Exam 1 Solutions

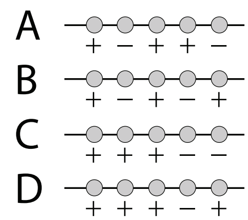
Answer 1 is correct for all problems.

1. Two uniformly charged spheres, A and B, are placed at a large distance from each other, with their centers on the x axis. The charge on sphere A is q, whereas that on sphere B is $2q$. If the electrostatic force on sphere A is $F\hat{\imath}$, that on sphere B is

(1) $-F\hat{\imath}$ (2) $F\hat{\imath}$ (3) $-2F\hat{\imath}$ (4) $2F\hat{\imath}$ (5) $-\frac{1}{2}F\hat{\imath}$

Recall Newton's third law.

2. In each of the four configurations labeled A, B, C, D in the figure, five charges are equally spaced along the x axis. All charges have the same magnitude, e, but some of them are $+e$ and some $-e$. Rank the magnitude of the force on the middle charge for the different configurations, with the largest first and the smallest last.



(1) C, D, A, B (2) C, A, D, B (3) C, B, D, A (4) A, C, D, B (5) D, C, A, B

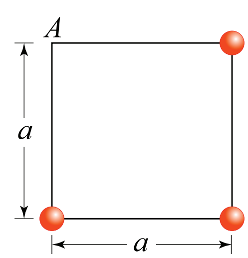
In C, the forces by the four charges add up, whereas they completely cancel out in B. Therefore, C is the first, and B the last. Of the remaining two configurations, D comes before A, because the two outermost charges do not contribute to the net force in D, whereas in A they compete with the forces by the two inner charges.

3. A particle with positive charge Q is on the y axis a distance a from the origin, and a particle with positive charge q is on the x axis a distance d from the origin. What is the value of d for which the x component of the electrostatic force on the second particle is the largest?

(1) $a/\sqrt{2}$ (2) a (3) $\sqrt{2}a$ (4) $2a$ (5) $2\sqrt{2}a$

For convenience, let's call the second distance x instead of d. The magnitude of the force is $F = kQq/(a^2 + x^2)$. The similarity of the two triangles, one defined by the two particles and the origin and the other defined by the force vector and its x and y components, leads to $F_x = Fx/(a^2+x^2)^{1/2}$ for the x component of the force. Hence $F_x = kQqx/(a^2+x^2)^{3/2}$. To maximize this, set $dF_x/dx = 0$, which gives $1/(a^2 + x^2)^{3/2} - \frac{3}{2}x(2x)/(a^2 + x^2)^{5/2} = 0$. Solve this for x.

4. Three particles, each charged with 1.0 nC, are placed on three corners of a square of edge lengths $a = 1.0$ m, as shown. What is the magnitude of the electric field at the empty corner, labeled A?

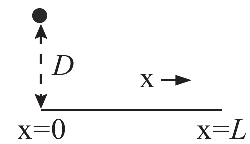


(1) 17 N/C (2) 27 N/C (3) 12 N/C (4) 24 N/C (5) 31 N/C

Find the x and y components of each force, then add the components of the three forces separately to obtain F_x and F_y of the net force F. Finally, find F by computing $(F_x^2 + F_y^2)^{1/2}$.

Alternatively, notice that the sum of the forces by the two nearest charges, $\sqrt{2}(kq^2/a^2)$, is in exactly the same direction as the force by the third charge on the diagonal corner, $kq^2/(2a^2)$. So just add the two expressions and plug in numbers.

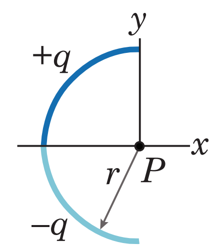
5. A line of charge of length L and charge per unit length, λ, is located on the x axis, as shown in the figure. Which expression below gives the x component of the electric field, E_x, at a distance D from the left end of the line?



(1) $-\int_0^L \frac{kx\lambda}{(x^2+D^2)^{3/2}}dx$ (2) $-\int_0^L \frac{kD\lambda}{(x^2+D^2)^{3/2}}dx$ (3) $-\int_0^L \frac{kx\lambda}{x^2+D^2}dx$ (4) $-\int_0^L \frac{kD\lambda}{x^2+D^2}dx$ (5) $-\int_0^L \frac{k\lambda}{x^2+D^2}dx$

The magnitude of the electric field due to an infinitesimal line element dx located at x is $dE = k(\lambda dx)/(x^2 + D^2)$. Use the similarity of two normal triangles, as was done in the solution to Problem 3, to find that the x component of this force is given by $dE_x = -dEx/(x^2 + D^2)^{1/2}$. Integrate this from $x = 0$ to $x = L$.

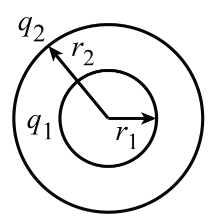
6. A glass rod forms a semi-circle of radius $r = 5$ cm with a charge of $+q$ distributed uniformly in the upper quadrant and $-q$ distributed uniformly in the lower quadrant. What is the direction of the electric field at the center P of the semi-circle?



(1) $-\hat{j}$
(2) $\hat{j}$
(3) $\hat{i}$
(4) $-\hat{i}$
(5) The electric field is zero.

The electric field due to the positively charged part of the rod points in the $\hat{i} - \hat{j}$ direction, whereas the negatively charged part produces an electric field pointing in the $-\hat{i} - \hat{j}$ direction. Since the magnitudes of the two fields are exactly the same, their x components will cancel out, leaving only the component in the $-\hat{j}$ direction.

7. Two thin concentric spherical shells made of copper have net charges of $q_1 = -46$ nC, $q_2 = 69$ nC and radii of $r_1 = 15$ cm, $r_2 = 34$ cm as shown in the figure. What is the surface charge density on the *inner* surface of the smaller shell, with radius r_1, in units of C/m^2?

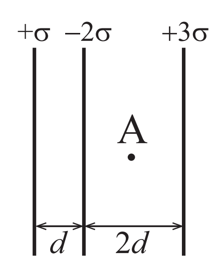


(1) 0 (2) -6.51×10^{-7} (3) 6.51×10^{-7} (4) -3.25×10^{-7} (5) 3.25×10^{-7}

At equilibrium, no charge will appear on the inner surface of a conductor unless the cavity contains a charged body.

Alternatively, in the present geometry, you may draw a spherical, concentric Gaussian surface whose radius is slightly larger than the inner radius of the inner shell. Since the electric field in a conductor is zero at equilibrium, the field on the Gaussian surface will be zero. Gauss' theorem tells that the enclosed charge, namely the charge on the inner surface, is zero.

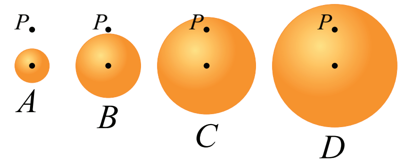
8. Three very large parallel planes are separated by distances d and $2d$. They carry charges, whose densities are $+\sigma$, -2σ, and $+3\sigma$, as shown in the figure. What is the magnitude of the electric field at point A in the middle between the second and third plates? (Hint: Electric fields are vectors that add as such.)



(1) $\frac{2\sigma}{\epsilon_0}$ (2) $\frac{\sigma}{\epsilon_0}$ (3) $\frac{\sigma}{2\epsilon_0}$ (4) $\frac{3\sigma}{2\epsilon_0}$ (5) $\frac{3\sigma}{\epsilon_0}$

The magnitudes of the fields produced by the three plates are $\sigma/(2\epsilon_0)$, $2\sigma/(2\epsilon_0)$, and $3\sigma/(2\epsilon_0)$. The first one points to the right, whereas the other two point to the left. So the magnitude of the net field is $(-1 + 2 + 3)\sigma/(2\epsilon_0)$,

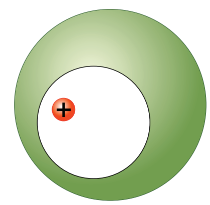
9. The figure shows four solid spheres, A, B, C, D, each with charge Q uniformly distributed throughout its volume. For all spheres, point P is at the same distance from the center of the sphere, as shown. Rank the spheres according to the magnitude of the electric field they produce at point P, greatest first.



(1) A and B, C, D
(2) The field is equally strong in all of the cases.
(3) B, A, C and D (with C and D equal)
(4) B, A, C, D
(5) D, C, A and B (with A and B equal)

Around the center of each sphere, draw a concentric spherical Gaussian surface that goes through point P. Since the areas of these surfaces are identical, the electric field at P depends only on the amount of the enclosed charge, not on the area of the Gaussian surface. The Gaussian surfaces for A and B completely enclose Q, so these spheres produce the strongest field. The Gaussian surface for C encloses a larger fraction of Q than does the Gaussian surface for D. So sphere C comes second, and sphere D the last.

10. A small ball, with a charge of 4 μC on it, is placed in a spherical cavity inside a spherical conductor, as illustrated in the figure. The net charge on the conductor is -3 μC. What is the amount of charge on the *outer* surface of the conductor?



(1) 1 μC (2) -7 μC (3) 0 (4) -3 μC (5) 4 μC

The 4 μC charge on the ball attracts a charge of -4 μC to the inner surface of the conductor. As a result, $-3-(-4)$ μC will appear on the outer surface so that the net charge on the conductor will remain -3 μC.

11. A solid spherical conductor is given a net positive charge. The electrostatic potential of the conductor is

(1) constant throughout the volume.
(2) largest at the center.
(3) largest on the surface.
(4) largest somewhere between the center and surface.
(5) zero at the center.

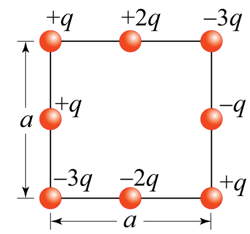
At equilibrium, the electric field inside a conductor is always zero and, consequently, the electrostatic potential is constant throughout.

12. Two conducting spheres are placed far apart from each other. The smaller of the two carries a total charge Q. The larger one, whose radius is three times that of the smaller one, carries no net charge. After the two spheres are connected by a thin conducting wire, the amounts of charge on the smaller and larger spheres are, respectively:

(1) $Q/4$ and $3Q/4$ (2) $Q/10$ and $9Q/10$ (3) $Q/3$ and $2Q/3$ (4) $Q/2$ and $Q/2$ (5) $-Q$ and $2Q$

Some of the Q will move to the larger sphere so that the electrostatic potentials V of the two sphere will be equal. Since V of a given sphere is proportional to the charge on it and inversely proportional to its radius, the charges on the two spheres must be in the ratio of 1 to 3, the ratio of the two radii. The smaller sphere gets $Q/4$ and the larger sphere gets $3Q/4$.

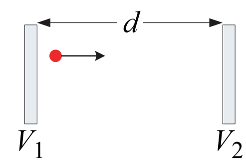
13. Eight particles are placed on a square of side length a, as shown in the figure. Each particle is charged with an amount indicated. What is the electric potential at the center of the square, if $V = 0$ at infinity?



(1) $-4\sqrt{2}kq/a$ (2) $\sqrt{5}kq/a$ (3) $3kq/a$ (4) $-kq/(\sqrt{5}a)$ (5) $kq/(\sqrt{2}a)$

The electrostatic potentials produced by the four particles on the edges will cancel out at the center. Adding the potentials produced by the four particles at the corners gives $kq/(a/\sqrt{2}) - k(3q)/(a/\sqrt{2}) + kq/(a/\sqrt{2}) - k(3q)/(a/\sqrt{2})$.

14. An electron is moving rightward between two parallel charged plates separated by distance $d = 1.0$ cm, as shown in the figure. The plate potentials are $V_1 = -30$ V and $V_2 = -20$ V. If the initial speed of the electron at the left plate is 1.0 km/s, what is its speed just as it reaches the right plate?

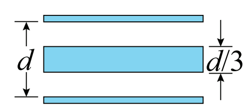


(1) 1,900 km/s (2) 230 km/s (3) 64 km/s (4) 7.6 km/s (5) The electron will never reach the right plate.

Since the electron is negatively charged whereas the potential of the right plate is higher than that of the left plate, the electron will reach the right plate. Let V be the potential difference, 10 V. According to the conservation of energy, $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = eV$, where v_f and v_i are the final and initial speeds of the electron. After plugging in numbers, you will immediately find that the second term on the left side is much smaller than the right side of the equation. So just solve the simplified, approximate equation $\frac{1}{2}mv_f^2 = eV$ for v_f.

A slower, error-prone way of solving this problem is to first find the electric field $E = V/d$, then the acceleration $a = eE/m$. Next find the travelling time t of the electron by solving the kinetic equation, which is quadratic (yikes), and compute $v_f = v_i + at$.

15. A parallel-plate capacitor has plate separation d and capacitance C. If a slab of metal of thickness $d/3$ is inserted in the middle of the gap between the two plates, as shown, what will be the capacitance between the plates?

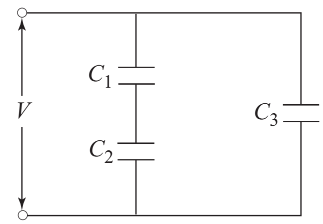


(1) $3C/2$ (2) $2C$ (3) C (4) $C/2$ (5) 0

The original capacitance is given by $C = \epsilon_0 A/d$, where A is the area of each plate. After the insertion of the slab, let us charge the capacitor with charge q — $+q$ on the top plate and $-q$ on the bottom plate. Then $-q$ will appear on the top surface of the slab and $+q$ on the bottom surface, attracted to the charges on the plates. This means that the arrangement is equivalent to two capacitors, each with a plate separation of $d/3$, in series. Each of these capacitors has a capacitance of $\epsilon_0 A/(d/3)$, which is $3C$. Two such capacitances in series make $3C/2$.

Alternatively, compare the original capacitor with the final capacitor, both charged with q, and notice that the electric field in the gap(s) is the same for the two. Since the total gap size for the second capacitor is 2/3 of the first one, the potential difference across it is also 2/3 of that of the first one. (Note that the potential difference across the slab is zero.) The same q but 2/3 of V — this means that its capacitance, $C = q/V$, is 3/2 of the original capacitance.

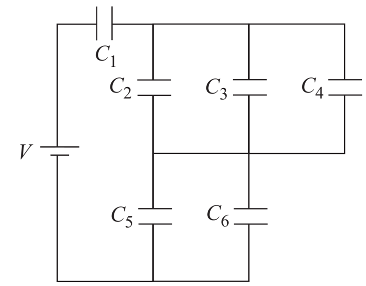
16. In the figure, a potential difference of $V = 24$ V is applied across the arrangement of capacitors with capacitances $C_1 = C_2 = 4.0\ \mu$F and $C_3 = 1.0\ \mu$F. What is the charge on one of the plates of capacitor C_1?



(1) 48 μC
(2) 96 μC
(3) 72 μC
(4) 12 μC
(5) 6.0 μC

The equivalent capacitance of C_1 and C_2 in series is 2.0 μF. The equivalent capacitor will therefore have a charge of 24 V $\times$ 2.0 μF = 48 μC. "Peek" inside the equivalent capacitor and find that this amount of charge appears on C_1, as well as C_2 for that matter.

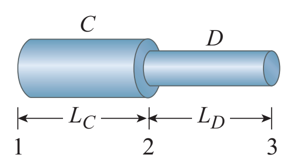
17. What is the equivalent capacitance, in μF, of the circuit shown if each capacitor has a capacitance of 1 μF?



(1) 6/11
(2) 11/6
(3) 5/6
(4) 6
(5) 1/6

The circuit consists of three groups of capacitors, in series. The first group comprises C_1 alone, the second group three capacitors in parallel, and the third group two capacitors in parallel. Therefore, the equivalent capacitance is that of 1 μF, 3 μF, and 2 μF in series — 6/11 μF because $1 + 1/3 + 1/2 = 11/6$.

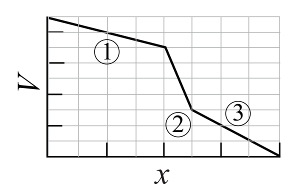
18. Wires C and D are made of different materials and have length $L_C = L_D = 1.0$ m. The resistivity and radius of wire C are 4.0×10^{-6} Ωm and 1.0 mm, and those of wire D are 1.0×10^{-6} Ωm and 0.30 mm. The wires are joined as shown, and a current of 2.0 A is passed through the combo. What is the rate at which energy is dissipated in wire C?



(1) 5 W (2) 10 W (3) 3 W (4) 14 W (5) 21 W

The resistance of wire C is $R = (4.0 \times 10^{-6})(1.0)/[\pi(1.0 \times 10^{-3})^{-2}] = 4.0\pi\ \Omega$. The rate at which energy is dissipated is the power, $P = i^2R$, which is about 5 W.

19. A 9 V battery is connected to a resistive strip consisting of three sections with the same cross-sectional area but different conductivities. The figure gives the electric potential V versus position x along the strip. The conductivity of section 3 is 3.0×10^7 $(\Omega\text{m})^{-1}$. What is the conductivity of section 1, in 10^7 $(\Omega\text{m})^{-1}$?



(1) 6.0 (2) 3.0 (3) 1.5 (4) 4.5 (5) 0

Recall $E = \rho J$, where the three symbols are the electric field, conductivity, and current density. Since the cross-sectional areas of the three sections are the same, J is the same throughout. Therefore, the E of each section is proportional to ρ, inversely proportional to its conductivity. Noting that E is the slope of the curve of V versus x, find that the E in section 1 is one half the E in section 3. This means that the conductivity of section 1 is twice the conductivity of section 3.

Alternatively, solve $V = iR = i(\rho L/A) = iL(\sigma A)$ for conductivity σ to find $\sigma = iL/(AV)$. Since i and A are constant throughout, σ is proportional to L/V, which is twice as large in section 1 as in section 3 according to the graph.

20. A resistor dissipates 0.5 W when a potential difference of 3.0 V is applied. When the potential difference is changed to 1.0 V, what is the power dissipated in this resistor?

(1) 0.056 W (2) 4.5 W (3) 1.5 W (4) 0.5 W (5) 0.17 W

$P = V^2/R$. As V is decreased by a factor of 3, P will decrease by a factor of 9.