Volume

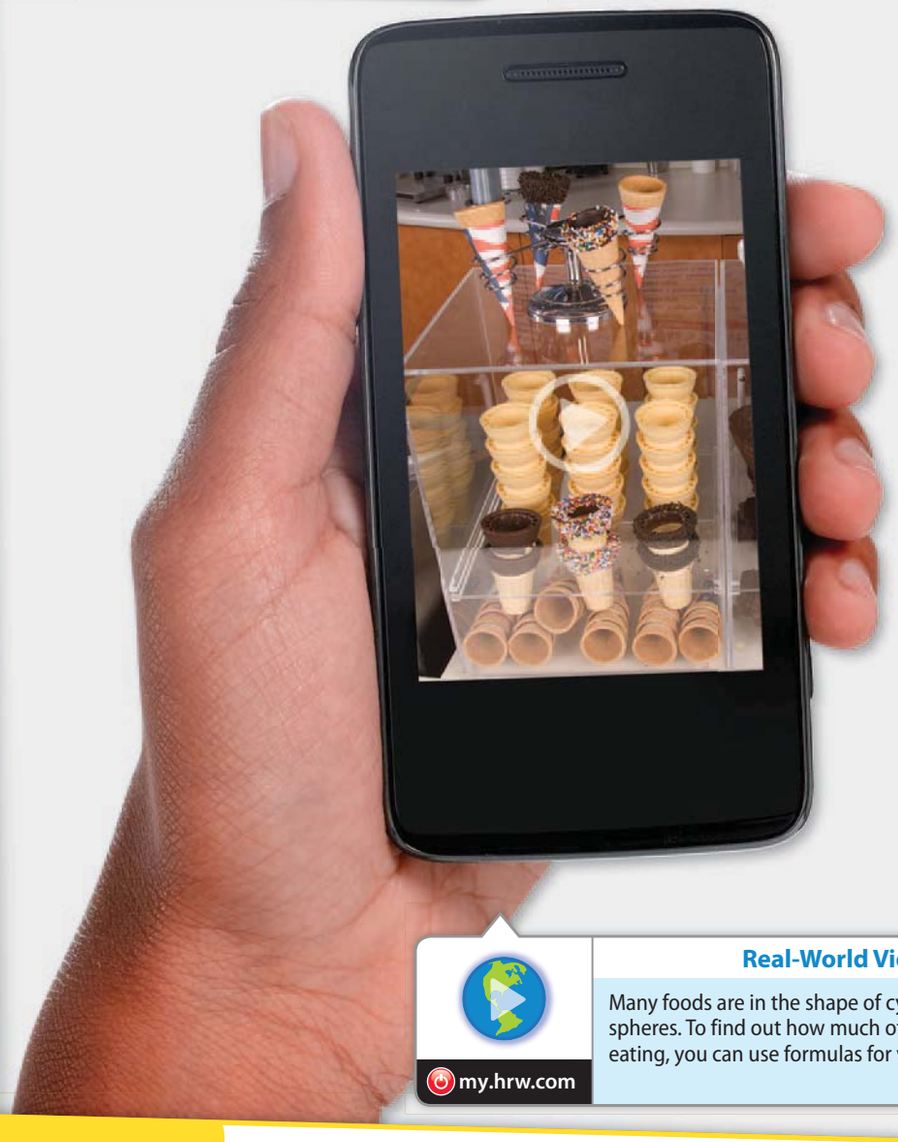
MODULE 13

COMMON CORE

ESSENTIAL QUESTION

How can you use volume to solve real-world problems?

You can use formulas to find the volumes of real-world objects shaped like cylinders, cones, and spheres.



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Real-World Video

Many foods are in the shape of cylinders, cones, and spheres. To find out how much of the food you are eating, you can use formulas for volume.

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Are You Ready?

Assess Readiness

Use the assessment on this page to determine if students need intensive or strategic intervention for the module's prerequisite skills.

Response to Intervention

Personal Math Trainer
Online Assessment and Intervention
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Intervention	Enrichment

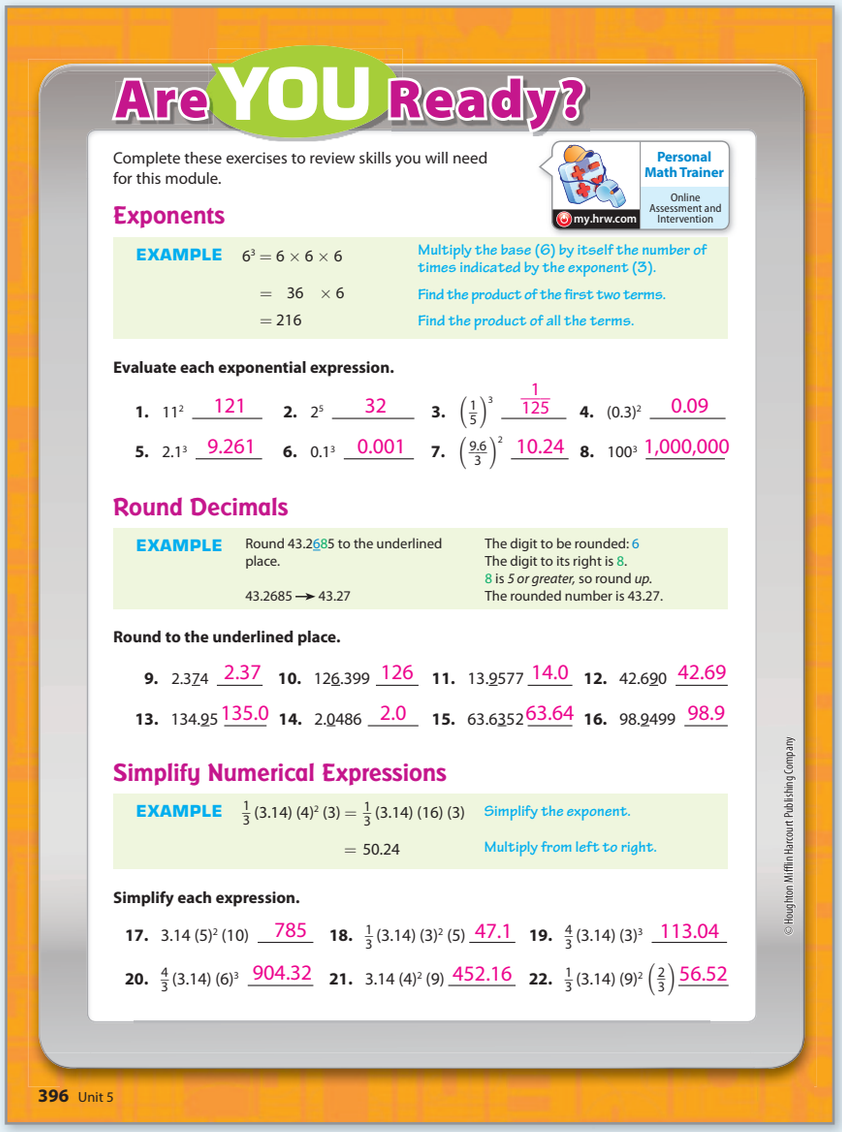
Access Are You Ready? assessment online, and receive instant scoring, feedback, and customized intervention or enrichment.

Online and Print Resources

Skills Intervention worksheets
- Skill 12 Exponents
- Skill 16 Round Decimals
- Skill 52 Simplify Numerical Expressions

Differentiated Instruction
- Challenge worksheets **PRE-AP**

Extend the Math **PRE-AP** Lesson Activities in TE



Are YOU Ready?

Complete these exercises to review skills you will need for this module.

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Exponents

EXAMPLE $6^3 = 6 \times 6 \times 6$ — Multiply the base (6) by itself the number of times indicated by the exponent (3).

$= 36 \times 6$ — Find the product of the first two terms.

$= 216$ — Find the product of all the terms.

Evaluate each exponential expression.

1. 11^2 121
2. 2^5 32
3. $\left(\frac{1}{5}\right)^3$ $\frac{1}{125}$
4. $(0.3)^2$ 0.09
5. 2.1^3 9.261
6. 0.1^3 0.001
7. $\left(\frac{9.6}{3}\right)^2$ 10.24
8. 100^3 1,000,000

Round Decimals

EXAMPLE Round 43.2685 to the underlined place. — The digit to be rounded: 6. The digit to its right is 8. 8 is *5 or greater*, so round *up*. The rounded number is 43.27.

43.2685 → 43.27

Round to the underlined place.

9. 2.374 2.37
10. 126.399 126
11. 13.9577 14.0
12. 42.690 42.69
13. 134.95 135.0
14. 2.0486 2.0
15. 63.6352 63.64
16. 98.9499 98.9

Simplify Numerical Expressions

EXAMPLE $\frac{1}{3}(3.14)(4)^2(3) = \frac{1}{3}(3.14)(16)(3)$ — Simplify the exponent.

$= 50.24$ — Multiply from left to right.

Simplify each expression.

17. $3.14(5)^2(10)$ 785
18. $\frac{1}{3}(3.14)(3)^2(5)$ 47.1
19. $\frac{4}{3}(3.14)(3)^3$ 113.04
20. $\frac{4}{3}(3.14)(6)^3$ 904.32
21. $3.14(4)^2(9)$ 452.16
22. $\frac{1}{3}(3.14)(9)^2\left(\frac{2}{3}\right)$ 56.52

PROFESSIONAL DEVELOPMENT VIDEO

Author Juli Dixon models successful teaching practices as she explores the concept of volume of curved-surface solids in an actual eighth-grade classroom.

Professional Development
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Easily plan your classes and access all your resources online.

Interactive Answers and Solutions
Customize answer keys to print or display in the classroom. Choose to include answers only or full solutions to all lesson exercises.

Interactive Whiteboards
Engage students with interactive whiteboard-ready lessons and activities.

Personal Math Trainer: Online Assessment and Intervention
Assign automatically graded homework, quizzes, tests, and intervention activities. Prepare your students with updated practice tests aligned with Common Core.

Reading Start-Up

Have students complete the activities on this page by working alone or with others.

Visualize Vocabulary

The chart helps students review the terms related to volume of three-dimensional figures. Students should write one or more review words in each box.

Understand Vocabulary

Use the following explanation to help students learn the preview words.

> A birthday hat with a pointed top is similar to a **cone**. A can of vegetables is a **cylinder**, and a basketball is a **sphere**. In this module, you will learn how to find the volume of all of these shapes.

Active Reading

Integrating Language Arts

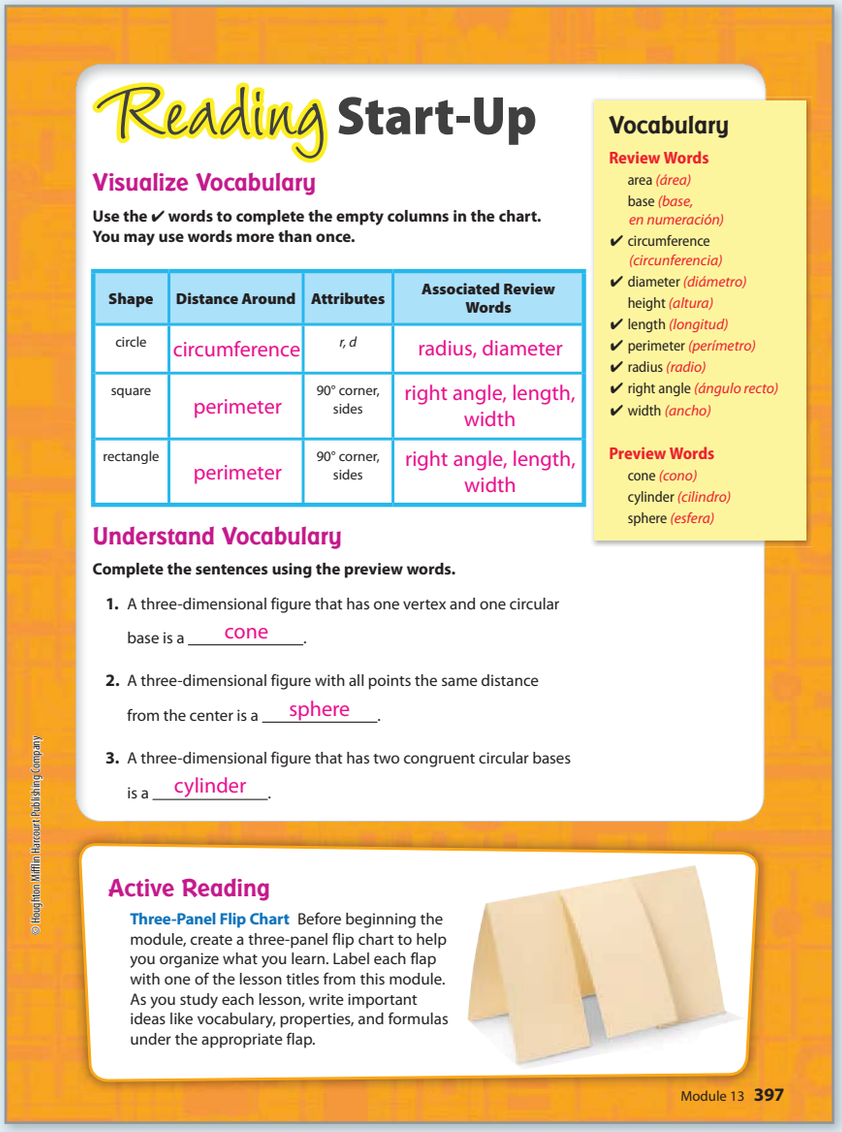
Students can use these reading and note-taking strategies to help them organize and understand new concepts and vocabulary.

COMMON CORE **ELA-Literacy.RST.6-8.7** Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).

Additional Resources

Differentiated Instruction

- Reading Strategies ELL



Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the empty columns in the chart. You may use words more than once.

Shape	Distance Around	Attributes	Associated Review Words
circle	circumference	*r*, *d*	radius, diameter
square	perimeter	90° corner, sides	right angle, length, width
rectangle	perimeter	90° corner, sides	right angle, length, width

Vocabulary

Review Words
area (*área*)
base (*base, en numeración*)
✔ circumference (*circunferencia*)
✔ diameter (*diámetro*)
height (*altura*)
✔ length (*longitud*)
✔ perimeter (*perímetro*)
✔ radius (*radio*)
✔ right angle (*ángulo recto*)
✔ width (*ancho*)

Preview Words
cone (*cono*)
cylinder (*cilindro*)
sphere (*esfera*)

Understand Vocabulary

Complete the sentences using the preview words.

1. A three-dimensional figure that has one vertex and one circular base is a cone.
2. A three-dimensional figure with all points the same distance from the center is a sphere.
3. A three-dimensional figure that has two congruent circular bases is a cylinder.

Active Reading

Three-Panel Flip Chart Before beginning the module, create a three-panel flip chart to help you organize what you learn. Label each flap with one of the lesson titles from this module. As you study each lesson, write important ideas like vocabulary, properties, and formulas under the appropriate flap.

Module 13 397

Before	In this module	After
Students understand how to use formulas: • find the circumference of a circle • find the area of a circle • find the volume of rectangular prisms and pyramids, and of triangular prisms and pyramids	Students represent and solve for the volumes of three-dimensional curved figures: • describe the volume formula $V = Bh$ of a cylinder in terms of its base area and height • model the relationship between the volume of a cylinder and a cone having both congruent bases and height and connect that relationship to their volume formulas • solve problems involving the volume of cylinders, cones, and spheres	Students will connect: • the effect on volume when the dimensions of a solid change proportionally • capacity and volume

Unpacking the Standards

Use the examples on this page to help students know exactly what they are expected to learn in this module.

Common Core Standards

Content Areas

COMMON CORE **Geometry—8.G**

Understand and apply the Pythagorean Theorem.

Go online to see a complete unpacking of the Common Core Standards.

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MODULE 13

Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

COMMON CORE **8.G.9**

Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Key Vocabulary

volume *(volumen)*
The number of cubic units needed to fill a given space.

cylinder *(cilindro)*
A three-dimensional figure with two parallel, congruent circular bases connected by a curved lateral surface.

What It Means to You

You will learn the formula for the volume of a cylinder.

UNPACKING EXAMPLE 8.G.9

The Asano Taiko Company of Japan built the world's largest drum in 2000. The drum's diameter is 4.8 meters, and its height is 4.95 meters. Estimate the volume of the drum.

$d = 4.8 \approx 5$
$h = 4.95 \approx 5$
$r = \frac{d}{2} \approx \frac{5}{2} = 2.5$

$V = (\pi r^2)h$ Volume of a cylinder
$\approx (3)(2.5)^2 \cdot 5$ Use 3 for π.
$= (3)(6.25)(5)$
$= 18.75 \cdot 5$
$= 93.75 \approx 94$

The volume of the drum is approximately 94 m^3.

COMMON CORE **8.G.9**

Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Key Vocabulary

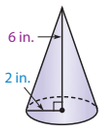
cone *(cono)*
A three-dimensional figure with one vertex and one circular base.

sphere *(esfera)*
A three-dimensional figure with all points the same distance from the center.

What It Means to You

You will learn formulas for the volume of a cone and a sphere.

UNPACKING EXAMPLE 8.G.9



Find the volume of the cone. Use 3.14 for π.

$B = \pi(2^2) = 4\pi$ in^2
$V = \frac{1}{3} \cdot 4\pi \cdot 6$ $V = \frac{1}{3}Bh$
$V = 8\pi$ Use 3.14 for π.
≈ 25.1 in^3

The volume of the cone is approximately 25.1 in^3.

The volume of a sphere with the same radius is $V = \frac{4}{3}\pi r^3 \approx \frac{4}{3}(3)(2)^3 = 32$ in^3.

Visit my.hrw.com to see all the Common Core Standards unpacked.

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Common Core Standards	Lesson 13.1	Lesson 13.2	Lesson 13.3
8.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	COMMON CORE	COMMON CORE	COMMON CORE

LESSON

13.1 Volume of Cylinders

Common Core Standards

The student is expected to:

COMMON CORE **Geometry—8.G.9**

Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Mathematical Practices

COMMON CORE **MP.3** Logic

Engage

ESSENTIAL QUESTION

How do you find the volume of a cylinder? Sample answer: You multiply the area of the base by the height.

Motivate the Lesson

Ask: What three-dimensional objects with two congruent circular bases do you see around you in the classroom? What examples of this shape might you find in a kitchen? How could you find out how much food an object like this might hold?

Explore

EXPLORE ACTIVITY

Connect to Daily Life

Ask students to think of a circular pancake or cookie (B), and then to think of stacking up enough of these pancakes (h) to fill the cylinder. Relate this to the formula $V = Bh$.

Explain

EXAMPLE 1

Questioning Strategies CC Mathematical Practices

- Compare and contrast cylinders and prisms; how are they alike and how are they different? Both have two parallel congruent bases. For cylinders the bases are circles; for prisms the bases are polygons. The bases of a cylinder are connected by a curved surface; the bases of a prism are connected by faces that are polygons.
- Why do the steps in Example 1 use the symbol $\approx$ instead of $=$ after values are substituted in the formula? The value substituted for π is approximate, so the answer must also be approximate.
- Which will increase the volume by a greater amount, doubling the radius or doubling the height? Explain. Doubling the radius; the radius is squared when calculating the volume, so doubling the radius will increase the volume four-fold; doubling the height will double the volume.

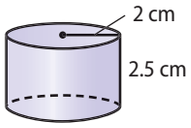
Engage with the Whiteboard

Have students find the area of the circular base for each cylinder, and have a student label each base with its area. Complete the calculation of the volume by multiplying the area of the base by the height. Students should see that the volume calculations in the Examples could have included a step where the base area is found first and then multiplied by h. Point out that the final answer may differ slightly between the two methods if the area of the base is rounded before multiplying by the height.

ADDITIONAL EXAMPLE 1

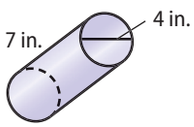
Find the volume of each cylinder. Round your answers to the nearest tenth if necessary. Use 3.14 for π.

A



31.4 cm^3

B



87.9 in^3

Interactive Whiteboard
Interactive example available online

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Animated Math
Volume of Cylinders

Students explore the effect of radius and height on the volume of a cylinder using a dynamic model.

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LESSON

13.1 Volume of Cylinders

COMMON CORE 8.G.9

Know the formulas for the volumes of…cylinders…and use them to solve real-world and mathematical problems.

ESSENTIAL QUESTION How do you find the volume of a cylinder?

EXPLORE ACTIVITY COMMON CORE 8.G.9

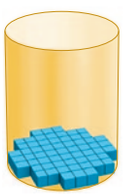
Modeling the Volume of a Cylinder

A **cylinder** is a three-dimensional figure that has two congruent circular bases that lie in parallel planes. The volume of any three-dimensional figure is the number of cubic units needed to fill the space taken up by the solid figure.

One cube represents one cubic unit of volume. You can develop the formula for the volume of a cylinder using an empty soup can or other cylindrical container. First, remove one of the bases.

A Arrange centimeter cubes in a single layer at the bottom of the cylinder. Fit as many cubes into the layer as possible. How many cubes are in this layer?

Answers will vary. Check students' work.

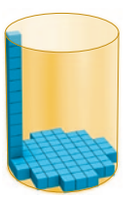


B To find how many layers of cubes fit in the cylinder, make a stack of cubes along the inside of the cylinder. How many layers fit in the cylinder?

Answers will vary. Check students' work.

C How can you use what you know to find the approximate number of cubes that would fit in the cylinder?

Sample answer: Multiply the number of cubes in the bottom layer times the number of layers.



Reflect

1. **Make a Conjecture** Suppose you know the area of the base of a cylinder and the height of the cylinder. How can you find the cylinder's volume?

Multiply the area of the base times the height.

2. Let the area of the base of a cylinder be B and the height of the cylinder be h. Write a formula for the cylinder's volume V. $V = Bh$

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My Notes

Finding the Volume of a Cylinder Using a Formula

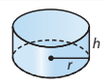
Finding volumes of cylinders is similar to finding volumes of prisms. You find the volume V of both a prism and a cylinder by multiplying the height h by the area of the base B, so $V = Bh$.

The base of a cylinder is a circle, so for a cylinder, $B = \pi r^2$.

Volume of a Cylinder

The volume V of a cylinder with radius r is the area of the base B times the height h.

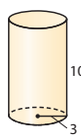
$V = Bh$ or $V = \pi r^2 h$



EXAMPLE 1 COMMON CORE 8.G.9

Find the volume of each cylinder. Round your answers to the nearest tenth if necessary. Use 3.14 for π.

A



$V = \pi r^2 h$

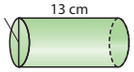
$\approx 3.14 \cdot 3^2 \cdot 10$ Substitute.

$\approx 3.14 \cdot 9 \cdot 10$ Simplify.

≈ 282.6 Multiply.

The volume is about 282.6 in^3.

B 6.4 cm, 13 cm



Since the diameter is 6.4 cm, the radius is 3.2 cm.

Recall that the diameter of a circle is twice the radius, so $2r = d$ and $r = \frac{d}{2}$.

$V = \pi r^2 h$

$\approx 3.14 \cdot 3.2^2 \cdot 13$ Substitute.

$\approx 3.14 \cdot 10.24 \cdot 13$ Simplify.

≈ 418 Multiply.

The volume is about 418 cm^3.

Reflect

3. **What If?** If you want a formula for the volume of a cylinder that involves the diameter d instead of the radius r, how can you rewrite it?

Write $\frac{d}{2}$ in place of r, so $V = \pi\left(\frac{d}{2}\right)^2 h$.

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices MP.3

This lesson provides an opportunity to address this Mathematical Practices standard. It calls for students to construct viable arguments by making conjectures and building a logical progression of statements. Students explore ways to find the volume of a cylinder, working from descriptions or diagrams. Students then represent the volume in symbolic form as an equation.

Math Background

The general formula $V = Bh$ can be applied to all prisms and cylinders. However, the formula used to calculate B, the area of the base, will differ due to the shape of the base.

The cylinders in this lesson are *right cylinders* (with an axis perpendicular to the base) with circular bases. However, the same formula applies to *oblique* prisms and cylinders, based on Cavalieri's Principle: if two three-dimensional figures have the same height and the same cross-sectional area at every level, then they have the same volume.

ADDITIONAL EXAMPLE 2

A cylindrical silo that stores grain has a diameter of 16 feet and is 40 feet tall. Find the volume of the silo to the nearest tenth. Use 3.14 for π. 8038.4 ft^3

Interactive Whiteboard
Interactive example available online

my.hrw.com

YOUR TURN

Avoid Common Errors

Remind students to always consider the question, "Do I know the radius, *r*, or only the diameter?" In Exercise 4, students may try to use the diameter instead of the radius in the formula.

EXAMPLE 2

Questioning Strategies CC Mathematical Practices

- When is step 1 unnecessary? when you know the radius
- Why is 8 divided by 2? The formula calls for the radius, which is found by dividing the diameter by 2.
- Explain why you must square 4 before you multiply by 3.14. Sample answer: The correct order of operations (PEMDAS) requires evaluating the exponent before multiplying.

Talk About It

Check for Understanding

Ask: Which words in the problem will you use to find the value of *r*? diameter of 8 feet Which words in the problem will you use to find the value of *h*? 4.5 feet deep

YOUR TURN

Focus on Critical Thinking

To calculate the answer for Exercise 6, Elsa multiplied 6^2 by 3.14 and then multiplied that product by 4. Tommie multiplied 6^2 by 4 and then multiplied that product by 3.14. Which process gives the correct answer? Explain. Both; factors can be multiplied in any order.

Elaborate

Talk About It

Summarize the Lesson

Ask: What measurements do you need to know in order to find the volume of a cylinder? What formula will you use? You need to know either the radius or the diameter of the bases and the height of the cylinder. $V = Bh$

GUIDED PRACTICE

Engage with the Whiteboard

In Exercise 3, have a student write the variables for the radius *r* and the height *h* next to the measurements on the cylinder. In Exercise 4, have a student draw and label a cylinder next to the statement of the problem.

Avoid Common Errors

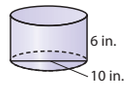
Exercises 2–4 Ask students to predict the units they will write for the final answer before they begin the problem. Make sure students remember that volume requires cubed units.

Exercises 3–4 Have students circle the key words *radius* and *diameter* before they begin to substitute values into the volume formula. Students may carelessly use the diameter for the radius in Exercise 4.

YOUR TURN

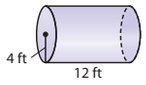
Find the volume of each cylinder. Round your answers to the nearest tenth if necessary. Use 3.14 for π.

4.



471 in^3

5.



602.9 ft^3

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Finding the Volume of a Cylinder in a Real-World Context

The Longhorn Band at the University of Texas at Austin has one of the world's largest bass drums, known as Big Bertha.

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EXAMPLE 2 Real World COMMON CORE 8.G.9

Big Bertha has a diameter of 8 feet and is 4.5 feet deep. Find the volume of the drum to the nearest tenth. Use 3.14 for π.

STEP 1 Find the radius of the drum.

$r = \frac{d}{2} = \frac{8}{2} = 4$ ft

STEP 2 Find the volume of the drum.

$V = \pi r^2 h$

$\approx 3.14 \cdot 4^2 \cdot 4.5$ Substitute.

$\approx 3.14 \cdot 16 \cdot 4.5$ Simplify the exponent.

≈ 226.08 Multiply.

The volume of the drum is about 226.1 ft^3.

YOUR TURN

6. A drum company advertises a snare drum that is 4 inches high and 12 inches in diameter. Find the volume of the drum to the nearest tenth. Use 3.14 for π.

452.2 in^3

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Guided Practice

1. **Vocabulary** Describe the bases of a cylinder. (Explore Activity)

two congruent circles that lie in parallel planes

2. Figure 1 shows a view from above of inch cubes on the bottom of a cylinder. Figure 2 shows the highest stack of cubes that will fit inside the cylinder. Estimate the volume of the cylinder. Explain your reasoning. (Explore Activity)

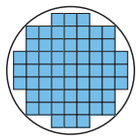


Figure 1

Figure 2

Sample answer: 427 in^3; there are 61 cubes on the bottom of the cylinder. The height is 7 cubes.

$V = 61 \times 7 = 427$ in^3

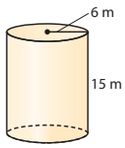
3. Find the volume of the cylinder to the nearest tenth. Use 3.14 for π. (Example 1)

$V = \pi r^2 h$

$V = \pi \cdot (6)^2 \cdot 15$

$\approx 3.14 \cdot 36 \cdot 15$

≈ 1695.6



The volume of the cylinder is approximately 1695.6 m^3.

4. A Japanese odaiko is a very large drum that is made by hollowing out a section of a tree trunk. A museum in Takayama City has three odaikos of similar size carved from a single tree trunk. The largest measures about 2.7 meters in both diameter and length, and weighs about 4.5 metric tons. Using the volume formula for a cylinder, approximate the volume of the drum to the nearest tenth. (Example 2)

The radius of the drum is about 1.35 m.

The volume of the drum is about 15.5 m^3.

ESSENTIAL QUESTION CHECK-IN

5. How do you find the volume of a cylinder? Describe which measurements of a cylinder you need to know.

You need to know the height and know or be able to calculate the radius of the base. Then you can substitute into the volume formula $V = \pi r^2 h$.

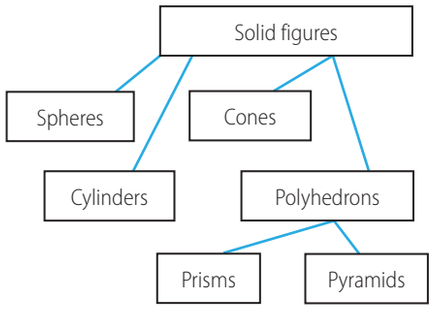
DIFFERENTIATE INSTRUCTION

Manipulatives

Help students understand the component parts of a cylinder by having them take a cardboard tube and cut it apart to see the the rectangle that forms the side, or lateral surface. Alternatively, have them build and tape together a cylinder from two circles and a rectangle. Have them label various measurements to discover and confirm that the width of the rectangle is the height of the cylinder and the length of the rectangle is the circumference of the circle.

Graphic Organizers

Use this web to help students see how various three-dimensional figures relate. Discuss why cylinders, cones, and spheres are not polyhedrons.



Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners ELL
- Reteach
- Challenge PRE-AP

Personal Math Trainer

Online Assessment and Intervention

Online homework assignment available

my.hrw.com

13.1 LESSON QUIZ

COMMON CORE **8.G.9**

Use 3.14 for π. Round answers to the nearest tenth if necessary.

1. A can of chili has a radius of 5.25 cm and a height of 13 cm. Find the volume.
2. A cylindrical carton of oatmeal has a diameter of 13 cm and is 24 cm tall. Find the volume.
3. Which has a greater volume?
   Cylinder A: $r = 3$ m, $h = 1.2$ m
   Cylinder B: $d = 4$ m, $h = 2.5$ m
4. Daren uses rice to fill a cylindrical glass measuring 6 inches high with a radius of 2.5 inches. He pours this rice into a cardboard cylinder that is 3.5 inches high with a diameter of 8 inches. Will he have enough rice to fill the cardboard cylinder? Explain.

Lesson Quiz available online

my.hrw.com

Answers

1. 1125.1 cm^3
2. 3184.0 cm^3
3. Cylinder A
4. The glass has a volume of 117.75 in^3. The cardboard cylinder has a volume of 175.84 in^3. He will not have enough rice to fill the cardboard cylinder, since 117.75 in^3 is less than 175.84 in^3.

Evaluate

GUIDED AND INDEPENDENT PRACTICE

COMMON CORE **8.G.9**

Concepts & Skills	Practice
Explore Activity Modeling the Volume of a Cylinder	Exercises 1–2
Example 1 Finding the Volume of a Cylinder Using a Formula	Exercises 3, 6–11
Example 2 Finding the Volume of a Cylinder in a Real-World Context	Exercises 4, 12–17

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
6–11	**2** Skills/Concepts	**MP.5** Using Tools
12–17	**2** Skills/Concepts	**MP.4** Modeling
18	**3** Strategic Thinking H.O.T.	**MP.3** Logic
19	**3** Strategic Thinking H.O.T.	**MP.6** Precision
20	**3** Strategic Thinking H.O.T.	**MP.3** Logic

Additional Resources

Differentiated Instruction includes:

- Leveled Practice worksheets

Name _____ Class _____ Date _____

13.1 Independent Practice

COMMON CORE 8.G.9

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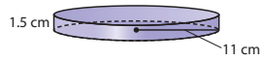
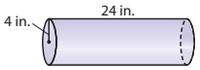
Find the volume of each figure. Round your answers to the nearest tenth if necessary. Use 3.14 for π.

6.



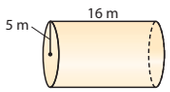
569.9 cm^3

7.



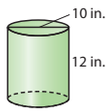
1205.8 in^3

8.



1256 m^3

9.



942 in^3

10. A cylinder has a radius of 4 centimeters and a height of 40 centimeters.

2009.6 cm^3

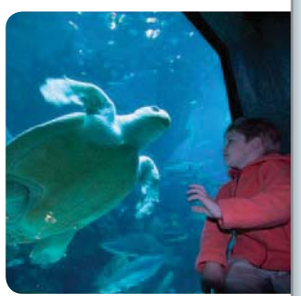
11. A cylinder has a radius of 8 meters and a height of 4 meters.

803.8 m^3

Round your answer to the nearest tenth, if necessary. Use 3.14 for π.

12. The cylindrical Giant Ocean Tank at the New England Aquarium in Boston is 24 feet deep and has a radius of 18.8 feet. Find the volume of the tank.

26,635.2 ft^3



13. A standard-size bass drum has a diameter of 22 inches and is 18 inches deep. Find the volume of this drum.

6838.9 in^3

14. Grain is stored in cylindrical structures called silos. Find the volume of a silo with a diameter of 11.1 feet and a height of 20 feet.

1934.4 ft^3

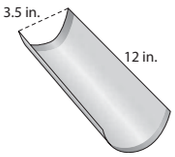
15. The Frank Erwin Center, or "The Drum," at the University of Texas in Austin can be approximated by a cylinder that is 120 meters in diameter and 30 meters in height. Find its volume.

339,120 m^3

16. A barrel of crude oil contains about 5.61 cubic feet of oil. How many barrels of oil are contained in 1 mile (5280 feet) of a pipeline that has an inside diameter of 6 inches and is completely filled with oil? How much is "1 mile" of oil in this pipeline worth at a price of $100 per barrel?

184.7 barrels; $18,470

17. A pan for baking French bread is shaped like half a cylinder. It is 12 inches long and 3.5 inches in diameter. What is the volume of uncooked dough that would fill this pan?



57.7 in^3

H.O.T. FOCUS ON HIGHER ORDER THINKING

18. Explain the Error A student said the volume of a cylinder with a 3-inch diameter is two times the volume of a cylinder with the same height and a 1.5-inch radius. What is the error?

Sample answer: The volumes are equal because a cylinder with a 3-inch diameter has a 1.5-inch radius.

19. Communicate Mathematical Ideas Explain how you can find the height of a cylinder if you know the diameter and the volume. Include an example with your explanation.

Divide the diameter by 2 to find the radius. Then substitute the volume and radius in $V = \pi r^2 h$ and solve for h. Sample example: For a cylinder with volume 72 m^3 and a diameter of 6 m, $72 = \pi \cdot 3^2 h$, so $h = \frac{72}{9\pi} = \frac{8}{\pi} \approx 2.5$ m.

20. Analyze Relationships Cylinder A has a radius of 6 centimeters. Cylinder B has the same height and a radius half as long as cylinder A. What fraction of the volume of cylinder A is the volume of cylinder B? Explain.

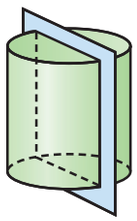
$\frac{1}{4}$; for any height h, cylinder A has a volume of $36\pi \cdot h$. Cylinder B has a volume of $9\pi \cdot h$. Since 9 is $\frac{1}{4}$ of 36, the volume of cylinder B is $\frac{1}{4}$ the volume of cylinder A.

Work Area

EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity When a three-dimensional figure and a plane intersect, the intersection is called a cross section. A three-dimensional figure can have many different cross sections. For example, when you cut a cylinder in half, the cross section that is exposed depends on the direction of the cut. Have students explore the cross sections of a right circular cylinder, either by using drawings or making a cylinder of clay or plastic foam, and cutting it in various ways to form a rectangle, a circle, or an ellipse (oval). Students may also obtain a partial ellipse if the cut enters the side and exits through a base.



LESSON

13.2 Volume of Cones

Common Core Standards

The student is expected to:

COMMON CORE **Geometry—8.G.9**

Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Mathematical Practices

COMMON CORE **MP.4** Modeling

Engage

ESSENTIAL QUESTION

How do you find the volume of a cone? Sample answer: Find one-third of the product of the area of the circular base and the height of the cone.

Motivate the Lesson

Ask: If you had a paper cone and paper cylinder with the same base and height, which would hold more popcorn? How much more? Begin the Explore Activity to find out.

Explore

EXPLORE ACTIVITY

Avoid Common Errors

Some students may try to measure the height of a cone from the vertex to a point on the circumference of the base, which is the slant height. Point out that the height is a segment perpendicular to the base from the vertex to the center of the circular base.

Explain

EXAMPLE 1

Questioning Strategies CC Mathematical Practices

- Compare and contrast a cone and a pyramid. A cone has one circular base and a curved lateral surface; a pyramid has one base that is a polygon and sides that are triangles.
- Compare and contrast a cone and a cylinder. A cone has one circular base and a curved lateral surface; a cylinder has two congruent circular bases and a curved lateral surface.
- If these cones were cylinders with the same base and height, how would you use the cone volumes to find the cylinder volumes? Multiply them by 3.

Integrating Language Arts ELL

Encourage English learners to use the active reading strategies as they encounter new terms and concepts.

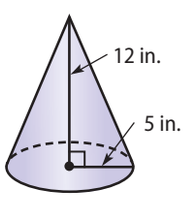
Engage with the Whiteboard

In part B, have a student color the line that is labeled as 8 feet long, and then write the name of that part (either *d* or *diameter*). Have a student circle the 4 in the formula and explain the source of this value.

ADDITIONAL EXAMPLE 1

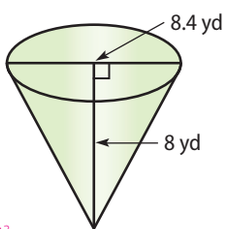
Find the volume of each cone. Round your answers to the nearest tenth. Use 3.14 for π.

A



314 in^3

B



147.7 yd^3

Interactive Whiteboard
Interactive example available online

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LESSON

13.2 Volume of Cones

COMMON CORE 8.G.9

Know the formulas for the volumes of cones. . . and use them to solve real-world and mathematical problems.

ESSENTIAL QUESTION How do you find the volume of a cone?

EXPLORE ACTIVITY COMMON CORE 8.G.9

Modeling the Volume of a Cone

A cone is a three-dimensional figure that has one vertex and one circular base.

To explore the volume of a cone, Sandi does an experiment with a cone and a cylinder that have congruent bases and heights. She fills the cone with popcorn kernels and then pours the kernels into the cylinder. She repeats this until the cylinder is full.

Sandi finds that it takes 3 cones to fill the volume of the cylinder.

STEP 1 What is the formula for the volume V of a cylinder with base area B and height h? $V = Bh$

STEP 2 What is the area of the base of the cone? B, or πr^2

STEP 3 Sandi found that, when the bases and height are the same, 3 times $V_{cone} = V_{cylinder}$.

STEP 4 How does the volume of the cone compare to the volume of the cylinder?

Volume of the cone: $V_{cone} = \frac{1}{3} \cdot V_{cylinder}$

Reflect

1. Use the conclusion from this experiment to write a formula for the volume of a cone in terms of the height and the radius. Explain.
   $V_{cylinder} = \pi r^2 h$, so $V_{cone} = \frac{1}{3}\pi r^2 h$

2. How do you think the formula for the volume of a cone is similar to the formula for the volume of a pyramid?
   Both are one third the area of the base times the height.

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Finding the Volume of a Cone Using a Formula

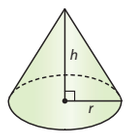
The formulas for the volume of a prism and the volume of a cylinder are the same: multiply the height h by the area of the base B, so $V = Bh$.

In the **Explore Activity**, you saw that the volume of a cone is one third the volume of a cylinder with the same base and height.

Volume of a Cone

The volume V of a cone with radius r is one third the area of the base B times the height h.

$V = \frac{1}{3}Bh$ or $V = \frac{1}{3}\pi r^2 h$

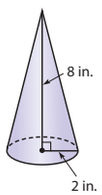


My Notes

EXAMPLE 1 COMMON CORE 8.G.9

Find the volume of each cone. Round your answers to the nearest tenth. Use 3.14 for π.

A



$V = \frac{1}{3}\pi r^2 h$

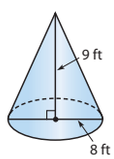
$\approx \frac{1}{3} \cdot 3.14 \cdot 2^2 \cdot 8$ Substitute.

$\approx \frac{1}{3} \cdot 3.14 \cdot 4 \cdot 8$ Simplify.

≈ 33.5 Multiply.

The volume is about 33.5 in^3.

B Since the diameter is 8 ft, the radius is 4 ft.



$V = \frac{1}{3}\pi r^2 h$

$\approx \frac{1}{3} \cdot 3.14 \cdot 4^2 \cdot 9$ Substitute.

$\approx \frac{1}{3} \cdot 3.14 \cdot 16 \cdot 9$ Simplify.

≈ 150.7 Multiply.

The volume is about 150.7 ft^3.

Reflect

3. How can you rewrite the formula for the volume of a cone using the diameter d instead of the radius r? $V = \frac{1}{3}\pi\left(\frac{d}{2}\right)^2 h$

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices MP.4

This lesson provides an opportunity to address this Mathematical Practices standard. It calls for students to model with mathematics. Students use models to explore the relationship between the volume of a cone and a cylinder with congruent bases and heights. They use this activity to write a rule for the volume of a cone.

Math Background

A cone has two aspects that are referred to as *height*.

The *height* of the cone is the length of the segment from the vertex perpendicular to the center of the base. For a right circular cone, this height joins the vertex and the center of the base.

The *slant height* is the length of the segment from the vertex to any point on the circumference of the circular base. The slant height is used to find the lateral surface area of a cone.

ADDITIONAL EXAMPLE 2

A model of a volcano is in the shape of a cone. The model has a circular base with a diameter of 48 centimeters and a height of 12 centimeters. Find the volume of the cone in the model to the nearest tenth. Use 3.14 for π.

7234.6 cm^3

Interactive Whiteboard
Interactive example available online

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YOUR TURN

Avoid Common Errors

If students use a calculator, remind them to enter 3.14 instead of using the π key.

Focus on Technology

Discuss the use of the π key if the instruction to use 3.14 were not given. Have students consider how much error is introduced by using 3.14 for π. Remind students that π is irrational, and even the value used by the calculator is inexact. When students are expected to use the π key, they will still need to round the answer to a specified precision.

EXAMPLE 2

Questioning Strategies CC Mathematical Practices

- Why is the first step to divide 12 by 2? The value of 12 is for the diameter; the formula uses the radius.
- After you substitute the values, what do you do first in the calculation? Explain. Begin by applying the exponent, following the PEMDAS rule for the order of operations.

Engage with the Whiteboard

Have a student draw and label a figure for this Example next to step 1.

YOUR TURN

Avoid Common Errors

Students may get arithmetic errors when calculating with large numbers. Before they solve the problem, have them estimate the answer. The radius is about 200 meters, so the radius squared is about 40,000 m^2. Multiplying this value by the height, which is about 400 meters, gives 16,000,000 m^3. Multiplying by π and then multiplying by $\frac{1}{3}$ roughly cancel out. Since all the measurements in this estimate were rounded down, the volume should be larger than but within two-fold of 16,000,000 m^3.

Elaborate

Talk About It

Summarize the Lesson

Ask: What step is similar when you find the volume of a cone, a cylinder, or a prism? You have to find the area of the base and multiply by the height. What do you think is the most important difference between a cone and a cylinder or prism when you need to find the volume? The volume for a cone is one-third of the product of the base area and height.

GUIDED PRACTICE

Engage with the Whiteboard

Exercise 1 Have students draw both figures and label them. Note that they do not need to know the radius of the base for each figure because the base area is given.

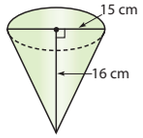
Avoid Common Errors

Exercises 3–4 Have students, before they substitute or calculate, write the formula and write the value of the radius. This will help students who may use the diameter instead of the radius or who may forget to include the $\frac{1}{3}$ in the formula.

YOUR TURN

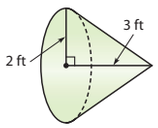
Find the volume of each cone. Round your answers to the nearest tenth. Use 3.14 for π.

4.



942 cm³

5.



12.6 ft³

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Finding the Volume of a Volcano

The mountain created by a volcano is often cone–shaped.

Math On the Spot
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EXAMPLE 2 Real World COMMON CORE 8.G.9

For her geography project, Karen built a clay model of a volcano in the shape of a cone. Her model has a diameter of 12 inches and a height of 8 inches. Find the volume of clay in her model to the nearest tenth. Use 3.14 for π.

STEP 1 Find the radius.

$r = \frac{12}{2} = 6$ in.

STEP 2 Find the volume of clay.

$V = \frac{1}{3}\pi r^2 h$

$\approx \frac{1}{3} \cdot 3.14 \cdot 6^2 \cdot 8$ Substitute.

$\approx \frac{1}{3} \cdot 3.14 \cdot 36 \cdot 8$ Simplify.

≈ 301.44 Multiply.

The volume of the clay is about 301.4 in³.

YOUR TURN

6. The cone of the volcano Parícutin in Mexico had a height of 410 meters and a diameter of 424 meters. Approximate the volume of the cone.

about 19,286,968.5 m³

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Guided Practice

1. The area of the base of a cylinder is 45 square inches and its height is 10 inches. A cone has the same area for its base and the same height. What is the volume of the cone? (Explore Activity)

$V_{cylinder} = Bh = 45 \cdot 10 = 450$

$V_{cone} = \frac{1}{3} V_{cylinder}$

$= \frac{1}{3} 450$

$= 150$

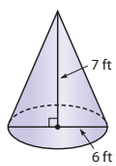
The volume of the cone is 150 in³.

2. A cone and a cylinder have congruent height and bases. The volume of the cone is 18 m³. What is the volume of the cylinder? Explain. (Explore Activity)

54 m³; the volume of a cylinder is 3 times the volume of a cone with a congruent base and height.

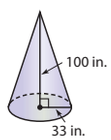
Find the volume of each cone. Round your answer to the nearest tenth if necessary. Use 3.14 for π. (Example 1)

3.



65.9 ft³

4.



113,982 in³

5. Gretchen made a paper cone to hold a gift for a friend. The paper cone was 15 inches high and had a radius of 3 inches. Find the volume of the paper cone to the nearest tenth. Use 3.14 for π. (Example 2)

141.3 in³

6. A cone-shaped building is commonly used to store sand. What would be the volume of a cone-shaped building with a diameter of 50 meters and a height of 20 meters? Round your answer to the nearest tenth. Use 3.14 for π. (Example 2)

13,083.3 m³

ESSENTIAL QUESTION CHECK-IN

7. How do you find the volume of a cone?

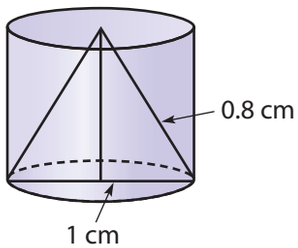
You can find one third of the volume of a cylinder with the same base and height, or you can use the formula $V = \frac{1}{3}\pi r^2 h$.

DIFFERENTIATE INSTRUCTION

Modeling

The tip of a sharpened pencil is shaped like a cone. How much of the pencil is lost after the tip is formed?

To answer this question, you should know that the volume of a cone is $\frac{1}{3}$ the volume of the cylinder from which it was formed.



The tip of this pencil was formed out of a cylinder with a height of 0.8 cm and a diameter of 1 cm.

The cylinder had a volume of approximately 0.63 cm³.

volume of cone $= \frac{1}{3} \cdot$ volume of cylinder

volume of cone $= \frac{1}{3} \cdot 0.63 = 0.21$

Since the tip of the pencil has a volume of 0.21 cm³, 0.42 cm³ was lost when the tip of the pencil was formed.

Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners ELL
- Reteach
- Challenge PRE-AP

Personal Math Trainer

Online Assessment and Intervention

Online homework assignment available

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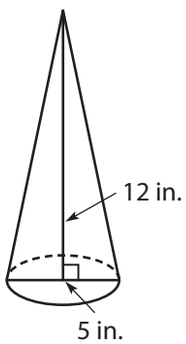
13.2 LESSON QUIZ

COMMON CORE **8.G.9**

Round your answers to the nearest tenth. Use 3.14 for π.

1. The volume of a cone is 20 cm^3. What is the volume of a cylinder with the same base and height?

2. Find the volume of the cone.



3. Find the volume of a cone with a radius of 20 inches and a height of 25 inches.

4. A paper cup in the shape of a cone has a diameter of 6 centimeters and is 7 centimeters high. Ken needs to add about 264 cm^3 of water to his plaster mixture. How many paper cups of water will he need to use?

Lesson Quiz available online

my.hrw.com

Answers

1. 60 cm^3

2. 78.5 in^3

3. 10,466.7 in^3

4. 4 cups

Evaluate

GUIDED AND INDEPENDENT PRACTICE

COMMON CORE **8.G.9**

Concepts & Skills	Practice
Explore Activity Modeling the Volume of a Cone	Exercises 1–2
Example 1 Finding the Volume of a Cone Using a Formula	Exercises 3–4, 8–11, 16–17
Example 2 Finding the Volume of a Volcano	Exercises 5–6, 12–15

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
8–11	**2** Skills/Concepts	**MP.5** Using Tools
12–15	**2** Skills/Concepts	**MP.4** Modeling
16–18	**2** Skills/Concepts	**MP.2** Reasoning
19	**3** Strategic Thinking H.O.T.	**MP.3** Logic
20	**3** Strategic Thinking H.O.T.	**MP.2** Reasoning
21–22	**3** Strategic Thinking H.O.T.	**MP.3** Logic

Additional Resources

Differentiated Instruction includes:

- Leveled Practice worksheets

Name _____ Class _____ Date _____

13.2 Independent Practice

COMMON CORE 8.G.9

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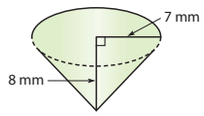
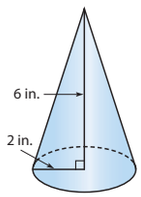
Find the volume of each cone. Round your answers to the nearest tenth if necessary. Use 3.14 for π.

8.



410.3 mm³

9.



25.1 in³

10. A cone has a diameter of 6 centimeters and a height of 11.5 centimeters.

108.3 cm³

11. A cone has a radius of 3 meters and a height of 10 meters.

94.2 m³

Round your answers to the nearest tenth if necessary. Use 3.14 for π.

12. Antonio is making mini waffle cones. Each waffle cone is 3 inches high and has a radius of $\frac{3}{4}$ inch. What is the volume of a waffle cone?

1.8 in³

13. A snack bar sells popcorn in cone-shaped containers. One container has a diameter of 8 inches and a height of 10 inches. How many cubic inches of popcorn does the container hold?

167.5 in³

14. A volcanic cone has a diameter of 300 meters and a height of 150 meters. What is the volume of the cone?

3,532,500 m³

15. **Multistep** Orange traffic cones come in a variety of sizes. Approximate the volume, in cubic inches, of a traffic cone that has a height of 2 feet and a diameter of 10 inches. Use 3.14 for π.

628 in³

Find the missing measure for each cone. Round your answers to the nearest tenth if necessary. Use 3.14 for π.

16. radius = 4 in.

height = 6 in.

volume = 100.48 in³

17. diameter = 6 cm

height = 6 cm

volume = 56.52 cm³

18. The diameter of a cone-shaped container is 4 inches, and its height is 6 inches. How much greater is the volume of a cylinder-shaped container with the same diameter and height? Round your answer to the nearest hundredth. Use 3.14 for π.

50.24 in³

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

19. Alex wants to know the volume of sand in an hourglass. When all the sand is in the bottom, he stands a ruler up beside the hourglass and estimates the height of the cone of sand.

a. What else does he need to measure to find the volume of sand?

either the diameter or the radius of the base

b. **Make a Conjecture** If the volume of sand is increasing at a constant rate, is the height increasing at a constant rate? Explain.

No; the cone is tapered as it goes from top to bottom. An equal volume of sand has a smaller radius and a greater height as the sand rises.

20. **Problem Solving** The diameter of a cone is x cm, the height is 18 cm, and the volume is 301.44 cm³. What is x? Use 3.14 for π.

$x = 8$ cm

21. **Analyze Relationships** A cone has a radius of 1 foot and a height of 2 feet. How many cones of liquid would it take to fill a cylinder with a diameter of 2 feet and a height of 2 feet? Explain.

Since the radius and height of the cones and cylinder are the same, it will take 3 cones to equal the volume of the cylinder.

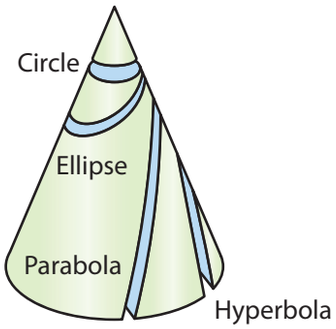
22. **Critique Reasoning** Herb knows that the volume of a cone is one third that of a cylinder with the same base and height. He reasons that a cone with the same height as a given cylinder but 3 times the radius should therefore have the same volume as the cylinder, since $\frac{1}{3} \cdot 3 = 1$. Is Herb correct? Explain.

No; the volume of the cone will be 3 times that of the cylinder because the radius is squared in the formula. For example, for a cylinder with radius 1 and height 5, $V = \pi\,(1)^2(5) = 5\pi$. For a cone with radius 3 and height 5, $V = \frac{1}{3}\pi\,(3)^2(5) = 15\pi$, or 3 times as much.

EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity The activity in the previous lesson explored the cross sections formed when a plane intersects a cylinder. The three shapes that can be obtained are a rectangle, a circle, and an ellipse.



Have students explore the cross sections of a right circular cone, either by using drawings or making a cone of clay or plastic foam, and cutting it in various ways. What two cross sections of a cylinder can also be obtained from a cone? circle, ellipse What shape cannot be obtained? rectangle What is the new shape obtained if the cut is through the vertex and perpendicular to the base? triangle Students will also find that they can generate curved shapes that are either parabolas or hyperbolas. Collectively, these cross sections are known as the *conic sections*.

LESSON

13.3 Volume of Spheres

Common Core Standards

The student is expected to:

COMMON CORE **Geometry—8.G.9**

Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Mathematical Practices

COMMON CORE **MP.6** Precision

Engage

ESSENTIAL QUESTION

How do you find the volume of a sphere? Sample answer: Find the product of $\frac{4}{3}$, the cube of the radius, and π.

Motivate the Lesson

Ask: What are some sports that are played with a ball? What is the mathematical name for this shape? How can you find the volume of a ball?

Explore

EXPLORE ACTIVITY

Engage with the Whiteboard

Have a student use a red marker to circle the height for each figure (to emphasize that *h* is the same for each). Then have a student use a blue marker to circle the three radii.

Explain

EXAMPLE 1

Connect Vocabulary ELL

Students may mispronounce *sphere* as *spere*. Remind them that the *ph* sound is an *f*, so this word is correctly pronounced *sfere*. Have them consider other examples of this sound, such as phone, graph and sphinx.

Questioning Strategies CC Mathematical Practices

- How many variables are used when finding the volume of a sphere? one: the radius, *r*
- Describe the difference between the information in part A and that in part B. A shows the length of the radius; B shows the length of the diameter.
- What mathematical symbols change when you substitute values into the formula for the volume of a sphere? Numbers take the place of variables and, because the value for π is approximate, the $=$ becomes $\approx$.

YOUR TURN

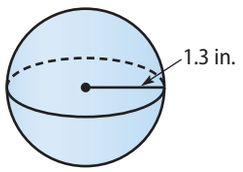
Avoid Common Errors

To avoid using the wrong value for the radius, have students circle the word *radius* or underline the word *diameter* twice in the problem before they substitute values.

ADDITIONAL EXAMPLE 1

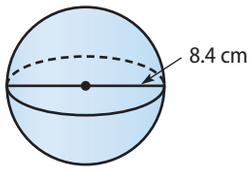
Find the volume of each sphere. Round your answers to the nearest tenth if necessary. Use 3.14 for π.

A



9.2 in^3

B



310.2 cm^3

Interactive Whiteboard
Interactive example available online

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LESSON

13.3 Volume of Spheres

COMMON CORE 8.G.9

Know the formulas for the volumes of…spheres and use them to solve real-world and mathematical problems.

ESSENTIAL QUESTION

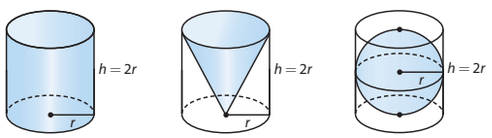
How do you find the volume of a sphere?

EXPLORE ACTIVITY COMMON CORE 8.G.9

Modeling the Volume of a Sphere

A **sphere** is a three-dimensional figure with all points the same distance from the center. The **radius** of a sphere is the distance from the center to any point on the sphere.

You have seen that a cone fills $\frac{1}{3}$ of a cylinder of the same radius and height h. If you were to do a similar experiment with a sphere of the same radius, you would find that a sphere fills $\frac{2}{3}$ of the cylinder. The cylinder's height is equal to twice the radius of the sphere.



STEP 1 Write the formula $V = Bh$ for each shape. Use $B = \pi r^2$ and substitute the fractions you know for the cone and sphere.

Cylinder	Cone	Sphere
$V = \pi r^2 h$	$V = \frac{1}{3}\pi r^2 h$	$V = \frac{2}{3}\pi r^2 h$

STEP 2 Notice that a sphere always has a height equal to twice the radius. Substitute $2r$ for h. $V = \frac{2}{3}\pi r^2(2r)$

STEP 3 Simplify this formula for the volume of a sphere. $V = \boxed{\frac{4}{3}}\pi r^3$

Reflect

1. **Analyze Relationships** A cone has a radius of r and a height of $2r$. A sphere has a radius of r. Compare the volume of the sphere and cone.

The cone's volume is $\frac{1}{3}$ of a cylinder with radius r and height $2r$. The sphere's volume is $\frac{2}{3}$ of the volume of this cylinder.

So, the sphere's volume is twice the cone's volume.

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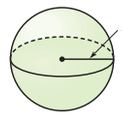
Finding the Volume of a Sphere Using a Formula

The Explore Activity illustrates a formula for the volume of a sphere with radius r.

Volume of a Sphere

The volume V of a sphere is $\frac{4}{3}\pi$ times the cube of the radius r.

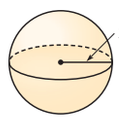
$V = \frac{4}{3}\pi r^3$



EXAMPLE 1 COMMON CORE 8.G.9

Find the volume of each sphere. Round your answers to the nearest tenth if necessary. Use 3.14 for π.

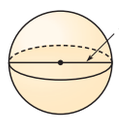
A



$V = \frac{4}{3}\pi r^3$
$\approx \frac{4}{3} \cdot 3.14 \cdot 2.1^3$ Substitute.
$\approx \frac{4}{3} \cdot 3.14 \cdot 9.26$ Simplify.
≈ 38.8 Multiply.

The volume is about 38.8 cm^3.

B



Since the diameter is 7 cm, the radius is 3.5 cm.

$V = \frac{4}{3}\pi r^3$
$\approx \frac{4}{3} \cdot 3.14 \cdot 3.5^3$ Substitute.
$\approx \frac{4}{3} \cdot 3.14 \cdot 42.9$ Simplify.
≈ 179.6 Multiply.

The volume is about 179.6 cm^3.

Math Talk
Mathematical Practices

If you know the diameter of a sphere, how would the formula for the volume of a sphere be written in terms of d?

$V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$

YOUR TURN

Find the volume of each sphere. Round your answers to the nearest tenth. Use 3.14 for π.

2. A sphere has a radius of 10 centimeters. 4186.7 cm^3
3. A sphere has a diameter of 3.4 meters. 20.6 m^3

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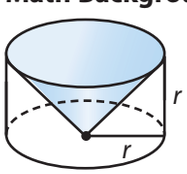
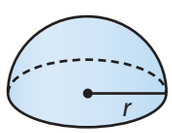
PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices MP.6

This lesson provides an opportunity to address this Mathematical Practices standard. It calls for students to communicate mathematics precisely. Students explore the relationship between the volumes of cylinders, cones, and spheres. Students learn to express the volumes through formulas and to explain the differences and similarities in the coefficients and variables in the formulas.

Math Background





Archimedes derived the formula for the volume of a sphere by showing that a hemisphere with radius r and the solid remaining when the cone is removed from a cylinder with radius and height r have equal volumes.

$(\pi r^2)r - \frac{1}{3}(\pi r^2)r = \frac{2}{3}(\pi r^2)r = \frac{2}{3}\pi r^3$.

The volume of the sphere is thus twice that of the hemisphere, or $2\left(\frac{2}{3}\pi r^3\right) = \frac{4}{3}\pi r^3$.

ADDITIONAL EXAMPLE 2

A steel ball bearing has a diameter of 1.6 centimeters. What is the volume of this steel ball? Round your answer to the nearest tenth if necessary. Use 3.14 for π.

2.1 cm^3

Interactive Whiteboard
Interactive example available online

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EXAMPLE 2

Questioning Strategies CC Mathematical Practices

- Explain whether cubing the diameter first and then dividing by 2 would give the same answer. No; 11 cubed is 1331, but $22^3 = 10{,}648$, and half of that (5324) is much greater than 1331.
- How could you write the answer so that it is an exact value? $(1774\frac{2}{3})\pi$

Engage with the Whiteboard

Have a student draw and label a sphere to represent the ball described.

YOUR TURN

Focus on Critical Thinking

One student found the volume to be 7234.6 in^3 instead of the correct answer. Describe an error that might have led to this result. This answer could result from using the diameter instead of the radius to calculate the volume.

Elaborate

Talk About It

Summarize the Lesson

Ask: What do you need to know about a sphere to be able to calculate its volume? You need to know the radius or be able to determine the radius from the diameter or the circumference.

GUIDED PRACTICE

Engage with the Whiteboard

In Exercises 5–8, draw diagrams of the spheres and label the diameter or radius as appropriate.

Avoid Common Errors

Exercise 8 Students may not find the radius correctly when given the circumference. Have students find the diameter first, and then find the radius from the diameter. Students should use a calculator when finding the radius. Answers may vary slightly, depending on how students choose to round the value found for the radius.

Finding the Volume of a Sphere in a Real-World Context

Many sports, including golf and tennis, use a ball that is spherical in shape.

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EXAMPLE 2

COMMON CORE 8.G.9

Soccer balls come in several different sizes. One soccer ball has a diameter of 22 centimeters. What is the volume of this soccer ball? Round your answer to the nearest tenth. Use 3.14 for π.

STEP 1 Find the radius.

$r = \frac{d}{2} = 11$ cm

STEP 2 Find the volume of the soccer ball.

$V = \frac{4}{3}\pi r^3$

$\approx \frac{4}{3} \cdot 3.14 \cdot 11^3$ Substitute.

$\approx \frac{4}{3} \cdot 3.14 \cdot 1331$ Simplify.

≈ 5572.4533 Multiply.

The volume of the soccer ball is about 5572.5 cm^3.

Reflect

4. What is the volume of the soccer ball in terms of π, to the nearest whole number multiple? Explain your answer.

1,775π; Sample answer: I multiplied $\frac{4}{3}$ and 11^3 and rounded.

5. **Analyze Relationships** The diameter of a basketball is about 1.1 times that of a soccer ball. The diameter of a tennis ball is about 0.3 times that of a soccer ball. How do the volumes of these balls compare to that of a soccer ball? Explain.

Basketball: about 1.3 times as big; tennis ball: about 0.03 times as big. The radius is cubed in the formula, so compared to the soccer ball, $1.1^3 \approx 1.3$, and $0.3^3 \approx 0.03$.

YOUR TURN

6. Val measures the diameter of a ball as 12 inches. How many cubic inches of air does this ball hold, to the nearest tenth? Use 3.14 for π.

904.3 in^3

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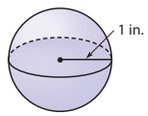
Guided Practice

1. **Vocabulary** A sphere is a three-dimensional figure with all points the same distance from the center. (Explore Activity)

2. **Vocabulary** The radius is the distance from the center of a sphere to a point on the sphere. (Explore Activity)

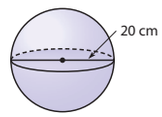
Find the volume of each sphere. Round your answers to the nearest tenth if necessary. Use 3.14 for π. (Example 1)

3.



4.2 in^3

4.



4,186.7 cm^3

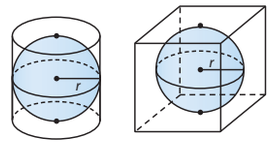
5. A sphere has a radius of 1.5 feet. 14.1 ft^3

6. A sphere has a diameter of 2 yards. 4.2 yd^3

7. A baseball has a diameter of 2.9 inches. Find the volume of the baseball. Round your answer to the nearest tenth if necessary. Use 3.14 for π. (Example 2) 12.8 in^3

8. A basketball has a radius of 4.7 inches. What is its volume to the nearest cubic inch. Use 3.14 for π. (Example 2) 435 in^3

9. A company is deciding whether to package a ball in a cubic box or a cylindrical box. In either case, the ball will touch the bottom, top, and sides. (Explore Activity)



a. What portion of the space inside the cylindrical box is empty? Explain.

$\frac{1}{3}$; the ball takes up $\frac{2}{3}$ of the space, so $\frac{1}{3}$ is empty.

b. Find an expression for the volume of the cubic box. $(2r)^3 = 8r^3$

c. About what portion of the space inside the cubic box is empty? Explain.

Almost $\frac{1}{2}$; the empty space is $8r^3 - \left(\frac{4}{3}\right)\pi r^3$, or about $3.81r^3$, and $\frac{3.81}{8} \approx 0.48$.

ESSENTIAL QUESTION CHECK-IN

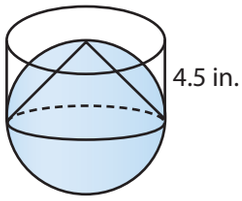
10. Explain the steps you use to find the volume of a sphere.

Find the radius. Then substitute r into the formula $V = \frac{4}{3}\pi r^3$ and simplify.

DIFFERENTIATE INSTRUCTION

Modeling

The radius of a basketball is about 4.5 inches. To find the volume of a basketball, imagine the ball is sliced into two halves. Then find the volume of one half and multiply times 2.



The volume of a hemisphere is exactly halfway between the volume of a cone and the volume of a cylinder, which both have the same radius r as the hemisphere and a height equal to r.

Cognitive Strategies

Ask students in small groups to brainstorm which mnemonic devices they might use to remember the different formulas for the volumes of a cylinder, a cone, and a sphere. Invite them to create rhymes, raps, cartoons, etc. to help them remember these three different but related formulas and the shapes for which they apply.

Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners ELL
- Reteach
- Challenge PRE-AP

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Online Assessment and Intervention

Online homework assignment available

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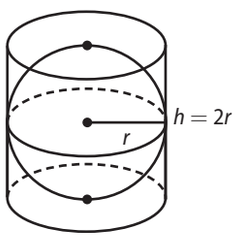
13.3 LESSON QUIZ

COMMON CORE **8.G.9**

Round your answers to the nearest tenth if necessary. Use 3.14 for π.

1. A ball fits exactly into a cylinder as shown in the figure. The volume of the cylinder is 30 cm^3. What is the volume of the sphere?



2. Find the volume of a sphere with a radius of 2.6 inches.

3. Find the volume of a sphere with a diameter of 8.2 meters.

4. Jen has a silver charm on her bracelet in the shape of a soccer ball with a radius of 1 centimeter. What is the volume of this charm?

5. A ball has a circumference of 37.68 inches. What is the volume of the ball?

Lesson Quiz available online

my.hrw.com

Answers

1. 20 cm^3

2. 73.6 in^3

3. 288.5 m^3

4. 4.2 cm^3

5. 904.3 in^3

Evaluate

GUIDED AND INDEPENDENT PRACTICE

COMMON CORE **8.G.9**

Concepts & Skills	Practice
Explore Activity Modeling the Volume of a Sphere	Exercises 1–2, 9
Example 1 Finding the Volume of a Sphere Using a Formula	Exercises 3–6, 11–16
Example 2 Finding the Volume of a Sphere in a Real-World Context	Exercises 7–8, 17–20

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
11–16	**2** Skills/Concepts	**MP.5** Using Tools
17–19	**2** Skills/Concepts	**MP.4** Modeling
20	**3** Strategic Thinking H.O.T.	**MP.1** Problem Solving
21	**2** Skills/Concepts	**MP.6** Precision
22	**3** Strategic Thinking H.O.T.	**MP.7** Using Structure
23	**2** Skills/Concepts	**MP.2** Reasoning
24	**3** Strategic Thinking H.O.T.	**MP.3** Logic
25	**3** Strategic Thinking H.O.T.	**MP.4** Modeling
26	**3** Strategic Thinking H.O.T.	**MP.2** Reasoning
27	**3** Strategic Thinking H.O.T.	**MP.3** Logic

Additional Resources

Differentiated Instruction includes:

- Leveled Practice worksheets

CC CLUSTER CONNECTION **Exercise 23** combines concepts from the Common Core cluster "Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres."

Name _____ Class _____ Date _____

13.3 Independent Practice

COMMON CORE 8.G.9

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Find the volume of each sphere. Round your answers to the nearest tenth if necessary. Use 3.14 for π.

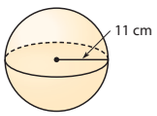
11. radius of 3.1 meters 124.7 m^3

12. diameter of 18 inches 3,052.1 in^3

13. $r = 6$ in. 904.3 in^3

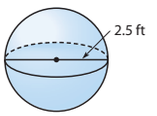
14. $d = 36$ m 24,416.6 m^3

15.



5572.5 cm^3

16.



8.2 ft^3

The eggs of birds and other animals come in many different shapes and sizes. Eggs often have a shape that is nearly spherical. When this is true, you can use the formula for a sphere to find their volume.

17. The green turtle lays eggs that are approximately spherical with an average diameter of 4.5 centimeters. Each turtle lays an average of 113 eggs at one time. Find the total volume of these eggs, to the nearest cubic centimeter.

5389 cm^3

18. Hummingbirds lay eggs that are nearly spherical and about 1 centimeter in diameter. Find the volume of an egg. Round your answer to the nearest tenth.

0.5 cm^3

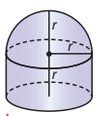
19. Fossilized spherical eggs of dinosaurs called titanosaurid sauropods were found in Patagonia. These eggs were 15 centimeters in diameter. Find the volume of an egg. Round your answer to the nearest tenth.

1766.3 cm^3

20. Persevere in Problem Solving An ostrich egg has about the same volume as a sphere with a diameter of 5 inches. If the eggshell is about $\frac{1}{12}$ inch thick, find the volume of just the shell, not including the interior of the egg. Round your answer to the nearest tenth.

6.3 in^3

21. Multistep Write the steps you would use to find a formula for the volume of the figure at right. Then write the formula.



Find the volume of a sphere with radius r: $V = \frac{4}{3}\pi r^3$. Divide by 2 to find the volume of the hemisphere: $V = \frac{2}{3}\pi r^3$. Find the volume of the cylinder: $V = \pi r^2 h = \pi r^3$. Note that $h = r$. Add the volume of the hemisphere and the volume of the cylinder: $V = \frac{2}{3}\pi r^3 + \pi r^3 = \frac{5}{3}\pi r^3$.

22. Critical Thinking Explain what happens to the volume of a sphere if you double the radius.

The volume is multiplied by 8 (or 2 cubed).

23. Multistep A cylindrical can of tennis balls holds a stack of three balls so that they touch the can at the top, bottom, and sides. The radius of each ball is 1.25 inches. Find the volume inside the can that is not taken up by the three tennis balls.

12.3 in^3

H.O.T. FOCUS ON HIGHER ORDER THINKING

24. Critique Reasoning A sphere has a radius of 4 inches, and a cube-shaped box has an edge length of 7.5 inches. J.D. says the box has a greater volume, so the sphere will fit in the box. Is he correct? Explain.

No; the box has a greater volume, but it would need an edge length of 8 inches, the diameter of the sphere, for the sphere to fit inside the box.

25. Critical Thinking Which would hold the most water: a bowl in the shape of a hemisphere with radius r, a cylindrical glass with radius r and height r, or a cone-shaped drinking cup with radius r and height r? Explain.

The cylindrical glass; the cylinder has a volume of πr^3, while the hemisphere's volume is $\frac{2}{3}\pi r^3$, and the cone's volume is $\frac{1}{3}\pi r^3$.

26. Analyze Relationships Hari has models of a sphere, a cylinder, and a cone. The sphere's diameter and the cylinder's height are the same, $2r$. The cylinder has radius r. The cone has diameter $2r$ and height $2r$. Compare the volumes of the cone and the sphere to the volume of the cylinder.

The volume of the cone is one-sixth the volume of the cylinder. The volume of the sphere is two-thirds the volume of the cylinder.

27. A spherical helium balloon that is 8 feet in diameter can lift about 17 pounds. What does the diameter of a balloon need to be to lift a person who weighs 136 pounds? Explain.

About 16 feet; 136 is 8 times 17, so the volume must be 8 times as big. Because $2^3 = 8$, this means that the radius, and thus the diameter, must be twice as big.

Work Area

EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

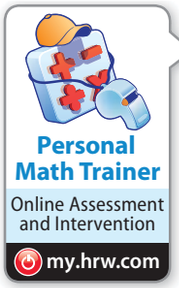
Activity Use a sphere of clay or plastic foam, and intersect, or cut, it with a plane passing through the center of the sphere. The cross section formed is a *great circle* of the sphere, the largest cross section for that sphere. The distance along the edge of a great circle is the shortest distance between two points on the sphere. Examine a globe. What is the name for the special great circle on Earth from which latitude is measured? equator How many different intersections are possible that result in a cross section that is a great circle? infinitely many Are all great circles of a given sphere congruent? yes Examine a flat map of the world. What is the shortest route from New York to Beijing, China? Now look at a globe. Do you see a shorter route? Help students see that the shortest route from New York to Beijing is a great circle route that passes over the Arctic regions.

Ready to Go On?

Assess Mastery

Use the assessment on this page to determine if students have mastered the concepts and standards covered in this module.

RtI Response to Intervention



Intervention	Enrichment

Access Ready to Go On? assessment online, and receive instant scoring, feedback, and customized intervention or enrichment.

Online and Print Resources

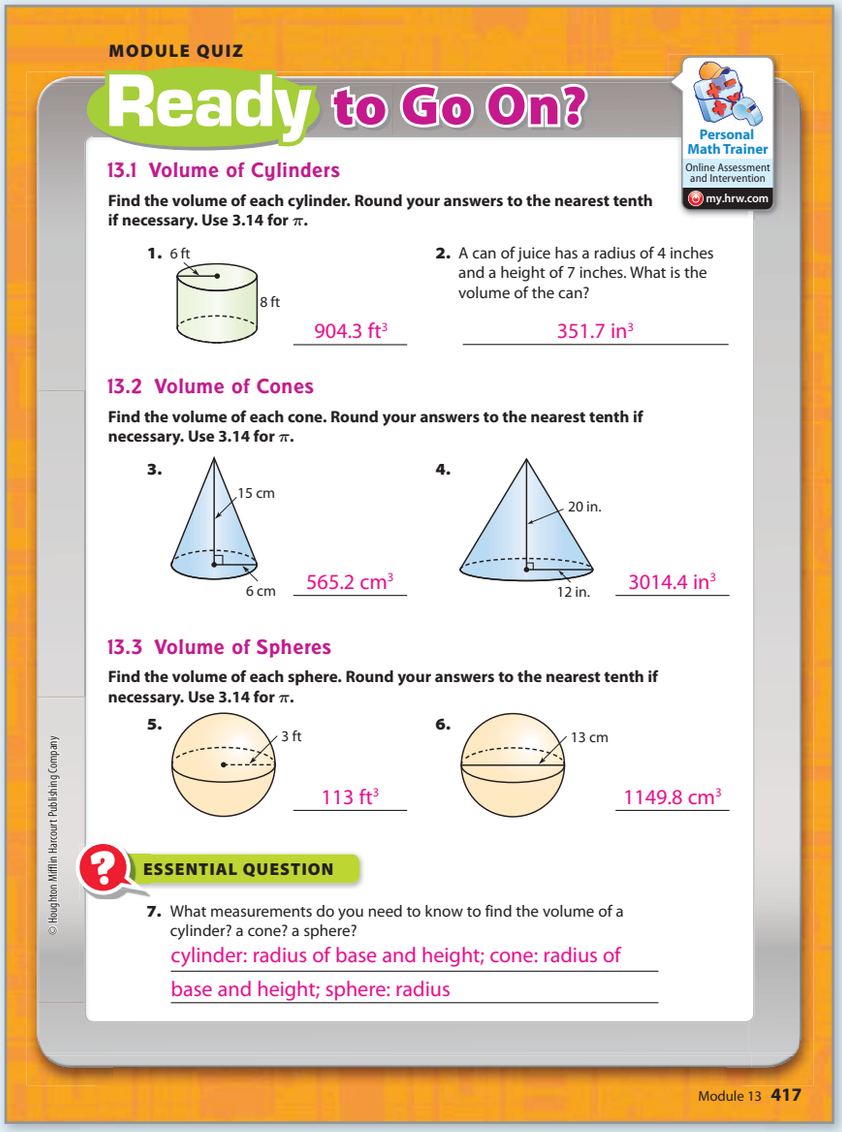
Differentiated Instruction
- Reteach worksheets
- Reading Strategies ELL
- Success for English Learners ELL

Differentiated Instruction
- Challenge worksheets PRE-AP

Extend the Math PRE-AP Lesson Activities in TE

Additional Resources

Assessment Resources include:
- Leveled Module Quizzes



MODULE QUIZ

Ready to Go On?

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13.1 Volume of Cylinders

Find the volume of each cylinder. Round your answers to the nearest tenth if necessary. Use 3.14 for π.

1. 6 ft, 8 ft — 904.3 ft³

2. A can of juice has a radius of 4 inches and a height of 7 inches. What is the volume of the can? — 351.7 in³

13.2 Volume of Cones

Find the volume of each cone. Round your answers to the nearest tenth if necessary. Use 3.14 for π.

3. 15 cm, 6 cm — 565.2 cm³

4. 20 in., 12 in. — 3014.4 in³

13.3 Volume of Spheres

Find the volume of each sphere. Round your answers to the nearest tenth if necessary. Use 3.14 for π.

5. 3 ft — 113 ft³

6. 13 cm — 1149.8 cm³

ESSENTIAL QUESTION

7. What measurements do you need to know to find the volume of a cylinder? a cone? a sphere?

cylinder: radius of base and height; cone: radius of base and height; sphere: radius

Module 13 417

Common Core Standards

Lesson	Exercises	Common Core Standards
13.1	1–2	**8.G.9**
13.2	3–4	**8.G.9**
13.3	5–6	**8.G.9**

Assessment Readiness

Assessment Readiness Tip Students can use estimation to eliminate some of the answer choices.

Item 2 Students can round 11.4 down to 10, 10.7 down to 10, and 3.14 down to 3. This makes the estimate for the volume of the cylinder $3(5)^2(10) = 750$. Because all numbers were rounded down, the estimate is less than the actual volume. Answer choices A and B are less than the estimate, and D is several times larger than the estimate. C is the only reasonable answer.

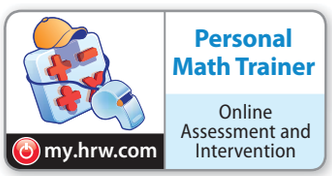
Item 4 Students can round 17 to 20, 6 to 5, and 3.14 to 3, and then plug into the formula to find $\frac{1}{3}(3)(5)^2(20) = 500$. The only answer choice close to 500 is B, 640.6.

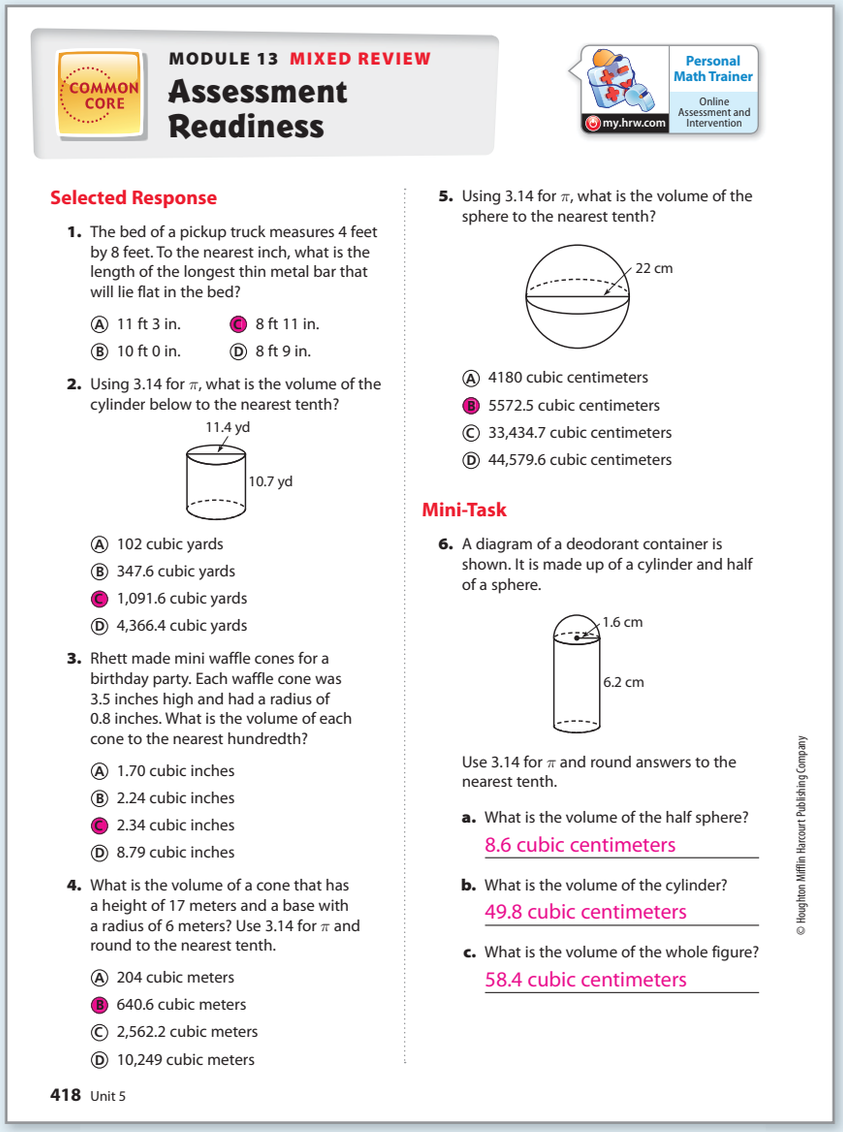
Avoid Common Errors

Item 3 Remind students that the volumes of both cylinders and cones are defined by radius and height, and they need to read the problem carefully to decide which formula to use. Some students may use the formula for the volume of a cylinder rather than the volume of a cone.

Item 6 Students may correctly find the volume of the sphere but forget to divide it by 2 to make it a half-sphere. Remind students to confirm that their numerical answer corresponds to the question that was asked.

Additional Resources





MODULE 13 MIXED REVIEW

Assessment Readiness

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Selected Response

1. The bed of a pickup truck measures 4 feet by 8 feet. To the nearest inch, what is the length of the longest thin metal bar that will lie flat in the bed?

Ⓐ 11 ft 3 in. Ⓒ 8 ft 11 in.
Ⓑ 10 ft 0 in. Ⓓ 8 ft 9 in.

2. Using 3.14 for π, what is the volume of the cylinder below to the nearest tenth?

11.4 yd
10.7 yd

Ⓐ 102 cubic yards
Ⓑ 347.6 cubic yards
Ⓒ 1,091.6 cubic yards
Ⓓ 4,366.4 cubic yards

3. Rhett made mini waffle cones for a birthday party. Each waffle cone was 3.5 inches high and had a radius of 0.8 inches. What is the volume of each cone to the nearest hundredth?

Ⓐ 1.70 cubic inches
Ⓑ 2.24 cubic inches
Ⓒ 2.34 cubic inches
Ⓓ 8.79 cubic inches

4. What is the volume of a cone that has a height of 17 meters and a base with a radius of 6 meters? Use 3.14 for π and round to the nearest tenth.

Ⓐ 204 cubic meters
Ⓑ 640.6 cubic meters
Ⓒ 2,562.2 cubic meters
Ⓓ 10,249 cubic meters

5. Using 3.14 for π, what is the volume of the sphere to the nearest tenth?

22 cm

Ⓐ 4180 cubic centimeters
Ⓑ 5572.5 cubic centimeters
Ⓒ 33,434.7 cubic centimeters
Ⓓ 44,579.6 cubic centimeters

Mini-Task

6. A diagram of a deodorant container is shown. It is made up of a cylinder and half of a sphere.

1.6 cm
6.2 cm

Use 3.14 for π and round answers to the nearest tenth.

a. What is the volume of the half sphere?
8.6 cubic centimeters

b. What is the volume of the cylinder?
49.8 cubic centimeters

c. What is the volume of the whole figure?
58.4 cubic centimeters

418 Unit 5

Common Core Standards

Items	Grade 8 Standards	Mathematical Practices
1*	8.G.7	MP.4
2	8.G.9	MP.4
3	8.G.9	MP.4
4	8.G.9	MP.2
5	8.G.9	MP.4
6	8.G.9	MP.4

* Item integrates mixed review concepts from previous modules or a previous course.