The Pythagorean Theorem

MODULE 12

COMMON CORE

ESSENTIAL QUESTION

How can you use the Pythagorean Theorem to solve real-world problems?

You can use the Pythagorean Theorem to find the length of any side of a right triangle if you know the lengths of the other two sides.

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Real-World Video

The sizes of televisions are usually described by the length of the diagonal of the screen. To find this length of the diagonal of a rectangle, you can use the Pythagorean Theorem.

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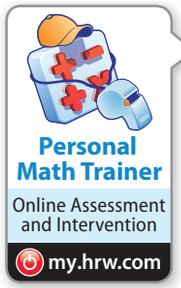
Are You Ready?

Assess Readiness

Use the assessment on this page to determine if students need intensive or strategic intervention for the module's prerequisite skills.

RtI Response to Intervention



Intervention	Enrichment

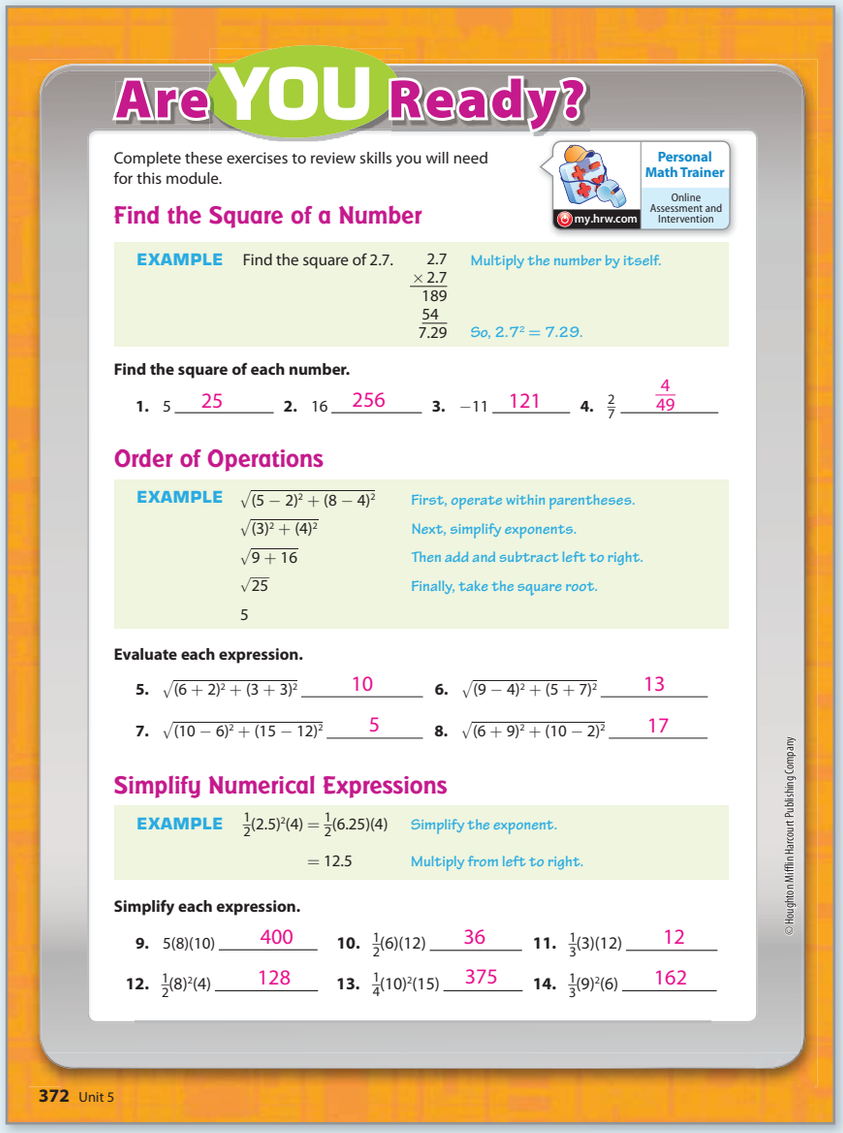
Access Are You Ready? assessment online, and receive instant scoring, feedback, and customized intervention or enrichment.

Online and Print Resources

Skills Intervention worksheets
- Skill 11 Find the Square of a Number
- Skill 51 Order of Operations
- Skill 52 Simplify Numerical Expressions

Differentiated Instruction
- Challenge worksheets **PRE-AP**

Extend the Math **PRE-AP** Lesson Activities in TE



Are YOU Ready?

Complete these exercises to review skills you will need for this module.

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Find the Square of a Number

EXAMPLE Find the square of 2.7.

$$\begin{array}{r} 2.7 \\ \times\, 2.7 \\ \hline 189 \\ 54 \\ \hline 7.29 \end{array}$$

Multiply the number by itself.

So, $2.7^2 = 7.29$.

Find the square of each number.

1. 5 ___25___
2. 16 ___256___
3. −11 ___121___
4. $\frac{2}{7}$ ___$\frac{4}{49}$___

Order of Operations

EXAMPLE

$\sqrt{(5-2)^2+(8-4)^2}$ — First, operate within parentheses.

$\sqrt{(3)^2+(4)^2}$ — Next, simplify exponents.

$\sqrt{9+16}$ — Then add and subtract left to right.

$\sqrt{25}$ — Finally, take the square root.

5

Evaluate each expression.

5. $\sqrt{(6+2)^2+(3+3)^2}$ ___10___
6. $\sqrt{(9-4)^2+(5+7)^2}$ ___13___
7. $\sqrt{(10-6)^2+(15-12)^2}$ ___5___
8. $\sqrt{(6+9)^2+(10-2)^2}$ ___17___

Simplify Numerical Expressions

EXAMPLE

$\frac{1}{2}(2.5)^2(4) = \frac{1}{2}(6.25)(4)$ — Simplify the exponent.

$= 12.5$ — Multiply from left to right.

Simplify each expression.

9. $5(8)(10)$ ___400___
10. $\frac{1}{2}(6)(12)$ ___36___
11. $\frac{1}{3}(3)(12)$ ___12___
12. $\frac{1}{2}(8)^2(4)$ ___128___
13. $\frac{1}{4}(10)^2(15)$ ___375___
14. $\frac{1}{3}(9)^2(6)$ ___162___

372 Unit 5

PROFESSIONAL DEVELOPMENT VIDEO

Author Juli Dixon models successful teaching practices as she explores the Pythagorean Theorem in an actual eighth-grade classroom.

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Access a full suite of teaching resources online—plan, present, and manage classes and assignments.

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Customize answer keys to print or display in the classroom. Choose to include answers only or full solutions to all lesson exercises.

Interactive Whiteboards
Engage students with interactive whiteboard-ready lessons and activities.

Personal Math Trainer: Online Assessment and Intervention
Assign automatically graded homework, quizzes, tests, and intervention activities. Prepare your students with updated practice tests aligned with Common Core.

Reading Start-Up

Have students complete the activities on this page by working alone or with others.

Visualize Vocabulary

The decision tree helps students review vocabulary associated with right triangles. Students should write one review word in each box.

Understand Vocabulary

Use the following explanation to help students learn the preview words.

> Right triangles have special properties. One angle in a right triangle always measures 90°, which is a right angle.
>
> The side of a right triangle that is opposite the right angle, is called the **hypotenuse**. The two sides that form the right angle are called the **legs**. Naming these sides of the triangle can help us understand and talk about a triangle and its properties.

Active Reading

Integrating Language Arts

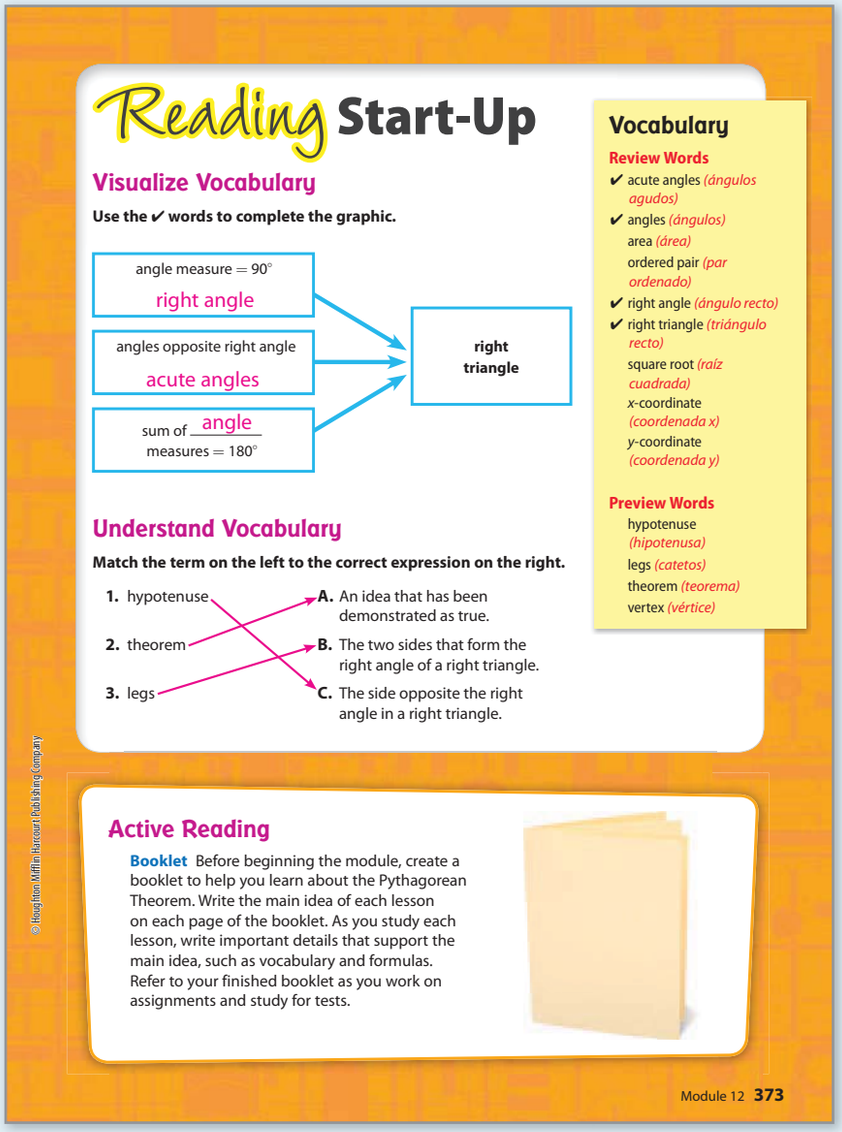
Students can use these reading and note-taking strategies to help them organize and understand new concepts and vocabulary.

COMMON CORE **ELA-Literacy.RST.6-8.7** Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).

Additional Resources

Differentiated Instruction

- Reading Strategies ELL



Before	In this module	After
Students understand: • how to write and solve an equation • how to use exponents and the order of operations • how to graph points on the coordinate plane	Students represent and solve right triangles using the Pythagorean Theorem: • use models and diagrams to explain the Pythagorean Theorem • use the Pythagorean Theorem and its converse to solve problems • determine the distance between two points on a coordinate plane using the Pythagorean Theorem	Students will connect: • right triangles and the Pythagorean triples • sum of the interior angles of a triangle and sum of the interior angles of a polygon

Unpacking the Standards

Use the examples on this page to help students know exactly what they are expected to learn in this module.

Common Core Standards

Content Areas

COMMON CORE **Geometry—8.G**

Understand and apply the Pythagorean Theorem.

Go online to see a complete unpacking of the Common Core Standards.

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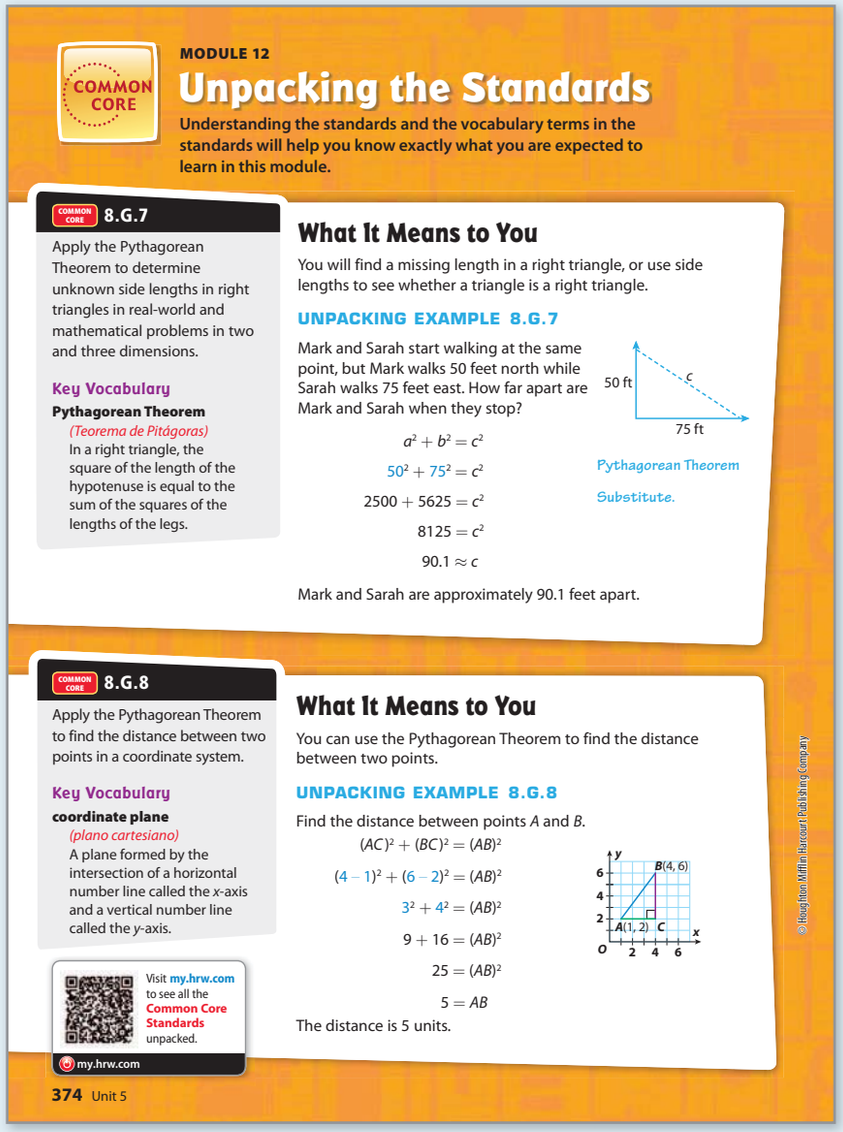

MODULE 12

Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

COMMON CORE **8.G.7**

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Key Vocabulary

Pythagorean Theorem
(Teorema de Pitágoras)
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

What It Means to You

You will find a missing length in a right triangle, or use side lengths to see whether a triangle is a right triangle.

UNPACKING EXAMPLE 8.G.7

Mark and Sarah start walking at the same point, but Mark walks 50 feet north while Sarah walks 75 feet east. How far apart are Mark and Sarah when they stop?

50 ft, c, 75 ft

$a^2 + b^2 = c^2$ Pythagorean Theorem

$50^2 + 75^2 = c^2$ Substitute.

$2500 + 5625 = c^2$

$8125 = c^2$

$90.1 \approx c$

Mark and Sarah are approximately 90.1 feet apart.

COMMON CORE **8.G.8**

Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Key Vocabulary

coordinate plane
(plano cartesiano)
A plane formed by the intersection of a horizontal number line called the *x*-axis and a vertical number line called the *y*-axis.

Visit my.hrw.com to see all the Common Core Standards unpacked.

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What It Means to You

You can use the Pythagorean Theorem to find the distance between two points.

UNPACKING EXAMPLE 8.G.8

Find the distance between points *A* and *B*.

$(AC)^2 + (BC)^2 = (AB)^2$

$(4 - 1)^2 + (6 - 2)^2 = (AB)^2$

$3^2 + 4^2 = (AB)^2$

$9 + 16 = (AB)^2$

$25 = (AB)^2$

$5 = AB$

The distance is 5 units.

Common Core Standards	Lesson 12.1	Lesson 12.2	Lesson 12.3
8.G.6 Explain a proof of the Pythagorean Theorem and its converse.	COMMON CORE	COMMON CORE	
8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	COMMON CORE		
8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.			COMMON CORE

LESSON

12.1 The Pythagorean Theorem

Common Core Standards

The student is expected to:

COMMON CORE **Geometry—8.G.7**

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

COMMON CORE **Geometry—8.G.6**

Explain a proof of the Pythagorean Theorem and its converse.

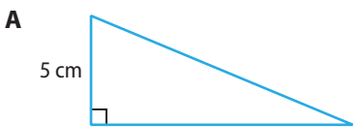
Mathematical Practices

COMMON CORE **MP.5** Using Tools

ADDITIONAL EXAMPLE 1

Find the length of the missing side.

A



B

48 in.

Interactive Whiteboard
Interactive example available online

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Animated Math
The Pythagorean Theorem

Students explore an interactive model of a dynamic proof of the Pythagorean Theorem.

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Engage

ESSENTIAL QUESTION

How can you prove the Pythagorean Theorem and use it to solve problems? Sample answer: You can use the formulas for the area of squares and triangles to prove the Pythagorean Theorem. You can use the Pythagorean Theorem to find the missing lengths of sides of right triangles and to solve real-world problems.

Motivate the Lesson

Ask: If you draw a triangle on a coordinate grid with horizontal and vertical legs and with the vertices at the intersection of grid lines, it is easy to count grid lines to find the length of the legs. How do you find the length of the hypotenuse? Begin the Explore Activity to find out.

Explore

EXPLORE ACTIVITY

Focus on Reasoning

Guide students through the reasoning to see why the unshaded regions of the two congruent squares have the same area. Then guide them to see why $a^2 + b^2 = c^2$.

Explain

EXAMPLE 1

Questioning Strategies CC Mathematical Practices

- How can you identify the hypotenuse of a right triangle? The hypotenuse is the side opposite the right angle.
- In part B, how do you get from $a^2 + 144 = 225$ to $a^2 = 81$? What is the justification for that? You subtract 144 from both sides of the equation. If $a = b$, then $a - c = b - c$.

Focus on Technology CC Mathematical Practices

Support students in using calculators to do the calculations in Example 1, including squaring numbers and taking the square roots of numbers.

Avoid Common Errors

Make sure that students square the side lengths before adding (in part A) or subtracting (in part B). Students may sometimes add or subtract the side lengths and then square the result.

Engage with the Whiteboard

Label the sides of the triangles a, b, and c to help students substitute the values of the side lengths into the formula. Emphasize that it does not matter which leg is a and which leg is b. After completing the problem, write the missing side length on the triangle and point out that the hypotenuse is always the longest side.

LESSON 12.1

The Pythagorean Theorem

COMMON CORE 8.G.7

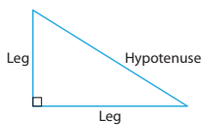
Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. *Also 8.G.6*

ESSENTIAL QUESTION How can you prove the Pythagorean Theorem and use it to solve problems?

EXPLORE ACTIVITY COMMON CORE 8.G.6

Proving the Pythagorean Theorem

In a right triangle, the two sides that form the right angle are the **legs**. The side opposite the right angle is the **hypotenuse**.



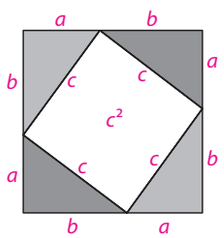
The Pythagorean Theorem

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

If a and b are legs and c is the hypotenuse, $a^2 + b^2 = c^2$.

A Draw a right triangle on a piece of paper and cut it out. Make one leg shorter than the other.

B Trace your triangle onto another piece of paper four times, arranging them as shown. For each triangle, label the shorter leg a, the longer leg b, and the hypotenuse c.



C What is the area of the unshaded square?

c^2 square units

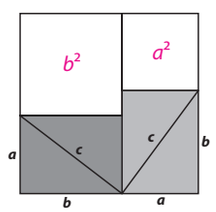
Label the unshaded square with its area.

D Trace your original triangle onto a piece of paper four times again, arranging them as shown. Draw a line outlining a larger square that is the same size as the figure you made in B.

E What is the area of the unshaded square at the top right of the figure in D? at the top left?

a^2 square units; b^2 square units

Label the unshaded squares with their areas.



F What is the total area of the unshaded regions in D?

$a^2 + b^2$ square units

EXPLORE ACTIVITY *(cont'd)*

Reflect

1. Explain whether the figures in B and D have the same area.

Yes, the outlines of the figures are the same size.

2. Explain whether the unshaded regions of the figures in B and D have the same area.

Yes, the shaded regions have the same area. Subtracting the area of the shaded region from the total area gives the same area for the unshaded region in each figure.

3. **Analyze Relationships** Write an equation relating the area of the unshaded region in step B to the unshaded region in D.

$a^2 + b^2 = c^2$

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Using the Pythagorean Theorem

You can use the Pythagorean Theorem to find the length of a side of a right triangle when you know the lengths of the other two sides.

EXAMPLE 1 COMMON CORE 8.G.7

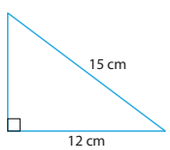
Find the length of the missing side.

A

$a^2 + b^2 = c^2$	
$24^2 + 7^2 = c^2$	Substitute into the formula.
$576 + 49 = c^2$	Simplify.
$625 = c^2$	Add.
$25 = c$	Take the square root of both sides.

The length of the hypotenuse is 25 inches.

B



$a^2 + b^2 = c^2$	
$a^2 + 12^2 = 15^2$	Substitute into the formula.
$a^2 + 144 = 225$	Simplify.
$a^2 = 81$	Use properties of equality to get a^2 by itself.
$a = 9$	Take the square root of both sides.

The length of the leg is 9 centimeters.

Math Talk Mathematical Practices

If you are given the length of the hypotenuse and one leg, does it matter whether you solve for a or b? Explain.

No, the length of the leg can be substituted for either a or b since both a and b represent the lengths of legs.

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices MP.5

This lesson provides an opportunity to address this Mathematical Practices standard. It calls for students to use appropriate tools strategically to solve problems. Students use paper and pencil to create models to prove the Pythagorean Theorem. They go on to solve problems using the Pythagorean Theorem with the aid of number sense to recognize reasonable answers and calculators to determine squares and square roots. They then find the diagonal of a box, an exercise which will be aided by examining a real box.

Math Background

The Pythagorean Theorem can also be proved algebraically as follows:

The area of the largest square in part B of the Explore Activity is $(a + b)^2$. The area of each triangle is $\frac{1}{2}ab$.

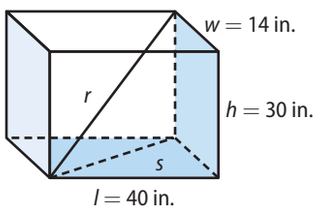
To find the area of the inner white square in terms of a and b, subtract the area of the four triangles from the area of the larger square.

$$c^2 = (a + b)^2 - 4\left(\tfrac{1}{2}ab\right)$$
$$= a^2 + 2ab + b^2 - 2ab$$
$$= a^2 + b^2$$

Therefore, $c^2 = a^2 + b^2$.

ADDITIONAL EXAMPLE 2

A bee is in a box shaped as a rectangular prism. The box measures 30 inches by 14 inches by 40 inches. The bee flies from one corner of the box to the corner that is the farthest away. To the nearest inch, how far does the bee fly?
52 in.



Interactive Whiteboard
Interactive example available online

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YOUR TURN

Avoid Common Errors

Make sure students correctly identify the hypotenuse before applying the Pythagorean Theorem. Students may assume that the two given side lengths are the legs. In Exercise 5, make sure that students substitute 41 for c in $a^2 + b^2 = c^2$ and then solve the equation correctly to find the length of the vertical leg.

EXAMPLE 2

Questioning Strategies CC Mathematical Practices

- How can a box hold an object that is longer than the length of the box? By positioning the object along a diagonal of the box
- Why is it necessary to find the length of the diagonal across the bottom before finding the diagonal from a bottom corner to the opposite top corner? The diagonal from the bottom corner to the opposite top corner is the hypotenuse of a right triangle with only one known side length, the shorter leg. The length of the other leg is the length of the diagonal across the bottom of the box.

Focus on Modeling

Use a box and a yardstick to illustrate to students the locations of the diagonals and their relative lengths. It is easier to show the location of the internal diagonal with a clear plastic box or storage bin, even if the box is an imperfect rectangular prism with rounded edges.

YOUR TURN

Focus on Communication

Students may object that an object such as a part for a desk may be too thick to be able to fit into the corner of a box. Point out that Exercise 6 asks what is the greatest length the part could be and that students have to treat this as a theoretical maximum.

Elaborate

Talk About It

Summarize the Lesson

Have students complete the graphic organizer below.
Let a, b, and c be the side lengths of a right triangle, with c the length of its hypotenuse.

a	*b*	*c*
3	4	5
5	12	13
7	24	25
8	15	17
9	40	41

GUIDED PRACTICE

Engage with the Whiteboard

In Exercise 1, label the sides of the triangle a, b, and c. Either leg can be labeled a or b. After completing the problem, write the missing side length on the triangle.

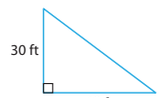
Avoid Common Errors

Exercise 1 Students may forget to find the square root of c^2. Remind them that the hypotenuse is the longest side of the triangle and that $a + b$ must be greater than c.

YOUR TURN

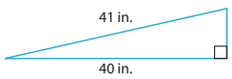
Find the length of the missing side.

4.



50 ft

5.



9 in.

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Pythagorean Theorem in Three Dimensions

You can use the Pythagorean Theorem to solve problems in three dimensions.

Math On the Spot
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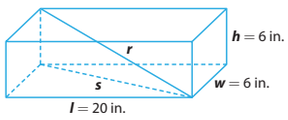
EXAMPLE 2 Real World COMMON CORE 8.G.7

A box used for shipping narrow copper tubes measures 6 inches by 6 inches by 20 inches. What is the length of the longest tube that will fit in the box, given that the length of the tube must be a whole number of inches?



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STEP 1 You want to find r, the length from a bottom corner to the opposite top corner. First, find s, the length of the diagonal across the bottom of the box.

$w^2 + l^2 = s^2$

$6^2 + 20^2 = s^2$ Substitute into the formula.

$36 + 400 = s^2$ Simplify.

$436 = s^2$ Add.

STEP 2 Use your expression for s to find r.

$h^2 + s^2 = r^2$

$6^2 + 436 = r^2$ Substitute into the formula.

$472 = r^2$ Add.

$\sqrt{472} = r$ Take the square root of both sides.

$21.7 \approx r$ Use a calculator to round to the nearest tenth.

The length of the longest tube that will fit in the box is 21 inches.

Math Talk
Mathematical Practices
Looking at Step 2, why did the calculations in Step 1 stop before taking the square root of both sides of the final equation?

By stopping with an expression for s^2, it could be substituted directly into the formula for r^2.

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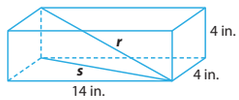
YOUR TURN

6. Tina ordered a replacement part for her desk. It was shipped in a box that measures 4 in. by 4 in. by 14 in. What is the greatest length in whole inches that the part could have been?



$4^2 + 14^2 = s^2$ $4^2 + 212 = r^2$

$16 + 196 = s^2$ $228 = r^2$

$212 = s^2$ $\sqrt{228} = r$

$15.1 \approx r$

The greatest length is 15 in.

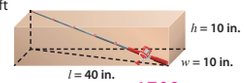
Guided Practice

1. Find the length of the missing side of the triangle. (Explore Activity 1 and Example 1)

$a^2 + b^2 = c^2 \rightarrow 24^2 + \boxed{10^2} = c^2 \rightarrow \boxed{676} = c^2$

The length of the hypotenuse is $\boxed{26}$ feet.

2. Mr. Woo wants to ship a fishing rod that is 42 inches long to his son. He has a box with the dimensions shown. (Example 2)



a. Find the square of the length of the diagonal across the bottom of the box.

1700

b. Find the length from a bottom corner to the opposite top corner to the nearest tenth. Will the fishing rod fit?

42.4 in.; yes

ESSENTIAL QUESTION CHECK-IN

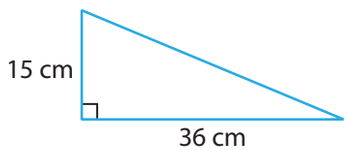
3. State the Pythagorean Theorem and tell how you can use it to solve problems.

Sample answer: For a right triangle with legs of lengths a and b and hypotenuse of length c, $a^2 + b^2 = c^2$. You can use it to find the length of a side of a right triangle when the lengths of the other two sides are known.

DIFFERENTIATE INSTRUCTION

Communicating Math

Have students write out a summary of the steps they would use to find (or approximate) the missing length of 39 cm for the triangle below.



Manipulatives

Have groups of students carefully draw several right triangles, using a protractor or angle template to draw the right angles. Then have them swap triangles and measure the lengths of the sides to the nearest millimeter. Have them verify the Pythagorean relationship between the side lengths of the right triangles. (Because the measurements are approximate, $a^2 + b^2$ may be close to but not exactly c^2 in some cases.)

Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners ELL
- Reteach
- Challenge PRE-AP

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Online Assessment and Intervention

Online homework assignment available

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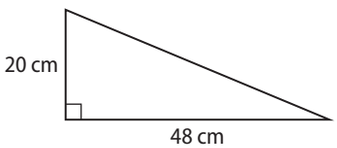
12.1 LESSON QUIZ

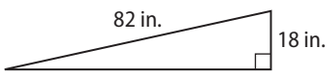
COMMON CORE **8.G.6, 8.G.7**

Find the length of the missing side of each triangle.

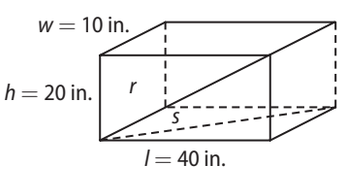
1.



2.



3. A box used for shipping a volleyball set measures 10 inches by 20 inches by 40 inches. What is the longest length of support pole that will fit into the box, rounded to a tenth of an inch?



Lesson Quiz available online

my.hrw.com

Answers

1. 52 cm

2. 80 in.

3. 45.8 in.

Evaluate

GUIDED AND INDEPENDENT PRACTICE

COMMON CORE **8.G.6, 8.G.7**

Concepts & Skills	Practice
Explore Activity Proving the Pythagorean Theorem	Exercise 1
Example 1 Using the Pythagorean Theorem	Exercises 1, 4–8, 10–11
Example 2 Pythagorean Theorem in Three Dimensions	Exercises 2, 9

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
4–5	**2** Skills/Concepts	**MP.5** Using Tools
6–10	**2** Skills/Concepts	**MP.4** Modeling
11–12	**3** Strategic Thinking H.O.T.	**MP.4** Modeling
13	**3** Strategic Thinking H.O.T.	**MP.3** Logic
14	**3** Strategic Thinking H.O.T.	**MP.1** Problem Solving

Additional Resources

Differentiated Instruction includes:

- Leveled Practice worksheets

Name _____ Class _____ Date _____

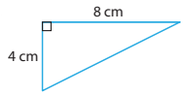
12.1 Independent Practice

COMMON CORE 8.G.6, 8.G.7

Personal Math Trainer — Online Assessment and Intervention — my.hrw.com

Find the length of the missing side of each triangle. Round your answers to the nearest tenth.

4.



8.9 cm

5.

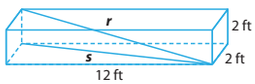
11.5 in.

6. The diagonal of a rectangular big-screen TV screen measures 152 cm. The length measures 132 cm. What is the height of the screen? 75.4 cm

7. Dylan has a square piece of metal that measures 10 inches on each side. He cuts the metal along the diagonal, forming two right triangles. What is the length of the hypotenuse of each right triangle to the nearest tenth of an inch? 14.1 in.

8. **Represent Real-World Problems** A painter has a 24-foot ladder that he is using to paint a house. For safety reasons, the ladder must be placed at least 8 feet from the base of the side of the house. To the nearest tenth of a foot, how high can the ladder safely reach? 22.6 ft

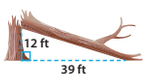
9. What is the longest flagpole (in whole feet) that could be shipped in a box that measures 2 ft by 2 ft by 12 ft? 12 feet



10. **Sports** American football fields measure 100 yards long between the end zones, and are $53\frac{1}{3}$ yards wide. Is the length of the diagonal across this field more or less than 120 yards? Explain.

Less than; $\sqrt{12844} \cong 113$

11. **Justify Reasoning** A tree struck by lightning broke at a point 12 ft above the ground as shown. What was the height of the tree to the nearest tenth of a foot? Explain your reasoning.



52.8 ft; $12^2 + 39^2 = c^2$, so $144 + 1521 = c^2$, $1665 = c^2$, and $40.8 \approx c$. Add this length to the height of the bottom of the tree: $40.8 + 12 = 52.8$ ft.

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

12. **Multistep** Main Street and Washington Avenue meet at a right angle. A large park begins at this corner. Joe's school lies at the opposite corner of the park. Usually Joe walks 1.2 miles along Main Street and then 0.9 miles up Washington Avenue to get to school. Today he walked in a straight path across the park and returned home along the same path. What is the difference in distance between the two round trips? Explain.

1.2 mi; today Joe walked $\sqrt{1.2^2 + 0.9^2} = 1.5$ mi each way. He usually walks $1.2 + 0.9 = 2.1$ mi each way. So the difference is $2.1 - 1.5 = 0.6$ mi each way, or 1.2 mi.

13. **Analyze Relationships** An isosceles right triangle is a right triangle with congruent legs. If the length of each leg is represented by x, what algebraic expression can be used to represent the length of the hypotenuse? Explain your reasoning.

$\sqrt{x^2 + x^2}$ (or $\sqrt{2x^2}$ or $x\sqrt{2}$); if $a = x$ and $b = x$, then $x^2 + x^2 = c^2$. Thus, $c = \sqrt{x^2 + x^2}$.

14. **Persevere in Problem Solving** A square hamburger is centered on a circular bun. Both the bun and the burger have an area of 16 square inches.

a. How far, to the nearest hundredth of an inch, does each corner of the burger stick out from the bun? Explain.

0.57 in.; side of burger is 4 in., so distance from center to side is $4 \div 2 = 2$ in. The distance from center of burger to corner is $2\sqrt{2} \approx 2.83$ in. The bun radius is $\sqrt{\frac{16}{\pi}} \approx 2.26$ in. The burger corner extends $2.83 - 2.26 \approx 0.57$ in.

b. How far does each bun stick out from the center of each side of the burger?

about $2.26 - 2 = 0.26$ in.

c. Are the distances in part **a** and part **b** equal? If not, which sticks out more, the burger or the bun? Explain.

No; the burger sticks out a little more than a half inch, and the bun sticks out about a quarter inch, so the corners of the burger stick out more.

EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity A common right triangle is called the "3–4–5 triangle." This set of numbers is called a "primitive Pythagorean triple" because the side lengths have no factors other than 1 in common. If you multiply each length in a Pythagorean triple by a whole number like 2, 3, or 4, you will get another Pythagorean triple. So, 6–8–10, 9–12–15, and 12–16–20 are also Pythagorean triples. Find as many Pythagorean triples as you can. Remember—once you find a primitive triple, find whole-number multiples of the triple to get other triples! Hint: There are 50 Pythagorean triples where each length is less than 100, with 16 of them primitive triples.

Primitive Pythagorean Triples Less Than 100	
3–4–5	16–63–65
5–12–13	20–21–29
8–15–17	28–45–53
7–24–25	33–56–65
9–40–41	36–77–85
11–60–61	39–80–89
12–35–37	48–55–73
13–84–85	65–72–97

LESSON

12.2 Converse of the Pythagorean Theorem

Common Core Standards

The student is expected to:

COMMON CORE **Geometry—8.G.6**

Explain a proof of the Pythagorean Theorem and its converse.

Mathematical Practices

COMMON CORE **MP.7** Structure

Engage

ESSENTIAL QUESTION

How can you test the converse of the Pythagorean Theorem and use it to solve problems?
Sample answer: Test whether triangles whose side lengths are a, b, and c satisfy $a^2 + b^2 = c^2$. You can use the converse of the Pythagorean Theorem to help you determine whether real-world triangles are right triangles.

Motivate the Lesson

Ask: How can you tell whether a triangle is a right triangle? Make a conjecture. Begin the Explore Activity to find out.

Explore

EXPLORE ACTIVITY

Talk About It

Check for Understanding

Ask: How do you know if a triangle is a right triangle? If the sum of the squares of the lengths of the two shorter sides of a triangle is equal to the square of the length of the longest side, then the triangle is a right triangle.

Explain

EXAMPLE 1

Questioning Strategies CC Mathematical Practices

- In part A, how do you know which length should be the value of c in the equation $a^2 + b^2 = c^2$? The longest side length is always the value of c.
- Does the converse of the Pythagorean Theorem only apply to triangles with rational numbers as side lengths? No; the side lengths can be irrational numbers.

Focus on Math Connections CC Mathematical Practices

Point out that if $a^2 + b^2 > c^2$ or if $a^2 + b^2 < c^2$, then $a^2 + b^2 \neq c^2$, and the triangle made by the side lengths a, b, and c is not a right triangle. For part B of this example, $a^2 + b^2 > c^2$.

Avoid Common Errors

Students may be confused about when to use the Pythagorean Theorem and when to use its converse. Make sure that students understand that if they want to determine whether a triangle is a right triangle and have all three side lengths, they should use the converse of the Pythagorean Theorem. If they want to find a missing side length of a right triangle, they should use the Pythagorean Theorem.

YOUR TURN

Focus on Math Connections CC Mathematical Practices

Since the side lengths of the triangle in Exercise 3 have a common factor of 2, $16^2 + 30^2 = 34^2$ can be rewritten as $(2 \cdot 8)^2 + (2 \cdot 15)^2 = (2 \cdot 17)^2$. The converse of the Pythagorean Theorem shows that side lengths of 8 in., 15 in., and 17 in. also form a right triangle.

ADDITIONAL EXAMPLE 1

Tell whether each triangle with the given side lengths is a right triangle.

A 16 inches, 30 inches, 34 inches yes

B 14 feet, 49 feet, 51 feet no

Interactive Whiteboard
Interactive example available online

my.hrw.com

LESSON 12.2

Converse of the Pythagorean Theorem

COMMON CORE 8.G.6

Explain a proof of the Pythagorean Theorem and its converse.

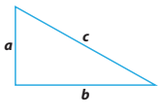
ESSENTIAL QUESTION How can you test the converse of the Pythagorean Theorem and use it to solve problems?

EXPLORE ACTIVITY COMMON CORE 8.G.6

Testing the Converse of the Pythagorean Theorem

The Pythagorean Theorem states that if a triangle is a right triangle, then $a^2 + b^2 = c^2$.

The *converse* of the Pythagorean Theorem states that if $a^2 + b^2 = c^2$, then the triangle is a right triangle.



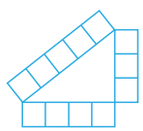
Decide whether the converse of the Pythagorean Theorem is true.

A Verify that the following sets of lengths make the equation $a^2 + b^2 = c^2$ true. Record your results in the table.

a	b	c	Is $a^2 + b^2 = c^2$ true?	Makes a right triangle?
3	4	5	yes	yes
5	12	13	yes	yes
7	24	25	yes	yes
8	15	17	yes	yes
20	21	29	yes	yes

B For each set of lengths in the table, cut strips of grid paper with a width of one square and lengths that correspond to the values of *a*, *b*, and *c*.

C For each set of lengths, use the strips of grid paper to try to form a right triangle. An example using the first set of lengths is shown. Record your findings in the table.



Reflect

1. **Draw Conclusions** Based on your observations, explain whether you think the converse of the Pythagorean Theorem is true.

Students should note that each set of lengths satisfies $a^2 + b^2 = c^2$ and forms a right triangle, and conclude that the converse is true.

Math On the Spot my.hrw.com

Identifying a Right Triangle

The converse of the Pythagorean Theorem gives you a way to tell if a triangle is a right triangle when you know the side lengths.

EXAMPLE 1 COMMON CORE 8.G.6

Tell whether each triangle with the given side lengths is a right triangle.

A 9 inches, 40 inches, and 41 inches

Let $a = 9$, $b = 40$, and $c = 41$.

$a^2 + b^2 = c^2$

$9^2 + 40^2 \stackrel{?}{=} 41^2$ Substitute into the formula.

$81 + 1600 \stackrel{?}{=} 1681$ Simpify.

$1681 = 1681$ Add.

Since $9^2 + 40^2 = 41^2$, the triangle is a right triangle by the converse of the Pythagorean Theorem.

B 8 meters, 10 meters, and 12 meters

Let $a = 8$, $b = 10$, and $c = 12$.

$a^2 + b^2 = c^2$

$8^2 + 10^2 \stackrel{?}{=} 12^2$ Substitute into the formula.

$64 + 100 \stackrel{?}{=} 144$ Simpify.

$164 \neq 144$ Add.

Since $8^2 + 10^2 \neq 12^2$, the triangle is not a right triangle by the converse of the Pythagorean Theorem.

YOUR TURN

Tell whether each triangle with the given side lengths is a right triangle.

2. 14 cm, 23 cm, and 25 cm — not a right triangle

3. 16 in., 30 in., and 34 in. — a right triangle

4. 27 ft, 36 ft, 45 ft — right triangle

5. 11 mm, 18 mm, 21 mm — not a right triangle

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PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices MP.7

This lesson provides an opportunity to address this Mathematical Practices standard. It calls for students to look for structure. Students determine whether triangle side lengths fulfill the Pythagorean Theorem. In Example 2 and in many of the exercises, students determine whether triangles in real-world situations are right triangles.

Math Background

Previously, students learned that if a triangle is a right triangle, then $a^2 + b^2 = c^2$, where *a*, *b*, and *c* are side lengths and *c* is the longest side. In this lesson, students learn that the converse of this statement is also true. When a conditional statement and its converse are both true, then the two statements can be combined to form a *biconditional* statement. For the Pythagorean Theorem and its converse, the biconditional statement can be stated this way: a triangle is a right triangle if and only if $a^2 + b^2 = c^2$, where *a*, *b*, and *c* are side lengths and *c* is the longest side.

ADDITIONAL EXAMPLE 2

A small triangular park at the intersection of 3 streets has side lengths 19 feet, 80 feet, and 82 feet. Does the park have the shape of a right triangle? Explain. No; $19^2 + 80^2 \neq 82^2$, since $a^2 + b^2 \neq c^2$, the park is not the shape of a right triangle.

Interactive Whiteboard
Interactive example available online

my.hrw.com

EXAMPLE 2

Questioning Strategies CC Mathematical Practices

- If the shortest edges of the garden must be 10 feet and 7 feet and the shape must be a right triangle, how long will the longest side be? 12.2 ft
- If the longest side of the garden must be 13 feet and the shape must be a right triangle, what are possible lengths for the other two sides? Sample answer: 5 ft and 12 ft

Engage with the Whiteboard

Have students sketch the triangle described in Example 2 in the margin of the page and then label the sides with the side lengths. Finally, label the sides *a*, *b*, and *c*.

YOUR TURN

Focus on Technology CC Mathematical Practices

Use geometry software to create the triangles in Exercises 6 and 7. Use the measurement tools of the software to find out if the triangle has an angle of 90°.

Elaborate

Talk About It

Summarize the Lesson

Ask: How can you use the converse of the Pythagorean Theorem to classify a triangle as a right triangle or not a right triangle? Find the sum of the squares of the two shortest sides of a triangle. If the sum is equal to the square of the longest side, the triangle must be a right triangle. Otherwise, it is not.

GUIDED PRACTICE

Engage with the Whiteboard

Have students sketch the triangles described in Exercises 2–3 in the margin and then label the sides with the side lengths. Then have them label the sides *a*, *b*, and *c*.

Avoid Common Errors

Exercises 1–3 Remind students to let *c* be the length of the longest side of the triangle. Also remind students to square the side lengths before adding.

Using the Converse of the Pythagorean Theorem

You can use the converse of the Pythagorean Theorem to solve real-world problems.

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EXAMPLE 2

Real World

COMMON CORE 8.G.6

Katya is buying edging for a triangular flower garden she plans to build in her backyard. If the lengths of the three pieces of edging that she purchases are 13 feet, 10 feet, and 7 feet, will the flower garden be in the shape of a right triangle?

Use the converse of the Pythagorean Theorem. Remember to use the longest length for c.

Let $a = 7$, $b = 10$, and $c = 13$.

$a^2 + b^2 = c^2$

$7^2 + 10^2 \stackrel{?}{=} 13^2$ Substitute into the formula.

$49 + 100 \stackrel{?}{=} 169$ Simplify.

$149 \neq 169$ Add.

Since $7^2 + 10^2 \neq 13^2$, the garden will not be in the shape of a right triangle.

Math Talk
Mathematical Practices
To what length, to the nearest tenth, can Katya trim the longest piece of edging to form a right triangle?

approximately 12.2 feet

YOUR TURN

6. A blueprint for a new triangular playground shows that the sides measure 480 ft, 140 ft, and 500 ft. Is the playground in the shape of a right triangle? Explain.

Yes; $140^2 + 480^2 = 250{,}000$; $500^2 = 250{,}000$; $250{,}000 = 250{,}000$

7. A triangular piece of glass has sides that measure 18 in., 19 in., and 25 in. Is the piece of glass in the shape of a right triangle? Explain.

No; $18^2 + 19^2 = 685$, $25^2 = 625$, $685 \neq 625$

8. A corner of a fenced yard forms a right angle. Can you place a 12 foot long board across the corner to form a right triangle for which the leg lengths are whole numbers? Explain.

No; there are no pairs of whole numbers whose squares add to $12^2 = 144$.

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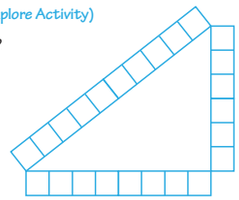
Guided Practice

1. Lashandra used grid paper to construct the triangle shown. (Explore Activity)



a. What are the lengths of the sides of Lashandra's triangle?

6 units, 8 units, 10 units

b. Use the converse of the Pythagorean Theorem to determine whether the triangle is a right triangle.

$a^2 + b^2 = c^2$

$6^2 + 8^2 \stackrel{?}{=} 10^2$

$36 + 64 \stackrel{?}{=} 100$

$100 \stackrel{?}{=} 100$

The triangle that Lashandra constructed (is) / is not a right triangle.

2. A triangle has side lengths 9 cm, 12 cm, and 16 cm. Tell whether the triangle is a right triangle. (Example 1)

Let $a =$ 9, $b =$ 12, and $c =$ 16.

$a^2 + b^2 = c^2$

$9^2 + 12^2 \stackrel{?}{=} 16^2$

$81 + 144 \stackrel{?}{=} 256$

$225 \stackrel{?}{=} 256$

By the converse of the Pythagorean Theorem, the triangle is / (is not) a right triangle.

3. The marketing team at a new electronics company is designing a logo that contains a circle and a triangle. On one design, the triangle's side lengths are 2.5 in., 6 in., and 6.5 in. Is the triangle a right triangle? Explain. (Example 2)

Yes; $2.5^2 + 6^2 = 42.25$, $6.5^2 = 42.25$, $42.25 = 42.25$

ESSENTIAL QUESTION CHECK-IN

4. How can you use the converse of the Pythagorean Theorem to tell if a triangle is a right triangle?

Test the side lengths in $a^2 + b^2 = c^2$, using the longest side for c. If the equation is true, the triangle is a right triangle. Otherwise, it isn't.

DIFFERENTIATE INSTRUCTION

Cooperative Learning

Have pairs of students create one real-world problem that can be solved using the Pythagorean Theorem and one real-world problem that can be solved using the converse of the Pythagorean Theorem. Have students find and justify their solutions.

Manipulatives

Have students make two triangles on a geoboard, one that is a right triangle and one that is not a right triangle. Then have them measure the lengths of the sides of each triangle to the nearest millimeter. Ask them to use the converse of the Pythagorean Theorem to verify that the right triangle is indeed a right triangle and the other triangle is not a right triangle. (Because the measurements are approximate, for the right triangle, $a^2 + b^2$ and c^2 may be close but not exactly the same.)

Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners ELL
- Reteach
- Challenge PRE-AP

Personal Math Trainer

Online Assessment and Intervention

Online homework assignment available

my.hrw.com

12.2 LESSON QUIZ

COMMON CORE **8.G.6**

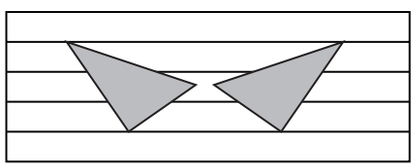
Tell whether each triangle with the given side lengths is a right triangle.

1. 36 cm, 48 cm, 60 cm

2. 12 ft, 35 ft, 37 ft

3. 60.5 ft, 63 ft, 87.5 ft

4. A club at school designed a banner consisting of two congruent triangles surrounded by stripes. The lengths of the sides of each of the triangles were 1.5 feet, 2.0 feet, and 2.5 feet. Are the triangles right triangles? Explain.



Lesson Quiz available online

my.hrw.com

Answers

1. right triangle

2. right triangle

3. not a right triangle

4. Yes; $1.5^2 + 2.0^2 = 6.25 = 2.5^2$

Evaluate

GUIDED AND INDEPENDENT PRACTICE

COMMON CORE **8.G.6**

Concepts & Skills	Practice
Explore Activity Testing the Converse of the Pythagorean Theorem	Exercise 1
Example 1 Identifying a Right Triangle	Exercises 2, 5–14
Example 2 Using the Converse of the Pythagorean Theorem	Exercises 3, 15–17

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
5–14	**2** Skills/Concepts	**MP.5** Using Tools
15–17	**2** Skills/Concepts	**MP.4** Modeling
18–19	**3** Strategic Thinking H.O.T.	**MP.2** Reasoning
20–21	**3** Strategic Thinking H.O.T.	**MP.3** Logic
22	**3** Strategic Thinking H.O.T.	**MP.2** Reasoning
23	**3** Strategic Thinking H.O.T.	**MP.6** Precision

Additional Resources

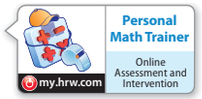
Differentiated Instruction includes:

- Leveled Practice worksheets

Name _____ Class _____ Date _____

12.2 Independent Practice

COMMON CORE 8.G.6



Tell whether each triangle with the given side lengths is a right triangle.

5. 11 cm, 60 cm, 61 cm
right triangle

6. 5 ft, 12 ft, 15 ft
not a right triangle

7. 9 in., 15 in., 17 in.
not a right triangle

8. 15 m, 36 m, 39 m
right triangle

9. 20 mm, 30 mm, 40 mm
not a right triangle

10. 20 cm, 48 cm, 52 cm
right triangle

11. 18.5 ft, 6 ft, 17.5 ft
right triangle

12. 2 mi, 1.5 mi, 2.5 mi
right triangle

13. 35 in., 45 in., 55 in.
not a right triangle

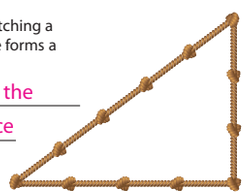
14. 25 cm, 14 cm, 23 cm
not a right triangle

15. The emblem on a college banner consists of the face of a tiger inside a triangle. The lengths of the sides of the triangle are 13 cm, 14 cm, and 15 cm. Is the triangle a right triangle? Explain.
No; $13^2 + 14^2 = 365$, $15^2 = 225$, and $365 \neq 225$.

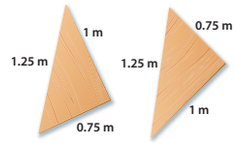
16. Kerry has a large triangular piece of fabric that she wants to attach to the ceiling in her bedroom. The sides of the piece of fabric measure 4.8 ft, 6.4 ft, and 8 ft. Is the fabric in the shape of a right triangle? Explain.
Yes; $4.8^2 + 6.4^2 = 64$, and $8^2 = 64$.

17. A mosaic consists of triangular tiles. The smallest tiles have side lengths 6 cm, 10 cm, and 12 cm. Are these tiles in the shape of right triangles? Explain.
No; $6^2 + 10^2 = 136$, $12^2 = 144$, and $136 \neq 144$.

18. **History** In ancient Egypt, surveyors made right angles by stretching a rope with evenly spaced knots as shown. Explain why the rope forms a right angle.
Sample answer: The knots are evenly spaced, so the side lengths are 3 units, 4 units, and 5 units. Since $3^2 + 4^2 = 5^2$, the sides form a right triangle.



19. **Justify Reasoning** Yoshi has two identical triangular boards as shown. Can he use these two boards to form a rectangle? Explain.
Yes; since $0.75^2 + 1^2 = 1.25^2$, the triangles are right triangles. Adjoining them at their hypotenuses will form a rectangle with sides 1 m and 0.75 m.



20. **Critique Reasoning** Shoshanna says that a triangle with side lengths 17 m, 8 m, and 15 m is not a right triangle because $17^2 + 8^2 = 353$, $15^2 = 225$, and $353 \neq 225$. Is she correct? Explain.
No, she did not use the longest length for *c*.
$8^2 + 15^2 = 17^2$, so it is a right triangle.

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

21. **Make a Conjecture** Diondre says that he can take any right triangle and make a new right triangle just by doubling the side lengths. Is Diondre's conjecture true? Test his conjecture using three different right triangles.
Yes. His conjecture is true. Students' work will vary but should show that they've used the converse of the Pythagorean Theorem to test whether the new triangles are right triangles.

22. **Draw Conclusions** A diagonal of a parallelogram measures 37 inches. The sides measure 35 inches and 1 foot. Is the parallelogram a rectangle? Explain your reasoning.
Yes; 1 ft = 12 in. Since $12^2 + 35^2 = 37^2$, each half of the parallelogram is a right triangle. Therefore, the sides of the parallelogram meet at right angles, making the parallelogram a rectangle.

23. **Represent Real-World Problems** A soccer coach is marking the lines for a soccer field on a large recreation field. The dimensions of the field are to be 90 yards by 48 yards. Describe a procedure she could use to confirm that the sides of the field meet at right angles.
She could measure the diagonal of the field to see if the sides of the field and the diagonal form a right triangle. The diagonal should measure 102 yards if the sides of the field meet at right angles.

EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity Draw three different acute triangles (triangles with all three angles less than 90°) and three different obtuse triangles (triangles with one angle greater than 90°). Label the longest side of each triangle *c* and the other two sides *a* and *b*. Measure each side of each triangle to the nearest millimeter and record the results in a table like the one shown here. Calculate a^2, b^2, and c^2 for each triangle. Compare c^2 to $a^2 + b^2$ for each triangle. Make a conjecture about the type of triangle that has $a^2 + b^2 > c^2$ and about the type of triangle that has $a^2 + b^2 < c^2$. Use your conjecture to predict whether a triangle with side lengths 12 cm, 60 cm, and 61 cm is acute, right, or obtuse.

Type of triangle	*a*	*b*	$a^2 + b^2$	*c*	c^2	$a^2 + b^2$ greater than or less than c^2
acute						
acute						
acute						
obtuse						
obtuse						
obtuse						

If the triangle is acute, then $a^2 + b^2 > c^2$. If the triangle is obtuse, then $a^2 + b^2 < c^2$; acute

LESSON

12.3 Distance Between Two Points

Common Core Standards

The student is expected to:

COMMON CORE **Geometry—8.G.8**

Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Mathematical Practices

COMMON CORE **MP.2** Reasoning

Engage

ESSENTIAL QUESTION

How can you use the Pythagorean Theorem to find the distance between two points on a coordinate plane? Sample answer: Draw a segment connecting the two points and a right triangle with that segment as the hypotenuse. Then use the Pythagorean Theorem to find the length of the hypotenuse.

Motivate the Lesson

Ask: How would you verify, without measuring, that three points on a coordinate plane form a right triangle? Take a guess.

Explore

To find the slope of a line on a coordinate plane, you can draw a horizontal line to represent the run and a vertical line to represent the rise. These two lines and the original line form a right triangle. Can you use the Pythagorean Theorem to find the distance between the two points on the hypotenuse? See also Explore Activity in student text.

Explain

EXAMPLE 1

Questioning Strategies CC Mathematical Practices

- What is the rise and run between the endpoints of the hypotenuse? rise = 4; run = 2
- What is the slope of the line segment that is the hypotenuse? 2

Focus on Math Connections CC Mathematical Practices

Remind students that they were drawing right triangles on the coordinate plane when they drew lines representing the rise and run between two points. Point out that the legs of the right triangle represent the rise and run between the endpoints of the hypotenuse.

YOUR TURN

Engage with the Whiteboard

Label the lengths of the legs. Label one leg *a* and the other leg *b* before substituting the values into the Pythagorean Theorem.

EXPLORE ACTIVITY

Questioning Strategies CC Mathematical Practices

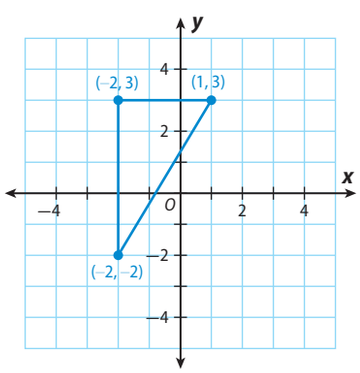
- In the Distance Formula, what does $x_2 - x_1$ represent? the length of the horizontal leg
- The variable *c* in the Pythagorean Theorem is represented by what variable in the Distance Formula? *d*

Focus on Math Connections CC Mathematical Practices

Point out to students that finding the length of the hypotenuse of a right triangle is the same as finding the distance between the endpoints of the hypotenuse in the coordinate plane.

ADDITIONAL EXAMPLE 1

The figure shows a right triangle. Approximate the length of the hypotenuse to the nearest tenth using a calculator. about 5.8 units



Interactive Whiteboard
Interactive example available online

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LESSON 12.3

Distance Between Two Points

COMMON CORE 8.G.8

Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

ESSENTIAL QUESTION How can you use the Pythagorean Theorem to find the distance between two points on a coordinate plane?

Pythagorean Theorem in the Coordinate Plane

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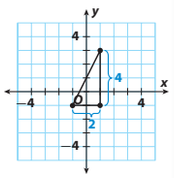
EXAMPLE 1

COMMON CORE 8.G.8

The figure shows a right triangle. Approximate the length of the hypotenuse to the nearest tenth using a calculator.



STEP 1 Find the length of each leg.

The length of the vertical leg is 4 units.

The length of the horizontal leg is 2 units.

STEP 2 Let $a = 4$ and $b = 2$. Let c represent the length of the hypotenuse. Use the Pythagorean Theorem to find c.

$a^2 + b^2 = c^2$

$4^2 + 2^2 = c^2$ Substitute into the formula.

$20 = c^2$ Add.

$\sqrt{20} = c$ Take the square root of both sides.

$\sqrt{20} \approx 4.5$ Use a calculator and round to the nearest tenth.

STEP 3 Check for reasonableness by finding perfect squares close to 20.

$\sqrt{20}$ is between $\sqrt{16}$ and $\sqrt{25}$, or $\sqrt{16} < \sqrt{20} < \sqrt{25}$.

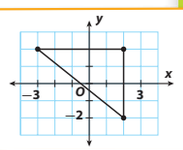
Simplifying gives $4 < \sqrt{20} < 5$.

Since 4.5 is between 4 and 5, the answer is reasonable.

The hypotenuse is about 4.5 units long.

YOUR TURN

1. Approximate the length of the hypotenuse to the nearest tenth using a calculator.



6.4 units

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EXPLORE ACTIVITY COMMON CORE 8.G.8

Finding the Distance Between Any Two Points

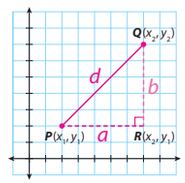
The Pythagorean Theorem can be used to find the distance between any two points (x_1, y_1) and (x_2, y_2) in the coordinate plane. The resulting expression is called the Distance Formula.

Distance Formula

In a coordinate plane, the distance d between two points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Use the Pythagorean Theorem to derive the Distance Formula.



A To find the distance between points P and Q, draw segment $\overline{PQ}$ and label its length d. Then draw horizontal segment $\overline{PR}$ and vertical segment $\overline{QR}$. Label the lengths of these segments a and b. Triangle PQR is a right triangle, with hypotenuse $\overline{PQ}$.

B Since $\overline{PR}$ is a horizontal segment, its length, a, is the difference between its x-coordinates. Therefore, $a = x_2 - x_1$.

C Since $\overline{QR}$ is a vertical segment, its length, b, is the difference between its y-coordinates. Therefore, $b = y_2 - y_1$.

D Use the Pythagorean Theorem to find d, the length of segment $\overline{PQ}$. Substitute the expressions from **B** and **C** for a and b.

$d^2 = a^2 + b^2$

$d = \sqrt{a^2 + b^2}$

$$d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$$

Math Talk
Mathematical Practices
What do $x_2 - x_1$ and $y_2 - y_1$ represent in terms of the Pythagorean Theorem?

They represent the lengths of the legs of the right triangle formed by P, Q, and R.

Reflect

2. Why are the coordinates of point R the ordered pair (x_2, y_1)?

Since R lies on the same vertical line as Q, their x-coordinates are the same. Since R lies on the same horizontal line as P, their y-coordinates are the same.

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices MP.2

This lesson provides an opportunity to address this Mathematical Practices standard. It calls for students to reason abstractly and quantitatively. Students connect the Pythagorean Theorem with finding the distance between two points in the coordinate plane and then derive the Distance Formula.

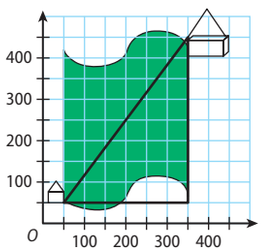
Math Background

The Distance Formula presented here is an application of the Pythagorean Theorem in two-dimensional space. In Lesson 8.1, the Pythagorean Theorem was used in two sequential operations to find the distance between opposite corners of a box of known length, width, and height. Points on a box or rectangular prism can be represented in a three-dimensional coordinate space using the (x, y, z) coordinate system. The distance between two points in three-dimensional space can be found by extending the Distance Formula to the (x, y, z) system:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

ADDITIONAL EXAMPLE 2

Dominic wants to find the distance between his house on one side of the park and his school on the other side. He marks off a third point forming a right triangle, as shown in the diagram. The distances in the diagram are measured in yards. Use the Pythagorean Theorem to find the distance from Dominic's house to the school. 500 yd



Interactive Whiteboard
Interactive example available online

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EXAMPLE 2

Questioning Strategies CC Mathematical Practices

- Why is $|280 - 10| = 270$? because the absolute value of a positive number is the number itself
- What would you get if you found $|10 - 280|$? Is it the same as $|280 - 10|$? Explain. 270; yes; because the absolute value of a negative number is the additive inverse (opposite) of the number.

Avoid Common Errors

The absolute value of the difference between coordinates is used here to emphasize that the length of a segment, or the distance between two points, is a positive number. If students subtract the larger coordinate from the smaller coordinate they may be confused by the negative number. Tell students to use the absolute value of the difference.

Integrating Language Arts ELL

Encourage a broad class discussion on the Math Talk. English learners will benefit from hearing and participating in classroom discussions.

YOUR TURN

Connect to Daily Life

Discuss with students why in real life the distance across the lake might be more easily calculated from the length of the legs of the triangle instead of directly. The distance over land can be measured with an odometer or a GPS device, or estimated by counting steps and measuring the length of a step. Only a GPS device could be used while crossing the lake, and only if a boat is available. Remind students that they have already learned another method to measure the length of inaccessible areas using similar triangles.

Elaborate

Talk About It

Summarize the Lesson

Ask: What are two ways you can find the distance between points (x_1, y_1) and (x_2, y_2) in the coordinate plane? Draw a segment connecting the two points and complete a right triangle with the segment as the hypotenuse. Then find the lengths of the sides of the triangle and use the Pythagorean Theorem to find the length of the hypotenuse. You can also find the distance between the two points directly by using the Distance Formula.

GUIDED PRACTICE

Engage with the Whiteboard

Sketch a graph for Exercise 2. Label the points, and then add vertical and horizontal segments that connect at (15, 7). Label the legs of the triangle with their length (horizontal leg is 12 units long; vertical leg is 5 units long).

Avoid Common Errors

Exercise 2 Watch for students who confuse the x- and y-coordinates. Have them write (x_1, y_1) and (x_2, y_2) over the given coordinates to help them substitute the correct values into the Distance Formula.

Exercise 3 Some students may be unsure what is meant by north and east. Help them to interpret the diagram.

Finding the Distance Between Two Points

The Pythagorean Theorem can be used to find the distance between two points in a real-world situation. You can do this by using a coordinate grid that overlays a diagram of the real-world situation.

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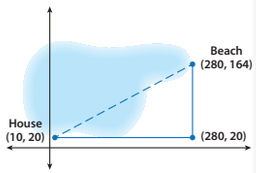
EXAMPLE 2

Real World

COMMON CORE 8.G.8

Francesca wants to find the distance between her house on one side of a lake and the beach on the other side. She marks off a third point forming a right triangle, as shown. The distances in the diagram are measured in meters.



Use the Pythagorean Theorem to find the straight-line distance from Francesca's house to the beach.

STEP 1 Find the length of the horizontal leg.

The length of the horizontal leg is the absolute value of the difference between the *x*-coordinates of the points (280, 20) and (10, 20).

$$|280 - 10| = 270$$

The length of the horizontal leg is 270 meters.

STEP 2 Find the length of the vertical leg.

The length of the vertical leg is the absolute value of the difference between the *y*-coordinates of the points (280, 164) and (280, 20).

$$|164 - 20| = 144$$

The length of the vertical leg is 144 meters.

STEP 3 Let $a = 270$ and $b = 144$. Let c represent the length of the hypotenuse. Use the Pythagorean Theorem to find c.

$a^2 + b^2 = c^2$

$270^2 + 144^2 = c^2$ Substitute into the formula.

$72{,}900 + 20{,}736 = c^2$ Simplify.

$93{,}636 = c^2$ Add.

$\sqrt{93{,}636} = c$ Take the square root of both sides.

$306 = c$ Simplify.

The distance from Francesca's house to the beach is 306 meters.

Math Talk
Mathematical Practices
Why is it necessary to take the absolute value of the coordinates when finding the length of a segment?

Sample answer: You take the absolute value because the length of a segment cannot be a negative number.

Reflect

3. Show how you could use the Distance Formula to find the distance from Francesca's house to the beach.

Let $(x_1, y_1) = (10, 20)$ and $(x_2, y_2) = (280, 164)$.

$d = \sqrt{(280 - 10)^2 + (164 - 20)^2} = \sqrt{(270)^2 + (144)^2}$

$d = \sqrt{72{,}900 + 20{,}736} = \sqrt{93{,}636} = 306$

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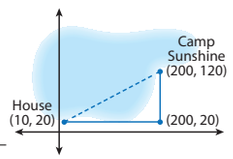
YOUR TURN

4. Camp Sunshine is also on the lake. Use the Pythagorean Theorem to find the distance between Francesca's house and Camp Sunshine to the nearest tenth of a meter.

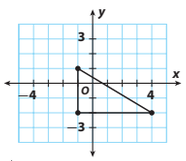
approximately 214.7 meters



Guided Practice

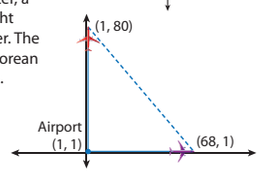
1. Approximate the length of the hypotenuse of the right triangle to the nearest tenth using a calculator. (Example 1) 5.8 units

2. Find the distance between the points (3, 7) and (15, 12) on the coordinate plane. (Explore Activity) 13 units



3. A plane leaves an airport and flies due north. Two minutes later, a second plane leaves the same airport flying due east. The flight plan shows the coordinates of the two planes 10 minutes later. The distances in the graph are measured in miles. Use the Pythagorean Theorem to find the distance shown between the two planes.

(Example 2) 103.6 miles



ESSENTIAL QUESTION CHECK-IN

4. Describe two ways to find the distance between two points on a coordinate plane.

Sample answer: Draw a right triangle whose hypotenuse is the segment connecting the two points and then use the Pythagorean Theorem to find the length of that segment, or use the Distance Formula.

DIFFERENTIATE INSTRUCTION

Visual Cues

Show students how two points that do not form a vertical segment or a horizontal segment on the coordinate plane form a right triangle with a third point. Then lead them through the derivation of the Distance Formula using the points (x_1, y_1) and (x_2, y_2) as the endpoints of a nonvertical, nonhorizontal segment.

Cooperative Learning

Have groups of students graph various nonvertical, nonhorizontal segments in the coordinate plane. Have some students in the group find the length of each segment using the Pythagorean Theorem and other students find the length of the same segments using the Distance Formula. Ask them to compare and contrast their methods and solutions.

Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners ELL
- Reteach
- Challenge PRE-AP

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Online Assessment and Intervention

Online homework assignment available

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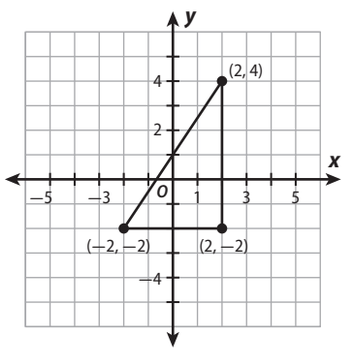
12.3 LESSON QUIZ

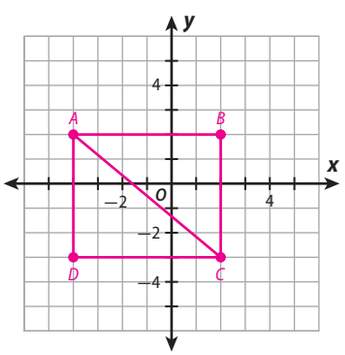
COMMON CORE **8.G.8**

1. Approximate the length of the hypotenuse of the right triangle to the nearest tenth of a unit.



2. Find the distance between points (1, 3) and (9, 18) on the coordinate plane.

3. The coordinates of the vertices of a rectangle are $A(-4, 2)$, $B(2, 2)$, $C(2, -3)$, and $D(-4, -3)$. Plot these points on the coordinate plane and connect them to draw a rectangle. Connect points A and C. Find the exact length of the diagonal $\overline{AC}$.



Lesson Quiz available online

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Answers

1. 7.2 units

2. 17 units

3. $\sqrt{61}$ units

Evaluate

GUIDED AND INDEPENDENT PRACTICE

COMMON CORE **8.G.8**

Concepts & Skills	Practice
Example 1 Pythagorean Theorem in the Coordinate Plane	Exercises 1, 5
Explore Activity Finding the Distance Between Any Two Points	Exercises 2, 7
Example 2 Finding the Distance Between Two Points	Exercises 3, 6, 7–8

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
5–6	**2** Skills/Concepts	**MP.4** Modeling
7	**2** Skills/Concepts	**MP.5** Using Tools
8	**2** Skills/Concepts	**MP.1** Problem Solving
9	**3** Strategic Thinking H.O.T.	**MP.8** Patterns
10	**2** Skills/Concepts	**MP.2** Reasoning
11	**3** Strategic Thinking H.O.T.	**MP.5** Using Tools
12	**3** Strategic Thinking H.O.T.	**MP.4** Modeling

Additional Resources

Differentiated Instruction includes:

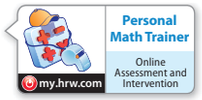
- Leveled Practice worksheets

CC CLUSTER CONNECTION **Exercise 12** combines concepts from the Common Core cluster "Understand and apply the Pythagorean Theorem."

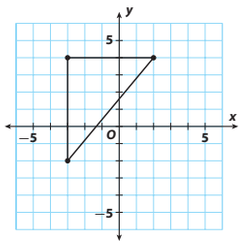
Name _____ Class _____ Date _____

12.3 Independent Practice

COMMON CORE 8.G.8



5. A metal worker traced a triangular piece of sheet metal on a coordinate plane, as shown. The units represent inches. What is the length of the longest side of the metal triangle? Approximate the length to the nearest tenth of an inch using a calculator. Check that your answer is reasonable.

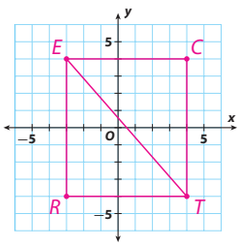


7.8 in.

6. When a coordinate grid is superimposed on a map of Harrisburg, the high school is located at (17, 21) and the town park is located at (28, 13). If each unit represents 1 mile, how many miles apart are the high school and the town park? Round your answer to the nearest tenth.

13.6 miles

7. The coordinates of the vertices of a rectangle are given by $R(-3, -4)$, $E(-3, 4)$, $C(4, 4)$, and $T(4, -4)$. Plot these points on the coordinate plane at the right and connect them to draw the rectangle. Then connect points E and T to form diagonal $\overline{ET}$.



a. Use the Pythagorean Theorem to find the exact length of $\overline{ET}$.

$ET = \sqrt{113}$ units

b. How can you use the Distance Formula to find the length of $\overline{ET}$? Show that the Distance Formula gives the same answer.

Let $(x_1, y_1) = (-3, 4)$ and $(x_2, y_2) = (4, -4)$ and then substitute the coordinates into the Distance Formula.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(4 - (-3))^2 + (-4 - 4)^2}$

$d = \sqrt{(7)^2 + (-8)^2} = \sqrt{49 + 64} = \sqrt{113}$

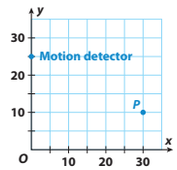
8. Multistep The locations of three ships are represented on a coordinate grid by the following points: $P(-2, 5)$, $Q(-7, -5)$, and $R(2, -3)$. Which ships are farthest apart?

the ships at points P and Q

9. Make a Conjecture Find as many points as you can that are 5 units from the origin. Make a conjecture about the shape formed if all the points 5 units from the origin were connected.

(5, 0), (4, 3), (3, 4), (0, 5), (−3, 4), (−4, 3), (−5, 0), (−4, −3), (−3, −4), (0, −5), (3, −4), (4, −3); The points would form a circle.

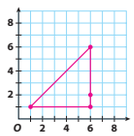
10. Justify Reasoning The graph shows the location of a motion detector that has a maximum range of 34 feet. A peacock at point P displays its tail feathers. Will the motion detector sense this motion? Explain.



Yes; the distance from the motion detector to the peacock is $\sqrt{30^2 + 15^2} \approx 33.5$ ft, which is less than 34 ft.

H.O.T. FOCUS ON HIGHER ORDER THINKING

11. Persevere in Problem Solving One leg of an isosceles right triangle has endpoints (1, 1) and (6, 1). The other leg passes through the point (6, 2). Draw the triangle on the coordinate plane. Then show how you can use the Distance Formula to find the length of the hypotenuse. Round your answer to the nearest tenth.

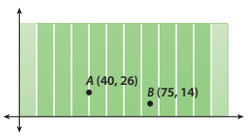


Let $(x_1, y_1) = (6, 6)$, and $(x_2, y_2) = (1, 1)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - 1)^2 + (6 - 1)^2}$

$d = \sqrt{(5)^2 + (5)^2} = \sqrt{25 + 25} = \sqrt{50}$; $d \approx 7.1$

12. Represent Real-World Problems The figure shows a representation of a football field. The units represent yards. A sports analyst marks the locations of the football from where it was thrown (point A) and where it was caught (point B). Explain how you can use the Pythagorean Theorem to find the distance the ball was thrown. Then find the distance.



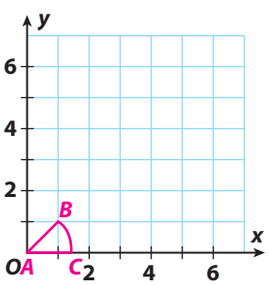
Create a right triangle with hypotenuse $\overline{AB}$. The vertex at the right angle is either (40, 14) or (75, 26), and the lengths of the legs of the triangle are 12 yards and 35 yards. The distance between A and B is about 37 yards.

Work Area

EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity Draw and label a line segment from $A(0, 0)$ to $B(1, 1)$. Use the Pythagorean Theorem to find the length of $\overline{AB}$. Use a protractor with the compass point on A and the opening of the compass set to $\overline{AB}$. With the point of the compass on A, rotate the pencil point from B to a point on the x-axis. Label the point C. What are the exact coordinates of C? What is the length of $\overline{AC}$? What kind of number did you graph?



$AB = \sqrt{2}$; $C = (\sqrt{2}, 0)$; $AC = \sqrt{2}$; irrational number

Ready to Go On?

Assess Mastery

Use the assessment on this page to determine if students have mastered the concepts and standards covered in this module.

RtI Response to Intervention

Personal Math Trainer
Online Assessment and Intervention
my.hrw.com

Intervention	Enrichment

Access Ready to Go On? assessment online, and receive instant scoring, feedback, and customized intervention or enrichment.

Online and Print Resources

Differentiated Instruction
- Reteach worksheets
- Reading Strategies ELL
- Success for English Learners ELL

Differentiated Instruction
- Challenge worksheets PRE-AP

Extend the Math PRE-AP Lesson Activities in TE

Additional Resources

Assessment Resources includes
- Leveled Module Quizzes

MODULE QUIZ

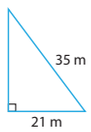
Ready to Go On?

Personal Math Trainer
Online Assessment and Intervention
my.hrw.com

12.1 The Pythagorean Theorem

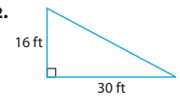
Find the length of the missing side.

1.



28 m

2.



34 ft

12.2 Converse of the Pythagorean Theorem

Tell whether each triangle with the given side lengths is a right triangle.

3. 11, 60, 61 yes

4. 9, 37, 40 no

5. 15, 35, 38 no

6. 28, 45, 53 yes

7. Keelie has a triangular-shaped card. The lengths of its sides are 4.5 cm, 6 cm, and 7.5 cm. Is the card a right triangle? yes

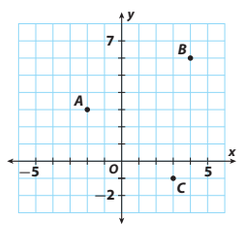
12.3 Distance Between Two Points

Find the distance between the given points. Round to the nearest tenth.

8. *A* and *B* 6.7

9. *B* and *C* 7.1

10. *A* and *C* 6.4



ESSENTIAL QUESTION

11. How can you use the Pythagorean Theorem to solve real-world problems?

Sample answer: You can use the Pythagorean Theorem to find missing lengths in objects that are right triangles.

Module 12 393

Common Core Standards

Lesson	Exercises	Common Core Standards
12.1	1–2	**8.G.6, 8.G.7**
12.2	3–7	**8.G.6**
12.3	8–10	**8.G.8**

Assessment Readiness

Assessment Readiness Tip Quickly sketching the problem situation can help students to organize information and solve correctly.

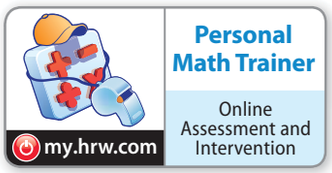
Item 3 Sketching the triangle and labeling the sides *a*, *b*, and *c* can help students to correctly use the Pythagorean Theorem to find the answer.

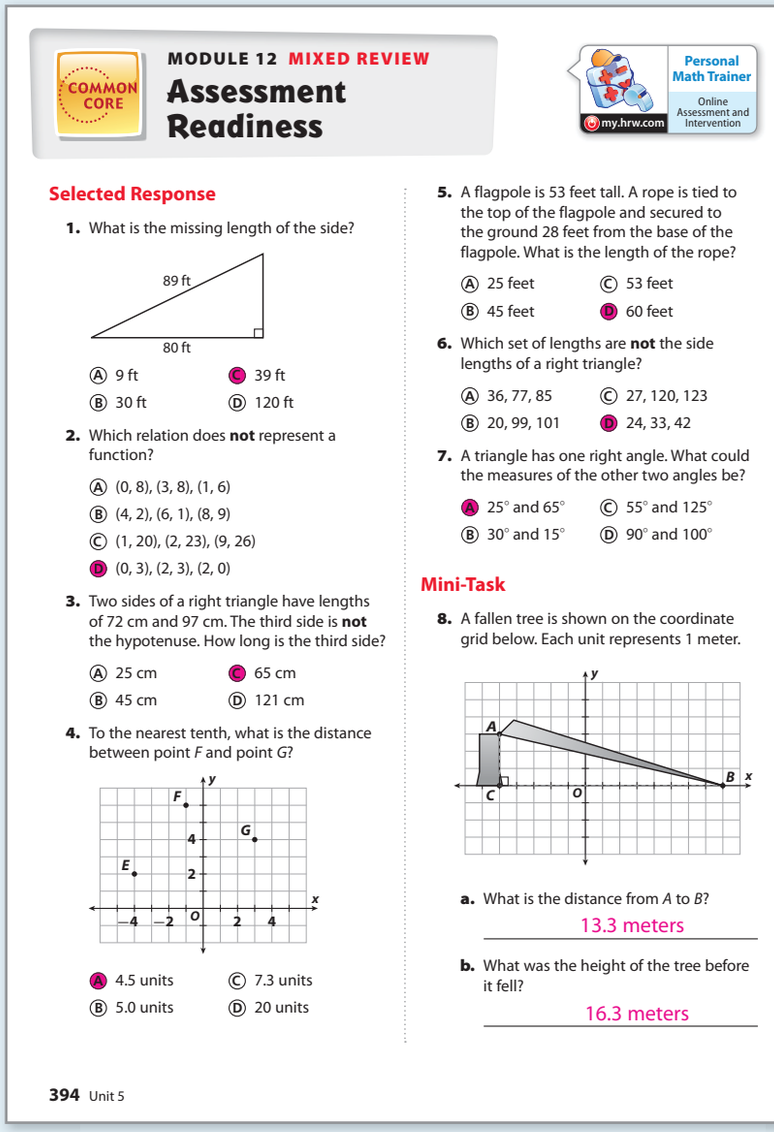
Item 5 Drawing a sketch of the flagpole, rope, and ground and labeling it with the given measurements can help students to see that they are finding the hypotenuse of a right triangle.

Avoid Common Errors

Item 4 Remind students to use the Distance Formula to find the distance between two points. Some students may try to count the boxes between *F* and *G*. Remind them that counting the boxes only works when a line is horizontal or vertical. If it is angled in any way, the Distance Formula must be used.

Additional Resources





MODULE 12 MIXED REVIEW

COMMON CORE

Assessment Readiness

Personal Math Trainer
Online Assessment and Intervention
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Selected Response

1. What is the missing length of the side?

89 ft
80 ft

Ⓐ 9 ft Ⓒ 39 ft
Ⓑ 30 ft Ⓓ 120 ft

2. Which relation does **not** represent a function?

Ⓐ (0, 8), (3, 8), (1, 6)
Ⓑ (4, 2), (6, 1), (8, 9)
Ⓒ (1, 20), (2, 23), (9, 26)
Ⓓ (0, 3), (2, 3), (2, 0)

3. Two sides of a right triangle have lengths of 72 cm and 97 cm. The third side is **not** the hypotenuse. How long is the third side?

Ⓐ 25 cm Ⓒ 65 cm
Ⓑ 45 cm Ⓓ 121 cm

4. To the nearest tenth, what is the distance between point *F* and point *G*?

Ⓐ 4.5 units Ⓒ 7.3 units
Ⓑ 5.0 units Ⓓ 20 units

5. A flagpole is 53 feet tall. A rope is tied to the top of the flagpole and secured to the ground 28 feet from the base of the flagpole. What is the length of the rope?

Ⓐ 25 feet Ⓒ 53 feet
Ⓑ 45 feet Ⓓ 60 feet

6. Which set of lengths are **not** the side lengths of a right triangle?

Ⓐ 36, 77, 85 Ⓒ 27, 120, 123
Ⓑ 20, 99, 101 Ⓓ 24, 33, 42

7. A triangle has one right angle. What could the measures of the other two angles be?

Ⓐ 25° and 65° Ⓒ 55° and 125°
Ⓑ 30° and 15° Ⓓ 90° and 100°

Mini-Task

8. A fallen tree is shown on the coordinate grid below. Each unit represents 1 meter.

a. What is the distance from *A* to *B*?

13.3 meters

b. What was the height of the tree before it fell?

16.3 meters

394 Unit 5

Common Core Standards

Items	Grade 8 Standards	Mathematical Practices
1	8.G.7	MP.4
2*	8.F.1	MP.6
3	8.G.7	MP.2
4	8.G.8	MP.2
5	8.G.7	MP.4
6	8.G.7	MP.4
7*	8.G.5	MP.2
8	8.G.7, 8.G.8	MP.4

* Item integrates mixed review concepts from previous modules or a previous course.