Name: ____

Essential Skills Practice for students entering Algebra One and Accelerated Algebra One

Use this document to review the mathematics that you have learned previously. Completion of the essential skills practice and comprehension of these topics will help you to be more successful in Algebra 1 or Accelerated Algebra 1.

A. operations with rational numbers
B. graphing points
C. slope
D. graphing lines
E. combining like terms
F. distribution
G. solving basic equations
H. proportions
I. basic perimeter and area
J. the real number system

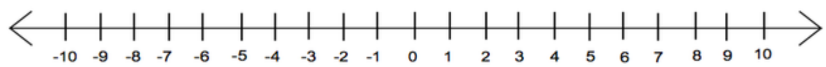
A. operations with positive and negative numbers

- Every real number has an opposite on the number line. The opposite of a number is shown using the negative symbol. For example, the opposite of 5 is –5.
- When evaluating –(–5) we are saying the opposite of –5. Thus, – (–5) is 5.

*Adding/subtracting

Using a number line, we can easily add and subtract positive and negative numbers.

• If we need to do –4 –3 we will start at –4 and move three spaces left to show that –4 – 3 = –7.



• If we need to do –4 + 9 we will start at –4 and move 9 spaces to the right to show that –4 + 9 = 5.

• What about 7 –(–2)? Remember that –(–2) means a positive 2 and change our problem to reflect this. Now 7 + 2 means we will start at 7 on the number line and move 2 right to show that 7 –(–2) = 9.

• However, what if we need to do –157 + 263? It is not reasonable to make a number line for this. So, what are the rules that the number line shows us? We can apply the rules to any problem we need to solve.

• Look at –4 – 3, both numbers are negative. The answer is also negative and is the sum of the numbers. Thus:

***If the signs are the same, add the numbers and use the common sign.**

• Look at –4 + 9, the signs are different. In this case, the answer is positive and is the difference of the numbers. Thus:

***If the signs are different, subtract the numbers and use the sign of the larger number.**

*Multiplying/dividing

• When multiplying or dividing we use the following rules:

(positive) × or ÷ (positive) =	positive number
(positive) × or ÷ (negative) =	negative number
(negative) × or ÷ (negative) =	positive number

Examples:

–9 • 7 = –63

–8 • –8 = 64

–4 • –5 • –2 =

20 • –2 = –40

Practice with positive and negative numbers

Students entering Algebra 1 should be able to do these problems without the use of a calculator.

1. –12 + 17 =

2. 11 • –6 =

3. –15 • –10 =

4. 100 –(–25) =

5. –20 • 4 =

6. –15 – 10 =

7. –12 • –12

8. –6 • 3 • –2 =

9. 22 + (–8) =

10. –20 –(–25) =

(fr)Action Packed Math!

What is a fraction? A fraction is a quantity that cannot be represented by a whole number.
Why do we need fractions you may ask….. Consider this:

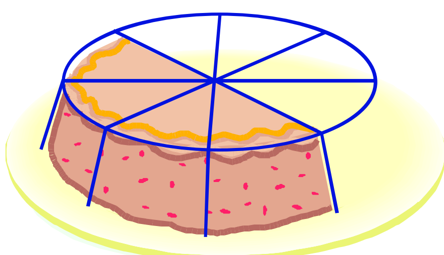
Can you finish this all the cookies?
If not, how many of them could you eat?
Your answer to this question is a fraction, or a portion, of the set of cookies.

$$\frac{a}{b}$$

The **numerator** tells us how many pieces are in the portion of the set.

The **denominator** tells us how many equal pieces are in the total set, thus this number cannot be zero.

Equivalent fractions
A fraction can have many different appearances. Fractions that look different but have the same values are called equivalent fractions.



Consider this cake. We have 4 pieces out of 8 equal pieces. However we also could have divided the cake into 4 pieces and we would have two of these left. We could even say we have half of the cake left, or ½. Whether we say we have $\frac{4}{8}$, $\frac{2}{4}$, or ½, they all represent the same amount of cake and are equivalent fractions.

$$\frac{4}{8} = \frac{2}{4} = \frac{1}{2}$$

How do your simplify a fraction?
A fraction is in its <u>simplest form</u> if we cannot find a whole number (other than 1) that can divide into both its numerator and denominator evenly.   The original fraction and its simplified form are equivalent.

Example: $\frac{54}{68}$ To simplify both numbers are divisible by 2

$$\frac{54 \div 2}{68 \div 2} = \frac{27}{34}$$

An improper fraction is a fraction whose value is larger than one, the numerator is greater than the denominator, for example: $\frac{5}{3}$.   It is acceptable to leave answers as *simplified* improper fractions.

Adding and Subtracting Fractions:

Subtract the numerators.

Simplify.

$$\frac{1}{8} - \frac{7}{8} = -\frac{6}{8} = -\frac{3}{8}$$

Denominators stay the same.

When adding and subtracting fractions, you must ALWAYS have common denominators.
What do you do if the denominator is NOT the same?

$$\frac{4}{7} + \frac{2}{3} =$$

The denominators are NOT the same, so you need to multiply to make them the same. The smallest number that both 7 and 3 will go into (the LCM) is 21. So, we will multiply the first fraction by $\frac{3}{3}$. The second fraction will be multiplied by $\frac{7}{7}$. Then we can add the fractions.

$$\frac{4}{7} + \frac{2}{3} = \frac{4 \cdot 3}{7 \cdot 3} + \frac{2 \cdot 7}{3 \cdot 7} = \frac{12}{21} + \frac{14}{21} = \frac{26}{21}$$

****Practice adding/subtracting fractions***

Students entering Algebra 1 should be able to do these problems without the use of a calculator.

1. $\frac{2}{5} + \frac{1}{5} =$

2. $\frac{2}{5} - \frac{3}{4} =$

3. $\frac{4}{9} - \frac{7}{9} =$

4. $\frac{8}{9} + \frac{1}{3} =$

Multiplying and Dividing Fractions:
To multiply or divide you do NOT need common denominators.

- Multiplication: Multiply the numerators, multiply the denominators and simplify your answer.
- Division: Flip the second fraction and multiply.

Example: Multiplication

Multiply the numerators, 4 ● 3 = 12

$$\frac{4}{7} \bullet \frac{3}{8} = \frac{12}{56} = \frac{3}{14}$$

$\frac{12}{56}$ simplifies to $\frac{3}{14}$ by dividing 12 and 56 by dividing 12 and 56 by 4

Multiply the denominators, 7 ● 8 = 56

Example: Division

Flip the second fraction and change to multiplication

$$\frac{4}{7} \div \frac{3}{8} = \frac{4}{7} \bullet \frac{8}{3} = \frac{32}{21}$$

Multiply the numerators and denominators to get $\frac{32}{21}$, which cannot be reduced further.

or

Convert fractions to have common denominators. Divide the numerators.

$$\frac{4}{7} \div \frac{3}{8} = \frac{32}{56} \div \frac{21}{56} = \frac{\frac{32}{21}}{\frac{56}{56}} = \frac{32}{21}$$

****Practice multiplying/dividing fractions***

Students entering Algebra 1 should be able to do these problems without the use of a calculator.

1. $\frac{8}{9} \bullet \frac{3}{4} =$

2. $\frac{8}{9} \div \frac{3}{4} =$

3. $\frac{1}{9} \bullet \frac{1}{18} =$

4. $\frac{1}{9} \div \frac{1}{18} =$

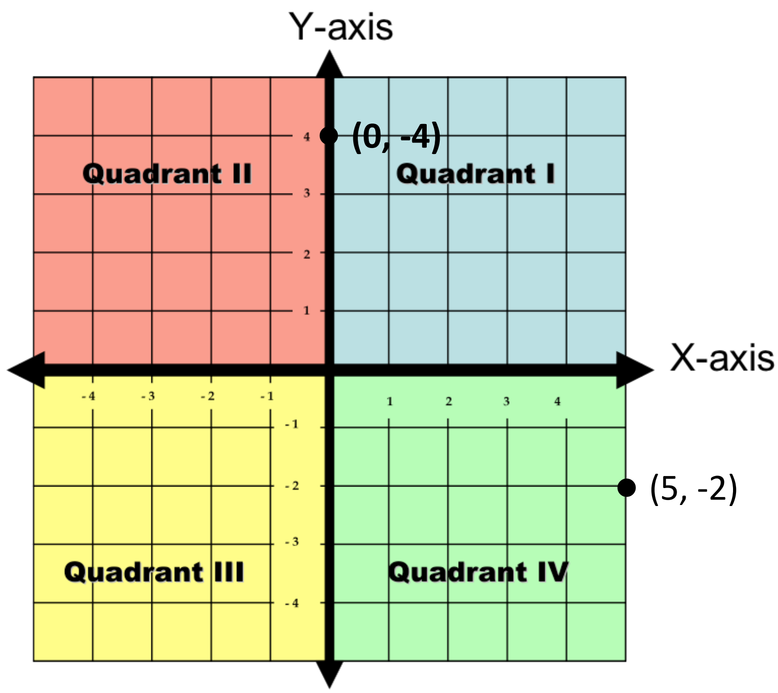
5. $\frac{2}{7} \bullet \frac{4}{7} =$

6. $\frac{2}{7} \div \frac{4}{7} =$

B. graphing points

A point, or an **ordered pair**, gives us a location on the coordinate plane relative to the origin (the origin is where the x and y axes intersect.)

Ordered pairs are written to tell us how far to go left or right and then to tell us how far to go up or down. The first value is the x coordinate (left or right) and then the second value is the y coordinate (up or down.) We write this as (x, y)



Positive x values are to the right of the origin and negative x values are to the left of the origin.

Positive y values are above the origin while negative y values are below the origin.

Examples:

- The point (5, -2) tells us to go 5 right and 2 down from the origin.

- The point (0, 4) tells us to go 0 left or right, and 4 up from the origin. Thus, this point lies on the y-axis.

Practice graphing points

Graph the following points on the coordinate plane.

A. (-4, 6)

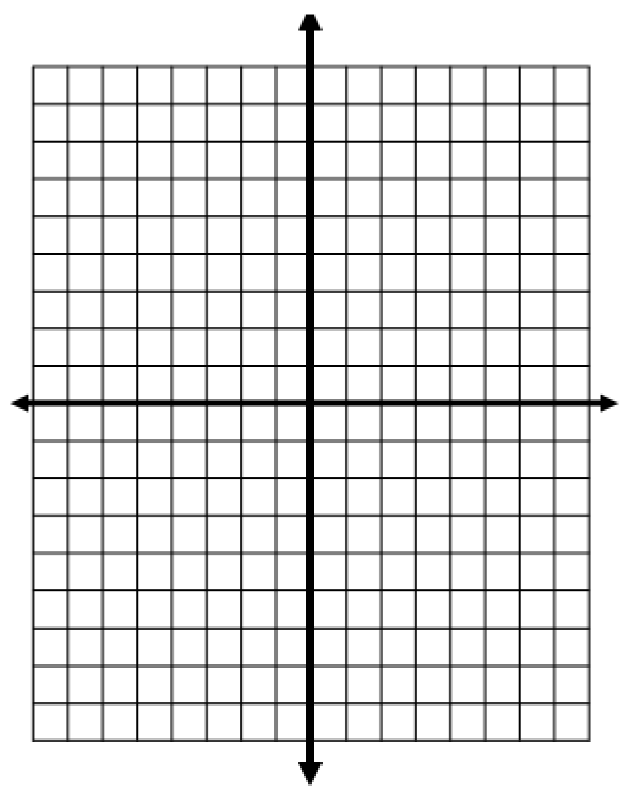
B. (7, 0)

C. (3, 8)

D. (0, -5)

E. (-7, - 2)

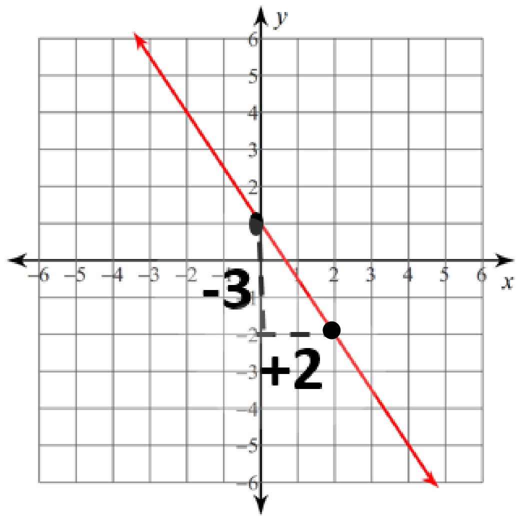
F. (5, -4)



C. slope

The **slope** of a line refers to how steep the line is. We represent this by evaluating $\frac{rise\ of\ the\ line}{run\ of\ the\ line}$. Rise is the difference in y-values from one point to another. Run in the difference in x-values from one point to another. We can count slope by counting the rise and run on the graph, or we can calculate it using the x and y values of two points on the line. We use the formula $\frac{y_2 - y_1}{x_2 - x_1}$. By subtracting the y values we find the rise of the line and by subtracting the x values we find the run of the line. You will see the letter *m* used to represent slope.

Example using a graph:



The slope of the line is $\frac{-3}{2}$.

We can determine the slope by counting the rise (down 3, so we use -3 as the rise) and counting the run (right 2 so we use positive 2 as the run.)

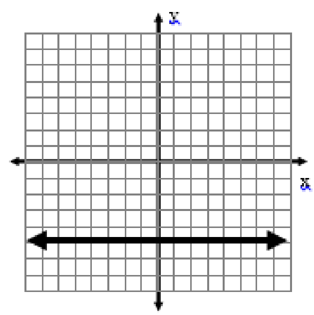
Example using the formula:

Find the slope of the line containing the points (5, -2) and (-10, 8).
(x_1, y_1) (x_2, y_2)

Make sure you simplify your fractions.

Substitute the points into our formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ so $m = \frac{8-(-2)}{-10-5} = \frac{\mathbf{10}}{\mathbf{-15}} = -\frac{\mathbf{2}}{\mathbf{3}}$

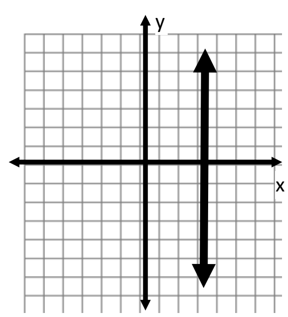
- **What about horizontal and vertical lines?**



A horizontal line will always have a slope of 0, since the rise of the line is 0.

$$m = \frac{0}{run} = 0$$

A vertical line will always have an undefined slope since the run of the line is 0 and division by zero is undefined.

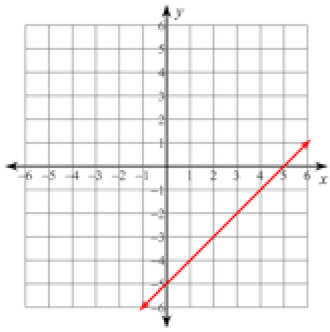


$$m = \frac{rise}{0} = \text{undefined}$$

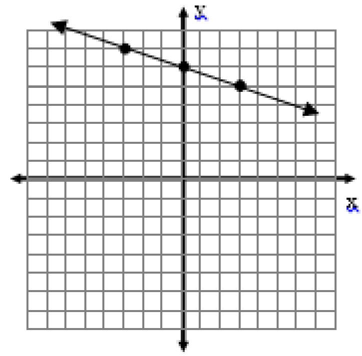
Practice finding slope

Find the slope for each of the following graphs.

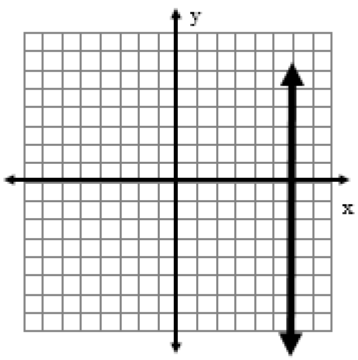
1.



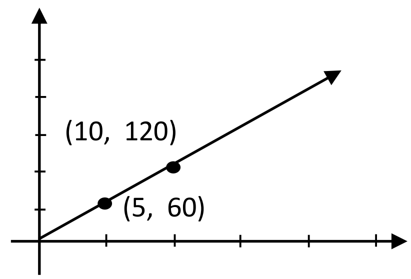
2.



3.



4.



Find the slope of each line using the points given.

5. (0, 0) and (-2, 8)

6. (-1, 2) and (-9, 6)

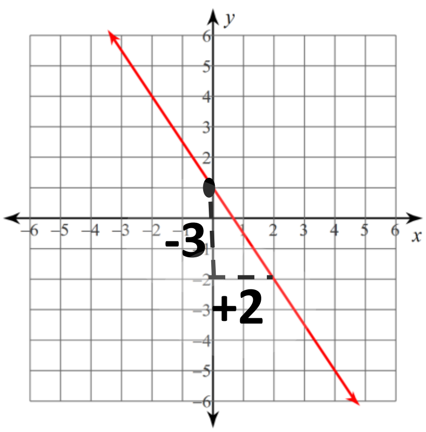
7. (1, -2) and (6, -2)

8. (1, -2) and (-6, 3)

D. graphing linear equations

Example 1: Sketch the graph given the information provided.

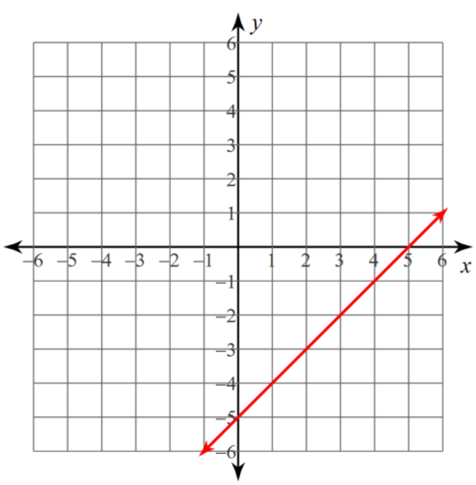
$$y = -\frac{3}{2}x + 1$$



When an equation is given in slope-intercept form (y = mx + b) we will graph using the y-intercept (the b value) and the slope (the m value.)
We start by plotting the y-intercept, which is (0,1).
Use the slope's numerator to move up or down and denominator to move left or right. Since the slope for this line is $\frac{-3}{2}$ we moved 3 down and 2 right from the y-intercept.

Example 2: Sketch the graph given the information provided.

$$x - y = 5$$



2x - 2y = 10 is in standard form. Convert the equation into slope-intercept form by solving for y.

$$2x - 2y = 10$$
$$-2x \qquad -2x$$

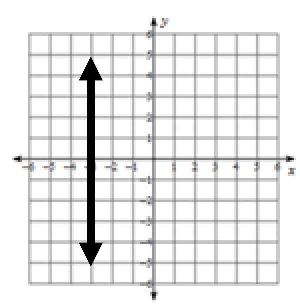
$$-2y = -2x + 10$$

$$-\tfrac{1}{2}(-2y) = -\tfrac{1}{2}(-2x + 10)$$

$$\boldsymbol{y = x - 5}$$

Now graph as slope intercept form.

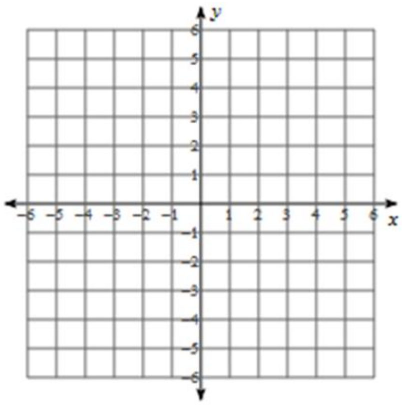
Example 3: Graph the line x = -3.
Since this equation is not in y = mx + b form, consider what the equation tells you about the line. All points on the line have an x-value of -3. The line is vertical.



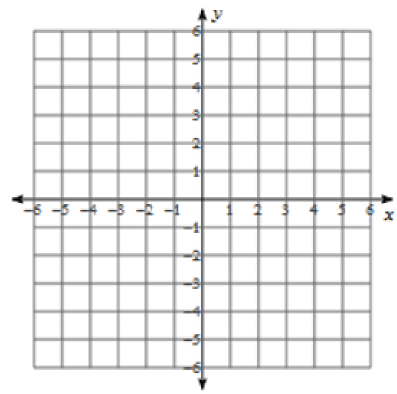
*** Practice Graphing Equations***

Sketch the graph of each line.

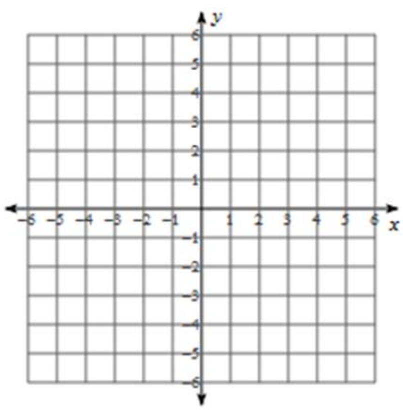
1. through (2, 5) with a slope of -3



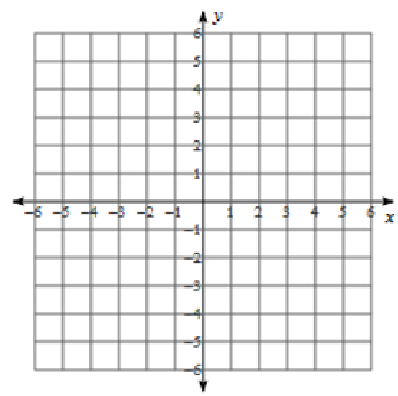
2. through (-1, 7) with an undefined slope



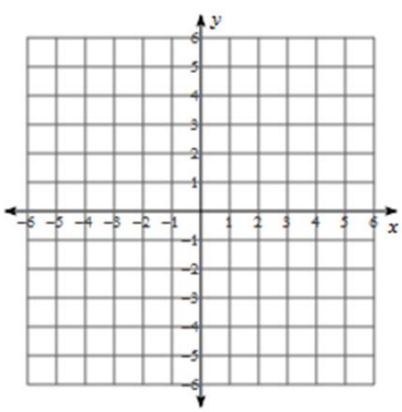
3. $y = 8x + 4$



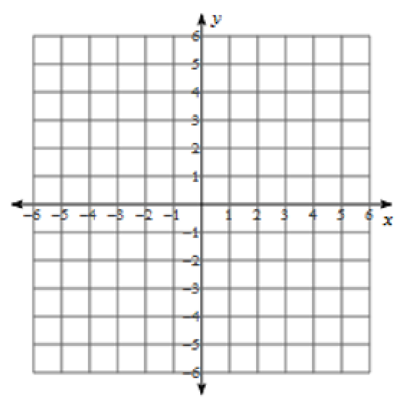
4. $y = -\frac{5}{4}x + 1$



5. $x - 4y = 16$



6. $y = 5$



E. combining like terms

Terms in an expression or equation are **like terms** when they have all the same variable and the same exponents.

Examples:

- $5x^2$ and $-2x^2$ are like terms, they both have an x^2
- $3xy^4$ and $12xy^4$ are like terms, they both have xy^4
- $6x^2y$ and $6xy$ are NOT like terms since the exponents on the x are not the same

When terms are like, they can be added and subtracted. When adding and subtracting like terms, we add/subtract the coefficient (number in front of the variable) while leaving the variable and exponent the same.

Examples:

- $5x^2 - 2x^2 = 3x^2$

How many x^2 do we have? We had 5, subtracted 2, and are left with $3x^2$.

- $12x + 2y - 15x = -3x + 2y$

12x can be combined with 15x. We use the sign in front of the 15x, thus we calculate 12x – 15x and get -3x. The 2y cannot be combined since the variable is not the same and the 2y remains in our final answer.

- $6 + 6x - 10 + x = 7x - 4$

6x + x is actually 6x + 1x. When there is not a coefficient in front of a variable, the coefficient is a 1. Thus, 6x + 1x combines to 7x. Then, we combine the numbers without a variable using the signs in front of them. 6 – 10 is -4. Put these together for our final answer.

Practice combining like terms

1. $x - 2y + 6y =$
2. $-12x^4 + 3x^4 =$
3. $-x - 11x =$
4. $4x^3 - 2 + 7 - 3x^3 =$
5. $3x - y + 17x + y =$
6. $14x - 2 + 3x^2 + 5 - 6x =$

F. distribution

The distributive property says that the product of an individual term and a sum of numbers is the same as the individual products added together, or $a(b + c) = ab + ac$.

Example:

We can't subtract the 3 from the x since they are not like terms. But, we can use the distributive property. We multiply the 5 outside the parentheses to both the x and the -3.

- $5(x - 3) = 5x - 15$

Sometimes we will distribute, then combine like terms.

Example:

First, we distribute the -2 (the sign in front of the 2 belongs to it!) Remember that -2 times –x becomes positive.

- $3x - 2(4 - x) + 7 = 3x - \underline{8} + 2x + \underline{7}$

Now we can combine our like terms.

$= 5x - 1$

What if there is a negative in front of the parentheses? This means we distribute a negative 1.

Example:

First, we distribute the negative one to the x and the 2.

- $5 - (x + 2) + 6x = 5 - x - 2 + 6x$

Now we can combine our like terms.

$= 5x + 3$

Practice distributing*

Simplify the following expressions.

1. $4(-3x - 7) =$

2. $-9(3 - x) =$

3. $8x + 6(2x - 4) =$

4. $6 - (2x + 4) + 10x =$

5. $x + 3(4 - x) - 2(7x - 3) =$

6. $10 + 3(x - 4) - (x - 2) =$

G. solving basic equations

When we solve an equation we are looking for the value of the variable that will make the equation true. We use opposite operations to isolate the variable. Sometimes, we will need to distribute, combine like terms, and move variables from one side to another in order to solve.

Example: Solve 3x – 12 = 72

+12 + 12 add 12 to each side of the equation

$\frac{3x}{3} = \frac{84}{3}$ divide each side of the equation by 3

x = 28

To check our answer we can plug 28 into our original equation for x and show that it makes the equation true.

3(28) – 12 = 72
84 – 12 = 72
72 = 72 The x value of 28 makes the equation true, thus our solution is correct.

Example: $\frac{1}{3}(6x - 12) - 10x = 20$

2x – 4 – 10x = 20 distribute the 1/3

-8x – 4 = 20 combine the like terms

+4 = + 4 add 4 to each side

$\frac{-8x}{-8} = \frac{24}{-8}$ divide both sides by -8

x = -3

Check the answer: $\frac{1}{3}(6(-3) - 12) - 10(-3) = 20$

$\frac{1}{3}(-18 - 12) + 30 = 20$

$\frac{1}{3}(-30) + 30 = 20$

-10 + 30 = 20 The x value of -3 makes the equation true,

20 = 20 thus our solution is correct.

Example: 5x + 34 = -2 (1 – 7x)

5x + 34 = -2 + 14x distribute the -2

-5x -5x subtract 5x from each side to move the variables onto the same side

34 = -2 + 9x combine the like terms

+2 = +2 add 2 to each side of the equation

$\frac{36}{9} = \frac{9x}{9}$ combine like terms and then divide both sides by 9

4 = x

Practice solving basic equations*

Solve each equation.

1. $-10 = -14x - 6x$

2. $x - 1 = 5x + 3x - 8$

3. $-8 = -(x + 4)$

4. $-(7 - 5x) = 9$

5. $2(4x - 3) - 8 = 4 + 2x$

6. $-3(4x + 3) + 4(6x + 1) = 43$

H. proportions

Students will need to be able to write and solve proportions. A proportion is used when two ratios (fractions) are equal. There are numerous ways to solve a proportion, but most students will remember that when a situation is proportional, the cross products are equal. These cross products can be used to solve for missing pieces of the proportion.

Examples:

A. $\frac{35}{21}=\frac{20}{x}$ Cross multiply so 35x = 20 • 21 simplify 35x = 420 solve x = 12	B. $\frac{9}{x+1}=\frac{5}{x}$ cross multiply so 9x = 5(x+1) simplify 9x = 5x + 5 solve 4x = 5 x = 1.25
C. Jill decides to bring cookies from Vito's Bakery for all of the students in her Geometry class. She knows that last time she bought cookies her bill was \$3.95 and she bought 5 cookies. There are 32 people in her Geometry class and everyone wants a cookie. Write and solve a proportion to calculate what Jill's bill will be. This is a proportion because the cost per cookie will be equal. So, set up a proportion making sure to match the units. $\frac{\$3.95}{5\ cookies}=\frac{x}{32\ cookies}$ Now, cross multiply and solve! 5x = 126.4 x = 25.28 So, Jill's bill will be \$25.28 for 32 cookies	

Practice for proportions

Solve each proportion.

1. $\frac{5}{4}=\frac{k}{8}$

2. $\frac{5}{9}=\frac{2}{a}$

3. $\frac{8}{10}=\frac{2}{p+3}$

4. $\frac{x-1}{6}=\frac{6}{7}$

Write and solve a proportion to solve each problem.

5. Jack is going to Florida! The first 325 miles took 5 hours. Jack knows the entire trip is about 1170 miles. If they can travel at the same rate, how many hours should the trip take?

6. Mr. Jones wants to buy his geometry students protractors to start the year. He found a pack of 12 protractors for \$9.66. The company said they will also sell him the protractors individually for the same unit price. If Mr. Jones has \$75 to spend, how many protractors can he buy?

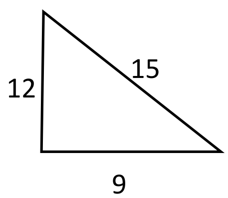
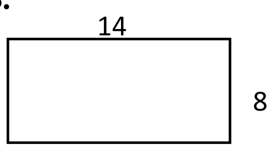
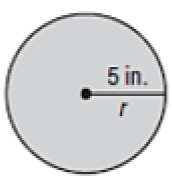
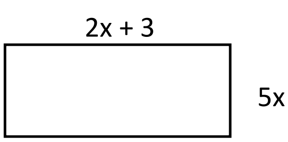
I. basic perimeter and area

The **perimeter** of a figure refers to the distance around a two-dimensional figure. The **area** of a figure refers to the amount of space inside a two-dimensional figure.

Students should be able to calculate the perimeter of any polygon and be able to calculate the circumference of a circle. Incoming algebra students should be able to calculate the area of rectangles, triangles and circles proficiently. The following table includes information about how to calculate these measurements.

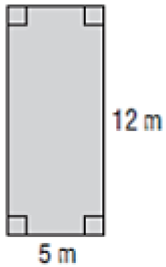
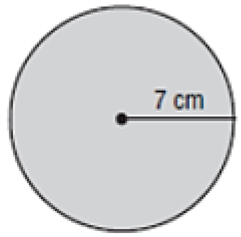
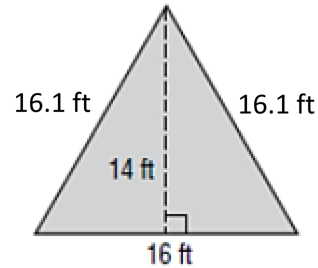
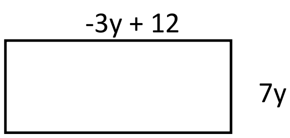
shape	**perimeter or circumference**	**area**
rectangle	add all sides	length times width
triangle	add all sides	½ length times width
circle	2π times the radius	π times the radius squared

Examples:

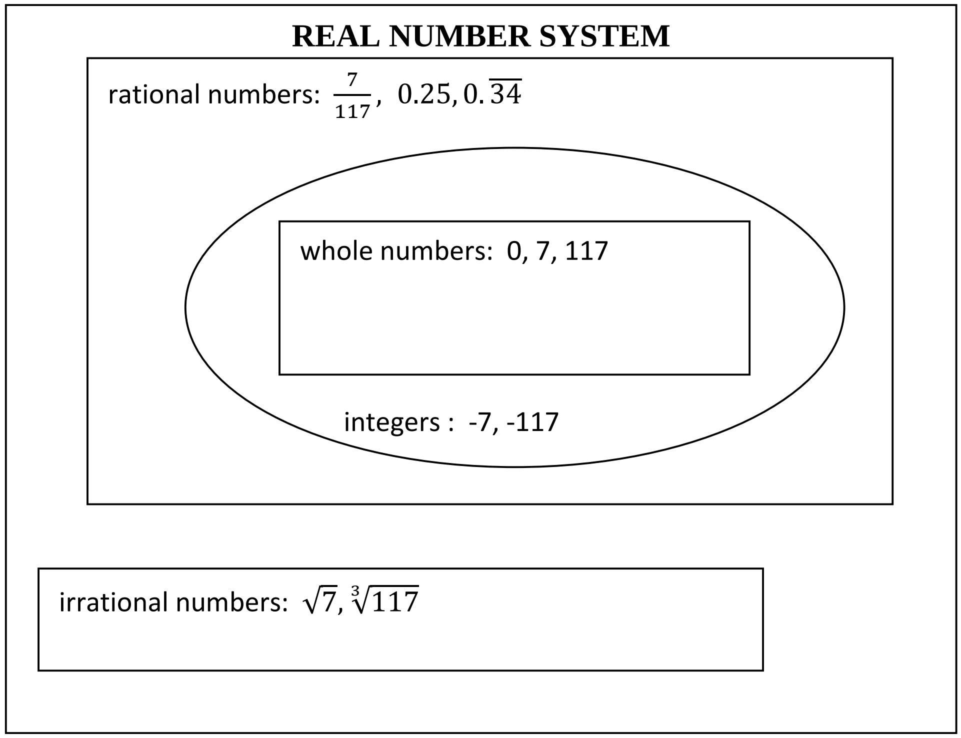
A. 	**perimeter = add all sides** $P = 9 + 12 + 15$ $P = 36$	**area = ½ l • w** $A = \frac{1}{2} \cdot 12 \cdot 9$ $A = 54$
B. 	**perimeter = add all sides** $P = 14 + 8 + 14 + 8$ $P = 44$	**area = l • w** $A = 14 \cdot 8$ $A = 112$
C. 	**circumference = 2πr** $C = 2 \cdot \pi \cdot 5$ $C = 10\pi$ inches	**area = πr^2** $A = \pi \cdot 5^2$ $A = 25\pi$ inches squared
D. 	**perimeter = add all sides** $P = 2x + 3 + 5x + 2x + 3 + 5$ $P = 14x + 6$	**area = l • w** $A = 5x(2x+ 3)$ $A = 10x^2 + 15x$

Practice for basic perimeter and area

Find the perimeter or circumference **and** the area. Express answers for circles in terms of π.

1.  Perimeter = Area=	2  Circumference = Area =	3.  Perimeter = Area=
4. 	Perimeter =	Area =

J. the real number system



Match the definitions and numbers below with their appropriate position in the diagram above. Students entering Algebra 1 should be able to simplify the numerical expressions without the use of a calculator.

1. Any number that can be represented by a fraction. Numbers in decimal form will repeat or terminate.
2. Zero and all positive numbers without decimals or fractions.
3. All the whole numbers and their negatives.
4. Numbers that cannot be expressed in fraction form. If expressed as a decimal, will not repeat nor terminate.

5. $\frac{-36}{6}$
6. $\frac{6}{36}$
7. $\sqrt{36}$
8. $\sqrt{6}$
9. $\frac{0}{\sqrt{36}}$
10. $\sqrt[3]{-8}$
11. $\frac{\sqrt{100}}{\sqrt{25}}$
12. $\frac{\sqrt{25}}{\sqrt{100}}$
13. $\sqrt[3]{27} + \sqrt[3]{64}$
14. $\sqrt[3]{27} - \sqrt[3]{64}$
15. $\sqrt{6}\bullet\sqrt{2}$
16. $\sqrt{8}\bullet\sqrt{2}$

Algebra Essential Skills Practice - Answer Key

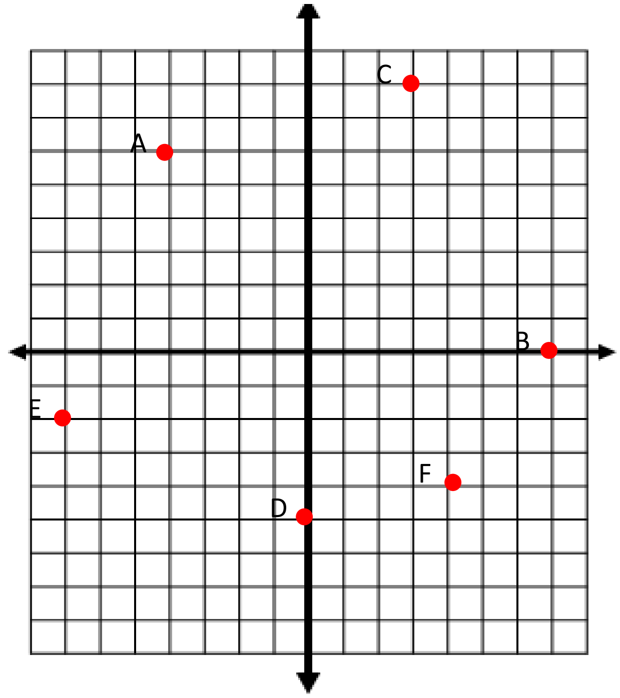
positive and negative numbers
1. 5
2. -66
3. 150
4. 125
5. -80
6. -25
7. 144
8. 36
9. 14
10. 5

fraction basics
1. both simplify to $\frac{2}{3}$
2. $\frac{26}{3}$

adding fractions
1. $\frac{3}{5}$
2. $-\frac{7}{20}$
3. $-\frac{1}{3}$
4. $\frac{11}{9}$

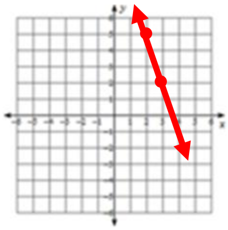
multiplying fractions
1. $\frac{2}{3}$ 2. $\frac{32}{27}$
3. $\frac{1}{162}$ 4. 2
5. $\frac{8}{49}$ 6. $\frac{1}{2}$

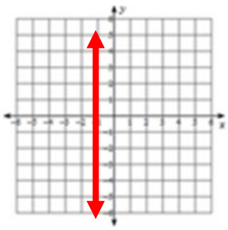
graphing points

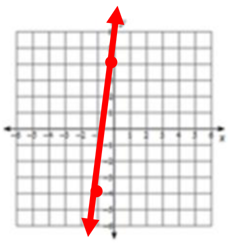


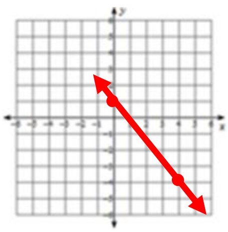
slope
1. 1
2. $-\frac{1}{3}$
3. undefined
4. 12
5. -4
6. $-\frac{1}{2}$
7. 0
8. $-\frac{5}{7}$

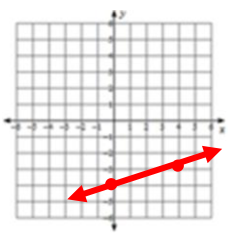
graphing lines

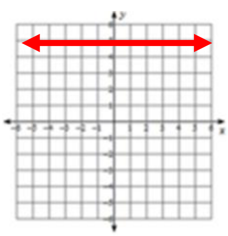
1. 

2. 

3. 

4. 

5. 

6. 

combining like terms

1. x + 4y
2. $-9x^4$
3. -12x
4. $x^3 + 5$
5. 20x
6. $3x^2 + 8x + 3$

distribution

1. -12x - 28
2. -27 + 9x
3. 20x - 24
4. 2 + 8x
5. -16x + 18
6. 2x

solving basic equations

1. 0.5
2. 1
3. 4
4. 3.2
5. 3
6. 4

proportions

1. 10
2. 3.6
3. -0.5
4. 6.14
5. 18 hrs
6. 92 protractors… use proportion with unit price $\frac{\$0.81}{1\ protractor} = \frac{\$75.00}{x\ protractors}$

basic perimeter and area

1. 34 m, 60 m^2
2. 14π cm, 49π cm^2
3. 48.2 ft, 112 ft^2
4. $8y + 24$, $-21y^2 + 84y$

the real number system

1. rational
2. whole
3. integer
4. irrational
5. integer
6. rational
7. whole
8. irrational
9. whole
10. integer
11. whole
12. rational
13. whole
14. integer
15. irrational
16. whole