3$, and the rate of depreciation of capital to $\delta = 0.025$, which corresponds to an annual decline of productive use of capital of 10%. The elasticity of the investment adjustment cost is 0.025, within a range of values used in different studies (e.g., Baxter and Crucini, 1995, Ambler, Cardia and Zimmermann, 2002, and Baxter and Farr, 2005), and is chosen to match the relative standard deviation of aggregate investment in the data. Finally, the parameter $\nu$ in the Armington aggregator is chosen to imply an elasticity of substitution between the foreign and domestic firms' goods of 1.5.

To calibrate parameters related to foreign direct investment, it is useful to let the theory shed some light on the data. Recall the foreign capital accumulation equation

$$k_{t+1}^{fdi} = (1-\delta)k_t^{fdi} + inflow_t - outflow_t.$$

As the Balance of Payments provide information on foreign direct investment gross inflows and outflows, given a rate of capital depreciation one can compute the implied foreign capital stock in the host economy, using the steady state property $k^{fdi} = [inflow - outflow]/\delta$ to initiate the capital stock. It is then possible, using the time series on outflows, to obtain a time series for the reallocation rate as

$$s_t = \frac{outflow_t}{(1-\delta)k_t^{fdi}};$$

resulting in a mean rate of $s = 0.0602$, an H.-P. filtered standard deviation relative to output of 1.45 and contemporaneous correlation with output of 0.16.

Next it is assumed that it takes on average a little more than a quarter before liquid capital is allocated and becomes productive; i.e. $q(\theta) = 0.75$, and the household's bargaining weight is set to $\eta = 0.5$, in the mid-range of possible values.[26] The final parameter left to calibrate is the elasticity of the matching function, which is of the form $m(v_t, l_t) = (v_t)^{\epsilon}(l_t)^{1-\epsilon}$. This parameter only influences the dynamics of the model but does not affect the steady state, and is therefore selected such that the relative volatility of the reallocation rate $s$ is close to the data, leading to a choice of $\epsilon = 0.8$. A sensitivity analysis of results to variations in these parameters is performed below.

With these calibrations there is sufficient information to endogenously determine the rest of the parameters (i.e. $\theta$, $\kappa$, $\sigma$) such that the system of steady state equations is

[26] As it is well know that the results of the search and matching model of equilibrium unemployment are sensitive to the value of this parameter (see Hagedorn and Manovskii, 2008), a series of sensitivity test will be performed below.

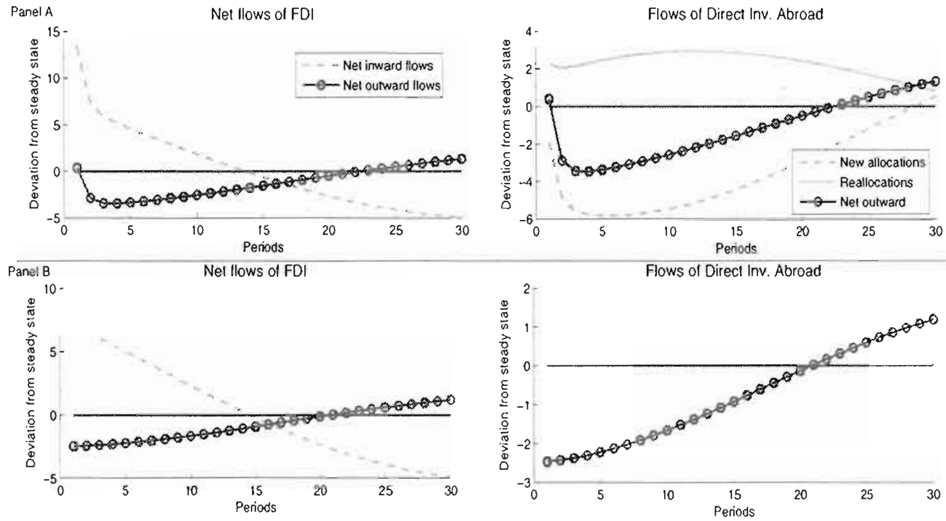


Figure 3.3: IRFs to a positive "Home" sourced technology shock.

satisfied.[27] The resulting long-run ratios of interest are the following: the consumption-output ratio equals 76.52% in line with King and Rebelo (1999); the labor share of income amounts to 0.67, which lies in the range reported by Gollin (2002). Furthermore, this calibration implies that the steady state ratio of net FDI to aggregate investment is 23%, that the average initiation cost relative to output equals $\upsilon\kappa/y^{fdi} = 1\%$ and that the standard deviation of the idiosyncratic productivity shock equals $\sigma_a = 0.27$.

### 3.4.2 Flows of foreign direct investment

Figure 3.3 plots the impulse responses to a positive technology shock in the Home economy of net inward and outward foreign direct investment for that expanding economy. The significant difference between the responses of the proposed model (Panel A) and an IRBC model with investment adjustment costs (Panel B), beyond their magnitude, is the behavior of net outward flows (see circled line of panels A and B). In the search model, outward flows drop progressively, whereas in the standard model the drop occurs on impact. It is this difference that generates the positive cyclical correlation of net inward and outward flows that is a characteristic of the data.

To detail the response of net outward direct investment flows, it is useful to recall its definition as the difference between gross outflows and gross inflows from the Home

[27] The details concerning the procedure for computing the steady state are available in the appendix.

to the Foreign economy:

$$\text{Net outward } : l_t^* q(\theta_t^*) - (1-\delta) s_t^* k_t^{fdi*}$$

The second column of Figure 3.3 decomposes the response of net outward flows into these new allocations (dashed line) and reallocations (solid line) of capital abroad.[28]

Consider now what matters for the initial response of net outward FDI: the response of gross outflows $l_t^* q(\theta_t^*)$ and the reallocation rate $s_t^*$, $k_t^{fdi*}$ being predetermined. As the opportunity cost of capital abroad increases, households diminish their pool of liquid capital $l^*$, shifting resources to domestic firms, causing a drop in the Home country's pool of capital available for investment abroad. This the only source of change in net outflows in the model without allocation frictions, and therefore the drop in outward FDI is immediate. When allocation frictions are present, however, the decline in the pool of liquid capital is larger than the initial decline in project initiations at foreign affiliates for two periods after the shock, leading to a short lived increase in the capital allocation probability $q(\theta_t^*)$.[29] This reduction in market congestion counters some of the drop in $l^*$ upon impact, and is seen in the muted initial decline in new allocations abroad (see the upper right quadrant of Figure 3.3). As $q(\theta_t^*)$ declines thereafter, new allocations attain their lowest 6 quarters following the shock.

The second distinction comes from the effects on reallocation of capital already abroad. As illustrated in the second quadrant of Figure 3.4, a positive innovation in the Home country causes the reallocation rate in the foreign country to drop, reducing the gross flow $(1-\delta)s_t^* k_t^{fdi*}$ on impact.[30] The drop pulls net outward flows from the Home the Foreign country upward, such that on impact net outward flows change very little (again, see Figure 3.3). Thus the key to understanding the response of net foreign direct investment flows are the time-varying congestion, and to a lesser extent reallocation rates, effects that are absent in the standard model.

Making this point clearer. Table 3.3 present unconditional second moments for flows of foreign direct investment in the data and generated by the competing models. The standard IRBC model with investment adjustment costs, for the reasons just outlined, generates a strong negative contemporaneous correlation between net inflows and outflows of FDI. A model with constant, exogenous reallocation goes a long way in improving

[28]Note that the negative of gross inflows is plotted so as to better illustrate the positive impact of net outflows.

[29]Kurmann and Petrosky-Nadeau (2009) show that under relatively weak conditions, if preferences are additive and concave in consumption for example, that congestion on the investment market will be increasing in the expected growth rate of the marginal utility of consumption.

[30]The contemporaneous correlation with the host economy's business cycle is 0.85 and its standard deviation relative to aggregate output 1.13. These numbers relate to Hodrick-Prescott filtered moments.

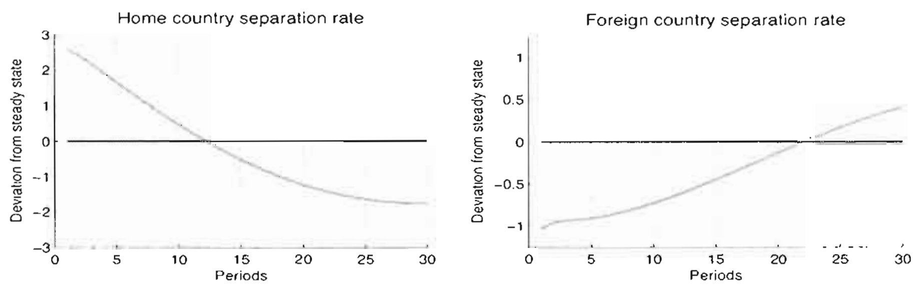


Figure 3.4: IRFs of the separation rates to a Home country positive technology shock.

Table 3.3: 2nd moments for flows of foreign direct investment.

| 1976:1 - 2005:4 | Canadian data | | Search in FDI model | | | | IRBC with FDI | |
|---|---|---|---|---|---|---|---|---|
| | | | End. reall. | | Exog. reall. | | | |
| | a | b | a | b | a | b | a | b |
| Net FDI in Canada | 14.19 | 0.36 | 10.73 | 0.75 | 10.41 | 0.83 | 7.17 | 0.91 |
| Net Canadian Direct Inv abroad | 8.8 | 0.40 | 8.83 | 0.06 | 7.08 | -0.04 | 2.73 | -0.54 |
| | c | d | c | d | c | d | c | d |
| Net outward / Net inward FDI | 0.62 | 0.27 | 0.82 | 0.34 | 0.68 | 0.23 | 0.38 | -0.58 |

a: Standard deviation relative to output; b: Contemporaneous correlation with output;
c: Ratio of outward to inward FDI flow standard deviations; d: contemporaneous corr.,
inward-outward net flows. All moments are Hodrick-Prescott filtered.

this correlation, almost perfectly matching the data with a contemporaneous correlation of 0.32 compared to 0.27 in the data. Contrasting this result, the IRBC model generates a correlation of -0.58. The contribution of endogenous reallocation is to increase the correlation even further. Thus the model is able to replicate the fact that periods of increased net investment abroad are also characterized by increased net inflow of foreign investment.

Another feature of the data concerns the relative volatility of net FDI outflows and inflows of approximately two thirds, and that net outflows are as procyclical as net inflows. The standard IRBC model fails on both these counts. The ratio of H.-P. filtered standard deviations is only 0.38, and the correlation of net outflows with the source country's business cycle is strongly negative at -0.54. On the other hand, the proposed model of search in FDI performs very well on the relative volatility of net inflows and outflows, the ratio being 0.68 compared to 0.62 in the data in the presence of exogenous reallocation, and 0.82 with endogenous reallocation. However, while the model raises the correlation of net outflows with the domestic business cycle, under the present calibration it is insufficient to be in-line with the data.

Table 3.4: Sensitivity to search parameters

| | | Reall. rate $s$ | | Alloc. rate $q(\theta)$ | | Bargaining weight $\eta$ | |
|---|---|---|---|---|---|---|---|
| | Baseline | 0.04 | 0.08 | 0.5 | 0.9 | 0.3 | 0.7 |
| $\sigma(net\ inward)/\sigma(Y)$ | 10.73 | 9.58 | 11.95 | 9.71 | 11.36 | 12.29 | 10.67 |
| $\sigma(net\ outward)/\sigma(net\ inward)$ | 0.82 | 0.76 | 0.85 | 0.98 | 0.77 | 1.40 | 0.78 |
| $corr(inward, outward)$ | 0.35 | 0.27 | 0.40 | 0.38 | 0.34 | 0.50 | 0.34 |

### 3.4.3 Robustness of results

This section performs two sets of sensitivity checks, first to the values of search related parameters, second to the specification of the exogenous technological process.

#### 3.4.3.1 Sensitivity to search parameters

First, given the lack of direct evidence on the mean allocation rate $q(\theta)$, the effects of its variation on the main results, along with the consequence of varying the mean reallocation rate $s$, are presented in Table 3.4.[31] Second, as the work of Hagedorn and Manovskii (2008) has shown that the dynamics of search unemployment models are sensitive to variations in the bargaining weight $\eta$, the results are examined along this dimension as well.

Beginning with the mean allocation rate, its principle effect is to change the relative standard deviation of net foreign direct investment flows. The change to the correlation between net inward and outward flows of FDI when decreasing the degree of congestion in the allocation of capital abroad (i.e., increasing the mean $q(\theta)$) between approximately a year and a half and just over a quarter is very small.

The results are very robust to changes in the reallocation rate $s$, the volatility of net inward FDI and the relative volatility of net inward and outward flows changing little. The main effect is to increase the correlation between inward and outward flows, from 0.27 when $s = 0.04$ to 0.40 when $s = 0.08$. This is to be expected as the mechanism generating a positive correlation that is endogenous reallocation becomes more important.

Increasing the lender's bargaining weight from 0.3 to 0.7 reduces the relative volatility of net inward foreign investment from 12.29 to 10.67. This occurs because, for lower values of the bargaining weight, the expected benefit of a new unit of capital allocated abroad is more elastic to changes in productivity.[32] It also has the effect of reducing both

[31] The mean rate $s$ may be affected by the initial foreign capital stock. It is therefore worth while exploring the sensitivity of the results to its calibration.

[32] The mechanism is similar to that which allows Hagedorn and Manovskii (2008) to generate greater

Table 3.5: Sensitivity specification of exogenous process

| 1976:1 - 2005:4 | Baseline | | Search in FDI model | | | | IRBC with FDI | |
|---|---|---|---|---|---|---|---|---|
| | calibration | | End. reall. | | Exog. reall. | | | |
| | a | b | a | b | a | b | a | b |
| Net FDI in Canada | 10.73 | 0.75 | 9.72 | 0.70 | 9.38 | 0.787 | 5.30 | 0.88 |
| Net Canadian Direct Inv abroad | 8.83 | 0.06 | 8.26 | 0.35 | 6.11 | 0.26 | 3.41 | 0.07 |
| | c | d | c | d | c | d | c | d |
| Net outward / Net inward FDI | 0.82 | 0.35 | 0.85 | 0.35 | 0.65 | 0.25 | 0.64 | -0.29 |

a: Standard deviation relative to output; b: Contemporaneous correlation with output;
c: Ratio of outward to inward FDI flow standard deviations; d: contemporaneous corr., inward-outward net flows. All moments are Hodrick-Prescott filtered.

the relative volatility of net inward to outward investment and the correlation between both flows, suggesting that the model is a better fit of the data with a higher bargaining weight with the exception of the relative volatility of net inward investment.

#### 3.4.3.2 Sensitivity to the specification of the technological process

An alternative specification of the exogenous process for technology cuts off cross-country spill-overs while fixing identical persistence parameters.

$$\begin{bmatrix} \rho_c & \rho_{c.us} \\ \rho_{us.c} & \rho_{us} \end{bmatrix} = \begin{bmatrix} 0.9747 & 0 \\ 0 & 0.9747 \end{bmatrix} \qquad \text{Covariance matrix}: \begin{bmatrix} 0.079 & 0.02 \\ 0.02 & 0.05 \end{bmatrix}$$

The results are presented in Table 3.5. Changing the specification of the exogenous process has little effect on the relative volatilities of net inward and outward flows of FDI, but increases the contemporaneous correlation of net investment abroad with the domestic business cycle in all models. The ratio of volatilities of inward and outward flows hardly changes except for the IRBC with investment adjustment costs model, for which the ratio becomes close to the data. However, the contemporaneous correlation of net inward of outward flows remains strongly negative, at -0.29, while for the proposed model it almost exactly matches the data.

### 3.4.4 Aggregate variables

Figure 3.5 plots the impulse response functions of output, hours and capital, at domestic firms, foreign firms and in the aggregate, to a Home sourced positive technology shock.

elasticity of labor market tightness to changes in productivity. The lower bargaining weight induces rigidity in the rental rate of capital (as opposed to wages), increasing the elasticity of the expected benefit side of the "project creation" condition (6) to changes in productivity.

Panel A presents results for the proposed search model with endogenous reallocation, while Panel B reports results for a standard IRBC with investment adjustment costs.

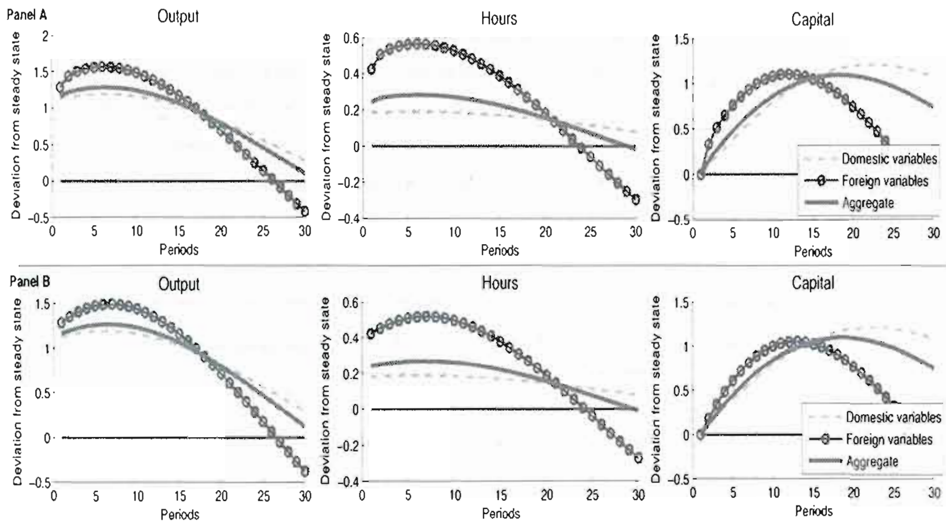


Figure 3.5: IRFs to a positive "Home" country technology shock.

The first observation is that the responses of aggregate variables are quite similar for both models. The differences arise in the response of foreign firms when investing in capacity abroad is subject to a time consuming search process. On impact, hours at foreign firms rise more than at domestic firms in both models. However, the ensuing additional increase in hours at foreign firms is more pronounced in the proposed model, and stems from the different capital stock dynamics: the stock of foreign capital rises more quickly than in the standard case, pushing up further the labor demand of foreign establishments. The model thus implies that hours at foreign firms are more volatile than at domestic firms over the business cycle. There is recent empirical evidence from Europe that foreign controlled firms tend to make larger and more frequent employment adjustments (Checchi et al. 2003), but no direct evidence of systematic differences in the response of hours to the business cycle.

A second dimension along which the model's performance is evaluated is a series of H.-P. filtered second moments. Table 3.6 presents the 2nd moments of prominent macroeconomic variables for the three models and the data. Both in terms of standard deviations and correlations with output, all three models are similar in being close to the data, with the well known exception of the volatility of hours. Thus, the ability of the model to generate high volatility in flows of FDI does not come at the expense of creating too much volatility in aggregate investment.

Table 3.6: 2nd moments for prominent macro variables.

| 1976:1 - 2005:4 | Canadian data | | | Search in FDI model | | | | | | IRBC with FDI | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | End. reall. | | | Exog. reall. | | | | | |
| variable: | a | b | c | a | b | c | a | b | c | a | b | c |
| *Consumption* | 0.80 | 0.87 | 0.60 | 0.55 | 0.99 | 0.26 | 0.55 | 0.99 | 0.30 | 0.56 | 0.99 | 0.29 |
| *Hours* | 0.80 | 0.83 | 0.63 | 0.23 | 0.98 | 0.43 | 0.31 | 0.98 | 0.43 | 0.30 | 0.98 | 0.41 |
| *Investment* | 3.11 | 0.69 | 0.45 | 3.79 | 0.93 | 0.13 | 4.93 | 0.93 | 0.10 | 4.33 | 0.95 | -0.09 |
| *Output* | 1.53 | | 0.75 | 1.40 | | 0.50 | 1.40 | | 0.51 | 1.37 | | 0.50 |

a: Standard deviation relative to output; b: Contemporaneous correlation with output

c: Cross country contemporaneous correlations. All moments are Hodrick-Prescott filtered

A well know deficiency of standard IRBC models, the quantity problem, concerns the ordering of cross country correlations of consumption, output, investment and hours. The problem of the ordering of consumption and output cross correlations is the most known of the quantity problems in the IRBC literature, as raised in the work of Backus, Kehoe and Kydland (1995), while the shortcomings related to the cross correlation of hours and investment have been raised in papers such as Ambler, Cardia and Zimmermann (2004). Table 3.6 shows the performance of the search in FDI model with this respect. All models get the ordering of a higher output than consumption cross-correlations right, although the cross-correlation of consumption is lower than in the data. This time on labor markets the three models perform quite well.[33] However, where aggregate investment is concerned, the cross-correlation is positive in both models with search frictions in foreign direct investment, while an IRBC model with investment adjustment costs generates a negative correlation. By altering the dynamics of a component of aggregate investment, the time varying-congestion on foreign investment markets resolves part of the investment quantity puzzle. Driving the point further, raising the size of the foreign sector (as a fraction of total hours) from one quarter to one half, raises the cross-country correlation of aggregate investment from 0.13 to 0.20 in the model with search in FDI, while the correlation is reduced from -0.09 to -0.21 in the standard IRBC model.

[33]This is due, essentially, to the correlation structure to innovations and the presence of investment adjustment costs. This was first pointed out by Backus, Kehoe and Kydland (1992), but made more explicit in Baxter and Crucini (1995).

## 3.5 Conclusion

A commonly used measure of the rate at which foreigners gain control over a domestic economy, flows of foreign direct investment (FDI), represent an increasingly important share of aggregate investment in industrialized economies as they further integrate. Given the importance of the foreign sector for aggregate outcomes and the relatively high volatility of direct investment flows, quantitative models of open economies need to be consistent with their dynamics.

As this paper has shown, a combination of frictions in the allocation of physical capital to production abroad, and allowing for the endogenous reallocation of this capital, can replicate the positive correlation between net inflows and outflows of FDI that is a feature of the data. In addition the model can generate the higher volatilities of inward and outward net FDI, while the implication for prominent macroeconomic variables are similar to a standard IRBC model with investment adjustment costs. However, there are important sectoral differences worth mentioning in conclusion. The model implies that, for example, hours worked at foreign establishments are more volatile than hours worked at domestic establishments. An interesting question, and most relevant for economic policy, is whether this is the case in the data. In particular, if one considers the extensive margin of labor adjustments, are jobs at foreign establishments more elastic to the business cycle? If so, this might offer a rationalization for the public's skepticism toward the benefits of increased foreign control of a domestic economy as employment at these firms would be more fragile.

# Conclusion

It has been argued that the standard model of equilibrium unemployment cannot generate sufficient propagation as productivity shocks, by inducing a rise in wages, have little effect on firm profits from a new employee and, hence, on the incentive to post new job vacancies. The first chapter of this thesis has shown that when vacancies must be funded in part on frictional credit markets, agency problems can lead to higher, time-varying, unit costs that greatly increase the elasticity of vacancies to productivity. This propagation mechanism operates through two distinct channels: (i) a cost channel - lowered unit costs during an upturn as credit constraints are relaxed increase the incentive to post vacancies; (ii) a wage channel - the improved bargaining position of firms afforded by the lowered cost of vacancies limits part of the upward pressure of market tightness on wages. The quantitative exercise has shown that the cost channel is largely dominant in allowing the model to match the observed volatility of unemployment, vacancies and labor market tightness. Moreover, the progressive easing of financing constraints to innovations as firms accumulate assets generates persistence in the response of market tightness and vacancies, a robust feature of the data and shortcoming of the standard model. The paper thus concludes that the dynamics of vacancy creation costs are an essential element in understanding the cyclical behavior of job creation and the dynamics of the labor market. Extending the model to allow for endogenous job separation improved its ability to match gross labor flows statistics while preserving the propagation properties.

Two questions remain and warrant further investigation in subsequent research. First, how general these results are to the type of friction present on credit markets is an open question. This can, however, be partially addressed by considering that any friction which will generate a counter-cyclical cost external funds will have the same qualitative implications. Second, if hiring is conditional on the state of credit markets, it may be that worker flows, as opposed to investment in new capital goods, are an alternative channel for the transmission of monetary policy shocks that affect the cost of credit. This avenue seems particularly promising as the propagation mechanism in the paper can be interpreted as increasing the rigidity of the firm's marginal cost to changes in production. Often referred in the New Keynesian literature as a greater degree of real rigidity, this property is known to be essential for the dynamics of inflation and for allowing any scope for monetary policy.

The second chapter examined the business cycle consequences of search frictions for the allocation of physical capital. The investigation was motivated by firm- and industry-level evidence on market imperfections in the allocation of physical capital. Despite the fundamentally different nature of physical capital and labor, it was argued that the market imperfections involved in the allocation of these two factors are quite similar. This research is thus a first step towards analyzing capital allocation with the same type of search frictions that have proven fruitful for our understanding of labor markets. By the same token, it is a complementary view to existing models of investment that focus on aggregate adjustment costs and building delays in a world with perfect markets.

The capital search model generates countercyclical congestion in physical capital markets, in line with the data, yet the analysis in a modern DSGE context suggests that for reasonable calibrations, the internal propagation effects of these search frictions are modest. The main reason for this lack of internal propagation is quantitative: separation and reallocation flows of physical capital are too small for the search friction to play a significant role. This conclusion remains intact when the model is extended to credit market frictions that result in countercyclical capital separations. While the combination of countercyclical separations and imperfect capital (re-) allocation increases internal propagation, almost all of these effects stem from a general equilibrium income effect that these frictions have on labor supply. Once the model is tied down to generate consumption dynamics in line with the data, capital separations due to financial distress are simply not important and volatile enough for them to generate significant internal propagation.

These results provide an interesting contrast to Den Haan, Ramey and Watson (2000) who show that the introduction of countercyclical job destruction in a labor search model substantially magnifies and prolongs the business cycle effects of small shocks. This difference in results is mainly due to the fact that labor is twice as important of an input to production as capital and that job destructions fluctuate on average much more over the business cycle than capital separations. Furthermore, job destructions overall are countercyclical while for capital separations, only the part linked to financial distress is countercyclical. This part makes up only a small fraction of all capital reallocations, which explains why its impact is so limited.

The final chapter has shown, a combination of frictions in the allocation of physical capital to production abroad, and allowing for the endogenous reallocation of this capital, can replicate the positive correlation between net inflows and outflows of FDI that is a feature of the data. In addition the model can generate the higher volatilities of inward and outward net FDI, while the implication for prominent macroeconomic variables are similar to a standard IRBC model with investment adjustment costs. However, there are important sectoral differences worth mentioning in conclusion. The model implies that, for example, hours worked at foreign establishments are more volatile than hours worked at domestic establishments. An interesting question, and most relevant for economic policy, is whether this is the case in the data. In particular, if one considers the extensive margin of labor adjustments, are jobs at foreign establishments more elastic to the business cycle? If so, this might offer a rationalization for the public's skepticism toward the benefits of increased foreign control of a domestic economy as employment at these firms would be more fragile.

# Appendix A

# Credit, Vacancies and Unemployment Fluctuations

## A.1 Data sources

Table A.1: Data sources

| | |
|---|---|
| Job vacancies | Conference Board Help-Wanted Index |
| Unemployment rate | B.L.S. series LNS14000000 |
| Job finding rate | Fujita and Ramey (2008) based on C.P.S. data |
| Output | Expenditure based, 2000 chained dollars, B.E.A. |
| Yield Spreads | Moody's Seasoned Aaa and Baa Corporate Bond Yield, DRI database with 3-month U.S. Treasury bills, from FRED II |

## A.2 Solving the wage under Nash bargaining

Define the surplus to the worker-firm relationship as $S_t = J_n(t) + \frac{V_n(t)-V_u(t)}{\lambda_t}$. Using the definitions of marginal values :

$$\begin{aligned}
S_t &= \Omega(\overline{x}_t)(X_t - W_t) + (1-\delta)\beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_n(N_{t+1}, A_{t+1}) \\
&\quad + W_t + \frac{\beta}{\lambda_t} E_t \left[(1-\delta)V_n(N_{t+1}, U_{t+1}) + \delta V_u(N_{t+1}, U_{t+1})\right] \\
&\quad - b - \frac{\beta}{\lambda_t} E_t \left[(1-f(\theta_t))V_u(N_{t+1}, U_{t+1}) + f(\theta_t)V_n(N_{t+1}, U_{t+1})\right] \\
S_t &= \Omega(\overline{x}_t)X_t + (1-\Omega(\overline{x}_t))W_t - b \\
&\quad + (1-\delta)\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left[J_n(N_{t+1}, A_{t+1}) + \frac{V_n(N_{t+1}, U_{t+1}) - V_u(N_{t+1}, U_{t+1})}{\lambda_{t+1}}\right] \\
&\quad - f(\theta_t)\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left[\frac{V_n(N_{t+1}, U_{t+1}) - V_u(N_{t+1}, U_{t+1})}{\lambda_{t+1}}\right]
\end{aligned}$$

Under Nash bargaining the surplus is split as $J_n(t) = (1-\eta)S_t$ and $\frac{V_n(t)-V_u(t)}{\lambda_t} = \eta S_t$. As a result, the above expression can be rewritten as

$$S_t = \Omega(\overline{x}_t)X_t + (1-\Omega(\overline{x}_t))W_t - b + (1-\delta)\beta E_t \frac{\lambda_{t+1}}{\lambda_t} S_{t+1} - f(\theta_t)\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\eta S_{t+1}$$

Since the optimality condition for vacancy posting can be expressed as $\frac{\gamma\phi_t}{p(\theta_t)} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}(1-\eta)S_{t+1}$, we now have

$$\begin{aligned}
S_t &= \Omega(\overline{x}_t)X_t + (1-\Omega(\overline{x}_t))W_t - b + (1-\delta)\frac{\gamma\phi_t}{p(\theta_t)(1-\eta)} - \eta f(\theta_t)\frac{\gamma\phi_t}{p(\theta_t)(1-\eta)} \\
(1-\eta)S_t &= (1-\eta)\left[\Omega(\overline{x}_t)X_t + (1-\Omega(\overline{x}_t))W_t - b\right] + (1-\delta)\frac{\gamma\phi_t}{p(\theta_t)} - \eta\gamma\phi_t\theta_t
\end{aligned}$$

Equation this expression with the marginal value of an addition worker $(1-\eta)S_t = \Omega(\overline{x}_t)(X_t - W_t) + (1-\delta)\frac{\gamma\phi_t}{p(\theta_t)}$ obtains

$$[1 + \eta(\Omega(\overline{x}_t) - 1)]W_t = \eta\left[\Omega(\overline{x}_t)X_t + \gamma\phi_t\theta_t\right] + (1-\eta)b$$

Finally, by defining $\omega_t = \frac{1}{[1+\eta(\Omega(\overline{x}_t)-1)]}$ yields the wage rule

$$W_t = \eta\omega_t\left[\Omega(\overline{x}_t)X_t + \gamma\phi_t\theta_t\right] + (1-\eta)\omega_t b$$

## A.3 Extension to endogenous job separation

Assume that each job within a firm draws an i.i.d. productivity $z$, where $z \in [0, \infty($ with cdf $H(z)$, pdf $h(z)$ and $E(z) = 1$, and that this job productivity is observed by both the worker and the firm before the idiosyncratic productivity $x$ is known. Firms and individual workers negotiate a wage conditional on the productivity of the job, $W(z)$, and a job drawing productivity $z < \overline{z}$ is not profitable and terminated. Given frictional credit markets, this threshold is defined such that current net revenues are non negative, or $\overline{z}$ is such that $\overline{z}X - W(\overline{z}) = 0$. This job destruction margin differs for the efficient separation rule in Mortensen and Pissarides (1994) in which the value of the cut off corresponds to that for which the job yields no surplus to either the worker or the firm. A separation rule that is efficient from the point of view of both parties involves a cut off for which the losses in current revenue are equal to the expected value of the job in the future

$$z_t^* X_t - W_t(z_t^*) = -\beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_N(N_{t+1})$$

where $z_t^*$ is the job productivity threshold in the absence of credit market frictions. The restriction that firms cannot run current period losses implies that $\overline{z}_t > z_t^*$. In other words the cut-off productivity is higher in the presence of credit market frictions resulting in a higher rate of endogenous separations, and part of these separations will be inefficient.

The timing assumption affords the following two benefits. First, ex-ante all firm face identical problems and make identical choices such that the analysis continues in a representative firm framework. Second, expected net revenues can are $\int_{\overline{z}}^{\infty} (zX - W(z))\, dH(z)\widetilde{N}$ which leaves the expected gross shares of net revenue under the debt contract unchanged, and the optimal contracting problem is naturally expressed as

$$\begin{aligned} J(N_t, A_t) &= \max_{V_t, \overline{x}_t, \overline{z}_t} [1 - \Gamma(\overline{x}_t)] \int_{\overline{z}_t}^{\infty} (zX_t - W_t(z))\, dH(z)\widetilde{N}_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J(N_{t+1}, A_{t+1}) \\ &\quad \text{subject to } [\Gamma(\overline{x}_t) - \mu G(\overline{x})] \int_{\overline{z}_t}^{\infty} (zX_t - W_t(z))\, dH(z)\widetilde{N}_t = (\gamma V_t - A_t) \\ &\quad A_{t+1} = \varsigma [1 - \Gamma(\overline{x}_t)] \int_{\overline{z}_t}^{\infty} (zX_t - W_t(z))\, dH(z)\widetilde{N}_t \\ &\quad N_{t+1} = (1 - \delta_t)N_t + V_t p(\theta_t) \\ &\quad \widetilde{N}_t = \int_{\overline{z}_t}^{\infty} N_t dH(z) \end{aligned}$$

with the appropriately modified participation constraint and law of motion for aggregate assets. An advantage of the current set up its that the optimality conditions for vacancy postings and the monitoring threshold retain the same form as earlier. Closing the extension to endogenous job separation, the intertemporal vacancy condition is now

$$\frac{\gamma\phi_t}{p(\theta_t)} = \frac{1}{1+r_t}E_t\left[[1+\Omega(\overline{x}_{t+1})]\int_{\overline{z}_{t+1}}^{\infty}(zX_{t+1}-W_{t+1}(z))\,dH(z)+(1-\delta_{t+1})\frac{\gamma\phi_{t+1}}{p(\theta_{t+1})}\right]$$

and the negotiated wage

$$W_t(z) = \eta\omega_t\left[(1+\Omega(\overline{x}_t))\,zX_t+\gamma\theta_t\phi_t\right]+(1-\eta)\omega_t b$$

Had the cut-off value of the job productivity corresponded to that for which the value of the job to the firm is equal to zero, the job destruction margin would have been expressed as

$$(\overline{z}_t):\,[1-\Gamma(\overline{x}_t)+\phi_t\left[\Gamma(\overline{x}_t)-\mu G(\overline{x}_t)\right]]\left[-\overline{z}_tX_t+W_t(\overline{z}_t)\right]-\beta E_t\frac{\lambda_{t+1}}{\lambda_t}J_N(N_{t+1})=0$$

Using the same notation as earlier, $\Omega(\overline{x}_t)=\phi_t\left[\Gamma(\overline{x}_t)-\mu G(\overline{x}_t)\right]-\Gamma(\overline{x}_t)$, the separation condition would have been

$$\overline{z}_tX_t-W_t(\overline{z}_t)=\frac{\gamma\phi_t}{p(\theta_t)\left[1+\Omega(\overline{x}_t)\right]}$$

and is interpreted as the job productivity at which the current revenue generated by the worker net of his cost is equal the present discounted value of the worker in the future. Using results from earlier, in the absence of monitoring cost this collapses to the familiar

$$\overline{z}_tX_t-W_t(\overline{z}_t)=\frac{\gamma}{p(\theta_t)}$$

# Appendix B

# Search in Physical Capital Markets as a Propagation Mechanism

This technical appendix presents the full details of the extended capital search model with endogenous separation due to credit market imperfections. It also describes a scenario with the matching friction removed. Results for an extensive sensitivity analysis of the models with exogenous and endogenous separation are provided. The propagation potential of endogenous separation is illustrated before discussing the data used in this paper.

## B.1 Model

As in the frictionless RBC benchmark, our model is populated by two agents: firms that produce using capital and labor; and households who decide on optimal consumption, leisure and investments in either risk less bonds or productive capital.

We add two frictions to this benchmark. First, the allocation of capital from households to firms involves a costly and time-consuming matching process. Second, the capital lending relationship between households and firms is subject to a credit market friction. Following the costly state verification literature initiated by Townsend (1979), this credit market friction takes the form of an idiosyncratic productivity shock that households can only observe at a cost. This asymmetric information assumption gives rise to an agency problem that results in a debt contract with endogenous separation of capital from firms whose productivity falls below a state-dependent threshold.

For the sake of simplicity, the model abstracts from a number of potentially important factors that deserve to be mentioned. First, there is no distinct sector for capital allocation. Instead, households act directly as capital lenders. Second, firms transfer all of their profits to households at the end of the period. Hence, net worth – the channel through which credit frictions affect investment in the existing financial accelerator models – is absent. Third, the same matching friction applies to the allocation of both new and used (i.e. previously separated) capital. This assumption simplifies our model because we do not need to keep track of different types of capital. Fourth, we do not distinguish among different firms because, as discussed below, our modeling assumptions imply that all firms are identical and that firm size is indeterminate.

### B.1.1 Search and matching in the capital market

Capital is either in a productive state or in a liquid state. We define by $K_t$ the capital stock that enters the production function of a representative firm in period $t$. Liquid capital $L_t$, in turn, is made up of two components: used capital that has been separated previously from other firms and new capital made available by households.

To undertake investments, firms must post projects and search for liquid capital at a cost of $\kappa$ per project. We denote by $V_t$ the number of posted projects of a representative firm in period $t$. Actual investment (i.e. new capital allocations) in period $t$ then is the result of a matching process $m(L_t, V_t)$ that is a positive function of the total amount of liquid capital $L_t$ and the total number of project postings $V_t$. A firm's probability to find capital is therefore given by $p(\theta_t) = \frac{m(V_t, L_t)}{V_t}$ with $\partial p(\theta_t)/\partial\theta_t > 0$, where $\theta_t = \frac{L_t}{V_t}$ is a measure of capital market liquidity. Likewise, the probability of liquid capital being matched to a firm equals $q(\theta_t) = \frac{m(V_t, L_t)}{L_t}$ with $\partial q(\theta_t)/\partial\theta_t < 0$.[1]

Capital matched to a firm in period $t-1$ enters production in period $t$. This relationship between firm and capital continues to hold in $t+1$ with probability $(1-s_t)$ and so on for the periods thereafter. If the relationship is terminated, which happens with probability $s_t$, the capital is separated and returned to the household net of depreciation $\delta$. Both the matching probability and the separation rate are taken as exogenous by the firm but depend on the state of the economy, as will be described below. Given these assumptions, the evolution of the productive capital stock is described by

$$K_{t+1} = (1-\delta)(1-s_t)K_t + m(L_t, V_t). \tag{B.1}$$

[1] In addition, to ensure that $p(\theta_t)$ and $q(\theta_t)$ are between 0 and 1, we require that $m(L_t, V_t) \leq min[L_t, V_t]$

### B.1.2 Firms

At the beginning of each period, firms and households observe exogenous aggregate technology $X_t$. Given the existing capital stock $K_t$, a firm then posts new projects $V_t$ at unit cost $\kappa$ and hires labor $N_t$ at wage rate $W_t$ to produce output $Y_t$ with technology

$$Y_t = a_t f(X_t N_t, K_t), \tag{B.2}$$

with $f_N$, $f_K > 0$ and $f_{NN}$, $f_{KK} < 0$. The variable $a_t > 0$ denotes the realization of an idiosyncratic productivity shock that is independently distributed over time with probability density $h(a)$, cumulative density $H(a)$ and mean $E(a) = 1$. This shock occurs after all optimal decisions have taken place and after the factor price equilibria are established.

Given these assumptions, the profit maximization problem of the firm is described by the following Bellman equation

$$\begin{aligned} J(K_t) &= \max_{N_t, V_t} \left\{ \int_{\bar{a}_t}^{\infty} \left[\psi a f(X_t N_t, K_t) - \rho_t K_t - W_t N_t - \kappa V_t\right] dH(a) + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} J(K_{t+1}) \right\} \\ \text{s.t. } K_{t+1} &= (1-\delta)(1-s_t)K_t + p(\theta_t)V_t, \end{aligned}$$

where $\rho_t$ is the rental rate of capital; and $\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}$ is the discount factor of future cash flows. Several comments are in order about this expression. First, the discount factor is a function of the marginal utility of consumption $\Lambda$ because the firm transfers all profits to the household. Second, the firm maximizes only over the portion of the revenue net of capital rental costs for which it is expected to retain profits, $\int_{\bar{a}_t}^{\infty} \left[\psi a f(X_t N_t, K_t) - \rho_t K_t - W_t N_t - \kappa V_t\right] dH(a)$, where $\bar{a}_t$ is defined as the break-even point associated with zero profits; i.e. $\bar{a}_t$ such that $\bar{a}_t f(X_t N_t, K_t) = \rho_t K_t + W_t N_t + \kappa V_t$. As explained below, this is because under optimal contracting, any revenues associated with productivity shocks below $\bar{a}_t$ are absorbed either by the capital lender (in case of continuation of the capital match) or by an insurance (in case of capital separation). Third, we assume that firms are monopolistic competitors and apply a constant markup $1/\psi \geq 1$ on their optimal decisions. This addition is necessary because the firm's optimization over the range $[\bar{a}_t, \infty]$ by itself would result in substantial over hiring relative to the RBC benchmark and thus a labor share that is too high. Fourth, the firm takes both $W_t$ and $\rho_t$ as exogenous. The exogeneity of $W_t$ is a direct consequence of our assumption of competitive labor markets. The exogeneity of $\rho_t$, in turn, implies that firms in our model do not internalize the effects of their capital stock on the marginal productivity of capital and thus on the negotiation of $\rho_t$ discussed below.

The first-order conditions of the optimization problem are

$$(N_t): \int_{\bar{a}_t}^{\infty} \psi a f_N(X_t N_t, K_t) dH(a) = w_t[1 - H(\bar{a}_t)] \tag{B.3}$$

$$(V_t): \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} J_K(K_{t+1}) = \frac{\kappa[1 - H(\bar{a}_t)]}{p(\theta_t)} \tag{B.4}$$

where $J_K(K)$ is the marginal value to the firm of an additional matched unit of capital. Differentiating the firm's value function with respect to capital, the definition of $J_K(K_t)$ is

$$J_K(K_t) = \int_{\bar{a}_t}^{\infty} [\psi a f_K(X_t N_t, K_t) - \rho_t] \, dH(a) + (1-\delta)(1-s_t)\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} J_K(K_{t+1}). \tag{B.5}$$

This equation states that the value to the firm of an additional unit of capital is worth today's marginal product of capital net of the rental rate (in case the firm posts positive profits) plus its expected future value net of depreciation in case the project is continued.

### B.1.3 Households

Before considering the household's optimal program, it is useful to define the rental contract that leads to optimal capital separation. Specifically, we assume that capital matches are discontinued either for exogenous reasons or for reasons associated with credit frictions. Hence,

$$s_t = s^x + s_t^e,$$

where $s^x$ denotes (constant) exogenous separation and $s_t^e$ denotes endogenous separation.

To model endogenous separation, we postulate that while firms perfectly observe the realization of the idiosyncratic shock $a_t$, households can only do so at an auditing cost. This asymmetric information assumption creates an agency problem because in the absence of auditing, the firm would always want to miss report $a_t$. The debt contract to deal with this problem is structured as follows.[2] Households and firms negotiate the rental rate $\rho_t$ per unit of matched capital *prior to* the realization of the idiosyncratic productivity shock $a_t$. Then, if $a_t \geq \bar{a}_t$ the firm pays $\rho_t K_t$, the household refrains from auditing and the capital match continues. If, on the other hand, $a_t < \bar{a}_t$ the firm is unable to pay the negotiated capital rental because we assume that the wage bill $W_t N_t$ and the cost of posting vacancies $\kappa V_t$ need to be covered in order for the firm to continue

[2]Conditional on selecting a debt contract the proposed monitoring and separation scheme is optimal. The firm would not gain anything from reporting output below what it actually produced because in case of monitoring, it will loose all output anyway. Likewise, the household would not gain anything from negotiating a higher or lower auditing cutoff $\bar{a}_t$.

operating next period. In this situation, the household pays the auditing cost to verify the firm's production and decides on the continuation of the capital match. If $a_t$ is above some threshold $\underline{a}_t$ that is associated with the household's choice of optimal separation, the household takes all of the firm's production and covers for the totality of $W_tN_t$ and $\kappa V_t$ so as to continue the capital match. Note that if $a_t$ is sufficiently low, this may entail injecting additional funds. If instead $a_t$ is below the optimal threshold $\underline{a}_t$, the household separates the match and takes back its capital stock without receiving nor paying anything. In this case, the firm bankrupts and the difference between production and the cost of $W_tN_t$ and $\kappa V_t$ is picked up by an insurance that is funded with the dividends from profit-making firms.[3]

In addition, we incorporate what Fleisher and Vishny (1992) term asset illiquidity by assuming that the loss of value applied to the separated unit of capital is increasing in the rate of endogenous separation:

$$\varphi_t = \varsigma(s_t^e), \tag{B.6}$$

where $\varsigma'(\cdot) < 0$ and $\varsigma'(\cdot) < 0$. As an equilibrium phenomenon, households take this loss of value as exogenous when making their optimal decisions.

Given these assumption, the endogenous part of separation is defined as

$$s_t^e = H(\underline{a}_t) \tag{B.7}$$

and the household's expected revenue from matched capital equals

$$\begin{aligned} R_t^K &= \int_{\bar{a}_t}^{\infty} \rho_t K_t dH(a) + \int_{\underline{a}_t}^{\bar{a}_t} \left[af(X_tN_t, K_t) - W_tN_t - \kappa V_t\right] dH(a) \\ &\quad -\tau \int_0^{\bar{a}_t} \left[af(X_tN_t, K_t)\right] dH(a) + (1-\delta)\varphi_t s_s K_t, \end{aligned} \tag{B.8}$$

where the term $\tau \int_0^{\bar{a}_t} \left[af(X_tN_t, K_t)\right] dH(a)$ denotes the auditing cost paid by the household, which we assume to be a fixed proportion $\tau > 0$ of output. The final term $(1-\delta)\varphi_t s_t K_t$ is the value of capital separated from firms and returned into the budget constraint, net of depreciation and a loss of value due to specificity.

Households maximize the expected discounted flow of utility $u(C_t, 1 - N_t)$ over consumption $C_t$, leisure $1 - N_t$, risk-free bond holdings $B_{t+1}$, the amount of liquid capital $L_t$ destined for matching with firms, and the optimal separation threshold $\underline{a}_t$. Time spent

[3]See Section A.5 for more details on this insurance. Suffice to say here that we implicitly assume that firms or capital lenders on their own cannot contract a similar insurance to cover for potential shortfalls in case the firm does not disappear.

working yields revenue $W_tN_t$ while risk free bond holdings carry a net rate of return $r_t$ in the following period. Matched capital, in turn, yields expected revenue $R_t^K$, while any capital unmatched is carried over into the next period with zero net return. Formally, this problem is described by the following Bellman equation

$$\begin{aligned} V(U_t, K_t, B_t) &= \max_{C_t, N_t, L_t, B_{t+1}, \underline{a}_t} [u(C_t, 1-N_t) + \beta E_t V(U_{t+1}, K_{t+1}, B_{t+1})] \\ &+ \Lambda_t[W_tN_t + R_t^K + U_t + B_t + D_t - C_t - L_t - \frac{B_{t+1}}{(1+r_t)}] \\ \text{s.t. } K_{t+1} &= (1-\delta)(1-s_t)K_t + q(\theta_t)L_t \end{aligned}$$

where $U_t = (1 - q(\theta_{t-1}))L_{t-1}$ is the quantity of unmatched liquid capital in $t-1$; $D_t$ are firm profits transferred to households. Similar to the firm's optimization problem, we assume that the household considers the wage rate $W_t$, the rental rate $\rho_t$ and its matching probability $q(\theta_t)$ as exogenous.

The first-order conditions of this optimization problem are

$$(C_t) : u_C(C_t, 1-N_t) = \Lambda_t \tag{B.9}$$

$$(N_t) : -u_N(C_t, 1-N_t) = \Lambda_t W_t \tag{B.10}$$

$$(B_{t+1}) : \beta E_t[\Lambda_{t+1}(1+r_t)] = \Lambda_t \tag{B.11}$$

$$(L_t) : \beta E_t[V_U(U_{t+1}, K_{t+1}, B_{t+1})(1-q(\theta_t)) + V_K(U_{t+1}, K_{t+1}, B_{t+1})q(\theta_t)](\text{B.12})$$

$$(\underline{a}_t) \quad : \quad \Lambda_t \left[\frac{\partial R_t^K}{\partial \underline{a}_t}\right] = (1-\delta)h(\underline{a}_t)K_t\beta E_t V_K(U_{t+1}, K_{t+1}, B_{t+1}) \tag{B.13}$$

The first three conditions are standard. The fourth condition for the household's choice of $L_t$ states that the discounted expected utility of the marginal unit of liquid capital available for investment must equal the expected discounted return from investing in the risk less bond. With probability $(1-q(\theta_t))$ liquid capital remains unmatched and is worth $V_U(U_{t+1}, K_{t+1}, B_{t+1})$ to the household, while with probability $q(\theta_t)$ it is matched with a project and turned into productive capital with marginal value $V_K(U_{t+1}, K_{t+1}, B_{t+1})$. From the above Bellman equation and the definition of $R_t^K$ in (B.8), we can work out these marginal values as

$$V_U(U_t, K_t, B_t) = \Lambda_t \tag{B.14}$$

$$\begin{aligned} &V_K(U_t, K_t, B_t) = \Lambda_t\{\rho_t[1 - H(\bar{a}_t)] + (\underline{\mu}_t - \bar{\mu}_t - \tau(1-\bar{\mu}_t))f_K(X_tN_t, K_t) + (1-\delta)\varphi_t s_t\} \\ &+(1-\delta)(1-s_t)\beta E_t V_K(U_{t+1}, K_{t+1}, B_{t+1}), \end{aligned} \tag{B.15}$$

where $1 - H(\bar{a}_t) = \int_{\bar{a}_t}^{\infty} dH(a)$ and $\underline{\mu}_t = \int_{\underline{a}_t}^{\infty} adH(a)$, $\bar{\mu}_t = \int_{\bar{a}_t}^{\infty} adH(a)$ denote partial expectations. Note that $V_K$ is forward-looking because with probability $1 - s_t$ the investment relationship between household and firm continues into the next period.

Finally, the fifth condition states that the optimal separation threshold $\underline{a}_t$ is such that the marginal utility from capital revenue plus the last unit of capital separated equals the expected discounted value of the last unit of matched capital carried over into the next period. Applying the fundamental theorem of calculus on $\partial R_t^K/\partial \underline{a}_t$, we can write this condition more explicitly as

$$\begin{aligned}\Lambda_t(1-\delta)\varphi_t K_t &= \Lambda_t\left[\underline{a}_t f(X_t N_t, K_t) - W_t N_t - \kappa V_t\right] \\ &\quad +(1-\delta)K_t \beta E_t V_K(U_{t+1}, K_{t+1}, B_{t+1})\end{aligned}$$

This condition implicitly defines the optimal separation threshold $\underline{a}_t$. It can be shown that $\underline{a}_t f(X_t N_t, K_t) < W_t N_t + \kappa V_t$; i.e. the household is willing to refinance distressed firms up to a certain point so as to continue the capital match. This is because separated capital yields zero return in the next period and comes with the risk that rematching takes time.

### B.1.4 Rental rate of capital

To determine the rental rate of capital, we assume that once matched, households and firms split the surplus of their relationship according to a Nash bargaining process. As discussed above, this bargaining process takes place before the idiosyncratic shock $a_t$ is realized. The surplus is the sum of marginal benefits to each party, $S_t = J_K(K_t) + \frac{V_K(U_t, K_t, B_t) - V_U(U_t, K_t, B_t)}{\Lambda_t}$. Define $\eta$ as the household's relative bargaining power. The household then receives $\frac{V_K(U_t, K_t, B_t) - V_U(U_t, K_t, B_t)}{\Lambda_t} = \eta S_t$, while the firm's share is $J_K(K_t) = (1-\eta)S_t$. Using the first order condition on project postings from the firm's problem (B.4) together with the definition for the marginal value to the firm of an additional unit of capital (B.5), and a result from Nash bargaining that the firm's share of the total surplus is $J(K_t) = (1-\eta)S_t$, we obtain

$$(1-\eta)S_t = \overline{\mu}_t \psi f_K(X_t N_t, K_t) - \rho_t[1 - H(\bar{a}_t)] + (1-\delta)(1-s_t)\frac{\kappa}{p(\theta_t)}[1 - H(\bar{a}_t)] \quad \text{(B.16)}$$

By definition $S_t = J(K_t) + \frac{V_k(U_t,K_t,B_t) - V_u(U_t,K_t,B_t)}{\Lambda_t}$, thus

$$\begin{aligned} S_t &= \bar{\mu}_t \psi f_K(X_t N_t, K_t) - \rho_t[1 - H(\bar{a}_t)] + (1-\delta)(1-s_t)\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} J_K(K_{t+1}) \\ &\quad + \rho_t[1 - H(\bar{a}_t)] + (\underline{\mu}_t - \bar{\mu}_t - \tau(1-\bar{\mu}_t)) f_K(X_t N_t, K_t) + (1-\delta)\varphi_t s_t \\ &\quad + (1-\delta)(1-s_t)\beta E_t \frac{V_K(U_{t+1}, K_{t+1}, B_{t+1})}{\Lambda_t} - \frac{V_U(U_t, K_t, B_t)}{\Lambda_t} \\ S_t &= f_K(X_t N_t, K_t)\left[\psi\bar{\mu}_t + (\underline{\mu}_t - \bar{\mu}_t - \tau(1-\bar{\mu}_t))\right] + (1-\delta)\varphi_t s_t - 1 \\ &\quad + (1-\delta)(1-s_t)\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}[J_k(K_{t+1}) + \frac{V_k(U_{t+1}, K_{t+1}, B_{t+1}) - V_u(U_{t+1}, K_{t+1}, B_{t+1})}{\Lambda_{t+1}}] \\ &\quad + (1-\delta)(1-s_t)\beta E_t \frac{V_u(U_{t+1}, K_{t+1}, B_{t+1})}{\Lambda_t} \end{aligned}$$

From the first order condition for liquid capital (B.12) and the household's share of the total surplus, $\beta E_t \frac{V_u(U_{t+1},K_{t+1},B_{t+1})}{\Lambda_t}$ can be written as $\left[1 - q(\theta_t)\eta\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} S_{t+1}\right]$. Thus,

$$\begin{aligned} S_t &= f_K(X_t N_t, K_t)\left[\psi\bar{\mu}_t + (\underline{\mu}_t - \bar{\mu}_t - \tau(1-\bar{\mu}_t))\right] + (1-\delta)\varphi_t s_t - 1 \\ &\quad + (1-\delta)(1-s_t)\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} S_{t+1} + (1-\delta)(1-s_t)\left[1 - \eta q(\theta_t)\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} S_{t+1}\right] \\ S_t &= f_K(X_t N_t, K_t)\left[\psi\bar{\mu}_t + (\underline{\mu}_t - \bar{\mu}_t - \tau(1-\bar{\mu}_t))\right] + (1-\delta)\varphi_t s_t - 1 \\ &\quad + (1-\delta)(1-s_t)\frac{\kappa[1 - H(\bar{a}_t)]}{p(\theta_t)(1-\eta)} + (1-\delta)(1-s_t)\left[1 - \eta q(\theta_t)\frac{\kappa[1 - H(\bar{a}_t)]}{p(\theta_t)(1-\eta)}\right] \\ (1-\eta)S_t &= (1-\eta)\left\{f_K(X_t N_t, K_t)\left[\psi\bar{\mu}_t + (\underline{\mu}_t - \bar{\mu}_t - \tau(1-\bar{\mu}_t))\right] + (1-\delta)\varphi_t s_t - 1. + (1-\delta)(1-s_t)\right. \\ &\quad + (1-\delta)(1-s_t)\frac{\kappa[1 - H(\bar{a}_t)]}{p(\theta_t)} - \eta q(\theta_t)\frac{\kappa[1 - H(\bar{a}_t)]}{p(\theta_t)} \end{aligned} \quad \text{(B.1}$$

Finally, equating (B.16) and (B.17), and recalling that $\frac{p(\theta_t)}{q(\theta_t)} = \theta_t$, yields the repayment rule

$$\begin{aligned} \rho_t &= \eta\left[\bar{\mu}_t \psi f_K(X_t N_t, K_t) + (1-\delta)(1-s_t)\frac{\kappa}{\theta_t}[1 - H(\bar{a}_t)]\right] + (1-\eta)\left[\delta + (1-\delta)(1-\varphi_t)s_t\right] \\ &\quad + \left[\rho_t H(\bar{a}_t) - (1-\eta)(\underline{\mu}_t - \bar{\mu}_t - \tau(1-\bar{\mu}_t)) f_K(X_t N_t, K_t)\right]. \end{aligned} \quad \text{(B.18)}$$

The first term in brackets is the maximum amount the firm is willing to pay per unit of capital. It equals the marginal product of capital conditional on making a profit plus the average cost that is saved by entering the proposed capital match rather than continuing the search. The second term in brackets is the household's the cost of capital depreciation, $\delta$, and specificity, $\varphi_t$. Finally, the third term in brackets represents the default risk-premium that arises because households need to pay auditing costs and do not receive the full contractual payment $\rho_t$ when the firm's idiosyncratic shock drops below $\bar{a}_t$ (zero profit).

The Nash bargaining approach to determine the rental rate conditional on an optimal separation threshold $\underline{a}_t$ differs from existing financial accelerator models such as Carlstrom and Fuerst (1997) or Bernanke, Gertler and Gilchrist (1998) where the price of capital and the separation threshold are chosen under the assumption that lenders make zero profit. In our framework, this assumption would obtain for the special case $\eta = 0$.

### B.1.5 Aggregation and equilibrium

The micro literature on firm dynamics usually assumes decreasing returns to scale production (see for example Cooley and Quadrini, 2001 or Esteban-Rossi and Wright, 2005). Here, for reasons of tractability, we follow the traditional macro literature and assume that the production function $f(\cdot)$ exhibits constant returns to scale. Under this assumption, it is straightforward to show that the capital labor ratio of all firms is the same. Hence, all optimality conditions are independent of firm size and the rental rate is identical for all firms.

The constant returns assumption justifies our derivation of the optimality conditions in a representative firm framework, but at the same time bypasses any issues that arise from firm size heterogeneity. These issues are admittedly important but taking them into account would render our model less tractable and complicate the quantitative analysis. In particular, we would no longer be able to draw direct comparisons with other representative agent models such as the frictionless RBC benchmark or the financial accelerator model of Bernanke, Gertler and Gilchrist (1998).

A second simplifying assumption to keep our model tractable is the existence of a state contingent insurance that covers for any shortfall in wage payments and costs of project postings left over by bankrupt firms (firms with productivity below $\underline{a}_t$ for which the household refuses to inject funds to continue the lending relationship). We assume that this insurance is financed by the profits of firms with productivity above $\bar{a}_t$.

The remainder of the profits is transferred at the end of each period to the household in the form of dividends. Hence, we also bypass any net worth considerations that are at the center of the financial accelerator models of credit market frictions. Aggregating over the different firms, the total amount of dividends is defined as

$$\begin{aligned} D_t &= \int_{\bar{a}_t}^{\infty} \left[af(X_tN_t, K_t) - \rho_t K_t - W_tN_t - \kappa V_t\right] dH(a) \\ &+ \int_{0}^{\underline{a}_t} \left[af(X_tN_t, K_t) - W_tN_t - \kappa V_t\right] dH(a). \end{aligned} \tag{B.19}$$

The equilibrium of this economy is defined by the capital accumulation equation (B.1), aggregate production $\int_0^\infty af(X_tN_t, K_t)dH(a) = f(X_tN_t, K_t)$ and the system of equations (B.3)-(B.19).

In addition, by defining new capital investments as $I_t^{new} = L_t - U_t - (1-\delta)\varphi_t s_t K_t$, a familiar aggregate resource constraint can be derived:

$$Y_t(1 - \tau(1 - \bar{\mu}_t)) = C_t + I_t^{new} + \kappa V_t.$$

where $\tau(1 - \bar{\mu}_t)$ is the resource cost of monitoring, and $\kappa V_t$ that of project postings.

## B.2 Proof of Proposition 1

**Proof:** Consider the first order condition for liquid capital (B.12) in the presence of exogenous capital separation only. Combined with the firms' optimality condition for project postings and the Nash bargaining results regarding the division of the match surplus, this may be rewritten as $\Lambda_t = \beta E_t \Lambda_{t+1} + \frac{\eta}{1-\eta}\frac{\kappa}{\theta_t}\Lambda_t$. By rearranging terms, congestion in the physical capital market can be expressed, in log deviations around the steady state, as increasing function of the expected growth rate of the marginal utility of consumption:

$$\widehat{\theta}_t = \theta \frac{1-\eta}{\eta\kappa} \beta E_t \left[\widehat{\Lambda}_{t+1} - \widehat{\Lambda}_t\right].$$

## B.3 Equilibrium system

After normalizing by the deterministic trend to the labor augmenting technological growth, the equilibrium system comprises 22 equations for the variables $y_t, c_t; n_t, k_t; l_t; u_t; v_t; \theta_t; p(\theta_t), q(\theta_t)$; $r_t; \rho_t; w_t, \lambda_t; R_t^k, d_t; s_t; \bar{a}_t, \underline{a}_t; \bar{\mu}_t, \underline{\mu}_t, \varphi_t$, where a lower case variable is defined as $y_t \equiv \frac{Y_t}{X_t}$.

$$
\begin{aligned}
\frac{1}{c_t} &= \lambda_t \\
\omega(1-n_t)^{-\xi} &= \lambda_t w_t \\
\lambda_t &= \beta E_t \frac{\lambda_{t+1}}{g} + \frac{\eta}{1-\eta}\frac{\kappa}{\theta_t}[1-H(\bar{a}_t)]\lambda_t \\
\lambda_t &= \beta E_t \frac{\lambda_{t+1}}{g}\left[1+r_t\right] \\
[1-H(\bar{a}_t)]w_t &= \overline{\mu}_t(1-\alpha)\psi\frac{y_t}{n_t} \\
\frac{\kappa[1-H(\bar{a}_t)]}{p(\theta_t)} &= \beta E_t \frac{\lambda_{t+1}}{g\lambda_t}\left\{\overline{\mu}_{t+1}\psi\alpha\frac{y_{t+1}}{k_{t+1}} - \rho_{t+1}\left[1-H(\bar{a}_{t+1})\right] + (1-\delta)(1-s_{t+1})\frac{\kappa[1-H(\bar{a}_{t+1})]}{p(\theta_{t+1})}\right\} \\
\rho_t &= \eta\left[\bar{\mu}_t\psi\alpha\frac{y_t}{k_t} + (1-\delta)(1-s_t)\frac{\kappa[1-H(\bar{a}_t)]}{\theta_t}\right] + (1-\eta)\left[\delta+(1-\delta)(1-\varphi_t)s_t\right] \\
&\quad + \left[\rho_t H(\bar{a}_t) - (1-\eta)(\underline{\mu}_t - \bar{\mu}_t - \tau(1-\bar{\mu}_t))\alpha\frac{y_t}{k_t}\right] \\
gk_{t+1} &= (1-\delta)(1-s_t)k_t + q(\theta_t)l_t \\
y_t\left[1-\tau(1-\bar{\mu}_t)\right] &= c_t + [l_t + v_t\kappa] - [(1-\delta)s_t\varphi_t k_t + u_t] \\
gu_{t+1} &= [1-q(\theta_t)]l_t
\end{aligned}
$$

$$
\begin{aligned}
R_t^k &= \dot{\rho}_t k_t\left[1-H(\bar{a}_t)\right] + (\underline{\mu}_t - \bar{\mu}_t - \tau(1-\bar{\mu}_t))y_t - (w_t n_t + v_t\kappa)\left[H(\bar{a}_t) - H(\underline{a}_t)\right] \\
d_t + R_t^k &= y_t\left[1-\tau(1-\underline{\mu}_t)\right] - w_t n_t - v_t\kappa + (1-\delta)\varphi_t s_s K_t \\
\theta_t &= \frac{l_t}{v\,t} \\
p(\theta_t) &= \theta_t^{1-\epsilon} \\
q(\theta_t) &= \theta_t^{-\epsilon} \\
y_t &= A_t n_t^{1-\alpha}k_t^{\alpha} \\
s_t &= H(\bar{a}_t) + s^x \\
\bar{a}_t &= (w_t n_t + \rho_t k_t + v_t\kappa)/y_t \\
\underline{a}_t y_t &= w_t n_t + v_t\kappa + (1-\delta)k_t\left\{\varphi_t - \frac{1}{q(\theta_t)}\left[1-\frac{\beta}{g}E_t\frac{\lambda_{t+1}}{g\lambda_t}(1-q(\theta_t))\right]\right\} \\
\varphi_t &= \varsigma(s_t^e) \\
\overline{\mu}_t &= \int_{\bar{a}_t}^{\infty} a\,dH(a) \\
\underline{\mu}_t &= \int_{\underline{a}_t}^{\infty} a\,dH(a)
\end{aligned}
$$

## B.4 Steady state system of equations

$$
\begin{aligned}
\frac{1}{c} &= \lambda \\
\omega(1-n)^{-\xi} &= \lambda w \\
1 &= \frac{\beta}{g} + \frac{\eta}{1-\eta}\frac{\kappa}{\theta}[1-H(\bar{a})] && \text{(B.20)} \\
r &= \frac{g}{\beta} - 1 && \text{(B.21)} \\
[1-H(\bar{a})]w &= \overline{\mu}\psi(1-\alpha)\frac{y}{n} && \text{(B.22)} \\
\frac{\kappa[1-H(\bar{a})]}{p(\theta)} &= \frac{\beta}{g}\left\{\overline{\mu}\psi\alpha\frac{y}{k} - \rho\,[1-F(\overline{a})] + (1-\delta)(1-s)\frac{\kappa[1-H(\bar{a})]}{p(\theta)}\right\} && \text{(B.23)} \\
[1-H(\bar{a})]\rho &= \eta\left[\bar{\mu}\psi\alpha\frac{y}{k} + (1-\delta)(1-s)\frac{\kappa[1-H(\bar{a})]}{\theta}\right] + (1-\eta)\,[\delta + (1-\delta)(1-\varphi)s] \\
&\quad -(1-\eta)(\underline{\mu} - \bar{\mu} - \tau(1-\bar{\mu}))\alpha\frac{y}{k} && \text{(B.24)} \\
gk &= (1-\delta)(1-s)k + q(\theta)l && \text{(B.25)} \\
y\,[1-\tau(1-\bar{\mu})] &= c + [l + v\kappa] - [(1-\delta)\varphi sk + u] \\
gu &= [1-q(\theta)]l \\
R^k &= \rho k\,[1-H(\overline{a})] + (\underline{\mu} - \bar{\mu} - \tau(1-\bar{\mu}))y - (wn + v\kappa)\,[H(\overline{a}) - H(\underline{a})] + (1-\delta)\varphi sk \\
d + R^k &= y\left[1-\tau(1-\underline{\mu})\right] - wn - v\kappa + (1-\delta)\varphi sk \\
\theta &= \frac{l}{v} \\
p(\theta) &= \theta^{1-\epsilon} \\
q(\theta) &= \theta^{-\epsilon} \\
y &= An^{1-\alpha}k^{\alpha} && \text{(B.26)} \\
s &= H(\underline{a}) + s^x \\
\bar{a} &= (wn + \rho k + v\kappa)/y && \text{(B.27)} \\
\underline{a}y &= wn + v\kappa + (1-\delta)k\left\{\varphi - \frac{1}{q(\theta)}\left[1 - \frac{\beta}{g}(1-q(\theta))\right]\right\} && \text{(B.28)} \\
\varphi &= \varsigma(s^e) && \text{(B.29)} \\
\overline{\mu} &= \int_{\bar{a}}^{\infty} a\,dH(a) \\
\underline{\mu} &= \int_{\underline{a}}^{\infty} a\,dH(a)
\end{aligned}
$$

## B.5 Computing the steady state

Using (B.20), (B.21) and (B.23), the equation for the repayment in the steady state can be written as

$$\rho = (r+\delta)+\left(\frac{1-q}{q}\right)\left\{r-(1-\frac{\beta}{g})(1-\delta)(1-s)\right\}+(1-\delta)(1-\varphi)s-(\underline{\mu}-\bar{\mu}-\tau(1-\bar{\mu}))\alpha\frac{y}{k}+\rho H(\bar{a})$$

Now, use (B.27) and(B.28) to form $(\bar{a}-\underline{a}) = [\rho-(1-\delta)\Psi]\frac{k}{y}$, from which $\frac{y}{k} = \frac{[\rho-(1-\delta)\Psi]}{(\bar{a}-\underline{a})}$, where $\Psi = \left\{\varphi - \frac{1}{q(\theta)}\left[1-\frac{\beta}{g}(1-q(\theta))\right]\right\}$. The repayment is then

$$\begin{aligned}\rho &= (r+\delta)+\left(\frac{1-q}{q}\right)\left\{r-(1-\frac{\beta}{g})(1-\delta)(1-s)\right\}+(1-\delta)(1-\varphi)s \\ &\quad -(\underline{\mu}-\bar{\mu}-\tau(1-\bar{\mu}))\alpha\frac{[\rho-(1-\delta)\Psi]}{[\bar{a}-\underline{a}]}+\rho H(\bar{a}).\end{aligned}$$

Equation (B.23) defines the ratio of capital to output as

$$\frac{k}{y} = \frac{\bar{\mu}\alpha}{\left[\frac{\kappa}{q(\theta)\theta}\left(\frac{g}{\beta}-(1-\delta)(1-s)\right)+\rho\left[1-H(\bar{a})\right]\right]}.$$

using first equation (B.20) to determine the value $\frac{\kappa}{\theta} = \frac{1-\eta}{\eta}\left(1-\frac{\beta}{g}\right)$. Since $E(a) = 1$, the mean of the lognormal distribution is $-\frac{\sigma^2_{\log(a)}}{2}$. For a given $\sigma^2_{\log(a)}$ the cutoff threshold $\underline{a}$ is given by $\underline{a}= H^{-1}(s^e)$, where $s^e$ is the proportion of separations occurring endogenously. Given a value of $H(\bar{a})$ one obtains values for $\rho$ and $\frac{k}{y}$. The numerical strategy is then to iterate over values of $H(\bar{a})$ such that the equation relating the two thresholds, $(\bar{a}-\underline{a}) = [\rho-(1-\delta)\Psi]\frac{k}{y}$, is respected.

Using the production function the steady state capital stock is then simply

$$k = \left(\frac{y}{Ak}\right)^{\frac{1}{\alpha-1}} n.$$

Equations (B.26) and (B.22) give us the level of output and steady state wage. Liquid capital is then computed using the law of motion of capital (B.25)

$$l = \frac{k[g-(1-\delta)(1-s)]}{q(\theta)};$$

and unmatched liquid capital is $u = (1-q(\theta))\frac{l}{g}$. Now both (B.20) and (B.27) each imply a value for $v\kappa$. We thus iterate over values of $\sigma$ such that these two correspond.

Finally, the elasticity of separations to the cutoff $\underline{a}$, $\Psi$, is given as

$$\Psi = \frac{\underline{a}h(\underline{a})}{s^e},$$

and the steady state values for profits, consumption, the Lagrange multiplier, and the weight $\omega$ are pinned down by the remaining steady state equations. The elasticity of the loss of capital value to changes in the cut-off idiosyncratic productivity, $\Gamma$, is chosen to match the relative standard deviation of the rate of capital separations due to financial distress observed in the data.

## B.6 Log-Linear system

$$\widehat{c}_t = -\widehat{\lambda}_t$$
$$\hat{n}_t = \frac{1-n}{\xi n}[\hat{\lambda}_t + \hat{w}_t]$$
$$\frac{\beta}{g}E_t[\widehat{\lambda}_{t+1}] = \frac{\beta}{g}\widehat{\lambda}_t + \frac{\eta}{(1-\eta)}\frac{\kappa}{\theta}\left[[1-H(\bar{a})]\hat{\theta}_t + \bar{a}f(\bar{a})\widehat{\bar{a}}_t\right]$$
$$E_t\widehat{\lambda}_{t+1} + \frac{\beta}{g}(r_t - r) = \widehat{\lambda}_t$$
$$\widehat{w}_t = \widehat{y}_t - \widehat{n}_t + \widehat{\bar{\mu}}_t + \frac{\bar{a}f(\bar{a})}{[1-H(\bar{a})]w}\widehat{\bar{a}}_t$$
$$gk\widehat{k}_{t+1} = (1-\delta)(1-s)k\widehat{k}_t - (1-\delta)sk\widehat{s}_t + lq(\theta)\left[\widehat{l}_t + \widehat{q(\theta_t)}\right]$$
$$g\widehat{u}_{t+1} = \widehat{l}_t - \frac{q(\theta)}{1-q(\theta)}\widehat{q(\theta_t)}$$
$$y\left[1-\tau(1-\bar{\mu})\right]\widehat{y}_t = c\widehat{c}_t + l\widehat{l}_t + \upsilon\kappa\widehat{\upsilon}_t - (1-\delta)s\varphi k\left[\widehat{s}_t + \widehat{k}_t + \widehat{\varphi}_t\right] - u\widehat{u}_t + \tau y\overline{\mu}\widehat{\overline{\mu}}_t$$
$$d\widehat{d}_t + R^k\widehat{R^k_t} = y\left[1-\tau(1-\bar{\mu})\right]\widehat{y}_t - wn[\widehat{w}_t + \widehat{n}_t] - \upsilon\kappa\widehat{\upsilon}_t - \tau y\overline{\mu}\widehat{\overline{\mu}}_t + (1-\delta)s\varphi k\left[\widehat{s}_t + \widehat{k}_t + \widehat{\varphi}_t\right]$$
$$\bar{a}y(\widehat{\bar{a}}_t + \widehat{y}_t) = k(\rho_t - \rho) + \rho k\widehat{k}_t + wn(\widehat{w}_t + \widehat{n}_t) + \upsilon\kappa\widehat{\upsilon}_t$$
$$\hat{y}_t = \widehat{A}_t + (1-\alpha)\hat{n}_t + \alpha\widehat{k}_t$$
$$\hat{\theta}_t = \hat{l}_t - \hat{\upsilon}_t$$
$$\widehat{p(\theta_t)} = (1-\epsilon)\widehat{\theta}_t$$
$$\widehat{q(\theta_t)} = -\epsilon\widehat{\theta}_t$$
$$\widehat{s}_t = \frac{s^a}{s}\widehat{s^a}_t$$
$$\widehat{s^a}_t = \varphi\widehat{\bar{a}}_t$$
$$\widehat{\varphi}_t = \Gamma\widehat{\underline{a}}_t$$
$$\underline{\mu}\widehat{\underline{\mu}} = -\underline{a}^2h(\underline{a})\widehat{\underline{a}}_t$$
$$\overline{\mu}\widehat{\overline{\mu}}_t = -\overline{a}^2h(\overline{a})\widehat{\overline{a}}_t$$

$$\begin{aligned}
\widehat{\lambda}_t - \widehat{p(\theta_t)} - \frac{\bar{a}f(\bar{a})}{[1-H(\overline{a})]}\widehat{\bar{a}}_t &= E_t\widehat{\lambda}_{t+1} \\
&+\frac{\beta}{g}\frac{p(\theta)}{\kappa}\{\frac{\overline{\mu}\psi\alpha y}{[1-H(\overline{a})]\,k}\left[\widehat{y}_{t+1} - \widehat{k}_{t+1} + \widehat{\overline{\mu}}_{t+1}\right] - (\rho_{t+1} - \rho) - (1-\delta)s\frac{\kappa}{p(\theta)}\widehat{s}_{t+1} \\
&-(1-\delta)(1-s)\frac{\kappa}{p(\theta)}\widehat{p(\theta_{t+1})} + \left[\rho - (1-\delta)(1-s)\frac{\kappa}{p(\theta)}\frac{\bar{a}f(\bar{a})}{[1-H(\overline{a})]}\right]\widehat{\bar{a}}_{t+1}\}
\end{aligned}$$

$$[1-H(\bar{a})](\rho_t-\rho)=\alpha\frac{y}{k}\left(\eta\psi\overline{\mu}-(1-\eta)(\underline{\mu}-\bar{\mu}-\tau(1-\bar{\mu}))\right)\left[\widehat{y}_t-\widehat{k}_t\right]$$
$$-\left[\eta(1-\delta)\frac{\kappa[1-H(\bar{a})]}{\theta}-(1-\eta)(1-\delta)(1-\varphi)\right]s\widehat{s}_t-\eta(1-\delta)(1-s)\frac{\kappa[1-H(\bar{a})]}{\theta}\widehat{\theta}_t$$
$$-\left[\rho-\eta(1-\delta)\frac{\kappa}{\theta}\right]\bar{a}h(\bar{a})\widehat{\bar{a}}_t+\alpha\frac{y}{k}[\eta\psi-(1-\eta)(1-\tau)]\overline{\mu}\widehat{\overline{\mu}}_t-(1-\eta)(1-\tau)\alpha\frac{y}{k}\underline{\mu}\widehat{\underline{\mu}}_t$$
$$-(1-\eta)(1-\delta)\varphi s\widehat{\varphi}_t$$

$$R^k\widehat{R^k_t}=k[1-H(\bar{a})](\rho_t-\rho)+\rho k[1-H(\bar{a})]\widehat{k}_t+y(\underline{\mu}-\bar{\mu}-\tau(1-\bar{\mu}))\widehat{y}_t+y\underline{\mu}\widehat{\underline{\mu}}_t-(1-\tau)y\overline{\mu}\widehat{\overline{\mu}}_t$$
$$-wn(H(\bar{a})-H(\underline{a}))[\widehat{w}_t+\widehat{n}_t]-v\kappa(H(\bar{a})-H(\underline{a}))\widehat{v}_t-[wn+\rho k+v\kappa]\bar{a}h(\bar{a})\widehat{\bar{a}}_t+[wn+v\kappa]\underline{a}h(\underline{a})\widehat{\underline{a}}_t$$
$$+(1-\delta)s\varphi k\left[\widehat{s}_t+\widehat{k}_t+\widehat{\varphi}_t\right]$$

$$\underline{a}y(\widehat{\underline{a}}_t+\widehat{y}_t)-wn(\hat{w}_t+\hat{n}_t)-v\kappa\hat{v}_t=\left\{(1-\delta)k\left[\varphi-\frac{1}{q(\theta)}[1-\frac{\beta}{g}(1-q(\theta))]\right]\right\}\widehat{k}_t$$
$$+(1-\delta)k\frac{(1-q(\theta))}{q(\theta)}\frac{\beta}{g}[E_t\widehat{\lambda}_{t+1}-\widehat{\lambda}_t]-\frac{(1-\delta)k}{q(\theta)}\left(1-\frac{\beta}{g}+\frac{2\beta}{g}q(\theta)\right)q(\theta_t)$$
$$+(1-\delta)k\varphi\widehat{\varphi}_t$$

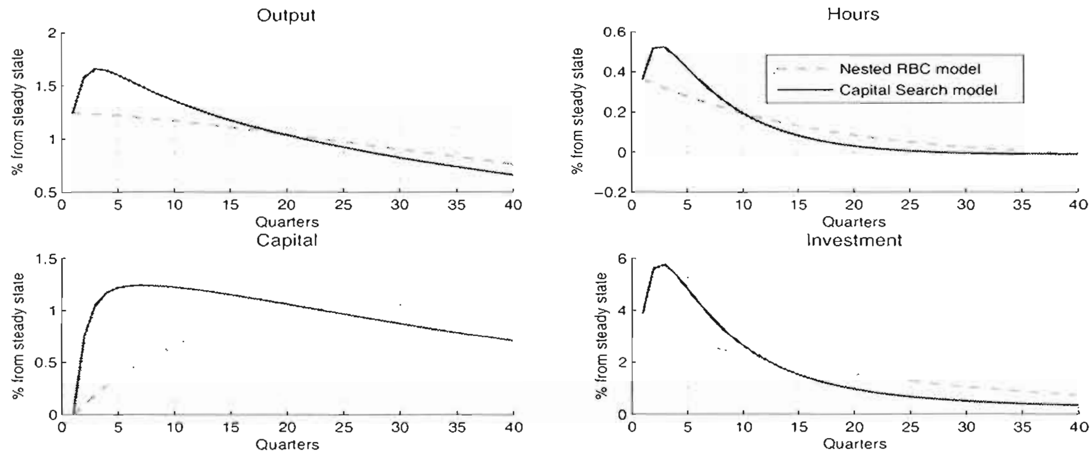


Figure B.1: IRFs to a persistent technology shock.

## B.7 Illustration of propagation potential

This sections illustrates the potential of the propagation mechanism that is the conjunction of search in the capital market with endogenous capital separations due to credit market frictions. The simulations are the result of assuming a constant loss of capital value following separation (i.e. $\varphi_t = \varphi \ \forall t$) and an elasticity in the capital matching function $\epsilon = 0.5$.

Figure B.1 plots the impulse response functions of output, hours, the capital stock and investment to a persistent technology shock. Table 1 presents Hodrick-Prescott filtered second moments for this scenario along with the second moments for the data, the RBC benchmark, the capital search model with exogenous separations and the extended capital search model :for the baseline calibration $((\partial\varphi/\partial s^e)/(s^e/\varphi)$ such that $\sigma(s^e)/\sigma(y) = 2.46$ and $\epsilon = 0.25)$.

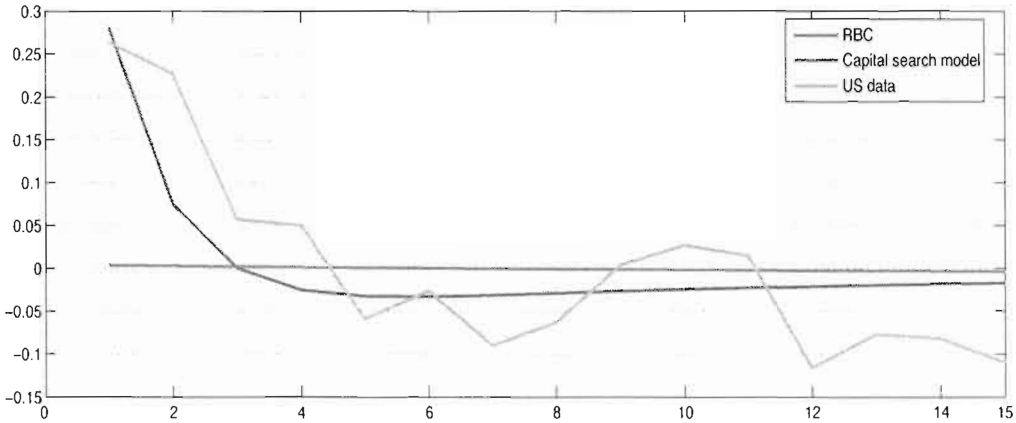


Figure B.2: Autocorrelation functions for output growth

Table 3: Second moments

| | U.S data | | RBC | | Capital search | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | benchmark | | Exogenous separation | | Endogenous separation | | | |
| | | | | | | | $\varphi$ constant, $\epsilon = 0.5$ | | $\varphi$ variable, $\epsilon = 0.25$ | |
| | a | b | a | b | a | b | a | b | a | b |
| $c$ | 0.58 | 0.69 | 0.45 | 0.96 | 0.48 | 0.97 | 0.46 | 0.92 | 0.33 | 0.85 |
| $n$ | 0.95 | 0.87 | 0.29 | 0.97 | 0.27 | 0.98 | 0.35 | 0.95 | 0.40 | 0.97 |
| $k$ | - | - | 0.28 | 0.10 | 0.28 | 0.14 | 0.69 | 0.77 | 0.25 | 0.10 |
| $i$ | 2.89 | 0.87 | 2.68 | 0.99 | 2.68 | 0.99 | 3.72 | 0.97 | 2.53 | 0.99 |
| $s^e$ | 2.46 | -0.31 | - | - | - | - | 1210.17 | -0.78 | 2.46 | -0.96 |
| *premium* | 0.54 | -0.59 | 0.004 | 0.003 | 0.003 | 0.98 | 0.04 | -0.90 | 0.03 | -0.97 |
| $\sigma(y)$ | 1.66 | | | | 1.16 | | 1.52 | | 1.28 | |

Notes: (a) Standard deviation relative to output; (b) contemporaneous correlation with output.

All moments are Hodrick-Prescott filtered. Data source from DRI Basic Economics 1953:2-2001:4

The extent of persistence in output growth generated by the extended capital search model with $\varphi$ constant and $\epsilon = 0.5$ is illustrated in Figure B.2.

## B.8 Data sources

All time series used in this paper are quarterly data taken from the DRI Basic Economics database (formerly Citibase). We restrict our statistical analysis to the sample 1953:2–2001:4. Data related to firm-level property, plant and equipment are from Standard and Poor's Compustat database, for the period spanning 1980-1993. Finally, bankruptcy filing data are from the Bankruptcy Research Database compiled by Prof. Lyn LoPucki at UCLA's law department. This covers filings by publicly traded firms in the U.S. since October 1979. The following table gives the definition and a short description of the different series (where the definition is given in actual DRI mnemonics):

Table 4: Data description

| Compustat | |
|---|---|
| **variable** | **data item** |
| Property, plant and equipment, gross | 7 |
| Property, plant and equipment, net | 8 |
| Sales | 107 |
| Retirements | 184 |
| Reason for deletion | aftnt35 |
| Year of deletion | aftnt34 |

| DRI Basic Economics database | | |
|---|---|---|
| **Variable** | **Definition** | **Description** |
| $y$ | ln(gdpq-gpbfq)-ln(p16) | real GDP (non-farm) per capita |
| $c$ | ln(gcnq+gcsq)-ln(p16) | real per capita private consumption of non-durables and servics |
| $i$ | ln(gifq)-ln(p16) | real per capita private fixed investment (incl. residential) |
| $n$ | ln(lpmhu)-ln(p16) | total hours (non-farm) per capita |

# Appendix C

# Endogenous Flows of Foreign Direct Investment and International Real Business Cycles

## C.1 Flows of FDI and Canada - U.S. business cycles

Table C.1: Data series and sources

| | Canada | | United States | |
|---|---|---|---|---|
| | source | series | source | series |
| Output | Statistics Canada | v1992067 | Fred II : B.E.A | GDP96 |
| Consumption | Statistics Canada | v1992044 | Fred II : B.E.A | PCESVC96+PCNDGC96 |
| Hours | Statistics Canada | v3443721 | Fred II : B.L.S | AWHI |
| Investment | Statistics Canada | v1992054 | Fred II · B.E.A | PNFIC96 |
| Investment deflator | Statistics Canada | Table 380-0003 | Fred II B.E.A | |
| FDI in Canada* | Statistics Canada | Table 376-0003 | | |
| gross inflows | | v113032 | | |
| gross outflows | | v113035 | | |
| Canadian Inv. abroad* | Statistics Canada | Table 376-0003 | | |
| gross inflows | | v113021 | | |
| gross outflows | | v113018 | | |

*: Excludes portfolio investment flows

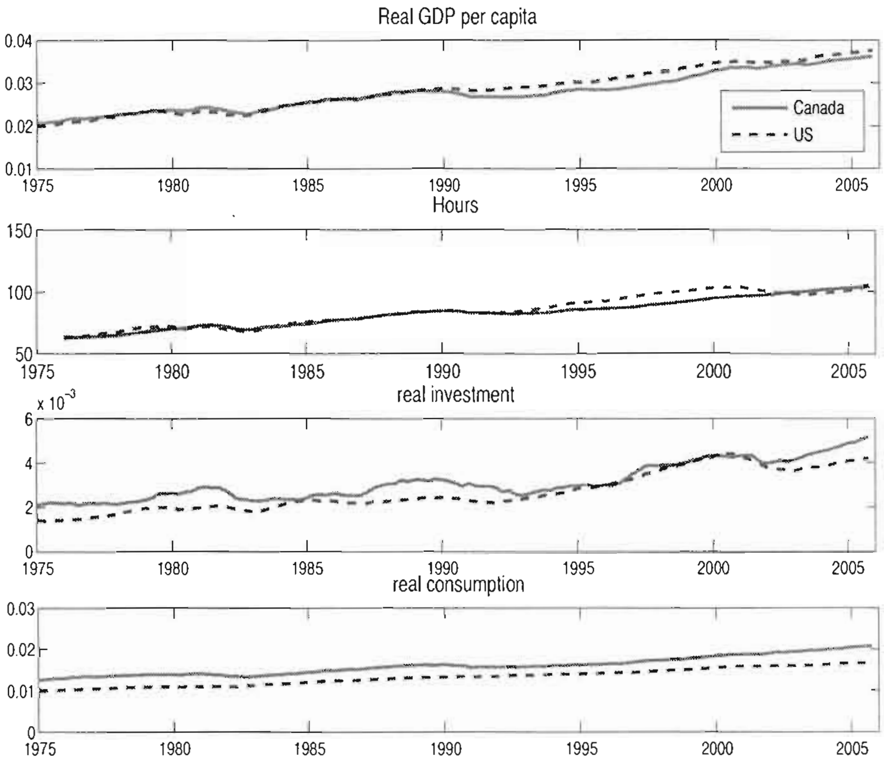


Figure C.1: Evolution of prominent macroeconomic variables, Canada and the U.S., 1976-2005.

### C.1.1 Canadian and US macro variables.

Figure C.1 shows the evolution of hours worked (indexed), real output, investment and consumption per capita, over the period 1976-2006, for Canada and the U.S.. The general observation is of a close similarity in per capita variables, with few episodes where each country follows a different path. One example is the output per capita gap appearing during the 1990s, which also shows up as a gap in average hours worked.

### C.1.2 Foreign controlled firms in Canada

The following table presents the shares of assets and operating revenue of foreign controlled non-financial firms in Canada. Manufacturing stands out as a sector with a large share of employment and high degree of foreign control, involving nearly one fifth of employment and where just over half of the revenues and assets are under foreign control.

| Industry | Percentage foreign | | total employment |
|---|---|---|---|
| | Operating revenue | Assets | |
| Oil and gas extraction and support activities | 55,2 | 44,9 (2) | 1,48* |
| Manufacturing | 51,7 | 50,3 (1) | 18,87 |
| Mining (except oil and gas) | 41,6 | 29,7 (4) | |
| Wholesale trade | 36,6 | 33,4 (3) | 7,04 |
| Utilities | 31 | 6,3 (13) | 1,15 |
| Transportation and warehousing | 20,5 | 26,9 (5) | 5,87 |
| Administrative and support, waste management and remediation services | 19,4 | 23,5 (6) | 6,02 |
| Retail trade | 16,2 | 21 (7) | 15,68 |
| Professional, scientific and technical services | 15,3 | 15,5 (9) | 6,26 |
| Real estate and rental and leasing | 11,7 | 13,8 (11) | 2,24 |
| Accommodation and food services | 10,5 | 14,7 (10) | 8,95 |
| Information and cultural industries | 9,3 | 6,5 (12) | 3,27 |
| Repair, maintenance and personal services | 6,5 | 18,4 (8) | 4,81 |
| Construction | 4,5 | 4,7 (14) | 6,35 |
| Educational, healthcare and social assistance services | 2,1 | 1,5 (17) | 9,04 |
| Arts, entertainment and recreation | 1,8 | 2,2 (15) | 2,35 |
| Agriculture, forestry, fishing and hunting | 1,7 | 1,7 (16) | 0,62 |
| Total non-financial industries | 30,7 | 28.5 | 100 |

Source: Statistics Canada
notes: * includes mining.

Figure C.2: Share of operating revenues and assets under foreign control - non-financial industries 2004

## C.2 Models

This section presents the full details of the three models simulated in the paper.

### C.2.1 IRBC with search in FDI, endogenous reallocation and investment adjustment costs

Each country is populated by domestic and foreign firms and a representative household. Households decide on optimal consumption, an aggregate of goods produced by both types of firms, and allocation of investment goods to firms located at home or abroad. Initiating a new investment project abroad required disbursing a flow cost $\kappa$ and is subject to a time consuming search and matching process. No such friction applies to changing production capacity at home. Thus domestic firms rent capital on spot markets while foreign firms chose the amount of new projects to initiate. As will be seen later, the frictionless capital market is a special case of the search environment with $\kappa = 0$. Firms, domestic and foreign, hire labor on competitive domestic markets. Finally, domestic and foreign firms produce intermediate goods aggregated into a final homogeneous consumption good by an Armington (1996) aggregator $y_t = G\left(y_t^d, y_t^{fdi}\right) = \left[\phi(y_t^d)^\nu + (1-\phi)(y_t^{fdi})^\nu\right]^{\frac{1}{\nu}}$, with elasticity of substitution $\psi = 1/(1-\nu)$ and relative shares determined by the parameter $\phi$. The relative price of the foreign firm's good is then simply $p_t^{fdi} = G_2\left(y_t^d, y_t^{fdi}\right)$ and that of the domestic firm's good $p_t^d = G_1\left(y_t^d, y_t^{fdi}\right)$.

#### C.2.1.1 Domestic and foreign producers

Domestic firms produce with technology $y_t^d = A_t(n_t^d)^{1-\alpha}(k_t^d)^\alpha$. Optimization yields the following two first order conditions:

$$\begin{aligned}(n_t^d) &: \quad w_t^d = (1-\alpha)\frac{p_t^d y_t^d}{n_t^d} \\ (k_t^d) &: \quad r_t^d = \alpha\frac{p_t^d y_t^d}{k_t^d}.\end{aligned}$$

The following is the dynamic problem for foreign firms:

$$\begin{aligned}J(k_t^{fdi}) &= \max_{n_t^{fdi}, v_t}\left[p_t^{fdi}A_t(n_t^{fdi})^{1-\alpha}(k_t^{fdi})^\alpha - w_t^{fdi}n_t^{fdi} - r_t^{fdi}k_t^{fdi} - \kappa v_t + \beta E_t\frac{\lambda_{t+1}^*}{\lambda_t^*}J(k_{t+1}^{fdi})\right] \\ \text{subject to } k_{t+1}^{fdi*} &= (1-\delta)(1-s_t)k_t^{fdi} + p(\theta_t)v_t\end{aligned}$$

Optimization yields the following two first order conditions:

$$\begin{aligned}(n_t^{fdi}) &: \quad w_t^{fdi} = (1-\alpha)\frac{p_t^{fdi}y_t^{fdi}}{n_t^{fdi}} \\ (v_t) &: \quad \beta E_t\frac{\lambda_{t+1}^*}{\lambda_t^*}J_{k^{fdi}}(k_{t+1}^{fdi}) = \frac{\kappa}{p(\theta_t)}.\end{aligned}$$

where $J_{k^{fdi}}(k_{t+1}^{fdi})$ is the marginal value of an additional unit of capital to the firm, defined as

$$J_{k^{fdi}}(k_{t+1}^{fdi}) = \alpha\frac{p_t^{fdi}y_t^{fdi}}{k_t^{fdi}} - r_t^{fdi} + (1-\delta)(1-s_{t+1})\beta E_t\frac{\lambda_{t+1}^*}{\lambda_t^*}J_{k^{fdi}}(k_t^{fdi})$$

In combination with the first order condition for project initiations, this yield the forward looking condition

$$\frac{\kappa}{p(\theta_t)} = \beta E_t\frac{\lambda_{t+1}^*}{\lambda_t^*}\left\{\alpha\frac{p_t^{fdi}y_{t+1}^{fdi}}{k_{t+1}^{fdi}} - r_{t+1}^{fdi} + (1-\delta)(1-s_t)\frac{\kappa}{p(\theta_{t+1})}\right\}.$$

#### C.2.1.2 Domestic households

Households choose a level of aggregate consumption (defined as the sum of both intermediate goods), hours to supply to both domestic and foreign employers, and have two capital investment options: investing in firms at home or investing in capacity abroad.

In addition, there are convex cost to new investments, domestic and foreign. The resulting dynamic program for the representative household is thus

$$V(k_t^d, k_t^{fdi*}, u_t^*) = \max_{n_t^d, n_t^{fdi}, i_t, fdi_t^*} \left[ u(c_t, 1 - n_t) + \beta E_t V(k_{t+1}^d, k_{t+1}^{fdi*}, u_{t+1}^*) \right]$$

$$\text{subject to } w_t^d n_t^d + w_t^{fdi} n_t^{fdi} + r_t^d k_t^d + r_t^{fdi*} k_t^{fdi*} + \Pi_t^* = c_t + q_t^d i_t^d + q_t^{fdi*} i_t^{fdi*}$$

$$\text{and} \qquad k_{t+1}^{fdi*} = (1-\delta)(1-s_t)k_t^{fdi} + q(\theta_t^*) l_t^*$$

where $n_t = n_t^d + n_t^{fdi}$, $i_t^d = k_{t+1}^d - (1-\delta)k_t^d$, and $q_t^d$ and $q_t^{fdi*}$ are, respectively, the cost of new investment goods destined for plants at home and abroad. This cost is given by $q_t = \left[\Phi'(\frac{i_t}{k_t})\right]^{-1}$, with $\Phi'(\bullet) > 0$ and $\Phi''(\bullet) < 0$, and such that in the steady state $q = 1$. New investment goods destined for foreign direct investment are defined as $i_t^{fdi*} = l_t^* - (1-\delta)s_t k_t^{fdi*} - u_t^*$, where $(1-\delta)s_t k_t^{fdi*}$ is capital recuperated from terminated operations abroad, net of depreciation, and $u_t$ are units of investments goods not yet allocated. Thus $l_t^*$ is the total amount of investment goods available for allocation to production abroad.

The optimality conditions are

$$\begin{aligned}
(c_t) &: u_c(c_t, 1-n_t) = \lambda_t \\
(n_t^d) &: u_{n^d}(c_t, 1-n_t) = \lambda_t w_t^d \\
(n_t^{fdi}) &: u_{n^{fdi}}(c_t, 1-n_t) = \lambda_t w_t^{fdi} \\
(k_{t+1}^d) &: \lambda_t q_t^d = \beta E_t \left[ r_{t+1}^d + q_{t+1}^d (1-\delta) \right] \\
(l_t^*) &: \lambda_t q_t^{fdi*} = \beta E_t \left[ q(\theta_t^*) V_{k^{fdi*}}(k_{t+1}^d, k_{t+1}^{fdi*}, u_{t+1}^*) + (1 - q(\theta_t^*)) V_u(k_{t+1}^d, k_{t+1}^{fdi*}, u_{t+1}^*) \right]
\end{aligned}$$

where the marginal values of allocated and non-allocated investment goods are

$$\begin{aligned}
V_u(k_t^d, k_t^{fdi*}, u_t^*) &= \lambda_t q_t^{fdi*} \\
V_{k^{fdi}}(k_t^d, k_t^{fdi*}, u_t^*) &= \lambda_t \left[ r_t^{fdi*} + q_t^{fdi*}(1-\delta)s_t \right] + (1-\delta)(1-s_t)\beta E_t V_{k^{fdi}}(k_{t+1}^d, k_{t+1}^{fdi*}, u_{t+1}^*)
\end{aligned}$$

The repayment on capital is determined by Nash bargaining over a surplus defined as

$S_t = J(k_t^{fdi}) + \frac{V_{k^{fdi}}(k_t^{d*},k_t^{fdi},u_t) - V_u(k_t^{d*},k_t^{fdi},u_t)}{\lambda_t^*}$ :

$$\begin{aligned} S_t &= \alpha\frac{p_t^{fdi}y_t^{fdi}}{k_t^{fdi}} - r_t^{fdi} + (1-\delta)(1-s_t)\beta E_t\frac{\lambda_{t+1}^*}{\lambda_t^*}J_{k^{fdi}}(k_{t+1}^{fdi}) \\ &+ r_t^{fdi} + q_t^{fdi}(1-\delta)s_t + (1-\delta)(1-s_t)\beta E_t V_{k^{fdi}}(k_{t+1}^{d*},k_{t+1}^{fdi},u_{t+1}) - q_t^{fdi} \\ S_t &= \alpha\frac{p_t^{fdi}y_t^{fdi}}{k_t^{fdi}} + q_t^{fdi}\left[(1-\delta)s_t - 1\right] \\ &+ (1-\delta)(1-s_t)\beta E_t\frac{\lambda_{t+1}^*}{\lambda_t^*}\left[J_{k^{fdi}}(k_{t+1}^{fdi}) + \frac{V_{k^{fdi}}(k_{t+1}^{d*},k_{t+1}^{fdi},u_{t+1}) - V_u(k_{t+1}^{d*},k_{t+1}^{fdi},u_{t+1})}{\lambda_{t+1}^*}\right] \\ &+ (1-\delta)(1-s_t)\beta E_t\frac{V_u(k_t^{d*},k_t^{fdi},u_t)}{\lambda_t^*} \end{aligned}$$

The first order condition on $l_t$ can be rewritten as $\beta E_t\frac{V_u(k_t^{d*},k_t^{fdi},u_t)}{\lambda_t^*} = q_t^{fdi} - \eta q(\theta_t)\beta E_t\frac{\lambda_{t+1}^*}{\lambda_t^*}S_{t+1}$ such that

$$\begin{aligned} S_t &= \alpha\frac{p_t^{fdi}y_t^{fdi}}{k_t^{fdi}} + q_t^{fdi}\left[(1-\delta)s_t - 1\right] + (1-\delta)(1-s_t)\beta E_t\frac{\lambda_{t+1}^*}{\lambda_t^*}S_{t+1} \\ &+ (1-\delta)(1-s_t)\left[q_t^{fdi} - \eta q(\theta_t)\beta E_t\frac{\lambda_{t+1}^*}{\lambda_t^*}S_{t+1}\right] \\ S_t &= \alpha\frac{p_t^{fdi}y_t^{fdi}}{k_t^{fdi}} + q_t^{fdi}\left[(1-\delta)s_t - 1 + (1-\delta)(1-s_t)\right] + (1-\delta)(1-s_t)\frac{\kappa}{p(\theta_t)(1-\eta)} \\ &- (1-\delta)(1-s_t)\frac{\eta}{1-\eta}\frac{\kappa}{\theta_t} \\ S_t &= \alpha\frac{p_t^{fdi}y_t^{fdi}}{k_t^{fdi}} + q_t^{fdi}\delta + (1-\delta)(1-s_t)\frac{\kappa}{p(\theta_t)(1-\eta)} \\ &- (1-\delta)(1-s_t)\frac{\eta}{1-\eta}\frac{\kappa}{\theta_t} \end{aligned}$$

From the definition of $J(k_t^{fdi})$ we have

$$(1-\eta)S_t = \alpha\frac{p_t^{fdi}y_t^{fdi}}{k_t^{fdi}} - r_t^{fdi} + (1-\delta)(1-s_t)\frac{\kappa}{p(\theta_t)}$$

Combining these two yields

$$r_t^{fdi} = \eta\alpha\frac{p_t^{fdi}y_t^{fdi}}{k_t^{fdi}} + (1-\eta)q_t^{fdi}\delta + \eta(1-\delta)(1-s)\frac{\kappa}{\theta_t}$$

#### C.2.1.3 Endogenous reallocation

We assume the existence of a random idiosyncratic productivity to the match, the realization of which occurs after production decision are made and factor price equilibria are establish. Denote this realization $a_t > 0$, where $a$ is independently distributed over time with probability density $h(a)$, cumulative density $H(a)$ and mean $E(a) = 1$. The surplus is then a function of this shock, $S(a_t)$, and a match is discontinued for realization of $a_t < \underline{a}_t$ where $\underline{a}_t$ is define as $S(\underline{a}_t) = 0$. Using $(1-\eta)S_t = a_t \alpha \frac{p_t^{fdi} y_t^{fdi}}{k_t^{fdi}} - r_t^{fdi} + (1-\delta)(1-s_t)\frac{\kappa}{p(\theta_t)}$ we have

$$\underline{a}_t \alpha \frac{p_t^{fdi} y_t^{fdi}}{k_t^{fdi}} = r_t^{fdi} - (1-\delta)(1-s_t)\frac{\kappa}{p(\theta_t)}.$$

#### C.2.1.4 Equilibrium system of equations

$$
\begin{aligned}
u_c(c_t; 1-n_t) &= \lambda_t \\
u_{n^d}(c_t; 1-n_t) &= \lambda_t w_t^d \\
u_{n^{fdi}}(c_t; 1-n_t) &= \lambda_t w_t^{fdi} \\
\lambda_t q_t^d &= \beta E_t\left[r_{t+1}^d + q_{t+1}^d(1-\delta)\right] \\
\lambda_t q_t^{fdi*} &= \beta E_t \lambda_{t+1} q_{t+1}^{fdi*} + \frac{\eta}{1-\eta}\frac{\kappa}{\theta_t^*}\lambda_t \\
q_t^d &= \left[\Phi'(\frac{i_t^d}{k_t^d})\right]^{-1} \\
q_t^{fdi*} &= \left[\Phi'(\frac{fdi_t^*}{k_t^{fdi*}})\right]^{-1} \\
i_t^d &= k_{t+1}^d - (1-\delta)k_t^d \\
i_t^{fdi*} &= l_t^* - (1-\delta)s_t k_t^{fdi*} - u_t^* \\
k_{t+1}^{fdi*} &= (1-\delta)(1-s_t)k_t^{fdi} + q(\theta_t^*)l_t^* \\
w_t^d &= (1-\alpha)\frac{p_t^d y_t^d}{n_t^d} \\
r_t^d &= \alpha\frac{p_t^d y_t^d}{k_t^d} \\
w_t^{fdi} &= (1-\alpha)\frac{p_t^{fdi} y_t^{fdi}}{n_t^{fdi}} \\
\frac{\kappa}{p(\theta_t)} &= \beta E_t \frac{\lambda_{t+1}^*}{\lambda_t^*}\left\{\alpha\frac{p_t^{fdi} y_{t+1}^{fdi}}{k_{t+1}^{fdi}} - r_{t+1}^{fdi} + (1-\delta)(1-s_{t+1})\frac{\kappa}{p(\theta_{t+1})}\right\} \\
r_t^{fdi} &= \eta\alpha\frac{p_t^{fdi} y_t^{fdi}}{k_t^{fdi}} + (1-\eta)q_t^{fdi}\delta + \eta(1-\delta)(1-s)\frac{\kappa}{\theta_t} \\
& \Pi_t^* + w_t^d n_t^d + w_t^{fdi} n_t^{fdi} + r_t^d k_t^d + r_t^{fdi*} k_t^{fdi*} = c_t + q_t^d i_t^d + q_t^{fdi*} i_t^{fdi*} \\
y_t^d &= A_t (n_t^d)^{1-\alpha}(k_t^d)^\alpha \\
y_t^{fdi} &= A_t (n_t^{fdi})^{1-\alpha}(k_t^{fdi})^\alpha \\
n_t &= n_t^d + n_t^{fdi} \\
\theta &= \frac{l_t}{v_t} \\
p(\theta_t) &= \theta_t^{1-\epsilon} \\
q(\theta_t) &= \theta_t^{-\epsilon}
\end{aligned}
$$

$$
\begin{aligned}
s_t &= H(\underline{a}_t) \\
\underline{a}_t \alpha \frac{p_t^{fdi} y_t^{fdi}}{k_t^{fdi}} &= r_t^{fdi} - (1-\delta)(1-s_t)\frac{\kappa}{p(\theta_t)} \\
\Pi_t^* &= p_t^{fdi} y_t^{fdi*} - w_t^{fdi*} n_t^{fdi*} - r_t^{fdi*} k_t^{fdi*} - \kappa v_t^* \\
y_t &= \left[\phi (y_t^d)^\nu + (1-\phi)(y_t^{fdi})^\nu\right]^{\frac{1}{\nu}} \\
p_t^d &= G_1\left(y_t^d, y_t^{fdi}\right) \\
p_t^{fdi} &= G_2\left(y_t^d, y_t^{fdi}\right)
\end{aligned}
$$

The Home and Foreign countries are modeled symmetrically. Equilibrium in each country (with an inversion and the * on the each variable) is defined for the variables $y$, $y^d$, $y^{fdi}$ $c$, $n$, $n^d$, $n^{fdi}$, $q^d$, $i^d$, $k^d$, $q^{fdi*}$, $fdi^*$, $k^{fdi*}$, $l^*$, $u^*$, $v$, $\theta$, $q(\theta^*)$, $p(\theta)$, $w^d$, $w^{fdi}$, $r^d$, $r^{fdi}$,$p^d$, $p^{fdi}$, $\lambda$, $s$, $\underline{a}$ , $\Pi^*$ and the system of equations.

#### C.2.1.5 Computing the steady state

Hours at domestic firms are determined for a given fraction of total hours worked in the foreign sector. To compute the steady state in the first country, first note that the remuneration of domestic capital is given by $r^d = 1/\beta - 1 + \delta$ since in steady state $q = 1$. For a given initial guess on the price of the domestic good $p^d$, the price of the foreign firms good can be computed as $p^{fdi} = \frac{1-\phi}{\phi} p^d$. Combined with the capital demand equation, the steady state capital-output ratio at domestic firms is $\frac{k^d}{y^d} = \frac{\alpha p^d}{r^d}$.

By combining the forward looking equation for project initiations, the first order condition for $l$ and the rental rate equation in the steady state, the steady state rental rate can be expressed as

$$
r^{fdi} = (\frac{1}{\beta} - 1 + \delta) + \left(\frac{1-q(\theta)}{q(\theta)}\right)\left\{\frac{1}{\beta} - 1 - (1-\beta)(1-\delta)(1-s)\right\}.
$$

The capital-output ratio at foreign firms is then defined as

$$
\frac{k^{fdi}}{y^{fdi}} = \frac{\alpha p^{fdi}}{\left[\frac{\kappa}{q(\theta)\theta}\left(\frac{1}{\beta} - (1-\delta)(1-s)\right) + r^{fdi}\right]}.
$$

where $\frac{\kappa}{\theta} = \frac{1-\eta}{\eta}(1-\beta)$.

Given the choice of production functions the capital stocks are then computed as $k^d = n^d (A^d \frac{k^d}{y^d})^{\frac{1}{1-\alpha}}$ and $k^{fdi} = n^{fdi}(A^{fdi}\frac{k^{fdi}}{y^{fdi}})^{\frac{1}{1-\alpha}}$. Output and wages in each sector are then given by their respective equations and the share parameter $\phi$ is chosen such that $w^d = w^{fdi}$ is satisfied.

Investment in domestic firms is given by $i^d = \delta k^d$ . To compute new investment goods $i^{fdi}$, first compute $l$ as $l = \frac{k[1-(1-\delta)(1-s)]}{q(\theta)}$ and $u = (1-q(\theta))l$. Then $i^{fdi} = l - (1-\delta)sk^{fdi} - u$.

The variance of the idiosyncratic shock is computed numerically to satisfy the cut-off value definition, $\underline{a}\alpha\frac{p^{fdi}y^{fdi}}{k^{fdi}} = r^{fdi} - (1-\delta)(1-s)\frac{\kappa}{p(\theta)}$, and that for the separation rate, $s = H(\underline{a})$.

The same procedures obtain the corresponding steady state values for the foreign country.

Finally, consumption is obtained using the budget constraint and $\lambda$ by using the first order condition on consumption.

## C.2.2 IRBC with FDI and investment adjustment costs

### C.2.2.1 Domestic and foreign producers

As before, domestic and foreign firms produce intermediate goods aggregated into a final homogeneous consumption good by an Armington (1996) aggregator $y_t = G\left(y_t^d, y_t^{fdi}\right) = \left[\phi(y_t^d)^\nu + (1-\phi)(y_t^{fdi})^\nu\right]^{\frac{1}{\nu}}$, with elasticity of substitution $\psi = 1/(1-\nu)$ and relative shares determined by the parameter $\phi$. The relative price of the foreign firm's good is then simply $p_t^{fdi} = G_2\left(y_t^d, y_t^{fdi}\right)$ and that of the domestic firm's good $p_t^d = G_1\left(y_t^d; y_t^{fdi}\right)$.Domestic firms face the same problem as above. When there is no search cost, firms post an infinity of projects and all capital is reallocated in the beginning of each period; i.e. $s = 1$, $q(\theta_t) = 1$ and $u_t = 0$. Thus the foreign firm problem is now static. Optimization yields the following two first order conditions:

$$(n_t^{fdi}) \quad : \quad w_t^{fdi} = (1-\alpha)\frac{p_t^{fdi} y_t^{fdi}}{n_t^{fdi}}$$

$$(k_t^{fdi}) \quad : \quad r_t^{fdi} = \alpha\frac{p_t^{fdi} y_t^{fdi}}{k_t^{fdi}}.$$

### C.2.2.2 Domestic households

The household's dynamic program is now[1]

$$V(k_t^d, k_t^{fdi*}) = \max_{n_t^d, n_t^{fdi}, i_t, i_t^{fdi*}} \left[u(c_t, 1-n_t) + \beta E_t V(k_{t+1}^d, k_{t+1}^{fdi*})\right]$$

$$\text{subject to } w_t^d n_t^d + w_t^{fdi} n_t^{fdi} + r_t^d k_t^d + r_t^{fdi*} k_t^{fdi*} = c_t + q_t^d i_t^d + q_t^{fdi*} fdi_t^*$$

Note that now $i_t^{fdi*} = k_{t+1}^{fdi*} - (1-\delta)k_t^{fdi*}$. The optimality conditions are

$$\begin{aligned}
(c_t) &\quad : \quad u_c(c_t; 1-n_t) = \lambda_t \\
(n_t^d) &\quad : \quad u_{n^d}(c_t; 1-n_t) = \lambda_t w_t^d \\
(n_t^{fdi}) &\quad : \quad u_{n^{fdi}}(c_t; 1-n_t) = \lambda_t w_t^{fdi} \\
(k_{t+1}^d) &\quad : \quad \lambda_t q_t^d = \beta E_t\left[r_{t+1}^d + q_{t+1}^d(1-\delta)\right] \\
(k_{t+1}^{fdi*} &\quad : \quad )\lambda_t q_t^{fdi*} = \beta E_t\left[r_{t+1}^{fdi*} + q_{t+1}^{fdi*}(1-\delta)\right]
\end{aligned}$$

### C.2.2.3 Equilibrium system of equations

The home and foreign countries are modeled symmetrically. Equilibrium in each country (with an inversion and the * on the each variable) is defined for the variables $y$, $y^d$, $y^{fdi}$ $c$,

[1] Note that taxes on capital flows have been omitted.

$n$, $n^d$, $n^{fdi}$, $q^d$, $i^d$, $k^d$, $q^{fdi*}$, $fdi^*$, $k^{fdi*}$, $w^d$, $w^{fdi}$, $r^d$, $r^{fdi}$, $p^d$, $p^{fdi}$, $\lambda$ and the system of equations,

$$\begin{aligned}
u_c(c_t, 1-n_t) &= \lambda_t \\
u_{n^d}(c_t, 1-n_t) &= \lambda_t w_t^d \\
u_{n^{fdi}}(c_t, 1-n_t) &= \lambda_t w_t^{fdi} \\
\lambda_t q_t^d &= \beta E_t \left[ r_{t+1}^d + q_{t+1}^d (1-\delta) \right] \\
\lambda_t q_t^{fdi*} &= \beta E_t \left[ r_{t+1}^{fdi*} + q_{t+1}^{fdi*} (1-\delta) \right] \\
q_t^d &= \left[ \Phi'(\frac{i_t^d}{k_t^d}) \right]^{-1} \\
q_t^{fdi*} &= \left[ \Phi'(\frac{fdi_t^*}{k_t^{fdi*}}) \right]^{-1} \\
i_t^d &= k_{t+1}^d - (1-\delta) k_t^d \\
fdi_t^* &= k_{t+1}^{fdi*} - (1-\delta) k_t^{fdi*} \\
w_t^d &= (1-\alpha) \frac{p_t^d y_t^d}{n_t^d} \\
r_t^d &= \alpha \frac{p_t^d y_t^d}{k_t^d} \\
w_t^{fdi} &= (1-\alpha) \frac{p_t^{fdi} y_t^{fdi}}{n_t^{fdi}} \\
r_t^{fdi} &= \alpha \frac{p_t^{fdi} y_t^{fdi}}{k_t^{fdi}} \\
w_t^d n_t^d + w_t^{fdi} n_t^{fdi} + r_t^d k_t^d + r_t^{fdi*} k_t^{fdi*} &= c_t + q_t^d i_t^d + q_t^{fdi*} fdi_t^* \\
y_t^d &= A_t (n_t^d)^{1-\alpha} (k_t^d)^\alpha \\
y_t^{fdi} &= A_t (n_t^{fdi})^{1-\alpha} (k_t^{fdi})^\alpha \\
n_t &= n_t^d + n_t^{fdi} \\
y_t &= \left[ \phi (y_t^d)^\nu + (1-\phi)(y_t^{fdi})^\nu \right]^{\frac{1}{\nu}} \\
p_t^d &= G_1 \left( y_t^d, y_t^{fdi} \right) \\
p_t^{fdi} &= G_2 \left( y_t^d, y_t^{fdi} \right)
\end{aligned}$$

#### C.2.2.4 Computing the steady state

The procedure is identical to previously with the exception that now the remuneration of domestic and foreign capital are given by $r^d = 1/\beta - 1 + \delta$ and $r^{fdi} = 1/\beta - 1 + \delta$, since in the steady state $q = 1$(the second rental rate uses the foreign household's first order condition on foreign investment).

### C.2.3 IRBC with search in FDI, exogenous separations and investment adjustment costs

The problem faced by domestic and foreign firms, and households, is the same as in the endogenous separation case. Computing the steady state involves the same procedures, save the iteration to pin down the variance of the idiosyncratic shock.

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