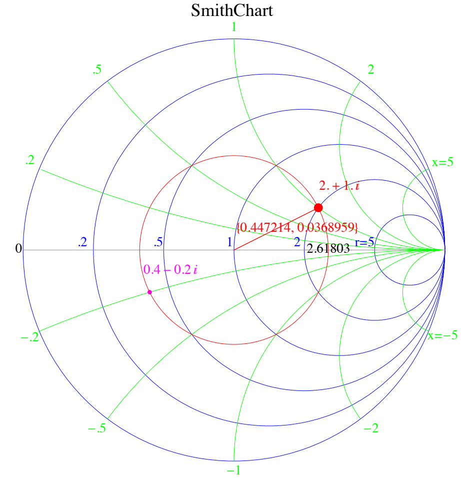
35 Smith Chart examples



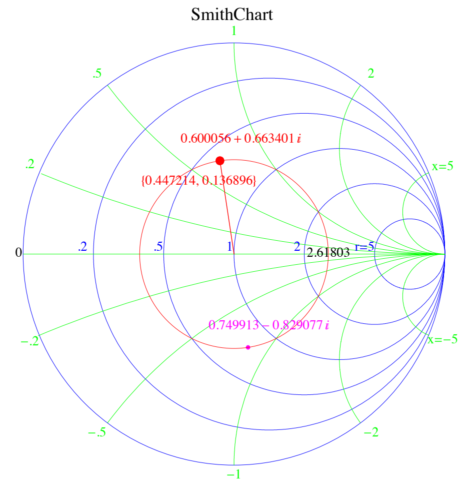
Example 1: A load $Z_L = 100 + j50\,\Omega$ is connected across a TL with $Z_o = 50\,\Omega$ and $l = 0.4\lambda$. At the generator end, $d = l$, the line is shunted by an impedance $Z_s = 100\,\Omega$. What are the input impedance Z_{in} and admittance Y_{in} of the line, including the shunt connected element.

Solution: Normalized load impedance

$$z(0) = \frac{Z_L}{Z_o} = \frac{100 + j50}{50} = 2 + j1$$

is entered in the SC shown in the margin on the top. Clockwise rotation (from load toward generator) at fixed $|\Gamma|$ (red circle) by

$$0.4\lambda \Leftrightarrow 0.8 \times 360^\circ = 288^\circ$$

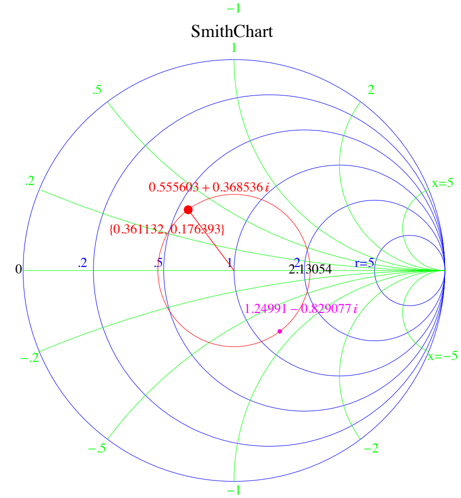


takes us to

$$z(l) \approx 0.6 + j0.66 \quad \text{and} \quad y(l) \approx 0.75 - j0.83$$

as shown on the SC in the middle. Hence, including the shunt element with normalized input impedance $z_{si} = 2$ and admittance $y_{si} = \frac{1}{2}$, we obtain

$$y_{in} = y(l) + y_{si} \approx 1.25 - j0.83$$



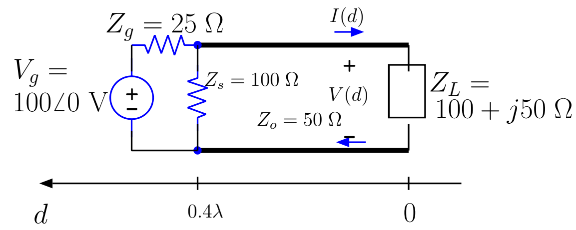
for the overall normalized input admittance of the shunted line as shown on the SC in the bottom — the corresponding normalized input impedance is

$$z_{in} = \frac{1}{y_i} \approx 0.56 + j0.37.$$

Hence, the unnormalized input impedance and admittance are

$$Z_{in} = Z_o z_{in} \approx 27.8 + j18.4\,\Omega \text{ and } Y_{in} = Y_o y_{in} \approx 0.025 - j0.017\,\text{S}.$$

Example 2: The TL network described in Example 1 is connected to a generator with open circuit voltage phasor $V_g = 100\angle 0$ V and internal impedance $Z_g = 25\,\Omega$. What is the average power (a) input of the shunted line, (b) delivered to the shunt element, delivered to the load.



Solution:

(a) Using the input impedance

$$Z_{in} \approx 27.8 + j18.4\,\Omega,$$

from Example 1, we can write

$$V_{in} = V_g \frac{Z_{in}}{Z_g + Z_{in}} \text{ and } I_{in} = \frac{V_g}{Z_g + Z_{in}}.$$

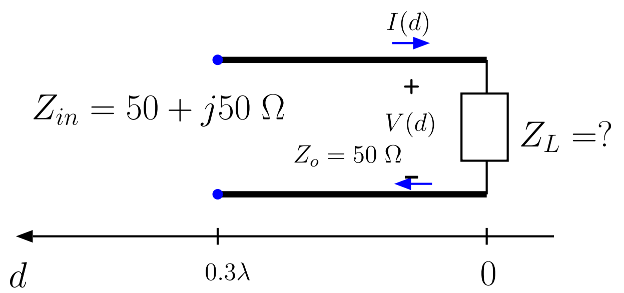
Therefore, the average power input of the shunted line is

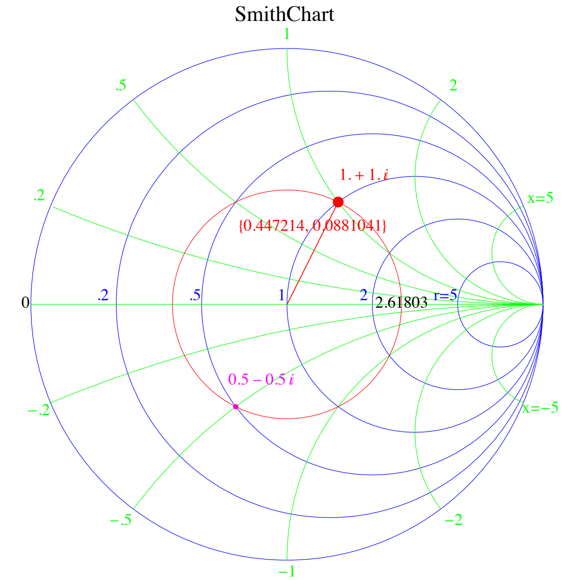
$$\begin{aligned} P &= \frac{1}{2}\mathrm{Re}\{V_{in} I_{in}^*\} = \frac{1}{2}\mathrm{Re}\{\frac{V_g Z_{in}}{Z_g + Z_{in}}(\frac{V_g}{Z_g + Z_{in}})^*\} \\ &= \frac{|V_g|^2}{2|Z_g + Z_{in}|^2}\mathrm{Re}\{Z_{in}\} = \frac{100^2}{2|25 + 27.8 + j18.4|^2} 27.8 \approx 44.44\,\mathrm{W}. \end{aligned}$$

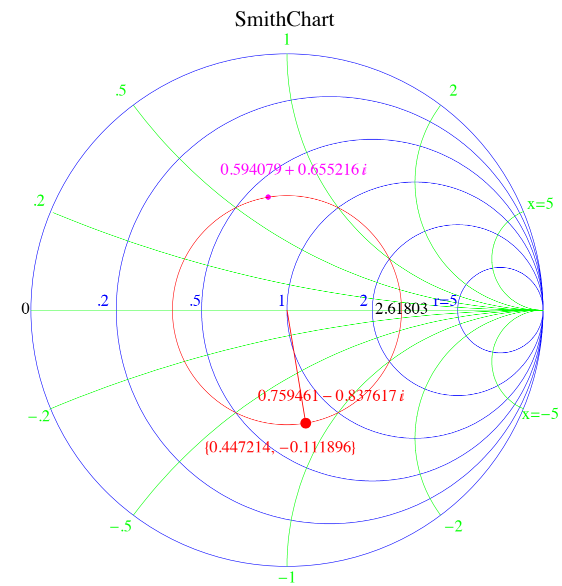
(b) The shunt element $Z_s = 100\,\Omega$ sees the same voltage V_{in} and conducts a current V_{in}/Z_s. Therefore it absorbs an average power of

$$\begin{aligned} P &= \frac{1}{2}\mathrm{Re}\{V_{in}(\frac{V_{in}}{Z_s})^*\} = \frac{|V_{in}|^2}{2Z_s} = \frac{|V_g Z_{in}|^2}{2Z_s|Z_g + Z_{in}|^2} \\ &\approx \frac{|100 \cdot (27.8 + j18.4)|^2}{2 \cdot 100 \cdot |25 + 27.8 + j18.4|^2} \approx 17.78\,\mathrm{W}. \end{aligned}$$

The remainder of 44.44 W will be absorbed in Z_L.







Example 3: A TL of length $l = 0.3\lambda$ has an input impedance $Z_{in} = 50 + j50\,\Omega$. Determine the load impedance $Z_L = Z(0)$ and $Y_L = Y(0)$ given that $Z_o = 50\,\Omega$ for the line.

Solution: First enter the normalized inpur impedance

$$z_{in} = \frac{Z_{in}}{Z_o} = \frac{50 + j50}{50} = 1 + j$$

in the SC as shown in the margin on the top. Counter-clockwise rotation (from generator toward load) at fixed $|\Gamma|$ (red circle) by

$$0.3\lambda \Leftrightarrow 0.6 \times 360^\circ = 216^\circ$$

takes us to

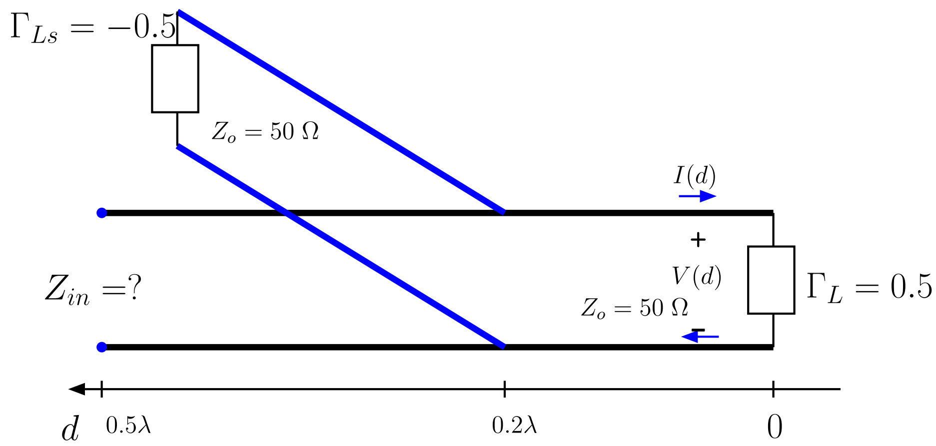
$$z(0) \approx 0.76 - j0.84 \quad \text{and} \quad y(0) \approx 0.59 + j0.66$$

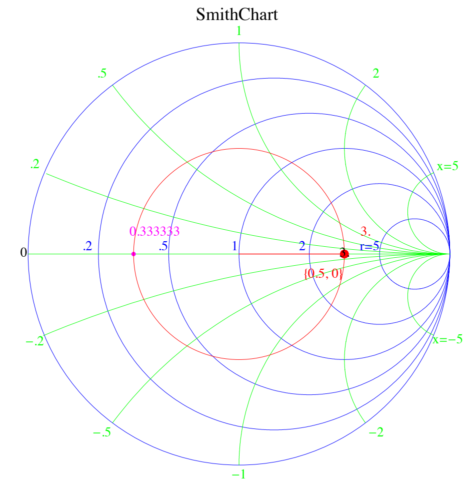
as shown on the next SC at the load point. Hence, we find

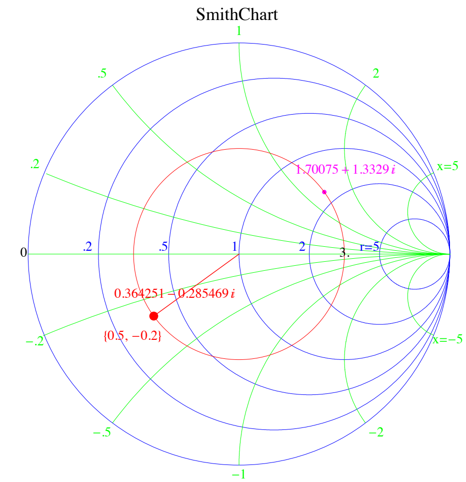
$$Z_L = Z_o z(0) \approx 50 \cdot (0.76 - j0.84) = 37.97 - j41.88\,\Omega$$

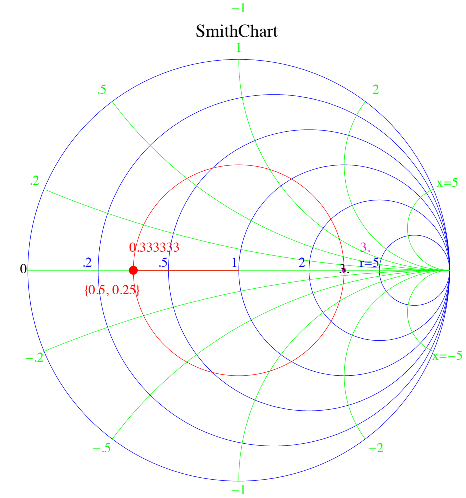
and

$$Y_L = Y_o y(0) \approx \frac{1}{50}(0.59 + j0.66) = 0.012 + j0.013\,\text{S}.$$









Example 4: A TL of length $l = 0.5\lambda$ and $Z_o = 50\,\Omega$ has a load reflection coefficient $\Gamma_L = 0.5$ and and a shunt connected TL at $d = 0.2\lambda$. The shunt connected TL has $l = 0.3\lambda$, $Z_o = 50\,\Omega$, and a load reflection coefficient $\Gamma_L = -0.5$. Determine the input impedance of the line.

Solution: Recall that the SC covers the unit circle of the complex plane and therefore the complex number

$$\Gamma_L = 0.5 + j0 = 0.5$$

can be entered directly in the SC as shown on the top SC in the margin. Clockwise rotation (from load toward generator) at fixed $|\Gamma|$ (red circle) by

$$0.2\lambda \Leftrightarrow 0.4 \times 360^\circ = 144^\circ$$

takes us to

$$z(0.2\lambda) \approx 0.36 - j0.29 \quad \text{and} \quad y(0.2\lambda) \approx 1.7 + j1.33$$

as shown on the SC in the middle. Likewise, entering

$$\Gamma_{Ls} = -0.5 + j0 = -0.5$$

for the shunt connected stub in the third SC and rotating clockwise by

$$0.3\lambda \Leftrightarrow 0.6 \times 360^\circ = 216^\circ$$

we obtain

$$z_s(0.3\lambda) \approx 1.7 - j1.33 \quad \text{and} \quad y_s(0.3\lambda) \approx 0.36 + j0.29.$$

We proceed by combining the normalized admittances as

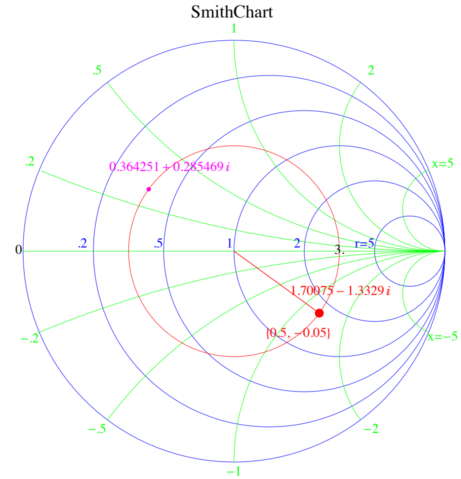
$$y_c = y(0.2\lambda) + y_s(0.3\lambda) \approx (1.7 + j1.33) + (0.36 + j0.29) = 2.065 + j1.61837,$$

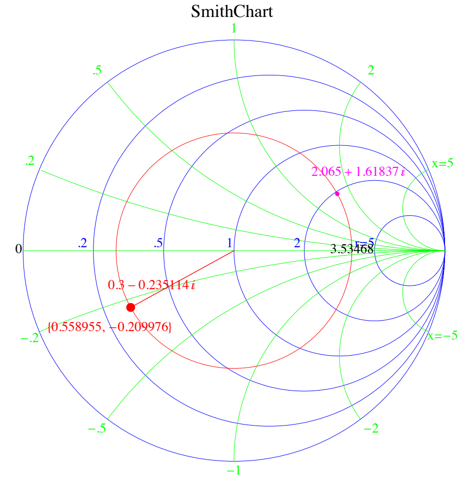
and entering it in the next SC. Finally rotating clockwise once again by

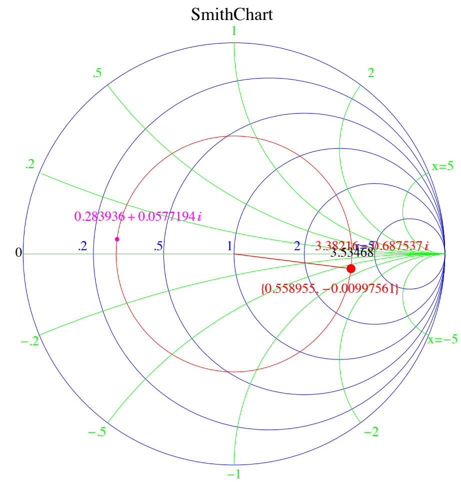
$$0.3\lambda \Leftrightarrow 0.6 \times 360^\circ = 216^\circ$$

we obtain, from the last SC

$$z_{in} \approx 3.38 - j0.69 \quad \Rightarrow \quad Z_{in} = z_{in} Z_o \approx 169 - j34.4\,\Omega.$$







Example 5: What is the load impedance Z_{Ls} terminating the shunt connected stub in Example 4?

Solution: Given that the corresponding reflection coefficient is

$$\Gamma_{Ls} = -0.5,$$

it follows from the bilinear transformation linking z_{Ls} and Γ_{Ls} that

$$z_{Ls} = \frac{1 + \Gamma_{Ls}}{1 - \Gamma_{Ls}} = \frac{1 - 0.5}{1 + 0.5} = \frac{1}{3}.$$

Hence, the impedance is

$$Z_{Ls} = Z_o z_{Ls} = \frac{50}{3}\,\Omega.$$

Example 6: What is the load impedance Z_L in Example 4?

Solution: This is similar to Example 5. Given that the load reflection coefficient is

$$\Gamma_L = 0.5,$$

it follows from the bilinear transformation linking z_L and Γ_L that

$$z_L = \frac{1 + \Gamma_L}{1 - \Gamma_L} = \frac{1 + 0.5}{1 - 0.5} = 3.$$

Hence, the impedance is

$$Z_L = Z_o z_L = 150\,\Omega.$$