



## **EVALUATION OF NONLINEAR STATIC PROCEDURES USING BUILDING STRONG MOTION RECORDS**

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### **SUMMARY**

The objective of this investigation is to evaluate the FEMA-356 Nonlinear Static Procedure (NSP), the Modal Pushover Analysis (MPA) procedure, and the “Sum-Difference” procedure using recorded motions of a building that was damaged during the 1994 Northridge earthquake. For this purpose, displacements and drifts from these procedures are compared with the values “derived” from the recorded motions. It is found that the FEMA-356 NSP and the “Sum-Difference” procedure typically underestimates the drifts in upper stories when compared to the recorded motions. Among the four FEMA-356 distributions considered, the “Uniform” distribution led to the most excessive underestimation indicating that this distribution may be unnecessary. The MPA procedure, in general, provides much-improved estimates of the response compared to the FEMA-356 NSP and the “Sum-Difference” procedure. In particular, the MPA procedure is able to capture the effects of higher modes.

### **INTRODUCTION**

Estimating seismic demands at low performance levels, such as life safety and collapse prevention, requires explicit consideration of inelastic behavior of the structure. While nonlinear response history analysis (RHA) is the most rigorous procedure to compute seismic demands, current civil engineering practice prefers to use the nonlinear static procedure (NSP) or pushover analysis specified in the FEMA documents. In early version of the NSP procedure [1, 2], the seismic demands are computed by nonlinear static analysis of the structure subjected to monotonically increasing lateral forces with an invariant height-wise distribution until a predetermined target displacement is reached. Both the force distribution and target displacement are based on the assumption that the response is controlled by the fundamental mode and that the mode shape remains unchanged after the structure yields.

In past few years, several researchers have discussed the underlying assumptions and limitations of the pushover analysis [3-8]. It has been found that satisfactory predictions of seismic demands are mostly

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restricted to low- and medium-rise structures for which higher mode effects are likely to be minimal and the inelastic action is distributed throughout the height of the structure [9].

None of the invariant force distributions can account for redistribution of inertia forces because of structural yielding and the associated changes in the vibration properties of the structure. To overcome this limitation, several researchers have proposed adaptive force distributions that attempt to follow more closely the time-variant distributions of inertia forces [10, 11]. The most recent version of the FEMA documents [12], denoted as FEMA-356, includes one adaptive distribution in the list of lateral load pattern from which two are selected (details are provided later). While these adaptive force distributions may provide better estimates of seismic demands [10], they are conceptually complicated, computationally demanding for routine application in structural engineering practice, and require special purpose computer program to carry out the step-by-step analysis.

Attempts have also been made to consider more than the fundamental vibration mode in pushover analysis. The Multi-Mode Pushover (MMP) procedure [13, 14] provided information on possible failure mechanisms due to higher modes, which may be missed by the standard NSP analyses. But other information of interest in the design process, such as story drifts and plastic rotations, could not be computed by the MMP procedure. The “Sum-Difference” procedure [15, 16] also provided “useful” information but was tested on a single building [16].

Recently, a modal pushover analysis (MPA) procedure has been developed based on structural dynamics theory that includes the contribution of several modes of vibration [17]. This procedure was further refined and systematically evaluated using six buildings, each analyzed for 20 ground motions [18]. It was found that with sufficient number of “modes” included, the height-wise distribution of story drifts estimated by MPA is generally similar to trends noted from nonlinear RHA. Furthermore, the additional error (or bias) in the MPA procedure applied to inelastic structures is small to modest compared to the bias in response spectrum analysis (RSA) applied to elastic structures – the standard analytical tool for the structural engineering profession – unless the building is deformed far into the inelastic region with significant stiffness and strength deterioration.

Most of the previous work on development and evaluation of the NSP and improved procedures are based on response of analytical models subjected to recorded and/or simulated earthquake ground motions. Recorded motions of buildings, especially those deformed into the inelastic range, provide a unique opportunity to evaluate such procedures. Therefore, the principal objective of this investigation is to evaluate the FEMA-356 NSP, the MPA, and the “Sum-Difference” procedures using recorded motions of a building that was deformed beyond the elastic limit during the 1994 Northridge earthquake. Although general results from evaluation of NSP procedures using strong motion records of buildings have been reported previously [19-22], this paper presents comprehensive results for one building including additional results for the “Sum-Difference” Procedure.

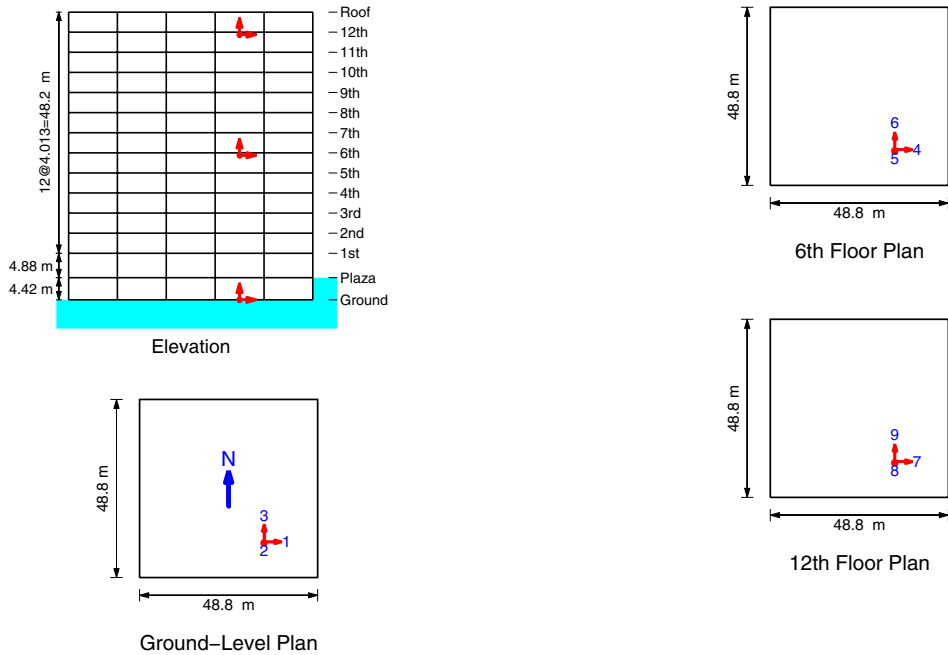
### **SELECTED BUILDING**

Recorded motions of buildings that were deformed beyond the elastic limit (or damaged) during the earthquake are required for this investigation. For this purpose, a 13-story welded special moment frame building located in Woodland Hills, California has been selected. This building was damaged and its motions were recorded during the 1994 Northridge earthquake.

The Woodland Hills 13-Story building was constructed in 1975. Its lateral load resisting system consists of four identical steel welded special moment frames along the building perimeter. The typical floor is square with 160-ft (48.8 m) sides. At the first floor above ground, the plan broadens on three sides to form

a plaza level while the fourth side abuts a landscape berm. These conditions provide a high degree of lateral restraint at this level. Basement perimeter walls are reinforced concrete and the foundation system consists of piles, pilecaps, and grade beams.

The Woodland Hills building is nominally instrumented as required by the local building code (Fig. 1). Motions were recorded during the 1994 Northridge earthquake at three levels: ground, 6<sup>th</sup> floor, and 12<sup>th</sup> floor [23]. The peak horizontal accelerations were 0.44g at the base and 0.33g in the structure. This building was damaged during the 1994 Northridge earthquake. The damage consisted of local fracture at the beam-to-column welded joints [24].



**Figure 1. Sensor location in Woodland Hills 13-Story building.**

## ANALYSIS OF RECORDED MOTIONS

### “Derived” Displacements and Drifts

Since buildings are typically instrumented at a limited number of floors, the motions of non-instrumented floors must be inferred from the instrumented floors for calculations of inter-story drifts in all stories. For this purpose, cubic spline interpolation procedure [25, 26] is used. The cubic spline interpolation procedure is preferred over the parametric model procedure because it automatically accounts for nonlinearities and time variance of the building parameters. This procedure has been tested [25] and found to be highly accurate in estimating the motions of non-instrumented floors.

The cubic spline interpolation is performed on the building deformation (relative to the base) instead of the floor accelerations as traditionally done. This is because splines satisfy conditions of continuity and differentiability of second order at the interpolation points (i.e., instrumented floors in this case) and hence provide smooth shapes, as it should be, for the displacement field of the building.

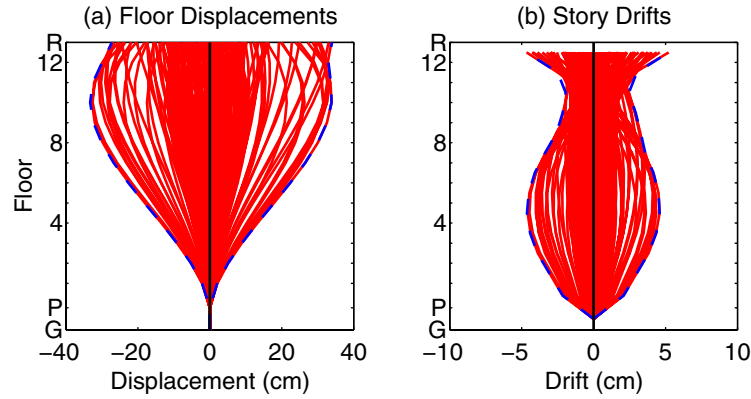
Once the time variation of deformations of all floors have been developed using the cubic spline interpolation procedure, inter-story drifts at each time instant is computed from

$$\delta_j(t) = u_j(t) - u_{j-1}(t) \quad (1)$$

in which  $\delta_j(t)$  is the inter-story drift in the  $j$ th story, and  $u_j(t)$  and  $u_{j-1}(t)$  are the deformations at the  $j$ th and  $j-1$ th floor levels at time  $t$ . Once the time histories of the inter-story drifts have been developed, peak values in the  $j$ th story,  $\delta_{j_o}$ , is computed as the absolute maximum value over time. These values, denoted as “derived” inter-story drifts, along with the peak floor displacements, would be used to evaluate the FEMA-356 NSP, the MPA, and the “Sum-Difference” procedures.

### Displacement and Drift Profile Histories

Histories of floor displacements and inter-story drifts at geometric center of the building were “derived” using the aforementioned procedure for each of the four selected buildings and are presented in Fig. 2. The displacement profile results indicate that although the first mode contribution is dominant, the second mode also contributes significantly (Fig. 2a). The contribution of second and higher modes is especially visible in the story drift profile (Fig. 2b).



**Figure 2. Histories of displacement and drift profiles for the selected building.**

### Modal Decomposition of Recorded Motions

The contributions of various natural modes of vibration of the building to the total displacement can be extracted from the recorded (or “derived”) motions by using the standard modal analysis method [27]; the procedure would lead to exact modal contributions for buildings that remain elastic but approximate for inelastic buildings. This procedure has been used previously [17] to investigate the contributions of higher modes in inelastic buildings.

The contribution of the  $n^{\text{th}}$  mode to total deformation at floor level  $j$  and time instant  $t$  is given by:

$$u_{jn}(t) = \frac{\phi_n^T \mathbf{m} \mathbf{u}(t)}{\phi_n^T \mathbf{m} \phi_n} \phi_{jn} \quad (2)$$

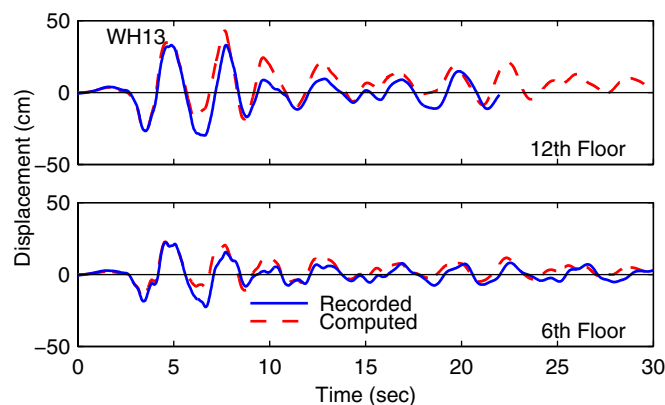
in which  $\phi_n$  is the  $n^{\text{th}}$  mode shape of the elastic building,  $\mathbf{m}$  is the mass matrix,  $\mathbf{u}(t)$  is the vector of displacements at all floor levels at time  $t$ , and  $\phi_{jn}$  is the  $n^{\text{th}}$  mode shape component at the  $j^{\text{th}}$  floor level. Once the contribution of the  $n^{\text{th}}$  mode to the floor displacements have been computed, its contribution to inter-story drift,  $\delta_{jn}(t)$ , can be computed using Eq. (1).

## ANALYTICAL MODELS

The computer program DRAIN-2DX [28] was used for analysis of the selected building. The analytical model was calibrated against the information from the recorded motions as follows. First, the fundamental mode period from eigen analysis of the analytical model and the “elastic” period obtained from system-identification analysis were compared to assess accuracy of the linear model. Second, the time history of the displacement response is computed from the analytical model using the acceleration recorded at the base as the input motion. The computed motions are then compared with the recorded motions to verify that the response from the analytical model correlates reasonably with the recorded motions. Following is a brief description of the analytical model and comparison of the computed and recorded motions; additional details are available elsewhere [20].

The computer model developed earlier [24] was adopted for analysis of this building. The moment frame in the north-south direction is modeled because it experienced significant damage, in the form of connection failures, during the 1994 Northridge earthquake [24]. The two-dimensional model consisted of beams and columns modeled by nonlinear beam-column element with 100% rigid-end offsets, 2% strain hardening for the beams, steel section P-M interaction curve for columns, panel zones modeled as semi-rigid with connection element, and Rayleigh damping of 5% for the first and third modes. The expected yield stress for steel members equal to 47.3 ksi is used, which is about 30% higher than the nominal value of 36 ksi. The two-dimensional model for this building is reasonable because of symmetric plan of this building.

The displacement response of above described model computed to the north-south component of the motions recorded at the base matched reasonably well with the recorded motions in this direction [24]. But pushover analysis (presented later in this paper) to the peak roof displacement recorded during the 1994 Northridge earthquake indicates that none of its elements yield. This behavior of the model is contrary to the physical observation during the post-earthquake inspection, which revealed numerous beam-column connection failures. Therefore, the model was modified by reducing the strengths of beams and panel zone elements by 25% compared to the original model. This brings the expected yield stress close to the nominal yield stress of 36 ksi. Furthermore, the Rayleigh damping was increased from 5% to 7% in the first and third modes.



**Figure 3. Comparison of displacements computed from analytical model with recorded displacements of the Woodland Hills 13-story building.**

The displacement response history of the analytical model was calculated using the north-south component of the motion recorded at the base during the 1994 Northridge earthquake. The comparison of displacements from the response history analysis with the recorded motions in the north-south direction at

the center of the building, shown in Fig. 3, indicates a reasonable match between the two. This implies that the simple model used in this study is adequate in representing the recorded motions. It may be possible to further improve the accuracy of the model by using more “accurate” connection behavior.

## NONLINEAR STATIC PROCEDURES

### FEMA-356 NSP

The nonlinear static procedure (NSP) specified in the FEMA-356 [12] document may be used for any structure and any rehabilitation objective except for structures with significant higher mode effects. To determine if higher mode effects are present, two linear response spectrum analyses must be performed: (1) using sufficient modes to capture 90% of the total mass, and (2) using only the fundamental mode. If shear in any story from the first analysis exceeds 130% of the corresponding shear from the second analysis, the higher mode effects are deemed significant. In case the higher mode effects are present, the NSP analysis needs to be supplemented by the Linear Dynamic Procedure (LDP); acceptance criteria for the LDP are relaxed but remain unchanged for the NSP.

The FEMA-356 NSP requires development of a pushover curve, which is defined as the relationship between the base shear and lateral displacement of a control node, ranging between zero and 150% of the target displacement. The control node is located at the center of mass at the roof of a building. For buildings with a penthouse, the floor of the penthouse (not its roof) is regarded as the level of the control node. Gravity loads are applied prior to the lateral load analysis required to develop the pushover curve.

The pushover curve is developed for at least two vertical distributions of lateral loads. The first pattern is selected from one of the following: (1) Equivalent lateral force (ELF) distribution:  $s_j^* = m_j h_j^k$  (the floor number  $j = 1, 2, \dots, N$ ) where  $s_j^* = m_j$  is the lateral force and  $s_j^* = m_j$  the mass at  $j$ th floor,  $h_j$  is the height of the  $j$ th floor above the base, and the exponent  $k = 1$  for fundamental period  $T_1 \leq 0.5$  sec,  $k = 2$  for  $T_1 \geq 2.5$  sec; and varies linearly in between; (2) Fundamental mode distribution:  $s_j^* = m_j \phi_{j1}$  where  $\phi_{j1}$  is the fundamental mode shape component at the  $j$ th floor; and (3) SRSS distribution:  $\mathbf{s}^*$  is defined by the lateral forces back-calculated from the story shears determined by linear response spectrum analysis of the structure including sufficient number of modes to capture 90% of the total mass. The second pattern is selected from either “Uniform” distribution:  $s_j^* = m_j$  in which  $s_j^* = m_j$  is the mass and  $s_j^* = m_j$  is the lateral force at  $j$ th floor; or Adaptive distribution that changes as the structure is displaced.

The target displacement is computed from

$$\delta_t = C_0 C_1 C_2 C_3 S_a \frac{T_e^2}{2\pi^2} g \quad (3)$$

where  $T_e$  = Effective fundamental period of the building in the direction under consideration,  $S_a$  = Response spectrum acceleration at the effective fundamental vibration period and damping ratio of the building under consideration and  $g$  is the acceleration due to gravity,  $C_0$  = Modification factor that relates the elastic response of an SDF system to the elastic displacement of the MDF building at the control node,  $C_1$  = Modification factor that relates the maximum inelastic and elastic displacement of the SDF system,  $C_2$  = Modification factor to represent the effects of pinched hysteretic shape, stiffness degradation, and strength deterioration, and  $C_3$  = Modification factor to represent increased displacement due to P-delta effects.

The deformation/force demands in each structural element is computed at the target displacement and compared against acceptability criteria set forth in the FEMA-356 document. These criteria depend on the material (e.g., concrete, steel etc.), type of member (e.g., beam, column, panel zones, connections etc.), importance of the member (e.g., primary, or secondary) and the structural performance levels (e.g., immediate occupancy, life safety, collapse prevention).

The FEMA-356 NSP procedure contains several approximations. These include those in estimating the target displacement from Eq. 3, and using the pushover curve to estimate the member demands imposed by the earthquake. In this investigation, the focus is primarily on the second source of approximation; the first approximation is a focus of numerous other investigations. For this purpose, the following analysis method is employed.

The target displacement is selected to be equal to that of the roof level recorded during the earthquake, as opposed to calculating it according to the FEMA-356 document (Eq. 3). The structure is pushed to this target displacement using the FEMA-356 lateral load patterns and floor displacements and inter-story drifts are computed. These computed values are then compared with the “derived” values, i.e., those computed directly from the recorded motions using the procedure described in the preceding section. Such a comparison enables evaluation of the adequacy of various lateral load patterns in the FEMA-356 NSP, in particular, if the FEMA-356 NSP is able to capture the higher mode effects, which are likely to be present in the selected buildings.

### MPA Procedure

Recently a MPA procedure has been developed to account for the higher mode effects and analytically tested for SAC buildings and ground motions [17, 18]. This procedure has been found to be highly accurate unless the building is deformed far into the region of stiffness and strength deterioration [18]. Following is a summary of this procedure.

1. Compute the natural frequencies,  $\omega_n$  and modes,  $\phi_n$ , for linearly elastic vibration of the building.
2. For the  $n$ th-mode, develop the base shear-roof displacement,  $V_{bn} - u_m$ , pushover curve for force distribution,  $\mathbf{s}_n^* = \mathbf{m}\phi_n$ , where  $\mathbf{m}$  is the mass matrix of the structure. Gravity loads, including those present on the interior (gravity) frames, are applied before the modal pushover analysis. The resulting P- $\Delta$  effects may lead to negative post-yielding stiffness in the pushover curve. Note the value of the lateral roof displacement due to gravity loads,  $u_{rg}$ .
3. Idealize the pushover curve as a bilinear curve. If the pushover curve exhibits negative post-yielding stiffness, the second stiffness (or post-yield stiffness) of the bilinear curve would be negative.
4. Convert the idealized  $V_{bn} - u_m$  pushover curve to the force-displacement,  $F_{sn}/L_n - D_n$ , relation for the  $n$ th “mode” inelastic SDF system by utilizing  $F_{sny}/L_n = V_{bny}/M_n^*$  and  $D_{ny} = u_{mny}/\Gamma_n\phi_m$  in which  $M_n^*$  is the effective modal mass,  $\phi_m$  is the value of  $\phi_n$  at the roof, and  $\Gamma_n = \phi_n^T \mathbf{m} \mathbf{l} / \phi_n^T \mathbf{m} \phi_n$ .
5. Compute the peak deformation  $D_n$  of the  $n$ th “mode” inelastic single-degree-of-freedom (SDF) system defined by the force-deformation relation developed in Step 4 and damping ratio  $\zeta_n$ . The elastic vibration period of the system is  $T_n = 2\pi(L_n D_{ny} / F_{sny})^{1/2}$ . For an SDF system with known  $T_n$  and  $\zeta_n$ ,  $D_n$  can be computed either by nonlinear RHA, from inelastic design spectrum, or by empirical equations for the ratio of deformations of inelastic and elastic systems [29].

6. Calculate peak roof displacement  $u_{rn}$  associated with the  $n$ th-“mode” inelastic SDF system from  $u_{rn} = \Gamma_n \phi_{rn} D_n$ .
7. From the pushover database (Step 2), extract values of desired responses  $r_n$  due to the combined effects of gravity and lateral loads at roof displacement equal to  $u_{rn} + u_{rg}$ .
8. Repeat Steps 3-7 for as many modes as required for sufficient accuracy.
9. Compute the dynamic response due to  $n$ th-“mode”:  $r_n$ , where  $r_g$  is the contribution of gravity loads alone.
10. Determine the total response (demand) by combining gravity response and the peak “modal” responses using the SRSS rule:  $r \approx \left( \sum_n r_n^2 \right)^{1/2}$ .

Steps 3 to 6 of the MPA procedure described above are used to compute the peak roof displacement associated with the  $n$ th-“mode” inelastic SDF system. However, these steps are not necessary for analysis of a building for which recorded motions are available. The contribution of the  $n$ th-“mode” to the total roof displacement,  $u_{rn}$ , can be computed from modal decomposition of recorded motion using Eq. (2).

#### “Sum-Difference” Procedure

The “Sum-Difference” Procedure requires development of the pushover curve for force distribution given by

$$s = s_n \pm s_r \quad (4)$$

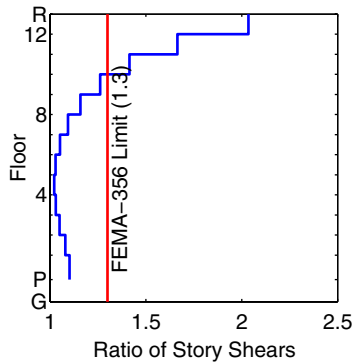
in which  $s_n = \Gamma_n m \phi_n A_n$ , ( $A_n$  = pseudo-acceleration of a linear elastic SDF system with period and damping ratio equal to that of corresponding to the  $n$ th mode of the building) and  $\Gamma_n = \phi_n^T m r / \phi_n^T m \phi_n$ . The original procedure suggested combinations with  $n = 1$  and  $r = 2$  [16], however, other combination in Eq. (4) may be considered with  $n = 1$  and  $r = 2, 3, \dots, N$ . The floor displacements and story drifts are computed in a manner similar to that in the FEMA-356 NSP but utilizing the pushover curves for force distributions of Eq. (4).

#### FEMA-356 CHECK FOR HIGHER MODES

The FEMA-356 criterion for checking presence of significant higher mode effects is applied to the selected building. For this purpose, story shears are computed from two elastic modal analyses: (1) considering sufficient number of modes to capture at least 90% of the total mass, and (2) considering the fundamental mode only. For the Woodland Hills building, five modes were needed to capture 90% of the total mass. The ratio of the story shears from the two analyses is computed and compared with the limiting value of 1.3 specified in the FEMA-356 document in Fig. 4 for the selected building. These results lead to the following conclusions.

The ratio of story shears from 5-mode analysis and 1-mode analysis exceeds the FEMA-356 limiting value of 1.3 in upper stories of the Woodland Hills building (Fig. 4). Clearly, this building is expected to respond significantly in higher modes. The displacement and drift profile histories of this building during the ground shaking (Fig. 2) also indicated presence of higher mode effects, especially in the story drifts. It is also apparent from the results of Fig. 4 that the largest value of the ratio of shears from 5-modes and 1-mode analyses occurs in upper stories of the selected buildings. This indicates that the higher mode effects are likely to be significant for responses, such as drifts, in upper stories.





**Figure 4. FEMA-356 check for presence of higher modes in the selected building.**

### **PUSHOVER CURVES**

The lateral force distributions corresponding to four FEMA-356 NSP, first three modes of the MPA procedure, and combination of first and second and first and third modes in the “Sum-Difference” procedure are used to generate pushover curves for the frame in the north-south direction of the Woodland Hills building. These pushover curves lead to the following observations.

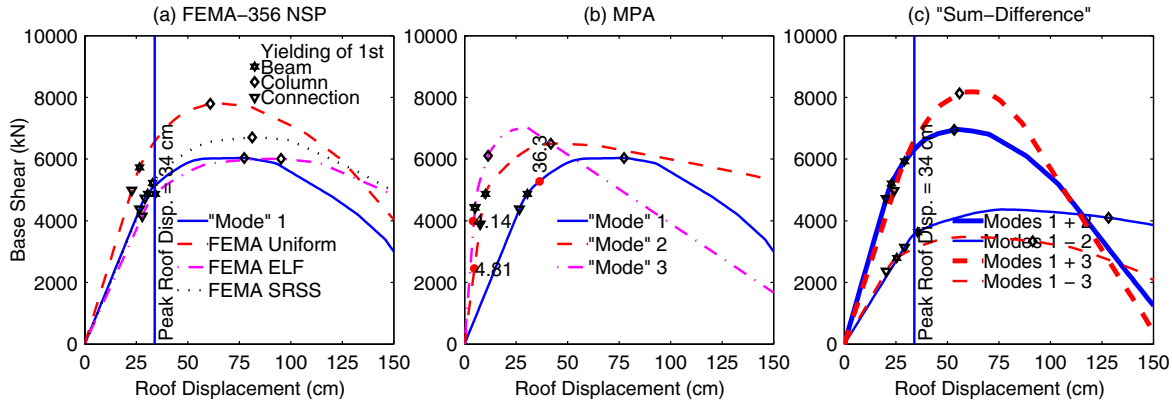
The characteristic – elastic stiffness, and yield strength and displacement – of the pushover curve depend on the lateral force distribution (Fig. 5a). The “Uniform” distribution generally leads to pushover curve with higher elastic stiffness, higher yield strength, and lower yield displacement compared to all other distributions. The ELF distribution, on the other hand, leads to pushover curve with lower elastic stiffness, lower yield strength, and higher yield displacement. The “Mode” 1 and SRSS distribution give pushover curves that are bounded by the pushover curves due to “Uniform” and ELF distributions.

The pushover curves for the Woodland Hills building (Fig. 5a) exhibit significant degradation in lateral load carrying capacity at large roof displacements. The onset of the degradation depends on the lateral force distribution: the “Uniform” distribution induces the earliest, the ELF distribution the latest, and the “Mode” 1 and SRSS distributions in between the “Uniform” and ELF distributions. The degradation in the lateral load carrying capacity occurs due to P-Delta effects arising from the gravity loads. These effects may lead to negative slope of the pushover curve at large roof displacements (Fig. 5a).

The first yielding in the Woodland Hills building occurs in the connection followed soon after by the first yielding of the beam (Fig. 5a). The columns start to yield at much higher deformation level, followed immediately by rapid deterioration of the lateral load carrying capacity of the building. The Woodland Hills building is deformed only slightly beyond the elastic limit during the 1994 Northridge earthquake.

The Woodland Hills building is deformed beyond the elastic limit only in the first mode during the 1994 Northridge earthquake (Fig. 5b), but remains elastic in the higher modes with the roof displacement during the 1994 Northridge earthquake being smaller than that required to induce yielding in any element. Since, the selected building did not responded beyond the elastic limit in modes higher than the fundamental mode, the Modified Modal Pushover Analysis (MMPA), wherein the response contributions of the modes higher than the fundamental mode are computed by assuming the building to be linearly elastic, may be used to estimate the seismic demands [30]. The MMPA procedure is an attractive alternative to the MPA procedure for these buildings because of reduced computational efforts; the pushover curves for higher modes are not needed in the MMPA procedure.

The “Sum-Difference” pushover curves (Fig. 5c) exhibit characteristics – elastic stiffness, yield strength, yield displacement, degradation in lateral load carrying capacity – similar to the FEMA-356 pushover curves (Fig. 5a). The pushover curves for modes 1+2 and 1+3 are essentially identical; similar trend applies to modes 1-2 and 1-3. However, pushover curves for modes 1+2 and 1-2 are quite different with the pushover curve for modes 1+2 exhibiting larger elastic stiffness, higher yield strength, and more rapid strength degradation. Similar trend applied to pushover curves for modes 1+3 and 1-3.



**Figure 5. Pushover curves for the four FEMA-356, three MPA, and four “Sum-Difference distributions.**

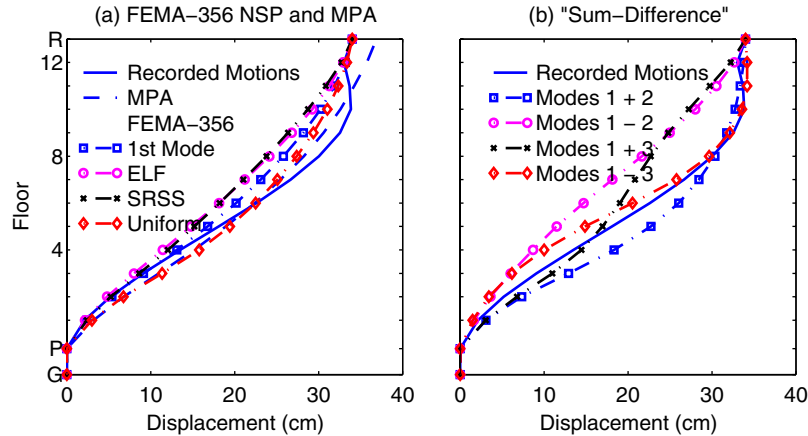
### EVALUATION OF NONLINEAR STATIC PROCEDURES

The nonlinear static procedures are evaluated in this section using recorded motions of the selected building. For this purpose, floor displacements and story drifts from the four FEMA-356 analyses, the four “Sum-Difference” analyses, and the MPA procedure are compared with the “derived” values from the recorded motions. The target roof displacement in the FEMA-356 and the “Sum-Difference” analyses was selected to be that “derived” from the motions recorded at the roof. Similarly, the  $n$ th-“mode” component of the roof displacement,  $u_m$ , required in the MPA procedure was taken to be the value obtained from the  $n$ th “modal” decomposition of the recorded motions. It is useful to emphasize again that two-dimensional model has been used in this investigation and the computed and recorded motions at the center of the selected building are examined in this section. Although FEMA-356 criterion for higher mode effects is exceeded for the selected building, results from the FEMA-356 NSP are included because such analysis is permitted if supplemented by the LDP analysis.

The results presented for the floor displacements (Fig. 6) show that all procedures – FEMA-356, “Sum-Difference”, and MPA – lead to floor displacements that are essentially similar to those “derived” from recorded motions with some minor discrepancies. Note that displacements at the roof level from the FEMA-356 and the “Sum-Difference” analyses and the recorded motions are the same because the target roof displacement in these analyses was selected to be the roof displacement during the ground motion. The displacements are underestimated slightly in middle few floors of the Woodland Hills building by the NSP procedures.

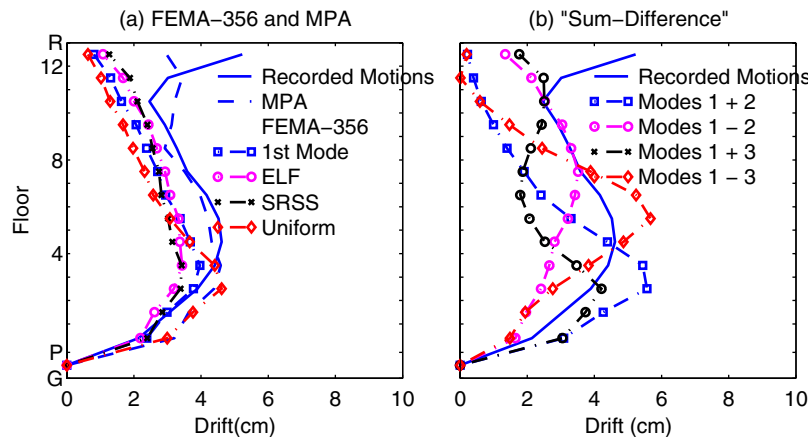
Although the selected building exceeded the FEMA-356 criterion for higher mode effects (Fig. 4), the FEMA-356 NSP, which is applicable for buildings responding primarily in the fundamental mode, provided reasonable estimate of the floor displacement. Furthermore, the MPA procedure or the “Sum-Difference” procedure, which are designed to capture higher mode effects, did not lead to displacements much different than the FEMA-356 NSP. This is the case because the fundamental mode is known to

dominate the floor displacements [27]; higher mode contributions are typically very small for floor displacements.



**Figure 6. Comparison of displacements from recorded motions, MPA procedure, FEMA-356 NSP for four distributions, and the “Sum-Difference” procedure for four distributions.**

However, the FEMA-356 NSP led to gross underestimation of drifts in the upper stories (Fig. 7). Since the larger drift demand in upper stories occurs due to higher mode effects, the FEMA-356 NSP is clearly unable to capture the higher mode effects. Results from the “Sum-Difference” Procedure (Fig. 7b) indicate that distribution corresponding to modes  $n+r$  and  $n-r$  give larger drifts in lower and upper stories, respectively. The envelop of the two sets is expected to give drift distribution similar to that from the nonlinear RHA [16], or the recorded motions in this case. However, the drifts are significantly underestimated in upper stories indicating that the “Sum-Difference” Procedure may also not be able to capture higher mode effects.

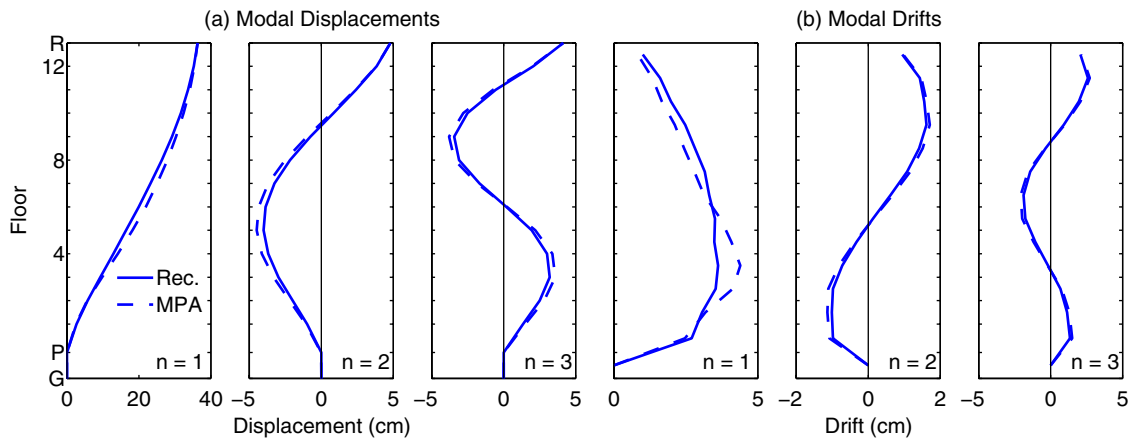


**Figure 7. Comparison of drifts from recorded motions, MPA procedure, FEMA-356 NSP for four distributions, and the “Sum-Difference” procedure for four distributions.**

Among the four FEMA-356 distributions, the “Uniform” force distribution leads to the worst estimates of story drifts (Fig. 7a). This distribution leads to underestimation of the drift in the top story by about 67% – the story drifts from recorded motions and FEMA-356 “Uniform” distributions are 3.01 cm and 1.02 cm, respectively, for the Woodland Hills building. Therefore, this distribution seems unnecessary in the FEMA-356 NSP, an observation which is consistent with that based on an earlier analytical study [18].

The MPA procedure for the selected building provides much better estimates of story drifts throughout the building height (Fig. 7a). In particular, the match between the story drifts from MPA and recorded motions is excellent in upper stories indicating that the MPA procedure is able to capture the higher mode effects for this building.

While the estimates of story drifts from the MPA procedure are much better compared to the FEMA-356 NSP or the “Sum-Difference” Procedure, differences exist, such as drifts in top story of Woodland Hills building (Fig. 7a). In order to understand the source of this discrepancy, peak displacements and drifts in each mode of the MPA procedure are compared with those obtained from modal decomposition of recorded motions (Fig. 8). This comparison shows that the match between the two is reasonably good. Therefore, the prime source of discrepancy appears to be from modal combination procedure. The modal combination rule was found to be deficient in an earlier study [18] even for elastic buildings.



**Figure 8. Comparison of displacements and drifts from MPA procedure with results derived from modal decomposition of recorded motions for first three modes ( $n = 1, 2,$  and  $3$ ).**

A fraction of the errors in the modal combination may be attributed to application of the modal combination rule, which is strictly valid for elastic buildings, for buildings responding beyond the elastic range. However, this fraction has been found to be small in an earlier study where errors in the MPA results of elastic and inelastic systems were compared [18].

The error in large part appears to be due to application of the modal combination rule for peak responses of a single ground motion. Note that the modal combination rules are based on random vibration theory and the combined peak response should be interpreted as the mean of the peak values of response to an ensemble of earthquake excitations. Thus, the modal combination rules are intended for use when the excitation is characterized by a smooth response (or design) spectrum. Although modal combination rules can also approximate the peak response to a single ground motion characterized by a jagged response spectrum, the errors are expected to be much larger in some cases, as noted in this investigation.

## CONCLUSIONS

This research investigation evaluated the FEMA-356 NSP, the MPA, and the “Sum-Difference” procedures using a building that was damaged and its motions recorded during the 1994 Northridge earthquake. Two-dimensional analytical model of this building was developed using computer program DRAIN-2DX and calibrated against information from the recorded motions. This model was analyzed using the FEMA-356 NSP, the MPA, and the “Sum-Difference” procedures.

The pushover curves for the four distributions – “Uniform”, ELF, SRSS, and 1<sup>st</sup> “Mode” – in the FEMA-356 NSP, for four distributions in the “Sum-Difference” Procedure, and for the first three modal distributions in the MPA procedure were generated for each of the selected buildings. These pushover curves show that the characteristic – elastic stiffness, and yield strength and displacement – of the pushover curve depend on the lateral force distribution. Among the FEMA-356 distributions, the “Uniform” distribution generally leads to pushover curve with higher elastic stiffness, higher yield strength, and lower yield displacement compared to all other distributions; the ELF distribution leads to pushover curve with lower elastic stiffness, lower yield strength, and higher yield displacement; and the “Mode” 1 and SRSS distribution pushover curves are bounded by the pushover curves due to “Uniform” and ELF distributions. The pushover curves exhibit significant degradation in lateral load carrying capacity at larger roof displacements due to P-Delta effects arising from the gravity loads. The trends for the “Sum-Difference” distributions are similar to those for the FEMA-356 distributions. The pushover curves for the MPA distribution indicated that the Woodland Hills building is deformed beyond the elastic limit only in the first mode during the 1994 Northridge earthquake.

The estimates of the floor displacements and story drifts were computed from the FEMA-356 NSP, the “Sum-Difference”, and the MPA procedures. These estimates were compared against the values “derived” from the recorded motions of the selected during the 1994 Northridge earthquake. This comparison showed that all procedures lead to floor displacements that are essentially similar to those “derived” from recorded motions. This is the case because the fundamental mode is known to dominate the floor displacements and higher mode contributions are typically very small for floor displacements. However, the FEMA-356 NSP and the “Sum-Difference” Procedure led to gross underestimation of drifts in upper stories of the selected building, indicating that these procedures are unable to account for higher mode effects in the selected building, which typically contribute significantly to the drifts in upper stories. The MPA procedure, on the other hand, provides much better estimates of drifts compared to the FEMA-356 NSP or the “Sum-Difference” Procedure, and is able to account for the higher mode effects. Although significant discrepancy was observed for drifts at a few locations in the selected buildings.

The response for each mode in the MPA procedure matched closely with the modal response obtained from decomposition of the recorded motions, indicating the observed discrepancy between the response from MPA and recorded response is due to limitations in the combination procedure. The modal combination rules are based on random vibration theory and the combined peak response should be interpreted as the mean of the peak values of response to an ensemble of earthquake excitations. Thus, the modal combination rules are intended for use when the excitation is characterized by a smooth response (or design) spectrum. Applied to the peak response to a single ground motion characterized by a jagged response spectrum, the errors are expected to be much larger in some cases, as noted in this investigation.

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