Exam 2 Solutions

Note that there are several variations of some problems, indicated by choices in parentheses.

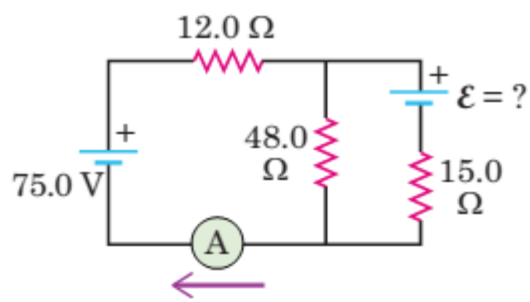
Problem 1: *Two light bulbs have resistances of* 100Ω *and* 300Ω. *They are connected in parallel across a* $120V$ *line. What is the total power dissipated by the two bulbs (or the* 100Ω *or* 300Ω *bulbs)?*

This problem was similar to Exercise 26-C done in class.

$$P = I\mathtt{E} = \mathtt{E}^2 R^{-1} = \boxed{\mathtt{E}^2(R_1^{-1} + R_2^{-1})} = (120)^2(\tfrac{1}{100} + \tfrac{1}{300})W = \boxed{192W}; \qquad (1.1)$$

or 144 W and 48 W individually.

Problem 2: *In the circuit shown in the figure both batteries have insignificant internal resistance and the idealized ammeter reads* $I_1 = 4.0A$ *in the direction shown. Find the E.M.F. of the battery (a negative answer indicates that the E.M.F. polarity is opposite to what is shown).*



This problem was similar to Exercise 26-D done in class.

$$75.0V = \mathcal{E}_1 = R_1 I_1 + R_2(I_1 - I_2); \quad -\mathcal{E} = R_3 I_2 + R_2(I_2 - I_1); \quad I_1 = 4.0A; \quad R_1 = 12.0\Omega; \quad R_2 = 48.0\Omega; \quad R_3 = 15.0\Omega; \quad (1.2)$$

We note that the first equation in (1.2) implies $I_2 = \frac{1}{R_2}((R_1 + R_2)I_1 - \mathcal{E}_1)$, which can be immediately put into the 2nd equation to yield $\mathcal{E}$, as,

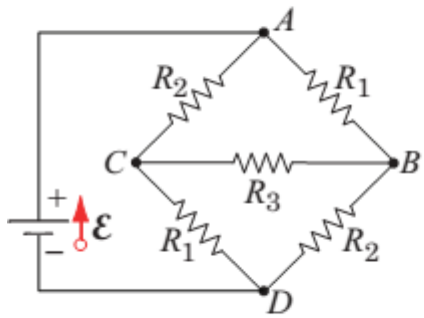
$$\mathcal{E} = R_2 I_1 - (R_2 + R_3)I_2 = \boxed{R_2 I_1 - \tfrac{R_2 + R_3}{R_2}((R_1 + R_2)I_1 - \mathcal{E}_1)} = \boxed{-24.5625V}; \qquad (1.3)$$

Problem 3: *A capacitor with an initial potential difference of* $V(0) = 150V$ *is discharged through a resistor when a switch between them is closed at* $t = 0$. *At* $t = 10.0$, *the potential difference across the capacitor is* $V_1 = 1.5V$. *What is the potential difference across the capacitor at* $t = 20s$ *?*

This is based on homework problem 27.64

Recalling that $q(t) = CV(t) = q_0 + q_1 e^{-t/\tau}$ in which the discharging-conditions $q(0) = C\mathcal{E} = CV(0)$ and $q(\infty) = 0$ give $q_0 = 0$ and $q_1 = CV(0)$, we have $V(t) = V(0)e^{-t/\tau}$. The third condition $V(10.0s) = 1.5V = (150V)e^{-(10.0s)/\tau}$ determines τ to be $\tau = 10.0s/(-\ln\frac{1.5}{150}) = 2.1715s$, which determines the voltage at *all* times, and so we calculate directly $V(20.0s) = (150V)e^{-(20.0s)/(2.1715s)} = \boxed{0.015V}$.

Problem 4: *In the figure* $\mathcal{E} = 14V$, $R_1 = R_3 = 1.00\Omega$, *and* $R_2 = 2\Omega$. *What is the potential difference* $V_A - V_B$?



(1.4)

This is based on a Ch.27 homework problem (27.35), and Exercise 26-F done in class

Let the current through the EMF be i, that through R_1 be i_1, and the current through R_2 be i_2.

Junction rules:

$i = i_1 + i_2$

$i_1 = i_2 + i_3 \Rightarrow i_3 = i_1 - i_2$

Left loop rule, substituting $R_1 = 1\ \Omega$ and $R_2 = 2\ \Omega$:

$14 - i_1 - 2i_2 = 0$

$i_1 = 14 - 2i_2$

Zig-zag loop rule:

$14 - i_1 - i_3 - i_1 = 0$

$14 - 2i_1 - (i_1 - i_2) = 0$

$14 - 3i_1 + i_2 = 0$

$14 - 3(14 - 2i_2) + i_2 = 0$

$-28 + 7i_2 = 0 \Rightarrow i_2 = 4A$

$\Rightarrow i_1 = 14 - 2i_2 = 6A$

$\Rightarrow i = i_1 + i_2 = 10A$

Now $V_A - V_B = i_1 R_1 = (6A)(1\Omega) = 6V$

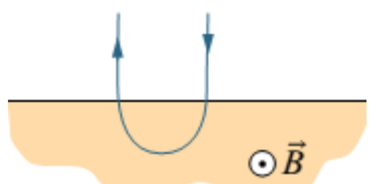
Problem 5: *A proton travels through uniform magnetic and electric fields. The magnetic field is* $\vec{B} = -2.50\hat{i}$ mT. *At one instant the velocity of the proton is* $\vec{v} = 2000\hat{j}$ m/s. *At that instant what is the net force acting on the proton if the electric field is* $4.00\hat{k}$ V/m*?*

This is problem 28.10, which was assigned in homework

$$\begin{aligned} \vec{F} &= q(\vec{E} + \vec{v}\times\vec{B}) = q(E\hat{k} + v\hat{j}\times B(-\hat{i})) = \boxed{q(E+vB)\hat{k}} \\ &= (1.602\times10^{-19})(4.00 + (2000)(2.50\times10^{-3}))\hat{k}N = \boxed{1.442\times10^{-18}\,N\hat{k}} \end{aligned} \quad (1.5)$$

Problem 6: *In the figure a charged particle moves into a region of uniform magnetic field* $\vec{B}$*, goes through half a circle, and then exits that region. The particle is either a proton or an electron (you must decide which). It spends 130 ns in the region. What is the magnitude of* $\vec{B}$*?*

This is problem 28.26, which was assigned in homework



(1.6)

We immediately notice that the particle velocity $\vec{v} = v(-\hat{j})$ is deflected in the $-\hat{i}$ direction by the magnetic field $\vec{B} = B\hat{k}$ pointing out of the page. Looking at the vector-value of the forces,

$$\vec{F}_B = F_B(-\hat{i}) = q\vec{v}\times\vec{B} = q(v(-\hat{j}))\times(B\hat{k}) = qvB(-\hat{i}) \to qvB > 0 \to q > 0 \to \begin{cases} q = +e = 1.602\times10^{-19}C; \\ m = m_p = 1.672\times10^{-27}kg; \end{cases} \quad (1.7)$$

While in the circular region, the charged particle has constant speed $v = |\vec{v}|$, and maintains this velocity. This is because the magnetic force $\vec{F}_B$ (due to magnetic field $\vec{B} = B\hat{k}$ and velocity $\vec{v} = v\cdot d\vec{\theta} = R\omega\cdot d\vec{\theta}$) is perpendicular to the displacement, and thus does no work. Letting the half-circle have radius R, and letting the particle have charge q, we have,

$$dK = dW = \vec{F}_B \bullet d\vec{s} = (q\vec{v}\times\vec{B})\bullet Rd\vec{\theta} = qvBR(\hat{r}\bullet d\vec{\theta}) = qvBR(0) = 0 \to dK = dW = 0 \to \boxed{K_i = K_f = K_a}; \quad (1.8)$$

Hence, no kinetic energy is added or subtracted to the particle of mass m. the velocity $\vec{v}$ is of constant magnitude. In the circular trajectory, Newton's 2nd Law then is,

$$\sum F = m_p a = m_p\frac{-v^2}{R} = -\vec{F}_B \to m_p\frac{-v^2}{R} = -q\vec{v}\times\vec{B} \to \left|m_p\frac{-v^2}{R}\right| = \left|-q\vec{v}\times\vec{B}\right| \to m_p\frac{v^2}{R} = qvB\overbrace{\sin\theta_{\vec{v}\vec{B}}}^{=\sin 90^\circ=1} \leftrightarrow B = \frac{m_p v}{Rq}; \quad (1.9)$$

In (1.9), the velocity is given by $v = \frac{2\pi R}{T} = \frac{\pi R}{(1/2)T}$, so,

$$v = \frac{2\pi R}{T} = \frac{\pi R}{\frac{1}{2}T} \rightarrow B = \frac{m_p(\frac{\pi R}{(1/2)T})}{R(+e)} = \boxed{\frac{m_p \pi}{e\frac{1}{2}T}} = \frac{(1.672\times10^{-27}\,kg)\pi}{(1.602\times10^{-19}\,C)(130\times10^{-9}\,s)} = \boxed{0.252\frac{kg}{C\cdot s} = 0.252\frac{N}{\frac{m}{s}\cdot C} = 0.252T};(1.10$$

In the last steps of (1.10), we illustrate the units[1] of magnetic field.

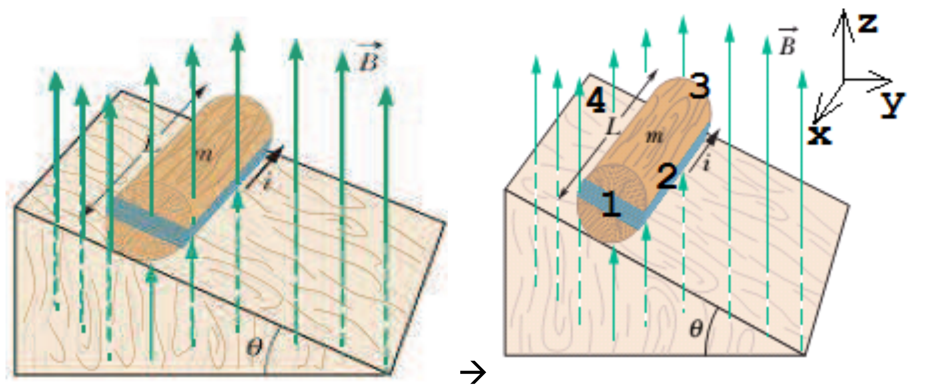
Problem 7: *In a certain cyclotron a proton moves in a circle of radius 0.5 m. The magnitude of the magnetic field is 1.2 T. What is the kinetic energy of the proton in million electron-volts (MeV)?*

This is problem 28-38, which was assigned in homework.

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m(\omega_C R)^2 = \boxed{\tfrac{1}{2}m(\tfrac{eB}{m}R)^2} = \tfrac{1}{2}(1.67\times10^{-27}\,kg)(\tfrac{(1.602\times10^{-19}\,C)(1.2T)}{1.67\times10^{-27}\,kg}0.5m)^2\frac{1}{1.602\times10^{-13}\frac{J}{MeV}} = \boxed{17.267MeV};(1.11)$$

Problem 8: *The figure shows a wood cylinder of mass m = 0.250 kg and length L = 0.100 m, with N = 10 turns of wire wrapped around it longitudinally, so that the plane of the wire contains the long central axis of the cylinder. The cylinder is released on a plane inclined at an angle θ to the horizontal, with the plane of the coil parallel to the incline plane. If there is a vertical uniform magnetic field of magnitude 0.500 T, what is the least current i through the coil that keeps the cylinder from rolling down the plane?*

This is problem 28-51, which was assigned in homework.



(1.12)

Let the wood-cylinder be of mass m, and have moment of inertia[2] I. Newton's 2nd Law for translational equilibrium between the force of static[3] friction f and magnetic force F_B and rotational equilibrium between the torque of static friction Rf and magnetic torque τ_B is,

$$ma = m\cdot 0 = 0 = \sum F = f - mg\sin\theta;\ \ I\alpha = 0 = \sum\tau = \sum_{i=1}^{4} R_i F_i \sin\theta_i - fR = \sum_{i=1}^{4} R_i F_i \sin 90° - fR = \sum_{i=1}^{4} R_i F_i - Rf;(1.13)$$

[1] A tesla is a unit of force per unit velocity per unit charge; essentially the units of electric field divided by velocity.

[2] The radius of the wooden cylinder is R, but the wooden-material may be inhomogeneous, so assume $I \neq \frac{1}{2}mR^2$.

[3] CAUTION: The force of static friction is a *reaction* force, and its magnitude is unknown. The *maximum* value the force of friction *could* take on, if we knew the static-friction-coefficient μ_S, is $\max f = \mu_S N$, where N is a reaction force which has a known contribution $mg\cos\theta$ from gravity, but an *unknown* contribution from the net magnetic force.

The problem is to find F_1, F_2, F_3, F_4: the magnetic forces upon the four sections of the square-loop shown in the Figure. The magnitude of the force upon a wire of length l making angle θ with a magnetic field $\vec{B}$ carrying current $I = dq/dt$ is given by $F = Il\left|\vec{B}\right|\sin\theta$. There are N such wires producing identical and superimposing forces. Thus, let the wood-cylinder be of radius R. Sections 1 and 3 are of length $2R$, while Sections 2 and 4 are of length L. Then,

$$\begin{aligned} R_1F_1 &= Ri_1 l_1 B\sin\theta_1 N = R(+i)(2R)B\sin(90° - \theta)N = +2iR^2B\cos\theta N; \\ R_3F_3 &= Ri_3 l_3 B\sin\theta_3 N = R(+i)(2R)B\sin(270° - \theta)N = -2iR^2B\cos\theta N = -R_1F_1; \\ R_2F_2 &= Ri_2 l_2 B\sin\theta_2 N = (+R)(+iLB\sin\theta)N = RiLB\sin\theta N; \\ R_4F_4 &= Ri_4 l_4 B\sin\theta_4 N = (-R)(-iLB\sin\theta)N = RiLB\sin\theta N; \end{aligned} \quad (1.14)$$

Combining (1.13) and the explicit forces (1.14), and noting the simplification $R_1F_1 = -R_3F_3$, we have,

$$\overbrace{Rf = Rmg\sin\theta}^{f = mg\sin\theta} = \sum_{i=1}^{4} R_iF_i = R_1F_1 + R_2F_2 + R_3F_3 + R_4F_4 = RiLB\sin\theta N + RiLB\sin\theta N + 0 = 2RiLB\sin\theta N; \quad (1.15)$$

Solving (1.15) for i, we have,

$$Rmg\sin\theta = 2RiLB\sin\theta N \xleftrightarrow{\text{solve for } i} i = \boxed{\frac{mg}{2LBN}} = \frac{(0.250kg)(9.81\frac{m}{s^2})}{2(0.100m)(0.500\frac{N}{(m/s)C})(10.0)} = \boxed{2.453\frac{C}{s}}; \quad (1.16)$$

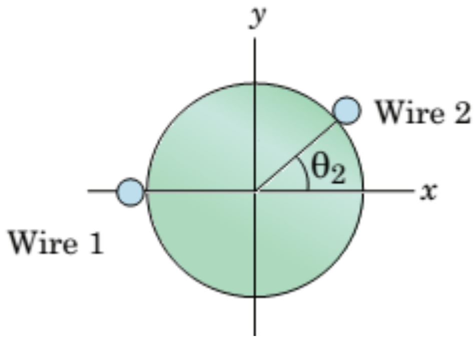
<u>Afterword</u>: We note that the net torque τ_B due to the four branches of the loop has the property,

$$\tau_B = \sum_{i=1}^{4} R_iF_i = 2RiLBN\sin\theta = \left|B\right|\overbrace{\left|2RL \cdot i \cdot N\right|}^{2RL = a = \text{area} = A/N}\sin\theta = \left|B\right|\left|a \cdot i \cdot N\right|\sin\theta = \left|B\right|\left|A \cdot i\right|\sin\theta \equiv \left|B\right|\left|\vec{\mu}\right|\sin\theta = \left|\vec{B} \times \vec{\mu}\right|; \quad (1.17)$$

We introduced the magnetic dipole moment vector, $\vec{\mu} = i\vec{A} = iN\vec{a} = iNa\hat{n}$, where $\hat{n} = \hat{k}\cos\theta + \hat{j}\sin\theta$ is the plane-normal defining the vector-area $\vec{A} = N\vec{a} = Na\hat{n} = N2LR\hat{n}$. Recall, also, that we encountered vector area in our study of the flux that naturally occurred in Gauss's law.

<u>Problem 9</u>: *The figure shows, in cross section, two long straight wires held against a plastic cylinder of radius $R = 20cm$. Wire 1 carries current $i_1 = 60mA$ out of the page and is fixed in place at the left side of the cylinder. Wire 2 carries current $i_2 = 40mA$ out of the page and can be moved around the cylinder. At what (positive) angle θ_2 should wire 2 be positioned such that, at the origin, the net magnetic field due to the two currents has magnitude $B = 80nT$?*

This is problem 29-34, which was worked in class on Oct.14



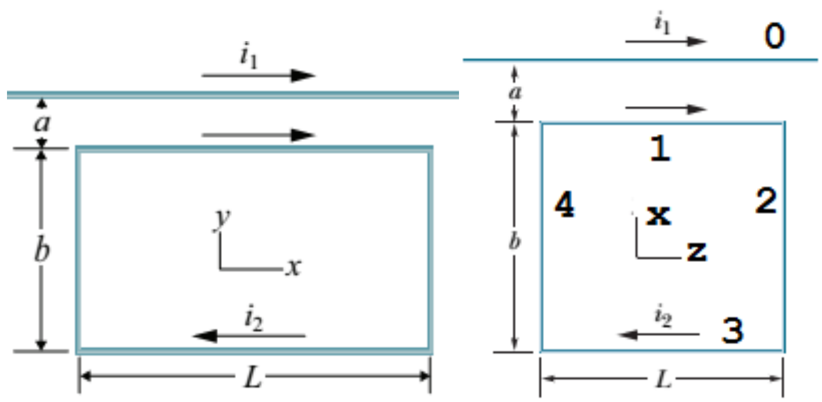
(1.18)

The two magnetic fields decompose as $\vec{B}_1 = B_1\hat{j}$ and $\vec{B}_2 = B_2(\sin\theta_2\hat{i} - \cos\theta_2\hat{j})$, so the resultant of this, using Ampère's law to say $B_1 = \frac{\mu_0 i_1}{2\pi R}$ and $B_2 = \frac{\mu_0 i_2}{2\pi R}$ (in which we clearly have $i_1 = \frac{3}{2}i_2$), is,

$$B = \sqrt{(B_2\sin\theta_2)^2 + (B_1 - B_2\cos\theta_2)^2} = \tfrac{\mu_0 i_2}{2\pi R}\sqrt{\sin^2\theta_2 + (\tfrac{3}{2} - \cos\theta_2)^2} = \tfrac{\mu_0 i_2}{2\pi R}\sqrt{\sin^2\theta_2 + (\tfrac{3}{2})^2 + \cos^2\theta_2 - 2\tfrac{3}{2}\cos\theta_2}$$
$$= \tfrac{\mu_0 i_2}{2\pi R}\sqrt{1 + \tfrac{9}{4} - 3\cos\theta_2} \leftrightarrow \theta_2 = \cos^{-1}\tfrac{1}{3}(\tfrac{13}{4} - (\tfrac{2\pi RB}{\mu_0 i_2})^2) = \cos^{-1}\tfrac{1}{3}(\tfrac{13}{4} - (\tfrac{0.2(80\times10^{-2}\,T)}{2(40\times10^{-3}\,A)})^2) = \boxed{104.4775^\circ}; \quad (1.19)$$

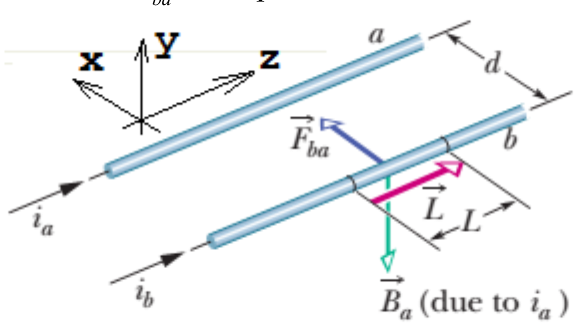
Problem 10: *In the figure a long straight wire carries a current* $i_1 = 30.0A$ *and a rectangular loop carries current* $i_2 = 20.0A$. *Take the dimensions to be* $a = 1.00cm$, $b = 8.00cm$, *and* $L = 30.0cm$. *In unit vector notation, what is the force on the loop due to* i_1*?*

This is problem 29-41, which was assigned in homework.



(1.20)

The horizontal wires: Consider two typical wires, a and b, a distance d apart, and carrying respective currents i_a and i_b. A differential element of force $d\vec{F}_{ba}$ acts upon wire-b and is due to wire-a,



(1.21)

By the Lorentz force law, the differential element of force $d\vec{F}_{ba}$ per unit length dx is due to magnetic field[4] $\vec{B}_b = \frac{\mu_0 i_a}{2\pi d}(-\hat{j})$, so we calculate the force per unit length $\vec{f}_{ba} \equiv \frac{d\vec{F}_{ba}}{dx}$ as,

$$\vec{f}_{ba} = \frac{d\vec{F}_{ba}}{dz} = \frac{dq_b \cdot \vec{v}_b \times \vec{B}_a}{dz} = \frac{dq_b \cdot \frac{d\vec{z}}{dt} \times \left(\frac{\mu_0 i_a}{2\pi d}(-\hat{j})\right)}{dz} = \frac{\frac{dq_b}{dt} \cdot (\hat{k} \cdot dz) \times \left(\frac{\mu_0 i_a}{2\pi d}(-\hat{j})\right)}{dz} = i_b \frac{\mu_0 i_a}{2\pi d} \hat{k} \times (-\hat{j}) = \frac{\mu_0 i_a i_b}{2\pi d} \hat{i}; \quad (1.22)$$

Evidently, the force between wires a and b is in the $+\hat{i}$ direction, and thus is *attractive*. Using this result (1.22) upon the two horizontal wires in the Figure (numbered 1 and 3 (note the different coordinates!)),

$$\vec{F}_1 = \int d\vec{F}_1 = \int_0^L \frac{d\vec{F}_1}{dx} dx = \int_0^L \left(\frac{\mu_0 i_1 i_2}{2\pi a} \hat{j} \right) dx = \frac{\mu_0 i_1 i_2 L}{2\pi a} \hat{j}; \; \vec{F}_3 = \int_0^L \frac{d\vec{F}_3}{dx} dx = \int_0^L \left(\frac{\mu_0 i_1 i_2}{2\pi (a+b)} (-\hat{j}) \right) dx = -\frac{\mu_0 i_1 i_2 L}{2\pi (a+b)} \hat{j}; (1.23)$$

<u>The vertical wires</u>: The total forces upon wires 2 and 4 due to wire 0 (of infinite length) are,

$$\vec{F}_2 = \int d\vec{F}_2 = \int_a^{a+b} \frac{d\vec{F}_2}{dy} dy = \int_a^{a+b} \frac{dq_2 \cdot \vec{v}_2 \times \vec{B}_0}{dy} dy; \; \vec{F}_4 = \int_a^{a+b} \frac{d\vec{F}_2}{dy} dy = \int_{a+b}^{a} \frac{dq_4 \cdot \vec{v}_4 \times \vec{B}_0}{dy} dy; \quad (1.24)$$

The integrands in (1.24) (i.e., the forces per unit y-length) are,

$$\frac{d\vec{F}_2}{dy} = \frac{dq_2 \cdot \vec{v}_2 \times \vec{B}_0}{dy} = \frac{i_2 \cdot (-\hat{j} \cdot dy) \times \frac{\mu_0 i_1}{2\pi y}(-\hat{k})}{dy} = \frac{\mu_0 i_1 i_2}{2\pi y} \hat{i}; \; \frac{d\vec{F}_4}{dy} = \frac{dq_4 \cdot \vec{v}_4 \times \vec{B}_0}{dy} = \frac{i_2 \cdot (+\hat{j} \cdot dy) \times \frac{\mu_0 i_1}{2\pi y}(-\hat{k})}{dy} = \frac{-\mu_0 i_1 i_2}{2\pi y} \hat{i}; (1.25)$$

Combining (1.24) and (1.25), we have,

$$\vec{F}_2 = \int_a^{a+b} \frac{\mu_0 i_1 i_2}{2\pi y} \hat{i} \cdot dy = \frac{\mu_0 i_1 i_2}{2\pi} \hat{i} \int_a^{a+b} \frac{dy}{y} = \frac{\mu_0 i_1 i_2}{2\pi} \hat{i} \ln \frac{a+b}{a}; \; \vec{F}_4 = \int_a^{a+b} \frac{-\mu_0 i_1 i_2}{2\pi y} \hat{i} \cdot dy = \frac{-\mu_0 i_1 i_2}{2\pi} \hat{i} \int_a^{a+b} \frac{dy}{y} = \frac{-\mu_0 i_1 i_2}{2\pi} \hat{i} \ln \frac{a+b}{a}; (1.26)$$

Looking at (1.26), we see $\vec{F}_2 = -\vec{F}_4$, so the superposition of these two forces make no contribution. Hence,

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 - \vec{F}_2 = \vec{F}_1 + \vec{F}_3 = \frac{\mu_0 i_1 i_2 L}{2\pi a} \hat{j} - \frac{\mu_0 i_1 i_2 L}{2\pi (a+b)} \hat{j} = \boxed{\frac{\mu_0 i_1 i_2 L}{2\pi} \hat{j} \left(\frac{1}{a} - \frac{1}{a+b} \right)}$$
$$= \frac{(4\pi \times 10^{-7} \frac{T \cdot m}{A})(30.0A)(20.0A)(30.0cm)}{2\pi} \hat{j} \left(\frac{1}{1.00cm} - \frac{1}{1.00cm + 8.00cm} \right) = \boxed{0.0032 N \cdot \hat{j}}; \quad (1.27)$$

<u>Problem 11</u>: *The current density $\vec{J}$ inside a long, solid, cylindrical wire of radius $a = 3.1 \times 10^{-3} m$ is in the direction of the central axis, and its magnitude varies linearly with radial distance r from the axis according to $\vec{J} = \vec{J}(r) = J_0 (r/a) \hat{k}$, where $J_0 = 310 \frac{A}{m^2}$. What is the magnitude of the magnetic field at $r/a = 1/2$? You may need the Jacobian term $r\,dr\,d\theta$ for integration in polar coordinates.*

This is problem 29-47, which was assigned in homework.

---

[4] <u>Proof</u>: from Ampère's law: $\int \vec{B}_{wire} \bullet d\vec{l} = \mu_0 i_{encl} \rightarrow \vec{B}_{wire} \bullet \hat{r}_\perp \int ds = \mu_0 i_{encl} \rightarrow B^\perp_{wire} 2\pi d = \mu_0 i_{encl} \rightarrow \vec{B}^\perp_{wire} = \frac{\mu_0 i_{encl}}{2\pi d} \hat{r}_\perp$. In this problem, $i_{encl} = i_a$ and (by the right hand rule) $\hat{r}_\perp = -\hat{j}$.

The current enclosed by an Ampèrian-loop of radius $0 \le b \le a$ is,

$$i_{encl} = i_{encl}(b) = \int di_{encl} = \int\limits_{wire\ x.s.} \frac{di}{dA} dA = \int_0^b \int_0^{2\pi} J_0 (r/a) \cdot dr \cdot r \cdot d\phi = \frac{2\pi J_0}{a} \int_0^b r^2 dr = \frac{2\pi J_0}{a} \frac{1}{3}(b^3 - 0^3) = \frac{2\pi J_0 b^3}{3a};(1.28)$$

The magnetic field at a distance b from the center of the wire, then, is given from the enclosed current (1.28) via the law of Ampère,

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{encl}(b) \rightarrow \vec{B} \bullet \oint d\vec{s} = \mu_0 \frac{2\pi J_0 b^3}{3a} \rightarrow B\hat{\phi} \bullet 2\pi b \cdot \hat{\phi} = \mu_0 \frac{2\pi J_0 b^3}{3a} \xleftrightarrow[\text{for B}]{\text{solve}} \boxed{B = \mu_0 \frac{J_0 b^2}{3a}}; \qquad (1.29)$$

From (1.29) follows,

$$B_b = B(\tfrac{1}{2}a) = \frac{\mu_0 J_0 \cdot (\tfrac{1}{2}a)^2}{3a} = \boxed{\tfrac{1}{12}\mu_0 a J_0} = \tfrac{1}{12}(4\pi \times 10^{-7} \tfrac{T \cdot m}{A})(3.1 \times 10^{-3} m)(310 \tfrac{A}{m^2}) = \boxed{1.0064 \times 10^{-7} T}; \quad (1.30)$$

<u>Problem 12</u>: *A solenoid that is $L = 95cm$ long has a radius $R = 2.0cm$ and a winding of $N = 1200$ turns; it carries a current of $i = 3.6A$. What is the magnitude of the magnetic field inside the solenoid?*

This is problem 29-50, which was assigned in homework.

The magnetic field $\vec{B}$ at the center of a solenoid made of a wire carrying current i with $n = N/L$ turns *per meter* is of magnitude $B = \mu_0 ni$. Therefore, $B = \boxed{\mu_0 (N/L) i} = 4\pi \times 10^{-7} \frac{T \cdot m}{A} \frac{1200}{0.95cm}(3.6A) = \boxed{5.7144 mT}$.

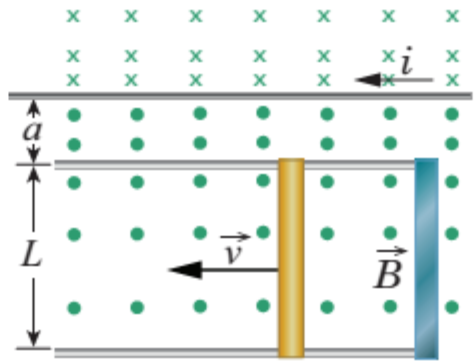
<u>Problem 13</u>: *A wire loop of lengths $L = 40cm$ and $W = 25cm$ lies in a magnetic field $\vec{B} = (0.08 \frac{T}{m \cdot s}) yt \cdot \hat{k}$. What are the magnitude and direction of the induced E.M.F.?*

This is problem 30-12, which was worked in class on Oct.19

$$\mathcal{E} = -\tfrac{d}{dt}\int_0^W dy \int_0^L b_0 yt \cdot dx = -b_0(\tfrac{1}{2}W^2 - \tfrac{1}{2}0^2)(L-0) = \boxed{-\tfrac{1}{2}b_0 W^2 L = -1.00mV; \ \text{clockwise;}}$$

<u>Problem 14</u>: *The figure shows a rod of length $L = 10cm$ that is forced to move at constant speed $v = 5m/s$ along the horizontal rails. The rod, rails and connecting strip at the right form a conducting loop. The rod has a resistance $R = 0.4\Omega$; the rest of the loop has negligible resistance. A current $i = 100A$ through the long straight wire at a distance $a = 10mm$ from the loop sets up a nonuniform magnetic field through the loop. At what rate (in µW) is thermal energy generated in the rod?*

This is problem 30-33, which was worked in class on Oct.21

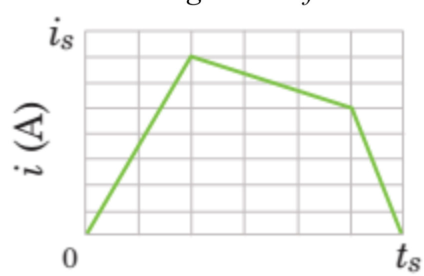


(1.31)

The magnetic field $\vec{B}$ due to the long straight wire at a distance $y \in [a, L+a]$ is $\vec{B} = \frac{\mu_0 i}{2\pi y}\hat{k}$, and the differential vector area through which it fluxes is $d\vec{A}(y) = dy(vt + x_0)\hat{k}$, and so the E.M.F. generated is $\mathcal{E} = -\frac{d}{dt}\int \vec{B} \bullet d\vec{A}$, yielding the power $P = I\mathcal{E} = \mathcal{E}^2 / R$. Putting all this together,

$$P = \frac{1}{R}\left(-\frac{d}{dt}\int_a^{L+a}(vt + x_0)\frac{\mu_0 i}{2\pi y}dy\right)^2 = \frac{1}{R}\left(\frac{\mu_0 iv}{2\pi}\right)^2\left(\int_a^{L+a}\frac{dy}{y}\right)^2 = \boxed{\frac{1}{R}\left(\frac{\mu_0 iv}{2\pi}\right)^2\left(\ln\frac{L+a}{a}\right)^2}$$
$$= \frac{1}{0.4\Omega}\left(\left(\tfrac{4\pi}{2\pi}\times 10^{-7}\,\tfrac{T\cdot m}{A}\right)(100A)(5\tfrac{m}{s})\right)^2\left(\ln\frac{0.11}{0.01}\right)^2 = \boxed{0.14375\mu W}; \tag{1.32}$$

<u>Problem 15</u>: *The current $i = i(t)$ through a $L = 4.6H$ inductor varies with time t as shown in the graph, where the vertical axis scale is set by $i_s = 8.0A$ and the horizontal axis scale is set by $t_s = 6.0\times 10^{-3}s$. The inductor has a resistance of $R = 12\Omega$. What is the magnitude of the induced E.M.F. during the time interval $2\times 10^{-3}s < t \le 5\times 10^{-3}s$?*



This is problem 30-46, which was assigned in homework.

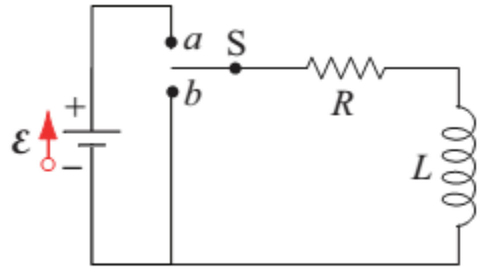
Recall the definition of inductance: L is the E.M.F.-magnitude $\mathcal{E}$ per unit rate-of-change of current $\frac{di}{dt}$, or Φ_B per unit i, and specialize it to the case of $\frac{di}{dt} = \frac{\Delta i}{\Delta t}$,

$$L \equiv \frac{\mathcal{E}}{\frac{d}{dt}i} \equiv \frac{\Phi_B}{i}; \quad \frac{di}{dt} = \frac{\Delta i}{\Delta t}; \tag{1.33}$$

Within the time-interval $2\times 10^{-3}s < t \le 5\times 10^{-3}s$, we have,

$$\left(\frac{\Delta i}{\Delta t}\right) = \frac{(5.0 - 7.0)A}{(5.0 - 2.0)\times 10^{-3}s} \to \mathcal{E} = L\left(\frac{\Delta i}{\Delta t}\right) = (4.6H)\frac{(5.0 - 7.0)A}{(5.0 - 2.0)\times 10^{-3}s} = \boxed{3.1\times 10^3 V}; \tag{1.34}$$

Problem 16: *The switch in the figure is closed on a at time t = 0. What fraction of the total voltage drop $\mathcal{E}$ occurs across the inductor at time $t = 2L/R$?*



(1.35)

This is problem 30-52, which was worked in class on Oct.23

Recall that an LR-circuit has a current $i(t) = i_0 + i_1 e^{-t/\tau}$, where $\tau = L/R$. The conditions $i(0) = 0$ and $i(\infty) = \mathcal{E}/R$ respectively yield $i_0 = -i_1$ and $i_0 = \mathcal{E}/R$, yielding $i(t) = (\mathcal{E}/R)(1 - e^{-t/\tau})$, meaning that at a time of $t = 2L/R = 2\tau$ the inductor's voltage in units of $\mathcal{E}$ is,

$$\frac{v_L}{\mathcal{E}} = \frac{1}{\mathcal{E}} L \frac{di}{dt} = \frac{L}{\mathcal{E}} \frac{\mathcal{E}}{R} \frac{d}{dt}(1 - e^{-t/\tau}) = \frac{L}{R}(0 - \frac{-1}{\tau} e^{-2\tau/\tau}) = \frac{L}{R}\frac{1}{\tau} e^{-2} = \frac{L}{R}\frac{1}{L/R} e^{-2} = \boxed{e^{-2} = 0.1353}; \qquad (1.36)$$

Problem 17:

An LC circuit has a capacitance of 20μF and an inductance of 10 mH. At time t = 0 the charge on the capacitor is 27μC and the current is 80 mA. What is the maximum possible charge in μC (or what is the maximum possible current)?

$$U_C = \int_0^q V(Q) \cdot dQ = \int_0^q \frac{Q}{C} \cdot dQ = \frac{q^2}{2C}; \quad U_L = \int P \cdot dt = \int \mathcal{E} I \cdot dt = \int L \frac{dI}{\cancel{dt}} I \cdot \cancel{dt} = \int_0^i (LI \cdot dI) = \frac{1}{2} L i^2; \qquad (1.37)$$

Thus, the total energy in the circuit, by energy-conservation, is $U = U_C + U_L = \frac{1}{2C} q^2 + \frac{1}{2} L i^2$, yielding a maximum charge on the capacitor of $\max U_C = U = \frac{1}{2C} {q_{\max}}^2 \leftrightarrow q_{\max} = \sqrt{2CU}$, which, explicitly, is,

$$q_{\max} = \sqrt{2CU} = \sqrt{2C(\tfrac{1}{2C} q^2 + \tfrac{1}{2} L i^2)} = \boxed{\sqrt{q^2 + LCi^2}} = \sqrt{(27\mu C)^2 + (10mH)(20\mu F)(80mA)^2} = \boxed{44.822\mu C}; (1.38)$$
$$i_{\max} = \sqrt{2U/L} = 100mA$$

Problem 18: *A sinusoidally varying source of E.M.F. with an amplitude of $10V$ and a cyclic frequency of $5GHz$ is applied across a $100\mu H$ inductor. What is the current amplitude through the inductor?*

This is based on a HITT clicker question given in class Oct.30

$$I_{\max} = \frac{V_{\max}}{X_L} = \frac{V_{\max}}{\omega_d L} = \frac{V_{\max}}{2\pi f_d L} = \frac{10V}{2\pi(5GHz)(100\mu H)} = \frac{1}{2\pi(5)(10)} \times 10^{-3} = \boxed{3.18 \times 10^{-6} A}; \qquad (1.39)$$

Problem 19: A 218Ω resistor, a $L = 0.775H$ inductor, and a $6.50\mu F$ capacitor are connected in series across a sinusoidally varying source of E.M.F. that has voltage amplitude $31.0V$ and a cyclic frequency of $37.5Hz$. What is the magnitude of the phase difference between the current in the resistor and the E.M.F.?

This is similar to Example 31-E done in class.

$$|\phi| = \left|\tan^{-1}\frac{X_L - X_C}{R}\right| = \boxed{\left|\tan^{-1}\frac{\omega_d L - (\omega_d C)^{-1}}{R}\right|} = \left|\tan^{-1}\frac{(235\,\tfrac{rad}{s})(0.775H) - ((235\,\tfrac{rad}{s})(6.50\mu F))^{-1}}{218\Omega}\right| = \boxed{|-65.234°|};$$

Problem 20: *A transformer connected to a* $V_p^{RMS} = 120V$ *AC line is to supply* $V_s^{RMS} = 12,000V$ *for a neon sign. To reduce shock hazard, a fuse is to be inserted in the primary circuit; the fuse is to blow when the R.M.S. current in the secondary circuit exceeds* $i_s^{RMS} = 3.0mA$. *What current rating should the fuse in the primary circuit have?*

The power delivered into the primary is the same as that on the secondary (or less for realistic transformers)

$$i_p^{RMS} = \frac{N_s}{N_p} i_s^{RMS} = \boxed{\frac{V_s^{RMS}}{V_p^{RMS}} i_s^{RMS}} = \frac{1.2\times10^4 V}{120V}(3.0mA) = \boxed{300mA}; \qquad (1.40)$$