education
Department:
Education
PROVINCE OF KWAZULU-NATAL

EUCLIDEAN GEOMETRY

TECHNICAL MATHEMATICS

GRADES 10-12

INSTRUCTIONS FOR USE:

This booklet consists of brief notes, Theorems, Proofs and Activities and should not be taken as a replacement of the textbooks already in use as it only acts as a supplement.

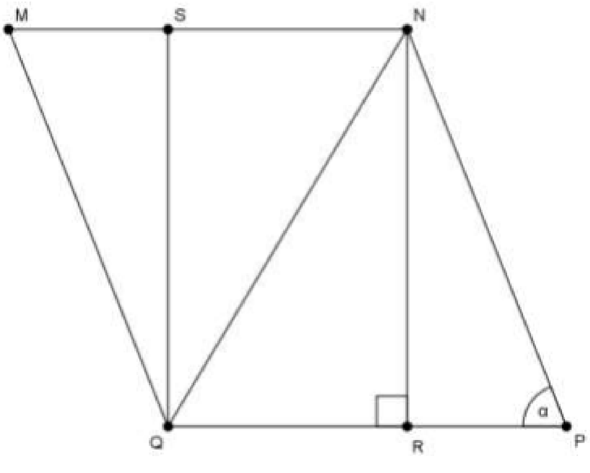
EUCLIDEAN GEOMETRY

Section A:
Grade 10

• Content covered in this section includes revision of lines, angles and triangles. The mid-point theorem is introduced.
Kites, parallelograms, rectangle, rhombus, square and trapezium are investigated.
•. The focus of this chapter is on introducing the
special quadrilaterals and revising content from earlier grades.
• Revision of triangles should focus on similar and congruent triangles.
• Sketches are valuable and important tools. Encourage learners to draw accurate diagrams to solve problems.
• It is important to stress to learners that proportion gives no indication of actual length. It only indicates the ratio
between lengths.
• Notation - emphasise to learners the importance of the correct ordering of letters, as this indicates which angles are equal and which sides are in the same proportion.

Euclidean Geometry Grade 10 Mathematics

1. Given below is the diagram of parallelogram MNPQ. NR ⊥ QP and $N\hat{P}R = \alpha$.



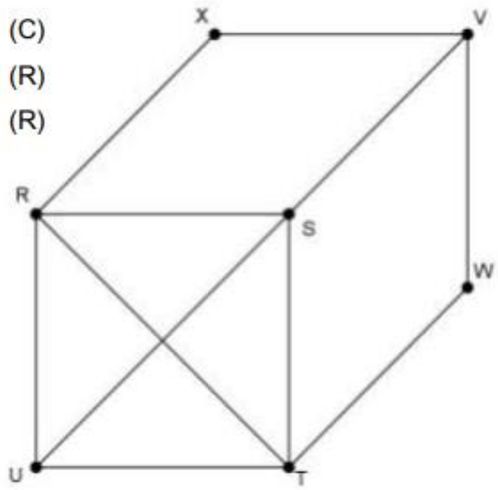
a) Prove that ΔMQN ≡ ΔNPQ (R)
b) Hence prove that ΔMSQ ≡ ΔPRN (C)
c) Prove that NRQS is a rectangle. (C)
d) What kind of shape is SNPQ, give reasons for your answer. (C)

2. In the diagram, TUVW is a trapezium with TW ∥ UV ∥ RX; RS ∥ XV and RT ⊥ US. UT = TW= SV and RSTU is a square.

a) Prove that STWV is a rhombus. (C)

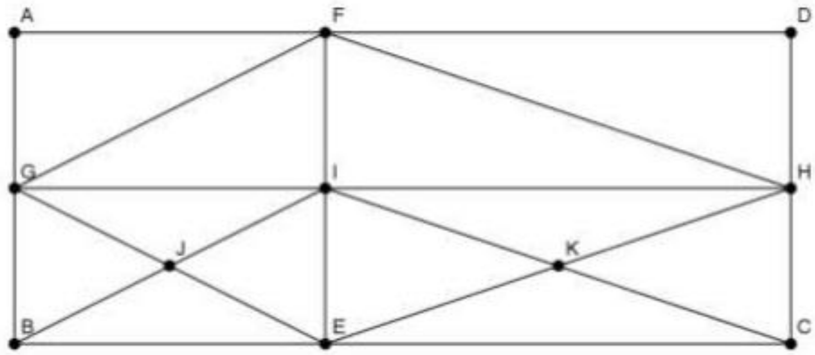
b) Hence, prove that RSVX is a rhombus. (R)

c) Show that RU = RX = XV (R)



3. Look at the shape given below. AD ∥ GH ∥ BC and AB∥ FE ∥ DH and GF∥ BI and FH ∥ IC.

Name:



a) all the rectangles

b) all the trapeziums

c) all the kites

d) all the squares

e) all the parallelograms

f) all isosceles triangles

4. Given below is quadrilateral BGHI, with ABCD a parallelogram inside BGHI. FE ∥ AB

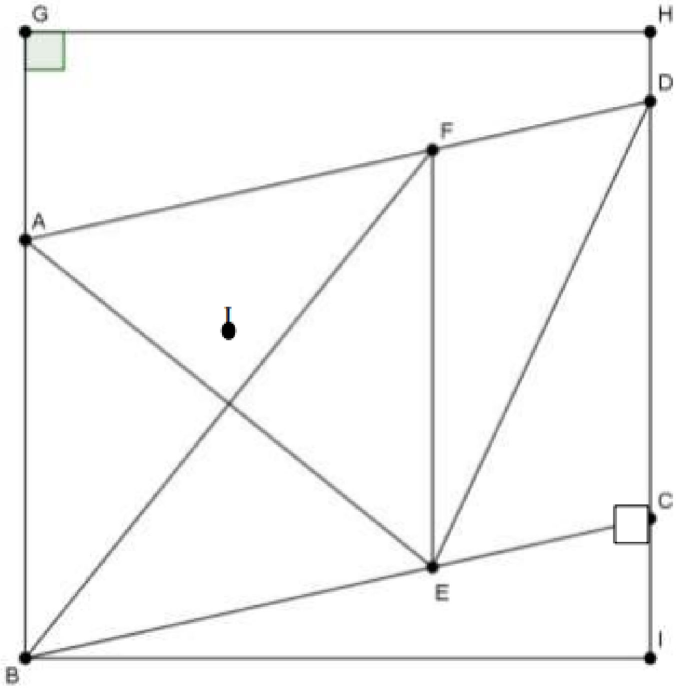
a) Prove that ΔDEF is congruent with ΔCDE. (R)

b) Hence, prove that CDFE is a parallelogram. (R)

c) Prove that ΔABE is congruent with ΔAFE. (C)

d) Prove that ABEF is a rhombus, given that $A\hat{B}J = A\hat{F}J$. (C)

e) Prove that GH ∥ BI. (P)



6. Given on the right is the diagram of rhombus JKLM.
KO ⊥ LM, KP ⊥ JM, MN ⊥ KJ, MQ ⊥ LK
and KO ⊥ KJ.

a) Show that ΔKLO and ΔMLQ are congruent. (R)

b) Prove that LOSQ is a kite. (C)

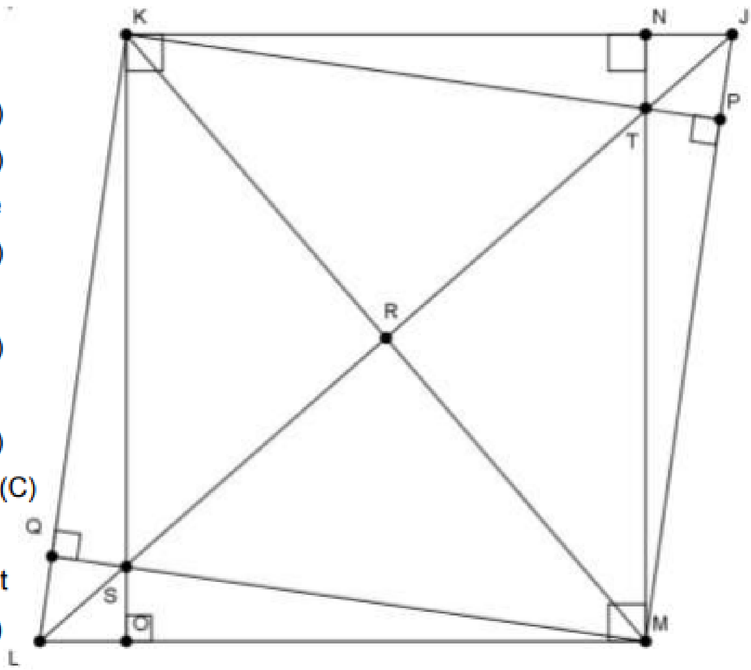
c) Prove that ΔKQS and ΔMOS are congruent. (R)

d) Prove that KSMT is a rhombus (C)

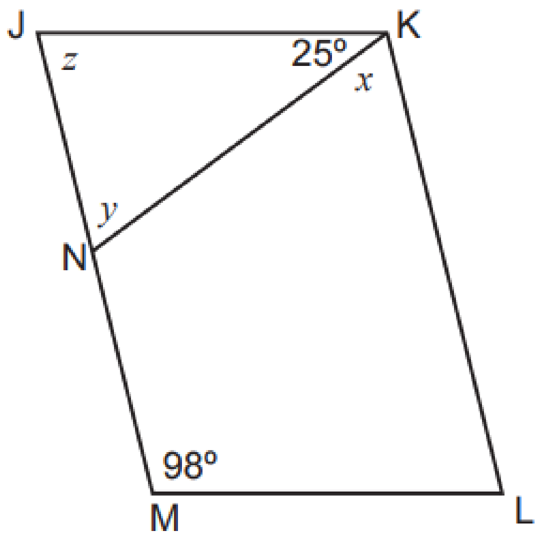
e) Hence, prove that KNMO is a rectangle. (C)

f) Prove that KPMQ is a rectangle.(C)

g) Are KNMO and KPMQ identical rectangles? Show all working out and reasons. (P)

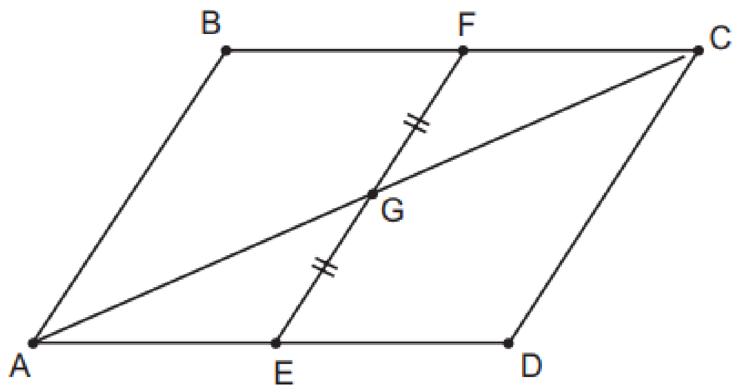


2.2 In the diagram below: *JKLM* is a parallelogram.



2.2.1 Determine the value of **ALL** the unknown variables (6)

2.3 In the diagram below: *E* is the midpoint of line *AD*, *F* is the midpoint of line *BC*, and *G* is the midpoint of line *AC*. *EG=FG*.



2.3.1 Prove that *ABCD* is a parallelogram. (7)

SECTION B

GRADE 11 : EUCLIDEAN GEOMETRY

THEOREMS

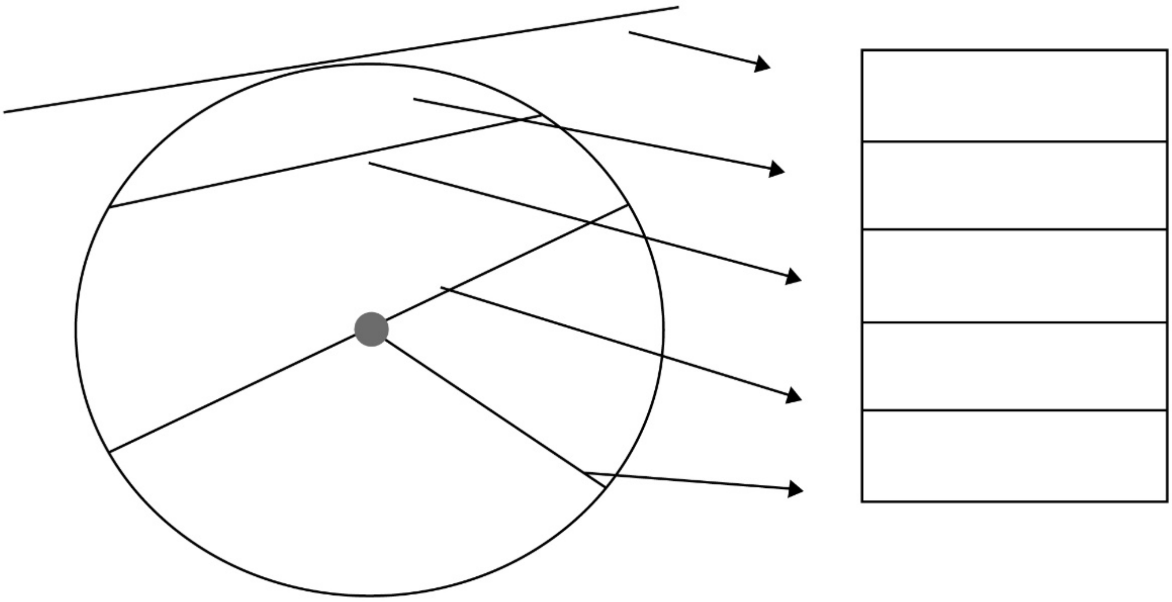
1. The line drawn from the centre of a circle perpendicular to a chord bisects the chord.
2. The perpendicular bisector of a chord passes through the centre of the circle.
3. The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle.
(On the same side of the chord as the centre)
4. Angles subtended by a chord of the circle, on the same side of the chord, are equal.
5. The opposite angles of a cyclic quadrilateral are supplementary.
6. Two tangents drawn to a circle from the same point outside the circle are equal in length.
7. The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.
8. The angle on circumference subtended by the diameter equals 90_o.
9. Exterior angle of a cyclic quadrilateral equals to the opposite interior angle.
10. A line from the centre of a circle to a tangent is perpendicular on tangent.

Circle Geometry

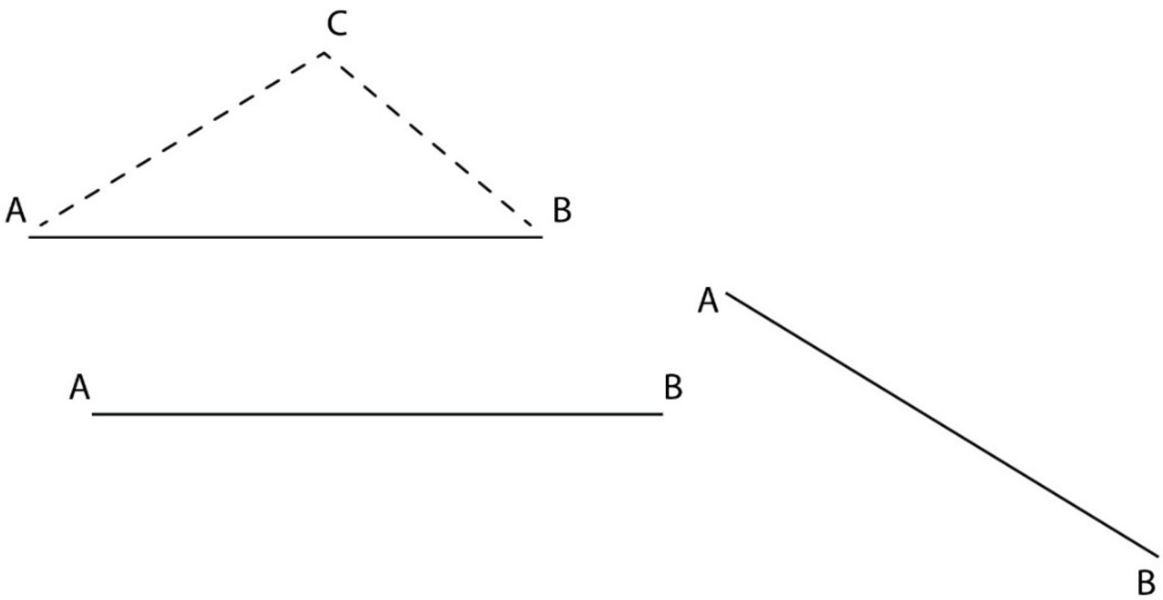
There are two parts to this investigation. Part A requires you to recall the parts of a circle that will be used in Part B. Part B will lead you, step by step, to discovering the new theorems that you are required to know in Grade 11.

PART A

1. Complete the following:
A chord cuts a circle into two ________________________
The 'perimeter' of a circle is called the _________________________
A portion of the circumference is called an _____________
2. Label the parts of a circle:



An important word to understand:
Form a triangle ABC from each of these line segments. The first one has been done for you.

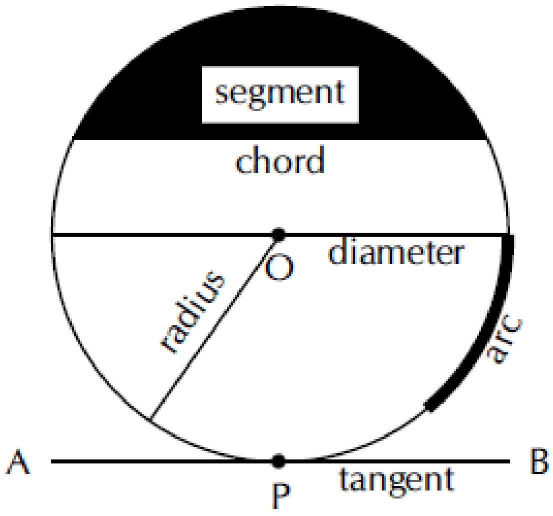


We say that AB **subtends** C ? *or* C ? is **subtended** from AB.

PART B

4. CIRCLES

4.1 TERMINOLOGY

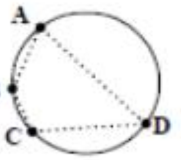


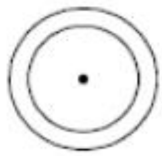
Arc	An arc is a part of the circumference of a circle
Chord	A chord is a straight line joining the ends of an arc.
Radius	A radius is any straight line from the centre of the circle to a point on the circumference
Diameter	A diameter is a special chord that passes through the centre of the circle. A Diameter is the length of a straight line segment from one point on the circumference to another point on the circumference, that passes through the centre of the circle.
Segment	A segment is the part of the circle that is cut off by a chord. A chord divides a circle into two segments
Tangent	A tangent is a line that makes contact with a circle at one point on the circumference (AB is a tangent to the circle at point P).

4.2 SUMMARY OF THEOREMS

4.2.1 Definitions

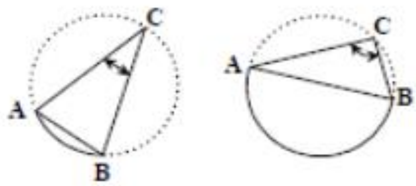
* Points are concyclic if they lie on the circumference of a circle.
* A quadrilateral is cyclic if all four vertices lie on the circumference of a circle.
* Concentric circles have the same centre.
* An arc (or chord) of a circle subtends an angle if the arms of the angle are joined by the arc (or chord)
* An angle is at the centre when its arms are radii.
* An angle is at the circumference (or in a segment) of a circle when its arms are chords.





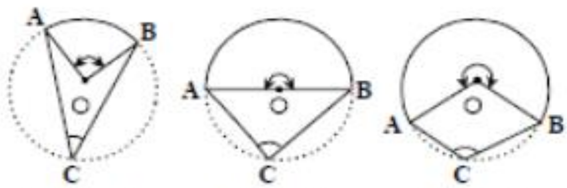
Concentric circles

Concyclic points
[*ABCD* is a cyclic quadrilateral]



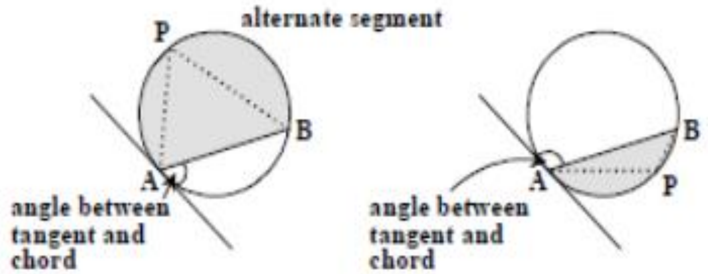
Arc (chord) *AB* subtends $\hat{C}$

Angles at centre and circumference subtended by . . .



Angles at centre and circumference subtended by . . .	minor arc	diameter	major arc
$\hat{C}$ is in the . . .	major segment	semi-circle	minor segment

* The chord *AB* subtends angle *P* in the segment opposite to the selected angle between the tangent and chord *AB*.



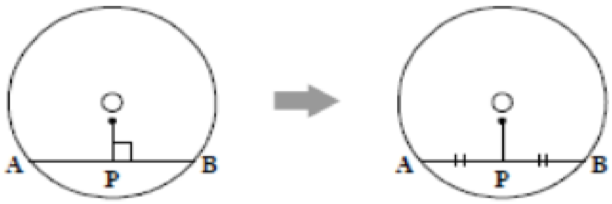
4.2.2 Chords and Midpoints

Note: O indicates the centre of the circle.

- **Theorem 1:**

A line drawn from the centre of a circle, perpendicular to a chord, bisects the chord.

($OP \perp AB$)

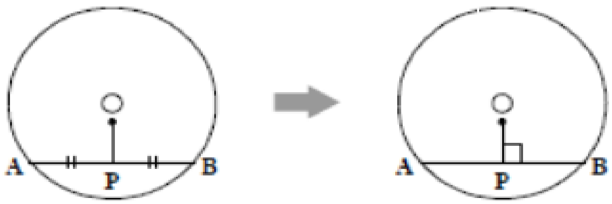


if $OP \perp AB$ **then** $AP = PB$

- **Theorem 2:**

(Converse of theorem 1).
The line joining the centre of a circle to the midpoint of a chord is perpendicular to the chord.

($AP = PB$)

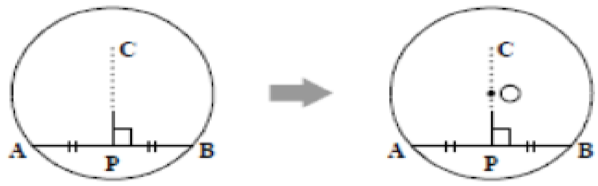


if $AP = PB$ **then** $OP \perp AB$

- **Theorem 3:**

The perpendicular bisector of a chord passes through the centre of the circle.

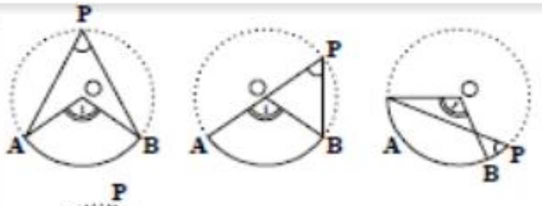
($AP = PB$ and $CP \perp AB$)



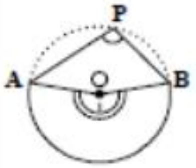
if $\begin{bmatrix} AP = PB \\ \text{and} \\ CP \perp AB \end{bmatrix}$ **then** $\begin{bmatrix} PC \text{ passes} \\ \text{throught } O \end{bmatrix}$

- **Theorem 4:**

The angle at the centre is twice the angle at the circumference subtended by the same arc.
($\angle$ at centre = $2 \times \angle$ at circumf.)



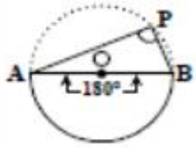
$\hat{O}_1 = 2\hat{P}$ subtended by minor arc AB



$\hat{O}_1 = 2\hat{P}$ subtended by major arc AB

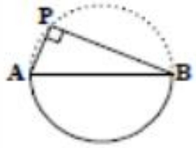
* **Important deductions:**

1. An angle in a semi-circle is a right angle



$2\hat{P} = A\hat{O}B = 180°$
$\therefore \hat{P} = 90°$

2. The chord that subtends a right angle at the circumference is a diameter.



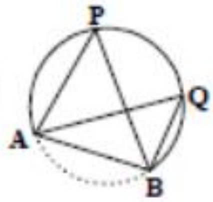
if $\hat{P} = 90°$ **then** AB is a diameter

- **Theorem 5:**

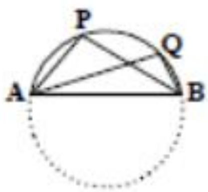
Angles subtended by a chord at the circumference of a circle, on the same side of the chord, are equal.
(In other words: Angles in the same segment of a circle are equal.)

($\angle s$ in same segment)

if



or



then $\hat{P} = \hat{Q}$

[$\hat{P}$ and $\hat{Q}$ are in the major segment]

[$\hat{P}$ and $\hat{Q}$ are in the minor segment]

- **Theorem 6:**

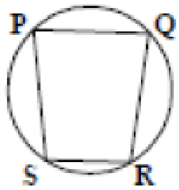
(Converse of theorem 5).
If a line segment joining two points subtends equal angles at two other points on the same side of it, the four points are concyclic.

(AB subtends equal $\angle s$)

if $\hat{P} = \hat{Q}$ **then** A, B, P **and** Q are concyclic
[i.e. $ABPQ$ is a cyclic quadrilateral]

- **Theorem 7:**

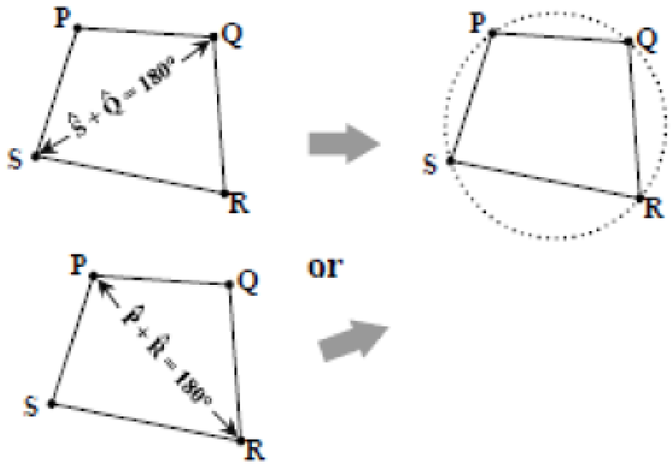
The opposite angles of a cyclic quadrilateral are supplementary.
(opp. ∠s of cyc. quad.)

if  then $\begin{bmatrix} \hat{P} + \hat{R} = 180° \\ \text{and} \\ \hat{Q} + \hat{A} = 180° \end{bmatrix}$

- **Theorem 8:**

(Converse of theorem 7).
If two opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.

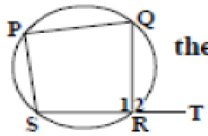
(opp. ∠s supp.)

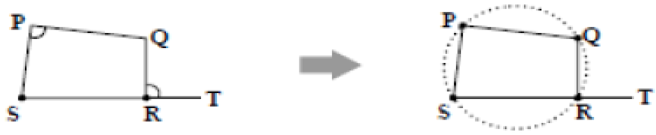


if $\begin{bmatrix} \hat{S} + \hat{Q} = 180° \\ \text{or} \\ \hat{P} + \hat{R} = 180° \end{bmatrix}$ then *PQRS* is a cyclic quadrilateral

* **Important deductions:**

1. The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.
   (ext. ∠ of cyc. quad.)
2. (Converse of deduction 1).
   If the exterior angle of a quadrilateral is equal to the interior opposite angle, the quadrilateral is cyclic.
   (ext. ∠ = opp. int. ∠)

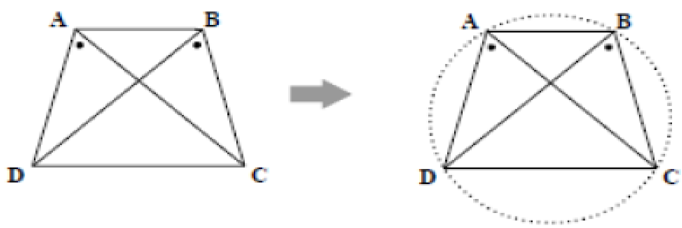
if  then $\hat{R}_2 = \hat{P}$



if $Q\hat{R}T = \hat{P}$ then *PQRS* is a cyclic quadrilateral.

* **Important reminder:**

1. A **third** way of proving that a quadrilateral is cyclic, is by using theorem 6.
2. Once you have proven a quadrilateral to be cyclic, draw a light circle around it for further use.



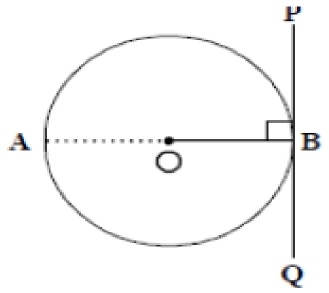
if $\hat{A} = \hat{B}$ then *ABCD* is a cyclic quadrilateral.

4.2.4 Tangents

- **Axiom:**

A tangent is perpendicular to the radius (or diameter) at the point of contact.

(radius OB ⊥ tangent PQ)

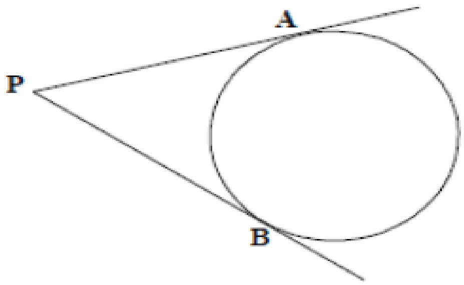


tangent $PBQ \perp$ radius OB

- **Theorem 9:**

Two tangents drawn to a circle from the same point outside the circle are equal in length.

(tangents from same point)



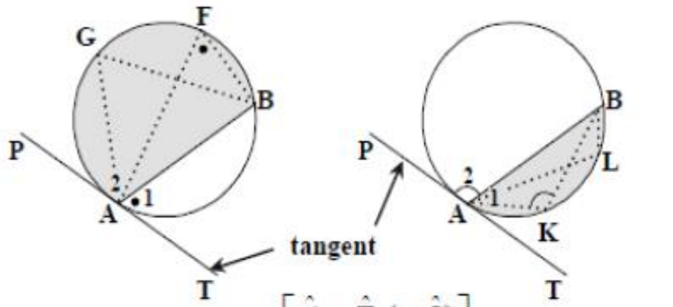
PA = PB

if PA and PB are tangents **then** $PA = PB$

- **Theorem 10:**

The angle between a tangent and a chord drawn to the point of contact is equal to the angles in the alternate segment.

(∠ between tangent and chord)

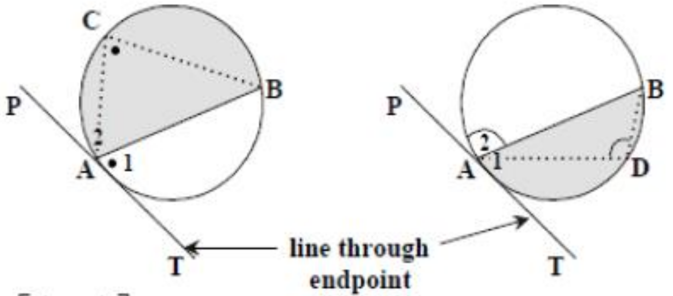


if PT is a tangent **then** $\left[\begin{matrix}\hat{A}_1 = \hat{F}\ (= \hat{G}) \\ \textbf{and} \\ \hat{A}_2 = \hat{K}\ (= \hat{L})\end{matrix}\right]$

- **Theorem 11:**

(Converse of theorem 10).
If the line through the endpoint of a chord makes an angle with the chord, equal to an angle in the alternate segment, then the line is a tangent to the circle.

(∠ between line and chord = ∠ in opp. segm.)



if $\left[\begin{matrix}\hat{A}_1 = \hat{C} \\ \textbf{or} \\ \hat{A}_2 = \hat{D}\end{matrix}\right]$ **then** PT is a tangent

<u>PROOFS OF SELECTED THEOREMS</u>

4.3 PROOF OF THEOREMS

All SEVEN theorems listed in the CAPS document must be proved. However, there are four theorems whose proofs are examinable (according to the Examination Guidelines 2018) in grade 12. In this guide, only FOUR examinable theorems are proved. These **four** theorems are written in **bold.**

1. The line drawn from the centre of a circle perpendicular to the chord bisects the chord.

2. The perpendicular bisector of a chord passes through the centre of the circle.

3. The angle subtended by an arc at the centre of a circle is double the angle subtended by the same arc at the circle (on the same side of the arc as the centre).

4. Angles subtended by an arc or chord of the circle on the same side of the chord are equal.

5. The opposite angles of a cyclic quadrilateral are supplementary.

6. Two tangents drawn to a circle from the same point outside the circle are equal in length (If two tangents to a circle are drawn from a point outside the circle, the distances between this point and the points of contact are equal).

7. The angle between the tangent of a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.

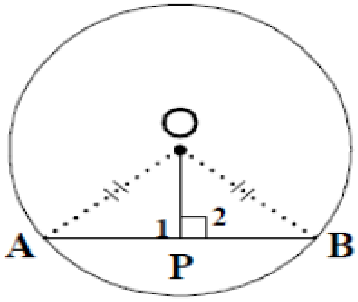
The above theorems and their converses, where they exist, are used to prove riders.

Theorem 1

The line drawn from the centre of a circle, perpendicular to a chord, bisects the chord.

Given:
Circle with centre O and chord $AB \perp OP$

To Prove:
$AP = PB$



Proof:
Draw OA and OB
In ΔOAP and ΔOBP

$OA = OB$ (radii)

$OP = OP$ (common)

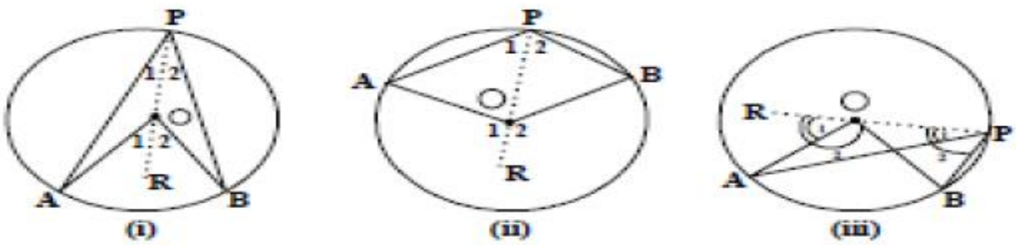
$\hat{P}_1 = \hat{P}_2 = 90°$ (given)

$\therefore \Delta OAP \equiv \Delta OBP$ $(90°, h, s)$

$\therefore \quad AP = BP$

Theorem 4

The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circumference of the circle.

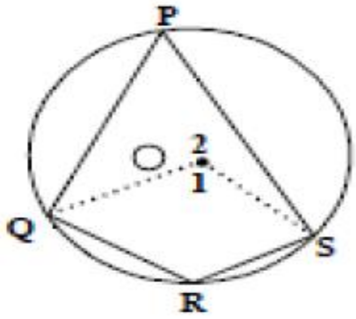


Given:
Circle centre O and arc AB subtending $A\hat{O}B$ at the centre and $A\hat{P}B$ at the circumference.
To Prove:
$A\hat{O}B = 2A\hat{P}B$
Proof:
Join PO and produce to R
$AO = PO$ (radii)
$\therefore \hat{A} = \hat{P}_1$ ($\angle s$ opposite equal sides)
$\hat{O}_1 = \hat{A} + \hat{P}_1$ (exterior $\angle$ of ΔAPO)
$\therefore \hat{O}_1 = 2\hat{P}_1$
Similarly $\hat{O}_2 = 2\hat{P}_2$

In figures (i) and (ii)
$\hat{O}_1 + \hat{O}_2 = 2\hat{P}_1 + 2\hat{P}_2$
$= 2(\hat{P}_1 + \hat{P}_2)$
$\therefore A\hat{O}B = 2A\hat{P}B$

In figure (iii)
$\hat{O}_2 - \hat{O}_1 = 2\hat{P}_2 - 2\hat{P}_1$
$= 2(\hat{P}_2 - \hat{P}_1)$
$\therefore A\hat{O}B = 2A\hat{P}B$

Theorem 7
The opposite angles of a cyclic quadrilateral are supplementary.

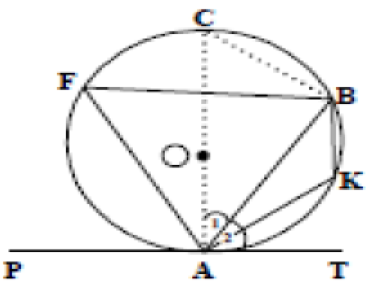


Given:
Cyclic quadrilateral *PQRS*
To Prove:
$\hat{P}+\hat{R}=180°$ and
$\hat{Q}+\hat{S}=180°$
Proof:
Join *OQ* and *OS*
$\hat{O}_1=2\hat{P}$ (∠ at centre = 2× ∠ at circumference)
$\hat{O}_2=2\hat{R}$ (∠at centre=2× ∠ at circumference)
$\therefore\quad \hat{O}_1+\hat{O}_2=2(\hat{P}+\hat{R})$
But $\hat{O}_1+\hat{O}_2=360°$ (∠s around point)
$\therefore\quad \hat{P}+\hat{R}=180°$
Similarly by drawing *OP* and *OR*
$\hat{Q}+\hat{S}=180°$

Theorem 10

The angle between a tangent and a chord, drawn at the point of contact, is equal to the angle which the chord subtends in the alternate segment.

Given:
Tangent PT touching circle O at A; chord AB with $\hat{F}$ in the major segment and $\hat{K}$ in the minor segment.



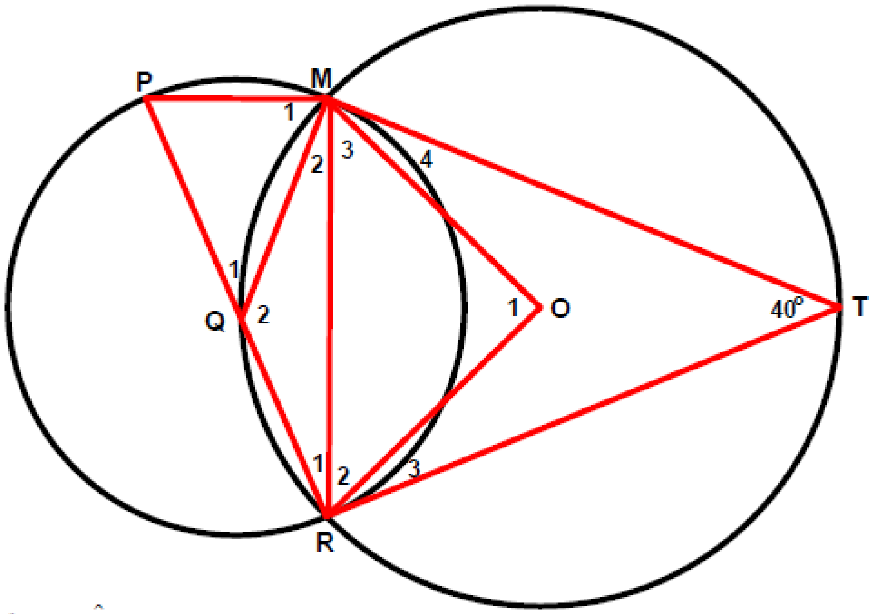
To Prove:
(i) $B\hat{A}T = \hat{F}$
(ii) $P\hat{A}B = \hat{K}$

Proof:
Draw diameter AC.
Join BC.
$\hat{A}_1 + \hat{A}_2 = 90°$ (radius $\perp$ tangent)
But $C\hat{B}A = 90°$ ($\angle$ in semi-circle)
$\therefore \quad \hat{A}_1 + \hat{C} = 90°$ ($\angle$s of ΔABC)
$\therefore \quad \hat{A}_2 = \hat{C}$

But $\hat{C} = \hat{F}$ (subtended by AB)
$\therefore \quad \hat{A}_2 = \hat{F}$
$\therefore (i)\ B\hat{A}T = \hat{F}$
$P\hat{A}B + \hat{A}_2 = 180°$ (adjacent $\angle$s on straight line)
and $\hat{K} + \hat{F} = 180°$ (opposite $\angle$s of cyclic quadrilateral)
But $\quad \hat{F} = \hat{A}_2$ (proved in (i))
$\therefore (ii)\ P\hat{A}B = \hat{K}$

ACTIVITIES

3. The diagram shows circles with centres Q and O, and $M\hat{T}R = 40^0$.
MT and RT are **not** necessarily tangents to the smaller circle.



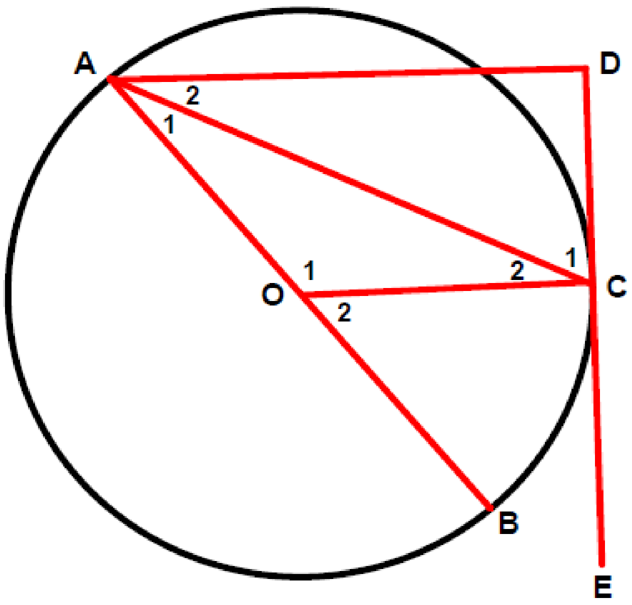
Determine: 3.1 $\hat{Q}_2$

3.2 $\hat{O}_1$

3.3 $P\hat{M}O$

3.4 $\hat{P}$

4. In the accompanying figure, AB is a diameter of the circle with centre O. DC is a tangent to the circle at point C. Chord AC is drawn. D is a point on the tangent DC so that $\hat{A}_1 = \hat{A}_2$.

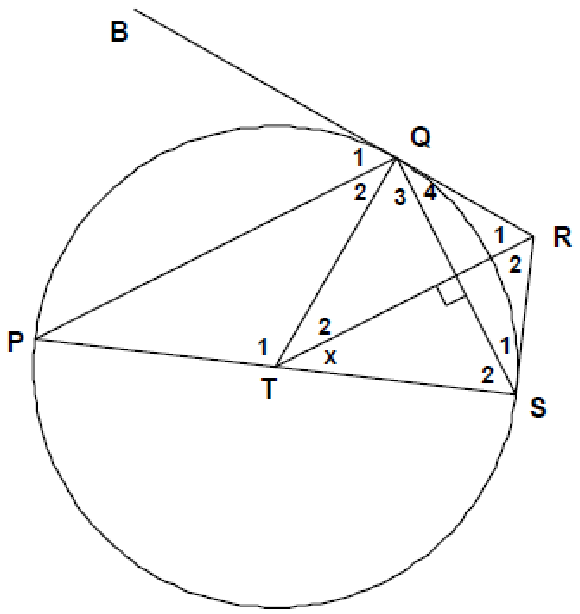


Prove that:

4.1 AD ǁ OC

4.2 $A\hat{D}C = 90°$

5. In the figure PS is a diameter of the circle with centre T. BQ is a tangent to the circle and TR is perpendicular to QS. $R\hat{T}S = x$.

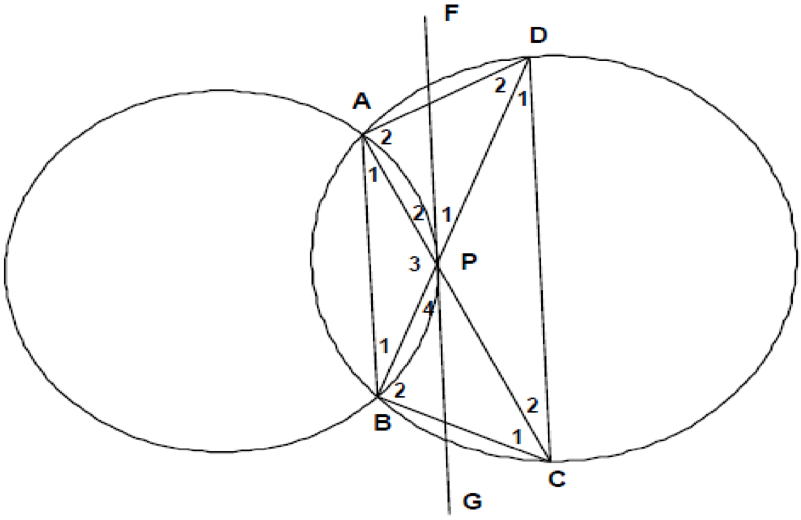


5.1 Prove that TR ǁ PQ.

5.2 Determine, with reasons, other four angles each equal to x.

5.3 Prove that TQRS is a cyclic quadrilateral.

6. In the figure below, diagonals AC and BD of cyclic quadrilateral ABCD intersect at P such that AP = PB. FPG is a tangent to circle

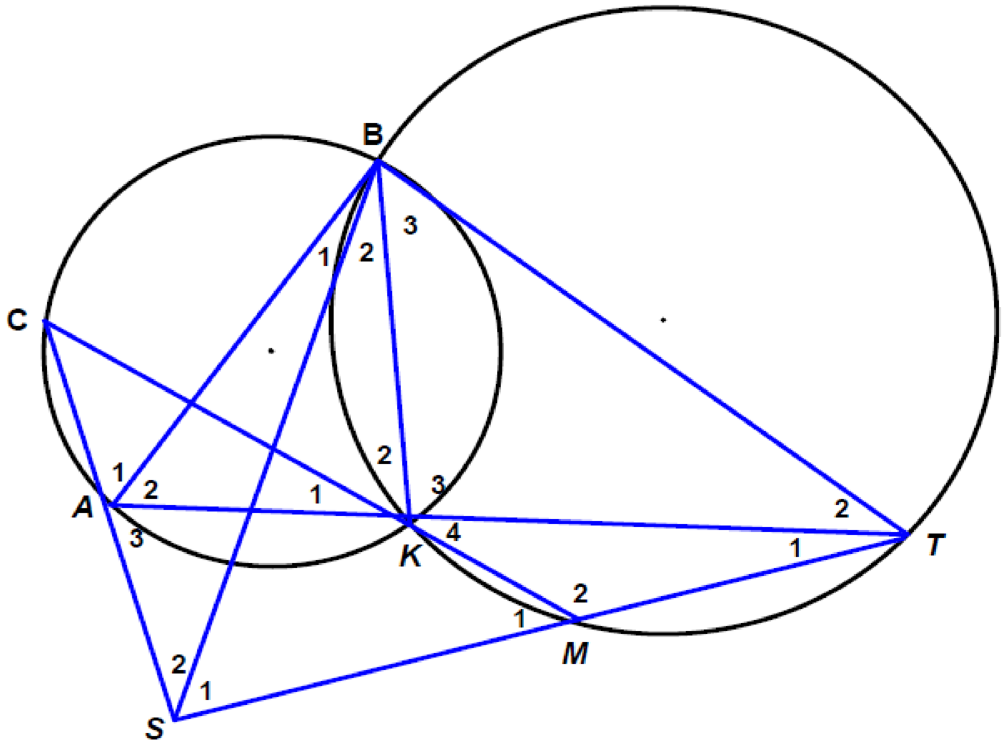


ABP.

Prove that:

6.1 FG ∥ DC

7. The sketch below shows circles BKAC and KMTB intersecting at K and B, and $A\hat{B}T = 90^{\circ}$. AB and BT are not diameters, BT is not a tangent to the smaller circle, and AB is not a tangent to the larger circle.



7.1 Prove that SABT is a cyclic quadrilateral.

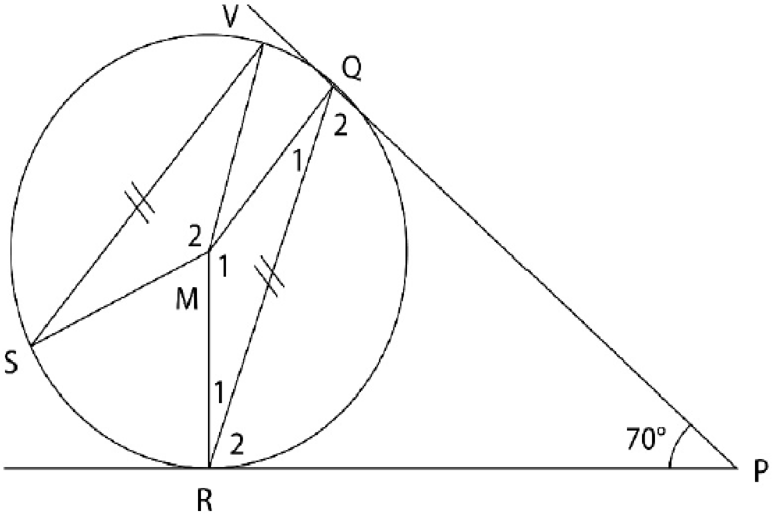
Question 8

M is the centre of the circle SVQR having equal chords SV and QR. RP and QP are tangents to the circle at R and Q respectively such that $R\hat{P}Q = 70^o$.

a) Calculate the size of $\hat{R}_2$.

b) Calculate the size of $\hat{Q}_1$.

c) Determine the size of $\hat{M}_2$.

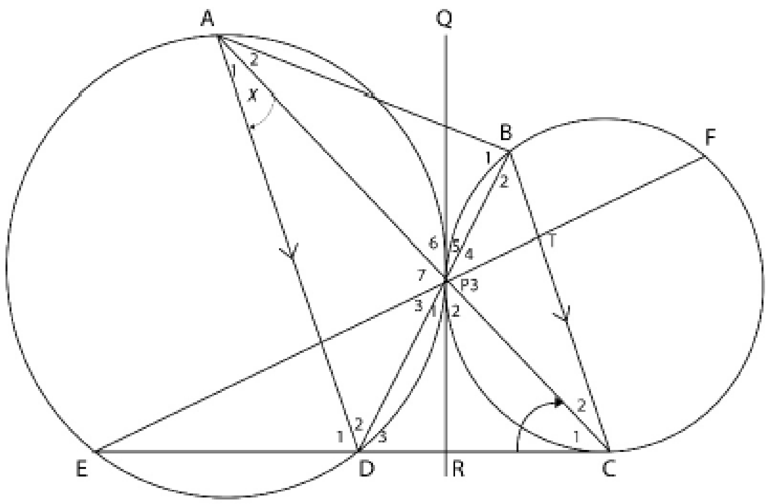


Question 9

In the diagram below two circles touch each other externally at point P. QPR is a common tangent to both circles at P. EDRC is a tangent to circle PBFC at C.

$R\hat{C}A = y$ and $D\hat{A}C = x$.

$AD//BC$.



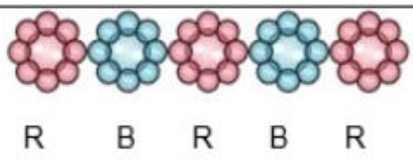
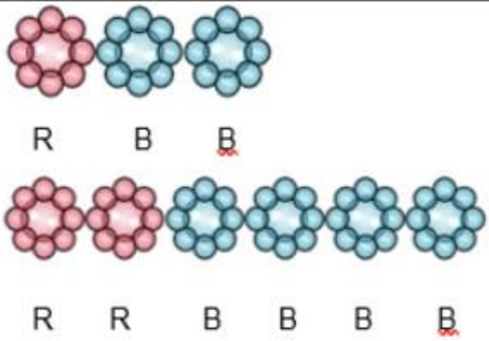
a) Name, with reasons, 4 other angles equal to x.

b) Show that $E\hat{P}A = x + y$.

c) Determine the numerical value of $x + y$, if it is given that DCTP is a cyclic quadrilateral.

SECTION C
GRADE 12 EUCLIDEAN GEOMETRY

Ratio and proportion

Ratio	Proportion
	
In the given figure, there are 3 red flowers to 2 blue flowers. In other words, the red to blue flowers are in the ratio 3: 2. 3 and 2 are two quantities of the same unit.	1 out of 3 flowers is red. Therefore, 2 out of 6 flowers are red. $1:3 = 2:6$

If:	Then:	Note:
$p : q = r : s$	$q : p = s : r$	the variables were inverted
$p : q = r : s$	$p : r = q : s$	The pairs were alternated but the first variable in one ratio went with the first variable in the other ratio
$p : q = r : s$	$p + q : q = r + s : s$ or $p - q : q = r - s : s$	If the 2nd variable (unit) is added/subtracted to the first in EACH ratio, the ratios remain in proportion.
$p : q = r : s$	$p + q : p - q = r + s : r - s$	This rule is a combination of the 2 previous rules.
$p : q = r : s$	$p + r : q + s$ or $p - r : q - s$	The first 2 units in each ratio are added/subtracted and the second two units in each ratio are added/subtracted.

Proportion Theorem

Draw two large triangles with a pen and ruler. They can be any type of triangle.
Label each one ABC – it is not important where you choose to put the A, B and C.

Triangle 1
Triangle 2

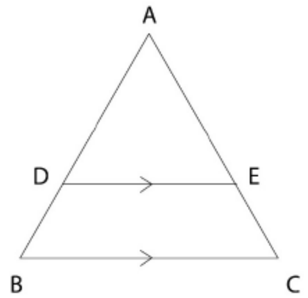
Lay your ruler on BC in the first triangle and draw a line IN PENCIL parallel to BC inside the triangle. It can be any distance from BC – try and avoid it being too close to the middle.
Label the new line DE.

Complete the following table by measuring all the sides (measure in mm for accuracy) and finding the ratio of the two sides. Round their answers to 3 decimal places.

$\frac{AD}{BD} =$	$\frac{AD}{AB} =$	$\frac{BD}{AB} =$
$\frac{AE}{EC} =$	$\frac{AE}{AC} =$	$\frac{CE}{AC} =$

What do you notice about your answers?

Conclusion:

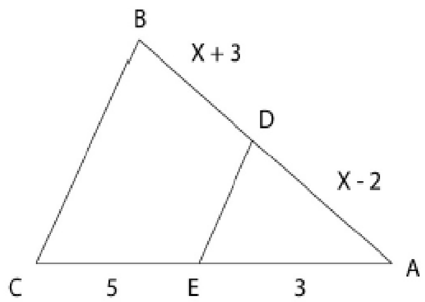


Theorem	**Acceptable abbreviated form**
A line drawn parallel to one side of a triangle divides the other two sides proportionally.	Line parallel one side of Δ

Converse:

Theorem	**Acceptable abbreviated form**
If a line divides two sides of a triangle in the same proportion, then the line is parallel to the third side	Line divides 2 sides of Δ in proportion

Example:

Given that $BC//DE$, find the value of x. 	

APPLICATION OF THE THEOREMS

Similar triangle theorems

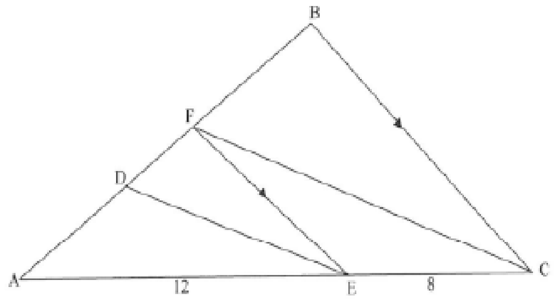
In the diagram, ABC is a triangle with F on AB and E on AC. $BC \parallel FE$.

D is on AF with $\frac{AD}{AF} = \frac{3}{5}$.

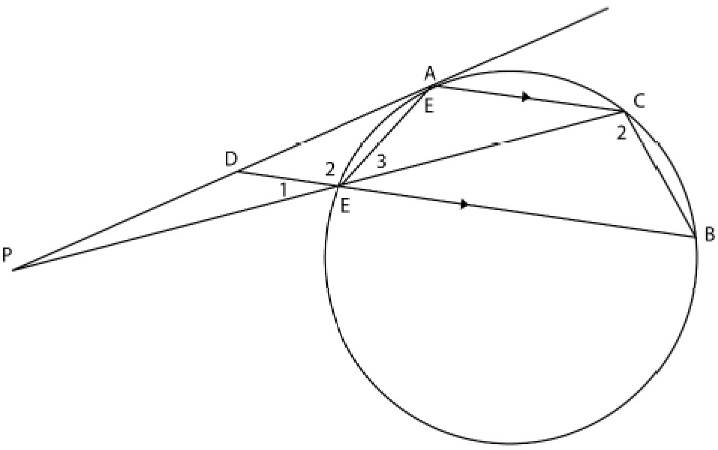
$AE = 12$ units and $EC = 8$ units.

a) Prove that $DE \parallel FC$.

b) If $AB = 14$ units, calculate the length of BF.

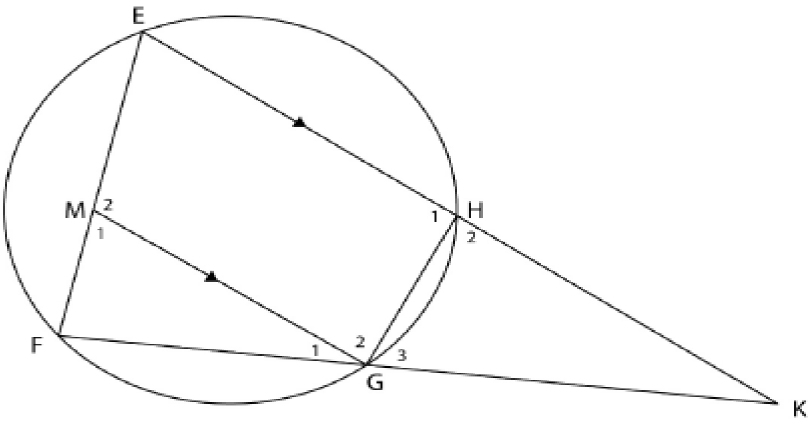


In the diagram below, ACBT is a cyclic quadrilateral having $AC \parallel TB$. CT is produced to P such that the tangent PA meets the circle at A. BT produced meets PA at D.



1. Prove that $\Delta PAT///\Delta PCA$.
2. If $PA = 6, TC = 5$ and $PT = x$,
   a) show that $PT = 4$
   b) calculate the length of PD.

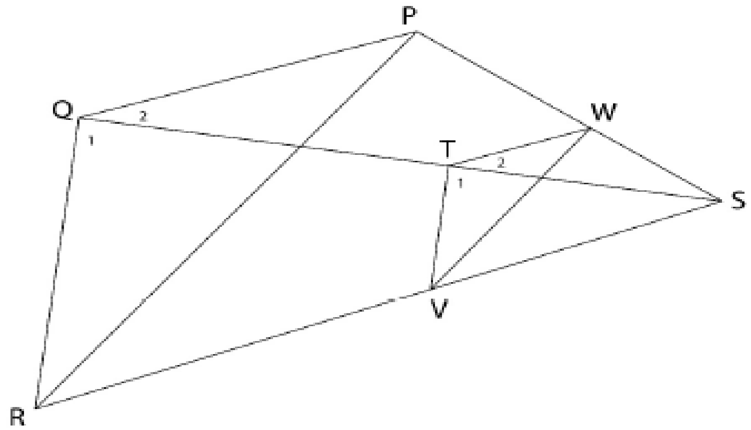
In the diagram below, cyclic quadrilateral EFGH is drawn. Chord EH produced and chord FG produced meet at K. M is a point on EF such that $MG \parallel EK$. Also, $KG = EF$.



1. Prove that:
   a) $\Delta KGH///\Delta KEF$
   b) $EF^2 = KE.GH$
   c) $KG^2 = EM.KF$
2. If it is given that $KE = 20$ units, $KF = 16$ units and $GH = 4$ units, calculate the length of EM.

In the diagram, PQRS is a quadrilateral with diagonals PR and QS drawn. W is a point on PS. WT is parallel to PQ with T on QS. WV is parallel to PR with V on RS. TV is drawn.

$PW : WS = 3 : 2$



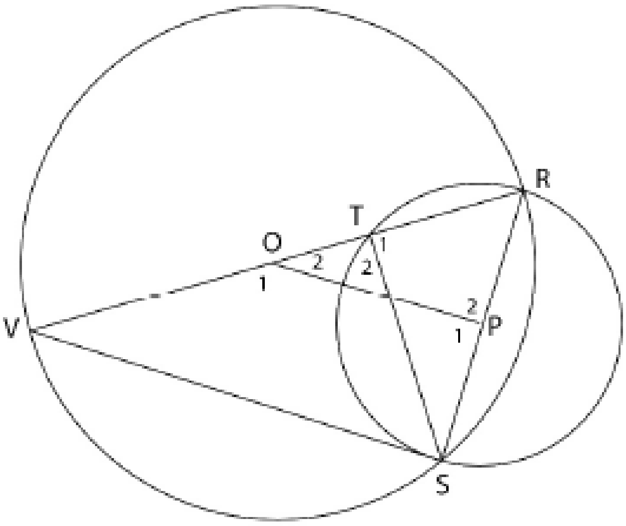
a) Write down the value of the ratios: (i) $\frac{ST}{TQ}$ (ii) $\frac{SV}{VR}$

b) Prove that $\hat{T}_1 = \hat{Q}_1$.

c) Complete the following statement: $\Delta VWS///$ ....

Determine $WV : PR$

In the diagram below, VR is a diameter of a circle with centre O. S is any point on the circumference. P is the midpoint of RS. The circle with RS as diameter cuts VR at T. ST, OP and SV are drawn. RP=PS



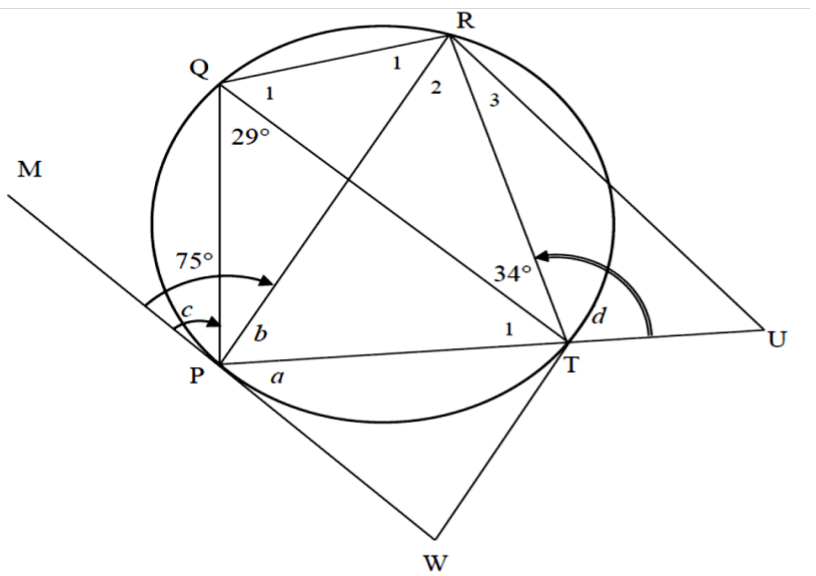
a) Why is $OP \perp PS$?

b) Prove that $\Delta ROP /// \Delta RVS$.

c) Prove that $\Delta RVS /// \Delta RST$.

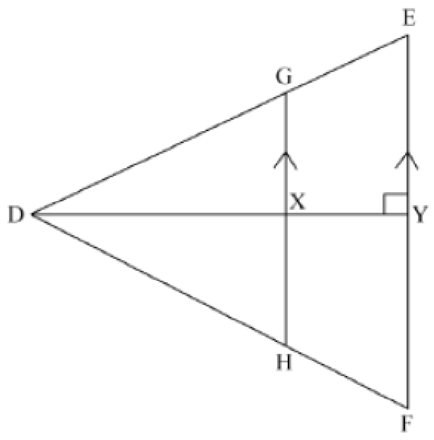
d) Prove that $ST^2 = VT.TR$

In the diagram points P, Q, R and T lie on the circumference of a circle. MW and TW are tangents to the circle at P and T respectively. PT is produced to meet RU at U.
MPR=75° ; PQT=29° ; QTR=34°



Calculate the values of the angles labelled: a,b,c,d (9)

In the figure, GH is drawn parallel to EF. DY is perpendicular to EF and cuts GH at X.



a. Prove:
   1. $\Delta\ DGH\ ///\ \Delta\ DEF$ (3)

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2. $\frac{DX}{DY}=\frac{GH}{EF}$ (4)

b. If the area of $\Delta\,GHD$ is equal to the area of quadrilateral $GHFE$:

1. Express the area of $\Delta\,DEF$ in terms of $\Delta\,GHD$ and $GHFE$. (1)

2. Hence, or otherwise, prove that $\frac{1}{2}EF.DY=GH.DX$ (4)

3. Prove that $\frac{DG}{DE}=\frac{1}{\sqrt{2}}$. (4)

[16]

............End