

SOCIETY OF ACTUARIES

EXAM IFM INVESTMENT AND FINANCIAL MARKETS

EXAM IFM SAMPLE QUESTIONS AND SOLUTIONS

FINANCE AND INVESTMENT

These questions and solutions are based on material from the *Corporate Finance* textbook by Berk/DeMarzo (Learning Outcomes 1-5 of the Exam IFM syllabus) and two study notes, IFM-21-18 and IFM-22-18. Questions 1-33 are from *Corporate Finance* and Questions 34-43 are from the study notes.

They are representative of the types of questions that might be asked of candidates sitting for Exam IFM. These questions are intended to represent the depth of understanding required of candidates. The distribution of questions by topic is not intended to represent the distribution of questions on future exams.

March 2018 update:

3: Typo in solution corrected

5: Typo in question corrected

8: Changed $E(X)$ to $E(Rx)$ in solution

11: Deleted correlation input, and now require correlation to be derived from the given table.

12: Changed answer C, which could have interpreted as correct

40: Arithmetic error in solution corrected

41: "Only" deleted from answer choice D

June 2018 update:

Edits have been made to questions/solutions 3, 7, 8, 9, 10, 14, 15, 16, 18, 28, 34, 42. These changes improve clarity and remove some possible ambiguities.

Question 44 has been added.

November 2019 update:

31: Answer E changed

37: A clarification that $p(t)$ and $c(t)$ are payoffs

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Finance and Investment Questions

- 1) You are given the following information about an asset.
 - i) Using 36 years of data, the average annual asset return is 10%.
 - ii) The volatility of the asset's return, over the same time period, was estimated to be 27%.
 - iii) The distributions of each year's returns are identically distributed and independent from each other year's returns.

Calculate the lower bound of the 95% confidence interval for the asset's annual expected return, using the approximation formula given in *Corporate Finance*.

- (A) 1.0%
- (B) 2.6%
- (C) 4.5%
- (D) 5.5%
- (E) 8.5%

Key: A

Because there are 36 years' worth of data points and the distributions are IID, the standard error is given by

$$SE = \frac{SD}{\sqrt{n}} = \frac{0.27}{\sqrt{36}} = 0.045$$

Berk/DeMarzo equation 10.9 for the 95% confidence interval is

Historical Average Return \pm (2 \times Standard Error).

Thus, the lower bound of the 95% confidence interval is

$$0.10 - 2 \times 0.045 = 0.10 - 0.09 = 0.01 = 1\%.$$

Reference: Berk/DeMarzo, Section 10.3

2) You are given the following information about a portfolio with four assets.

Asset	Market Value of Asset	Covariance of asset's return with the portfolio return
I	40,000	0.15
II	20,000	-0.10
III	10,000	0.20
IV	30,000	-0.05

Calculate the standard deviation of the portfolio return.

- (A) 4.50%
- (B) 13.2%
- (C) 20.0%
- (D) 21.2%
- (E) 44.7%

Key: D

Solution: Formula 11.10 in Berk/DeMarzo gives the equation for the variance of a portfolio in terms of assets and covariance with the entire portfolio (where x_i is the proportion of the portfolio invested in asset i):

$$\text{Var}(R_p) = \sum_i x_i \text{Cov}(R_i, R_p).$$

The standard deviation is the square root of the variance. Therefore, the standard deviation of the portfolio in the problem is:

$$SD = \sqrt{\frac{40,000}{100,000} \times (.15) + \frac{20,000}{100,000} \times (-.10) + \frac{10,000}{100,000} \times (.20) + \frac{30,000}{100,000} \times (-.05)} = 21.2\%$$

Reference: Berk/DeMarzo, Section 11.3

- 3) You are given the following information about the annual returns of two stocks, X and Y :
- i) The expected returns of X and Y are $E[R_X] = 10\%$ and $E[R_Y] = 15\%$.
 - ii) The volatilities of the returns are $V_X = 18\%$ and $V_Y = 20\%$.
 - iii) The correlation coefficient of the returns for these two stocks is 0.25.
 - iv) The expected return for a certain portfolio, consisting only of stocks X and Y , is 12%.

Calculate the volatility of the portfolio return.

- (A) 10.88%
- (B) 12.56%
- (C) 13.55%
- (D) 14.96%
- (E) 16.91%

Key: D

Let w be the weight of stock X and so $1 - w$ is the weight of stock Y .

Then, the expected return of the portfolio is:

$$0.12 = E[wR_X + (1 - w)R_Y]$$

$$0.12 = wE[R_X] + (1 - w)E[R_Y] = w(0.10) + (1 - w)(0.15) = 0.15 - 0.05w$$

$$0.05w = 0.03$$

$$w = 0.6$$

The variance of the return of the portfolio is:

$$\text{Var}[0.6R_X + 0.4R_Y]$$

$$= 0.6^2\text{Var}[R_X] + 0.4^2\text{Var}[R_Y] + 2(0.6)(0.4)\text{Cov}[R_X, R_Y]$$

$$= 0.6^2(0.18^2) + 0.4^2(0.20^2) + 2(0.6)(0.4)(0.25)(0.18)(0.20)$$

$$= 0.022384$$

The volatility of the return of the portfolio is:

$$\sqrt{0.022384} = 0.1496$$

Reference: Berk/DeMarzo, Section 11.2

- 4) You are given the following information about a portfolio consisting of stocks X, Y, and Z:

Stock	Investment	Expected Return
X	10,000	8%
Y	15,000	12%
Z	25,000	16%

Calculate the expected return of the portfolio.

- (A) 10.8%
- (B) 11.4%
- (C) 12.0%
- (D) 12.6%
- (E) 13.2%

Key: E

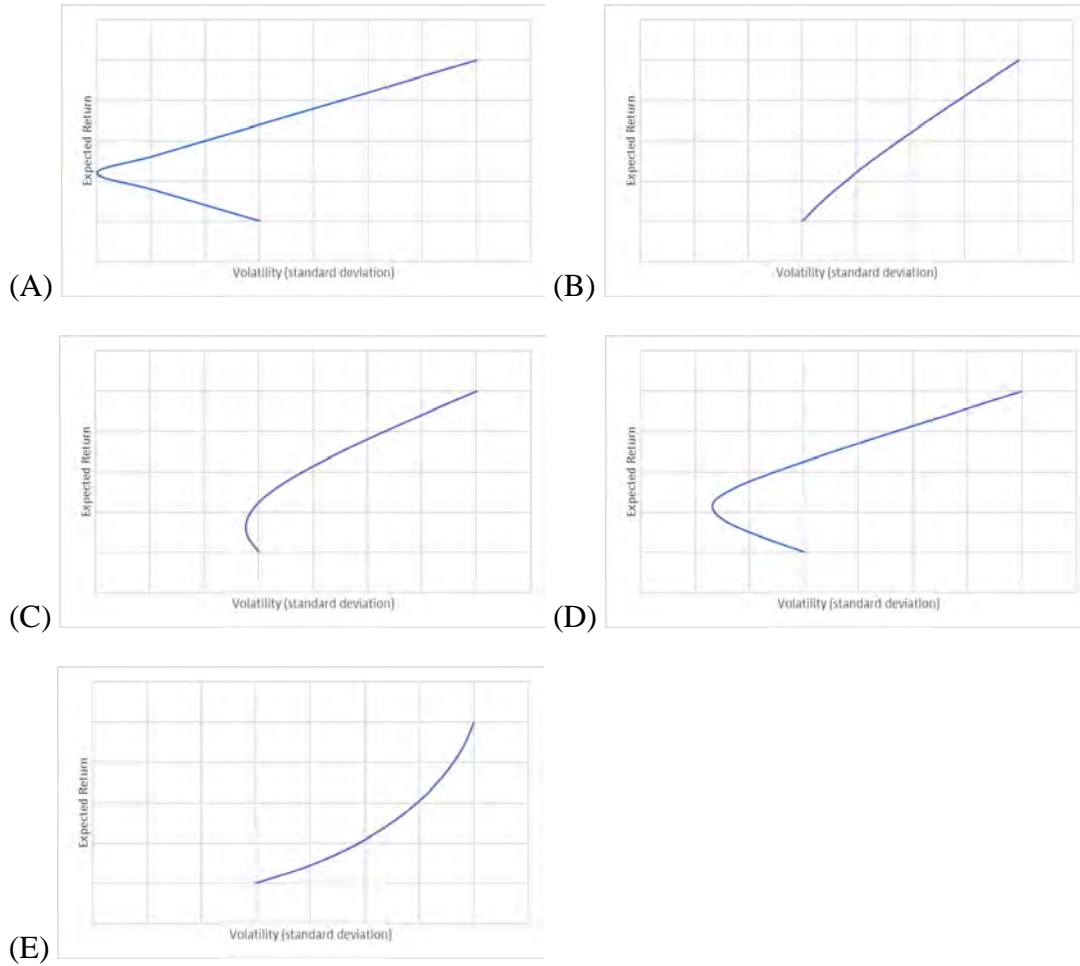
The expected return is the weighted average of the individual returns.

$$\frac{10,000}{50,000} \times 8\% + \frac{15,000}{50,000} \times 12\% + \frac{25,000}{50,000} \times 16\% = 13.2\%$$

Reference: Berk/DeMarzo, Section 11.1

- 5) You are given the following set of diagrams for a two-stock portfolio, with expected return on the vertical axis and volatility on the horizontal axis. These diagrams are meant to help investors identify the set of efficient portfolios.

Identify the diagram demonstrating the highest correlation between the two stocks.



Key: B

Based on Figure 11.4 of Berk/Demarzo, the figure closest to a straight line shows the highest correlation. Note that (E) is not a possible shape for an efficient portfolio since the curve is bending the wrong way.

Reference: Berk/DeMarzo, Section 11.4

6) You are given the following information about the four distinct portfolios:

Portfolio	Expected Return	Volatility
P	3%	10%
Q	5%	10%
R	5%	15%
S	7%	20%

Determine which two of the four given portfolios are NOT efficient.

- (A) P and Q
- (B) P and R
- (C) P and S
- (D) Q and R
- (E) Q and S

Key: B

P cannot be efficient because it has the same volatility as Q but lower expected return. R cannot be efficient because it has the same expected return as Q but higher volatility.

Thus, the answer is that P and R are not efficient.

Reference: Berk/DeMarzo, Section 11.4

7) Consider a portfolio of four stocks as displayed in the following table:

Stock	Weight	Beta
1	0.1	1.3
2	0.2	-0.6
3	0.3	β_3
4	0.4	1.1

Assume the expected return of the portfolio is 0.12, the annual effective risk-free rate is 0.05, and the market risk premium is 0.08.

Assuming the Capital Asset Pricing Model holds, calculate β_3 .

- A) 0.80
- B) 1.06
- C) 1.42
- D) 1.83
- E) 2.17

Key: C

First calculate the expected value of stocks 1, 2, and 4, noting that the market return must be 0.13 (since the market risk premium is $0.08 = 0.13 - 0.05$):

$$E(R_1) = 0.05 + 1.3 (0.13 - 0.05) = 0.154$$

$$E(R_2) = 0.05 - 0.6 (0.13 - 0.05) = 0.002$$

$$E(R_3) = 0.05 + 1.1 (0.13 - 0.05) = 0.138$$

Then,

$$0.12 = E(R_p) = 0.1(0.154) + 0.2(0.002) + 0.3[0.05 + X(0.13-0.05)] + 0.4(0.138), \text{ or}$$

$$0.12 = E(R_p) = 0.0154 + 0.0004 + 0.015 + 0.024X + 0.0552, \text{ or}$$

$$0.034 = 0.024X, \text{ or } X = 1.4167 \approx 1.42.$$

Reference: Berk/DeMarzo, Section 11.6

- 8) You are given the following information about a two-asset portfolio:
- (i) The Sharpe ratio of the portfolio is 0.3667.
 - (ii) The annual effective risk-free rate is 4%.
 - (iii) If the portfolio were 50% invested in a risk-free asset and 50% invested in a risky asset X, its expected return would be 9.50%.

Now, assume that the weights were revised so that the portfolio were 20% invested in a risk-free asset and 80% invested in risky asset X.

Calculate the standard deviation of the portfolio return with the revised weights.

- (A) 6.0%
- (B) 6.2%
- (C) 12.8%
- (D) 15.0%
- (E) 24.0%

Key: E

First, solve for the expected return of asset X:

$$E(R_p) = 9.5\% = 0.5 \times 4\% + 0.5 \times E(R_X)$$

$$E(R_X) = (9.5\% - 2\%) / 0.5 = 15\%.$$

A portfolio that is 20% risk-free and 80% invested in X has expected return

$$0.2 \times 4\% + 0.8 \times 15\% = 12.8\% .$$

All combinations of risky asset X and a risk-free asset will lie on the same line, and thus they will have the Sharpe ratio. Therefore, the Sharpe ratio is still 0.3667, and we have

$$0.3667 = \frac{0.128 - 0.04}{\sigma_p}$$

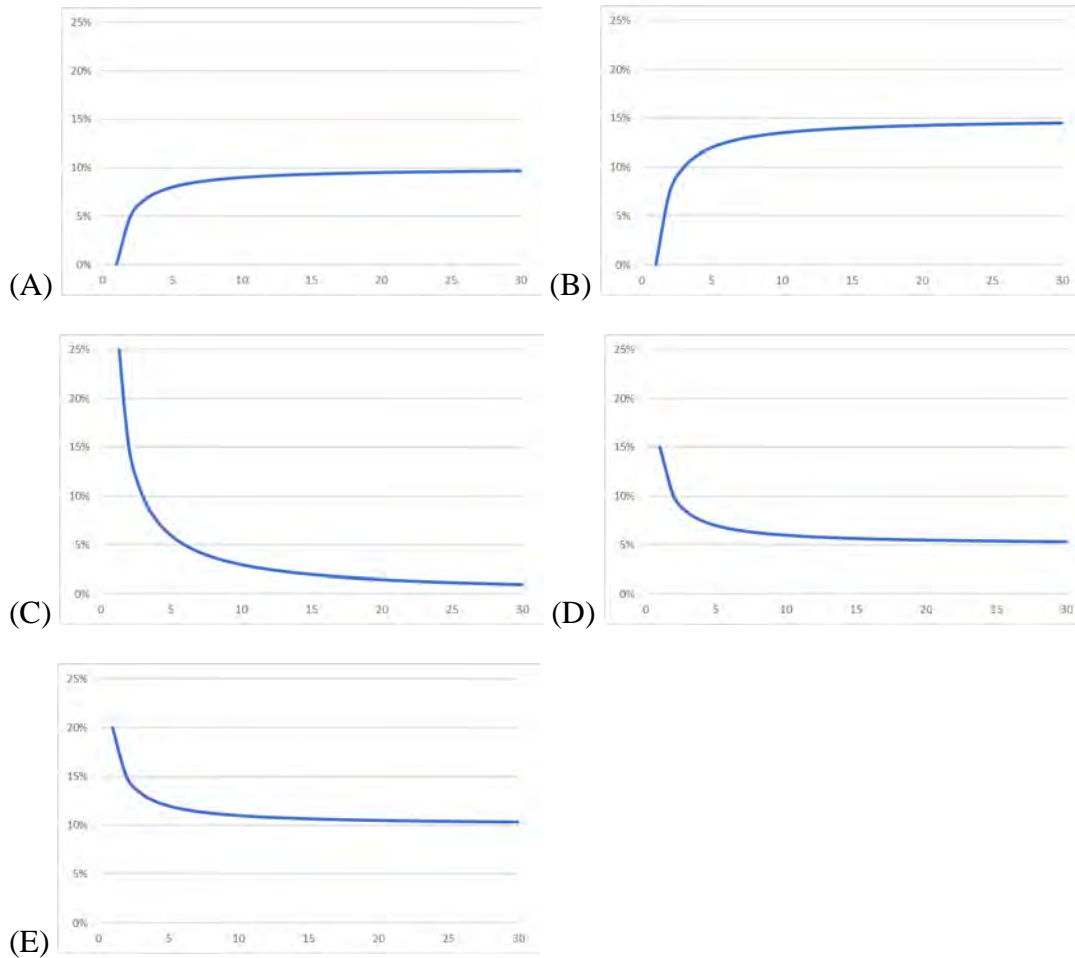
$$\sigma_p = 0.088 / 0.3667 = 0.240 = 24.0\%.$$

Reference: Berk/DeMarzo, Section 11.5

9) You are given the following information about an equally-weighted portfolio of n stocks:

- (i) For each individual stock in the portfolio, the variance is 0.20.
- (ii) For each pair of distinct stocks in the portfolio, the covariance is 0.10.

Determine which graph displays the variance of the portfolio as a function of n .



Key: E

Equation 11.12 gives the formula for the variance of an equally-weighted portfolio of n stocks as

$$\text{Var}(R_p) = \frac{1}{n} \times (\text{Average Variance of the Individual Stocks}) \\ + \left(1 - \frac{1}{n}\right) \times (\text{Average Covariance between the Stocks}).$$

Therefore, as n increases, the graph of portfolio variance should asymptotically become the average covariance between the stocks, which is 0.10. Thus only A and E can be correct.

Also, when $n = 1$, the variance is the average variance of the individual stocks, which is 0.20.

Of the two graphs, the only one that starts at 0.20 is E.

Reference: Berk/DeMarzo, Section 11.5

10) You are given the following information about three stocks (X, Y, and Z) in a portfolio:

(i) The covariance matrix for each stock with each other stock is given in the following table:

	X	Y	Z
X	0.040	-0.018	0.016
Y	-0.018	0.090	-0.021
Z	0.016	-0.021	0.010

(ii) The weighting of each stock in the portfolio is as follows:

Stock	Weighting
X	30%
Y	20%
Z	50%

Calculate the variance of this portfolio.

- (A) 0.0081
- (B) 0.0089
- (C) 0.0123
- (D) 0.0902
- (E) 0.0944

Key: A

Equation 11.11 gives the formula for the variance of a portfolio of stocks as

$$\text{Var}(R_p) = \sum_i \sum_j w_i w_j \text{Cov}(R_i, R_j)$$

where w_i is the portfolio weight for stock i and R_i is the return for that stock. Then,

$$\text{Var}(R_p) = 0.30^2(0.04) + 0.20^2(0.09) + 0.50^2(0.01) + 2(0.30)(0.20)(-0.018) + 2(0.30)(0.50)(0.016) + 2(0.20)(0.50)(-0.021) = 0.00814.$$

Reference: Berk/DeMarzo, Section 11.3

11) You are given the following information about a portfolio that has two equally-weighted stocks, P and Q.

- (i) The economy over the next year could be good or bad with equal probability.
- (ii) The returns of the stocks can vary as shown in the table below:

Stock	Return when economy is good	Return when economy is bad
P	10%	-2%
Q	18%	-5%

Calculate the volatility of the portfolio return.

- (A) 1.80%
- (B) 6.90%
- (C) 7.66%
- (D) 8.75%
- (E) 13.42%

Key: D

Let P be the random return of stock P and Q be the random return of stock Q. Then let R be the portfolio return.

Then,

$$E(P) = 0.5(0.10) + 0.5(-.02) = 0.04$$

$$E(Q) = 0.5(0.18) + 0.5(-.05) = 0.065$$

$$\text{Var}(P) = 0.5(0.10)^2 + 0.5(-.02)^2 - (.04)^2 = .0036$$

$$\text{Var}(Q) = 0.5(0.18)^2 + 0.5(-.05)^2 - (.065)^2 = .013225$$

$$\text{Cov}(P,Q) = E[PQ] - E[P]*E[Q] = [(0.10)(0.18)(0.5) + (-0.02)(-0.05)(0.5)] - 0.04*0.065 = 0.0069$$

$$\text{Then, } \text{Var}(R) = 0.5^2(.0036) + 0.5^2(.013225) + 2(0.5)(0.5)(.0069) = .00765625$$

$$\text{Finally, } \text{SD}(R) = (.00765625)^{0.5} = .0875 = 8.75\%.$$

Reference: Berk/DeMarzo, Section 11.3

- 12) Which of following statements represents the homogeneous expectations assumption that underlies the Capital Asset Pricing Model (CAPM)?
- (A) Investors can only buy and sell at competitive market prices.
 - (B) Investors can borrow or lend at the risk-free interest rate.
 - (C) There are no taxes or transaction costs.
 - (D) All investors have identical estimates for the volatilities, correlations, and expected returns of securities.
 - (E) Investors only hold portfolios that yield maximum expected return for a given level of volatility.

Key: D

If all investors rely on publicly available information, then they will have similar estimates regarding the volatilities, correlations, and expected returns of securities. A special case, in which all investors have the same estimates, is the definition for the homogeneous expectations assumption for CAPM.

Reference: Berk/DeMarzo, Section 11.7

13) The following table shows the beta and expected return for each of five stocks.

Stock (i)	β_i	$E(r_i)$
1	1.2	0.124
2	1.0	0.110
3	0.7	0.103
4	0.4	0.068
5	0.1	0.047

All of these stocks except one lie on the Security Market Line.

Calculate the alpha of the stock that does NOT lie on the Security Market Line.

- A) -0.026
- B) -0.014
- C) 0.000
- D) 0.014
- E) 0.026

Key: D

First, assume that Stocks 1 and 2 lie on the SML. Therefore,

$$0.124 = r_f + 1.2(r_m - r_f) \text{ and } 0.11 = r_f + 1.0(r_m - r_f) \text{ and by substituting,}$$

$$0.124 = r_f + 1.2(0.11 - r_f). \text{ Solving for } r_f \text{ and } r_m, r_f = 0.04 \text{ and } r_m = 0.11.$$

Checking CAPM for the other stocks:

$$\text{Stock 3: } r_f + 0.7(0.11 - r_f) = 0.04 + 0.7(0.11 - 0.04) = 0.089;$$

$$\text{Stock 4: } r_f + 0.4(0.11 - r_f) = 0.04 + 0.4(0.11 - 0.04) = 0.068; \text{ and}$$

$$\text{Stock 5: } r_f + 0.1(0.11 - r_f) = 0.04 + 0.1(0.11 - 0.04) = 0.047.$$

The required return of each of the stocks equals the expected return of those stocks except for Stock 3. Thus, Stock 3 does not lie on the SML

For Stock 3 the required return (determined by the CAPM) equals 0.089 and the expected return equals 0.103. As such, $\alpha = 0.103 - 0.089 = 0.014$.

Reference: Berk/DeMarzo, Sections 12.1 and 12.3

14) You are given the following information about Stock X, Stock Y, and the market:

- (i) The annual effective risk-free rate is 4%.
- (ii) The expected return and volatility for Stock X, Stock Y, and the market are shown in the table below:

	<u>Expected Return</u>	<u>Volatility</u>
Stock X	5.5%	40%
Stock Y	4.5%	35%
Market	6.0%	25%

- (iii) The correlation between the returns of stock X and the market is -0.25 .
- (iv) The correlation between the returns of stock Y and the market is 0.30 .

Assume the Capital Asset Pricing Model holds. Calculate the required returns for Stock X and Stock Y, and determine which of the two stocks an investor should choose.

- (A) The required return for Stock X is 3.20%, the required return for Stock Y is 4.84%, and the investor should choose Stock X.
- (B) The required return for Stock X is 3.20%, the required return for Stock Y is 4.84%, and the investor should choose Stock Y.
- (C) The required return for Stock X is 4.80%, the required return for Stock Y is 4.84%, and the investor should choose Stock X.
- (D) The required return for Stock X is 6.40%, the required return for Stock Y is 3.16%, and the investor should choose Stock Y.
- (E) The required return for Stock X is 3.50%, the required return for Stock Y is 3.16%, and the investor should choose both Stock X and Stock Y.

Key: A

For Stock X, $\beta = -0.25 \cdot (40\% / 25\%) = -0.40$.

The required return is $4\% + \beta \cdot (6\% - 4\%) = 4\% + (-0.40)(2\%) = 3.20\%$.

For Stock Y, $\beta = 0.30 \cdot (35\% / 25\%) = 0.42$.

The required return is $4\% + \beta \cdot (6\% - 4\%) = 4\% + (0.42)(2\%) = 4.84\%$.

The expected return for Stock X exceeds the required return, but the expected return for Stock Y is lower than its required return, so the investor should invest in stock X only.

Reference: Berk/DeMarzo, Section 11.6

15) You are given the following information about Stock X, Stock Y, and the market:

- (i) The expected return and volatility for Stock X, Stock Y, and the market are shown in the table below:

	<u>Required Return</u>	<u>Volatility</u>
Stock X	3.0%	50%
Stock Y	?	35%
Market	6.0%	25%

- (ii) The correlation between the returns of stock X and the market is -0.25 .
- (iii) The correlation between the returns of stock Y and the market is 0.30 .

Assume the Capital Asset Pricing Model holds. Calculate the required return for Stock Y.

- (A) 1.48%
- (B) 2.52%
- (C) 3.16%
- (D) 4.84%
- (E) 6.52%

Key: D

Begin by using information about Stock X to determine the risk-free rate.

For Stock X, $\beta = -0.25 \cdot (50\% / 25\%) = -0.50$.

The required return is $3\% = r_f - 0.50 \cdot (6\% - r_f) = -3\% + 1.5 \cdot r_f$.

Thus, $r_f = 6\% / 1.5 = 4\%$.

For Stock Y, $\beta = 0.30 \cdot (35\% / 25\%) = 0.42$.

The required return is $4\% + 0.42 \cdot (6\% - 4\%) = 4\% + 0.84\% = 4.84\%$.

Reference: Berk/DeMarzo, Section 11.6

16) You are given the following information about Stock X and the market:

- (i) The annual effective risk-free rate is 5%.
- (ii) The expected return and volatility for Stock X and the market are shown in the table below:

	<u>Expected Return</u>	<u>Volatility</u>
Stock X	5%	40%
Market	8%	25%

- (iii) The correlation between the returns of stock X and the market is -0.25 .

Assume the Capital Asset Pricing Model holds. Calculate the required return for Stock X and determine if the investor should invest in Stock X.

- (A) The required return is 1.8%, and the investor should invest in Stock X.
- (B) The required return is 3.8%, and the investor should NOT invest in stock X.
- (C) The required return is 3.8%, and the investor should invest in stock X.
- (D) The required return is 6.2%, and the investor should NOT invest in Stock X.
- (E) The required return is 6.2%, and the investor should invest in stock X.

Key: C

$$\beta = -0.25 \cdot (40\% / 25\%) = -0.40.$$

The required return is $5\% - (0.40) \cdot (8\% - 5\%) = 5\% - 1.2\% = 3.8\%$.

The expected return for Stock X is 5%, which exceeds the required return of 3.8%, so the investor should invest in stock X.

Reference: Berk/DeMarzo, Section 11.6

- 17) Determine which one of the following statements regarding multi-factor models is NOT true.
- (A) A collection of well-diversified portfolios, from which an efficient portfolio can be constructed, can be used to measure risk.
 - (B) These models are also referred to as the Arbitrage Pricing Theory.
 - (C) Taxes and transaction costs are incorporated when estimating the expected rate of return for a multi-factor model.
 - (D) The market portfolio of securities is not necessarily efficient.
 - (E) Small-Minus-Big (SMB) and High-Minus-Low (HML) portfolios are part of the Fama-French-Carhart multi-factor model.

Key: C

Neither taxes nor transaction costs need be considered in a multi-factor model.

Reference: Berk/DeMarzo, Section 13.7

- 18) You are given the following information about the return of a security, using a two-factor model.

Factors	Beta	Expected Return
T	0.10	25%
U	0.15	20%

The annual effective risk-free rate of return is 5%.

Calculate the expected return of this security using the given two-factor model.

- (A) 6.52%
- (B) 8.33%
- (C) 9.25%
- (D) 11.33%
- (E) 13.32%

Key: C

$$\begin{aligned} E(R_S) &= r_f + \beta_T[E(R_T) - r_f] + \beta_U[E(R_U) - r_f] \\ &= 0.05 + 0.1(0.25 - 0.05) + 0.15(0.2 - 0.05) = 0.0925 = 9.25\% \end{aligned}$$

Reference: Berk/DeMarzo, Section 13.7

- 19) Determine which of the following statements is most similar to the semi-strong version of the efficient markets hypothesis.
- (A) It should not be possible to consistently profit by selling winners and hanging on to losers.
 - (B) It should not be possible to consistently profit by trading on information in past prices.
 - (C) It should not be possible to consistently profit by trading on any public information, such as that found on the Internet or in the financial press.
 - (D) It should not be possible to consistently profit by trading on private information, such as that obtained from a thorough analysis of the company and its industry.
 - (E) It should not be possible to consistently profit by trading on inside information.

KEY: C

(A) and (B) refer to the weak form while (D) and (E) refer to the strong form.

Reference: Berk/DeMarzo, Section 13.6

- 20) Since the development of the CAPM model, it is not uncommon that practitioners use market capitalization, book-to-market ratio and past returns to form portfolios that have a positive alpha.

Thus, the market portfolio may not be efficient, and therefore a stock's beta is not an adequate measure of systematic risk.

Determine which of the following is NOT a reason why a market portfolio may not be efficient.

- (A) Alternative Risk Preferences
- (B) Non-Tradable Wealth
- (C) Proxy Error
- (D) Behavioral Biases
- (E) No portfolios are efficient

Key: E

Just because the market portfolio is not efficient, this does not rule out the possibility of another portfolio being efficient. This is the motivation for multi-factor models.

Reference: Berk/DeMarzo, Section 13.6

- 21) Determine which version of the efficient markets hypotheses is contradicted by a momentum strategy whereby investors can use past stock returns to form a portfolio with positive alpha.
- (A) Weak form only
 - (B) Weak form and semi-strong form only
 - (C) Weak form, semi-strong form, and strong form
 - (D) Strong form only
 - (E) It does not contradict any of the three forms of the efficient markets hypothesis.

Key: C

If one can realize consistent profits by trading on the record of past prices, this violates the weak-form of the EMH. However, in the process, this will also violate the semi-strong form and the strong form of the EMH (since both incorporate the weaker levels). Thus, a momentum strategy violates ALL three forms of the EMH.

Reference: Berk/DeMarzo, Section 13.6

- 22) Determine which of following is an example of a behavioral bias that might cause the market portfolio not to be efficient.
- (A) Investors are attracted to large growth stocks that receive greater news coverage.
 - (B) Investors are attracted to investments with skewed distributions that have a small probability of an extremely high payoff.
 - (C) The true market portfolio may be efficient, but the proxy an investor uses to mimic the market portfolio may be inaccurate.
 - (D) Investors are exposed to significant non-tradeable risks outside their portfolio, such as human capital.
 - (E) Investors systematically ignore positive-NPV investment opportunities.

Key: A

From Berk/DeMarzo p.468: "...some investors may be subject to systematic behavioral biases. For example, they may be attracted to large growth stocks that receive greater news coverage...By falling prey to these biases, these investors are holding inefficient portfolios. Thus, the answer is (A).

The other answer choices are more about alternative risk preferences (B), proxy error (C), non-tradable wealth (D), or a different implication (i.e., 1 instead of 2 on p.467) of positive-alpha trading strategies (E).

Reference: Berk/DeMarzo, Section 13.6

- 23) Determine which of the following situations demonstrates evidence that is contrary to the efficient markets hypothesis.
- (A) A takeover bid for a firm is announced at a higher price than the current market price. The firm's share price then increases sharply upon the announcement.
 - (B) By purchasing stocks with high returns over the past year, investors can earn positive excess returns over the next year.
 - (C) Skilled fund managers earn no excess returns relative to their benchmarks, even before fees and transaction costs are taken into account.
 - (D) In research studies completed several years after a severe market decline, many firms were determined to be overvalued prior to the decline.
 - (E) A firm announces that it will increase its dividend in the future, upon which its stock price increases immediately.

Key: B

(B) is an example of evidence against the EMH (pp. 466-467).

(A) is not a violation of the EMH because it occurs due to the uncertainty regarding timing or occurrence of the takeover. (C) is a prediction of the EHM so cannot be providing evidence. (D) refers to a situation where the information was not available or known prior to the event, so the EMH does not apply. (E) is also a prediction of the EMH.

Reference: Berk/DeMarzo, Sections 13.5-13.6

- 24) The following four observations were made about prices and/or returns:
- I. The annualized market return on perfectly sunny days in New York City is much higher than on perfectly cloudy days.
 - II. A company's stock price dropped sharply on the day it issued a warning that upcoming earnings would likely be lower than previously expected.
 - III. A company's stock price increased sharply on the day it was announced that they were a strong candidate to soon be taken over by a stronger company.
 - IV. Trader S consistently earned positive abnormal returns when using a momentum strategy that relied upon investing in stocks that had outperformed the S&P 500 index the previous year.

Determine which two of the four trends described above are consistent with the efficient markets hypothesis (EMH):

- (A) I and II
- (B) I and III
- (C) II and III
- (D) II and IV
- (E) III and IV

Key: C

I is contradictory as unrelated events should not affect the market. II and III are consistent with the semi-strong form because the change was based on publically available information. IV is contradictory, as the weak form of the EMH states that it should not be possible to achieve superior returns by studying the record of past returns.

Reference: Berk/DeMarzo, Section 13.5

- 25) Identify which of the following events has NOT been shown to affect market performance, according to *Corporate Finance* by Berk and DeMarzo.
- (A) Weather
 - (B) Sports
 - (C) Financial news
 - (D) Celebrity news
 - (E) Investor experience

Key: D

The topic of celebrity news is not mentioned in Berk/DeMarzo.

Reference: Berk/DeMarzo, Section 13.4

26) Consider the following four behavioral patterns of investors.

- I: Familiarity Bias
- II: Disposition Effect
- III: Overconfidence Bias
- IV: Herd Behavior

Determine which two of these behavioral patterns are NOT systematic trading biases, and are thus LESS likely to cause stock prices to deviate from their fundamental values.

- (A) I and II
- (B) I and III
- (C) II and III
- (D) II and IV
- (E) III and IV

Key: B

The disposition effect (II) is the tendency for investors to hang on to losers and sell winners. Herd behavior (IV) is when investors imitate each other's actions. Both are systematic trading biases, which will likely cause stock prices to deviate from their fundamental values.

Familiarity bias (I) occurs when investors show a preference toward companies for which they are familiar. Overconfidence bias (III) results from uninformed individuals overestimating the precision of their knowledge. These are LESS likely to cause stock prices to deviate from their fundamental values because (see p.453) "in order for the behavior of uninformed investors to have an impact on the market, there must be patterns to their behavior that lead them to depart from the CAPM in systematic ways." Here, the departures are mostly idiosyncratic, and may thus cancel out just like any other idiosyncratic risk; that is, uninformed investors may just be trading amongst themselves.

Reference: Berk/DeMarzo, Sections 13.3-13.4

27) Consider a two-year project, where the cost of capital is 10%.

There are only three cash flows for this project.

- The first occurs at $t = 0$, and is -100 .
- The second occurs at $t = 1$, and is 66 .
- The third occurs at $t = 2$, and is X .

Determine X , the level of the cash flow at $t = 2$, that leads to the project breaking even.

- (A) 34.0
- (B) 38.4
- (C) 44.0
- (D) 48.4
- (E) 54.0

Key: D

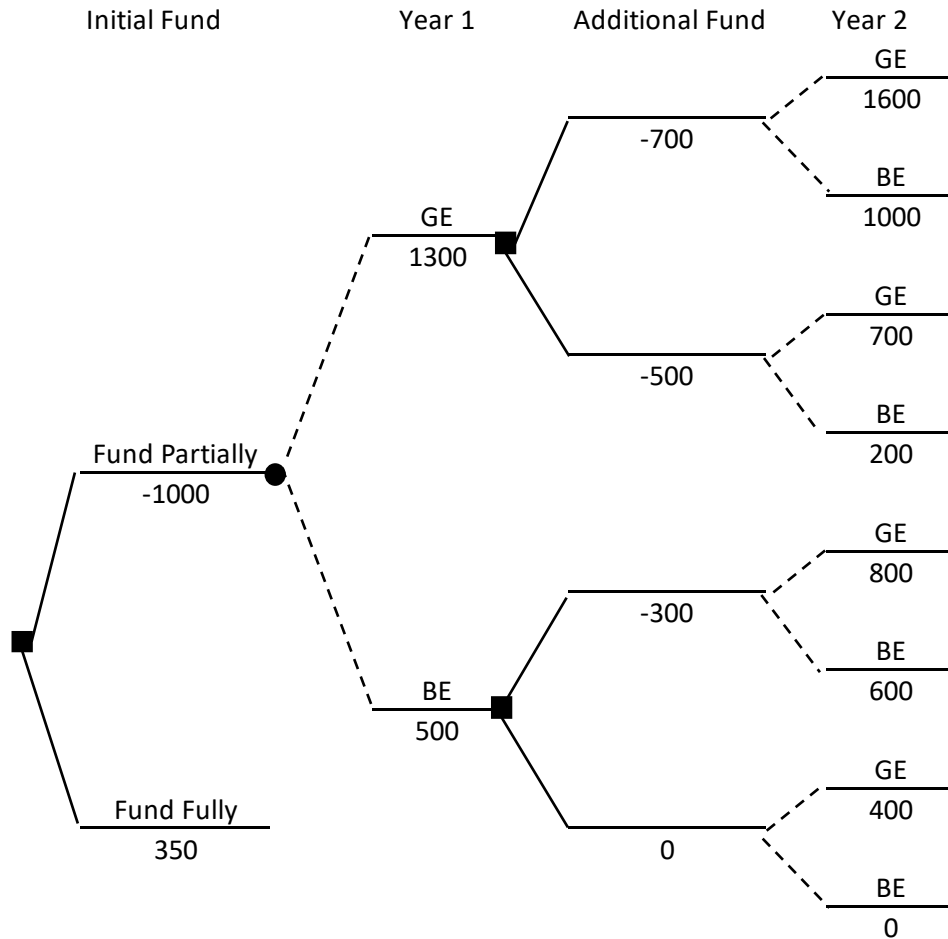
The break-even level is the level for which an investment has a Net Present Value equal to zero. In this case, the NPV can be derived by solving:

$$0 = -100 + 66(1.1)^{-1} + X(1.1)^{-2} = -100 + 60 + 0.82645X$$

$$X = 40 / 0.82645 = 48.4.$$

Reference: Berk/DeMarzo, Section 8.5

- 28) Consider a two-year project that when fully funded at time 0 has a net present value of \$350. The decision tree below shows the cash flows of the project when partially funded at the beginning of the Year 1 (at $t = 0$) with an option to provide different amounts of funding at the beginning of Year 2 (at $t = 1$). This tree reflects two economic states (GE = good economy, BE = bad economy) in each of the two years. For a given year, each economic state has a 50% probability.



Assume the discount rate is 0%.

Calculate the value of the option at $t = 0$.

- (A) 0
- (B) 50
- (C) 150
- (D) 200
- (E) 250

Key: B

We work backward from the end of the tree as discussed on p. 797. The upper branch following an information node represents a good year and the bottom branch a bad year.

We start with the top branch. At $t=1$, we have a choice to invest 700 or 500. If we invest 700 and the second year has a good economy, the project generates a cash flow of 1600. If we invest 700 and the second year has a bad economy, the project generates a cash flow of 1000. The expected value is $0.5(1600) + 0.5(1000) = 1300$ for the second year for a net gain of $1300 - 700 = 600$.

Similarly if we invest 500 and the economy in the second year is good, the project generates a cash flow of 700, and if bad a cash flow of 200. The expected cash flow is 450 for a net loss of 50. Hence we should invest the 700. Including the year one cash flow, a good year one economy leads to an expected value of $1300 + 600 = 1900$.

We do a similar analysis on the bottom branch. The expected value of cash flows at the end of the second year with an investment of 300 is 700 for a gain of 400 and with an investment of 0 the expected cash flow is 200 for a gain of 200. Hence, the investor will spend 300 at the end of the first year and with the year one gain in a bad economy, the value of this branch is $500 + 400 = 900$.

The expected value for year one is $(1900 + 900)/2 = 1400$ and the NPV is $1400 - 1000 = 400$.

The value of the option is the NPV of the project with the option – NPV of the project without the option (i.e., fully funded). That is $\text{Option Value} = 400 - 350 = 50$.

Reference: Berk/DeMarzo, Section 22.2

29) Determine which one of the following statements about debt financing is FALSE:

- A) Corporate notes are unsecured debt.
- B) Holders of mortgage backed securities face prepayment risk.
- C) The coupon of a floating-rate municipal bond is periodically adjusted.
- D) Income from U.S. Treasury securities is taxed at the state level.
- E) Most debentures contain clauses restricting the company from issuing new debt with equal or higher priority than existing debt.

Key: D

On p. 872, Berk/DeMarzo state: “All income from Treasury securities is taxable at the federal level. This income, however, is NOT taxable at the state or local level.” Thus, choice (D) is false. All of the other statements are true.

Reference: Berk/DeMarzo, Sections 24.1-24.2

30) Determine which one of the following statements about raising capital is TRUE:

- A) A private equity firm invests in publicly traded corporations.
- B) A venture capital firm is a corporation.
- C) Convertible notes held by angel investors are converted into equity at the price paid by new investors.
- D) A major disadvantage of an initial public offering is that the equity holders of the corporation become more widely dispersed.
- E) The lead underwriter of an initial public offering is generally the venture capital firm that has already provided funding to the firm.

Key: D

Statements A, B, C, and E are false (see below), while only statement D is true.

- A) P. 833 “A private equity firm...invests in the equity of existing privately held firms.”
- B) P. 830 “A venture capital firm is a limited partnership....”
- C) P. 829 “The note holders convert the value of their initial investment plus accrued interest into equity at a discount (often 20%) to the price paid by new investors.”
- D) P. 840 “The major advantage of undertaking an IPO is also one of the major disadvantages of an IPO: When investors diversify their holdings, the equity holders of the corporation become more widely dispersed.”
- E) P. 843 “Many IPOs, especially the larger offerings, are managed by a group of underwriters. The lead underwriter is the primary banking firm responsible for managing the deal.”

Reference: Berk/DeMarzo: Sections 23.1-23.2

- 31) Determine which one of the following statements is TRUE with respect to a perfect capital market:
- (A) Taxes and transaction costs can exist.
 - (B) A firm's choice of capital structure will have an effect on its cost of capital.
 - (C) A firm's choice of capital structure will always have an effect on the firm's value.
 - (D) Leverage has no effect on the risk of equity, even where there is no default risk.
 - (E) The total value of a levered firm is equal to the total value of the firm without leverage..

Key: E

- A) Perfect capital markets have no taxes, transactions costs, or other frictions.
- B) The choice of financing does not affect the cost of capital or NPV of a project. They are solely determined by its free cash flows.”
- C) Modigliani and Miller state that this is not so”
- D) Leverage increases the risk of equity even when there is no risk that the firm will default.
- E) This is true.

Reference: Berk/DeMarzo, Sections 12.6, 14.1, 14.2

- 32) Determine which one of the following statements about the agency costs of leverage is FALSE:
- (A) When a firm faces financial distress, shareholders have an incentive to withdraw cash from the firm.
 - (B) When an unlevered firm issues new debt, equity holders will bear any agency or bankruptcy costs via a discount in the price they receive for the new debt.
 - (C) When a levered firm issues new debt, existing debt holders will bear agency or bankruptcy costs.
 - (D) Equity holders may have an incentive to increase financial leverage, even if doing so would decrease the value of the firm.
 - (E) Agency costs are larger for short-term debt than for long-term debt.

Key: E

- A) When a firm faces financial distress, shareholders have an incentive to withdraw cash.
- B) When an unlevered firm issues new debt, equity holders will bear any anticipated agency or bankruptcy costs via a discount in the price they receive for that new debt.
- C) Once a firm has debt already in place, some of the agency or bankruptcy costs that result from taking on additional leverage will fall on existing debt holders.
- D) Once existing debt is in place, (1) shareholders may have an incentive to increase leverage even if it decreases the value of the firm.
- E) Agency costs are smallest for short-term debt.

Reference: Berk/DeMarzo, Section 16.5

- 33) The following table shows five potential investment opportunities for a firm, and the effect that the decision by shareholders about whether to invest or not invest in those opportunities has on bondholders.

Opportunity	NPV	Equity Holder	Debt Holder
I	100	Invest	Beneficial
II	50	Don't Invest	Not Beneficial
III	0	Don't Invest	Beneficial
IV	-50	Don't Invest	Not Beneficial
V	-100	Invest	Beneficial

Determine which one of the five opportunities best reflects the debt overhang problem.

- (A) I
- (B) II
- (C) III
- (D) IV
- (E) V

Key: B

When shareholders prefer not to invest in a positive-NPV project, there is a debt overhang or under-investment problem. This failure to invest is costly for debt holders. The opportunity in the table that best reflects this statement is opportunity 2. With opportunity 2, there is a positive NPV, but the shareholders decide not to invest, which in turn would be costly to bondholders.

Reference: Berk/DeMarzo, Section 16.5

- 34) Let X be the random gain from operations of a company. You are given:
- (i) X is normally distributed with mean 42 and variance 6400.
 - (ii) p is the probability that X is negative.
 - (iii) K is the amount of capital such that the Value-at-Risk (VaR) at the 5th percentile for $X + K$ is zero.

Calculate p and K .

- (A) $p = 0.7; K = 157$
- (B) $p = 0.7; K = 131$
- (C) $p = 0.5; K = 115$
- (D) $p = 0.3; K = 115$
- (E) $p = 0.3; K = 90$

Key: E

$$p = \Pr(X < 0) = \Pr(Z < (0 - 42) / 80) = \Pr(Z < -0.525) = 0.2998.$$

The 5th percentile of Z is -1.64485.

The 5th percentile of X is $42 - 1.64485 * 80 = -89.588$.

Thus, $K = -89.588$.

Reference: SN IFM-21-18

- 35) You own a share of a nondividend-paying stock and will hold it for a period of time. You want to set aside an amount of capital as a percentage of the initial stock price to reduce the risk of loss at the end of the holding period.

You are given:

- i) The stock price follows a lognormal distribution.
- ii) The annualized expected rate of return on the stock is 15%.
- iii) The annualized stock volatility is 40%.
- iv) The investment period is 4 years.
- v) The Value-at Risk (VaR) at the 3rd percentile for the capital plus the ending stock value equals the initial stock price.

Calculate the capital amount as a percentage of initial stock price.

- (A) 57%
- (B) 63%
- (C) 71%
- (D) 82%
- (E) 91%

Key: C

Let S_t be the stock price at time t .

Then, $X = \ln(S_t / S_0)$ is normally distributed with mean $(\mu - 0.5\sigma^2)t$ and variance $\sigma^2 t$.

From points (ii) and (iii), $\mu = 0.15$, $\sigma^2 = 0.4$, and $t = 4$.

Thus, X has a normal distribution with mean 0.28 and variance 0.64.

We then have, where C is the capital:

$$\Pr(C + S_4 \leq S_0) = 0.03$$

$$\Pr(C / S_0 + S_4 / S_0 \leq 1) = 0.03$$

$$\Pr(\ln(S_4 / S_0) \leq \ln(1 - C / S_0)) = 0.03$$

$$\Pr(Z \leq [\ln(1 - C / S_0) - 0.28] / 0.8) = 0.03$$

$$[\ln(1 - C / S_0) - 0.28] / 0.8 = -1.880794$$

$$\ln(1 - C / S_0) = -1.224635$$

$$1 - C / S_0 = \exp(-1.224635) = 0.293864$$

$$C / S_0 = 0.706136$$

Reference: SN IFM-21-18

- 36) Determine which one of the following statements regarding guarantees on variable annuity products is FALSE:
- (A) A guaranteed minimum death benefit (GMDB) with a return of premium guarantee is similar to a European put option with expiration contingent on the death of the policyholder or annuitant.
 - (B) A guaranteed minimum accumulation benefit (GMAB) with a return of premium guarantee is similar to a European put option with payment contingent on the policyholder surviving to the guarantee expiration date and the policy still being in force at that time.
 - (C) A guaranteed minimum withdrawal benefit (GMWB) provides a guarantee that the account value will not be less than the guaranteed withdrawal benefit base at any future time.
 - (D) A guaranteed minimum income benefit (GMIB) provides a guarantee on the future purchase rate for a traditional annuity.
 - (E) An earnings-enhanced death benefit is an optional benefit available with some variable annuity products that acts as a European call option with strike price equal to the original amount invested.

Key: C

- (A) True, see section 2.1.1.1, pp. 3-4 of the study note.
- (B) True, see section 2.1.4, p. 6 of the study note.
- (C) False, see section 2.1.1.3, p. 4 of the study note. The guaranteed minimum withdrawal benefit guarantees the size of **withdrawals**, not the size of the **account value**.
- (D) True, see section 2.1.1.4, p. 4 of the study note.
- (E) True, see section 2.1.3, p. 6 of the study note.

Reference: SN IFM-22-18

- 37) A policyholder owns a variable annuity contract with death benefit features defined as follows:
- Guaranteed minimum death benefit (GMDB) with return of premium: the greater of the account value and the initial investment will be paid when the policyholder dies.
 - Enhanced-income death benefit guarantee: 20% of the account value in excess of the initial investment amount will be added if the account value is greater than the initial investment when the policyholder dies.

Let T be the random variable denoting the future lifetime of the policyholder.

Let K be the initial investment amount of the variable annuity contract.

Let S_t be the value of the policyholder's account at time t .

You are given:

- T follows a distribution with probability density function $f(t), t > 0$.
- Given $T = t$, $p(t)$ is the payoff of a European put option based on account value S_t with strike price K .
- Given $T = t$, $c(t)$ is the payoff of a European call option based on account value S_t with strike price K .

Determine which one of the following statements is true.

- The total death benefit payout for death at time t can be expressed as $\max(S_t, K) + 0.2 \times \max(K - S_t, 0)$.
- The total death benefit payout for death at time t can be expressed as $\max(S_t - K, 0) + 0.2 \times \int_0^\infty p(t) f(t) dt$.
- The expected value of the death benefit can be expressed as $K + \int_0^\infty c(t) f(t) dt + 0.2 \times \int_0^\infty p(t) f(t) dt$.
- The expected value of the death benefit can be expressed as $K + \int_0^\infty p(t) f(t) dt + 0.2 \times \int_0^\infty c(t) f(t) dt$.
- The expected value of the death benefit can be expressed as $K + 1.2 \times \int_0^\infty c(t) f(t) dt$.

Key: E

The total benefit payout for death at time t is:

$$\begin{aligned}\max(S_t, K) + 0.2 \times \max(S_t - K, 0) &= K + \max(S_t - K, 0) + 0.2 \times \max(S_t - K, 0) \\ &= K + 1.2 \times \max(S_t - K, 0)\end{aligned}$$

This is neither (A) nor (B) and thus we must determine the expected death payout.

The total death benefit payout is equal to the initial investment plus 120% of a call option with strike price K and the time to expiration t . According to the given conditions (i) and (iii), the expected value of this call option is

$$\int_0^{\infty} c(t) f(t) dt .$$

Thus, the expected value of the death benefits is $K + 1.2 \times \int_0^{\infty} c(t) f(t) dt .$

Reference: SN IFM-22-18

- 38) An insurance company has a variable annuity linked to the S&P 500 index. A guaranteed minimum death benefit (GMDB) specifies the beneficiary will receive the greater of the account value and the original amount invested, if the policyholder dies within the first three years of the annuity contract. If the policyholder dies after three years, the beneficiary will receive the account value.

Out of every 1000 policies sold, the company expects 10 deaths in each of years one, two, and three. Thus they also expect that 970 will survive the first three years. Assume the deaths occur at the end of the year.

You are given the following at-the-money European call and put option prices, expressed as a percentage of the current value of the S&P 500 index.

Duration (years)	Call Price	Put Price
1	18.7%	15.8%
2	26.2%	20.6%
3	31.6%	23.4%

Calculate the expected value of the guarantee when the annuity is sold, expressed as a percentage of the original amount invested.

- (A) 0.23%
- (B) 0.32%
- (C) 0.52%
- (D) 0.60%
- (E) 0.76%

Key: D

The guarantee value is the sum of the three put prices multiplied by the probability of dying each year: $0.01(15.8) + 0.01(20.6) + 0.01(23.4) = 0.598 \approx 0.60\%$.

Reference: SN IFM-22-18

39) A variable annuity has the following guarantees:

- Guaranteed minimum death benefit with a return of premium guarantee.
- Guaranteed minimum accumulation benefit with a return of premium guarantee, effective 10 years from the date the policy is sold.
- Earnings-enhanced death benefit that pays the beneficiary an additional benefit equal to 20% of any increase in the account value.

The following notation is used:

- $P(T)$ denotes the value of a European put option on the annuity value, with the strike price equal to the original amount invested and time to expiration T .
- $C(T)$ denotes the value of a European call option on the annuity value, with the strike price equal to the original amount invested and time to expiration T .
- T_x denotes the future lifetime of the policyholder, and $f_{T_x}(t)$ denotes the probability density function of T_x .

Assuming no lapses, which expression below represents the combined value of all guarantees?

(A) $\int_0^{\infty} C(t)f_{T_x}(t)dt + \Pr(T_x \geq 10) \times P(10) + 0.2 \times \int_0^{\infty} C(t)f_{T_x}(t)dt$

(B) $\int_0^{\infty} C(t)f_{T_x}(t)dt + \Pr(T_x \geq 10) \times P(10) + 0.2 \times \int_0^{\infty} P(t)f_{T_x}(t)dt$

(C) $\int_0^{\infty} P(t)f_{T_x}(t)dt + \Pr(T_x \geq 10) \times P(10) + 0.2 \times \int_0^{\infty} C(t)f_{T_x}(t)dt$

(D) $\int_0^{\infty} P(t)f_{T_x}(t)dt + \Pr(T_x \leq 10) \times P(10) + 0.2 \times \int_0^{\infty} C(t)f_{T_x}(t)dt$

(E) $\int_0^{\infty} P(t)f_{T_x}(t)dt + \Pr(T_x \leq 10) \times P(10) + 0.2 \times \int_0^{\infty} P(t)f_{T_x}(t)dt$

Key: C

GMDBs are similar to put options, contingent on dying.

GMABs are similar to put options, contingent on surviving.

Earnings-enhanced death benefits are similar to call options, contingent on dying.

Reference: SN IFM-22-18

40) Several lookback options are written on the same underlying index. They all expire in 3 years.

Let S_t denote the value at time t of the index on which the option is written.

The initial index price, S_0 , is 150.

The index price when the option expires, S_3 , is 200.

The maximum index price over the 3-year period is 210.

The minimum index price over the 3-year period is 120.

Calculate the sum of the payoffs for the following three lookback options:

- Standard lookback call
- Extrema lookback call with a strike price of 100
- Extrema lookback put with a strike price of 100

(A) 180

(B) 190

(C) 200

(D) 210

(E) 220

Key: B

The payoffs for the three options are as follows:

Standard lookback call: $S_T - m$

Extrema lookback call: $\max(M - K, 0)$

Extrema lookback put: $\max(K - m, 0)$

Total payoff = $(200 - 120) + (210 - 100) + (0) = 190$

Reference: SN IFM-22-18

41) Which of the following statements about market anomalies is/are true?

- I. For IPOs, the high returns observed in the first few days after a new issue are often followed by relatively poor performance in the years ahead, suggesting that investors underreact to the initial news.
- II. Investors typically overreact to earning announcements, causing a subsequent price adjustment.
- III. The momentum effect is inconsistent with weak-form market efficiency.

- (A) I only
- (B) II only
- (C) III only
- (D) I, II, and III
- (E) None of (A), (B), (C), and (D) are correct

Key: C

Statement I is false. For IPOs, the subsequent poor performance, after the initial run-up, suggests that investors *overreact* to the initial news.

Statement II is false. Investors typically *underreact* to earning announcements, causing a subsequent price adjustment.

Statement III is true. The weak form of EMH states that prices reflect information contained in the record of past prices and investors cannot make consistent profits from analyzing past data. The momentum effect suggests that there is a positive serial correlation in stock prices. Thus, price patterns exist and can be exploited by analyzing historical price information, which contradicts with the weak form of EMH.

Reference: SN IFM-21-18

- 42) A firm's portfolio is currently valued at 50 million. The portfolio has an annualized expected rate of return of 7% and a volatility of 10.5%.

Assume the returns are normally distributed, and that the portfolio pays no dividends.

Calculate the 5% annual Value-at-Risk (VaR) for the firm's portfolio investment gain after one year.

- (A) -2,500,000
- (B) -5,135,000
- (C) -5,250,000
- (D) -12,135,462
- (E) -23,149,616

Key: B

Let X be the portfolio's annual return. Then, the VaR of X at the 5% level is the value $\pi_{0.05}$ such that:

$$\Pr(X \leq \pi_{0.05}) = 0.05$$

$$\Pr\left(Z \leq \frac{\pi_{0.05} - 0.07}{0.105}\right) = 0.05$$

$$\frac{\pi_{0.05} - 0.07}{0.105} = -1.64485$$

$$\pi_{0.05} = -0.1027$$

This means there is a 5% probability that the annual return for the portfolio will be less than or equal to -10.27% . Thus, the VaR, expressed in investment gain, is $50,000,000 * -10.27\% = -5,135,000$.

Reference: SN IFM-21-18

- 43) Which of the following statements about calendar/time anomalies is/are true?
- I. Stock returns have been observed to be lower in December than in other months.
 - II. Stock returns have been observed to be higher on Friday than on other days.
 - III. Stock returns have been observed to be more volatile during the middle of the trading day than toward either the beginning or the end of the trading day.
- (A) I only
- (B) II only
- (C) III only
- (D) I, II, and III
- (E) None of (A), (B), (C), and (D) are correct

Key: E

Statement I is true. Stock returns have been observed to be higher in January and lower in December than in other months.

Statement II is true. Stock returns have been observed to be lower on Monday and higher on Friday than in other days.

Statement III is false. Stock returns have been observed to be more volatile at times close to market open and to market close.

Thus, I and II only are correct, which is not consistent with answer choices A, B, C, or D.

Thus, E is correct.

Reference: SN IFM-21-18

- 44) A company is considering a franchise expansion project. It plans to estimate the project's net present value (NPV) to decide whether or not it should expand.

Below are 3 cases for the 6 different variables that affect the project's NPV:

Variable Name	Worst Case	Base Case	Best Case
Initial Investment	\$5,500,000	\$5,000,000	\$4,500,000
Pre-Tax Cash Flow at End-of-Year 1	\$900,000	\$1,000,000	\$1,100,000
Cash Flow Growth Rate after Year 1	2%	3%	4%
Tax Rate	35%	25%	15%
Cost of Capital	7%	6%	5%
Project Lifetime	8 years	10 years	12 years

You are given:

- The project has a finite time horizon and no shutdown costs. When the project expires, all cash flows cease.
- All six variables are mutually independent.
- The probability that the worst case occurs is 25%, the probability that the base case occurs is 50%, and the probability that the best case occurs is 25%. Also, assume these probabilities are the same across all 6 input variables.

Use the following 6 uniform random numbers u between 0 and 1 to simulate a value for each of the 6 variables (in order from the 1st row of the table down to the 6th row):

0.0277 0.1198 0.1724 0.4757 0.9617 0.1600

- If $0 \leq u < 0.25$, pull from the worst-case column.
- If $0.25 \leq u < 0.75$, pull from the base-case column.
- If $0.75 \leq u < 1$, pull from the best-case column.

Based on the simulated values, calculate the NPV for the project.

- (A) -1,778,422
- (B) -843,067
- (C) 161,971
- (D) 1,239,103
- (E) 5,643,381

Key: B

Solution:

Based on the 6 random numbers, select the appropriate values from the table:

As a result, we have:

Variable Name	<i>u</i>	Case	Value
Initial Investment	0.0277	Worst	\$5,500,000
Pre-Tax Cash Flow at End-of-Year 1	0.1198	Worst	\$900,000
Growth Rate in Cash Flows after Year 1	0.1724	Worst	2%
Tax Rate	0.4757	Base	25%
Cost of Capital	0.9617	Best	5%
Project Lifetime	0.1600	Worst	8 years

Based on the simulated values for the 6 input variables, we can compute the NPV:

$NPV = PV(\text{Future After-Tax Cash Flows}) - \text{Initial Investment}$

$$\begin{aligned} &= 900,000(1 - 0.25) \left[\frac{1}{1.05} + \frac{1.02}{1.05^2} + \dots + \frac{1.02^7}{1.05^8} \right] - 5,500,000 \\ &= 900,000(0.75) \frac{1 - \frac{1.02^8}{1.05}}{1 - \frac{1.02}{1.05}} - 5,500,000 = -843,067. \end{aligned}$$

Reference: IFM-21-18, 3.2.2 Example #2