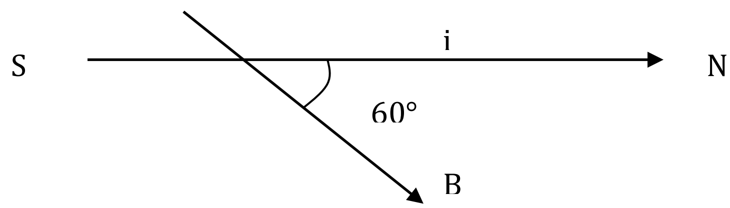
Exam 2 Solutions

1. A horizontal power line carries a current of 2900A from south to north. Earth's magnetic field, with a magnitude of 60 μ T, is directed toward the north with a dip angle 60° downward into the Earth relative to the horizontal. Find the magnitude and direction (use compass directions) of the magnetic force acting on a 100 m length of power line.

(1) 15 N, West (2) 15 N, East (3) 17.5 N, West (4) 8.7 N, East (5) 17.5 N, East

The magnetic field and the power line both point north, but the magnetic field points into the Earth at an angle of 60°.



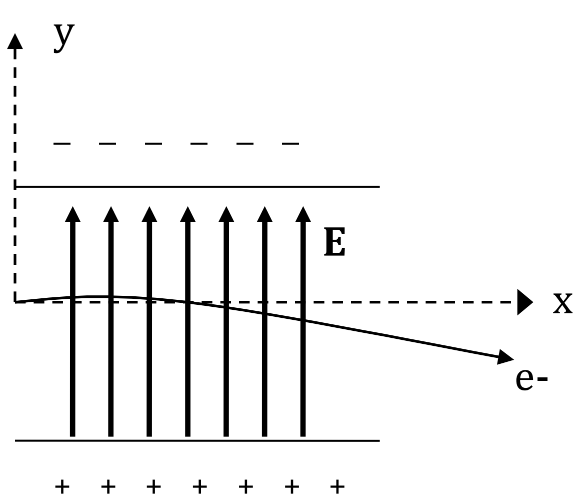
The force on a current carrying wire is given by:

$$\mathbf{F} = i\mathbf{L} \times \mathbf{B}$$

So the force points west with magnitude:

$$|\mathbf{F}| = iLB\sin\theta = (2900)(100)(60 \times 10^{-6})(\sin 60) = 15 \text{ N}$$

Answer: 15 N



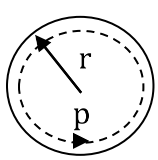
2. A beam of electrons ("cathode rays") is sent between two parallel electric plates with an electric field between them of 2x10^4 N/C j^ s . If the electron beam travels perpendicular to the electric field with a velocity of 4.2x 10^7 m/s in the +i direction, what magnetic field is necessary (direction and magnitude) so that the electrons continue traveling in a straight line without deflection by the electric field?

(1) $4.8\times10^{-4}\,\hat{k}$ T (2) $-4.8\times10^{-4}\,\hat{k}$ T (3) $2.0\times10^{4}\,\hat{j}$ T (4) $-2.0\times10^{4}\,\hat{j}$ T (5) $2.1\times10^{3}\,\hat{i}$ T

Balance forces:

$$\mathbf{F} = q(\mathbf{E}+\mathbf{v}\times\mathbf{B}) = 0$$
$$\Rightarrow \mathbf{E}+\mathbf{v}\times\mathbf{B} = 0$$
$$\Rightarrow \mathbf{B} = \frac{E}{v}\hat{\mathbf{k}} \quad \text{so that } \hat{\mathbf{j}}\times\hat{\mathbf{k}} = -\hat{\mathbf{i}}$$
$$E = 2\times10^{4}\ \text{N/C}$$
$$v = 4.2\times10^{7}\ \text{m/s}$$
$$\Rightarrow B = \frac{2\times10^{4}\ \text{N/C}}{4.2\times10^{7}} = 0.48\ \text{mT}$$

3. The magnetic field of a solenoid (long compared to its radius) is used to keep a proton in a perfectly circular orbit. The solenoid has 1000 windings per meter of length and has a radius of 1 m. If the proton has a velocity magnitude of $v = 1.5\times10^{6}$ m/s, what is the minimum current needed to keep the proton orbiting within the confines of the solenoid in a plane perpendicular to the solenoid axis? The proton mass is $m_p = 1.67\times10^{-27}$ kg and its charge is $q = +1.6\times10^{-19}$ C.



(1) 12.5 A (2) 78,000 A (3) 25,000 A (4) 12,500 A (5) 1.6x10^{-7} A

$$B = \mu_0 n i$$
$$mv = qBr = qr\mu_0 n i$$
$$\Rightarrow i = \frac{mv}{qr\mu_0 n} = \frac{(1.67\times10^{-27}\ \text{kg})(1.5\times10^{6}\ \text{m/s})}{(1.6\times10^{-19}\ \text{C})(1\text{m})(4\pi10^{-7})(1000)} = 12.5\ \text{A}$$

4. A series RLC circuit is driven by sinusoidally-varying EMF source with a maximum amplitude of 125V. The resistance R=100 Ohm, the inductance L = 2x10^-3 H, and the capacitance C = 0.1 uF. At what frequency (cycles/sec) will the amplitude of the current be a maximum?

(1) 11 kHz (2) 11 Hz (3) 7.1 kHz (4) $2\text{x}10^{-5}$ Hz (5) $1\text{x}10^{-5}$ Hz

$$i_m = \frac{\varepsilon_m}{\sqrt{R^2 + (X_L - X_C)^2}}$$

The amplitude of the current is a maximum when

$$X_L = X_C$$

$$\Rightarrow \omega = 2\pi f = \frac{1}{\sqrt{LC}}$$

$$f = \frac{1}{2\pi\sqrt{LC}} = 11.3 \text{ kHz}$$

5. An alternating EMF source drives a series RLC circuit with a maximum amplitude 9.0V. The phase angle of the current is +45º. When the potential difference across the capacitor reaches its maximum positive value of +6V, what is the potential difference across the inductor (including sign)?

(1) -12: 4 V (2) -8 V (3) 6.4 V (4) -0.4 V (5) 0 V

$$\varepsilon = \varepsilon_d \sin\omega t$$

$$i = i_m \sin(\omega t - \phi)$$

$$\Delta V_C = \frac{i_m}{\omega C}\sin\left(\omega t - \phi - 90^\circ\right)$$

This is a maximum when $\omega t - \phi - 90^\circ = 90^\circ, \omega t = \phi + 180^\circ$

So the potential across the resistor is:

$$\Delta V_R = \frac{i_m}{R}\sin(\omega t - \phi) = \frac{i_m}{R}\sin\left(180^\circ\right) = 0$$

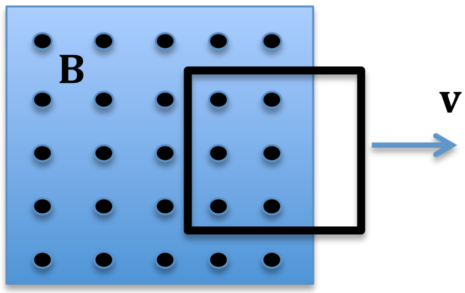
Now apply Kirchoff's loop rule to solve for the potential across the inductor:

$$\varepsilon - \Delta V_C - \Delta V_L - \Delta V_R = 0$$

$$\Delta V_L = \varepsilon(t) - \Delta V_C = \varepsilon_d \sin\left(180^\circ + \phi\right) - \Delta V_C$$

$$= 9(-0.707) - 6 = -12.4V$$

6. A square metal loop of side length l=0.5m is pulled out of a uniform magnetic field with a velocity of v=2 m/s. The magnetic field has a strength of B=0.25 T and is directed perpendicular to the surface of the loop. One side of the square is aligned with the edge of the field region when the pulling first starts. What is the magnitude of the induced EMF in the loop as it is pulled?



(1) 0.25 V (2) 5 V (3) 0.125 V (4) 0.0625 V (5) 1 V

The induced EMF is given by Faraday's Law:

$$|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = \left|\frac{d}{dt}(BA)\right| = \left|\frac{d}{dt}\left[BL(L - vt)\right]\right|$$

$$|\varepsilon| = BLv = 0.25V$$

7. An inductor of inductance L=10 H and resistance R = 2 Ohm is plugged into a DC source of EMF at t=0. How long does it take for the current through the inductor to reach 80% of its maximum?

(1) 8.0 s (2) 5.0 s (3) 1.1 s (4) 10 s (5) 2.0 s

The solution to Kirchoff's loop rule for LR circuits gives the current i(t):

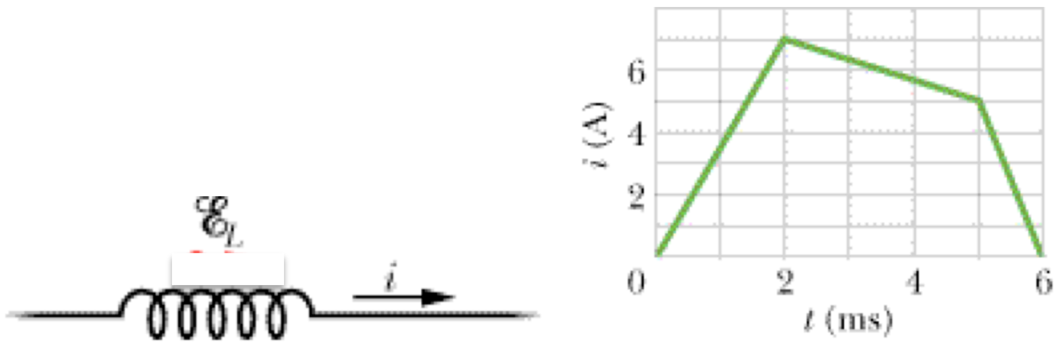
$$i(t) = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right) \qquad \tau_L = \frac{L}{R} = 5s$$

$$0.8\frac{\varepsilon}{R} = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right)$$

$$e^{-t/\tau_L} = 1 - 0.8$$

$$\frac{t}{\tau_L} = -\ln 0.2$$

$$t = -\tau_L \ln 0.2 = 8s$$



8. The current through an inductor with inductance L=0.1 H is shown by the graph, with the direction from left to right through the inductor as shown. What is the EMF across the inductor ($V_L = V_{right} - V_{left}$), including sign, at t = 1 ms?

(1) −350 V (2) 0.35 V (3) −7 V (4) 3500 V (5) −35 V

$$V_L = -L\frac{di}{dt} = -L\frac{\Delta i}{\Delta t}$$
$$= -(0.1\text{ H})\frac{7\text{ A}}{2\times10^{-3}\text{ s}} = -350\text{V}$$

9. A parallel plate capacitor has circular plates with a radius R=2cm and a time-dependent electric field between them of (3×10^6 V/m-s) t. What is the magnitude of the induced magnetic field at a radius of r = 3cm from the central axis connecting the centers of the plates, which is larger than the radius R covered by electric field?

(1) 2.2×10^{-13} T (2) 1.5×10^{-13} T (3) 3.3×10^{-13} T (4) 1.8×10^{-10} T (5) 0 T

We use Maxwell's Law of Induction:

$$\oint B\cdot ds = \mu_0\varepsilon_0\frac{d\Phi_E}{dt}$$
$$2\pi rB = \mu_0\varepsilon_0\left(\pi R^2\right)\left(3\times10^6\right)$$
$$B = \frac{\mu_0\varepsilon_0}{2\pi r}\left(\pi R^2\right)\left(3\times10^6\right) = 2.2\times10^{-13}\text{ T}$$

10. A constant current of i = 5A is used to charge a parallel plate capacitor with circular plates of radius R = 1cm. What is the magnitude of the magnetic field at a radius of r = 0.5 cm, which is less than *R*, in the region between the plates?

(1) 5×10^{-5} T (2) 1×10^{-4} T (3) 2×10^{-4} T (4) 6.3×10^{-6} T (5) 0 T

The total displacement current between the plates is i_d=i=5A. The effective displacement current density is $j_d = \frac{i_d}{\pi R^2}$

So using Maxwell's Law of Induction:

$$\oint B \cdot ds = \mu_0 i_d'$$

$$2\pi r B = \mu_0 j_d \pi r^2 = \mu_0 \frac{i_d}{\pi R^2} \pi r^2$$

$$B = \frac{\mu_0}{2\pi} \frac{i_d}{R^2} r = 5 \times 10^{-5} \text{ T}$$

11. A transformer is designed to provide a 5V output EMF when the input is 120V with a 60 Hz frequency. If there are 144 primary windings around the transformer core, how many secondary windings are necessary to provide the correct output voltage?

(1) 6 (2) 3450 (3) 144 (4) 24 (5) 30

The secondary EMF is given by the primary EMF times the ratio of secondary to primary windings:

$$\varepsilon_s = \frac{N_s}{N_p} \varepsilon_p$$

so the number of secondary windings necessary is:

$$N_s = \frac{\varepsilon_s}{\varepsilon_p} N_p = 6$$

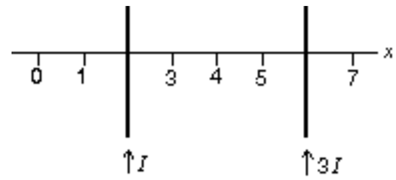
12. It takes an energy of 7.4×10^{-23} J to flip the alignment of the spin of an electron from parallel to anti-parallel to the direction of a magnetic field. What is the magnitude of the magnetic field? (μ_B = 9.27×10^{-24} J/T)

(1) 4 T (2) 8 T (3) 2 T (4) 3×10^{-17} T (5) 1.3×10^{4} T

The energy to flip the spin from parallel to anti-parallel is

$$U = 2\mu_B B$$

13. Two long straight current-carrying parallel wires cross the x axis and carry currents I and $3I$ in the same direction, as shown. At what value of x is the net magnetic field zero?



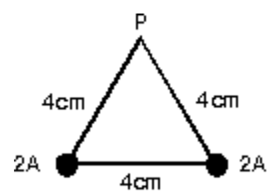
(1) 3 (2) 1 (3) 0 (4) 5 (5) 7

S: Between two wires, magnetic field produced by I points into the page and magnetic field produced by 3I points out of the page. The net magnetic field is zero when the two fields have same magnitude.

$$\frac{\mu_0 I}{2\pi(x-2)} = \frac{\mu_0 3I}{2\pi(6-x)}$$

Solve it to get x=3.

14. Two long straight wires pierce the plane of the paper at vertices of an equilateral triangle as shown below. They each carry 2 A, out of the paper. The magnetic field at the third vertex (P) has magnitude (in T):



(1) $1.7\text{x}10^{-5}$ (2) $1.0\text{x}10^{-5}$ (3) $2.0\text{x}10^{-5}$ (4) $5.0\text{x}10^{-6}$ (5) $8.7\text{x}10^{-6}$

S: At point P, vertical component of the net magnetic field vanishes. Horizontal component of the net magnetic field is

$$B = 2\frac{\mu_0 i}{2\pi r}cos30° = 2\frac{4\pi\times10^{-7}\times2}{2\pi\times4\times10^{-2}}cos30° = 1.7\times10^{-5}T.$$

B points toward left.

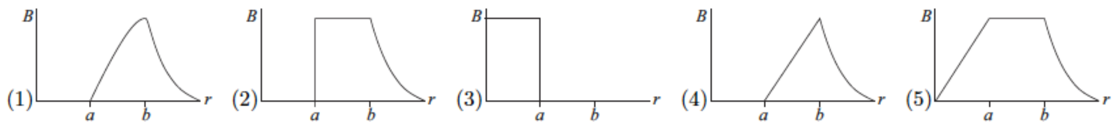
15. The magnetic field at any point is given by $\vec{B} = A\vec{r}\times\hat{k}$ where $\vec{r}$ is the position vector of the point and A is a constant. The net current through a circle of radius R, in the xy plane and centered at the origin is given by:

(1) $2\pi AR^2/\mu_0$ (2) $2\pi AR/\mu_0$ (3) $4\pi AR^3/3\mu_0$ (4) $\pi AR^2/\mu_0$ (5) $\pi AR^2/2\mu_0$

S: Consider circle of radius R. The magnetic field at any point on the circle is tangent to the circle with a magnitude AR. Use Ampere's law

$$\mu_0 i_{enc} = \oint \vec{B} \cdot d\vec{s} = \oint ARds = AR \oint ds = A2\pi R^2$$

16. A hollow cylindrical conductor (inner radius = *a*, outer radius = *b*) carries a current *i* uniformly spread over its cross section. Which graph below correctly gives *B* as a function of the distance *r* from the center of the cylinder?



S: Use Ampere's law and consider circle with radius r. B=0 for r<a and $B = \frac{\mu_0 i}{2\pi r}$ for r>b. For a<r<b

$$B = \frac{\mu_0 i_{enc}}{2\pi r} = \frac{\mu_0 i}{2\pi r} \cdot \frac{r^2 - a^2}{b^2 - a^2}$$

It is not a constant or a straight line.

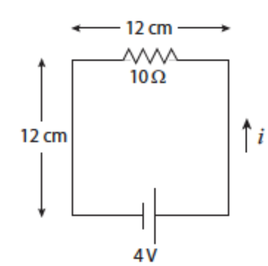
17. A loop of current-carrying wire has a magnetic dipole moment of 5×10^{-4} A · m^2. The moment initially is aligned with a 0.5-T magnetic field. To rotate the loop so its dipole moment is perpendicular to the field and hold it in that orientation, you must do work of:

(1) $2.5\text{x}10^{-4}$ J (2) 0 (3) $-2.5\text{x}10^{-4}$ J (4) $1.0\text{x}10^{-3}$ J (5) $-1.0\text{x}10^{-3}$ J

S: Applied work is equal to the change of potential energy

$$W_a = U_f - U_i = -\mu B cos\theta_f + \mu B cos\theta_i = 0 + 5\times10^{-4}\times0.5 = 2.5\times10^{-4}\,J$$

18. The circuit shown is in a uniform magnetic field that is into the page. The current in the circuit is 0.20 A. At what rate is the magnitude of the magnetic field changing: Is it increasing or decreasing?:



(1) 140 T/s, decreasing
(2) zero
(3) 140 T/s, increasing
(4) 420 T/s, decreasing
(5) 420 T/s, increasing

S: Induced emf is 2V in the clockwise direction. From Faraday's law

$$|\epsilon| = A\frac{dB}{dt};\ \frac{dB}{dt} = \frac{|\epsilon|}{A} = \frac{2}{(0.12)^2} = 144\ T/s$$

B is decreasing from Lenz's law.

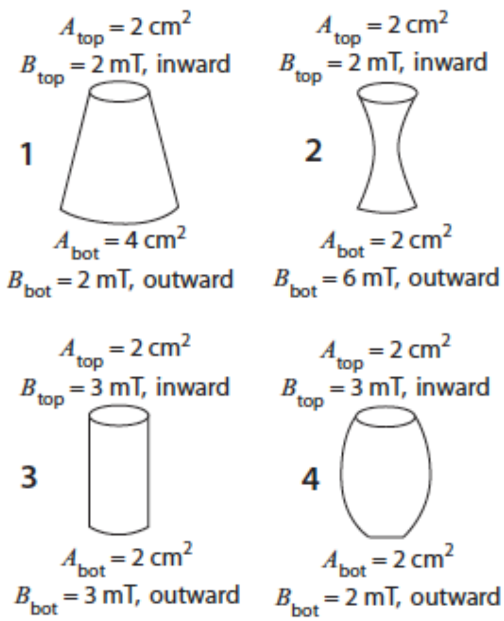
19. The total energy in an *LC* circuit is 5.0×10^{-6} J. If $L = 25$ mH and C = 15 μF the maximum current is:

(1) 20 mA (2) 14 mA (3) 10 mA (4) 28 mA (5) 40 mA

S: Maximum current occurs when all energy of the circuit is store in the inductor.

$$U = \frac{1}{2}Li^2;\ i = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2\times 5.0\times 10^{-6}}{25\times 10^{-3}}} = 2\times 10^{-2} A = 20\ mA$$

20. Four closed surfaces are shown. The areas A_{top} and A_{bot} of the top and bottom faces and the magnitudes B_{top} and B_{bot} of the uniform magnetic fields through the top and bottom faces are given. The fields are perpendicular to the faces and are either inward or outward. Rank the surfaces according to the magnitude of the magnetic flux through the curved sides, least to greatest.



(1) 3, 4, 1, 2 (2) 1, 2, 3, 4 (3) 1, 2, 4, 3 (4) 4, 3, 2, 1 (5) 2, 1, 4, 3

S: Use Gauss' law for magnetic field, magnetic flux through the sides is

$\int_s \vec{B} \cdot d\vec{A} = -\int_t \vec{B} \cdot d\vec{A} - \int_b \vec{B} \cdot d\vec{A}$.

We get -4, -8, 0 and 2 ($\times 10^{-7} Wb$) for 1, 2, 3 and 4 respectively. Their magnitudes are 4, 8, 0, and 2 ($\times 10^{-7} Wb$).