

Scatter Plots

MODULE

COMMON
CORE

14



ESSENTIAL QUESTION

How can you use scatter plots to solve real-world problems?

You can use scatter plots to find the relationships between two sets of real-world data.

LESSON 14.1

Scatter Plots and Association

COMMON CORE 8.SP.1

LESSON 14.2

Trend Lines and Predictions

COMMON CORE 8.SP.1, 8.SP.2, 8.SP.3



Real-World Video

An anthropologist measures dinosaur bones. To estimate a dinosaur's height based on the length of a bone, he can make a scatter plot comparing bone length and height of several dinosaurs.

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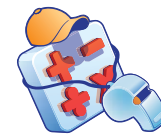
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Interactively explore key concepts to see how math works.



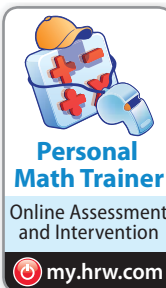
Personal Math Trainer

Get immediate feedback and help as you work through practice sets.

Are You Ready?

Assess Readiness

Use the assessment on this page to determine if students need intensive or strategic intervention for the module's prerequisite skills.



Intervention

Access Are You Ready? assessment online, and receive instant scoring, feedback, and customized intervention or enrichment.

Online and Print Resources

Skills Intervention worksheets

- Skill 54 Evaluate Expressions
- Skill 60 Solve Two-Step Equations

Enrichment

Differentiated Instruction

- Challenge worksheets

PRE-AP

Extend the Math **PRE-AP** Lesson Activities in TE

Are YOU Ready?

Complete these exercises to review skills you will need for this module.



Evaluate Expressions

EXAMPLE Evaluate $4x + 3$ for $x = 5$.

$$\begin{aligned} 4x + 3 &= 4(5) + 3 \\ &= 20 + 3 \\ &= 23 \end{aligned}$$

Substitute the given value for x .
Multiply.
Add.

Evaluate each expression for the given value of x .

- | | | |
|--|---|---|
| 1. $6x - 5$ for $x = 4$
<u>19</u> | 2. $-2x + 7$ for $x = 2$
<u>3</u> | 3. $5x - 6$ for $x = 3$
<u>9</u> |
| 4. $0.5x + 8.4$ for $x = -1$
<u>7.9</u> | 5. $\frac{3}{4}x - 9$ for $x = -20$
<u>-24</u> | 6. $1.4x + 3.5$ for $x = -4$
<u>-2.1</u> |

Solve Two-Step Equations

EXAMPLE $5x + 3 = -7$

$$\begin{aligned} 5x + 3 &= -7 \\ -3 &= -3 \\ 5x &= -10 \\ \frac{5x}{5} &= \frac{-10}{5} \\ x &= -2 \end{aligned}$$

Subtract 3 from both sides.

Divide both sides by 5.

Solve for x .

- | | | | |
|---------------------------------|---|---|---------------------------------------|
| 7. $3x + 4 = 10$
<u>2</u> | 8. $5x - 11 = 34$
<u>9</u> | 9. $-2x + 5 = -9$
<u>7</u> | 10. $8x + 13 = -11$
<u>-3</u> |
| 11. $4x - 7 = -27$
<u>-5</u> | 12. $\frac{1}{2}x + 16 = 39$
<u>46</u> | 13. $\frac{2}{3}x - 16 = 12$
<u>42</u> | 14. $0.5x - 1.5 = -6.5$
<u>-10</u> |

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PROFESSIONAL DEVELOPMENT VIDEO



Author Juli Dixon models successful teaching practices as she explores the concept of scatter plots in an actual eighth-grade classroom.



Professional Development

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Online Teacher Edition

Access a full suite of teaching resources online—plan, present, and manage classes and assignments.



ePlanner

Easily plan your classes and access all your resources online.



Interactive Answers and Solutions

Customize answer keys to print or display in the classroom. Choose to include answers only or full solutions to all lesson exercises.



Interactive Whiteboards

Engage students with interactive whiteboard-ready lessons and activities.



Personal Math Trainer: Online Assessment and Intervention

Assign automatically graded homework, quizzes, tests, and intervention activities. Prepare your students with updated practice tests aligned with Common Core.

Reading Start-Up

Have students complete the activities on this page by working alone or with others.

Visualize Vocabulary

The content chart helps students review slope to prepare them to graph trend lines. Students should write one or more review words in the cells in the right column of the chart. As a class, review the process for finding slope and define each of the terms represented in the chart.

Understand Vocabulary

Use the following explanation to help students learn the preview words.

In data analysis, a **cluster** is a set of closely grouped data. In a scatter plot, the dots showing a cluster are grouped close together. If they cluster in a linear fashion, you can draw a **trend line** to model the data. An **outlier** is a point that is very different from the others in the data set. On the scatterplot, an outlier is far away from the other dots and not close to the trend line.

Active Reading

Integrating Language Arts

Students can use these reading and note-taking strategies to help them organize and understand new concepts and vocabulary.

COMMON CORE

ELA-Literacy.RST.6-8.7 Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).

Additional Resources

Differentiated Instruction

• Reading Strategies **ELL**

Reading Start-Up

Visualize Vocabulary

Use the ✓ words to complete the right column of the chart.

Reviewing Slope	
Mathematical Representation	Review Word
$y = mx + b$	slope-intercept form of an equation, linear equation
y	y-coordinate
m	slope
x	x-coordinate
b	y-intercept

Understand Vocabulary

Match the term on the left to the correct expression on the right.

- | | | |
|---------------|---|---|
| 1. cluster | → | A. A data point that is very different from the rest of the data in a set |
| 2. outlier | → | B. A straight line that comes closest to the points on a scatter plot. |
| 3. trend line | → | C. A set of closely grouped data. |

Vocabulary

Review Words

- bivariate data (*datos bivariados*)
- data (*datos*)
- ✓ linear equation (*ecuación lineal*)
- ✓ slope (*pendiente*)
- ✓ slope-intercept form of an equation (*forma pendiente-intersección*)
- ✓ x-coordinate (*coordenada x*)
- ✓ y-coordinate (*coordenada y*)
- ✓ y-intercept (*intersección con el eje y*)

Preview Words

- cluster (*agrupación*)
- outlier (*valor extremo*)
- scatter plot (*diagrama de dispersión*)
- trend line (*línea de tendencia*)

Active Reading

Two-Panel Flip Chart Create a two-panel flip chart, to help you understand the concepts in this module. Label each flap with the title of one of the lessons in the module. As you study each lesson, write important ideas under the appropriate flap. Include any sample problems or equations that will help you remember the concepts later when you look back at your notes.



Module 14 431

Before

- Students understand:
- ways to display data
 - ways to solve problems using graphs of data
 - ways to compare two sets of data

In this module

- Students learn to:
- represent data in a scatter plot
 - describe associations in data in scatter plots
 - represent bivariate data in a scatter plot with a trend line
 - make predictions from a scatter plot or trend line

After

- Students will connect:
- scatter plots and linear and nonlinear associations
 - scatter plots and positive and negative associations
 - scatter plots and trend lines
 - trend lines and linear equations

Unpacking the Standards

Use the examples on the page to help students know exactly what they are expected to learn in this module.

Common Core Standards

Content Areas

COMMON CORE Statistics and Probability—8.SP

Investigate patterns of association in bivariate data.



Go online to see a complete unpacking of the Common Core Standards.

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MODULE 14

Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

COMMON CORE 8.SP.1

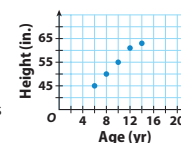
Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

What It Means to You

You will describe how the data in a scatter plot are related.

UNPACKING EXAMPLE 8.SP.1

The scatter plot shows Bob's height at various ages. Describe the type(s) of association between Bob's age and his height. Explain.



As Bob gets older, his height increases roughly along a straight line on the graph, so the association is positive and basically linear.

COMMON CORE 8.SP.2

Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

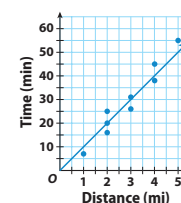
What It Means to You

You will use a trend line to show the relationship between two quantities.

UNPACKING EXAMPLE 8.SP.2

Joyce is training for a 10K race. For each of her training runs, she recorded the distance she ran and the time she ran. She made a scatter plot of her data and drew a trend line. Use the trend line to predict how long it would take Joyce to run 4.5 miles.

Distance (mi)	Time (min)
4	38
2	25
1	7
2	16
3	26
5	55
2	20
4	45
3	31



For a distance of 4.5 miles, the trend line shows a time of 45 minutes. So, it will take Joyce about 45 minutes to run 4.5 miles.



Visit my.hrw.com to see all the Common Core Standards unpacked.

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Common Core Standards

Lesson 14.1

Lesson 14.2

8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

COMMON CORE

COMMON CORE

8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

COMMON CORE

8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

COMMON CORE

LESSON

14.1 Scatter Plots and Association

Common Core Standards

The student is expected to:

COMMON CORE **Statistics and Probability—8.SP.1**

Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

Mathematical Practices

COMMON CORE **MP.7** Structure

Engage

ESSENTIAL QUESTION

How can you construct and interpret scatter plots? Sample answer: Plot bivariate data on a coordinate plane, with one variable represented by each axis. Look for positive or negative association, clusters, and outliers to interpret the data.

Motivate the Lesson

Ask: Do you think the grade you get on a test is related to how long you study for the test? Take a guess. Begin the Explore Activity to find out.

Explore

EXPLORE ACTIVITY 1

Focus on Math Connections **CC** Mathematical Practices

Emphasize that a scatter plot is used to investigate patterns of association between two quantities. This scatter plot shows hours spent studying as the independent variable and test grades as the dependent variable. Encourage students to discuss any trends they may see in the data in terms of these variables. It is important to keep in mind that an association between data sets does not mean that one data set causes the change in the other data set.

Explain

EXPLORE ACTIVITY 2

Questioning Strategies **CC** Mathematical Practices

- What are some reasons there may be clusters in the data? *When the variability of data is small, it tends to cluster. Multiple clusters may reflect the way a measurement was obtained or may indicate a relationship where a limited set of outcomes are more likely.*
- What is the maximum number of points that could constitute a cluster of points? *A cluster of points can be any number of points as long as they are grouped around a point or along a line.*
- Name a new point that would be considered an outlier for this data set. *Sample answer: A point at (30, 3) would be a clear outlier.*

Avoid Common Errors

A common error is to try to connect the points of a scatter plot. Point out that a scatter plot shows all of the collected data, and that it is incorrect and misleading to connect the points in a jagged line.

Talk About It

Check for Understanding

 **Ask:** How can you identify an outlier in a scatter plot? *An outlier can be identified in a scatter plot as a point that is separated from all other data points. It does not fall within any cluster.*

LESSON 14.1 Scatter Plots and Association

COMMON CORE 8.SP.1
Construct and interpret scatter plots. . . Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

ESSENTIAL QUESTION

How can you construct and interpret scatter plots?

EXPLORE ACTIVITY 1 **Real World** **COMMON CORE 8.SP.1**

Making a Scatter Plot

Recall that a set of bivariate data involves two variables. Bivariate data are used to explore the relationship between two variables. You can graph bivariate data on a *scatter plot*. A **scatter plot** is a graph with points plotted to show the relationship between two sets of data.

The final question on a math test reads, "How many hours did you spend studying for this test?" The teacher records the number of hours each student studied and the grade the student received on the test.

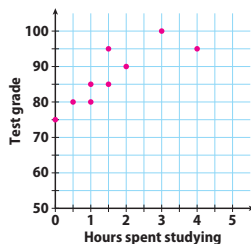
- A** Make a prediction about the relationship between the number of hours spent studying and test grades.
Sample answer: A greater number of study hours are likely to be associated with higher test grades.

- B** Make a scatter plot. Graph hours spent studying as the independent variable and test grades as the dependent variable.

Reflect

- What trend do you see in the data?
In general, test scores increase as the number of study hours increases.
- Justify Reasoning** Do you think the grade associated with studying for 10 hours would follow this trend?
No; the graph shows a general upward trend, but the grade cannot exceed 100.

Hours Spent Studying	Test Grade
0	75
0.5	80
1	80
1	85
1.5	85
1.5	95
2	90
3	100
4	90



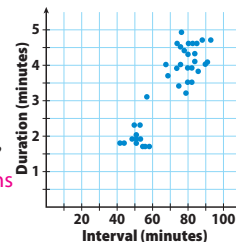
EXPLORE ACTIVITY 2 **Real World** **COMMON CORE 8.SP.1**

Interpreting Clusters and Outliers

A **cluster** is a set of closely grouped data. Data may cluster around a point or along a line. An **outlier** is a data point that is very different from the rest of the data in the set.

A scientist gathers information about the eruptions of Old Faithful, a geyser in Yellowstone National Park. She uses the data to create a scatter plot. The data show the length of time between eruptions (interval) and how long the eruption lasts (duration).

- A** Describe any clusters you see in the scatter plot.
There are clusters around the 50-minute and 80-minute intervals.
- B** What do the clusters tell you about eruptions of Old Faithful?
There are short wait times followed by short eruptions and longer wait times followed by longer eruptions.
- C** Describe any outliers you see in the scatter plot.
The point near (57, 3) appears to be an outlier because it does not fall into either cluster.



Math Talk

Mathematical Practices

Math Talk Anno: Yes; because it would not fall near either cluster, it would satisfy the conditions for being an outlier. Explain.

Reflect

- Suppose the geyser erupts for 2.2 minutes after a 75-minute interval. Would this point lie in one of the clusters? Would it be an outlier? Explain your answer.
No, the interval was too long for the first cluster, and the duration was too short for the second cluster. It might be considered an outlier because it is not very close to the rest of the data.
- Suppose the geyser erupts after an 80-minute interval. Give a range of possible duration times for which the point on the scatter plot would not be considered an outlier. Explain your reasoning.
Sample range: 3 to 5 minutes. The duration for other data points on the scatter plot that have an interval of 80 minutes are within this range.

Lesson 14.1 433

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PROFESSIONAL DEVELOPMENT

CC Integrate Mathematical Practices MP.7

This lesson provides an opportunity to address this Mathematical Practices standard. It calls for students to look closely to discern a pattern or structure. Students will look for patterns in scatter plots of bivariate data. They use a scatter plot to interpret clusters of data and identify any outliers. They also use scatter plots to identify how sets of data are associated.

Math Background

A scatter plot may suggest an association or a correlation between two variables. The terms *association* and *correlation* are often used interchangeably, but they do not mean exactly the same thing. An association can be linear or nonlinear, but a correlation always refers to a linear association. Like an association, a correlation can be negative or positive.



Animated Math Scatter Plots and Association

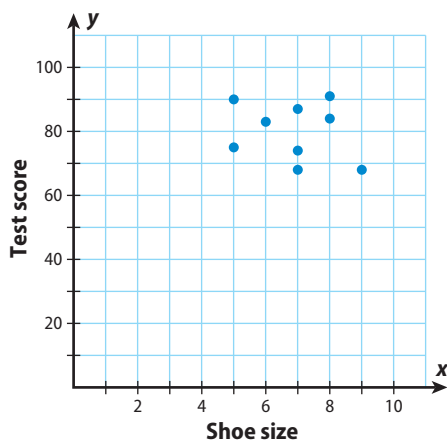
Students model real-world data by creating scatter plots; students then engage in exploring associations in the data.

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ADDITIONAL EXAMPLE 1

Juan recorded the shoe size and the math test scores of several students. The scatter plot shows what he recorded. Describe the association between test score and shoe size.

no association



Interactive Whiteboard

Interactive example available online

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EXAMPLE 1

Questioning Strategies Mathematical Practices

- How are slope and association alike? A positive association lies roughly along a line with a positive slope; a negative association lies roughly along a line with a negative slope.
- What does it mean for bivariate data to have no association? It means that changes in one variable have no predictable relationship to changes in the other variable.
- What does it mean for bivariate data to have a nonlinear association? It means that the data show a positive or negative relationship, but the points do not fall along a line.

Focus on Reasoning Mathematical Practices

Point out to students that they cannot make any conclusions about data that show no association when graphed as a scatter plot. Emphasize that changes in one data set do not affect the other data set.

Integrating Language Arts

Encourage English learners to ask for clarification on any terms or phrases that they don't understand.

YOUR TURN

Engage with the Whiteboard



Ask students if they see any clusters or outliers, and have them come up and circle them. They could say the points are clustered about a line with positive slope in Exercise 6, and the point (6, 45) could be recognized as an outlier.

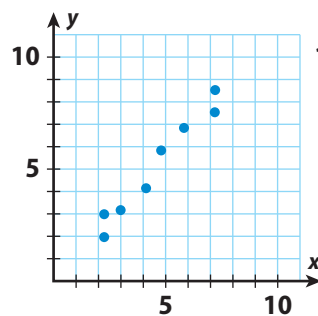
Elaborate

Talk About It

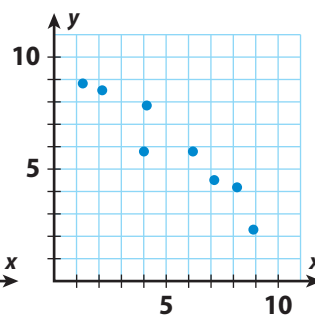
Summarize the Lesson



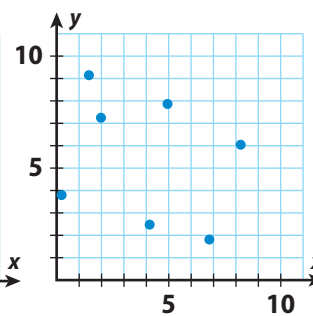
Ask: Which scatter plot shows a negative association? a positive association? no association? Label each plot with the correct type of association.



positive association



negative association



no association

GUIDED PRACTICE

Engage with the Whiteboard



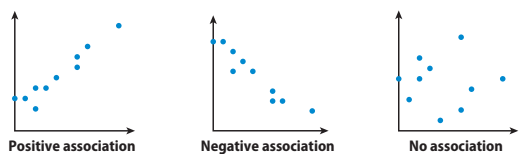
Ask student volunteers to complete Exercise 1. Discuss the type(s) of association between Bob's age in years and his height.

Avoid Common Errors

Exercise 3 Students may fail to identify the point at (35, 18) as an outlier because it fits the general trend of the data. Point out that if most of the data are clearly clustered, any point that lies outside the cluster or clusters is an outlier.

Determining Association

Association describes how sets of data are related. A *positive* association means that both data sets increase together. A *negative* association means that as one data set increases, the other decreases. *No association* means that there is no relationship between the two data sets.



Data that show a positive or negative association and lie basically along a line exhibit a *linear* association. Data that show a positive or negative association but do not lie basically along a line exhibit a *nonlinear* association.

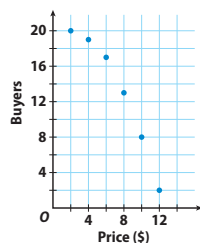
EXAMPLE 1



COMMON CORE 8.SP.1

Susan asked 20 people if they would buy a new product she developed at each of several prices. The scatter plot shows how many of the 20 said “yes” at a given price. Describe the association between price and the number of buyers.

As price increases, the number of buyers decreases. So, there is a negative association. Because the data points do not lie along a line, the association is nonlinear.



Reflect

5. **What If?** Based on the association shown in the scatter plot, what might happen if Susan increased the price to \$14?

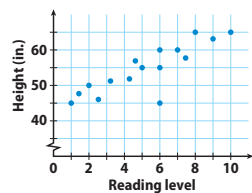
It is likely that no one would buy the product.

YOUR TURN

6. The plot shows the reading level and height for 16 students in a district. Describe the association and give a possible reason for it.

Positive and basically linear:

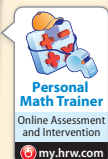
Older students would be taller and read at a higher level.



Math On the Spot
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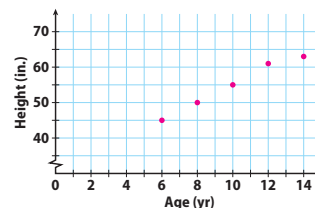
Lesson 14.1 435

Guided Practice

Bob recorded his height at different ages. The table below shows his data.

Age (years)	6	8	10	12	14
Height (inches)	45	50	55	61	63

1. Make a scatter plot of Bob's data. (Explore Activity 1)

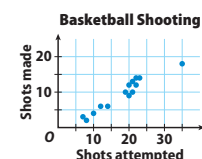


2. Describe the association between Bob's age and his height. Explain the association. (Example 1)

Positive and basically linear; as Bob gets older, his height increases. But if the data continued for increasing age, we would see that Bob's height stops increasing.

3. The scatter plot shows the basketball shooting results for 14 players. Describe any clusters you see in the scatter plot. Identify any outliers. (Explore Activity 2)

There is a cluster in the 20–23 shots attempted range, and a lesser one in the 7–14 shots attempted range. The point (35, 18) is an outlier.



ESSENTIAL QUESTION CHECK-IN

4. Explain how you can make a scatter plot from a set of bivariate data.

Let the numbers on the x-axis represent one variable and the numbers on the y-axis represent the other variable. Then plot points (x, y) for each pair of numbers in the bivariate data set.

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DIFFERENTIATE INSTRUCTION

Modeling

Ask students to make note cards showing graphs of scatter plots representing each type of association, positive, negative, and none, as well as linear and nonlinear. Tell them to include at least one scatter plot with clustering of data points and at least one scatter plot with one or more outliers. Have students circle any clusters and highlight any outliers.

Kinesthetic Experience

Clear space on a wall and make a large first quadrant coordinate plane using chalk or long strips of paper, with height as the vertical axis and months of the year as the horizontal axis. Then ask the students to place themselves on the scatter plot according to their birth month, and mark their heights. Depending on space, you may need to divide the class into groups and have them take turns. Have students describe the type(s) of association shown in the human scatter plot.

Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners **ELL**
- Reteach
- Challenge **PRE-AP**





Personal Math Trainer

Online Assessment and Intervention

Online homework assignment available

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14.1 LESSON QUIZ

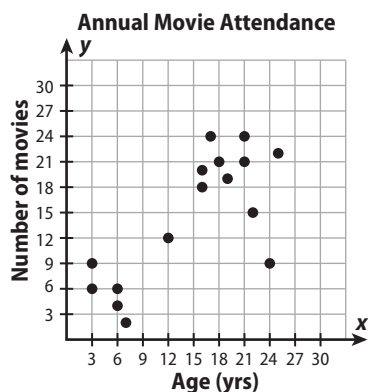
COMMON CORE 8.SP.1

Use the table for Exercises 1–2.

Height (in.)	Weight (lb)	Height (in.)	Weight (lb)
50	83	60	98
53	90	65	97
56	86	65	103
57	92	68	100

1. Make a scatter plot of the data.
2. Describe the type(s) of association you see between the height and the weight. Explain.

For Exercises 3 and 4, use the scatter plot for annual movie attendance by age.



3. Describe any clusters you see in the scatter plot.
4. Describe any outliers you see in the scatter plot.

Lesson Quiz available online

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Evaluate

GUIDED AND INDEPENDENT PRACTICE

COMMON CORE 8.SP.1

Concepts & Skills

Explore Activity 1

Making a Scatter Plot

Practice

Exercises 1, 10–11

Explore Activity 2

Interpreting Clusters and Outliers

Exercises 3, 8

Example 1

Determining Association

Exercises 2, 5–7, 9

Exercise

Depth of Knowledge (D.O.K.)

COMMON CORE

Mathematical Practices

5–9

2 Skills/Concepts

MP.7 Using Structure

10–11

2 Skills/Concepts

MP.6 Precision

12

3 Strategic Thinking **H.O.T.**

MP.7 Using Structure

13–14

3 Strategic Thinking **H.O.T.**

MP.3 Logic

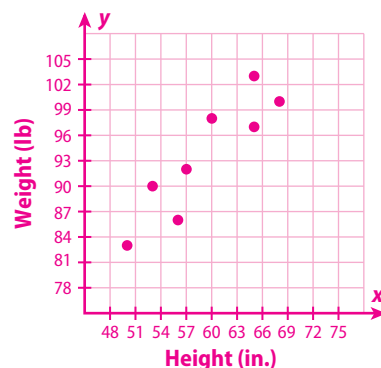
Additional Resources

Differentiated Instruction includes:

- Leveled Practice worksheets

Answers

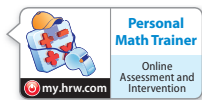
1.



2. Positive and linear; as the height increases, so does the weight.
3. There are clusters around the age of 6 and also around the age of 18.
4. There are outliers at (24, 9), and (12, 12).

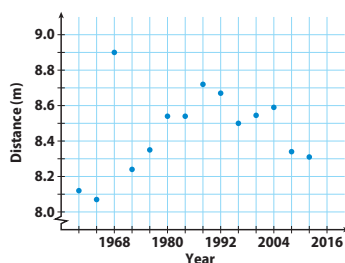
14.1 Independent Practice

COMMON CORE 8.SP.1



Sports Use the scatter plot for 5–8.

Olympic Men's Long Jump Winning Distances



5. Describe the association between the year and the distance jumped for the years 1960 to 1988.

The data generally show a positive linear association. As the years increase, so does the winning distance.

6. Describe the association between the year and the distance jumped for the years after 1988.

Overall, the data from 1988 to 2012 generally show a negative association, even though the 8 year period from 1996 to 2004 looked at by itself shows a slight rise in distance jumped over time.

7. For the entire scatter plot, is the association between the year and the distance jumped linear or nonlinear?

Nonlinear; the data points first rise as the years increased from 1960 to 1988 and then fall as the years increase from 1988 to 2012, so there is no overall linear pattern.

8. Identify the outlier and interpret its meaning.

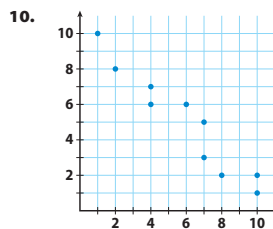
(1968, 8.9); the outlier represents a jump of 8.9 meters in 1968, a jump that far exceeds any jump made in prior or later years.

9. **Communicate Mathematical Ideas** Compare a scatter plot that shows no association to one that shows negative association.

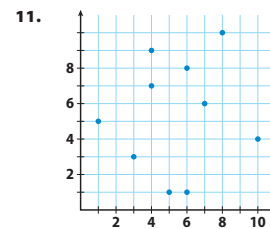
Sample answer: A plot with no association has randomly scattered data points. There does not appear to be any pattern in the association. On a plot with negative association, the data points fall from left to right. As one data set increases the other decreases.

Lesson 14.1 437

For 10–11, describe a set of real-world bivariate data that the given scatter plot could represent. Define the variable represented on each axis.



Sample answer: x-axis is number of people doing a job; y-axis is number of hours to do the job



Sample answer: x-axis is miles student lives from school; y-axis is student's score on a 10-pt quiz

H.O.T. FOCUS ON HIGHER ORDER THINKING

12. **Multiple Representations** Describe what you might see in a table of bivariate data that would lead you to conclude that the scatter plot of the data would show a cluster.

Sample answer: You would see a number of data items with x-values and y-values that are close to one another.

13. **Justify Reasoning** Is it possible for a scatter plot to have a positive or negative association that is not linear? Explain.

Yes; for example, the data points may appear to lie mostly along a rising or falling curve, or may generally rise or fall, but not in a way that suggests a linear association.

14. **Critical Thinking** To try to increase profits, a theater owner increases the price of a ticket by \$25 every month. Describe what a scatter plot might look like if x represents the number of months and y represents the profits. Explain your reasoning.

Sample answer: Initially, the number of tickets sold might decline a little, but the price increase would offset the loss in sales. So, profits would increase, showing a positive association. When the price got too high, ticket sales would decline more rapidly, so profits would fall, giving a negative association.

438 Unit 6

Work Area

EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity Use a graphing calculator to create a scatter plot of the height and points scored during the season for each player on the school basketball team. Then describe the type(s) of association you see between the height and number of points scored. There is no association in the data sets.

Height	Points
72, 68, 65, 73, 67, 78, 71, 72, 75, 77, 77, 78	85, 87, 62, 78, 78, 58, 24, 45, 52, 87, 79, 90

Press **STAT** and select "1: Edit" to enter the values for height and points into two lists

STAT PLOT
L1 and L2. Press **2nd** **Y=**, choose "1:," select "On," then choose the scatter plot (first style), L1, and L2. Press **ZOOM** and choose "9: ZoomStat" to see the scatter plot.

LESSON

14.2 Trend Lines and Predictions

Common Core Standards

The student is expected to:

COMMON CORE **Statistics and Probability—8.SP.3**

Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

COMMON CORE **Statistics and Probability—8.SP.1**

Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

COMMON CORE **Statistics and Probability—8.SP.2**

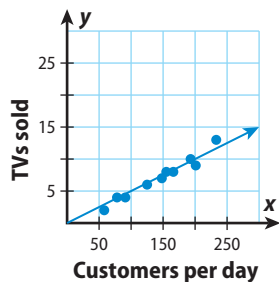
Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

Mathematical Practices

COMMON CORE **MP.6** Precision

ADDITIONAL EXAMPLE 1

The scatter plot and trend line show the relationship between the number of customers that enter an electronics store in a day and the number of TVs sold. Write an equation for the trend line. *Answers may vary; sample answer: $y = 0.05x$*



 **Interactive Whiteboard**
Interactive example available online

 my.hrw.com

Engage

ESSENTIAL QUESTION

How can you use a trend line to make a prediction from a scatter plot? **Sample answer:** Draw a trend line that fits the points as closely as possible. Then write an equation for that line, and use it to make predictions by substituting and solving.

Motivate the Lesson

Ask: Are you able to predict how far you can run in 5 minutes? in 30 minutes? Take a guess. Begin the Explore Activity to see how you could find out.

Explore

EXPLORE ACTIVITY 1

Focus on Patterns Mathematical Practices

Point out that trend lines should only be drawn if there is a clear linear association in the data once it is displayed in a scatter plot. Emphasize that you disregard the outliers when drawing a trend line because they do not fit the trend.

Explain

EXAMPLE 1

Questioning Strategies Mathematical Practices

- How can you use the slope to write an equation in the form $y = mx + b$ for the trend line? *Select two points on the line, determine the coordinates of those points, and calculate the slope m . Substitute m and the coordinates of one of the points into $y = mx + b$. Solve for b .*
- If chapters were on the vertical axis and pages were on the horizontal axis, what would be the value of the slope and what would it represent? *The slope of the line would be $\frac{1}{10}$, and it would represent a rate of 0.1 chapter per page.*

Focus on Modeling Mathematical Practices

Point out that drawing a trend line that goes through two data points makes finding the equation of the trend line easier if you know the coordinates of the points. However, it is not always possible to draw a good trend line that goes through two points.

Integrating Language Arts

Encourage English learners to take notes on new terms or concepts and to write them in familiar language.

Avoid Common Errors

A common error is to attempt to use the trend line to connect as many points of a scatter plot as possible. Point out that this may not be the best trend line, however, because it may make other data points too far away from the line and, therefore, not give the best fit.

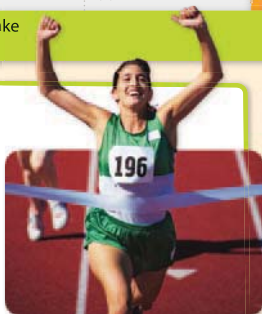
LESSON 14.2 Trend Lines and Predictions

COMMON CORE 8.SP.3

Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. Also 8.SP.1, 8.SP.2

ESSENTIAL QUESTION

How can you use a trend line to make a prediction from a scatter plot?



EXPLORE ACTIVITY 1



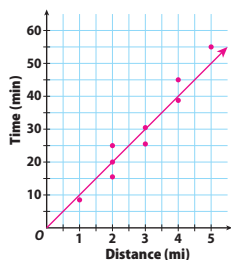
COMMON CORE 8.SP.2, 8.SP.1

Drawing a Trend Line

When a scatter plot shows a linear association, you can use a line to model the relationship between the variables. A **trend line** is a straight line that comes closest to the points on a scatter plot.

Joyce is training for a 10K race. For some of her training runs, she records the distance she ran and how many minutes she ran.

- A** Make a scatter plot of Joyce's running data.



Distance (mi)	Time (min)
4	38
2	25
1	7
2	16
3	26
5	55
2	20
4	45
3	31

- B** To draw a trend line, use a straight edge to draw a line that has about the same number of points above and below it. Ignore any outliers.

- C** Use your trend line to predict how long it would take Joyce to run 4.5 miles.

about 45 minutes

Reflect

1. How well does your trend line fit the data? Explain.

All the data points are close to the line. The data show a strong linear association, so the line should fit well.

Lesson 14.2 439

EXPLORE ACTIVITY 1 (cont'd)

2. Do you think you can use a scatter plot that shows no association to make a prediction? Explain your answer.

No; no association means that there is no relationship between the variables and the scatter plot shows no pattern.



Math On the Spot
my.hrwc.com

Finding the Equation of a Trend Line

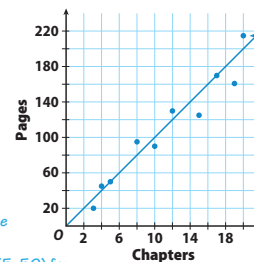
You can use two points on a trend line to write an equation in slope-intercept form for the trend line.

EXAMPLE 1



COMMON CORE 8.SP.3

The scatter plot and trend line show the relationship between the number of chapters and the total number of pages for several books. Write an equation for the trend line.



- STEP 1** Find the slope of the trend line. The line passes through points (5, 50) and (17, 170).

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Use the slope formula.}$$

$$m = \frac{170 - 50}{17 - 5} \quad \text{Substitute (5, 50) for } (x_1, y_1) \text{ and (17, 170) for } (x_2, y_2).$$

$$m = \frac{120}{12} = 10 \quad \text{Simplify.}$$

- STEP 2** Find the y-intercept of the trend line.

$$y = mx + b \quad \text{Slope-intercept form}$$

$$50 = 10 \cdot 5 + b \quad \text{Substitute 50 for } y, 10 \text{ for } m, \text{ and 5 for } x.$$

$$50 = 50 + b \quad \text{Simplify.}$$

$$50 - 50 = 50 - 50 + b \quad \text{Subtract 50 from both sides.}$$

$$0 = b \quad \text{Simplify.}$$

- STEP 3** Use your slope and y-intercept values to write the equation.

$$y = mx + b \quad \text{Slope-intercept form}$$

$$y = 10x + 0 \quad \text{Substitute 10 for } m \text{ and 0 for } y.$$

The equation for the trend line is $y = 10x$.

Math Talk

Mathematical Practices

Why are (5, 50) and (17, 170) the best points to use to draw the trend line?

Trend lines should have about the same number of points above the line as below it. By using (5, 50) and (17, 170), you get four points above the line and four points below it.

440 Unit 6

PROFESSIONAL DEVELOPMENT

CC Integrate Mathematical Practices MP.6

This lesson provides an opportunity to address this Mathematical Practices standard. It calls for students to communicate precisely to others. Students draw a trend line for a scatter plot of bivariate data with a positive linear association. Then students represent the trend line using an algebraic equation and use the equation to predict a value between data points that they already know or outside the data they know. In this way, students have used multiple representations, including symbols, graphs, and language, to communicate mathematical ideas precisely.

Math Background

A scatter plot may show a linear relationship between bivariate data. A line can be drawn to represent the data, called the trend line or the line of best fit. A trend line can be drawn by visualizing where a line would fall that best fits the data, usually with half the data points above the line and half below. The line of best fit is usually calculated using a statistical method of linear regression such as *least squares*. Least squares means the equation of the line chosen minimizes the sum of the squares of the *residuals* for the line. The residuals are the differences between the actual data values and the associated data values that fit the model. A graphing calculator automatically uses this method to give the line of best fit.

YOUR TURN

Avoid Common Errors

Suggest that students not pick two points that are close together to calculate the slope. Students may pick (0, 0) and (1, 1) and get a slope of 1. Point out that the line passes through (10, 9), which can be used with (0, 0) to get a more accurate value for the slope.

EXPLORE ACTIVITY 2

Questioning Strategies Mathematical Practices

- How can you use the equation of a trend line to make predictions? *You can substitute a value for either x or y , and solve for the value of the other variable. Then interpret the meaning for the context.*
- What is the difference between interpolation and extrapolation? *Interpolation involves the prediction of values that fall between known data points. Extrapolation involves the prediction of values that lie above or below the known range of data.*

Focus on Modeling Mathematical Practices

Point out to students that although extrapolation is generally less accurate than interpolation, any prediction about the future will involve extrapolation, so the technique is often necessary.

Engage with the Whiteboard



Invite a student volunteer to use the equation of the trend line for the data in Your Turn Exercise 6 to make predictions using interpolation and extrapolation. Discuss how they know whether they are using interpolation or extrapolation.

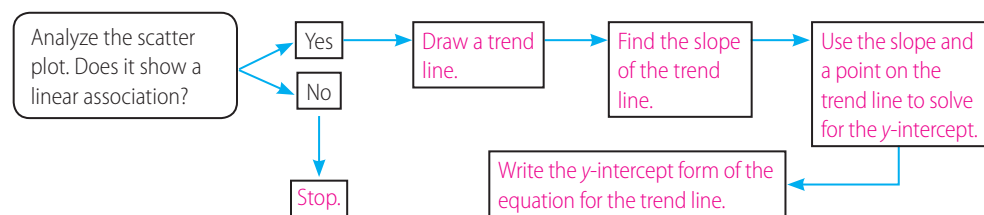
Elaborate

Talk About It

Summarize the Lesson



Ask students to fill in the blanks in the graphic organizer below to show the steps to find the equation of a trend line. *Sample answers are shown.*



GUIDED PRACTICE

Engage with the Whiteboard



Ask a student to complete Exercise 1 at the board. Have them mark two points on the line and use them to calculate the slope and y -intercept for Exercise 3. Have a student mark a point on the line to check the reasonableness of their answer for Exercise 4.

Avoid Common Errors

Exercise 1 A common error is to try to connect the points of the scatter plot. Point out that this would not give a single straight line. Emphasize that they cannot find the equation of a jagged line, and therefore cannot make predictions using it.

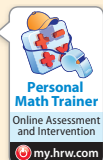
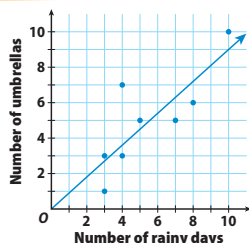
Exercise 2 A common error is to assume a good fit for the line is if the trend line goes through as many data values as possible. Point out that this may place more values farther away from the line than if equal numbers of points lie above and below the line.

Reflect

- What type(s) of association does the scatter plot show?
positive; linear
- What is the meaning of the slope in this situation?
There is an average of 10 pages per chapter.
- What is the meaning of the y-intercept in this situation?
the number of pages in a book with 0 chapters (0)

YOUR TURN

- The scatter plot and trend line show the relationship between the number of rainy days in a month and the number of umbrellas sold each month. Write an equation for the trend line.
Answers may vary.
Sample answer: $y = \frac{9}{10}x$



EXPLORE ACTIVITY 2



COMMON CORE 8.SP.3

Making Predictions

When you use a trend line or its equation to predict a value between data points that you already know, you *interpolate* the predicted value. When you make a prediction that is outside the data that you know, you *extrapolate* the predicted value.

Use the equation of the trend line in Example 1 to predict how many pages would be in a book with 26 chapters. extrapolation

Is this prediction an example of interpolation or extrapolation? extrapolation

$y =$ Write the equation for your trend line.

$y =$ Substitute the number of chapters for x .

$y =$ Simplify.

I predict that a book with 26 chapters will have 260 pages.

Lesson 14.2 441

EXPLORE ACTIVITY 2 (cont'd)

Reflect

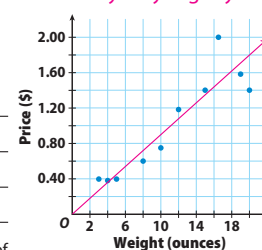
- Make a Prediction** Predict how many pages would be in a book with 14 chapters. Is this prediction an example of interpolation or extrapolation?
140 pages; interpolation
- Do you think that extrapolation or interpolation is more accurate? Explain.
Sample answer: Interpolation; the predicted value fits between known points where the trend is known. There is no guarantee that a trend will continue beyond the known data points.

Guided Practice

Angela recorded the price of different weights of several bulk grains. She made a scatter plot of her data. Use the scatter plot for 1–4.

Answers for 1–4 may vary slightly.

- Draw a trend line for the scatter plot. (Explore Activity 1)
- How do you know whether your trend line is a good fit for the data? (Explore Activity 1)
Most of the data points are close to the trend line and there is about the same number of points above and below the line.
- Write an equation for your trend line. (Example 1) $y = 0.09x$
- Use the equation for your trend line to interpolate the price of 7 ounces and extrapolate the price of 50 ounces.
(Explore Activity 2) \$0.63; \$4.50



ESSENTIAL QUESTION CHECK-IN

- A trend line passes through two points on a scatter plot. How can you use the trend line to make a prediction between or outside the given data points?
Use the two points to write the equation of the line. Substitute in the equation the value of x for which you want to make a prediction. The value of y that you obtain is the prediction.

442 Unit 6

DIFFERENTIATE INSTRUCTION

Auditory Cues

Ask students to think of interpolation as using the trend line to predict values in between existing data values. They should think of extrapolation as using the trend line to predict values outside existing values, like an extraterrestrial is outside Earth or an extracurricular activity is outside school.

Curriculum Integration

Ask students to work together to brainstorm some situations in science or social studies in which scatter plots and trend lines could be useful. For example, in social studies trend lines could be used to predict population growth, and in biology a scatter plot could show whether there is a relationship between average temperature and tree height.

Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners **ELL**
- Reteach
- Challenge **PRE-AP**





Personal Math Trainer

Online Assessment and Intervention

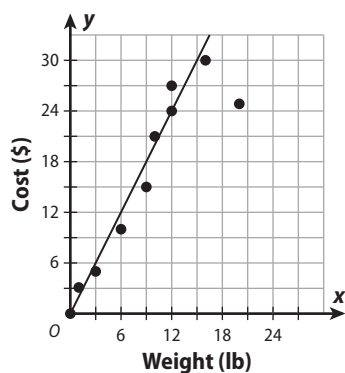
Online homework assignment available

my.hrw.com

14.2 LESSON QUIZ

COMMON CORE 8.SP.1, 8.SP.2, 8.SP.3

Marni recorded the cost of different weights of apples and made a scatter plot of her data. For Exercises 1–5, use the sample trend line drawn in the scatter plot.



1. What type(s) of association does the scatter plot show?
2. Find the slope of the trend line.
3. Find the equation of the trend line.
4. What is the meaning of the slope in this situation?
5. **a.** Use the equation of the trend line to predict the cost of buying 5 pounds of apples.
b. Is this prediction an example of interpolation or extrapolation?

Lesson Quiz available online

my.hrw.com

Answers

1. positive, linear
2. 2
3. $y = 2x$
4. The average cost of the price of apples is \$2 per pound.
5. **a.** \$10
b. interpolation

Evaluate

GUIDED AND INDEPENDENT PRACTICE

COMMON CORE 8.SP.1, 8.SP.2, 8.SP.3

Concepts & Skills	Practice
Explore Activity 1 Drawing a Trend Line	Exercises 1–2, 6, 11
Example 1 Finding the Equation of a Trend Line	Exercises 3, 8, 12
Explore Activity 2 Making Predictions	Exercises 4, 9, 13

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
6–14	2 Skills/Concepts	MP.4 Modeling
15–17	3 Strategic Thinking H.O.T.	MP.3 Logic

Additional Resources

Differentiated Instruction includes:

- Leveled Practice worksheets

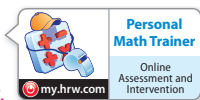


Exercises 11–14 combine concepts from the Common Core cluster “Investigate patterns of association in bivariate data.”

14.2 Independent Practice

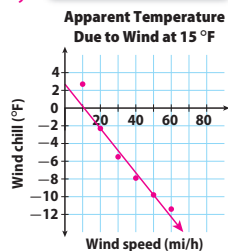
COMMON CORE 8.SP.1, 8.SP.2, 8.SP.3

Answers for 6–14 may vary slightly.



Use the data in the table for Exercises 6–10.

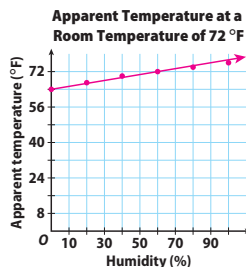
Wind speed (mi/h)	10	20	30	40	50	60
Wind chill (°F)	2.7	-2.3	-5.5	-7.9	-9.8	-11.4



- Make a scatter plot of the data and draw a trend line.
- What type of association does the trend line show?
negative; basically linear
- Write an equation for your trend line. **Sample answer: $y = -\frac{1}{4}x + 3$**
- Make a Prediction** Use the trend line to predict the wind chill at these wind speeds.
a. 36 mi/h **about -6°F** b. 100 mi/h **about -22°F**
- What is the meaning of the slope of the line?
The wind chill falls about 1 degree for every increase of 4 miles per hour in wind speed.

Use the data in the table for Exercises 11–14.

Humidity (%)	0	20	40	60	80	100
Apparent temperature (°F)	64	67	70	72	74	76



- Make a scatter plot of the data and draw a trend line.
- Write an equation for your trend line.
Sample answer: $y = \frac{2}{15}x + 64$
- Make a Prediction** Use the trend line to predict the apparent temperature at 70% humidity. **about 73°F**
- What is the meaning of the y-intercept of the line?
At 0% humidity, the apparent temperature is 64°F.

Lesson 14.2 443

HOT! FOCUS ON HIGHER ORDER THINKING

Work Area

- Communicate Mathematical Ideas** Is it possible to draw a trend line on a scatter plot that shows no association? Explain.
No; if the scatter plot shows no association, the data points have no relationship to one another. Unless there is a linear association you cannot draw a trend line.
- Critique Reasoning** Sam drew a trend line that had about the same number of data points above it as below it, but did not pass through any data points. He then picked two data points to write the equation for the line. Is this a correct way to write the equation? Explain.
No; although Sam drew the trend line correctly, he should use two points on the line to write the equation. Choosing two data points that are not on the line will result in an incorrect equation for the line.
- Marlene wanted to find a relationship between the areas and populations of counties in Texas. She plotted x (area in square miles) and y (population) for two counties on a scatter plot:
Kent County (903, 808) Edwards County (2118, 2002)
She concluded that the population of Texas counties is approximately equal to their area in square miles and drew a trend line through her points.
 - Critique Reasoning** Do you agree with Marlene's method of creating a scatter plot and a trend line? Explain why or why not.
No; two points are not sufficient for creating a scatter plot or a trend line. Marlene should have plotted data points for many more counties.
 - Counterexamples** Harris County has an area of 1778 square miles and a population of about 4.3 million people. Dallas County has an area of 908 square miles and a population of about 2.5 million people. What does this data show about Marlene's conjecture that the population of Texas counties is approximately equal to their area?
Sample answer: Marlene's conjecture is incorrect. Marlene chose counties whose areas are about equal to their populations. Harris and Dallas counties provide counterexamples for Marlene's original data.

444 Unit 6

EXTEND THE MATH PRE-AP Activity available online my.hrw.com

Activity Use a graphing calculator to find the equation of a trend line for a scatter plot of the data. Then use the trend line to predict the distance traveled in 12 hours.
701 mi

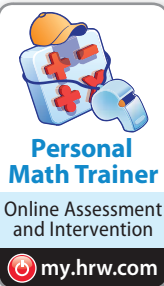
Time (h)	2	4	5.5	6	7	9	10
Distance (mi)	115	250	330	340	400	540	580

Follow the instructions given in Extend the Math for Lesson 14.1 to make a scatter plot of the data. Press **STAT**, select the "CALC" menu, then select "4: LinReg (ax+b)" and press **ENTER**. Press **Y=** and then press **VARS** and choose "5: Statistics". Select the EQ menu and choose "1: RegEQ". Press **ZOOM** and choose "9: ZoomStat" to see the trend line.

Ready to Go On?

Assess Mastery

Use the assessment on this page to determine if students have mastered the concepts and standards covered in this module.



Intervention

Access Ready to Go On? assessment online, and receive instant scoring, feedback, and customized intervention or enrichment.

Online and Print Resources

Differentiated Instruction

- Reteach worksheets
- Reading Strategies **ELL**
- Success for English Learners **ELL**

Differentiated Instruction

- Challenge worksheets
- PRE-AP**
- Extend the Math **PRE-AP**
- Lesson Activities in TE

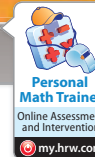
Additional Resources

Assessment Resources includes:

- Leveled Module Quizzes

MODULE QUIZ

Ready to Go On?



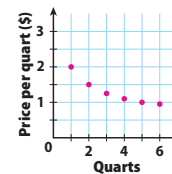
14.1 Scatter Plots and Association

An auto store is having a sale on motor oil. The chart shows the price per quart as the number of quarts purchased increases. Use the data for Exs. 1–2.

Number of quarts	1	2	3	4	5	6
Price per quart (\$)	2	1.50	1.25	1.10	1	0.95

1. Use the given data to make a scatter plot.
2. Describe the association you see between the number of quarts purchased and the price per quart. Explain.

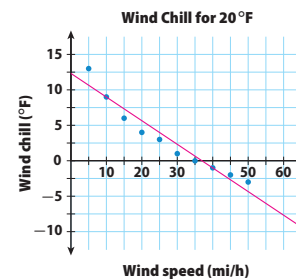
Negative but nonlinear; as number of quarts rises, price per quart decreases, but the data appear to lie along a curve.



14.2 Trend Lines and Predictions

The scatter plot below shows data comparing wind speed and wind chill for an air temperature of 20°F. Use the scatter plot for Exs. 3–5. Answers may vary slightly.

3. Draw a trend line for the scatter plot.
4. Write an equation for your trend line.
Sample ans.: $y = -\frac{1}{3}x + \frac{37}{3}$
5. Use your equation to predict the wind chill to the nearest degree for a wind speed of 60 mi/h.
-8°F



ESSENTIAL QUESTION

6. How can you use scatter plots to solve real-world problems?
Sample answer: You can plot data points and draw trend lines to make predictions.

Common Core Standards

Lesson	Exercises	Common Core Standards
14.1	1–2	8.SP.1
14.2	3–5	8.SP.1, 8.SP.2, 8.SP.3

Assessment Readiness

Assessment Readiness Tip Students should always be alert for the word *not*, and highlight or underline it to help them solve the problem correctly.


Item 3 If students overlook the word *not* in the question stem, any of the three incorrect answers will appear correct. Encourage them to read the question carefully.

Avoid Common Errors

Item 2 Some students are reluctant to choose an answer such as 'none' or 'not here.' Reinforce to the students that there isn't a relationship between every two quantities compared - sometimes, there might not be any relationship at all.

Item 6 For part c, remind students to go with the general trend, rather than simply with the last two points. If they look at only the last two points, they might believe that the graph has flattened out and will remain below 400.

Additional Resources



Personal Math Trainer

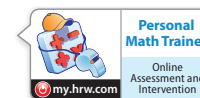
Online Assessment and Intervention

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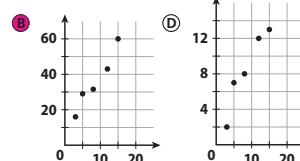
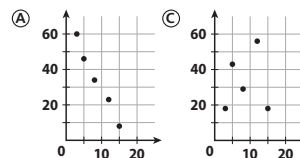
MODULE 14 MIXED REVIEW

Assessment Readiness



Selected Response

1. Which scatter plot could have a trend line whose equation is $y = 3x + 10$?

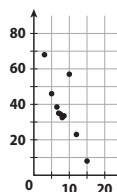


2. What type of association would you expect between a person's age and hair length?

- (A) linear (C) none
(B) negative (D) positive

3. Which is **not** shown on the scatter plot?

- (A) cluster (C) outlier
(B) negative association (D) positive association



4. A restaurant claims to have served 352,000,000 hamburgers. What is this number in scientific notation?

- (A) 3.52×10^6 (C) 35.2×10^7
(B) 3.52×10^8 (D) 352×10^6

5. Which equation describes the relationship between x and y in the table?

x	-8	-4	0	4	8
y	2	1	0	-1	-2

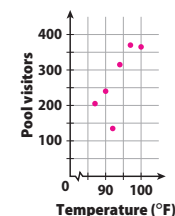
- (A) $y = -4x$ (C) $y = 4x$
(B) $y = -\frac{1}{4}x$ (D) $y = \frac{1}{4}x$

Mini-Task

6. Use the data in the table.

Temp (°F)	97	94	87	92	100	90
Pool visitors	370	315	205	135	365	240

- a. Make a scatterplot of the data.



- b. Which data point is an outlier?

(92, 135)

- c. Predict the number of visitors on a day when the high temperature is 102°F.

about 410 visitors

446 Unit 6

Common Core Standards

Items	Grade 8 Standards	Mathematical Practices
1	8.SP.2	MP.4
2	8.SP.1	MP.4
3	8.SP.2	MP.6
4*	8.EE.3	MP.4
5*	8.F.4	MP.4
6	8.SP.1, 8.SP.3	MP.4, MP.6

* Item integrates mixed review concepts from previous modules or a previous course.