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Examining How Students With Diverse Abilities Use Diagrams to Solve Mathematics Word Problems

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Abstract

This study examined students' understanding of diagrams and their use of diagrams as tools to solve mathematical word problems. Students with learning disabilities (LD), typically achieving students, and gifted students in Grades 4 through 7 (*N* = 95) participated. Students were presented with novel mathematical word problem-solving tasks and interviewed for their understanding of diagrams for solving problems. Scoring rubrics were designed to assess for levels of problem-solving performance, evidence of diagram use, type of diagram generated, how the diagram was used to solve problems, their definition of a diagram, and their reason for why a diagram can be used to solve problems. Results indicated that students with LD typically generated diagrams of a poorer nature and used them in a less strategic manner than their peers. Concerns regarding the impact of content knowledge on the quality of diagram generation as well as implications regarding instruction to use diagrams to understand and solve problems are discussed.

Keywords

special education, learning disabilities, representation, mathematics word problems, diagrams

Strategic competence is one of the many critical components necessary for students to be successful in mathematics. Broadly, strategic competence is the ability to "formulate mathematical problems, represent them, and solve them" (National Research Council, 2001, p. 124). This involves both the knowledge of strategies including representations that may be used to solve a problem and the ability to effectively and efficiently use strategies while flexibly switching between strategies in response to the demands of a problem situation (National Research Council, 2001). Central to strategic competence are representations.

While there are various definitions of a representation (e.g., Goldin, 2003; Kaput, 1987; Pimm, 1995), for this study, a *representation* is considered to be "a combination of something written on paper, something existing in the form of physical objects and a carefully constructed arrangement of idea in one's mind" (Davis, Young, & McLaughlin, 1982, as cited in Smith, 2003, p. 266). Clearly, many different representational forms (e.g., mental image, written language, oral language, action movements, symbols, manipulatives) exist (Zawojewski & Lesh, 2003). A critical idea about representations, however, is that they are not static end products but rather tools for cognitive activity (Pape & Tchoshanov, 2001). As the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 2000) noted, a representation refers "both to process and to product . . . to the act of capturing a mathematical concept or relationship in some form and to the form itself" (p. 67). Therefore, when viewed as a tool of cognitive activity for solving mathematical problems, representations can be used for analyzing problems and planning solutions, justifying and explaining actions, predicting consequences, monitoring and evaluating progress, and integrating and communicating results in forms that are useful to others (Pape & Tchoshanov, 2001).

Diagrams and Mathematical Word Problem Solving

Although all representational systems are important for developing mathematical understanding (Owens & Clements, 1998; Pape & Tchoshanov, 2001; Presmeg, 1986) and various representational systems can be used to solve word problems, the focus of this study is specifically on self-generated diagrams. According to Diezmann and English (2001), a

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diagram is "a visual representation that displays information in a spatial layout" (p. 77). It has been suggested that diagrams are powerful strategies to use in solving word problems because they can be used to unpack the structure of a problem, simplify a complex problem, and/or make abstract concepts concrete (Diezmann & English, 2001; Novick, Hurley, & Francis, 1999). Interestingly, the strategy of "draw a diagram" to solve word problems has been strongly advocated by many researchers (Diezmann, 2000; NCTM, 2000; Shigematsu & Sowder, 1994), particularly as it can be used for many different problem types involving various mathematical areas (van Garderen, 2007).

The use of diagrams can be an extremely powerful strategy for solving word problems; however, it is possible that a self-generated diagram can become a stumbling block, thus interfering with the solution process and resulting in poor problem-solving performance (Diezmann, 2000; Larkin & Simon, 1987). For example, Hegarty and Kozhenvikov (1999) correlated lower problem-solving performance with self-generated diagrams that were pictorial in nature. While poor conceptual understanding of mathematics has been cited as a reason for interfering with performance (e.g., Dufour-Janvier, Bednarz, & Belanger, 1987; van Garderen, 2007), poor performance may also be the result of difficulty with the representation. Thus, a representation can only be useful to the extent that it has been "grasped" by the child (Dufour-Janvier et al., 1987).

Representational Ability and Problem Solving

Although representations are important tools for students to use (Pape & Tchoshanov, 2001), without an adequate representational ability, the tools have limited usefulness. As Lesh and Zawojewski (2007) noted, good representational ability is critical to "enhancing the communication capability and conceptual flexibility that are important to the development of solutions to many real-life problem-solving situations" (pp. 791–792). Furthermore, representational ability has been linked as a mechanism for cognitive growth; if interactions with representations are poor, slower rates of learning and development are to be expected (Shafrir, 1999).

Representational ability, or "meta-representational competence" (diSessa & Sherin, 2000), is student knowledge *about* representations, including the "ability to select, produce and productively use representations but also the abilities to critique and modify representations and even to design completely new representations" (p. 386) within the context of a problem-solving situation. Clearly, the knowledge required regarding representations is extensive (Dufour-Janvier et al., 1987). Not only do students need to have a general knowledge base on representations (e.g., know that it should depict the relevant problem information), but they also need to have specific knowledge about each type of representation (e.g., several different diagrams, such as networks, matrices, and part–whole, exist, and each form carries its own individual distinguishing properties; Diezmann & English, 2001; Lehrer & Schauble, 2000; Novick et al., 1999; van Garderen, 2007).

Few studies have specifically focused on using diagrams with K–12 students in mathematics. Typically, the focus of these studies has been on either (a) the type or nature of diagrams students generate to solve various problem types (e.g., Hegarty & Kozhevnikov, 1999; Presmeg, 1986) or (b) the difficulties and challenges students experience when generating and using diagrams to solve problems (Diezmann, 2000). Research focusing specifically on students with learning disabilities (LD) and their representational abilities for using diagrams when solving mathematics problems is much more limited.

Representational Ability of Students With LD for Solving Word Problems

In investigations of students with LD in which strategies to solve mathematic word problems were used, the most salient difficulty identified involved the use of problem-representation strategies (Montague, 1997; Montague & Applegate, 1993a, 1993b). Two research studies provided findings related to strategic use of representations between students with LD and their peers. First, students with LD infrequently used representation strategies when solving word problems; rather, they often used solution strategies (e.g., computation, trial-and-error; Montague, 1997). Second, they often had difficulty translating linguistic information into an appropriate representation (Montague, 1997; van Garderen & Montague, 2003).

To date, only one study has focused specifically on how students with LD use diagrams. In their study, van Garderen and Montague (2003) examined how Grade 6 students used diagrams to solve nonroutine word problems. Two key findings emerged from this study. First, students with LD used significantly fewer visual images as a strategy to solve word problems than did the gifted students. Second, based on Hegarty and Kozhevnikov's (1999) classification scheme, it was found that students with LD used pictorial representations (images that primarily depict the visual appearance of the objects or persons described in the problem) significantly more frequently than the gifted students. In contrast, the gifted students used schematic representations (images that depict the spatial relations described in a problem) significantly more often than students with LD. These findings are of concern because the use of schematic imagery was positively correlated, whereas pictorial imagery was negatively correlated, with problem-solving performance.

Although interesting findings emerged from the van Garderen and Montague (2003) study, much more research is needed. First, this study focused on only Grade 6 students. Second, the researchers addressed only a small component of metarepresentational competence, specifically, the quality (in this case *type*) of diagram produced. The study did not examine what a student understood about a diagram and why a diagram is a useful strategy for solving word problems or how well a diagram was used as a strategy (e.g., to organize the information in the problem, to self-monitor) as they were solving the word problems.

Purpose of the Current Study

Research has documented that students with LD fail to use representations, including diagrams, in powerful and productive ways (Montague, 1997; Montague & Applegate, 1993a, 1993b; Montague, Bos, & Doucette, 1991; van Garderen & Montague, 2003); however, less clear is what may be contributing to these difficulties and why the students struggle to develop adequate representational abilities. This is of concern given that poor representational ability can hinder problem-solving performance and could potentially limit mathematical learning (National Research Council, 2001). Furthermore, researchers have suggested that the use of diagrams may provide additional advantages in addressing the characteristic learning difficulties that students with LD experience (Jones, Wilson, & Bhojwani, 1998; Montague, 2007; Swanson & Jerman, 2006). Specifically, they are a way to create a "visible" platform to self-monitor, examine progress, and increase motivation (van Garderen, 2007; van Garderen & Montague, 2003).

As a way to begin to understand what may be contributing to this difficulty, we selected as the purpose of this study to examine what both students with and without LD understand regarding diagrams and how they use diagrams as tools to solve mathematics word problems. To do this, the following questions were asked:

1. What type of diagrams do students generate to solve mathematics word problems?
2. How well are students using diagrams to solve mathematics word problems?
3. What do students understand about diagrams and their use for solving mathematics word problems?
4. What relationships exist between (a) diagram type and use, and student understanding of diagrams and (b) performance in solving mathematics word problems?

Method

Participants

A total of 95 students in 10 elementary and middle schools in Ohio and Missouri participated in this study. Of the 95 participants, 29 were in Grade 4, 28 were in Grade 5, 13 were in Grade 6, and 25 were in Grade 7. English was the primary language for all students, as determined by school records. Teachers distributed consent forms to students, and every student who returned a signed form and who met eligibility criteria participated.

Students represented three levels of mathematical ability: students with LD, typically achieving (TA) students, and high-achieving (HA) students. Students with LD had to have a Full-Scale IQ score of 80 or more on the *Wechsler Intelligence Scale for Children, Fourth Edition* (WISC-IV; Wechsler, 2003) as well as meet their local district eligibility criteria. To be classified as HA in mathematics, the students had to have a scale score of 14 or above on two or more subtests (Numeration, Applied Problem Solving, Addition and Subtraction, *or* Multiplication and Division) given from the *KeyMath3 Diagnostic Assessment* (KeyMath3; Connolly, 2007; see Note 1). TA students were not identified as having a LD and did not meet the criteria to be considered HA in mathematics. An ANOVA test was conducted comparing the scale scores for each of the four subtests on the KeyMath3 for the three levels of ability. A significant difference was found on all four subtests: Numeration, $F(2, 95) = 71.3$, $p < .00$; Addition and Subtraction, $F(2, 95) = 54.6$, $p < .00$; Multiplication and Division, $F(2, 95) = 47.3$, $p < .00$; and Applied Problem Solving, $F(2, 95) = 88.7$, $p < .00$. Follow-up tests using Tukey HSD were conducted to evaluate differences among the groups. The results indicated statistically significant differences among all three ability levels. The HA students outperformed both TA students and students with LD (HA > TA, HA > LD), and TA students outperformed students with LD (TA > LD) on all four subtests. Participant demographic data are provided in Table 1. Means and standard deviations for the KeyMath3 are given in Table 2.

Measures

Student use of diagrams and problem-solving performance was examined through a researcher-developed measure: *Nonroutine Word Problem Assessment* (NWPA). For each grade level, the measure consisted of eight mathematical problems that were taken from the *Mathematical Processing Instrument* (Hegarty & Kozhevnikov, 1999) and/or *Techniques of Problem Solving,* Problem Decks AA and D (Greenes, Immerzeel, Ockenga, Schulman, & Spungin, 1980). Each grade-level measure was divided into two sets (no-prompt and prompted), where each set contained two problems at grade level, one problem below grade level, and one above grade level. Where possible, problems were used on multiple measures (e.g., a problem one grade level below for the Grade 5 measure was also used in the Grade 4 measure).

To ensure validity (content and appropriate grade level) of the problems, two researchers in mathematics examined the problems. Based on their expert opinion, several problems were realigned for grade level. Following the changes, the experts reexamined the problems and confirmed the

Table 1. Demographics of Participants by Grade.

	Grade				
Variable	4 (*n* = 29)	5 (*n* = 28)	6 (*n* = 13)	7 (*n* = 25)	Overall (*N* = 95)
Age					
M (in years)	9.62	10.68	11.85	12.64	11.03
SD	0.56	0.61	0.56	0.57	1.33
Gender					
Male	13 (44.8%)	12 (42.9%)	4 (30.8%)	14 (56.0%)	43 (45.3%)
Female	16 (55.2%)	16 (57.1%)	9 (69.2%)	11 (44.0%)	52 (54.7%)
Ethnicity					
White	21 (72.4%)	24 (85.7%)	12 (92.3%)	19 (76.0%)	76 (80.0%)
African American	4 (13.8%)	4 (14.3%)	—	3 (12.0%)	11 (11.6%)
Hispanic	3 (10.3%)	—	—	1 (4.0%)	4 (4.2%)
Asian	1 (3.4%)	—	1 (7.7%)	1 (4.0%)	3 (3.2%)
Other	—	—	—	1 (4.0%)	1 (1.1%)
Free and/or reduced lunch					
Yes	14 (48.3%)	9 (32.1%)	4 (30.8%)	9 (36.0%)	36 (37.9%)
No	15 (51.7%)	19 (67.9%)	9 (69.2%)	16 (64.0%)	59 (62.1%)
Number of students each ability group					
LD	7 (24.1%)	3 (10.7%)	—	6 (24.0%)	16 (16.8%)
TA	13 (44.8%)	17 (60.7%)	8 (61.5%)	15 (60.0%)	53 (55.8%)
HA	9 (31.0%)	8 (28.6%)	5 (38.5%)	4 (16.0%)	26 (27.4%)
WISC-IV for students with LD[a]					
M	96.29	95.67	—	90.67	94.06
SD	9.03	2.89	—	8.59	8.11

Note: LD = learning disability; TA = typically achieving; HA = high achieving; WISC-IV = Wechsler Intelligence Scale for Children, Fourth Edition (Wechsler, 2003).
[a]*M* = 100, *SD* = 15.

Table 2. Means and Standard Deviations for KeyMath3 Subtests by Ability Group.

	Ability group			
Keymath3[a]	LD (*n* = 16)	TA (*n* = 53)	HA (*n* = 26)	Overall (*N* = 95)
Numeration				
M	6.81	10.58	14.23	10.95
SD	1.87	2.16	1.63	3.13
Addition and subtraction				
M	6.19	9.47	13.35	9.98
SD	2.34	2.09	2.40	3.25
Multiplication and division				
M	6.19	9.21	12.38	9.57
SD	1.56	1.88	2.58	2.89
Applied problem solving				
M	6.63	10.87	15.58	11.44
SD	1.82	2.53	1.42	3.67

Note: LD = learning disability; TA = typically achieving; HA = high achieving.
[a]Means calculated on scale score (*M* = 10, *SD* = 3).

final grade-level placement of the problems. The problems were then randomly assigned to a set (no-prompt or prompted). Although eight problems (four per set) were given to all students, only six problems were analyzed. Due to a transcription error and poor item reliability discovered after the measure had been administered, one problem was removed from the measure. To maintain measure integrity, the equivalent problem from the second set was also removed (e.g., if the problem was designated below-grade level in no-prompt, the equivalent below-grade level problem in prompted was removed). The final result was six problems (three per set) to be analyzed in this study. Cronbach's alphas for all items by grade were adequate (Grade 4 = .88, Grade 5 = .69, Grade 6 = .66, Grade 7 = .79).

The problems were printed on the top of a page, with room for the student to show his or her work. After each problem was solved, the examiner asked the student the following questions:

1. "Tell me how you solved this problem." *If the child could not solve the problem*, "What did you do to try to answer the problem?"
2. *After the no-prompt problems*, "Why did you solve the problem this way?" *After the prompted problems*, "In what way did you use a picture to solve this word problem?"

3. *When a diagram was generated*, "How did your picture help you solve the problem?"

If necessary, follow-up probes were asked to elicit further information for each question. For example, if the student in response to Question 1 stated, "That's the way I learned it," he or she was then asked, "How did you learn it?" or "What did you learn?"

Procedures

All measures were individually administered during two sessions, each 40 to 60 minutes long. During the first session, the KeyMath3 subtests were administered. The NWPA was administered during the second session. All problems were read to the students to control for reading achievement differences among the groups. All interview responses were audiotaped. Each student's sequence of actions was recorded in a researcher booklet similar to the student measure booklets. For the no-prompt problems of the NWPA, the students were simply instructed to solve the problems to the best of their ability. The following statement was read to the students to explain the goals of that portion of the interview:

> I am interested in how children think while solving mathematical problems. I am going to ask you to solve some math problems. I will read each problem to you and then I want you to solve them as best as you are able to. If you want me to re-read some or the entire word problem, let me know and I will read it for you. You may solve these problems on the paper that I have given to you. I will ask you several questions after you solve each problem. There are no wrong or right answers to my questions. I am interested only in how you solved the problems.

After the student solved the no-prompt problems, they were then asked the following questions: "In mathematics, what is a diagram?" "In mathematics, why would you use a diagram?"

For the prompted problems, the students were encouraged to use a picture/diagram to solve the problems. The statement above explaining the goals of the interview were reread to the students, with the following additional instructions inserted midway through: "As you solve these problems, you must use a picture or diagram. Use the space under the problem to draw your picture or diagram." Each time the students were presented with a problem, they were reminded to draw a diagram/picture as they solved the problem. If, after three problems, a student had not generated any diagrams, he or she was shown two word problems (not from the NWPA) that contained sample diagrams. Each word problem was read to the student, and the examiner pointed to the diagram, stating, "Here is a diagram a student drew to answer the problem." No details regarding when or why the student drew the diagram were provided. Because of time restrictions, the students were instructed to redo only the third problem and then complete the final problem.

Scoring of the NWPA and Interview Questions

For the word problems on the NWPA (student work, transcribed interview responses, and documented actions), six different scores were generated. To ensure reliability of each score, both the primary author of the study and a doctoral graduate student coded 23% to 41.1% of all participants' responses. Interrater agreement was determined by taking the number of agreements divided by the number of agreements plus disagreements. Disagreements were resolved through discussion.

Problem-solving performance. The first score generated was a performance score. For each problem, the student's solution and interview response was given a score of 0 to 4, for a total possible score of 24. The response was coded as a 0 if the solution was incorrect and no mathematical understanding of the problem was evident. A score of 1 was given if the solution was incorrect and demonstrated limited understanding (e.g., a main idea identified from the problem) or if the solution was correct but contained no evidence of mathematical understanding (e.g., got the answer correct by accident). A score of 2 was given if the answer was incorrect but mathematical understanding beyond one main idea was identified when attempting to solve the problem (e.g., able to identify a series of appropriate mathematical procedures but did not complete the process). A score of 3 was given if the solution was incorrect due to a minor calculation error (e.g., $6 \times 7 = 43$) but the student used a correct problem-solving process. A score of 4 was given for both a correct solution and an explanation that demonstrated understanding of the problem (see Figure 1 for samples). Interrater reliability for the performance score was 80% based on 39 (41.1%) of the participants.

Evidence of diagram use. The second score was the total number of times a student used a diagram to solve a word problem, and it had a total possible score of 6. Interrater reliability for the evidence of diagram use was 99.1% based on 26 (27.4%) of the participants.

Nature of the diagram generated. The third and fourth scores were the total number of pictorial and schematic diagrams generated, respectively, by each student. This coding was based on Hegarty and Kozhevnikov's (1999) and van Garderen and Montague's (2003) studies. These scores were generated using the student work samples and the interview responses. A diagram was scored as primarily pictorial if the student drew an image of objects or persons referred to in the problem and described the image as information in the corresponding interview (see Figure 2 for an

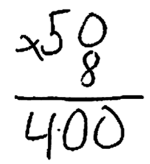
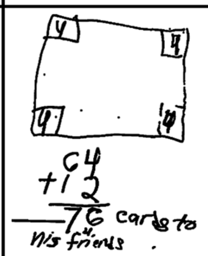
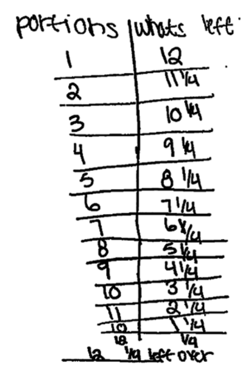
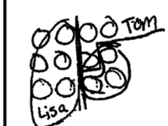
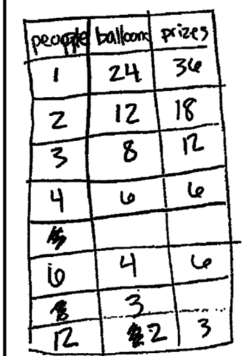
Score	0	1	2	3	4
Problem	Max bought 50 plants for his garden. He plans to put 8 plants in each row. How many complete rows of 8 plants can he make?	Greg had 64 baseball cards. He gave 12 cards to his sister. Then he divided the remaining cards equally among his four friends. How many cards did each of his friends get?	Adams Middle School students made an Italian submarine sandwich that was 12 ¾ feet long. After making it, they decided to divide the sandwich into smaller portions to share with other students. If each portion was ¾ of a foot long, how many students would get a portion?	There were 12 marbles on the floor. Lisa picked up ½ of the 12 marbles and Tom picked up ¼ of the 12 marbles. How many marbles were picked up?	Meredith is planning a Halloween party. She has 36 prizes and 24 balloons. What is the most number of children she can invite so each child gets an equal number of prizes and an equal number of balloons? She does not want any prizes or balloons left over.
Diagram/Work					
Transcript	S: 400 I: Tell me how you solved this problem. S: I did times, because 50 times 8 adds up to 400. I: Why did you solve it this way? S: Because it was the easiest for me. I: How is it the easiest? S: Because I'm really good at times.	I: Tell me how you solved this problem. S: I add. I: What did you add? S: 64 plus 12. I: Uh huh, well in what way did you use your picture to solve this problem? S: I used a table and I used a table graph I put 4 because I drew the friends in there and then 76 cards to his friends. I didn't draw 76.	I: Tell me how you solved this one. S: Since there's ¾ for every serving, I knew to take one off, there would be one portion, and then there would be 12. I thought ¼ minus ¾ would be negative ¼, but since I had more whole numbers, I just went down and subtracted ¾ from every time. Then I got 12 portions, but with ¼ leftover.	S: I drew the 12 marbles and I knew that Lisa picked up half, and I knew half of 12 was 6, so I cut it in half, and I wrote on one side Lisa. Then I knew that Tom picked up a quarter, or a fourth, and I knew that ¼ of 12 was 3, so I put 3 right there, and I put Tom, and the rest left were 3 marbles, so I put 3 marbles left.	S: Well, 24 and 26 are both even, so I started by making a chart with one person and how many balloons they would get and how many prizes they would get. And then 2, I divided them both by 2, and then 3, I divided them both by 3. And since 5 doesn't into even numbers, I skipped 5. Actually since 5 doesn't go into 24 and 26 I skipped 5. And then I basically just did the common factors of the balloons and prizes until I got to 12.

Figure 1. Sample items scored for problem-solving performance.

example). A diagram was scored as primarily schematic if the student drew the objects or persons referred to in the problem and depicted or explained some relationship expressed in the problem (see Figure 2 for an example). Diagrams were coded as either schematic or pictorial simultaneously; therefore, interrater reliability for the number of pictorial and schematic diagrams generated was 96% for 25 (27%) of the students who drew diagrams (n = 93; 2 students did not draw any diagrams).

Diagram use to solve problems. The fifth and sixth scores indicated the number of strategic ways the diagram was used and the quality of diagram use to solve word problems.

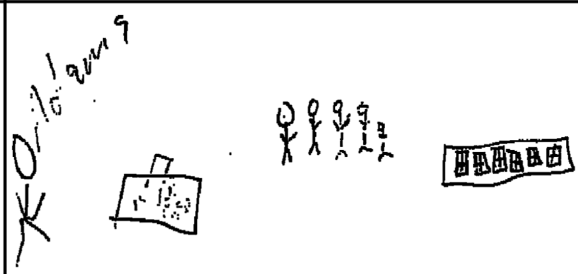
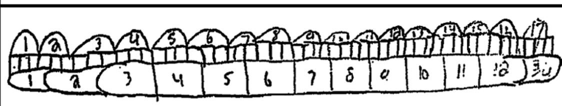
Problem	Adams Middle School students made an Italian submarine sandwich that was 12 ¾ feet long. After making it, they decided to divide the sandwich into smaller portions to share with other students. If each portion was ¾ of a foot long, how many students would get a portion?	
Pictorial		I: All right, how did you get that answer? Tell me about that. S: I don't know. I: Where did the 10 come from? S: People. I: How did you use this picture then to help solve it? Tell me about that. S: I don't know. I: Tell me about what's this and what's this. S: Those are the students and those are the sandwiches. I: OK, and then what were you counting to get to 10? S: I just guessed on the 10.
Schematic		I: How did you solve this one? S: I drew the sandwich. Then I divided it into 12 ¾ feet long. I drew little lines above it and counted all the little fourths, all the way up to 17 sets of 3's.

Figure 2. Sample items scored as pictorial and schematic for nature of the diagram.

To generate these scores, each diagram was examined to determine the way in which the student used it while solving the word problems. A list of initial codes was generated from descriptions in the literature (i.e., NCTM, 2000; Pape & Tchoshanov, 2001) on the different ways a diagram can be used when solving problems (e.g., organize, plan, monitor, compute, justify). The lead researcher and a graduate student then examined the data to find examples of each code. Based on this initial analysis, the data were categorized into five codes: (a) *image only,* where the information from the problem was depicted; (b) *organizer* where the data or information was depicted to help understand the problem; however, the calculation of the solution was conducted independent of the diagram; (c) *tracking technique* to keep count of objects or track computation while solving (e.g., tally marks); (d) *execute a solution,* including the computation; and (e) *check/monitor* work to verify whether the answer was correct or incorrect and/or to check the reasonableness of an answer.

To generate the scores, the codes were grouped into three main levels representing the three main components of problem solving (Mayer & Hegarty, 1996; Montague, 1997, 2007): *problem representation*—image only *or* organizer; *problem solution*—tracking technique *or* execute a solution; and *self-regulation*—check/monitor work. For the first two levels, only one of the two codes could be applied. See Figure 3 for a sample of each coding. Interrater reliability for the number of strategic ways a diagram was used and quality of diagram use was 80.8% based on 25% of the total number of diagrams generated ($n = 575$).

The fifth score—frequency-of-use—was generated by calculating the total number of ways each diagram was used (i.e., a combined score of the number of times a diagram was used to track, to organize, to check, etc.) when solving the problems. A higher score reflects a greater number of ways the student used diagrams to solve the word problems.

Although diagrams might have been used in a number of different ways to solve a problem, this does not necessarily mean they were used in a "quality" manner. Therefore, a sixth score that focused on the "quality" of diagram usage (i.e., poorer application vs. stronger application) was generated. To determine this, each way a diagram was used was given a value (i.e., image only and tracking technique = 0.5; organizer, execute a solution, and check/monitor = 1). For each participant, this resulted in three possible points for each diagram generated. Because not every student drew the same number of diagrams, these points were then divided by the total possible points (number of diagrams drawn × 3). This resulted in a weighted score for each student that could then be compared with others.

Diagram definition and reason for using. Two scores were generated from the interview questions regarding what a diagram is and why a diagram is used in mathematics. First, each response to "What a diagram is" was given a score of 0, 1, or 2. A response was scored as 0 if the student did not know what a diagram was or the answer did not relate in

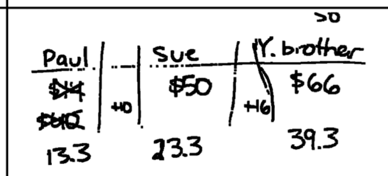
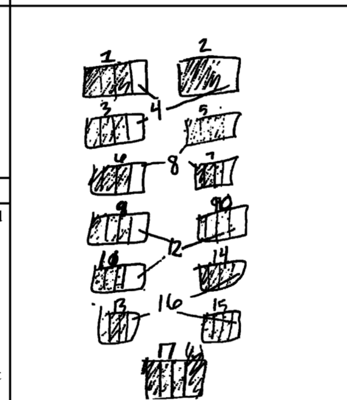
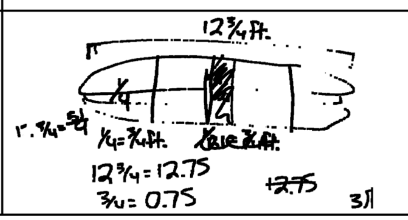
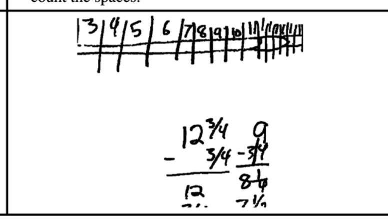
Code	Image	Organizer	Execute a Solution
Transcript	I: OK, so tell me about your picture here, what did you draw and why? S: I don't know, I was bored. I: You were bored and that's why you drew all that stuff? S: Yes. I: Did any of it help you to solve the problem? S: Not really.	I: Did you find it helpful to draw a table? S: Yes, because first I wrote this, but it wouldn't make as much sense, because I'd have to think of the connections back in my brain. But here, it's already written out. It organizes the data.	I: Talk to me about how you solved that problem. S: I drew a rectangle to represent each sandwich. I kind of divided the sandwich up into 12, and then one ¾. Then I divided each one into fourths, and counted the ¾. Then I saw a pattern that every three ¾, you get another fourth. So I just kept doing that until I got all of them.
Diagram			
Code	**Check/Monitor**	**Tracking Technique**	
Transcript	I: How was it helpful? S: I think it was helpful, but it's possible I could have made a mistake. I might have gotten the number wrong, and then it wouldn't make sense anymore	S: I drew a sandwich and cut of ¾ at a time and subtracted it from 12 and ¾. I: In what way did you use the picture to solve the problem? S: After I subtracted all the ¾ and whenever I got to 0 I counted them up. I: So did you find it helpful to draw this picture? S: Yes. I: How did it help you? S: Instead of having to count all of these numbers you just count the spaces.	
Diagram			

Figure 3. Sample items for each code for how diagrams are used.

any way to a diagram. A response was given a score of 1 if the student primarily believed a diagram to be a picture or way to display data. A response was scored as 2 if the student primarily believed a diagram involved a schema for solving (e.g., demonstrate a schematic use of the data to solve the problem). Interrater reliability for what a diagram is was 87.5% based on 24 (25%) of the participants.

Second, the response to why one should use a diagram to solve a word problem was assigned a score between 0 and 4. This overall score was generated with 1 point being assigned for each reference to the following: (a) tracking technique, (b) organizer, (c) check/monitor work, or (d) execute a solution. Interrater reliability for why a diagram is used was 76%, based on 32 (34%) of the total number of participants.

Data Analyses

To control for differences in grade-level performance, parametric statistics were used for all analyses (see Note 2). Where appropriate, one-way ANCOVA tests were used to detect differences across the three ability levels (HA, TA, LD) for all problems, as well as no-prompt and prompted problems. Where a statistically significant difference was detected, follow-up one-way ANOVA tests were conducted to evaluate pairwise differences among the groups, controlling for Type 1 error using the Bonferroni method (p value of .017 or less [.05/3]). To detect within-group (HA, TA, and LD) differences between scores on no-prompt and prompted problems, paired-sample t tests were used. Finally, Pearson product–moment correlation coefficients were used to determine relationships between variables examined and problem-solving performance on the NWPA.

Results

Research Question 1: Type of Diagrams Generated

To answer the question "What type of diagrams do students generate to solve mathematics word problems?" the dependent variables of (a) evidence for diagram use and (b) nature of the diagram use were analyzed across the ability levels as well as within each level.

Table 3. Means and Standard Deviations for Various Scores Generated From the NWPA by Ability Group.

	Ability group							
	Overall (N = 95)		LD (n = 16)		TA (n = 53)		HA (n = 26)	
Variable	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Problem-solving performance								
Total score (max 24)	14.24	6.73	5.81	5.01	14.11	5.74	19.69	3.21
No-prompt problems (max 12)	6.49	3.72	1.88	2.42	6.47	3.09	9.38	2.50
Prompted problems (max 12)	7.75	3.73	3.94	3.47	7.64	3.48	10.31	1.93
Evidence of diagram								
Total number (max 6)	3.43	1.24	3.31	0.70	3.34	1.41	3.69	1.12
No-prompt problems (max 3	0.81	1.01	0.50	0.89	0.83	1.07	0.96	0.96
Prompted problems (max 3)	2.62	0.69	2.81	0.40	2.51	0.72	2.73	0.72
Nature of the diagram								
Total pictorial (max 6)	0.46	0.85	0.94	1.29	0.40	0.74	0.31	0.62
Total schematic (max 6)	2.97	1.56	2.38	1.67	2.94	1.60	3.38	1.33
No-prompt pictorial (max 3)	0.05	0.27	0.00	0.00	0.08	0.33	0.04	0.20
Prompted pictorial (max 3)	0.41	0.82	0.94	1.29	0.32	0.70	0.27	0.53
No-prompt schematic (max 3)	0.76	0.98	0.50	0.89	0.75	1.02	0.93	0.93
Prompted schematic (max 3)	2.21	1.01	1.88	1.26	2.19	0.94	2.46	0.95
Diagram use to solve problems								
Total strategic use (max 18)[a]	6.63	2.85	5.69	2.39	6.52	3.01	7.48	2.65
No-prompt strategic use (max 9)[b]	3.58	1.79	3.00	2.00	3.72	1.97	3.53	1.46
Prompted strategic use (max 9)[a]	5.09	1.56	4.40	2.07	5.20	1.22	5.13	1.92
Total quality of use (max 1)[a]	0.58	0.17	0.50	0.21	0.59	0.15	0.62	0.15
No-prompt quality of use (max 1)[b]	0.65	0.15	0.57	0.22	0.65	0.16	0.68	0.12
Prompted quality of use (max 1)[a]	0.61	0.17	0.57	0.19	0.63	0.14	0.58	0.21
Diagram definition and reason for using								
What a diagram is (max 2)	1.09	0.60	0.69	0.60	1.26	0.62	1.00	0.40
Why use a diagram (max 4)	0.99	0.47	0.88	0.50	0.96	0.39	1.12	0.59

Note: NWPA = Nonroutine Word Problem Assessment; LD = learning disability; TA = typically achieving; HA = high achieving.
[a]Overall: n = 93, LD: n = 16, TA: n = 52, HA: n = 25.
[b]Overall: n = 45, LD: n = 5, TA: n = 25, HA: n = 15.

Type of Diagrams Generated Across Ability Levels

Evidence of diagram use. Overall, the average number of times a diagram was used to solve the six problems was 3.43 (SD = 1.24). Two students out of the 95 did not generate any diagrams despite prompting to use a diagram. The number of diagrams generated for all problems across the three ability groups was not statistically significant, ANCOVA $F(2, 91) = 0.44$, $p = .643$. Means and standard deviations are found in Table 3.

Nature of the diagram. Overall ($n = 93$), the average number of pictorial diagrams generated was 0.46 ($SD = 0.85$) whereas the average number of schematic diagrams generated was 2.97 (SD = 1.56). Statistically significant differences were found among the three ability levels for the number of pictorial diagrams generated, $F(2, 91) = 3.37$, $p = .039$, but not for schematic diagrams, $F(2, 91) = 2.22$, $p = .115$. Using the Bonferroni procedure to adjust the significance level (p = .017), follow-up (ANOVA) tests revealed no statistically significant differences among the three levels for the number of pictorial diagrams generated: TA to HA, $F(1, 91) = 0.049$, $p = .826$; TA to LD, $F(1, 91) = 5.86$, $p = .018$; and HA to LD, $F(1, 91) = 5.48$, $p = .21$.

Additional ANCOVA tests were conducted to detect differences in the number of pictorial and schematic diagrams generated among the ability groups for both no-prompt and prompted problems. No statistically significant differences were found among the three levels for the types of diagrams generated on the no-prompt problems: pictorial, $F(2, 91) = 0.48$, $p = .622$; schematic, $F(2, 91) = 1.13$, $p = .327$; or the number of schematic diagrams produced on the prompted problems, $F(2, 91) = 1.60$, $p = .207$. However, there was a statistically significant difference among the levels in the number of pictorial diagrams generated when prompted, $F(2, 91) = 4.47$, $p = .014$. Follow-up (ANOVA) tests on the pictorial diagrams produced when prompted revealed no statistically significant difference between HA and TA, $F(1, 91) = 0.004$, $p = .952$, but students with LD produced statistically significantly more pictorial diagrams when prompted than TA, $F(1, 91) = 8.15$, $p = .005$, and HA, $F(1, 91) = 6.80$, $p = .011$.

Type of Diagram Generated Within Each Ability Level

Evidence for diagram use. Overall ($N = 95$), there was a statistically significant difference between the number of diagrams used for the no-prompt problems compared with the prompted problems, $t(94) = -14.64$, $p < .001$. The average number of diagrams generated was lower for no-prompt problems ($M = 0.81$, $SD = 1.01$) than prompted problems ($M = 2.62$, $SD = 0.69$). A statistically significant difference was found for all ability levels: LD, $t(15) = -7.74$, $p < .001$; TA, $t(52) = -10.57$, $p < .001$; HA, $t(25) = -7.08$, $p < .001$, with all students using more diagrams for the prompted problems. Means and standard deviations are presented in Table 3.

Nature of the diagram. The sample as a whole ($N = 95$) used statistically significantly more schematic diagrams than pictorial diagrams, $t(94) = -11.19$, $p < .001$, to solve word problems. Based on the ability level of the group, a statistically significant difference was found for TA, $t(52) = -9.03$, $p < .001$, and HA, $t(25) = -9.01$, $p < .001$, with both groups generating, on average, more schematic diagrams than pictorial diagrams, but no statistically significant difference in diagrams generated for students with LD, $t(15) = -1.98$, $p = .066$. Means and standard deviations are presented in Table 3.

Further analysis examined the difference between pictorial and schematic diagrams produced on no-prompt and prompted problems. A statistically significant difference was found in the number of schematic diagrams, $t(94) = -11.54$, $p < .001$, and pictorial diagrams, $t(94) = -3.99$, $p < .001$, generated by all students for both no-prompt and prompted problems. Statistically significant differences were found for all ability levels for both schematic diagrams—LD, $t(15) = -3.91$, $p = .001$; TA, $t(52) = -9.20$, $p < .001$; HA, $t(25) = -5.88$, $p < .001$—and pictorial diagrams—LD, $t(15) = -2.91$, $p = .011$; TA, $t(52) = -2.21$, $p = .031$; HA, $t(25) = -2.29$, $p = .031$. For all the levels, the students generated more schematic and pictorial diagrams for the prompted problems than the no-prompt problems. Means and standard deviations are presented in Table 3.

Research Question 2: Diagram Use

To answer the question, "How well are students using diagrams to solve mathematics word problems?" the dependent variables of problem-solving performance on the NWPA, strategic diagram use, and quality of diagram use were analyzed.

Diagram Use Across Ability Levels

Performance on the NWPA. Overall, on the NWPA, the students received an average performance score of 14.24 (out of a possible 24 for six problems; $SD = 6.73$). Performance scores were found to be statistically significant, $F(2, 91) = 38.47$, $p < .001$. Follow-up (ANOVA) tests indicated statistically significant differences among all three levels of ability. Students in the HA group outperformed both the TA, $F(1, 91) = 19.51$, $p < .001$, and LD, $F(1, 91) = 76.93$, $p < .001$, groups. In addition, the TA group outperformed the LD group, $F(1, 91) = 36.47$, $p < .001$. The average score on the no-prompt problems was 6.49 ($SD = 3.72$), whereas the prompted problems average score was 7.75 ($SD = 3.73$). Performance scores for the three ability levels were found to be statistically significant for both sets of problems: no-prompt, $F(2, 91) = 34.14$, $p < .001$; prompted, $F(2, 91) = 22.08$, $p < .001$. Follow-up ANOVA tests indicated consistent statistically significant differences among all three levels of ability, regardless of the prompt, with the HA group outperforming the TA—no-prompt, $F(1, 91) = 17.42$, $p < .001$, and prompted, $F(1, 91) = 11.12$, $p = .001$—and LD—no-prompt, $F(1, 91) = 68.28$, $p < .001$, and prompted, $F(1, 91) = 44.15$, $p < .001$, groups; and the TA group outperforming the LD group—no-prompt, $F(1, 91) = 32.25$, $p < .001$, and prompted, $F(1, 91) = 21.02$, $p < .001$. Means and standard deviations are presented in Table 3.

Diagram use to solve problems. How diagrams were used when solving the problems was examined in two ways: (a) number of strategic ways the diagram was used and (b) the quality of diagram use for the total number generated. Overall ($n = 93$), the average strategic score was 6.63 out of 18 possible points ($SD = 2.86$) for all problems and 3.58 ($SD = 1.79$) and 4.88 ($SD = 1.60$) for the 9 possible points for no-prompt and prompted problems, respectively. The average weighted score for quality of use with a possible score of 1.00 was 0.58 ($SD = 0.17$) for all problems, 0.65 ($SD = 0.15$) for no-prompt, and 0.57 ($SD = 0.18$) for prompted.

For strategic use, on average, no statistically significant difference was found among the three ability levels for (a) the number of strategic ways the diagrams were used, $F(2, 89) = 1.97$, $p = .146$, or (b) regarding the quality of diagram use, $F(2, 89) = 2.67$, $p = .075$. Taken together, the findings suggest that all students, regardless of ability level, appeared to use diagrams in a similar manner; however, the means for the students with LD consistently lagged behind those of their peers for both variables. Means and standard deviations are presented in Table 3.

Diagram Use Within Each Ability Level

Performance on the NWPA. Overall ($N = 95$), there was a statistically significant difference between the scores on the no-prompt problems compared with the prompted problems, $t(94) = -3.83$, $p < .001$. The average score was lower for the no-prompt ($M = 6.49$, $SD = 3.72$) compared with the prompted ($M = 7.75$, $SD = 3.73$) problems.

A statistically significant difference was found for the LD, $t(15) = -2.52$, $p = .024$, and TA, $t(52) = -2.65$, $p = .011$, groups. In both cases, the students performed higher on the prompted problems. No statistically significant difference was found between the two sets of problems for the HA group, $t(25) = -1.51$, $p = .143$. Means and standard deviations are presented in Table 3.

Diagram use to solve problems. Across all ability levels, there was a statistically significant difference between the number of strategic uses of a diagram for the no-prompt and prompted problems, $t(44) = -4.52$, $p < .001$. The average number of strategic uses was higher for the prompted ($M = 5.01$, $SD = 1.56$) than the no-prompt ($M = 3.58$, $SD = 1.79$) problems. Within each group, a statistically significant difference in the strategic use of a diagram for no-prompt and prompted problems was found for the TA group, $t(24) = -3.34$, $p = .003$, and HA group, $t(14) = -2.67$, $p = .018$, but not for the LD group, $t(4) = -1.20$, $p = .296$. For the TA and HA students, the average strategic score was higher on the prompted problem than the no-prompted problems.

However, no statistically significant difference was found for the quality of diagram use when comparing the no-prompt with the prompted problems, $t(44) = 1.34$, $p = .186$). Interestingly, the quality score was higher ($M = 0.65$, $SD = 0.15$) for the no-prompt problems than the prompted problems ($M = 0.61$, $SD = 0.17$). A statistically significant difference for quality of diagrams was found for the HA group, $t(14) = 2.27$, $p = .040$, but not for the TA, $t(24) = 0.36$, $p = .723$, or LD, $t(4) = -0.06$, $p = .957$, groups. For all but the students with LD, the average quality score was higher for the no-prompt problems than the prompted problems, suggesting that TA and HA students used the diagrams in a more quality manner when not prompted. The average quality score for students with LD was the same in both situations. Means and standard deviations are found in Table 3.

Research Question 3: Definition and Reason for Using a Diagram

To answer the question, "What do students understand about diagrams and their use for solving mathematics word problems?" the following dependent variables, definition, and reason for use scores were analyzed.

What is a diagram? The average definition score (2 = highest score) for all the students ($N = 95$) was 1.09 ($SD = 0.60$). A statistically significant difference was found among the three ability groups, $F(2, 91) = 6.50$, $p = .002$. Follow-up ANOVA tests among the three groups indicated no significant difference between the HA and LD groups, $F(1, 91) = 3.16$, $p = .079$, or between the HA and TA groups, $F(1, 91) = 3.25$, $p = .075$. A statistically significant difference was found between the TA and LD groups, $F(1, 91) = 12.26$, $p = .001$, with the TA students outperforming the LD students in their ability to define a diagram. Means and standard deviations are presented in Table 3.

Why use a diagram? Overall ($N = 95$), the average reason-for-use score (4 = highest score) was 0.99 ($SD = 0.47$), with no statistically significant differences among the three ability levels, $F(2, 91) = 1.88$, $p = .159$. Means and standard deviations are noted in Table 3.

Research Question 4: Diagram Variables and Their Relationship to Performance on NWPA

To answer the question, "What relationships exist between diagram type, use, and student understanding of diagrams to performance for solving mathematics word problems?" Pearson product–moment correlations were computed between variables related to diagram evidence, nature, use (strategic and quality), definition and reason for using, and mathematic problem-solving performance on the NWPA. Overall and ability level results for the correlations can be found in Table 4.

Across all problems. First, a greater use of diagrams was positively and significantly correlated to higher performance on the NWPA, $r(95) = .36$, $p < .001$. Although not statistically significant for all groups, the pattern was similar to the overall correlation—LD, $r(16) = .40$, $p = .123$; TA, $r(53) = .38$, $p = .005$; HA, $r(26) = .49$, $p = .010$. Second, diagrams that were primarily pictorial were negatively and statistically significantly correlated to problem-solving performance, $r(95) = -.31$, $p = .002$. Conversely, diagrams that were primarily schematic were positively and statistically significantly correlated to problem-solving performance, $r(93) = .45$, $p < .001$. This pattern was evident across all ability levels for both diagrams that were pictorial—LD, $r(16) = -.72$, $p = .002$; TA, $r(53) = -.05$, $p = .715$; HA, $r(26) = -.03$, $p = .880$—and schematic—LD, $r(16) = .73$, $p = .001$; TA, $r(53) = .36$, $p = .008$; HA, $r(26) = .43$, $p = .027$. The strategic ways in which a diagram was used and the quality of the diagram use were positively and significantly correlated to higher performance on the NWPA: strategic, $r(93) = .41$, $p < .001$, and quality, $r(93) = .34$, $p = .001$. This pattern was also evident across ability levels for both strategic use—LD, $r(16) = .64$, $p = .007$; TA, $r(52) = .34$, $p = .014$; HA, $r(25) = .30$, $p = .144$—and quality—LD, $r(16) = .65$, $p = .006$; TA, $r(52) = .17$, $p = .236$; HA, $r(25) = .06$, $p = .77$. Fourth, a positive and statistically significant correlation was found between performance on the NWPA and the reason for using a diagram, $r(95) = .24$, $p = .018$;

Table 4. Diagram Variables and Relationship to Mathematical Problem Solving for Type of Problem on the NWPA by Ability Level.

Variable	LD (n = 16)	TA (n = 53)	HA (n = 26)	Overall (N = 95)
All six problems				
Evidence of diagram	.40	.38*	.49*	.36**
Nature of diagram				
Pictorial	−.72*	−.05	−.03	−.31*
Schematic	.73**	.36*	.43*	.45**
Diagram use to solve problems[a]				
Strategic use	.64*	.34*	.30	.41**
Quality use	.65*	.17	.06	.34**
Definition and reason for using				
What a diagram is	.33	.07	−.34	.15
Why use a diagram	.47	.23	−.11	.24*
No-prompt problems				
Evidence of diagram	.49	.33*	.11	.32*
Nature of the diagram				
Pictorial	—	−.07	.21	.02
Schematic	.49	.37*	.07	.33**
Diagram use to solve problems[b]				
Strategic use	.21	.48*	.04	.30*
Quality of use	.70	.17	.35	.36*
Prompted problems				
Evidence of diagram	−.01	.16	.29	.11
Nature of the diagram				
Pictorial	−.67*	−.20	−.12	−.40**
Schematic	.69*	.27	.29	.40**
Diagram use to solve problems[a]				
Strategic use	.48	.15	−.32	.18*
Quality of use	.59*	.20	−.28	.26*

Note: NWPA = Nonroutine Word Problem Assessment; LD = learning disability; TA = typically achieving; HA = high achieving.
[a]For variables related to diagram use, TA n = 52, HA n = 25, and overall n = 93.
[b]For variables related to diagram use, LD n = 5, TA n = 25, HA n = 15, and overall n = 45.
*$p < .05$ (two-tailed). **$p < .001$ (two-tailed).

however, a positive correlation was found only between diagram definition and performance on NWPA, $r(95) = .15$, $p = .150$. This pattern for both definition and reason, however, was only consistent for LD—what, $r(16) = .33$, $p = .208$, and why, $r(16) = .47$, $p = .067$—and TA—what, $r(53) = .07$, $p = .636$, and why, $r(53) = .23$, $p = .105$. Interestingly, negative correlations for students in the HA group and their definition and reason for use and performance on the NWPA were found: what, $r(26) = −.34$, $p = .087$, and why, $r(26) = −.11$, $p = .601$. See Table 4 for all the correlations.

No-prompt and prompted problems. Overall, the pattern of correlations, although not statistically significant in all cases, reflects the pattern of correlations discussed for all six problems. For the correlations by ability, the pattern is similar with three interesting opposite pattern exceptions. First, for the HA group on the no-prompt problems, a positive correlation was found between pictorial diagrams and performance on the NWPA, $r(26) = .21$, $p = .295$. Second, for students overall on the no-prompt problems, a positive correlation was found for pictorial diagrams and performance on the NWPA, $r(95) = .02$, $p = .875$. Third, for HA students on the prompted problems, negative correlations for performance on the NWPA in both strategic use, $r(25) = −.32$, $p = .123$, and quality of use, $r(25) = −.28$, $p = .172$, were found. All correlations are shown in Table 4.

Discussion

Typically, students have some representational knowledge base for solving mathematic problems (diSessa & Sherin, 2000; diSessa, Hammer, Sherin, & Kolpakowski, 1991; Lehrer & Schauble, 2000). However, it appears that not all students have similar or even adequate representational abilities for solving word problems (e.g., van Garderen & Montague, 2003). Less understood are the factors that may contribute to the difficulties students, in particular students with LD, may experience when using diagrams. Therefore, in this study we examined understanding of diagrams and use of diagrams as a tool to solve mathematic word problems by students of diverse abilities. Based on the results from this study, three main findings regarding diagram use for solving mathematical word problems emerged.

Main Findings

First, the students with LD did not differ from their peers in terms of the average number of diagrams they used to solve the word problems. Like their peers, they were able to generate diagrams that were both pictorial and schematic. However, when prompted, the students with LD drew more pictorial and less schematic diagrams than their peers. This is of concern because the results of this study indicate that students are more likely to solve the problem correctly when using schematic diagrams as opposed to pictorial diagrams; this finding has also been supported in other research (Hegarty & Kozhevnikov, 1999; van Garderen & Montague, 2003). Furthermore, this finding reflects findings of other studies (e.g., Montague & Applegate, 1993a, 1993b; Montague et al., 1991) in that students with LD do not necessarily differ in terms of the number of strategies they use but rather in the way in which they use strategies.

Although several studies (e.g., Hegarty & Kozhevnikov, 1999; Presmeg, 1986) have suggested that diagrams representing quantitative relationships (or schematic information) are more likely to result in correct solutions, it is important to note that depicting the quantitative relationships from a mathematical problem requires some understanding of the mathematical concepts embedded within the problem (e.g., addition, subtraction). For example, when examining student difficulties with representations, Diezmann (2000) found that limited mathematical knowledge, in particular measurement and number sense, was a hindrance for solving the word problems. Likewise, Brown and Presmeg (1993) found that students with a stronger schematic understanding of mathematics typically generated images more schematic in nature. Therefore, it may be possible that the students with LD in this study had difficulty generating schematic diagrams because they lacked understanding of the mathematical concepts necessary to solve the problems. Certainly, their mathematical understanding was lower than that of their peers as was evident on performance scores for both the KeyMath3 and the NWPA.

Second, although not statistically significant, in this study the students with LD consistently lagged behind their peers in both the frequency of the ways in which they used diagrams as a strategy and, more important, in the quality of their diagram use when solving mathematical word problems. However, it should also be noted that all students, regardless of ability, appeared to use diagrams poorly with overall quality scores ranging from 0.50 to 0.62 (with possible score of 1). As previously noted, students with LD may differ from their peers strategically in the way they use strategies when solving word problems (Montague, 1997). The poorer use of diagrams may be considered problematic because, in general, the findings of this study suggest a relationship between better quality diagrams and higher levels of performance. However, this finding should be interpreted cautiously because the relationship between diagram quality and performance appears to be complex.

For example, in comparing the diagram quality scores from the no-prompt to the prompted problems, we found that, although not statistically significant in all cases, the quality scores were lower on the prompted problems for the TA and HA groups and remained the same for the LD group. However, the performance scores improved from the no-prompt to the prompted problems. This appears to contradict the overall finding of the relationship between better quality diagrams and higher performance. Although speculative, one possible explanation for the higher scores may be that because the students were prompted to draw a diagram, they were "forced" to examine the problem more carefully, resulting in more thought about the problem itself and higher performance scores. Another example of the complexity between use of diagrams and performance was the correlations for the prompted problems: Although not significant, the correlations between performance and quality or frequency of strategy use for the HA group were not positive. In other words, in some cases, better quality diagrams or higher frequency of diagram use did not necessarily mean higher levels of performance. This appears to contradict the overall findings but may suggest the influence of other factors (e.g., conceptual knowledge) on performance.

It is also interesting to note that simply prompting students to generate a diagram does not necessarily mean they will generate a quality diagram. For all ability levels, the diagram quality score remained the same or dropped from when the students were prompted. In other words, telling the students to draw a diagram did not improve the diagram quality. Possibly the students either knew what they were doing or they did not, and prompting them to use a diagram was not enough to change their performance. Clearly, the findings of this study and others (e.g., Hegarty & Kozhevnikov, 1999; van Garderen & Montague, 2003) suggest that some form of instruction on diagrams may be required.

Third, the LD group had a poorer definition of what a diagram is when compared with their peers. Even more disconcerting, however, were the low scores across all students for their reasons to use a diagram. Again, the LD group members had a lower average, although not statistically significant, score for their reasons for using a diagram than did their peers. These low scores may explain the low-quality scores as well. It appears that these students had a limited perspective on what a diagram is and, more particularly, the ways in which a diagram could be used, thereby restricting their application of a diagram when solving word problems. Furthermore, a higher score for definition and reason for use correlated with higher scores for performance, suggesting that the knowledge of appropriate terminology as it relates to the use of representations is important and should not be underestimated (Dufour-Janvier et al., 1987). Many mathematical tasks are dependent on having the necessary vocabulary to carry them out as well as an understanding of the mathematical concepts being promoted (Gough, 2007). Again, however, this needs to be interpreted cautiously because the relationships between the definition and reason for use did not always positively correlate with performance. Like quality of use and performance, the relationship between problem solving and representational knowledge is complex.

Limitations and Future Research

Although the findings of this study are interesting, they need to be interpreted with caution, given the limitations of this study. First, the number of students with LD in this study was small and included students identified as having general LD and not specific mathematical LD. Although the students with LD were not classified using the KeyMath3, they did, however, clearly demonstrate lower performance on all the KeyMath3 subtests and the NWPA, suggesting possession of poorer math skills than their peers. In addition, this study did not include enough low-achieving non–special education students (i.e., having a scaled score of 6 or lower on two or more of the KeyMath3 subtests) to compare their performance with that of students with LD. Future research with larger numbers of students with LD and low-achieving students is required. Second, the number of mathematical word problems presented to the students was limited. The problems were given in two sets: no-prompt, followed by prompted. We deliberately chose not to counterbalance the prompting because exposure to a prompt might have influenced the students to use a diagram more frequently. Additional studies might consider using a greater number of mathematical word problems, more problem types, and varied problem placement to further validate the findings of this study. Third, the data were collected in one-on-one situations and thus are not representative of the instructional context of a typical classroom. It would be interesting to examine how students use diagrams when solving mathematical tasks within the classroom and how that may differ from an individual setting. Fourth, this study focused on only one of the many representational forms—diagrams—used in classrooms today, thus limiting the generalizability of these findings. It may be possible that students with LD are more effective and efficient when using other representational forms. Alternatively, they may have just as many challenges, if not more, using other representations. Clearly, this requires further research.

Conclusion and Implications for Practice

An overarching concern of this study is to better understand why students, in particular students with LD, may struggle to develop adequate representational ability. This is important given the strong recommendations by many researchers in special education (e.g., Gersten et al., 2009) regarding use of representations within instruction. The findings of the study suggest that students with LD have a limited representational ability.

Learning mathematics occurs through problem solving that happens through creating and being involved in mathematics (Lesh & Zawojewski, 2007). Likewise, representational ability develops when students are actively involved in the production, inspection, and use of representations (diSessa & Sherin, 2000). It should not, however, be assumed that for all students, in particular students with LD, engagement alone with representations will necessarily result in the development of an adequate representational ability. Explicit instruction specifically focused on helping students develop their representational ability may be necessary. Based on the findings of this study, that instruction is needed to (a) help students better understand what a diagram is and the different ways in which it can be used to solve word problems, (b) construct diagrams that represent the schematic elements of the problems, and (c) use diagrams (e.g., to organize data, to check and monitor their work) while solving word problems.

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Notes

1. The internal-consistency reliability score for the Total Test for Grades 1 through 12 was in the mid-to-upper .90s, and construct validity for subtests correlated with the Total Test score generally surpassed .90 (Connolly, 2007).
2. Due to assumption violations (nonrandom selected sample), nonparametric statistics were also run, with similar results. However, the parametric statistics are reported because they controlled for the possible influence of grade.

References

Brown, D. L., & Presmeg, N. C. (1993). Types of imagery used by elementary and secondary school students in mathematical reasoning. *Proceedings of the 17th annual meeting of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 137–144). Tsukuba, Japan.

Connolly, A. J. (2007). *KeyMath-3 diagnostic assessment*. San Antonio, TX: Pearson.

Diezmann, C. M. (2000). Making sense with diagrams: Students' difficulties with feature-similar problems. *Proceedings of the 23rd annual conference of Mathematics Education Research Group of Australasia* (pp. 228–234). Freemantle, Australia: MERGA.

Diezmann, C. M., & English, L. D. (2001). Promoting the use of diagrams as tools for thinking. In A. A. Cuoco & F. R. Curcio (Eds.), *The roles of representation in school mathematics: 2001 yearbook* (pp. 77–89). Reston, VA: National Council of Teachers of Mathematics.

diSessa, A. A., Hammer, D., Sherin, B. L., & Kolpakowski, T. (1991). Inventing graphing: Metarepresentational expertise in children. *Journal of Mathematical Behavior, 10*, 117–160. doi:10.1016/S0732-3123(01)00051-7

diSessa, A. A., & Sherin, B. L. (2000). Meta-representation: An introduction. *Journal of Mathematical Behavior, 19*, 385–398.

Dufour-Janvier, B., Bednarz, N., & Belanger, M. (1987). Pedagogical considerations concerning the problem of representation. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 109–122). Hillsdale, NJ: Erlbaum.

Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, D. (2009). *Assisting students struggling with mathematics: Response to intervention for elementary and middle schools (NCEE2009-4060)*. Washington, DC: U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance. Retrieved from http://ies.ed.gov/ncee/wwc/publications/practiceguides

Goldin, G. A. (2003). Representation in school mathematics: A unifying research perspective. In J. Kilpartrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 275–285). Reston, VA: National Council of Teachers of Mathematics.

Gough, J. (2007). Teaching square roots: Conceptual complexity in mathematics language. *Australian Senior Mathematics Journal, 21*(1), 53–57.

Greenes, C. E., Immerzeel, G., Ockenga, E., Schulman, L., & Spungin, R. (1980). *Techniques of problem solving: Problem Deck AA*. Palo Alto, CA: Dale Seymour Publications.

Hegarty, M., & Kozhevnikov, M. (1999). Types of visual-spatial representations and mathematical problem solving. *Journal of Educational Psychology, 91*, 684–689. doi:0022-0663/99/S3.00

Jones, E. D., Wilson, R., & Bhojwani, S. (1998). Mathematics instruction for secondary students with learning disabilities. In D. Pedrotty Rivera (Ed.), *Mathematics education for students with learning disabilities: Theory to practice* (pp. 155–176). Austin, TX: PRO-ED.

Kaput, J. J. (1987). Representation systems and mathematics. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 19–26.). Hillsdale, NJ: Erlbaum.

Larkin, J. H., & Simon, H. A. (1987). Why a diagram is (sometimes) worth ten thousand words. *Cognitive Science, 11*, 65–99. doi:10.1016/S0364-0213(87)80026-5

Lehrer, R., & Schauble, L. (2000). Inventing data structures for representational purposes: Elementary grade students' classification models. *Mathematical Thinking and Learning, 2*, 49–72. doi:10.1207/S15327833MTL0202_3

Lesh, R., & Zawojewski, J. (2007). Problem solving and modeling. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 2, pp. 763–804). Charlotte, NC: Information Age Publishing.

Mayer, R. E., & Hegarty, M. (1996). The process of understanding mathematical problems. In R. J. Sternberg & T. Ben-Zeev (Eds.), *The nature of mathematical thinking* (pp. 29–53). Mahwah, NJ: Erlbaum.

Montague, M. (1997). Cognitive strategy instruction in mathematics for students with learning disabilities. *Journal of Learning Disabilities, 30*, 164–177. doi:10.1177/002221949703000204

Montague, M. (2007). Self-regulation and mathematics instruction. *Learning Disabilities Research & Practice, 22*, 75–83. doi:10.1111/j.1540-5826.2007.00232.x

Montague, M., & Applegate, B. (1993a). Mathematical problem-solving characteristics of middle school students with learning disabilities. *Journal of Special Education, 27*, 175–201. doi:10.1177/002246699302700203

Montague, M., & Applegate, B. (1993b). Middle school students' mathematical problem solving: An analysis of think-aloud protocols. *Learning Disability Quarterly, 16*, 19–32. doi:10.2307/1511157

Montague, M., Bos, C., & Doucette, M. (1991). Affective, cognitive, and metacognitive attributes of eight-grade mathematical problem solvers. *Learning Disabilities Research & Practice, 6*, 145–151.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Research Council. (2001). Adding it up: Helping children learn mathematics. In J. Kilpatrick, J. Swafford, & B. Findell (Eds.), *Mathematics learning study committee, center for education, division of behavioral and social sciences and education*. Washington, DC: National Academy Press.

Novick, L. R., Hurley, S. M., & Francis, M. (1999). Evidence for abstract, schematic knowledge of three spatial diagram representations. *Memory & Cognition, 27*, 288–308. doi:10.3758/BF03211413

Owens, K. D., & Clements, M. A. K. (1998). Representations in spatial problem solving in the classroom. *Journal of Mathematical Behavior, 17*, 197–218.

Pape, S. J., & Tchoshanov, M. A. (2001). The role of representation(s) in developing mathematical understanding. *Theory Into Practice, 40*, 118–127. doi:10.1207/s15430421tip4002_6

Pimm, D. (1995). *Symbols and meanings in school mathematics*. New York, NY: Routledge.

Presmeg, N. C. (1986). Visualization in high school mathematics. *For the Learning of Mathematics, 6*, 42–46. doi:10.1007/BF00305075

Shafrir, U. (1999). Representational competence. In I. E. Siegel (Ed.), *Development of mental representation: Theories and applications* (pp. 371–389). Mahwah, NJ: Erlbaum.

Shigematsu, K., & Sowder, L. (1994). Drawings for story problems: Practices in Japan and the United States. *Arithmetic Teachers, 41*, 544–547.

Smith, S. P. (2003). Representation in school mathematics: Children's representations of problems. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 263–274). Reston, VA: National Council of Teachers of Mathematics.

Swanson, H. L., & Jerman, O. (2006). Math disabilities: A selective meta-analysis of the literature. *Review of Educational Research, 76*, 249–274. doi:10.3102/00346543076002249

van Garderen, D. (2007). Teaching students with LD to use diagrams to solve mathematical word problems. *Journal of Learning Disabilities, 40*, 540–553. doi:10.1177/00222194070400060501

van Garderen, D., & Montague, M. (2003). Visual-spatial representation, mathematical problem solving, and students of varying abilities. *Learning Disabilities Research & Practice, 18*, 246–254. doi:10.1111/1540-5826.00079

Wechsler, D. (2003). *Wechsler Intelligence Scale for Children* (4th ed.). San Antonio, TX: Pearson Assessment.

Zawojewski, J. S., & Lesh, R. (2003). A models and modeling perspective on problem solving. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism, models and modeling perspectives on mathematics problem solving* (pp. 317–328). Mahwah, NJ: Erlbaum.