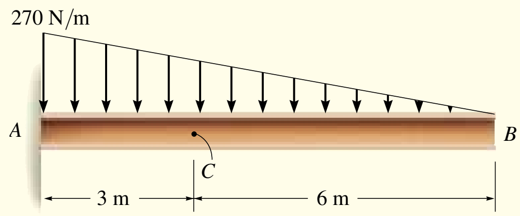
EXAMPLE 1.1

Determine the resultant internal loadings acting on the cross section at C of the beam shown in Fig. 1–4a.



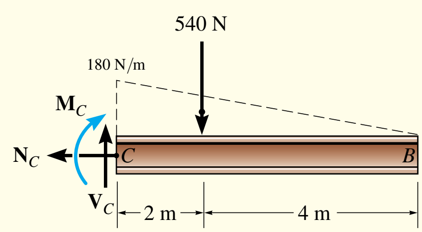
(a)

Fig. 1–4

Solution

Support Reactions. This problem can be solved in the most direct manner by considering segment CB of the beam, since then the support reactions at A do not have to be computed.

Free-Body Diagram. Passing an imaginary section perpendicular to the longitudinal axis of the beam yields the free-body diagram of segment CB shown in Fig. 1–4b. It is important to keep the distributed loading exactly where it is on the segment until *after* the section is made. Only then should this loading be replaced by a single resultant force. Notice that the intensity of the distributed loading at C is found by proportion, i.e., from Fig.1–4a, $w/6\text{ m} = (270\text{ N/m})/9\text{m}$, $w = 180\text{ N/m}$. The magnitude of the resultant of the distributed load is equal to the area under the loading curve (triangle) and acts throughthe centroid of this area. Thus, $F = \frac{1}{2}(180\text{ N/m})(6\text{ m}) = 540\text{ N}$, which acts $1/3(6\text{ m}) = 2\text{ m}$ from C as shown in Fig. 1–4b.



(b)

Equations of Equilibrium. Applying the equations of equilibrium we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -N_C = 0$$

$$N_C = 0 \qquad \textit{Ans.}$$

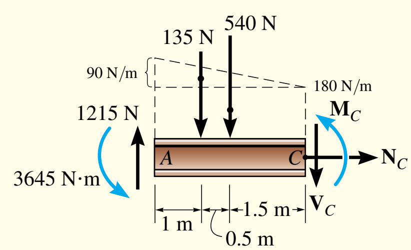
$$+\uparrow \Sigma F_y = 0; \qquad V_C - 540\text{ N} = 0$$

$$V_C = 540\text{ N} \qquad \textit{Ans.}$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad -M_C - 540\text{ N}(2\text{ m}) = 0$$

$$M_C = -1080\text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

The negative sign indicates that $\mathbf{M}_C$ acts in the opposite direction to that shown on the free-body diagram. Try solving this problem using segment AC, by first obtaining the support reactions at A, which are given in Fig. 1–4c.



(c)

EXAMPLE 1.2

Determine the resultant internal loadings acting on the cross section at C of the machine shaft shown in Fig. 1–5*a*. The shaft is supported by bearings at A and B, which exert only vertical forces on the shaft.

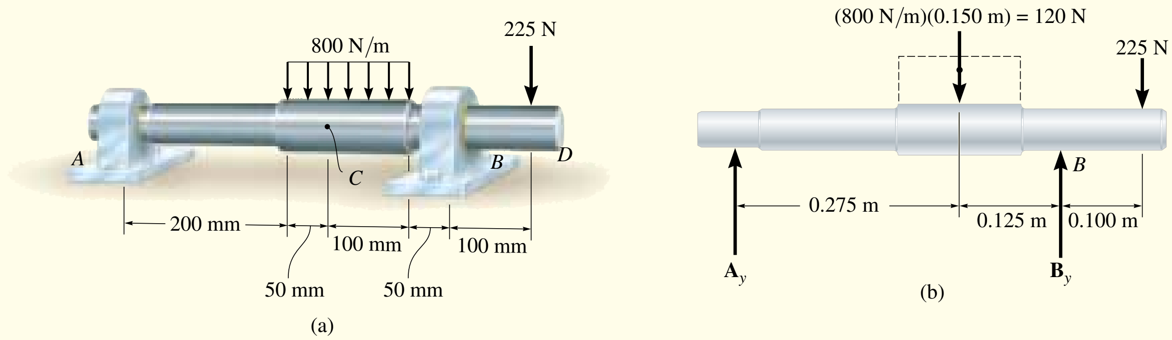


Fig. 1–5

Solution

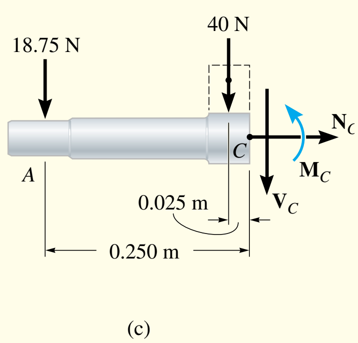
We will solve this problem using segment AC of the shaft.

Support Reactions. A free-body diagram of the entire shaft is shown in Fig. 1–5*b*. Since segment AC is to be considered, only the reaction at A has to be determined. Why?

$$\circlearrowleft + \Sigma M_B = 0; \quad -A_y(0.400 \text{ m}) + 120 \text{ N}(0.125 \text{ m}) - 225 \text{ N}(0.100 \text{ m}) = 0$$

$$A_y = -18.75 \text{ N}$$

The negative sign for A_y indicates that $\mathbf{A}_y$ acts in the *opposite sense* to that shown on the free-body diagram.



Free-Body Diagram. Passing an imaginary section perpendicular to the axis of the shaft through C yields the free-body diagram of segment AC shown in Fig. 1–5*c*.

Equations of Equilibrium.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N_C = 0 \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad -18.75 \text{ N} - 40 \text{ N} - V_C = 0$$

$$V_C = -58.8 \text{ N} \qquad \textit{Ans.}$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad M_C + 40 \text{ N}(0.025 \text{ m}) + 18.75 \text{ N}(0.250 \text{ m}) = 0$$

$$M_C = -5.69 \text{ N} \cdot \text{m} \qquad \textit{Ans.}$$

What do the negative signs for V_C and M_C indicate? As an exercise, calculate the reaction at B and try to obtain the same results using segment CBD of the shaft.

EXAMPLE 1.3

The hoist in Fig. 1–6*a* consists of the beam *AB* and attached pulleys, the cable, and the motor. Determine the resultant internal loadings acting on the cross section at *C* if the motor is lifting the 2000 N ($\approx$ 200 kg) load *W* with constant velocity. Neglect the weight of the pulleys and beam.

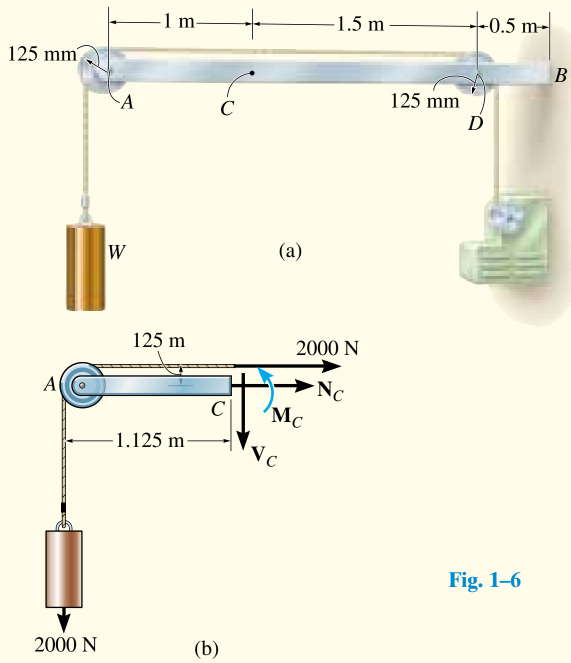


Fig. 1–6

Solution

The most direct way to solve this problem is to section both the cable and the beam at *C* and then consider the entire left segment.

Free-Body Diagram. See Fig. 1–6*b*.

Equations of Equilibrium.

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad 2000\text{ N} + N_C = 0 \qquad N_C = -2000\text{ N}$ *Ans.*

$+\uparrow \Sigma F_y = 0;\quad -2000\text{ N} - V_C = 0 \qquad V_C = -2000\text{ N}$ *Ans.*

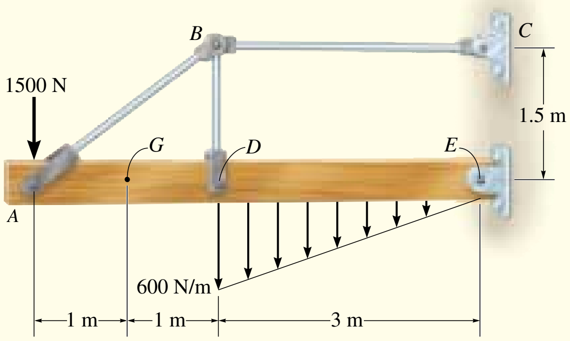
$\circlearrowleft + \Sigma M_C = 0;\quad 2000\text{ N}(1.125\text{ m}) - 2000\text{ N}(0.125\text{ m})\ 1\ M_C = 0$

$$M_C = -2000\text{ N}\cdot\text{m}$$ *Ans.*

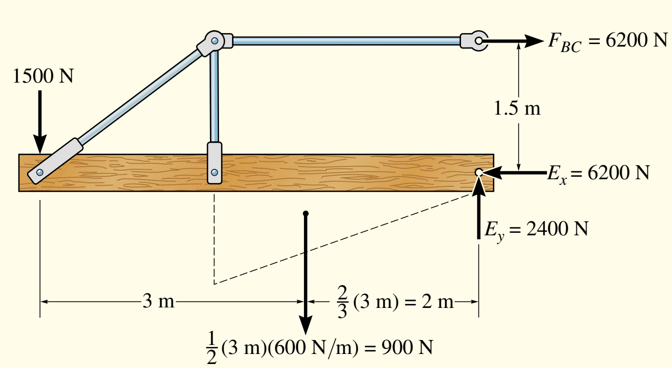
As an exercise, try obtaining these same results by considering just the beam segment *AC*, i.e., remove the pulley at *A* from the beam and show the 2000-N force components of the pulley acting on the beam segment *AC*. Also, this problem can be worked by first finding the reactions at B, ($B_x = 0$, $B_y = 4000$ N, $M_B = 7000$ N $\cdot$ m) and then considering segment CB.

EXAMPLE 1.4

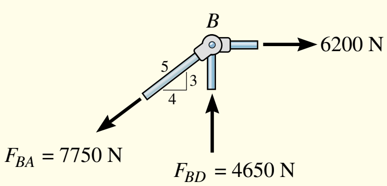
Determine the resultant internal loadings acting on the cross section at G of the wooden beam shown in Fig. 1–7*a*. Assume the joints at A, B, C, D, and E are pin connected.



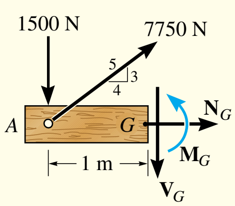
(a)



(b)



(c)



(d)

Fig. 1–7

Solution

Support Reactions. Here we will consider segment AG for the analysis. A free-body diagram of the *entire* structure is shown in Fig. 1–7*b*. Verify the computed reactions at E and C. In particular, note that BC is a *two-force member* since only two forces act on it. For this reason, the reaction at C must be horizontal as shown.

Since BA and BD are also two-force members, the free-body diagram of joint B is shown in Fig. 1–7*c*. Again, verify the magnitudes of the computed forces $\mathbf{F}_{BA}$ and $\mathbf{F}_{BD}$.

Free-Body Diagram. Using the result for $\mathbf{F}_{BA}$, the left section AG of the beam is shown in Fig. 1–7*d*.

Equations of Equilibrium. Applying the equations of equilibrium to segment AG, we have

$$\xrightarrow{+} \Sigma F_x = 0; \quad 7750\text{ N}\left(\tfrac{4}{5}\right) + N_G = 0 \quad N_G = -6200\text{ N} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad -1500\text{ N} + 7750\text{ N}\left(\tfrac{3}{5}\right) - V_G = 0$$

$$V_G = 3150\text{ N} \qquad \textit{Ans.}$$

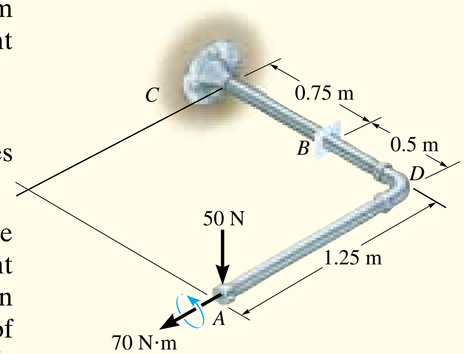
$$\circlearrowleft + \Sigma M_G = 0; \quad M_G - (7750\text{ N})\left(\tfrac{3}{5}\right)(1\text{ m}) + (1500\text{ N})(1\text{ m}) = 0$$

$$M_G = 3150\text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

As an exercise, compute these same results using segment GE.

EXAMPLE 1.5

Determine the resultant internal loadings acting on the cross section at B of the pipe shown in Fig. 1–8a. The pipe has a mass of 2 kg/m and is subjected to both a vertical force of 50 N and a couple moment of 70 N·m at its end A. It is fixed to the wall at C.



(a)

Solution

The problem can be solved by considering segment AB, which does *not* involve the support reactions at C.

Free-Body Diagram. The x, y, z axes are established at B and the free-body diagram of segment AB is shown in Fig. 1–8b. The resultant force and moment components at the section are assumed to act in the positive coordinate directions and to pass through the *centroid* of the cross-sectional area at B. The weight of each segment of pipe is calculated as follows:

$$W_{BD} = (2\text{ kg/m})(0.5\text{ m})(9.81\text{ N/kg}) = 9.81\text{ N}$$

$$W_{AD} = (2\text{ kg/m})(1.25\text{ m})(9.81\text{ N/kg}) = 24.525\text{ N}$$

These forces act through the center of gravity of each segment.

Equations of Equilibrium. Applying the six scalar equations of equilibrium, we have*

$\Sigma F_x = 0;\quad (F_B)_x = 0$ *Ans.*

$\Sigma F_y = 0;\quad (F_B)_y = 0$ *Ans.*

$\Sigma F_z = 0;\quad (F_B)_z - 9.81\text{ N} - 24.525\text{ N} - 50\text{ N} = 0$

$(F_B)_z = 84.3\text{ N}$ *Ans.*

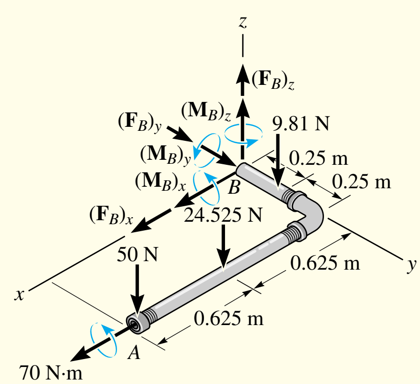
$\Sigma(M_B)_x = 0;\quad (M_B)_x + 70\text{ N}\cdot\text{m} - 50\text{ N}\,(0.5\text{ m}) - 24.525\text{ N}\,(0.5\text{ m}) - 9.81\text{ N}\,(0.25\text{ m}) = 0$

$(M_B)_x = -30.3\text{ N}\cdot\text{m}$ *Ans.*

$\Sigma(M_B)_y = 0;\quad (M_B)_y + 24.525\text{ N}\,(0.625\text{ m}) + 50\text{ N}\,(1.25\text{ m}) = 0$

$(M_B)_y = -77.8\text{ N}\cdot\text{m}$ *Ans.*

$\Sigma(M_B)_z = 0;\quad (M_B)_z = 0$ *Ans.*



(b)

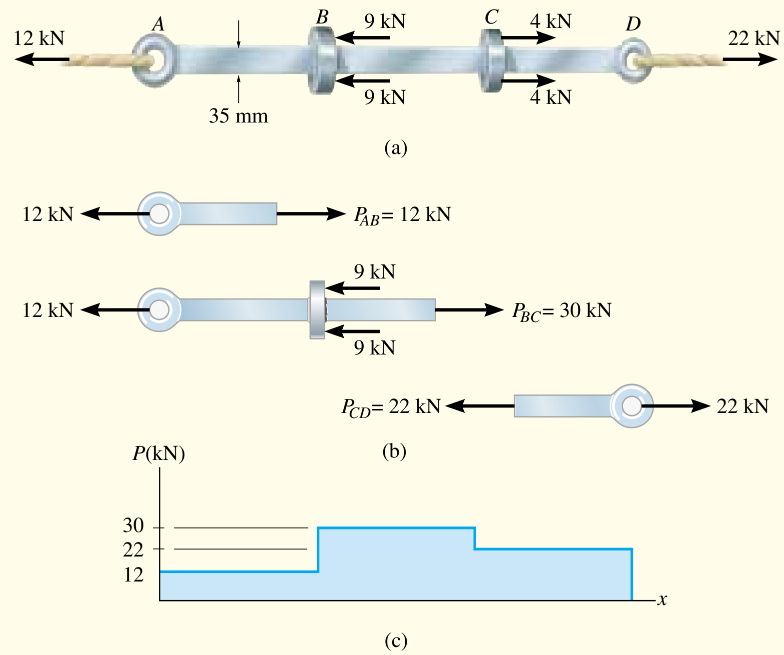
Fig. 1–8

What do the negative signs for $(M_B)_x$ and $(M_B)_y$ indicate? Note that the normal force $N_B = (F_B)_y = 0$, whereas the shear force is $V_B = \sqrt{(0)^2 + (84.3)^2} = 84.3$ N. Also, the torsional moment is $T_B = (M_B)_y = 77.8$ N·m and the bending moment is $M_B = \sqrt{(30.3)^2 + (0)} = 30.3$ N·m.

*The *magnitude* of each moment about an axis is equal to the magnitude of each force times the perpendicular distance from the axis to the line of action of the force. The *direction* of each moment is determined using the right-hand rule, with positive moments (thumb) directed along the positive coordinate axes.

EXAMPLE 1.6

The bar in Fig. 1–16*a* has a constant width of 35 mm and a thickness of 10 mm. Determine the maximum average normal stress in the bar when it is subjected to the loading shown.



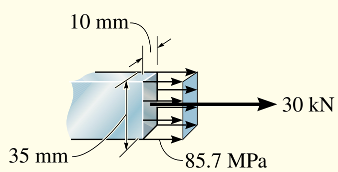
Solution

Internal Loading. By inspection, the internal axial forces in regions AB, BC, and CD are all constant yet have different magnitudes. Using the method of sections, these loadings are determined in Fig. 1–16*b*; and the normal force diagram which represents these results graphically is shown in Fig. 1–16*c*. By inspection, the largest loading is in region BC, where $P_{BC} = 30$ kN. Since the cross-sectional area of the bar is *constant*, the largest average normal stress also occurs within this region of the bar.

Average Normal Stress. Applying Eq. 1–6, we have

$$\sigma_{BC} = \frac{P_{BC}}{A} = \frac{30(10^3)\text{N}}{(0.035\text{ m})(0.010\text{ m})} = 85.7\text{ MPa} \qquad \textit{Ans.}$$

The stress distribution acting on an arbitrary cross section of the bar within region BC is shown in Fig. 1–16*d*. Graphically the *volume* (or "block") represented by this distribution of stress is equivalent to the load of 30 kN; that is, 30 kN = (85.7 MPa)(35 mm)(10 mm).



(d)

Fig. 1–16

EXAMPLE 1.7

The 80-kg lamp is supported by two rods AB and BC as shown in Fig. 1–17*a*. If AB has a diameter of 10 mm and BC has a diameter of 8 mm, determine the average normal stress in each rod.

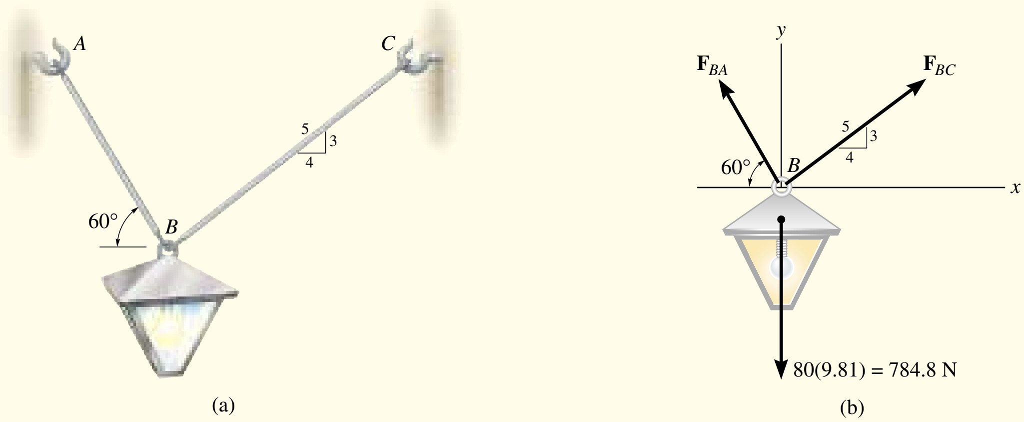


Fig. 1–17

Solution

Internal Loading. We must first determine the axial force in each rod. A free-body diagram of the lamp is shown in Fig. 1–17*b*. Applying the equations of force equilibrium yields

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{BC}\left(\tfrac{4}{5}\right) - F_{BA}\cos 60^\circ = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC}\left(\tfrac{3}{5}\right) + F_{BA}\sin 60^\circ - 784.8\text{ N} = 0$$

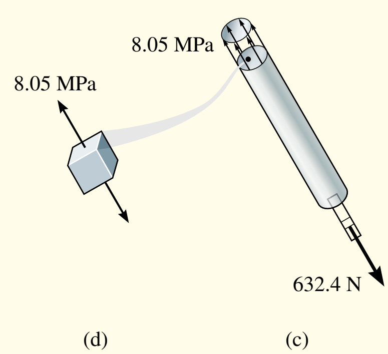
$$F_{BC} = 395.2\text{ N}, \qquad F_{BA} = 632.4\text{ N}$$

By Newton's third law of action, equal but opposite reaction, these forces subject the rods to tension throughout their length.

Average Normal Stress. Applying Eq. 1–6, we have

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{395.2\text{ N}}{\pi(0.004\text{ m})^2} = 7.86\text{ MPa} \qquad \textit{Ans.}$$

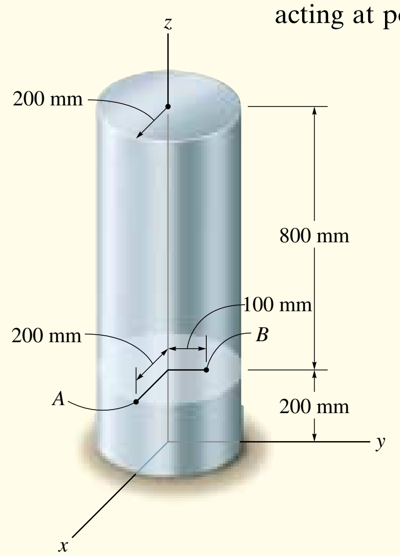
$$\sigma_{BA} = \frac{F_{BA}}{A_{BA}} = \frac{632.4\text{ N}}{\pi(0.005\text{ m})^2} = 8.05\text{ MPa} \qquad \textit{Ans.}$$



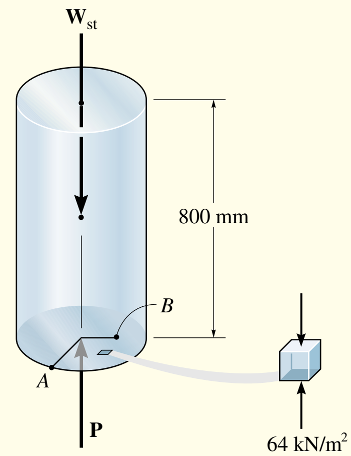
The average normal stress distribution acting over a cross section of rod AB is shown in Fig. 1–17*c*, and at a point on this cross section, an element of material is stressed as shown in Fig. 1–17*d*.

EXAMPLE 1.8

The casting shown in Fig. 1–18*a* is made of steel having a specific weight of $\gamma_{st} = 80\ \text{kN/m}^3$. Determine the average compressive stress acting at points A and B.



(a)



(b) (c)

Fig. 1–18

Solution

Internal Loading. A free-body diagram of the top segment of the casting where the section passes through points A and B is shown in Fig. 1–18*b*. The weight of this segment is determined from $W_{st} = \gamma_{st}V_{st}$. Thus the internal axial force P at the section is

$$+\uparrow \Sigma F_z = 0; \qquad P - W_{st} = 0$$

$$P - (80\ \text{kN/m}^3)(0.8\ \text{m})\pi(0.2\ \text{m})^2 = 0$$

$$P = 8.042\ \text{kN}$$

Average Compressive Stress. The cross-sectional area at the section is $A = \pi(0.2\ \text{m})^2$, and so the average compressive stress becomes

$$\sigma = \frac{P}{A} = \frac{8.042\ \text{kN}}{\pi(0.2\ \text{m})^2}$$

$$= 64.0\ \text{kN/m}^2 \qquad \textit{Ans.}$$

The stress shown on the volume element of material in Fig. 1–18*c* is representative of the conditions at either point A or B. Notice that this stress acts *upward* on the bottom or shaded face of the element since this face forms part of the bottom surface area of the cut section, and on this surface, the resultant internal force **P** is pushing upward.

EXAMPLE 1.9

Member AC shown in Fig. 1–19*a* is subjected to a vertical force of 3 kN. Determine the position x of this force so that the average compressive stress at the smooth support C is equal to the average tensile stress in the tie rod AB. The rod has a cross-sectional area of 400 mm^2 and the contact area at C is 650 mm^2.

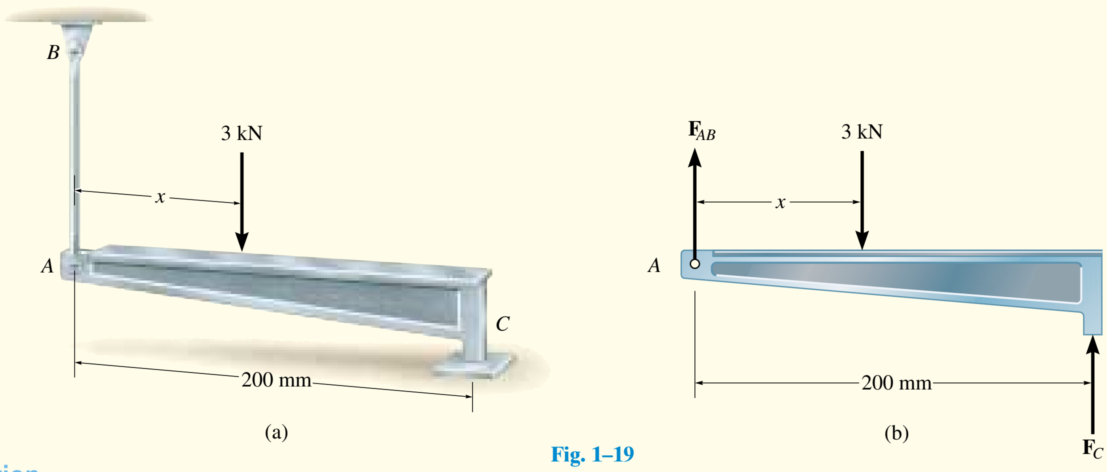


Fig. 1–19

Solution

Internal Loading. The forces at A and C can be related by considering the free-body diagram for member AC, Fig. 1–19*b*. There are three unknowns, namely, F_{AB}, F_C, and x. To solve this problem, we will work in units of newtons and millimeters.

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} + F_C - 3000\text{ N} = 0 \qquad (1)$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad -3000\text{ N}(x) + F_C\,(200\text{ mm}) = 0 \qquad (2)$$

Average Normal Stress. A necessary third equation can be written that requires the tensile stress in the bar AB and the compressive stress at C to be equivalent, i.e.,

$$\sigma = \frac{F_{AB}}{400\text{ mm}^2} = \frac{F_C}{650\text{ mm}^2}$$

$$F_C = 1.625 F_{AB}$$

Substituting this into Eq. 1, solving for F_{AB}, then solving for F_C, we obtain

$$F_{AB} = 1143\text{ N}$$

$$F_C = 1857\text{ N}$$

The position of the applied load is determined from Eq. 2,

$$x = 124\text{ mm} \qquad \textit{Ans.}$$

Note that $0 < x < 200$ mm, as required.

EXAMPLE 1.10

The bar shown in Fig. 1–24*a* has a square cross section for which the depth and thickness are 40 mm. If an axial force of 800 N is applied along the centroidal axis of the bar's cross-sectional area, determine the average normal stress and average shear stress acting on the material along (a) section plane *a–a* and (b) section plane *b–b*.

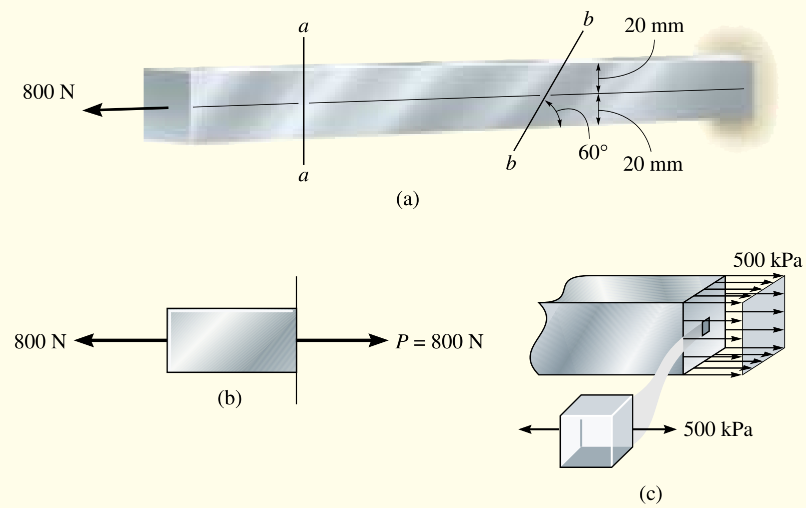


Fig. 1–24

Solution

Part (a)

Internal Loading. The bar is sectioned, Fig. 1–24*b*, and the internal resultant loading consists only of an axial force for which $P = 800$ N.

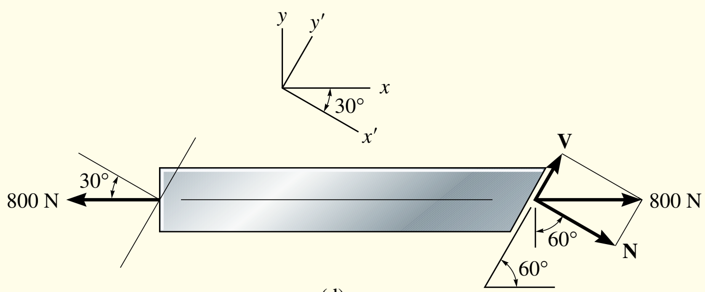
Average Stress. The average normal stress is determined from Eq.1–6.

$$\sigma = \frac{P}{A} = \frac{800\text{ N}}{(0.04\text{ m})(0.04\text{ m})} = 500\text{ kPa} \qquad \textit{Ans.}$$

No shear stress exists on the section, since the shear force at the section is zero.

$$\tau_{\text{avg}} = 0 \qquad \textit{Ans.}$$

The distribution of average normal stress over the cross section is shown in Fig. 1–24*c*.



(d)

Part (b)

Internal Loading. If the bar is sectioned along b–b, the free-body diagram of the left segment is shown in Fig. 1–24d. Here both a normal force (**N**) and shear force (**V**) act on the sectioned area. Using x, y axes, we require

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad -800\text{ N} + N \sin 60^\circ + V \cos 60^\circ = 0$$
$$+\uparrow \Sigma F_y = 0; \qquad V \sin 60^\circ - N \cos 60^\circ = 0$$

or, more directly, using x', y' axes,

$$+\searrow \Sigma F_{x'} = 0; \qquad N - 800\text{ N} \cos 30^\circ = 0$$
$$+\nearrow \Sigma F_{y'} = 0; \qquad V - 800\text{ N} \sin 30^\circ = 0$$

Solving either set of equations,

$$N = 692.8\text{ N}$$
$$V = 400\text{ N}$$

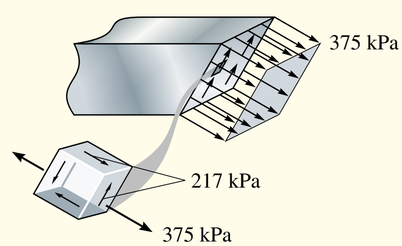
Average Stresses. In this case the sectioned area has a thickness and depth of 40 mm and 40 mm/sin 60° = 46.19 mm, respectively, Fig. 1–24a. Thus the average normal stress is

$$\sigma = \frac{N}{A} = \frac{692.8\text{ N}}{(0.04\text{ m})(0.04619\text{ m})} = 375\text{ kPa} \qquad \textit{Ans.}$$

and the average shear stress is

$$\tau_{\text{avg}} = \frac{V}{A} = \frac{400\text{ N}}{(0.04\text{ m})(0.04619\text{ m})} = 217\text{ kPa} \qquad \textit{Ans.}$$

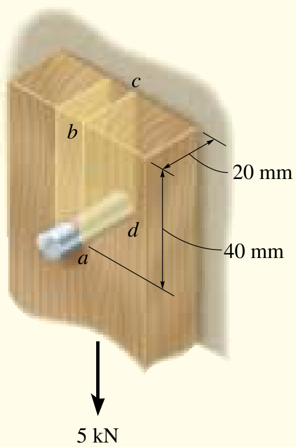
The stress distribution is shown in Fig. 1–24e.



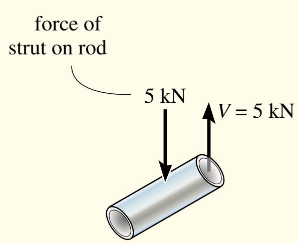
(e)

EXAMPLE 1.11

The wooden strut shown in Fig. 1–25*a* is suspended from a 10-mm-diameter steel rod, which is fastened to the wall. If the strut supports a vertical load of 5 kN, compute the average shear stress in the rod at the wall and along the two shaded planes of the strut, one of which is indicated as *abcd*.



(a)



(b)

Solution

Internal Shear. As shown on the free-body diagram in Fig. 1–25*b*, the rod resists a shear force of 5 kN where it is fastened to the wall. A free-body diagram of the sectioned segment of the strut that is in contact with the rod is shown in Fig. 1–25*c*. Here the shear force acting along each shaded plane is 2.5 kN.

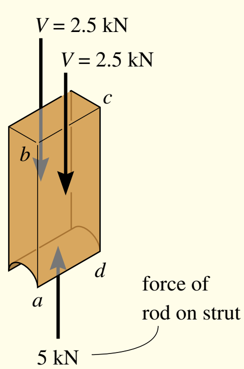
Average Shear Stress. For the rod,

$$\tau_{\text{avg}} = \frac{V}{A} = \frac{5000\ \text{N}}{\pi(0.005\ \text{m})^2} = 63.7\ \text{MPa} \qquad \textit{Ans.}$$

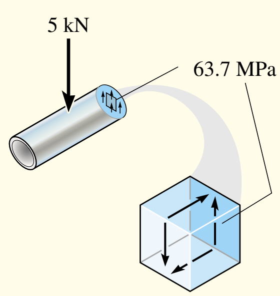
For the strut,

$$\tau_{\text{avg}} = \frac{V}{A} = \frac{2500\ \text{N}}{(0.04\ \text{m})(0.02\ \text{m})} = 3.12\ \text{MPa} \qquad \textit{Ans.}$$

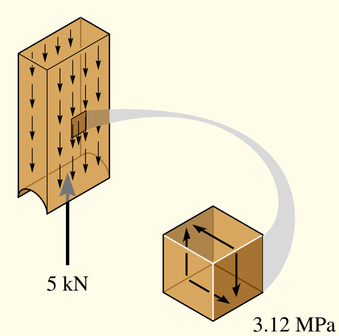
The average-shear-stress distribution on the sectioned rod and strut segment is shown in Figs. 1–25*d* and 1–25*e*, respectively. Also shown with these figures is a typical volume element of the material taken at a point located on the surface of each section. Note carefully how the shear stress must act on each shaded face of these elements and then on the adjacent faces of the elements.



(c)



(d)

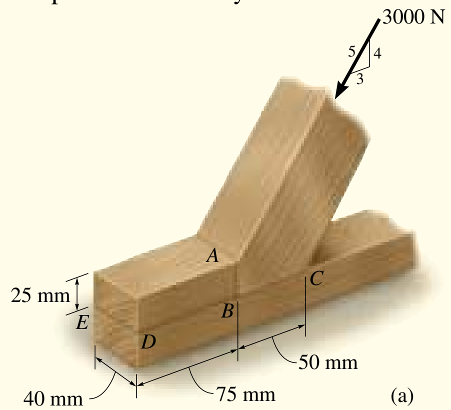


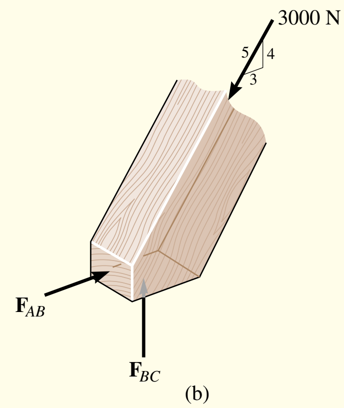
(e)

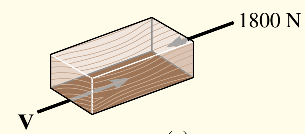
Fig. 1–25

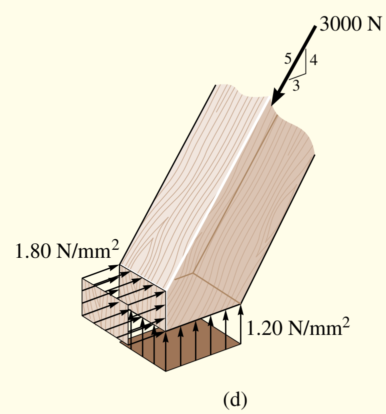
EXAMPLE 1.12

The inclined member in Fig. 1–26*a* is subjected to a compressive force of 3000 N. Determine the average compressive stress along the smooth areas of contact defined by *AB* and *BC*, and the average shear stress along the horizontal plane defined by *EDB*.









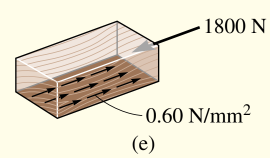


Fig. 1–26

Solution

Internal Loadings. The free-body diagram of the inclined member is shown in Fig. 1–26*b*. The compressive forces acting on the areas of contact are

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AB} - 3000\text{ N}\left(\tfrac{3}{5}\right) = 0 \qquad F_{AB} = 1800\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad F_{BC} - 3000\text{ N}\left(\tfrac{4}{5}\right) = 0 \qquad F_{BC} = 2400\text{ N}$$

Also, from the free-body diagram of the top segment of the bottom member, Fig. 1–26*c*, the shear force acting on the sectioned horizontal plane *EDB* is

$$\xrightarrow{+} \Sigma F_x = 0; \qquad V = 1800\text{ N}$$

Average Stress. The average compressive stresses along the horizontal and vertical planes of the inclined member are

$$\sigma_{AB} = \frac{1800\text{ N}}{(25\text{ mm})(40\text{ mm})} = 1.80\text{ N/mm}^2 \qquad \textit{Ans.}$$

$$\sigma_{BC} = \frac{2400\text{ N}}{(50\text{ mm})(40\text{ mm})} = 1.20\text{ N/mm}^2 \qquad \textit{Ans.}$$

These stress distributions are shown in Fig. 1–26*d*.

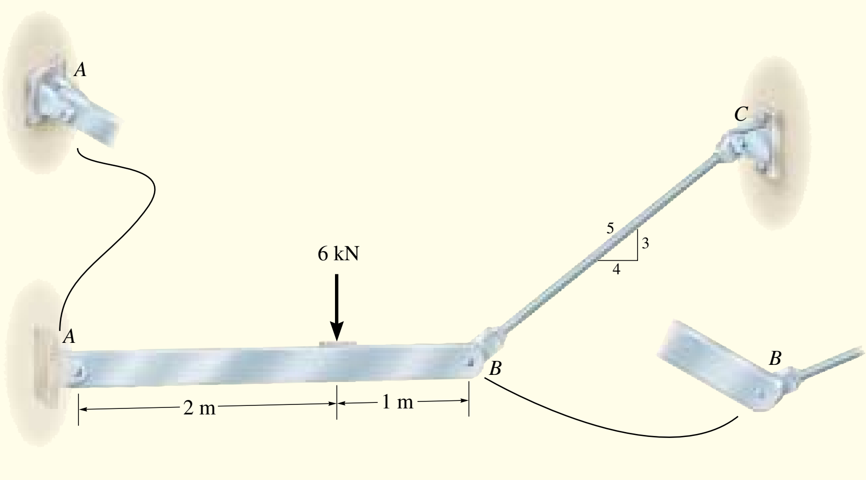
The average shear stress acting on the horizontal plane defined by *EDB* is

$$\tau_{\text{avg}} = \frac{1800\text{ N}}{(75\text{ mm})(40\text{ mm})} = 0.60\text{ N/mm}^2 \qquad \textit{Ans.}$$

This stress is shown distributed over the sectioned area in Fig. 1–26*e*.

EXAMPLE 1.13

The two members are pinned together at B as shown in Fig. 1–31*a*. Top views of the pin connections at A and B are also given in the figure. If the pins have an allowable shear stress of $\tau_{\text{allow}} = 90$ MPa and the allowable tensile stress of rod CB is $(\sigma_t)_{\text{allow}} = 115$ MPa, determine to the nearest mm the smallest diameter of pins A and B and the diameter of rod CB necessary to support the load.

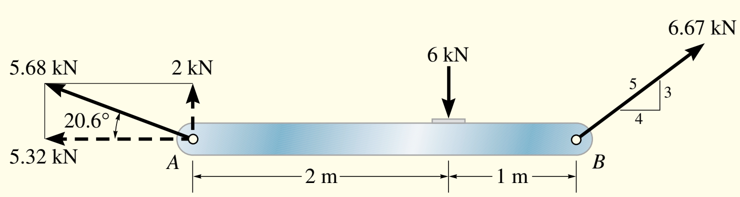


(a)

Fig. 1–31

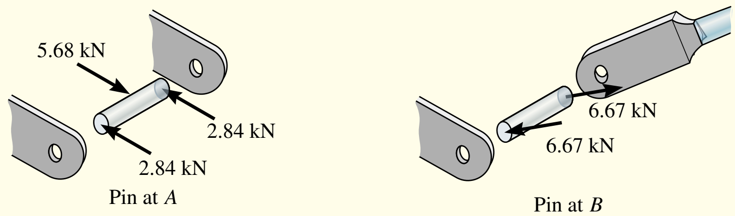
Solution

Recognizing CB to be a two-force member, the free-body diagram of member AB along with the computed reactions at A and B is shown in Fig. 1–31*b*. As an exercise, verify the computations and notice that the *resultant force* at A must be used for the design of pin A, since this is the shear force the pin resists.



(b)

Continued



(c)

Diameter of the Pins. From Fig. 1–31*a* and the free-body diagrams of the sectioned portion of each pin in contact with member AB, Fig. 1–31*c*, it is seen that pin A is subjected to double shear, whereas pin B is subjected to single shear. Thus,

$$A_A = \frac{V_A}{T_{\text{allow}}} = \frac{2.84\text{ kN}}{90 \times 10^3\text{ kPa}} = 31.56 \times 10^{-6}\text{ m}^2 = \pi\left(\frac{d_A{}^2}{4}\right) \qquad d_A = 6.3\text{ mm}$$

$$A_B = \frac{V_B}{T_{\text{allow}}} = \frac{6.67\text{ kN}}{90 \times 10^3\text{ kPa}} = 74.11 \times 10^{-6}\text{ m}^2 = \pi\left(\frac{d_B{}^2}{4}\right) \qquad d_B = 9.7\text{ mm}$$

Although these values represent the *smallest* allowable pin diameters, a *fabricated* or available pin size will have to be chosen. We will choose a size *larger* to the nearest millimeter as required.

$$d_A = 7\text{ mm} \qquad \textit{Ans.}$$

$$d_B = 10\text{ mm} \qquad \textit{Ans.}$$

Diameter of Rod. The required diameter of the rod throughout its midsection is thus,

$$A_{BC} = \frac{P}{(\sigma_t)_{\text{allow}}} = \frac{6.67\text{ kN}}{115 \times 10^3\text{ kPa}} = 58 \times 10^{-6}\text{ m}^2 = \pi\left(\frac{d_{BC}{}^2}{4}\right)$$

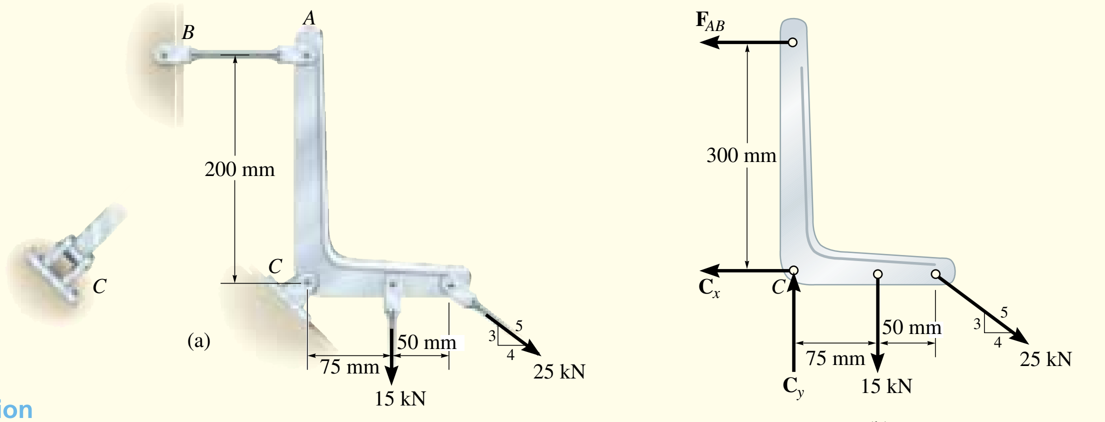
$$d_{BC} = 8.59\text{ mm}$$

We will choose

$$d_{BC} = 9\text{ mm} \qquad \textit{Ans.}$$

EXAMPLE 1.14

The control arm is subjected to the loading shown in Fig. 1–32*a*. Determine to the nearest 5 mm the required diameter of the steel pin at C if the allowable shear stress for the steel is $\tau_{allow} = 55$ MPa. Note in the figure that the pin is subjected to double shear.



(a)

(b)

Solution

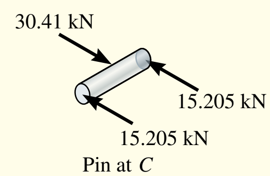
Internal Shear Force. A free-body diagram of the arm is shown in Fig. 1–32*b*. For equilibrium, we have

$$\circlearrowleft + \Sigma M_C = 0; \quad F_{AB}(0.2 \text{ m}) - 15 \text{ kN}(0.075 \text{ m}) - 25 \text{ kN}\left(\tfrac{3}{5}\right)(0.125 \text{ m}) = 0$$

$$F_{AB} = 15 \text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad -15 \text{ kN} - C_x + 25 \text{ kN}\left(\tfrac{4}{5}\right) = 0 \qquad C_x = 5 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad C_y - 15 \text{ kN} - 25 \text{ kN}\left(\tfrac{3}{5}\right) = 0 \qquad C_y = 30 \text{ kN}$$



(c)

Fig. 1–32

The pin at C resists the resultant force at C. Therefore,

$$F_C = \sqrt{(5 \text{ kN})^2 + (30 \text{ kN})^2} = 30.41 \text{ kN}$$

Since the pin is subjected to double shear, a shear force of 15.205 kN acts over its cross-sectional area *between* the arm and each supporting leaf for the pin, Fig. 1–32*c*.

Required Area. We have

$$A = \frac{V}{\tau_{allow}} = \frac{15.205 \text{ kN}}{55 \times 10^3 \text{ kN/m}^2} = 276.45 \times 10^{-6} \text{ m}^2$$

$$\pi\left(\frac{d}{2}\right)^2 = 276.45 \text{ mm}^2$$

$$d = 18.8 \text{ mm}$$

Use a pin having a diameter of

$$d = 20 \text{ mm} \qquad \textit{Ans.}$$

EXAMPLE 1.15

The suspender rod is supported at its end by a fixed-connected circular disk as shown in Fig. 1–33*a*. If the rod passes through a 40-mm-diameter hole, determine the minimum required diameter of the rod and the minimum thickness of the disk needed to support the 20-kN load. The allowable normal stress for the rod is $\sigma_{\text{allow}} = 60$ MPa, and the allowable shear stress for the disk is $\tau_{\text{allow}} = 35$ MPa.

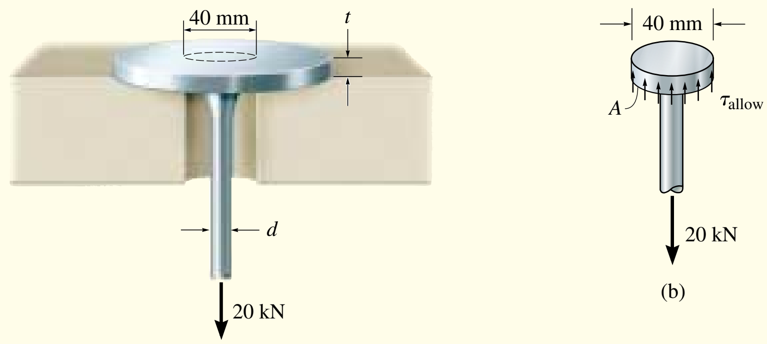


Fig. 1–33

Solution

Diameter of Rod. By inspection, the axial force in the rod is 20 kN. Thus, the required cross-sectional area of the rod is

$$A = \frac{P}{\sigma_{\text{allow}}} = \frac{20(10^3)\text{ N}}{60(10^6)\text{ N/m}^2} = 0.3333(10^{-3})\text{ m}^2$$

So that

$$A = \pi\left(\frac{d^2}{4}\right) = 0.3333(10^{-2})\text{ m}^2$$

$$d = 0.0206\text{ m} = 20.6\text{ mm} \qquad \textit{Ans.}$$

Thickness of Disk. As shown on the free-body diagram of the core section of the disk, Fig. 1–33*b*, the material at the sectioned area must resist *shear stress* to prevent movement of the disk through the hole. If this shear stress is *assumed* to be distributed uniformly over the sectioned area, then, since $V = 20$ kN, we have

$$A = \frac{V}{\tau_{\text{allow}}} = \frac{20(10^3)\text{ N}}{35(10^6)\text{ N/m}^2} = 0.571(10^{-3})\text{ m}^2$$

Since the sectioned area $A = 2\pi(0.02\text{ m})(t)$, the required thickness of the disk is

$$t = \frac{0.5714(10^{-3})\text{ m}^2}{2\pi(0.02\text{ m})} = 4.55(10^{-3})\text{ m} = 4.55\text{ mm} \qquad \textit{Ans.}$$

EXAMPLE 1.16

An axial load on the shaft shown in Fig. 1–34*a* is resisted by the collar at *C*, which is attached to the shaft and located on the right side of the bearing at *B*. Determine the largest value of *P* for the two axial forces at *E* and *F* so that the stress in the collar does not exceed an allowable bearing stress at *C* of $(\sigma_b)_{\text{allow}} = 75$ MPa and the average normal stress in the shaft does not exceed an allowable tensile stress of $(\sigma_t)_{\text{allow}} = 55$ MPa.

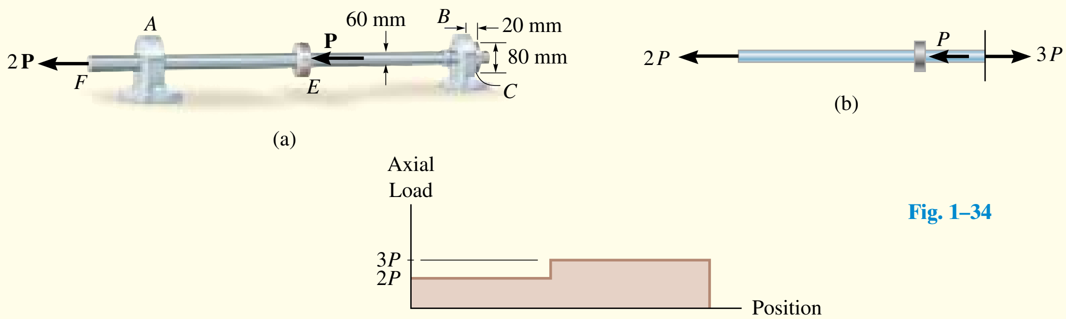


Fig. 1–34

Solution

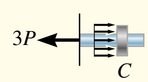
To solve the problem we will determine *P* for each possible failure condition. Then we will choose the *smallest* value. Why?

Normal Stress. Using the method of sections, the axial load within region *FE* of the shaft is $2P$, whereas the *largest* axial load, $3P$, occurs within region *EC*, Fig. 1–34*b*. The variation of the internal loading is clearly shown on the normal-force diagram, Fig. 1–34*c*. Since the cross-sectional area of the entire shaft is constant, region *EC* will be subjected to the maximum average normal stress. Applying Eq. 1–11, we have

$$\sigma_{\text{allow}} = \frac{P}{A} \qquad 55(10^6)\ \text{N/m}^2 = \frac{3P}{\pi(0.03\ \text{m})^2}$$

$$P = 51.8\ \text{kN}$$

Bearing Stress. As shown on the free-body diagram in Fig. 1–34*d*, the collar at *C* must resist the load of $3P$, which acts over a bearing area of $A_b = [\pi(0.04\ \text{m})^2 - \pi(0.03\ \text{m})^2] = 2.199(10^{-3})\ \text{m}^2$. Thus,



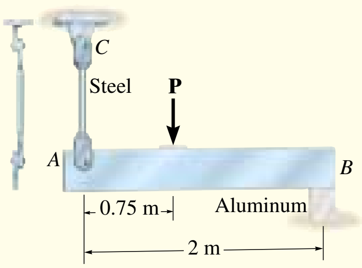
(d)

$$A = \frac{P}{\sigma_{\text{allow}}}; \qquad 75(10^6)\ \text{N/m}^2 = \frac{3P}{2.199(10^{-3})\ \text{m}^2}$$

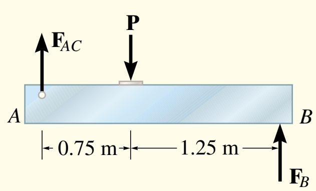
$$P = 55.0\ \text{kN}$$

By comparison, the largest load that can be applied to the shaft is $P = 51.8$ kN, since any load larger than this will cause the allowable normal stress in the shaft to be exceeded.

EXAMPLE 1.17



(a)



(b)

Fig. 1–35

The rigid bar AB shown in Fig. 1–35*a* is supported by a steel rod AC having a diameter of 20 mm and an aluminum block having a cross-sectional area of 1800 mm^2. The 18-mm-diameter pins at A and C are subjected to *single shear*. If the failure stress for the steel and aluminum is $(\sigma_{st})_{fail} = 680$ MPa and $(\sigma_{al})_{fail} = 70$ MPa, respectively, and the failure shear stress for each pin is $\tau_{fail} = 900$ MPa, determine the largest load P that can be applied to the bar. Apply a factor of safety of F.S. $= 2$.

Solution

Using Eqs. 1–9 and 1–10, the allowable stresses are

$$(\sigma_{st})_{allow} = \frac{(\sigma_{st})_{fail}}{\text{F.S.}} = \frac{680 \text{ MPa}}{2} = 340 \text{ MPa}$$

$$(\sigma_{al})_{allow} = \frac{(\sigma_{al})_{fail}}{\text{F.S.}} = \frac{70 \text{ MPa}}{2} = 35 \text{ MPa}$$

$$\tau_{allow} = \frac{\tau_{fail}}{\text{F.S.}} = \frac{900 \text{ MPa}}{2} = 450 \text{ MPa}$$

The free-body diagram for the bar is shown in Fig. 1–35*b*. There are three unknowns. Here we will apply the equations of equilibrium so as to express F_{AC} and F_B in terms of the applied load P. We have

$$\circlearrowleft + \Sigma M_B = 0; \qquad P(1.25 \text{ m}) - F_{AC}(2 \text{ m}) = 0 \qquad (1)$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_B(2 \text{ m}) - P(0.75 \text{ m}) = 0 \qquad (2)$$

We will now determine each value of P that creates the allowable stress in the rod, block, and pins, respectively.

Rod AC. This requires

$$F_{AC} = (\sigma_{st})_{allow}(A_{AC}) = 340(10^6) \text{ N/m}^2[\pi(0.01 \text{ m})^2] = 106.8 \text{ kN}$$

Using Eq. 1,

$$P = \frac{(106.8 \text{ kN})(2 \text{ m})}{1.25 \text{ m}} = 171 \text{ kN}$$

Block B. In this case,

$$F_B = (\sigma_{al})_{allow} A_B = 35(10^6) \text{ N/m}^2[1800 \text{ mm}^2(10^{-6}) \text{ m}^2/\text{mm}^2] = 63.0 \text{ kN}$$

Using Eq. 2,

$$P = \frac{(63.0 \text{ kN})(2 \text{ m})}{0.75 \text{ m}} = 168 \text{ kN}$$

Pin A or C. Here

$$V = F_{AC} = \tau_{allow} A = 450(10^6) \text{ N/m}^2[\pi(0.009 \text{ m})^2] = 114.5 \text{ kN}$$

From Eq. 1,

$$P = \frac{114.5 \text{ kN}(2 \text{ m})}{1.25 \text{ m}} = 183 \text{ kN}$$

By comparison, when P reaches its *smallest value* (168 kN), it develops the allowable normal stress in the aluminum block. Hence,

$$P = 168 \text{ kN} \qquad \textit{Ans.}$$